

1 RESEARCH ARTICLE

2 Running head: COMPARING COLOURS

3 Comparing colours using visual models

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13 Abstract

- 14 1. Colour in nature presents a striking dimension of variation, though under-
15 standing its function and evolution largely depends on our ability to capture
16 the perspective of relevant viewers. This goal has been radically advanced
17 by the development and widespread adoption of perceptual colour spaces,
18 which allow for the viewer-subjective estimation of colour appearance. Most
19 studies of colour in camouflage, aposematism, sexual selection, and other
20 signalling contexts draw on these colour spaces, with the shared analytical
21 objective of estimating how similar (or dissimilar) colour samples are to a
22 given viewer.
- 23 2. We summarise popular approaches for estimating the separation of samples
24 in colour space, and use a simulation-based approach to test their efficacy
25 with common data structures. We show that these methods largely fail to
26 estimate the separation of colour samples by neglecting (i) the statistical dis-
27 tribution and within-group variation of the data, and/or (ii) the perceptual
28 separation of groups relative to the observer's visual capabilities.
- 29 3. Instead, we formalize the two questions that must be answered to establish
30 both the statistical presence and perceptual magnitude of colour differences,
31 and propose a two-step, permutation-based approach that achieves this goal.
32 Unlike previous methods, our suggested approach accounts for the mul-
33 tidimensional nature of visual model data, and is robust against common
34 colour-data features such as heterogeneity and outliers.
- 35 4. We demonstrate the pitfalls of current methods and the flexibility of our
36 suggested framework using heuristic examples drawn from the literature,
37 with recommendations for future inquiry.

38 Introduction

39 Humans, as most primates, are an extremely visually-oriented species (Vorobyev,
40 2004), and the study of colour in nature has driven fundamental advances in ecol-
41 ogy and evolutionary biology (Cuthill *et al.*, 2017). Colour is a subjective expe-
42 rience, however, so substantial effort has been dedicated to measuring colours
43 “objectively” (Garcia *et al.*, 2014; Johnsen, 2016) through visual models in order
44 to explicitly consider the perspective of ecologically relevant viewers (Kemp *et al.*,
45 2015; Renoult *et al.*, 2017). These models have radically advanced the study of
46 colour traits by allowing researchers to account for the factors influencing the
47 generation and perception of visual information, such as the structure of signals
48 and viewing backgrounds, the properties of veiling and incident light, and the
49 attributes of receiver visual systems (Chittka, 1992; Endler & Mielke, 2005; Kelber
50 *et al.*, 2003; Vorobyev & Osorio, 1998).

51 Several forms of visual models exist and are currently used, which vary in their
52 assumptions about the psychophysical properties of visual systems and visual pro-
53 cessing (Chittka, 1992; Endler & Mielke, 2005; Vorobyev & Osorio, 1998). Despite
54 this variation, all models invariably attempt to delimit a colour space informed by
55 the number and sensitivity of photoreceptors in an animal’s retina (Renoult *et al.*,
56 2017). Individual colours can then be represented in this space, with their location
57 being determined by the degree to which reflected light differentially stimulates
58 the viewers’ receptors. Our own trichromatic vision, for example, can be repre-
59 sented by a triangle with the “red”, “green”, and “blue” cones as its vertices. Any
60 coloured stimulus can then appear as a point in this space, such that a colour that
61 exclusively triggers one of these receptors will fall on that vertex, while a white
62 or black (i.e. achromatic) colour that stimulates all cones equally will lie at the
63 geometric centre.

64 Representing colours in dedicated spaces is convenient for several reasons.
65 First, it offers an intuitive way of analysing phenotypes that we cannot measure

66 directly: we can instead estimate how animals with different visual systems “see”
67 different colours by representing them in a Cartesian coordinate system, thereby
68 producing a receiver-dependent morphospace (Kelber *et al.*, 2003; Renoult *et al.*,
69 2017). Second, it allows for the estimation of how similar or dissimilar colours are
70 to a given observer, by estimating the distance between colour points in its colour
71 space (Endler & Mielke, 2005; Vorobyev *et al.*, 1998; Vorobyev & Osorio, 1998). Fi-
72 nally, we can integrate behavioural and psychophysical data into models in order
73 to predict whether an observer could effectively discriminate pairs of colours, or
74 if they would instead be perceptually indistinguishable (Chittka, 1992; Vorobyev
75 *et al.*, 2001; Vorobyev & Osorio, 1998). This final point is critical to many tests of
76 ecological and evolutionary hypotheses pertaining to, for example, the efficacy of
77 camouflage (Pessoa *et al.*, 2014; Troscianko *et al.*, 2016), the precision of mimicry
78 (O’Hanlon *et al.*, 2014; White *et al.*, 2017), the extent of signal variability among
79 populations or species (Delhey & Peters, 2008; Rheindt *et al.*, 2014), the presence
80 of polymorphism or dichromatism (Schultz & Fincke, 2013; Whiting *et al.*, 2015),
81 or the effect of experimental treatments (Barry *et al.*, 2015). At the heart of these
82 diverse inquiries lies the same question: how different are these colours (or more
83 precisely, these samples of colours) to the animal viewing them? Note that while
84 a further distinction is often drawn between questions dealing with colours that
85 are very similar (‘discriminability’ near-threshold) and very different (‘perceptual
86 distance’ supra-threshold; Kemp *et al.*, 2015), accruing empirical evidence sug-
87 gests this is largely artificial (Fleishman *et al.*, 2016; van der Kooi *et al.*, 2016). We
88 thus refer to both as questions of discriminability hereafter, and while the below
89 discussion largely centres on near-threshold scenarios, the presented methods are
90 broadly applicable.

91 *Challenges in estimating the discriminability of colour samples*

92 The receptor noise-limited model of Vorobyev & Osorio (1998) has proven par-
93 ticularly useful for addressing questions of discriminability. The model assumes

94 that that chromatic and achromatic channels operate independently and that the
95 limits to colour discrimination are set by noise arising in receptors and during
96 subsequent neural processing (Vorobyev *et al.*, 1998; Vorobyev & Osorio, 1998).
97 This noise is dependent on the receptor type and abundance on the retina, as
98 well as being more generally defined by Weber's law of just noticeable differences
99 $k = \Delta I/I$ — that is, the difference threshold k is a constant determined by the
100 difference between two stimuli ΔI relative to the intensity of the baseline stimulus
101 I . For example, if the difference threshold is $k = 0.1$, then a stimulus I_A will only
102 be perceived as different from I_B in that channel if it is at least 10% greater than
103 I_A (the value of k for any species and receptor usually being determined from
104 behavioural experiments; Vorobyev *et al.*, 2001). Further, the noise e in receptor i
105 is defined by $e_i = k/\sqrt{N}$, where N is the relative abundance of receptor i in the
106 retina. The more abundant a receptor is in the retina relative to the other receptor
107 types, then, the lower the relative noise will be on that channel.

108 The Weber fraction thus establishes a unit of Just Noticeable Differences (JND's),
109 and distances in colour space can be weighted by photoreceptor noise and ex-
110 pressed in what are essentially units of signal:noise. Values lower than 1 JND
111 represent situations where $\frac{signal}{noise} < 1$ and are predicted to be indistinguishable,
112 while distances close to but greater than 1 JND lie at the very threshold of colour
113 discrimination (conventionally interpreted as the point in which two colours can
114 be distinguished 75% of the time when simultaneously presented against a neutral
115 background; Vorobyev & Osorio, 1998). Values greatly above this threshold (say,
116 above 2 or 4 JND's) are likely so different that they can be told apart with virtually
117 no errors. This provides a useful guide to estimating the similarity of points or
118 groups of points in colour space: the greater the distance between colours, the less
119 alike they are to a given viewer. It follows that if differences between sample A and
120 sample B are, on average, above an established threshold, then we can consider
121 the groups different: sexes dichromatic, subspecies distinct, mimetism imperfect,
122 and so on. This powerful approach allows for a clear link between variation and

123 classification within a perceptual framework, and has for that reason been used
124 in a vast number of studies seeking to answer such questions (Barry *et al.*, 2015;
125 Delhey & Peters, 2008; O’Hanlon *et al.*, 2014; Schultz & Fincke, 2013; White *et al.*,
126 2017).

127 This framework, however, raises an important methodological issue: how to
128 adequately compare samples of colours, and estimate if the average distance be-
129 tween them is both statistically and perceptually meaningful (i.e. above-threshold;
130 Endler & Mielke, 2005). Two methods are commonly used. In the first, an “aver-
131 age colour” for each group is derived by averaging their reflectance spectra before
132 modelling the visual system, or by averaging their location in colour space. In
133 either case, this mean quantum catch per-receptor per-group — the centroid for
134 that group in multivariate space — is then used to calculate the colour distance
135 between groups (Fig. 1, bold arrow). There are two issues with this approach.
136 First, since colour distances are perceived in a ratio scale, the centroid obtained
137 from arithmetic means of receptor coordinates is not an appropriate measure of
138 central tendency. Instead, the geometric mean (or the average reflectance of log-
139 transformed spectra, as suggested by Cardoso & Gomes, 2015) must be used. This
140 can be demonstrated by converting perceptual distances into Cartesian coordi-
141 nates (Pike, 2012), in which case the distance between arithmetic means in this
142 perceptual space matches the distance between geometric means in the untrans-
143 formed scale. Second, since the result is a single value representing the multivari-
144 ate distance between group means, there is no associated measure of uncertainty
145 or precision that would allow for the statistical testing of differences between sam-
146 ples (e.g. Avilés *et al.*, 2011; Burns & Shultz, 2012; Maia *et al.*, 2016).

147 The second approach calculates the pairwise distances between all points in
148 group A and group B (or between group A and the mean of group B if, for ex-
149 ample, group B consists of samples from the background), then using the average
150 of these distances to represent the mean distance between groups (Fig. 1, thin
151 arrows; e.g. Barry *et al.*, 2015; Dearborn *et al.*, 2012). In cluster analyses, this is

152 called the “average linkage” between groups (Hair *et al.*, 1998). This is an ap-
153 pealing method because it allows the calculation of measures of variation such as
154 the standard error of the distances, and thus a t-test or equivalent can be used to
155 test if these differences are greater than the threshold value. The average linkage,
156 however, is also inadequate because it conflates within- and among-group varia-
157 tion. This is because Euclidean distances (and by extension JND’s) are *translation-*
158 *invariant*: they ignore the position of points in colour space and the direction of the
159 distance vector, reflecting only the magnitude of differences between two points.
160 Therefore, the average linkage reduces to a measure of spread, rather than one of
161 relative position, and will scale with both within- and between-group distances.
162 We can demonstrate this by considering the Euclidean distance in one dimension,
163 in which the distance between points x_1 and x_2 is $\sqrt{(x_1 - x_2)^2}$, which reduces to
164 the absolute difference $|x_1 - x_2|$. The mean absolute difference, in turn, is a mea-
165 sure of dispersion, so if we have two $N = 2$ identical samples $A = B = \{10, 20\}$,
166 the average linkage between these samples would be 5, not zero as expected from
167 comparing two identical samples (Fig. 1).

168 These methods highlight the fact that appraising hypotheses of discriminabil-
169 ity has centred on tests of whether the difference between samples is above a
170 perceptual threshold. However, the ready convenience of such threshold value be-
171 lies fact that simply comparing mean distances between groups is not sufficient to
172 infer, statistically, whether the samples being compared are different. In order to
173 answer if two groups are different, one must compare the level of between-group
174 variation relative to within-group variation. This is particularly problematic in the
175 case of colours that function as signals, such as those used in social interactions
176 (e.g. Kemp & Rutowski, 2011). For a trait to function as a signal in this context, an
177 observer must be able to tell the difference between signals of “low quality” and
178 “high quality”. This means, by definition, that *individuals within a statistical popu-*
179 *lation should be readily distinguishable* — they must be highly variable and colour
180 distances between them should be above the threshold of discrimination (Delhey

181 *et al.*, 2017), otherwise no information can be extracted by a viewer when compar-
182 ing phenotypes.

183 This is readily appreciable by considering a hypothetical species that uses
184 colour in mate choice, but is not sexually dichromatic (Fig. 1). In this species
185 colour will be highly variable and, on average, differences among individuals of
186 the same sex will fall above the threshold of discrimination, but there is no *con-*
187 *sistent* difference between males and females. Therefore, if a researcher took a
188 sample from this species and calculated the average distance between all pairs
189 individuals, regardless of sex, these differences should be largely greater than 1
190 JND. However, it also follows that if the researcher took separate samples of males
191 and females from that species, then all pairwise distances (the average link dis-
192 tance) between sexes will be also greater than 1 JND, despite them being sampled
193 from the same (statistical) population.

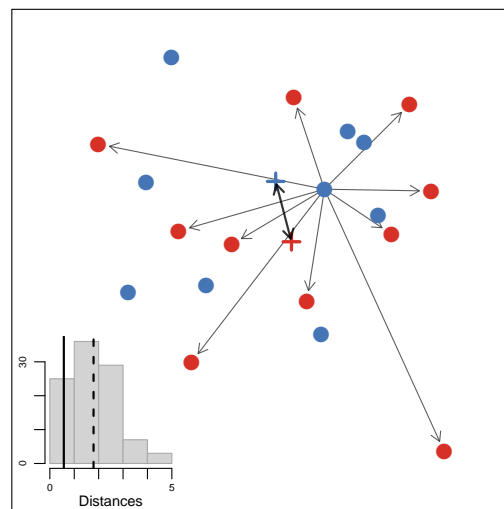


Figure 1: The link distance (i.e. average pairwise distance between groups in a colour space) conflates among- and within-group variation. Here, two samples were drawn from the same distribution. Thin arrows indicate the distances between a random point in the first sample (blue) and all points from the second sample (red), almost all of which are much greater than the distance between the geometric means of the two samples (“x”, bold line). The inset shows a histogram of all pairwise distances among groups, and how their average (dashed line) is greater than the mean distance (bold line).

194 *The limitations of current methods for comparing colour space distributions*

195 The issues raised above highlight the fact that identifying differences between
196 groups as lying above or below a perceptual threshold, even with an adequate
197 measure of uncertainty, is still not sufficient to tackle questions of discriminabil-
198 ity — it is also essential to consider how the sample are distributed relative to
199 one another in colour space. The importance of considering the distribution of
200 colour points relative to each other when comparing colours has been noted be-
201 fore (Eaton, 2005; Endler & Mielke, 2005). Eaton (2005), for example, noted that
202 within-group variation would influence his conclusions about the extent of avian
203 dichromatism using avian visual models. His solution to the problem was to test
204 for intersexual differences in photon catches separately for each receptor. How-
205 ever, this approach ignores the multivariate nature of visual model data, and may
206 thus inflate Type I errors by failing to account for multiple comparisons and ignor-
207 ing correlations among receptor catches (which is critical, since any visual system
208 defined by n receptors can be represented in $n - 1$ dimensions; Kelber *et al.*, 2003).
209 Further, unless quantum catches are made relative to their sum, this univariate
210 approach also fails to consider that visual models ignore the achromatic dimen-
211 sion of colour and the absolute value of receptor-specific quantal catches (Endler
212 & Mielke, 2005; Vorobyev & Osorio, 1998). That is, a colour that stimulates a bird's
213 four receptors $\{u, s, m, l\}$ by $\{1, 2, 3, 4\}$ should have a distance of zero to a colour
214 $\{10, 20, 30, 40\}$, but univariate analyses that ignore the multivariate structure of
215 colour spaces might conclude otherwise. (Note that this only holds if the Weber
216 fraction is constant, such as under the assumption of a bright illuminant; other-
217 wise the noise is not constant and is indeed influenced by the irradiant intensity.
218 However, even in such case differences would result from signals evoking differ-
219 ent noises in the receptor, and thus still making a univariate approach problematic
220 Osorio *et al.*, 2004).

221 An alternative, multivariate metric suggested by Stoddard & Prum (2008) is
222 the volume overlap. In this approach, the volume occupied by a sample of colours

223 is estimated from the convex hull defined by all its points, and the separation be-
224 tween is inferred from their overlap. Stoddard & Stevens (2011) used this metric
225 to show that a greater overlap in colour volume between cuckoo and host eggs is
226 associated with lower rejection in this nest parasite interaction. This approach is
227 interesting because it considers the entire distribution of colour points in multi-
228 variate space, though there are limits to its interpretation: (i) there is a lower bound
229 to group separation (i.e. if samples do not overlap, there is no measure of their
230 distance, offering no distinction between non-overlapping samples that are near
231 or far from each other in colour space) and (ii) it is unclear how varying degrees of
232 volume overlap should be interpreted biologically (e.g. how biologically meaning-
233 ful is the difference between 20% or 40% overlap?). It is also particularly sensitive
234 to outliers such that, for example, if two samples largely overlap but one or both
235 include extreme values that “stretch out” their volumes, the overlap between these
236 groups will be underestimated. Likewise, there is no distinction between cases in
237 which there is a small overlap between two samples due to close proximity *versus*
238 two samples that are largely separated but either has extreme values that “reach
239 into” each others’ volume. These problems arise because the volume as defined
240 by a convex hull does not lend itself to a probabilistic interpretation — leading
241 to the often unacknowledged assumption that the sampled data reflects the true
242 boundaries of the population (however, “loose wrap” hypervolumetric methods
243 exist, but to our knowledge these have not been applied to colour studies; Blonder
244 *et al.*, 2017). Finally, in its original implementation this method does not consider
245 receptor noise or discrimination thresholds (but doing so is straightforward; see
246 below).

247 The most robust attempt at comparing distributions of colours was proposed
248 by Endler & Mielke (2005), who devised a non-parametric rank distance-based
249 approach based on the least sum of Euclidean distances, compared through multi-
250 response permutation procedures (LSED-MRPP). This approach is powerful due
251 to its multivariate nature and the fact that it calculates a measure of effect size

252 based on the relationship of between- and within-group distances. However, this
253 approach calculates a single effect size statistic that captures differences between
254 samples not only in their means, but also in their dispersion and correlation struc-
255 ture (i.e. shape; Endler & Mielke, 2005). In other words, like many other multi-
256 variate distance-based methods, this method is sensitive to confounding hetero-
257 geneity among samples when attempting to test for differences in *location* between
258 samples (Anderson & Walsh, 2013; Warton *et al.*, 2012). Further, like the volume
259 overlap, this approach does not consider discrimination thresholds (though, again,
260 it would be straightforward to substitute Euclidean distance for distances in JNDs
261 to obtain receptor-noise limited statistics). Despite its strengths, this method has
262 seen little adoption in the discipline over the last decade, largely due to limitations
263 in implementation and accessibility.

264 The shortcomings of the methods described above reflect the fundamental fact
265 that the question of discriminability actually represents a test of two hypothe-
266 ses that are seldom formally distinguished: (i) that the focal samples are statis-
267 tically distinct, and (ii) that the magnitude of their difference is greater than a
268 psychophysiological threshold of detection. Most approaches will test one, but
269 not both, of these hypotheses through their respective nulls, and more often than
270 not with no estimate of variation, measurement error, or uncertainty in their es-
271 timates. Below we use a simulation-based approach to quantify these issues by
272 testing the efficacy of popular methods in detecting the separation of groups in
273 colour space. We then propose a flexible solution that avoids these problems, and
274 demonstrate its utility using heuristic examples drawn from the literature.

275 **Methods**

276 **Simulation procedures**

277 To test methods for detecting group separation in colour space, we simulated
278 data analogous to that obtained from applying an avian visual model to spectral

279 reflectance data. Birds are tetrachromatic (Hart, 2001) and colours will thus be
280 represented by the quantum catches in each of its four photoreceptors (though
281 the procedure followed here can be applied to visual systems with any number
282 of receptors). For each replicate, we simulated two groups of colours ($N = 50$ per
283 group) defined by four variables (*usml* photoreceptors) sampled from a multivari-
284 ate log-normal distribution (given that quantum catches are non-negative and per-
285 ceptual distances follow a ratio scale, as defined by the Weber fraction described
286 above). We generated samples according to two different scenarios: first, we sim-
287 ulated groups with varying degrees of separation (i.e. effect sizes) to evaluate
288 the power and Type I error rates of the approaches tested. Second, we simulated
289 threshold conditions to evaluate the performance of different approaches in cor-
290 rectly classifying whether samples are perceptually distinct.

291 For the first set of simulations focused on testing power and error-rates we
292 sought to consider a wide range of positions in colour space and intra-group vari-
293 ances. We therefore simulated the quantal catch of each photoreceptor i for the
294 first sample (group A) by drawing from a log-normal distribution with mean μ_{iA}
295 seeded from a uniform distribution $\mathcal{U}(0, 10)$, and standard deviation proportional
296 to the mean $\sigma_i = a_i \mu_{iA}$, with $a_i \sim \mathcal{U}(0, 0.5)$ (note that, for these simulations, μ
297 and σ refer to the mean and standard deviation of the random variable itself,
298 not in log scale). In order to generate two samples with varying degrees of sep-
299 aration proportional to the within-group variance, we used a multivariate effect
300 size S obtained by calculating a constant $k_i = \frac{S}{\sqrt{n}} \bar{\sigma}_i$, where n is the number of
301 photoreceptors (in this case, 4) and $\bar{\sigma}_i$ is the standard deviation of the sample.
302 We then drew a second sample (group B) from a log-normal distribution with
303 $\mu_{iB} = \mu_{iA} + k_i$ and standard deviation σ_i . Thus, our simulations effectively pro-
304 duced two samples with Mahalanobis Distance $D_M \sim S$. We simulated data for
305 $S = \{0, 0.1, 0.25, 0.5, 0.75, 1.0, 1.5, 2, 2.5, 3.0\}$ (Fig. 2), each replicated 200 times for
306 group sample sizes $N = \{10, 20, 50, 100\}$.

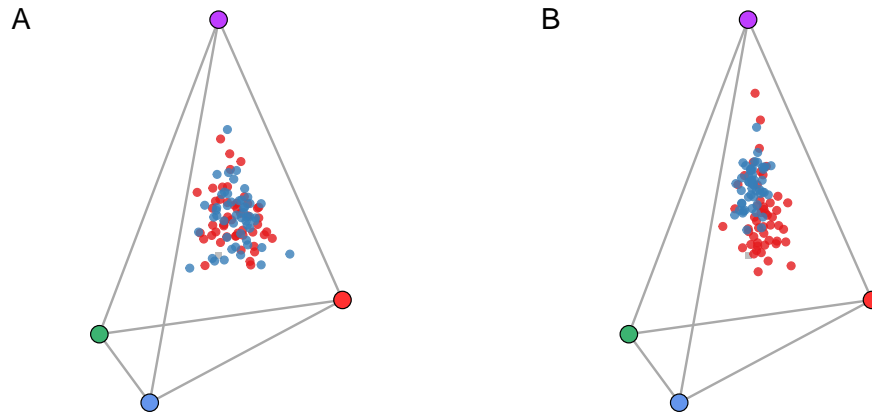


Figure 2: Example simulated data for the two groups (red, blue) in a tetrahedral colourspace. Shown here are data with sample size $N = 50$, and effect size $S = 0$ (left) and $S = 3$ (right).

307 For the second set of simulations we focused on threshold conditions across
308 a range of within-sample variation, following a similar procedure as described
309 above. Group A was sampled from a log-normal distribution with $\mu_{iA} \sim \mathcal{U}(0,10)$,
310 while σ_i was taken from an exponential distribution $\sigma_i \sim \text{Exp}(\lambda = 1)$. To obtain
311 a second sample, group B, that was separated from group A with an average ap-
312 proximate distance of ~ 1 JND given a Weber fraction of 0.1 (as often assumed
313 for the long-wavelength photoreceptor for birds; Vorobyev *et al.*, 1998), we would
314 need to draw from a distribution that differed in geometric mean quantal catch
315 by $\frac{0.1}{\sqrt{n}}$ (as described above for the relationship between S and k). However, due
316 to variation in relative receptor densities, the Weber fraction will also vary among
317 receptors, even if assuming a constant single receptor noise-to-signal ratio. There-
318 fore, to simplify simulations, we drew group B from a log-normal distribution
319 with $\mu_{iB} = k_i \mu_{iA}$, where $k_i \sim \mathcal{U}(0.88, 1.12)$, resulting in a distance between geo-
320 metric means (hereafter, “mean distance”) of 1.11 (95% quantiles: 0.35 – 2.77) and
321 within-group average pairwise distance of 4.46 (95% quantiles: 1.03 – 11.10 after
322 1000 simulation replicates.

323 After the two groups were simulated, we used the R package *pavo* (Maia
324 *et al.*, 2013) to calculate the colour distances between each pair of points in colour
325 space. We used the default function parameters (relative receptor densities for
326 $\{u, s, m, l\} = \{1, 2, 2, 4\}$ and Weber fraction for $l = 0.1$). Based on these distances,
327 we calculated the average within-group pairwise distance across both groups, as
328 well as the average between-group pairwise distance (average link distance). Fur-
329 ther, we estimated the geometric mean for both groups and the distance between
330 them.

331 We then used four procedures to statistically test for a difference between the
332 two groups. First, we used a distance-based PERMANOVA (hereafter “distance
333 PERMANOVA”) using the *adonis* function in the R package *vegan* (Oksanen *et al.*,
334 2007). This non-parametric approach uses distances in JND to directly calculate a
335 pseudo-F statistics based on the ratio of among:within distances between groups,
336 and obtains a null distribution by randomizing distances between observations
337 (Anderson, 2005). We recorded if the analysis was significant using 999 permuta-
338 tions for the null distribution, as well as the R^2 (the proportion of dispersion ex-
339 plained by the grouping factor) as an estimate of the effect size of the test. Second,
340 we obtained XYZ Cartesian coordinates based on perceptually-scaled distances
341 Delhey & Peters (2008); Pike (2012), and conducted a MANOVA on these variables,
342 again recording if the analysis was significant (hereafter “Cartesian MANOVA”).
343 For simplicity, we used a sum of squares and cross-products matrix approach and
344 calculated Pillai’s trace and its associated P-value, but see discussion for exten-
345 sions of this approach that allow for more complex parametrizations and relaxed
346 assumptions. Third, we calculated the volume overlap between the two groups
347 of points (relative to their combined volumes) in a tetrahedral colour space de-
348 fined by the receptors’ relative quantum catches (and thus not considering recep-
349 tor noise; Stoddard & Prum, 2008). Finally, we repeated the volume overlap for
350 the XYZ Cartesian coordinates based on perceptual distances, thereby generating
351 a colour volume overlap that also accounts for receptor noise.

352 Simulation results

353 Power and error rates

354 Both the distance PERMANOVA and the Cartesian MANOVA showed appropriate
355 Type-I error rates, with about 5% of our simulations producing significant results
356 when groups were sampled from the same population ($S = 0$), even when sample
357 sizes are small (Fig. 3). As expected, the power to detect small effects steadily
358 increased as a function of sample size, with the distance PERMANOVA being
359 slightly more conservative than the Cartesian MANOVA across both sample and
360 effect sizes (Fig. 3,4).

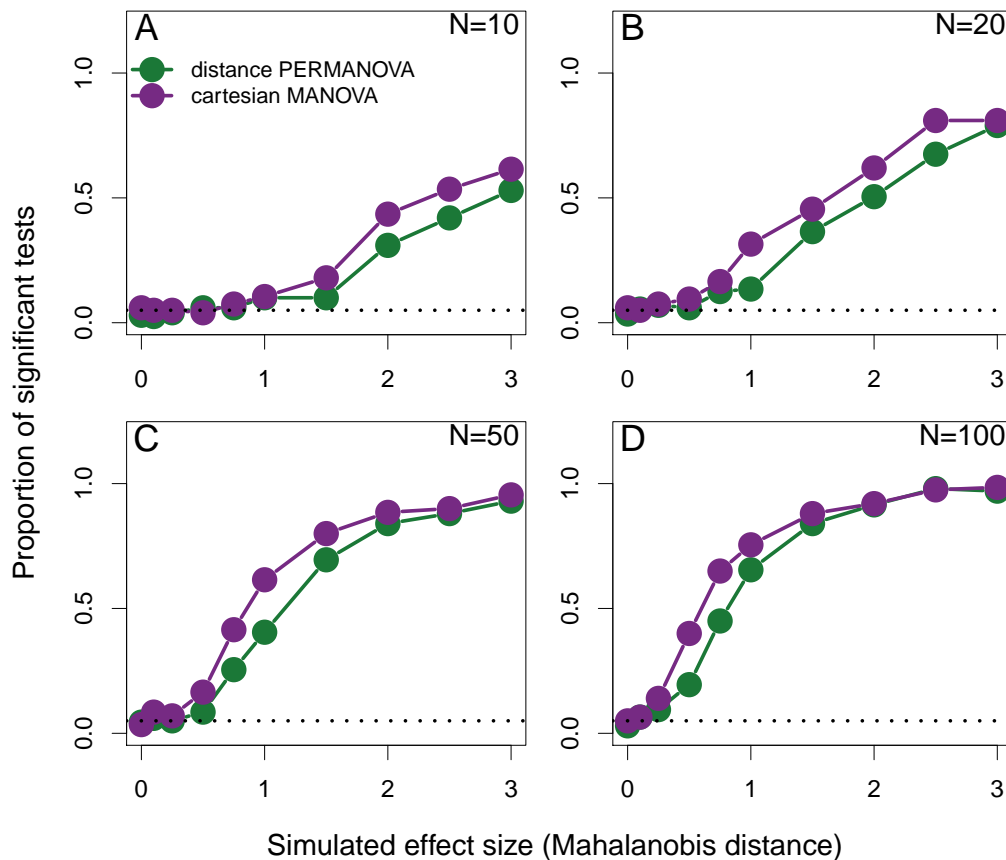


Figure 3: Power and Type I error rate of the distance PERMANOVA (green) and Cartesian MANOVA (purple). Panels show the proportion of simulations yielding significant results for each approach for simulations with varying sample sizes and effect sizes.

361 As a result, the two approaches showed some disagreement, with between
362 10 – 15% of the simulations being significant only in one of the two approaches
363 (Fig.4). This disagreement was not random, being concentrated at smaller effect
364 sizes with increasing sample sizes, and also with the Cartesian MANOVA being
365 more likely to consider a comparison significant when it was not significant under
366 the distance PERMANOVA than vice-versa, at an approximately constant rate
367 regardless of the sample size (Fig.4).

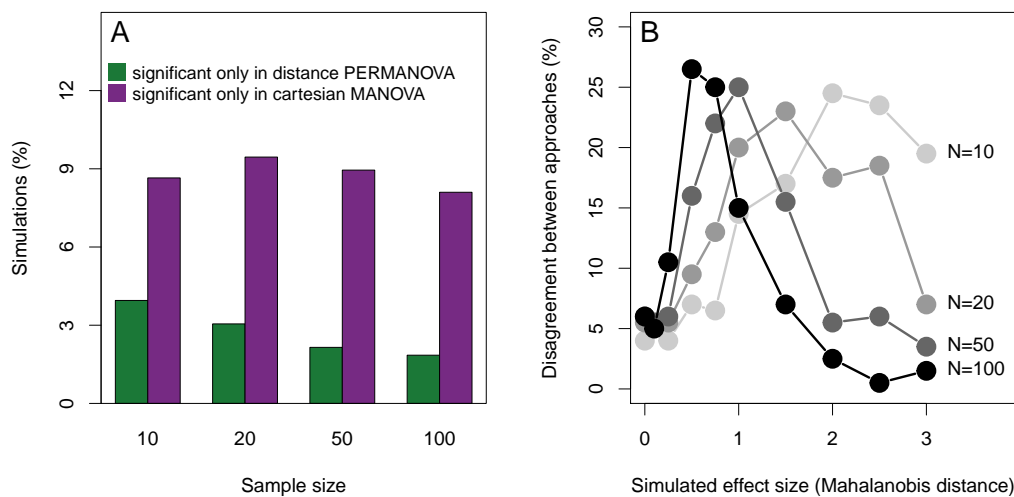


Figure 4: The disagreement between multivariate statistical approaches when testing for separation between samples in colour space.

368 Focusing on the $N = 50$ simulations, our results show that about half (46.5%) of
369 our simulations produced samples with mean difference greater than $1JND$ (Fig.
370 5). In these simulations, mean distance was positively associated with the effect
371 size, and the threshold of significance using the distance PERMANOVA fell ap-
372 proximately at the $1JND$ mark (Fig. 5A; equivalent results are observed with the
373 Cartesian MANOVA, not shown). Still, even around the $1JND$ mark significance
374 is variable, showing that large within-group variation can lead to non-significant
375 differences between groups despite among-group differences being, on average,
376 above the perceptual threshold. Colour volume overlap also showed a (negative)

377 association with the effect size, but despite being both an estimate of relative po-
378 sition and overlap between groups, no specific threshold for significance is iden-
379 tifiable (for example, both significant and non-significant results are observed for
380 values of overlap between 20 and 60%; Figure 5B).

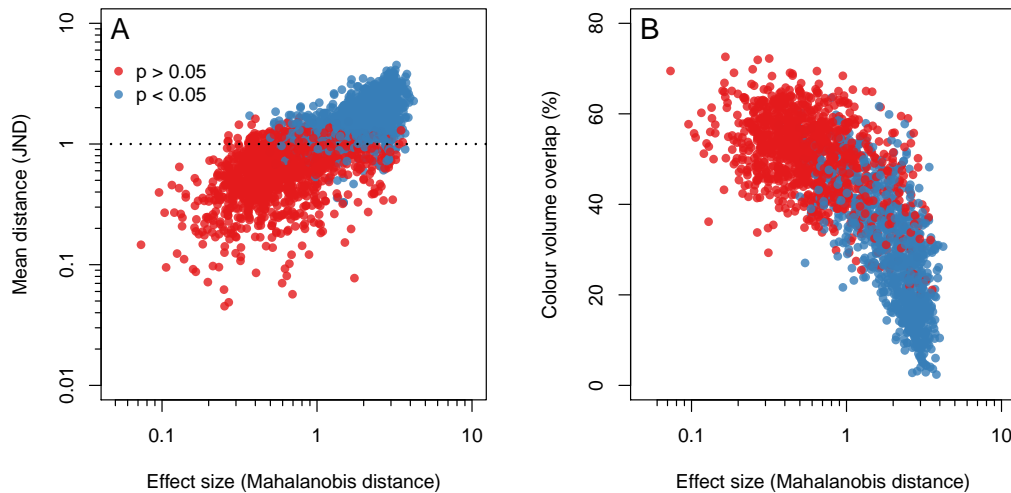


Figure 5: The association between effect size and (A) mean distance and (B) colour volume overlap. Significant distance PERMANOVA results are in blue, whereas non-significant results are in red. Dotted line indicates the threshold of 1JND.

381 **Threshold scenarios**

382 Our second set of simulations evaluated scenarios in which among-group differ-
383 ences were near the threshold of detectability (ca. 1 JND) across a wide range of
384 within-group variation. Since both the distance PERMANOVA and the Cartesian
385 MANOVA produced similar results, we focus on the former due to the conve-
386 nience of the resulting R^2 statistic in approximating the degree of among-group
387 separation. Our simulations generated a wide range of outcomes as desired, with
388 non-significant and significant tests both above and below the perceptual thresh-
389 old of 1 JND (Fig. 6). Thus, in contrast with our power simulations above (Fig 5),
390 the significance threshold did not match the perceptual threshold in these simula-
391 tions. Thus, as in the hypothetical example discussed in the introduction, some of

392 our simulated groups were statistically inseparable despite having mean distances
393 above the perceptual threshold (Fig. 6, dark red points). Likewise, some of our
394 simulations produced scenarios in which the samples were statistically different,
395 but that difference was below the perceptual threshold and therefore biologically
396 undetectable to this observer (Fig. 6, dark blue points). These results highlight the
397 importance of considering both among-group separation and perceptual thresh-
398 olds when testing the hypothesis that samples are perceptually discriminable.

399 Figure 6A shows that, intuitively, tests were significant when within-group dif-
400 ferences were proportionally small relative to among-group differences. However,
401 when measuring within-group difference using the link distance (i.e. the average
402 pairwise distance between all pairs of colours) nearly all simulations—including
403 most significant results—fell below the 1:1 line, indicating that the link distance is
404 a poor approximation of the variance-covariance structure of the data, overestimat-
405 ing it by about $0.5JND$ (grey line in Fig. 6A: mean distance = mean within-group
406 distance $- 0.05$). We can further see that significant results can be obtained for
407 fairly low levels of among-group separation, with R^2 as small as 3 or 4% (Fig. 6B,
408 horizontal line at 3%).

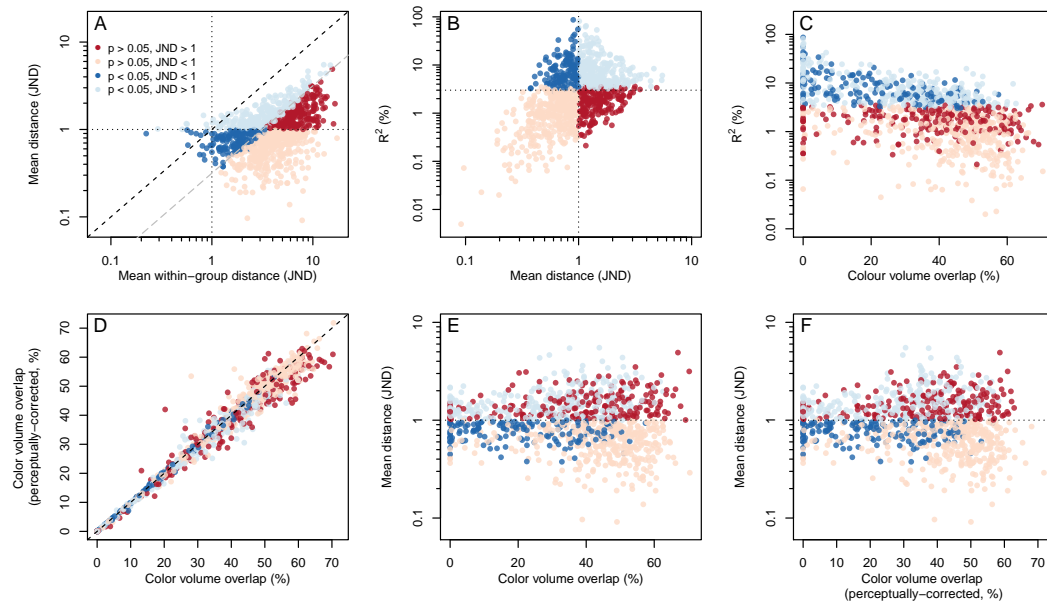


Figure 6: Results from threshold simulation. Red and blue denote non-significant and significant PERMANOVA tests, respectively, and light colours denote when that approach would yield the same inference as comparing mean distances to a threshold of 1JND. Thus, dark blue points indicate a significant statistical test that does not reach the threshold of discriminability of 1JND, whereas dark blue points indicate a non-significant statistical test that nonetheless has a mean distance greater than 1JND.

409 Though there is a negative association between R^2 and the overlap between
410 colour volumes, our results indicate a low overall consistency between these two
411 approaches: for any given level of volume overlap, all possible combinations of
412 results are observed — even when there overlap between samples is zero (Fig.
413 6C, E-F). In other words, even complete separation in colour volumes can re-
414 sult in non-significant, below-perceptual threshold cases, since samples can be
415 in close proximity in perceptual space (Fig. 6E-F) and have high within-group
416 variance. Likewise, samples can have high overlap but still be statistically and
417 perceptually distinguishable, because their overall distributions are nonetheless
418 discernible in multivariate space. Further, there is no association between volume
419 overlap and mean distance between groups (Fig. 6E,F). Importantly, these results
420 were unaltered by calculating volumes in perceptual colour space, since these are
421 still strongly and positively correlated with their non-perceptual counterparts (Fig.

422 6D,F).

423 **A flexible method for estimating statistical and perceptual separation**

424 As described conceptually and shown through simulations above, testing for dis-
425 criminability between two samples of colours actually requires testing two distinct
426 hypotheses: (i) are samples statistically distinct, and (ii) are samples perceptually
427 distinct. We therefore propose a two-step approach to answering the question of
428 discriminability between groups, which explicitly formalizes these hypotheses.

429 For the first question — are the samples of colours *statistically separate* in colour
430 space? — both a PERMANOVA using perceptual colour distances (Anderson,
431 2005; Cornuault *et al.*, 2015), or a MANOVA using perceptually-calibrated Carte-
432 sian coordinates (Delhey & Peters, 2008; Pike, 2012) are well suited to the task
433 (demonstrated above). Both approaches exclude achromatic variation, properly
434 account for the multivariate nature of visual model data, and perform well when
435 facing heterogeneity and outliers. There are also minimal difference in results
436 between the two (Fig. 3.4), so the decision between them may be informed by
437 convenience and the structure of the data at hand. The conceptual simplicity and
438 non-parametric robustness of the PERMANOVA has seen it widely adopted in
439 community ecology and genomics, and it has been shown to be the least sensitive
440 distance-based non-parametric approach to within-group dispersion and correla-
441 tion structure heterogeneity (Anderson & Walsh 2013; though see Warton *et al.*
442 2012 for broader limitations of distance-based methods, which are relatively com-
443 mon among colour space data; Endler & Mielke 2005).

444 Once the separation of samples is established statistically, a second question
445 must be answered: is this separation *perceptually discriminable*? Statistics calculated
446 as part of the first question will not generally be applicable here, since measures of
447 effect size desirably account for the ratio of among:within variation. We therefore
448 suggest this be tested independently by estimating the distance in colour space

449 between group geometric means rather than by calculating the average distance of
450 all pairwise comparisons or volume-overlap based metrics, which fail to accurately
451 estimate group separation (Fig. 6). However, this approach still has the limitation
452 of generating a single measure of distance for each pair of groups being com-
453 pared, with no measure of uncertainty. We thus suggest a bootstrap procedure
454 in which new samples for each group (of the same size as the original groups)
455 are produced through a re-sampling procedure (with replacement) of individuals
456 from that group, from which geometric means and the distance between them are
457 calculated. This procedure generates a distribution of mean distances, from which
458 the confidence interval for the observed distance can be estimated. If the groups
459 being compared are statistically different and this bootstrapped confidence inter-
460 val does not include the perceptual threshold of adequate biological significance,
461 one can conclude that the samples being compared are distinct and estimate their
462 degree of dissimilarity.

463 **Empirical examples**

464 We present two brief examples of this two-step approach to the analysis of visual
465 model data below, centred on questions of near-threshold discrimination drawn
466 from the literature. As above, we used the R package *pavo* for visual modelling,
467 and the *adonis* function in the R package *vegan* for PERMANOVAs.

468 **Sexual dichromatism in the leaf-nosed lizard *Ceratophora tennentii***

469 Visually signalling animals often use distinct body parts for different purposes,
470 such as social signalling to mates, or warning predators of available defences
471 (Barry *et al.*, 2015; Grether *et al.*, 2004; Johnstone, 1995). The nature of intraspecific
472 variation in colour traits can thus act as a guide to their putative function, since
473 selection may act differentially on signals used in different contexts. Aposematic
474 signals, for example, may be relatively invariable within species, by virtue of their

475 reliance on the formation of learning rules by predators (Endler, 1992; Guilford,
476 1990). Traits subject to strong sexual selection in one of the sexes, in contrast, are
477 often characterised by dimorphism, in which one sex (typically males) expresses
478 a conspicuous colour pattern that is greatly reduced or absent in the other (Bell &
479 Zamudio, 2012; Kemp & Rutowski, 2011).

480 Dragon lizards (Agamidae) are well known for variable colouration that is
481 used in both social and anti-predator contexts (Johnston *et al.*, 2013; Somaweera &
482 Somaweera, 2009). The leaf-nosed lizard *Ceratophora tennentii* has multiple discrete
483 colour patches, with apparent sex differences between body parts (Fig. 7). Here we
484 draw on the data of Whiting *et al.* (2015), who recorded the spectral reflectance of
485 29 male and 27 female *C. tennentii* from four body regions (throat, labials, mouth-
486 roof, and tongue). We used a tetrachromatic model of agamid vision to test for
487 sexual dichromatism among lizard body regions to test which colour patches, if
488 any, are sexually dimorphic from the perspective of conspecifics.

489 Following standard calculations for the log-linear receptor-noise model, we
490 estimated cone quantum catch as the log-transformed integrated product of stim-
491 ulus reflectance, ambient illumination, and photoreceptor absorbance across the
492 300-700 nm waveband (Vorobyev *et al.*, 1998). We used the spectral sensitivity of
493 *Ctenophorus ornatus* ($\lambda_{max} = 360, 440, 493, 571$ nm) as modelled according to a vi-
494 tamin A1 template (Barbour *et al.*, 2002; Govardovskii *et al.*, 2000). We assumed
495 a relative photoreceptor density of 1:1:3.5:6, and a photoreceptor signal-to-noise
496 ratio that yielded a Weber fraction of 0.1 for the long-wavelength cone (Fleishman
497 *et al.*, 2011; Vorobyev & Osorio, 1998). We tested each body region separately using
498 PERMANOVAs with 999 permutations.

499 We found a statistical difference between male and female throats (PERMANOVA:
500 $F_{1,58} = 14.84$, $P < 0.01$) and labials (PERMANOVA: $F_{1,57} = 13.96$, $P < 0.01$) as
501 modelled in agamid colourspace (Fig. 7a, b), but not for tongues (PERMANOVA:
502 $F_{1,58} = 1.63$, $P = 0.22$) or mouth-roofs (PERMANOVA: $F_{1,55} = 0.52$, $P = 0.50$;
503 Fig. 7C,D). However, subsequent bootstrap-based analysis of group separation

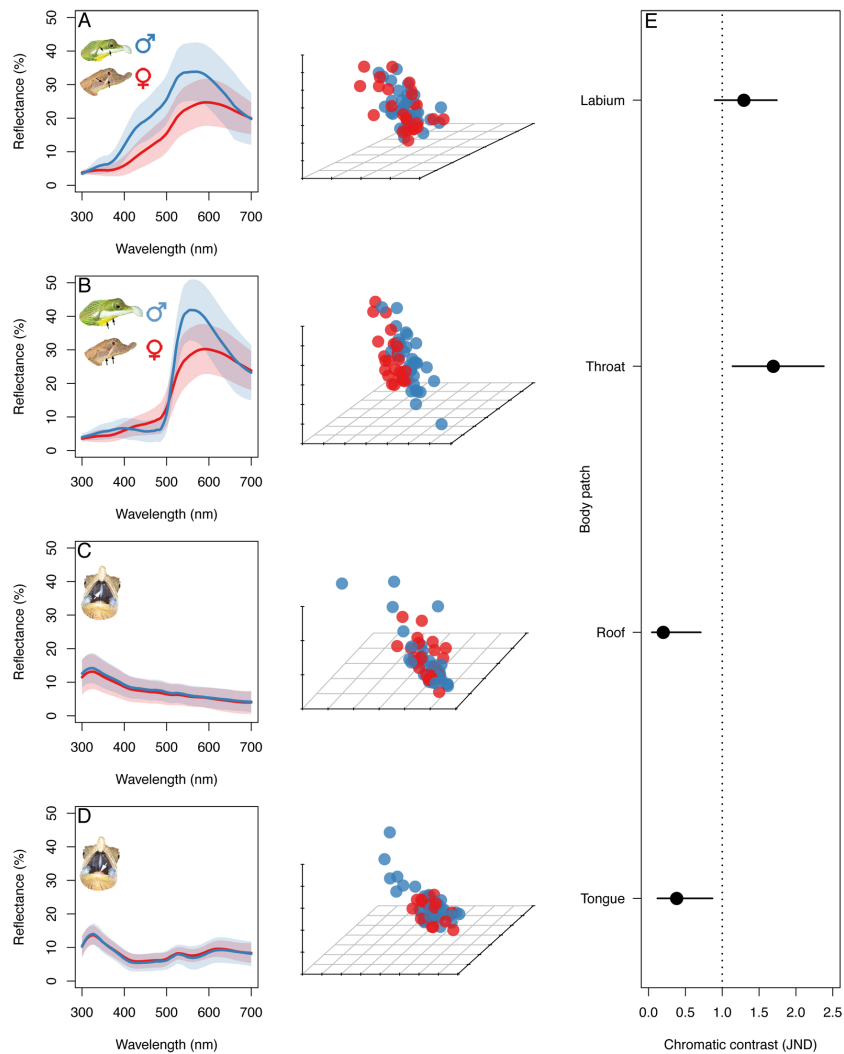


Figure 7: The mean (\pm SD) spectral reflectance of female (red) and male (black) (A) labial, (B) throat, (C) mouth-roof, and (D) tongue (left panels), and their colourspace distribution according in a tetrachromatic model of agamid vision (middle panels). Inset images indicate approximate sampling regions. The bootstrapped 95 % C.I's for mean distances between groups in colour space (right panels). Partly reproduced, with permission, from [Whiting *et al.* 2015](#).

504 suggested that intersexual differences in labial colour are likely imperceptible to
 505 conspecifics, and throat colour differences only barely so (Fig. 7; note that we
 506 present mean JND differences between all body parts for illustrative purposes,
 507 and it would generally be unnecessary to pursue this step for tests that were non-
 508 significant in the first step). Our results therefore suggest the absence of dichroma-
 509 tism in most measured body regions from the perspective of conspecifics despite

510 statistically significant differences in their colours, with only a subtle (predicted)
511 perceivable difference on the labials of males and females. Thus these results do
512 not implicate sexual selection as a strong driver of intersexual colour differences
513 in these body regions of *C. ornatus*.

514 **Floral mimicry in the spiny spider *Gasteracantha fornicata***

515 Biases in sensory perception offer opportunities for the evolution of deception
516 (Endler & Basolo, 1998). This is often showcased in predator-prey interactions,
517 wherein predators induce maladaptive responses in prey by using signals that
518 exploit innate preferences for certain visual cues, or learned preferences for oth-
519 erwise rewarding stimuli (e.g. Heiling *et al.*, 2003; O'Hanlon *et al.*, 2014). Many
520 sit-and-wait predators, such as orb-web spiders, use conspicuous colouration to
521 visually lure prey (reviewed in White & Kemp, 2015). While the attractant nature
522 of these signals is well documented (e.g. Chuang *et al.*, 2008; Tso *et al.*, 2007, 2002),
523 the ultimate basis of their effectiveness — that is, the nature of the sensory or
524 perceptual pathways being exploited in prey — remains unclear. A long-standing
525 hypothesis is that lures have evolved to mimic flowers (Chiao *et al.*, 2009; Tso
526 *et al.*, 2004), in which case theory predicts that the signals of mimics (lures) and
527 sympatric models (flowers) should be largely indistinguishable to their shared re-
528 ceivers (Christy, 1995; Endler & Basolo, 1998).

529 We tested this hypothesis using *Gasteracantha fornicata*, a conspicuously coloured
530 orb-web spider found in tropical and sub-tropical forests of Australia. Females of
531 this species are stably polymorphic and exhibit either 'white' or 'yellow' (UV-)
532 bands against a black outline (Fig. 8 inset), which serve to attract insect prey
533 (Hauber, 2002; Kemp *et al.*, 2013). We used a subset of data from several recent
534 studies Dalrymple *et al.* (2015); White *et al.* (2017); White & Kemp (2016), which
535 included reflectance spectra from the coloured bands of 33 yellow and 29 white *G.*
536 *fornicata*, and the flowers of 36 sympatric angiosperm species. We tested whether
537 spiders were discretely polymorphic from the perspective of a representative taxon

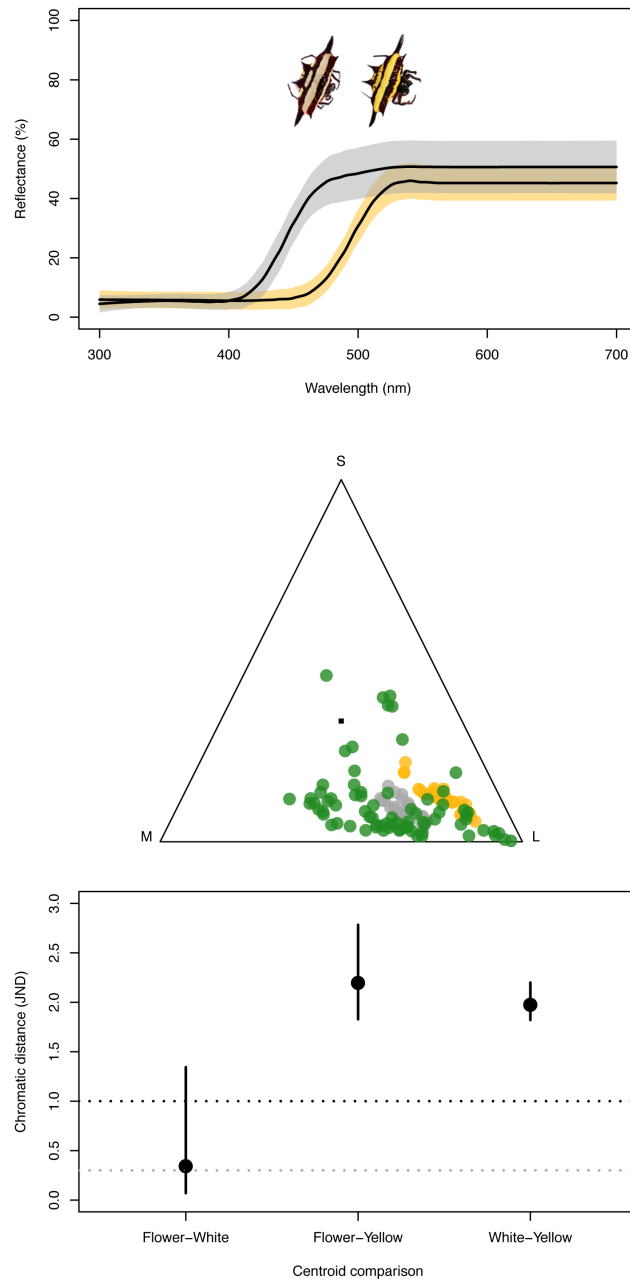


Figure 8: The mean (\pm SD) spectral reflectance of both morphs of the spiny spider *Gasteracantha fornicata* (top), and the location of spectra in a trichromatic colourspace representing Honeybee (*Apis mellifera* vision (middle), where yellow and grey points denote 'yellow' and 'white' spider morphs, respectively, and green points are sympatric flowers. The bootstrapped 95 % C.I. for mean distances between groups in honeybee colour space (bottom). Dotted lines indicate thresholds of 1JND (black) and 0.3JND (grey).

538 of insect prey (as given their conspicuous differences to human viewers; Fig. 8),
539 and whether spiders were indistinguishable from sympatric flowers, as predicted

540 by a floral-mimicry hypothesis (Christy, 1995; White & Kemp, 2015).

541 To estimate discriminability according to the receptor-noise limited model, we
542 used the visual phenotype of the honeybee *Apis mellifera*, with $\lambda_{max} = 340,440,536$
543 nm based on a vitamin A1 template (Govardovskii *et al.*, 2000). We estimated
544 photoreceptor noise using a Weber fraction for the long wavelength receptor of
545 0.13, and a ratio of 1:0.5:4.4 for the relative density of "ultraviolet", "blue", and
546 "green" photoreceptors. To test our hypotheses, we specified a PERMANOVA
547 with *a priori* two statistical contrasts: in the first, we considered yellow-versus-
548 white spider morphs (to test for polymorphism), and in the second, each morph-
549 versus-flowers (to test for mimicry).

550 We found spider morphs to be statistically distinct from one another in hon-
551 eybee colour space (PERMANOVA: $F_{1,60} = 32.13$, $P < 0.01$), but only the yellow
552 morph was statistically different from sympatric flowers (PERMANOVA: white vs.
553 flowers: $F_{1,134} = 1.61$, $P = 0.21$; yellow vs. flowers: $F_{1,134} = 9.30$, $P < 0.01$). Our
554 bootstrapped group distances predict that the statistically significant differences
555 are also perceptually distinct to a honeybee observer (Fig. 8). At a glance, these re-
556 sults offer relatively weak support for the hypotheses of polymorphism and floral
557 mimicry. As in all such analyses it is of course necessary to consider the assump-
558 tions of the model used. This includes consideration of the relevant "threshold"
559 value for focal species, which—even when based on an accurately parametrised
560 model (e.g. using precise Weber fractions)—may not strongly predict an animal's
561 realised abilities (Dyer, 2012; Dyer & Neumeyer, 2005). Honeybees, for example,
562 are capable of discriminating between simultaneously presented colour-stimuli
563 that differ by only 0.3 JND's under laboratory conditions (Dyer & Neumeyer,
564 2005). In this case, our results would be largely unaffected, because the statis-
565 tical test for the difference between white morphs and flowers is non-significant,
566 and the confidence interval of the white-vs-flower comparison also encompasses
567 the laboratory-ideal value of 0.3 JND's. Further interpreting the biological impli-
568 cations of such results demands a nuanced consideration of the assumptions of

569 the underlying model, as well as our broader understanding of the ecology of the
570 focal question. For example, future analyses may also consider the perspectives of
571 different viewers, or the distance estimates of alternate visual models (e.g. [Chittka,](#)
572 [1992](#)), all of which may be incorporated in our suggested framework.

573 **Discussion**

574 Visual models offer a powerful tool for quantifying the subjective perception of
575 colour which—as the ultimate canvas for colour-signal evolution—affords us di-
576 rect insight into a breadth of biological phenomena. It is therefore essential that
577 statistical considerations of biological hypotheses take into account both natural
578 variation in the samples being compared as well as the limits to perception that
579 observers experience. In this study, we highlight the importance of partitioning
580 these two facets. We show that contemporary methods typically consider only one
581 of these aspects, with undesirable consequences, and propose a flexible, robust al-
582 ternative that explicitly addresses both.

583 The use of relatively simpler visual models that do not consider the role of
584 receptor noise when defining colour spaces (and colour differences) is often jus-
585 tified on the basis of relaxing assumptions about the role noise plays in colour
586 perception; a phenomenon that requires intricate empirical work to estimate these
587 parameters with precision and identify the level of naturally occurring variation
588 ([Kelber *et al.*, 2017](#); [Olsson *et al.*, 2015](#); [Vorobyev & Osorio, 1998](#)). However, we
589 contend that these simplifying models often make very strong implicit assump-
590 tions, which are not necessarily supported by the empirical evidence: namely
591 that all cones contribute equally to colour perception, that colour discrimination
592 is unequivocal (i.e. there is no threshold of detectability) and that colour differ-
593 ences follow an interval scale (as opposed to a ratio scale). Thus, we argue that
594 detectability relative to a threshold of detection is essential for tests of discrim-
595 inability.

596 Our simulations show that both the distance PERMANOVA and the Carte-
597 sian MANOVA perform similarly well in statistically differentiating colours in
598 a perceptual space. As expected, the distribution-free non-parametric approach
599 showed slightly inferior power as a consequence of the relaxed assumptions about
600 underlying distributions. Both approaches are very flexible and can accommo-
601 date complex models, such as multiple predictors, interactions and hierarchical
602 designs. Several studies have pointed out that distance-based methods perform
603 poorly when the experimental design is unbalanced or when there are mean-
604 variance relationships or other sources of heteroscedasticity (Anderson & Walsh,
605 2013; Warton *et al.*, 2012). However, this might still be the most robust option
606 for high-dimensional visual systems (e.g. Arikawa *et al.*, 1987; Cronin & Marshall,
607 1989), by reducing the number of variables to a distance alone.

608 Multivariate generalizations of generalized linear models might offer a flexible
609 alternative, though they are still subject to assumptions of multivariate normal-
610 ity and equality of covariance matrices (though the latter can be relaxed when
611 the largest sample size group has greater absolute values in its covariance ma-
612 trix; Delhey & Peters, 2008; Tabachnick *et al.*, 2001). These models can also be
613 easily extended to include various error and model structures, such as hierarchi-
614 cal and phylogenetic models (Hadfield & Nakagawa, 2010; O'Hara & Kotze, 2010;
615 Warton *et al.*, 2012), and multi-response models can also relax the assumptions
616 of heteroscedasticity by estimating the variance-covariance of response variables
617 (Hadfield, 2010). When using a Bayesian approach, the mean distance bootstrap
618 can also be substituted by estimating distance credible intervals from the pos-
619 terior distribution of perceptually-corrected Cartesian coordinate estimated cen-
620 troids, though this will also be influenced by the priors adopted. This approach
621 also allows for the straightforward inclusion of luminance (achromatic) differ-
622 ences as another axis of variation in multivariate analyses (Pike, 2012), and can
623 be parametrized to reflect specific aspects of colour perception and discriminabil-
624 ity. For example, when comparing males and females of a species of bird with

625 multiple coloured patches, we may be interested in whether there is total plumage
626 differences between the sexes or, on the other hand, if there is at least a plumage
627 patch that distinguishes the sexes. By parametrizing a hierarchical model with
628 body patch as a random or fixed effect, one can model precisely what is meant
629 when asking if the species is dichromatic.

630 [Delhey & Peters \(2008\)](#) have recently advocated a similar approach to answer
631 the first question in our two-step approach, by suggesting the application of a
632 Principal Component Analysis (PCA) to the perceptually-corrected Cartesian co-
633 ordinates as an intermediate step before a MANOVA. However, if all the princi-
634 pal components are used in the multivariate analysis, results will be numerically
635 identical to simply using the XYZ coordinates directly. Further, Since it is often
636 tempting to discard PC axes of low variance in downstream analyses, which could
637 be problematic given the roll that residual variance may play in among-group dif-
638 ferentiation, we recommend using the Cartesian coordinates directly. Still, using
639 PCA's that preserve colour distances may be particularly useful when investigat-
640 ing differences in the orientation of axis of variation, and may be more readily
641 interpretable when tested against continuous variables ([Delhey & Peters, 2008](#)).
642 Of course while we have focused on tests of differences in the multivariate lo-
643 cation of colours in colour space, we recognise that other characteristics — such
644 as differences in dispersion and correlation structure — might themselves be of
645 biological interest.

646 These approaches will only test the degree of separation between groups in
647 colour space, and so it is still necessary to provide an estimate of the magnitude
648 of that separation. The bootstrap approach we present here provides a simple
649 solution, by adding an easy to interpret measure of accuracy to the mean distance
650 estimate. It is essential, however, to parametrize the underlying visual model
651 appropriately. The Weber fraction chosen for the receptor noise will strongly affect
652 perceptual distances ([Bitton *et al.*, 2017](#)) since it directly scales with the JND unit.
653 Further, even when adequate values of the Weber fraction are used, it is important

654 to realize that the unit JND value usually reflects psychophysiological limits under
655 extremely controlled conditions (Kelber *et al.*, 2003; Olsson *et al.*, 2015), and that a
656 more conservative estimate of two, four or even greater may be more appropriate
657 for ecological and evolutionary questions (Osorio *et al.*, 2004; Schaefer *et al.*, 2007).

658 Our results show that insight into the biology of colour and its role in commu-
659 nication is best achieved by disentangling the assumptions implicit in questions of
660 discriminability. By rendering these assumptions explicit, our two-step approach
661 offers a simple, flexible procedure for examining the statistical presence and per-
662 ceptual magnitude of differences between colour samples. We expect it will bring
663 exciting new perspectives on the role of colour in intra- and interspecific interac-
664 tions, and provide an efficient analytical framework for the study of colour in
665 nature.

666 **Implementation**

667 All analyses conducted here can be found in [the project's GitHub page](#), and will
668 be implemented in a release of the R package `pavo` to accompany the publication
669 of this manuscript. For now, they can be found in the GitHub page, under the `/R/`
670 folder. The function `bootcentroiddS` conducts the bootstrap for the calculation of
671 confidence intervals for mean distances, and the function `jnd2xyz` converts chro-
672 matic distances in JNDs to perceptually-corrected Cartesian coordinates. These
673 functions require the bleeding edge version of `pavo` to work, which can be found
674 on GitHub: <https://github.com/rmaia/pavo/>.

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681 **Author contributions**

682 RM and TEW conceived the ideas, designed methodology, analysed the data, and
683 wrote the manuscript. Both authors contributed critically to the drafts and gave
684 final approval for publication.

685 **Data accessibility**

686 Leaf-nosed lizard colour data from Whiting *et al.* (2015) is openly available at
687 <http://dx.doi.org/10.6084/m9.figshare.1452908>. Floral and spiny spider data
688 from Dalrymple *et al.* (2015) and White *et al.* (2017) is available from [http://dx.](http://dx.doi.org/10.6084/m9.figshare.1517656.v1)
689 [doi.org/10.6084/m9.figshare.1517656.v1](http://dx.doi.org/10.6084/m9.figshare.1517656.v1) Simulation data is available from the
690 project GitHub page at <https://github.com/rmaia/msdichromatism/>.

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