

The equivalent arc ratio for auditory space

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1 Abstract

2 The minimum audible movement angle increases as a function of source azimuth. If listeners do not
3 perceptually compensate for this change in acuity, then sounds rotating around the head should
4 appear to move faster at the front than at the side. We examined whether judgments of relative
5 amounts of acoustic motion depend on signal center angle and found that the azimuth of two signals
6 strongly affects their point of subjective similarity for motion. Signal motion centered at 90° had to
7 be roughly twice as large as motion centered at 0° to be judged as equivalent. This distortion of
8 acoustic space around the listener suggests that the perceived velocity of moving sound sources
9 changes as a function of azimuth around the head. The “equivalent arc ratio,” a mathematical
10 framework based on these results, is used to successfully provide quantitative explanations for
11 previously documented discrepancies in spatial localization, motion perception, and head-to-world
12 coordinate transformations.

13 1 Introduction

14 The binaural cues that are used to construct our internal representation auditory space are
15 interaural time difference (ITD) and interaural level difference (ILD). Both of these arise from the
16 physical structure of the head, as sounds arrive at the ear closest to the sound source both earlier
17 and at a higher level than at the further ear. Measurements of these binaural cues as a function of
18 sound source angle demonstrate that they change most rapidly at the front of a listener [1, 2]. If the
19 internal representation of space were based purely on these cues, then listeners would have
20 increased spatial resolution near the sagittal plane. Indeed this is well supported: both the threshold
21 measurements of minimum audible angle (MAA) and minimum audible movement angle (MAMA)
22 are known to change as a function of source azimuth [3, 4].

23 The study described here examined a potential consequence of this representation of auditory
24 space: namely that if listeners do not perceptually compensate for it, then the expansion of
25 resolution at the front and contraction at the side would dictate that a sound rotating at a constant
26 angular velocity around the head would not appear to do so, but would instead appear to move
27 faster at the front than at the side. Correspondingly, listeners turning their heads at a constant
28 velocity would experience an angle-dependent change in apparent source movement. The literature
29 on auditory motion is unclear on this subject and to our knowledge no studies have directly
30 examined the perceived difference in auditory motion at different angles and directions relative to
31 the head. That said, a number of curious discrepancies in spatial auditory perception have been
32 described over the years. Some of these have been classed as ‘incomplete coordinate
33 transformations’ [5], suggesting that a person’s head angle may affect the direction from which they
34 perceive a sound to emanate. Similarly, studies have reported discrepancies in listeners’ subtraction
35 of their own active head movements from the movement of the auditory scene [6, 7].

36 Although the latter two studies used Bayesian inference as a description of their observations, no
37 mathematical framework has been suggested to account for these findings. We propose instead that

38 these and a number of other discrepancies in spatial hearing could be explained via an angle-
39 dependent distortion in apparent sound location and motion. The underlying hypotheses to be
40 tested can be more formally stated as follows: first, that relative velocity judgements should change
41 as a function of azimuth, and second, that a quantitative model based on any observed expansion of
42 space should capture static phenomena such sound source eccentricity overestimation [8-11] and
43 dynamic phenomena like inconsistent self-motion subtraction [6, 7]. We further argue that the
44 MAMA is not simply a measure of acuity, rather it underlies a constant perceptual unit, changing in
45 absolute magnitude across space. One could argue that as a threshold measurement, roughly 1-2
46 degrees at the front of a listener may be considered equivalent to roughly 4 degrees at the side of
47 the listener in that these are the minimal amounts of motion required for a listener to change their
48 judgement from no motion to ‘some motion.’ Whether a similar azimuth-dependent expansion in
49 the perception of auditory space exists at suprathreshold levels has never been directly
50 demonstrated.

51 We term the proposed relationship of equal perceptual units across angle the “equivalent arc ratio”
52 for auditory space, borrowing terminology from the equivalent rectangular bandwidth [12]. Here we
53 quantify perceptual expansion by measuring the dependence of judgements of relative sound-source
54 motion on the angles with respect to the head from which the signals arrive. The consequences of
55 such a nonlinear representation of space are discussed, as is the potential for ‘hyper-stable’ virtual
56 acoustics. Finally, successful quantitative comparisons are made between the predictions of a
57 proposed mathematical framework and the results of a number of previously published studies.

58 2 Results

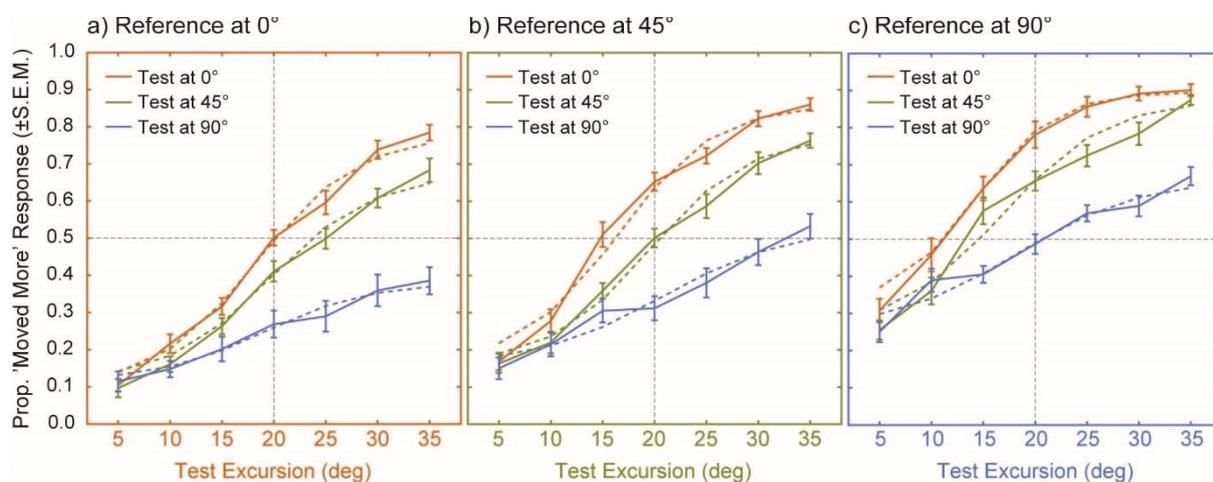
59 2.1 Relative motion judgements

60 Listeners were asked to make a judgement about which of two signals, a reference and a test signal,
61 “moved more.” The reference always moved 20° and the test moved less, the same, or more. Both
62 the test and reference signals could be centered at 0°, 45°, or 90°, and movement direction and the

63 order of presentation and condition were fully randomized. Across all conditions we found that the
64 center azimuth of both test and reference signals strongly affected subjects' comparison of relative
65 extents of motion, as expressed by a change in response as a function of test excursion. We found
66 main effects of test excursion, test azimuth, and reference azimuth ($F(1,6) = 338.11$, $p < 0.05$, $F(1,2) =$
67 71.34 , $p < 0.05$, and $F(1,2) = 97.74$, $p < 0.05$, respectively), an interaction between reference azimuth
68 and test excursion ($F(2,12) = 3.99$, $p < 0.05$), an interaction between test azimuth and test excursion
69 ($F(2,12) = 34.24$, $p < 0.05$), and a 3-way interaction between reference azimuth, test azimuth, and
70 test excursion ($F(2,24) = 1.92$, $p < 0.05$). The only insignificant comparison in the test was found in
71 the interaction between test and reference azimuths ($F(2,4) = 0.76$, $p = 0.55$).

72 Figure 1a illustrates this phenomenon for reference signals centered at 0° . Test signals centered at 0°
73 (orange line) were judged to move the same amount as the 20° reference motion when the test

FIGURE 1



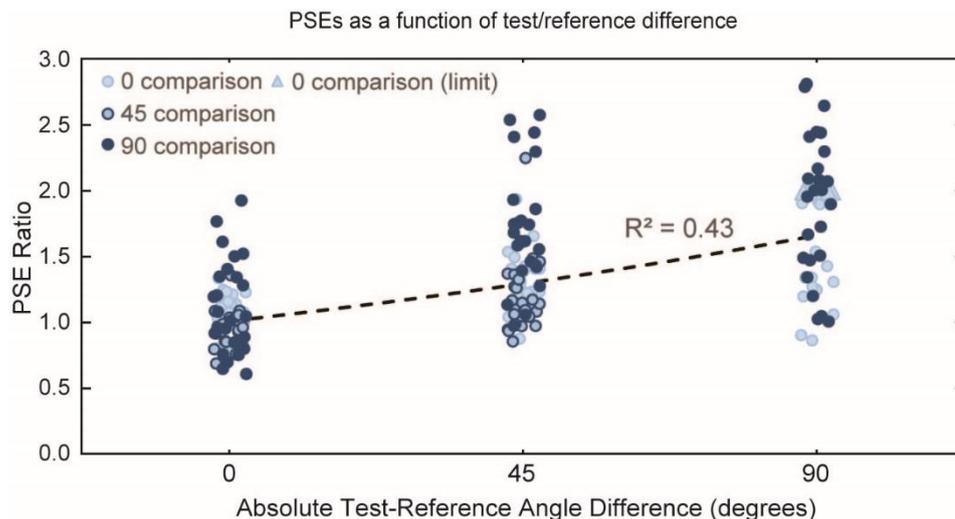
74 signals also moved by 20° . Thus the point of subjective equality (PSE) for this condition was roughly
75 $20/20 = 1$. The smooth psychometric function confirms that listeners were capable of making
76 judgements of relative motion, which is also reflected in the significant main effect of test excursion.
77 On the other hand, test signals centered at 45° (green line) had to move by about 25° to be judged as
78 moving the same amount as the 20° movement of the reference signal at the front. The range of
79 movement angles we used was not sufficient to estimate how much a 90° test signal (blue line) had

80 to be moved to reach the PSE with a reference at 0°, but the required excursion was likely much
81 greater than 35°. In Figure 1b it can be seen that when compared to reference signals at 45°, test
82 signals at 0° had to be moved significantly less to be perceived as moving the same amount, and test
83 signals at 90° had to be moved significantly more. Finally, reference signals at 90° (Figure 1c) showed
84 a pattern of expansion that was roughly inverted as compared to Figure 1a. Here it can be seen that
85 motion at 0° had to be roughly half as large to be judged as equivalent to motion at 90°.

86 2.2 Points of subjective equality for motion

87 PSE ratios were drawn from the individual logistic fits to the data (the *mean* of said fits are plotted
88 with dotted lines in Figure 1). The point at which the logistic fit crossed 0.5 probability was taken for
89 each condition for each listener and divided by the reference motion PSE. PSE ratios were also
90 computed for inverted pairs (i.e., the test/reference ratio 0/90 is accompanied by 1/the
91 test/reference ratio 90/0). A scatter plot of these PSE ratios, plotted as a function of the absolute
92 difference between the test and reference angles, is shown in Figure 2. The use of ‘comparison’ in

FIGURE 2



93 the legend is due to the mix of normal and inverted pairs (where reference and test are used
94 interchangeably). When the test and reference motions were identical the PSE ratios were clustered
95 around 1, albeit with a large degree of intersubject variability. PSE ratios for a difference of 45° were

96 on average larger than 1.0, and ratios for a 90° difference were still larger, reaching a value of
97 roughly 2. It should be noted that the triangle symbols in the plot represent measurements in which
98 the test signals at maximum excursion were still not judged to be moving by the same amount as the
99 20° reference motion. The true values for these data points cannot be reliably estimated as the
100 psychometric functions in question did not cross the PSE, but examining the individual data and
101 logistic fits makes it clear that the values are likely to be substantially larger than 2. The Pearson
102 correlation coefficient between the PSE ratio and the difference in test/reference angle is R^2 of 0.43.

103 3 Discussion

104 3.1 The non-uniformity of acoustic motion

105 The observed changes in the apparent amount of motion across azimuth are not subtle, making it
106 somewhat surprising that this effect has not been previously reported. Across all conditions we
107 found that the relative azimuth of two signals strongly affects their point of subjective similarity for
108 motion. Roughly speaking, 20° of motion at the front of the listeners is treated equivalently to 40° of
109 motion at the sides. This difference in PSE ratio over azimuth, which we will refer to as the
110 equivalent arc ratio from here onward, represents a perceptual expansion of space at the front and a
111 contraction at the sides. On one level, the equivalent arc ratio could be interpreted as a simple
112 relationship between acuity and perception, but this belies two perceptual consequences. One
113 consequence is that a sound rotating at a constant angular velocity around the head would appear to
114 accelerate towards the front of the listener, and decelerate towards the side. The second
115 consequence is that – from the perspective of a moving listener –the acoustic world not appear
116 stable as the head turns. Instead signals at the front should appear to counter-rotate as the listener
117 turns, and signals at the side should seem to be slightly dragged along with the listener's rotation.

118 3.2 Distortion in acoustic location

119 There are two possibilities for reconciling the observed change in perceived motion as a function of
120 angle with our current understanding of sound localization. The first requires a disassociation

121 between movement and location; in this case the apparent location of a signal at the end point of a
122 movement would have to be different from its apparent position(s) during the movement. There is
123 evidence in the visual system of just such disassociation [13]. It is conceivable that a similar process
124 occurs in the auditory system, but the disassociation in the visual system is thought to arise from
125 specialized motion-sensitive neurons in the middle temporal visual and medial superior temporal
126 areas [14], brain regions known for motion selectivity [15, 16]. Currently, however, there is no
127 physiological evidence for auditory neurons that exhibit motion selectivity while being agnostic to
128 spatial location.

129 If we assume, on the other hand, that auditory motion and spatial location are intrinsically linked
130 with each other, then the second possibility is that both the motion and the perceptual location of
131 *static* sound sources would be subtly distorted as a function of head angle. This framework prevents
132 any jump in perceived location after a movement (as would be found above), but requires that
133 listeners mislocalize sound sources. The function and its constants were chosen so that its slope at 0°
134 and its slope at 90° were related to each other in the same manner as the equivalent arc ratio
135 between these two angles. We used a hyperbolic tangent (Equation 1) because it is readily invertible,
136 although one could in principle also use a sine-expansion, or some other mathematical construct.

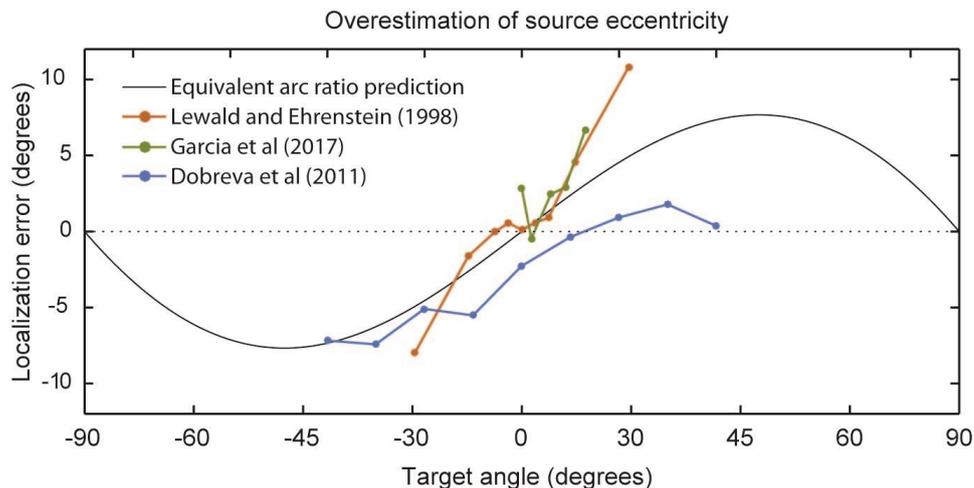
137 Equation 1:

$$138 \theta_p = \frac{\tanh(\theta_a \log(R t - c))}{\tanh(\log(R t - c))}$$

139 where all angles are degrees / 90 (including θ_a), \ln is the natural logarithm, t is a constant equal to
140 7.08, c is a constant equal to 5.97, R is the ratio between the PSE at 90° and at 0°, θ_a is the actual
141 position of a signal, and θ_p is the perceived position of that signal. The constants t and c were
142 empirically derived (using Matlab's `fminsearch` function) to ensure that the ratio between its 20°
143 slope (the amount of reference motion) at 0° and at 90° was closest to the ratio R between the PSE
144 at 0° and at 90° over a reasonable range of values of R .

145 For Θ_a angles larger than 0 and less than 90, the values of Θ_p generated by Equation 1 imply that
146 static acoustic targets would be perceived at larger eccentricities than they truly are. Precisely such a
147 phenomenon has been repeatedly demonstrated in the literature, as listeners have been shown to
148 regularly overestimate the angle of sound sources [11], particularly when fixating at the front and
149 using a laser pointer to indicate direction. Equation 1 provides a reasonable fit to the overestimation
150 of source angle measured in at least three separate laser-pointer studies [8-10] (laser pointing being
151 the most comparable task condition it does not involve a head movement). The data from the most
152 relevant portions of these studies are plotted in Figure 3 alongside predictions from the model. The

FIGURE 3

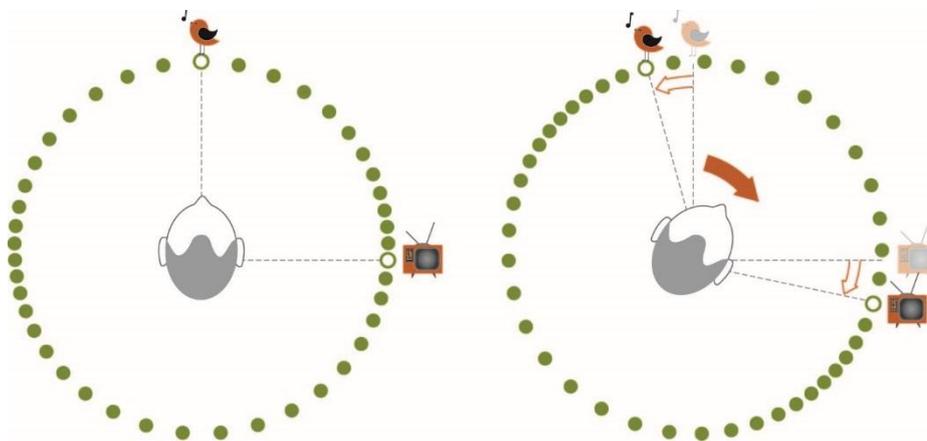


153 predictions are plotted as the difference between perceived and actual locations ($\Theta_p - \Theta_a$), and these
154 values fall well within the range of the data from the three studies. Physiological data on this subject
155 are somewhat limited, but predictions of a neural network trained on spike data from cat primary
156 auditory cortex also show a characteristic overestimation of target position that roughly follows the
157 predicted pattern [17]. The magnitude of this overestimation, however, is far larger than has been
158 observed behaviorally or predicted by the current mathematical framework.

159 3.3 Distortion in acoustic motion

160 When Equation 1 is used to examine *motion* (by examining the differences in the distortion between
161 θ_a and θ_p at different head and source angles) it becomes clear that the overestimation of static
162 signal angle must move with the head. The consequence of this is that signals appear to move in
163 different ways depending on their subtended angle with respect to the head during a turn. Figure 4
164 displays the way in which the apparent location of two sound sources (here a bird and a television)
165 should shift as the listener turns to the right. A signal at 0° should shift to the left and a signal at 90°

FIGURE 4



166 should shift to the right. Supplemental Figure S1 is an animation of this phenomenon depicting the
167 perceived locations of 32 static signals arranged around the head as it turns. The angle of the
168 listener's nose is depicted as a line along the radius of the circle. The expansion/contraction in Figure
169 4 and in the animation is exaggerated by a factor of 2 for clarity.

170 There are established phenomena that suggest there are perceptual distortions of auditory space
171 that depend on some interaction between stimulus angle and head angle. Genzel and colleagues [7]
172 demonstrated that, after an active head movement, a second sound source had to be shifted in
173 azimuth to be perceived as being at the same azimuth as a sound before the movement. Within the
174 framework of the equivalent arc ratio, this may be explainable as a distortion in the perceived
175 location of a static midline signal. Using Equation 1 (with a $90^\circ/0^\circ$ PSE ratio of the mean 1.82), a 35.3°

176 active rightward movement (the average reported in the study) should result in a 0° signal appearing
177 to be at -6.5° , which not only sign-correct, it is also reasonably close to the value of -5.5° from Genzel
178 et al (2016). Other systematic errors in movement compensation have also been documented. For
179 example, Freeman et al [6] demonstrated that signals at the front of the listener must be moved
180 with the head with a gain of $+0.17$ to be judged as being static. Here gain refers to amount of motion
181 with respect to the head, so if a listener turns 10° to the right, signals that move by $+1.7^\circ$ to the right
182 would be most consistently judged to be static. The corresponding value predicted by Equation 1 is
183 $+2.2^\circ$, which is at least sign-correct if not a perfect match.

184 Physiological data on the relationship between self motion and spatial receptive fields is virtually
185 non-existent, making comparisons with animal work problematic. Eye position has been shown to
186 clearly influence the apparent spatial location of auditory signals [18, 19], to modulate responses in
187 the inferior colliculus [20] and auditory cortex [21], and to actively shift spatial receptive fields in
188 superior colliculus [22], but little work has been done on head movements. Very recently, however,
189 experiments in ferret primary auditory cortex have revealed a subpopulation of neurons whose
190 spatial receptive fields appear to be specified in world-centric coordinates, rotated in opposition to
191 the animal's movement [23]. This finding represents a neural correlate of our percept of a stable
192 acoustic world. It is not currently possible, however to determine whether the shifts in receptive
193 field boundaries as a function of eccentricity match that predicted by the equivalent arc ratio
194 because the width and contralateral offset of cortical receptive fields make it difficult to assign
195 individual neurons to exact azimuths in space.

196 Returning to psychophysics work, results from the Freeman et al (2016) study are roughly line with
197 what the equivalent arc ratio model would predict, with one notable exception. According to the
198 model, the gain at which signals must be moved to be judged as static should change as a function of
199 azimuth, reducing to 0 when 45° is reached, and even changing to a *counter*-rotation for larger
200 eccentricities. The Freeman et al study did not find this, although they did find a decrease in gain and

201 a substantial increase in variance as a function of stimulus angle. It should be noted that these
202 authors' own Bayesian explanation for the non-unity gain *also* predicts a change in gain as a function
203 of azimuth. But the subjects in that study were blindfolded, so the discrepancy may point to an as-
204 yet unresolved role of eye position in this and related phenomena. The previously mentioned
205 dependence of neural and behavioral responses on eye position certainly attest to this possibility. A
206 related phenomenon was previously described by Lewald and Ehrenstein [5]; the displacement of
207 the subjective auditory midline (as measured using ILD) towards the trunk as a listener turns to more
208 eccentric head angles. This displacement was argued as being the result of an 'incomplete
209 coordinate transformation,' a failure of listeners to fully compensate for their own movement. Taken
210 together with the results from Freeman et al [6] this suggests that head-to-trunk angle may
211 represent a second unresolved factor that results in a shift in target location into a different region of
212 expansion / contraction of acoustic space.

213 Studies examining representational momentum have argued that the faster a signal is rotating
214 around the head, the further the perceived endpoint will be displaced in space [e.g., 24, 25]. This is
215 argued to be a consequence of a mental extrapolation of the signal's trajectory [26]. According to
216 the equivalent arc model, signals moving towards the midline would appear to accelerate, suggesting
217 their endpoints could seem more displaced than those of signals receding from the midline. An
218 advantage in direction discrimination has been demonstrated for signals approaching the median
219 plane [27], congruent with the equivalent arc model, but in the case of the first representational
220 motion study [24] all the motion trajectories used were across the midline. The analysis in the
221 second study [25] – while it did examine left versus right movements – collapsed the data across
222 different center azimuths, an averaging method that would prevent us from observing any
223 asymmetry predicted by the equivalent arc ratio model. Examination of the latter data set could
224 either lend support to or require a re-evaluation of the equivalent arc framework.

225 3.5 The relationship between the equivalent arc ratio and the MAMA

226 The equivalent arc ratio expansion observed appears to be related to – but not entirely dependent
227 on – the change in MAMA as a function of angle (the MAMA being roughly 1° in front of the listener
228 and increasing to about 4° at the side [4]). If the equivalent arc ratio were a simply the result of the
229 change in MAMA as a function of angle, then we might expect slightly larger equivalent arc ratios
230 between 0° and 90° than we observed. However, the two measurements may be linked with each
231 other on some level, as acoustic movement, whether a consequence of source or self motion, may
232 rely on similar underlying processing mechanisms [c.f. 28]. We did not test the MAMA at 0°, 45°, and
233 90° in our listeners, so we cannot at this point describe the correlation between the two measures.

234 3.6 Creating hyper-stable virtual acoustics

235 Because listeners may perceive signals to move at different velocities at different points in the arc
236 around the head, the equivalent arc ratio could be utilized alongside individualized head related
237 transfer functions and motion tracking to produce head-stabilized acoustic environments that
238 appear to be more stable than the real world. As seen from the scatter in Figure 2, the PSE ratio can
239 vary greatly from listener to listener. As such this must be measured or approximated through other
240 means to match a given listener's spatial distortion. Given the close relationship between the
241 equivalent arc predictions and previously described overestimations of target angle, it may be
242 sufficient simply to have a listener point to a few sound sources with a laser. Regardless of how this
243 is measured, an inverse of Equation 1 that is solved for θ_a would be necessary. This is included here
244 as Equation 2.

245 Equation 2:

$$246 \theta_a = \frac{\ln(10) \times \tanh^{-1} \left(\theta_p \tanh \left(\frac{\ln(R t - c)}{\ln(10)} \right) \right)}{\ln(R t - c)}$$

247 where the constants and definitions in the formula are the same as in Equation 1.

248 This formula allows one to determine the angles at a signal must be presented to be perceived at a
249 particular azimuth with respect to the head.

250 3.7 Caveats

251 The range of movement excursions in this study was not sufficient to compare references at 0° and
252 test signals at 90° for all listeners. We did not anticipate the magnitude of the spatial expansion that
253 we observed and so were not able to fully bracket the motion values and measure PSE ratios for all
254 movement pairs. We were able to measure PSE ratios for the inverse of these particular
255 reference/test pairs, but direct comparison between these makes the tacit assumption that the
256 amount of spatial expansion/contraction is a simply a multiple of the reference motion.

257 It should be also noted that Equation 1, while it may be reasonably applicable to perceptual
258 distortion of signal location in the listener's front hemifield they may not accurately reflect any
259 expansion or contraction of auditory space in the *rear* hemifield (and indeed Equation 1 is not
260 constructed to compute the perceived location of angles beyond $\pm 90^\circ$). We have no data that speaks
261 to this, so the expansion and contraction in the rear hemifield is depicted in Figure 4 and
262 Supplemental Figure S1 as a mirror reflection of the front, despite there being no reason to believe
263 this is necessarily the case. It remains for future studies to map out spatial distortions for 360°
264 around the head.

265 More generally speaking, since expansion estimates were only measured at three angles, it is unclear
266 whether a hyperbolic tangent expansion or some other function may be the most appropriate
267 mathematical descriptor of the change in equivalent arc ratio over all azimuths. Future work will be
268 required to determine what function best captures the observed phenomena but – provided the
269 function is readily invertible – such a technique could potentially increase the experience of
270 immersion for virtual reality systems.

271 3.8 Conclusions

272 Sound sources at the side of a listener must move at least twice as much as ones in front to be
273 judged as moving the same amount. This expansion of space in the front and compression at the side
274 that moves with the listener we term the equivalent arc ratio, and likely has real consequences for
275 spatial perception in dynamic listening situations. The prediction that the apparent location of static
276 sound sources may also be distorted suggests that this phenomenon is not limited to moving signals.
277 A mathematical model that mimics the equivalent arc ratio can be used to successfully predict
278 several previously unexplained phenomena in spatial auditory perception. We further suggest that
279 the inverse of this function could be utilized alongside individualized head related transfer functions
280 and motion tracking to produce head-stabilized virtual acoustic environments that appear to be
281 more stable than the real world.

282 4 Materials and Methods

283 4.1 Participants

284 We recruited 30 normal-hearing listeners, with normal hearing being defined as a four-frequency
285 average pure tone hearing threshold of less than 20 dB HL. Five listeners were excluded from the
286 analysis because they did not complete the full set of trials. We collected complete data sets for the
287 remaining 25 listeners, the result of two separate visits to the lab, with sessions of 60 minutes each.
288 The average age of the listeners was 27 (± 7.3 STD) years, ranging from 22 to 58 years old. We
289 received written and verbal informed consent from all subjects and the experiment was conducted
290 in accordance with procedures approved by the West of Scotland Research Ethics Service.

291 4.2 Stimuli and Presentation

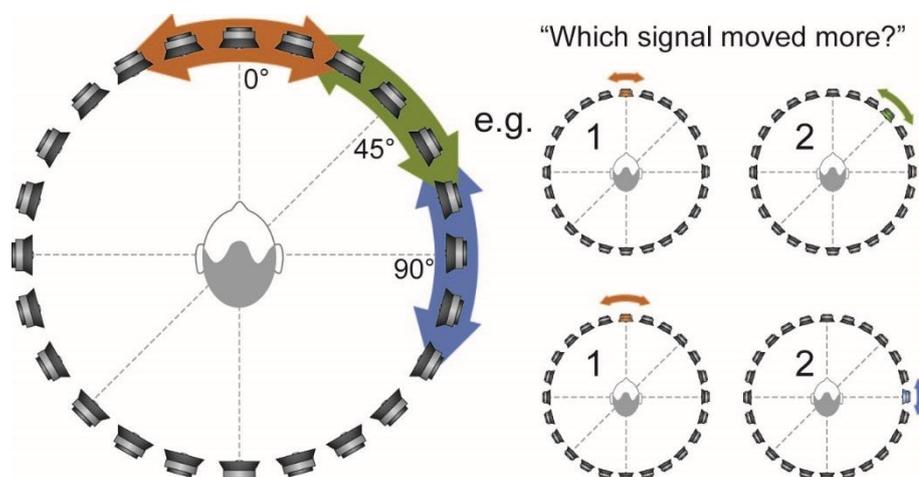
292 The experiment was conducted in a 4.8 x 3.9 x 2.75 m double walled, sound-attenuated chamber
293 that had 10 cm acoustic wedge foam lining the walls and ceiling, but not the floor, which was
294 carpeted. The listeners were seated in this chamber in the center of a 3.5 m diameter circular ring of
295 24 Tannoy VX-6 loudspeakers (Tannoy, Coatbridge, UK) placed at intervals of 15°. Because a forward
296 (towards the 0° loudspeaker) offset in listener position could yield an apparent expansion in space,

297 the listener's head was aligned with a spot on the ceiling and the floor. This method, while subject to
298 a few centimeters of error, prevented a misplacement that could explain the results observed (which
299 would require the listener to be at least an order of magnitude closer to the front loudspeaker). The
300 room was dimly lit, but the loudspeakers were visible, and listeners were asked to keep their head
301 still and their eyes open and fixated on the loudspeaker ahead of them at 0°. Signal sources were
302 moved around the ring using vector-based amplitude panning (performed on a sample-by-sample
303 basis in Matlab 2015b (The Mathworks, Natick, MA, USA) using the open source dynamic link library
304 "playrec" (www.playrec.co.uk)). The signals were played out using a MOTU 24 I/O (Mark of the
305 Unicorn, Cambridge, MA, USA) over ART SLA-4 amplifiers (Applied Research & Technology ProAudio,
306 Niagara Falls, NY, USA). The stimuli were unfrozen pink noise signals that were amplitude modulated
307 with a 10 Hz reverse sawtooth waveform at a depth of 50%. These signals provided sufficient high
308 frequency energy to provide robust interaural level differences as well as frequent sharp onset
309 transients to ensure that the signals were easily localizable. All signals were presented at a
310 comfortable listener-determined listening level (this ended up being between 70 and 75 dB SPL).

311 4.3 Experimental Paradigm

312 We measured the point of subjective equality (PSE) for amount of acoustic motion between "test"
313 and "reference" signals. The reference signal always moved 20° in a random direction and the test

FIGURE 5



314 signal moved either less, the same, or more (5, 10, 15, 20, 25, 30, or 35°), also in a random direction
315 (see Figure 5). The order of the test and reference signals was also randomized. In a two-alternative
316 forced choice paradigm, listeners were asked to report on a touchscreen whether the first or second
317 signal ‘moved more.’ If the listeners requested clarification on these instructions, they were told that
318 their task was to report whether the first or second noise moved over a larger distance in space,
319 regardless of its duration or apparent speed. Both the reference and the test signals could be
320 centred at either 0°, 45°, or 90° (See Figure 5) plus or minus a random value drawn from a uniform
321 distribution between -7.5 and 7.5°. The duration of the test and reference signals were individually
322 randomized on every trial to a value between 0.5 and 2 seconds. In this way, we mitigated velocity
323 and duration as potential cues, leaving total angular excursion as the variable that listeners were
324 asked to judge. Listeners were asked to complete a total of 10 blocks of 126 trials, each of which
325 contained 6 repeats of the 21 conditions.

326 The resulting psychometric functions for each listener were individually fitted with a logistic function
327 using Matlab’s `fminsearch` function. The resulting parameters were fed into an inverse logistic
328 equation to compute the test excursion value at which the function crossed the PSE (the reader may
329 roughly infer these values in Figure 1 as the point where the mean fit (dotted line) crosses 0.5 on the
330 y-axis). For logistic fits that did not cross the PSE before 40° (the next larger measurement point
331 step) we fixed the value at 40°. This likely underestimates the true PSE for many listeners, but avoids
332 excessive extrapolation. This value was divided by the reference excursion of 20° to yield a ratio
333 expressing the amount of expansion or contraction of auditory space.

334 4.4 Statistics

335 All statistics were performed with the Statistics Toolbox in Matlab 2016a. The analysis consisted of a
336 three-way repeated measures ANOVA with the dependent variable being the proportion of ‘moved
337 more’ responses, and the independent variables being reference angle, test angle, and test
338 excursion. Alpha was set to 0.05.

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413

414 Figure Legends

415 **Figure 1.** Psychometric functions for motion comparisons and points of subjective similarity (PSE)
416 for motion for moving signals centered at 0°, 45°, and 90°. A) For conditions with a reference signal
417 at 0°, the psychometric function for test signals also at 0° crossed the PSE at 20 degrees (orange line),
418 whereas test signals at 45° and 90° (green and blue, respectively) had to move more to be judged as
419 moving the same amount (rightward shift in the curves). B) Compared to 45° reference signals, 0°
420 test signals had to be moved less (orange) and 90° signals more (blue) to be judged as moving the
421 same amount. C). References of 90° required less motion to be judged the same as both the 0°
422 (orange) or the 45° (green) test signals.

423 **Figure 2.** Scatter plot of PSE ratios showing an expansion of auditory space. All x values are jittered
424 for visibility. PSE ratio increases as a function of the absolute difference between test and reference
425 angles. The different symbols represent actual angle comparisons, some values for which were
426 inverted from test/reference to reference/test. Triangle symbols represent measurements in which
427 the test signals at maximum excursion were still not judged to be moving by the same amount as the
428 20° reference motion.

429 **Figure 3.** Predictions of overestimation of source eccentricity as a function of target angle. Data
430 demonstrating that listeners overestimate target angles are displayed from three separate previous
431 studies (colored dot symbols) alongside predictions of angle overestimation from Equation 1 (black
432 line).

433 **Figure 4.** Illustration of the spatial distortion introduced by the equivalent arc ratio. The
434 dots represent the perceived locations of 32 static signals arranged around the head as it
435 turns to the right. The apparent location of a signal at the front moves leftward, whereas a

436 signal at the right should appear move further to the right. The expansion represented here is
437 exaggerated by a factor of 2 for the purpose of more clearly illustrating the phenomenon.

438 [Figure 5](#). Experimental Paradigm: Listeners were presented with moving reference (20°) and test
439 signals (variable °) at three possible center angles (0°, 45°, and 90°), randomized in order, and asked
440 to report which of the two signals moved more.

441 [Supplemental Figure S1](#). Animation of the spatial distortion introduced by the equivalent arc
442 ratio. The dots represent the perceived locations of 32 static signals arranged around the head
443 as it turns. The angle of the head is represented by the line along the radius of the circle. The
444 expansion represented here is exaggerated by a factor of 2 for the purpose of more clearly
445 illustrating the phenomenon.

SUPPLEMENTAL FIGURE S1

