Title: **Evolutionary rates for multivariate traits: the role of selection and**

genetic variation.

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Keywords: Quantitative genetics, evolution, haldanes, **G** matrix, natural selection,

evolutionary rates, constraint, evolvability, scaling.

Supplemental Material: Data & scripts available at DRYAD DOI:xxxxx

Summary

A fundamental question in evolutionary biology is the relative importance of selection

and genetic architecture in determining evolutionary rates. Adaptive evolution can be

described by the multivariate breeders' equation ($\Delta\bar{z} = \mathbf{G}\boldsymbol{\beta}$), which predicts evolutionary

change for a suite of phenotypic traits ($\Delta\bar{z}$) as a product of directional selection acting

on them ($\boldsymbol{\beta}$) and the genetic variance-covariance matrix for those traits (**G**). Despite

being empirically challenging to estimate, there are enough published estimates of **G**

and $\boldsymbol{\beta}$ to allow for synthesis of general patterns across species. We use published

estimates to test the hypotheses that there are systematic differences in the rate of

evolution among trait types, and that these differences are in part due to genetic

architecture. We find evidence that sexually selected traits exhibit faster rates of

evolution compared to life-history or morphological traits. This difference does not

appear to be related to stronger selection on sexually selected traits. Using numerous

proposed approaches to quantifying the shape, size and structure of **G** we examine how

these parameters relate to one another, and how they vary among taxonomic and trait

groupings. Despite considerable variation, they do not explain the observed differences

in evolutionary rates.

Introduction

Predicting the rate and direction of phenotypic evolution remains a fundamental

challenge in evolutionary quantitative genetics [1-4]. Empirical studies have

demonstrated that most traits are heritable [5-8] and can respond to selection – a

prediction borne out by an abundance of short- (e.g. [9-11] and long-term (e.g. [9,12-14]

artificial selection experiments targeting single traits. However, in most biological

systems, the targets of selection are suites of traits. Furthermore, different traits are

tied together by genetic associations (typically quantified as covariances), and

consequently selection on one trait can lead to evolutionary changes in other traits

[7,8,11,15-21]. Indeed, genetic covariation between traits appears to be ubiquitous and

has the potential to shape the evolution of associated traits [7,10,17,18,20,22,23].

Therefore, to improve our understanding of phenotypic evolution it is necessary to

invoke a multivariate perspective [5,17-19,24].

The evolutionary response of a suite of traits can be predicted by the multivariate

breeder's equation $\Delta\bar{z} = \mathbf{G}\boldsymbol{\beta}$ where $\Delta\bar{z}$ is the vector of responses in phenotypic means

for the suite of traits under consideration, $\mathbf{G}$ is the additive genetic variance-covariance

matrix and $\boldsymbol{\beta}$ is the vector of linear (directional) selection gradients [5-8]. The

importance of $\mathbf{G}$ to phenotypic evolution can be illustrated using the concept of "genetic

degrees of freedom" [9,11,15]. Whenever there is genetic covariance between them,

the number of trait "combinations" in $\mathbf{G}$ that can respond to selection can be

considerably smaller than the actual number of measured traits. This can be true even

when each trait in **G** is heritable and all pairwise genetic correlations between them are

less than one [1-3,9,11,25]. This reduced dimensionality constrains the population to

evolve in a genetic space with fewer dimensions than the number of traits (and trait

combinations) potentially under selection. A matrix whose variance is concentrated in

one or a few dimensions can exhibit "lines of least evolutionary resistance"; directions in

which the multivariate evolutionary response can proceed more rapidly than in others

[15]. The presence of these lines of least evolutionary resistance can have a major

influence in biasing the direction of evolutionary trajectories (Figure 1; ref.s [7,11,15-

20]), making the **G** matrix more informative about the short term capacity of a

population to respond to selection (i.e. its evolvability) than the heritabilities of

individual traits [7,10,17,18,20,22,23].

A variety of measures have been proposed as proxies for the evolutionary potential of a

population. Most current approaches represent a function of the components of the

multivariate breeder's equation: **G**, $\boldsymbol{\beta}$ and $\Delta \bar{z}$ [5,17-19,21,24]. Unfortunately, few

studies simultaneously estimate more than one of these components. The few notable

exceptions suggest that the structure of **G** plays an important role in directing

phenotypic evolution [26-29]. Even fewer studies provide direct estimates of observed

rates of evolution [30,31]. However, many individual estimates of selection and

evolutionary rates exist in the literature and evolutionary research has benefitted from

reviews that synthesize these parameters [30-38]. There is considerable variation in the

strength of selection across different trait types and fitness measures [33,34,38], as well

as over time (but see ref.s [36,39,40]. On average, linear selection appears stronger on

morphological than life-history traits and both linear and quadratic selection is stronger

when acting on mating success and fecundity compared to viability [1-4,33,38]. It should

be acknowledged, however, that such studies are subject to some methodological

debate [5-8,35] and potentially publication biases [9-11,40]. In particular, there has

been considerable disagreement about trait scaling, and how it influences estimates and

broader evolutionary conclusions [19,22,41].

Although they have not received the same attention as selection gradients, reviews

based on published genetic parameters also show clear differences across trait types.

Morphological traits generally have higher heritabilities than life-history traits, with

physiological and behavioural traits intermediate between these extremes [9,12-14,32],

but see [6-8,11,15-21]. Sexual traits have also been shown to have higher additive

genetic variances compared to non-sexually selected traits [7,10,17,18,20,22,23,42],

although this finding is based on few studies. As discussed above, trait scaling has been

shown to alter the observed patterns [19,22,41].

There have been even fewer attempts at synthesis from a multivariate perspective.

Notably, Kirkpatrick [20], Kirkpatrick & Lofsvold [9], Agrawal & Stinchcombe [23], and

Schluter [11,15] collected small samples of **G** matrices from the literature and found

that much of the available variance was concentrated in the first few dimensions. This

suggests that few genetic degrees of freedom may be the norm, but we know of no

systematic review that reveals how general this pattern is or whether it differs across

taxa or trait types. Likewise, although reviews on the rate of contemporary

microevolution suggest that rapid evolution should be viewed as the norm rather than

the exception [15,30,31], a comprehensive review of evolutionary rates across different

taxa and trait types does not currently exist.

We compiled a database of reported genetic parameters from the literature to ask

whether different types of traits evolve at different rates, and whether such differences

correlate with differences in selection, in patterns of genetic (co)variation or both. We

performed a quantitative literature review, to examine whether observed rates of

evolutionary response differ across trait types (morphological, life-history and sexual) in

plants and animals. We relate these observed rates of evolutionary response to

estimates of linear and quadratic selection, as well as measures that capture the size,

shape and structure of **G** [7,11,15-20], to determine whether there is an association

across trait types and taxa. We find evidence that sexual traits evolve faster than other

traits in animals but not in plants, where life-history traits evolve fastest. These

increased rates of evolution do not appear to be attributable to the same cause

however. In plants we find that selection also appears to be strongest on life-history

traits, whereas in animals selection on sexually selected traits appears to be stronger

than on life-history but indistinguishable from that on morphology. We then examined

how the measures used to capture the size, shape and structure of **G** vary among trait

types and between taxa, but find that this incompletely explains the observed pattern of

evolutionary rates. In addition, we compare the various measures based upon **G**, and

show that for these empirically observed matrices, many strongly co-vary.

Methods

Compilation of Database

We compiled our datasets by searching for publications on the ISI Web of Science

database between March 2006 and August 2012. We then refined this preliminary list of

references on the basis of their title, abstract and keywords and attempted to obtain

the full text for all papers included in the dataset.

Rates of evolution have been measured using a number of different units, most

prominently darwins [7,10,17,18,20,22,23,43,44] and haldanes [5,17-19,21,24,43,45].

Measurements in darwins have proved most appropriate for researchers studying

evolution on macro-evolutionary scales (e.g. paleontologists), since they express the

rate of evolution per million years (although there are known methodological issues

with making comparisons [44,46]). However, for our purposes rates expressed in

haldanes are the appropriate unit as they measure change per generation and are used

to measure evolution on a micro-evolutionary scale – the scale over which **G** may be

important. We therefore compiled a database of evolutionary rate measured in

haldanes *only*. We performed searches for the terms '*rate of evolution*', '*rate of*

adaptation', '*haldane*s', '*response to selection*' and '*experimental evolution*'. This

process was aided considerably by making use of the measurements from the studies

previously compiled by Hendry *et al* [26-29,47]. Where studies reported the results of

experimental evolution without explicitly reporting a rate of response, we contacted the

authors to ask for the data needed (e.g. generation time) to calculate a rate in haldanes,

standardizing traits as necessary. Previous work has shown that even with log

transformation of ratio scale data (where means and variances might co-vary), this had

little influence on overall estimates for haldanes [31].

For the database of selection gradients, we began with the database compiled by

Kingsolver *et al* [30,31,33,37], and supplemented this with additional measures from

work published after 2001 by searching for the terms '*natural selection*', '*sexual*

selection', '*selection gradient*' or '*selection differential*'. Unlike Kingsolver *et al* [30-38]

we included both field and laboratory studies. While there has been discussion about

the effects of trait scaling (mean vs. standard deviation) on estimates of selection

[19,35], we have only included estimates standardized using the approach as advocated

by Lande and Arnold [21], which has been broadly used.

For the **G** matrix dataset we searched the Web of Science database using the terms '*G*

matrix' (or '*G-matrix*'), '*covariance matrix*' (or '*co-variance matrix*' or '*(co)variance*

matrix') or '*quantitative genetics*'. We recorded **G** matrices expressed both as genetic

(co)variances (provided we were able to mean-standardize them, following [19]) and as

genetic correlations and narrow sense heritabilities. Where possible (i.e. where

estimates of phenotypic variance had been presented alongside genetic correlations and

heritabilities) we back-calculated the genetic variances and covariances as: $V_A = h^2 V_P$ and

$Cov_{(x,y)} = r_G \sqrt{V_{A(x)} V_{A(y)}}$ where V_A and V_P are the additive genetic and phenotypic variances,

h^2 is the narrow sense heritability and r_G is the genetic correlation between traits x and

y. In cases where matrices were incomplete we contacted the author(s) to request the

missing estimates. We thus have two **G** datasets; correlation matrices and covariance

matrices. Since we found correlations to be reported more often than covariances, the

correlation dataset is a superset of matrices that includes those in the covariance

dataset. Trait scaling for the co-variance matrices is discussed below. In a number of

cases matrices had component traits that had been measured in difficult-to-compare

units (e.g. both a length and a volume), or where traits were expressed as residuals (e.g.

from regression against size). In these cases we excluded these from the reported

analysis, but inclusion had little effect on the results. A number of matrices were also

found to include cells with correlations >1 and in these cases we rounded the offending

correlation down to 1.

Defining Trait Categories and Measures

Since we wished to make comparisons across different 'trait types' (*sensu* [33,34,38]), it

was necessary to assign our measurements from the literature into categories. We

chose three trait categories: life-history, morphological and sexually selected traits. It is

relatively straightforward to separate life-history from morphological traits and the

majority of measurements in the literature fall into these two categories. In animals, we

defined sexual traits as those where we were able to find at least one study

demonstrating the trait was subject to female preference or used in male-male

competition. For plants, we defined floral morphology as sexually selected

[36,39,40,48]. Thus, for both plants and animals, our sexually selected and morphology

categories are not mutually exclusive. In an attempt to reduce error in our study, traits

that did not fit clearly into one of our three categories were excluded from our dataset.

For **G** matrices whose component traits did not all fit the same category, we split the

matrix to produce sub-matrices relating to traits only within a single category. Where

matrices contained a single trait whose category differed from all others in the matrix

we removed that trait from the matrix.

When making comparisons across our trait categories, we acknowledge that our

classifications may not be directly equivalent in plants and animals. We therefore

included a 'taxon' category in our statistical models. The list of individual measures of

evolutionary rate was treated as a single response variable, as were the standardized

selection gradients.

In our analysis of the **G** data, we wished to capture those attributes of **G** that might be

expected to influence the rate of evolutionary change. Matrices vary principally in terms

of size and structure. While numerous studies suggest that the alignment of axes of **G**

with β is likely to be important, the nature of the data we were able to compile does not

allow us to quantify alignment. Instead (as outlined below) we utilized a number of

scalar measures derived from **G**, meant to capture aspects of the size and structure as a

means to express evolutionary potential, All of the measures we used are summarized in

Table 1. One general concern is that not all of the measured we used explicitly

accounted for the number of traits included in the matrix (i.e. n_D). While, in general the

number of traits seemed to have a small influence on these measures (Figures 4 & 5),

we also took several steps to account for these effects, such as including number of

traits as a co-variate in the models (below) and also by examining the effects of scaling

n_D by either trait number or its square ("effective subspace", as suggested by one of the

manuscript referees). In none of these cases did it substantially alter the results. While

we use the name "effective dimensionality" for n_D, as proposed by Kirkpatrick [20], this

measure actually captures aspects of matrix eccentricity, not dimensionality.

For the dataset of **G** as mean-standardized covariance matrices we used the three **G**-

structure measures suggested by Kirkpatrick [20]: 'total genetic variance' (*tgv*),

'maximum evolvability' (e_{max}) & 'effective number of dimensions' (n_D), and also Hansen

and Houle's [19] 'average evolvability' ($\bar{e}$). For the dataset of correlation matrices, we

calculated Pavlicev *et al.*'s [49] eigenvalue variance (Var(λ)) and relative eigenvalue

variance (Var$_{rel}$(λ)) and also Agrawal & Stinchcombe's [23] eigenvalue evenness (E_λ).

Both sets of **G** matrix measures are defined in Table 1.

While we present results from analyses of both the (co)variance and correlation matrix

datasets, it is important to note that results are not directly comparable between them,

since it is well known that different methods of scaling (i.e. mean-standardizing

(co)variance matrices vs. effectively variance-standardized correlation matrices)

produce fundamentally different results for genetic attributes [6,19,35]. Furthermore,

though the correlation matrix dataset is larger, we note that the covariance – not

correlation – matrix is part of the fundamental formulation for response to selection

[21].

Statistical Analyses

Analyses were performed using **R** (version 2.13.0; ref. [50]); we fit generalized linear

mixed-effect models using the MCMCglmm package (version 2.15; ref. [51]). A large

proportion of studies reporting selection gradients also reported standard errors or

confidence intervals (from which standard errors can be calculated). As noted by

Kingsolver *et al* [38], this allows for the application of formal meta-analyses, and we

followed their lead in modelling selection data with a meta-analysis including random-

effects to account for study- and species-level autocorrelation. We analysed estimates

of standardised selection gradients (β) expressed as absolute values.

We found that standard errors or confidence intervals were reported much less

frequently among studies of **G** or rates of evolution, and so we were unable to account

for measurement error variance in these analyses as we had for selection, though the

model structure we used was otherwise similar. We fit a set of models, and then

evaluated model fit by comparing Deviance Information Criterion values (DIC) [52], and

confirmed our selections by refitting the model set using reduced maximum likelihood

(lme4 package [53]) and comparing fits using Bayesian Information Criterion scores (BIC)

and likelihood ratio tests (Supplemental Methods) using a parametric bootstrap. The

selected models for each dataset are described in Table 2, and full model sets are

available with the data and scripts on Dryad. Since we modelled the magnitude

(absolute value) of our response variables, the appropriate distribution is the folded

normal [38]. We therefore extracted the posterior distributions of solutions, took the

mean and standard deviation from these distributions and applied these to the folded

normal distribution. We then report the mean and credible intervals from these

corrected distributions [38].

Our data contained 2571 estimates of the rate of evolutionary response (measured in

haldanes); there were comparatively few estimates for plants, with no estimates

available on the observed rate of evolution for sexually selected (floral) traits. This

imbalance caused our estimates to be unstable so we modelled plant and animal rates

separately. We had 776 estimates of β, but **G** is reported less frequently in the literature

(Table 3) and our sample size of **G** measures was 81 (co)variance matrices and 221

correlation matrices.

Results

Observed rates of evolution differ among trait types and between plants and animals

The overall posterior mean for evolutionary rate was 0.13 haldanes, with a 95% credible

interval from 0.08 – 0.17. Credible intervals for estimates in plants are quite wide

(Figure 2), most likely due to the comparatively low number of studies in these

categories. However there is a clear trend for faster rates in life-history traits, with the

life-history estimate being 2.02 times as large (95% credible interval 1.00 – 3.03) as that

for morphology (Table 3). In animals, life-history and morphology have similar

estimates, but the posterior mean estimate for sexually selected traits is higher – 1.48

times that for morphology (95% CI 0.78 – 2.11), and 1.51 times that for life-history (95%

CI 0.50 – 2.54). These results are consistent with similar rates of evolution for

morphology in both plants and animals, with higher rates for life-history traits in plants

and for sexually selected traits in animals.

Standardised selection gradients show different patterns between plants and animals

The overall posterior mean for absolute linear selection gradients was 0.21 (95% CI =

0.17 – 0.26), which was somewhat higher than the estimate reported by Kingsolver et al.

[38] (0.14, 95% CI = 0.13 – 0.16), most likely due to our inclusion of lab studies. The

credible intervals from our full model are again wider for plants, likely reflecting smaller

sample size (Table 3). For both plants and animals there is little difference between the

estimates for morphological and sexually selected traits. In plants, the model suggests

that selection is stronger on life-history traits, whose estimate is 40% larger than that for

morphology and approximately twice that for sexually selected traits. By contrast, in

animals selection appears to be weaker for life-history; the estimate for selection on

life-history traits is 0.43 times (95% CI 0.11 – 0.97) that for morphology, and 0.49 times

(95% CI 0.17 – 0.80) that for sexually selected traits (Figure 3).

The marginal utility of multiple measures

The magnitude, shape and alignment of the **G** matrix all have the potential to influence

the rate of evolution, but with the data available we are able to use measures intended

to quantify only the first two of these properties. Of the measures we report *tgv*, e_{max}

and $\bar{e}$ can be thought of as measures of magnitude, whereas n_D, Var(λ), Var$_{rel}$(λ) and E_λ

are intended to quantify the departure of the matrix from symmetricality (how

dissimilar variances are along the multiple axes of **G**). It is immediately obvious that the

magnitude measures are doing a good job of quantifying the same property of each

matrix (Table 1, Figures 4 & 5), since *tgv*, e_{max} and $\bar{e}$ are all very strongly inter-correlated

($r > 0.96$ in all cases). Given that these measures of magnitude are also strongly

correlated ($r > 0.93$ in all cases) with the magnitude of $\boldsymbol{g}_{max}$ (i.e. the principal eigenvalue

of **G**), it is perhaps unsurprising that they are only poorly predicted by the number of

traits measured, with which they are correlated only at $r = 0.15 - 0.19$.

With respect to the measures of matrix eccentricity, the first thing we note is that Var(λ)

and Var$_{rel}$(λ) are strongly correlated with each other ($r = 0.87$), and negatively correlated

with E_λ ($r = $ -0.32 & -0.55 respectively). Though E_λ was defined as a measure of

correlation matrices [23], when we applied the evenness formula to our dataset of

covariance matrices we find that the resulting measure is strongly correlated with

Kirkpatrick's [20] n_D ($r = 0.82$).

*The structure of **G***

We performed separate analyses and model selection procedures for each of our

measures describing the structure of **G**. Our models comparing covariance matrices

revealed very similar patterns of estimates for e_{max} , *tgv* and $\bar{e}$. Furthermore the pattern

of estimates among trait types was consistent between plants and animals (Figure 6). In

all cases the estimates for life-history and sexually selected traits were similar and those

for morphology were higher, but with much overlap in credible intervals our confidence

in these differences is low. Our results for n_D also show consistent patterns of estimates

between plants and animals, with the estimates showing a shallow increasing trend

from life-history to morphology to sexually selected traits (Figure 6(d)), but once again

there is wide overlap among credible intervals, indicating low confidence in this trend.

The results of our analyses of **G** matrices expressed as correlations were more diverse.

The pattern of estimates for Var(λ) showed a trend for values to increase from life-

history to morphology to sexually selected traits in both plants and animals, though the

estimates for animals were larger than those for plants (Figure 7(a)). A similar trend was

present in estimates for Var$_{rel}$(λ) for animals, thought the direction of the trend is

reversed in plants (Figure 7(b)). The wide overlap of credible intervals indicates low

confidence in both these trends however. Finally, our estimates for E_λ show a decreasing

trend from life-history to morphology to sexually selected traits in both plants and

animals, with higher estimates for plants than for animals (Figure 7(c)).

Discussion

Predicting the rate and direction of phenotypic evolution remains a fundamental

challenge in evolutionary genetics [1-4,54], with the multivariate breeders' equation

playing a central role. Researchers have published estimates of selection and of **G** from

many systems (though not commonly both in the same system). Here we have

integrated these data to ask if some traits generally evolve more rapidly than others,

and whether any differences correspond to differences in selection, **G** or both. Reviews

such as this are unavoidably limited by the availability of published genetic parameters,

and the resulting imbalances in the data. Nevertheless, we find that in animals, but not

plants, sexual traits evolve ~50% faster than morphological and life-history traits. We

find no evidence that the increased rate of evolution observed for sexual traits was due

to stronger selection operating on these traits relative to morphological and life-history

traits. We found weak evidence for differences in the evolutionary potential of **G** among

trait types, though this fails to provide a satisfactory explanation for the increased rates

of evolution associated with sexually selected traits. In addition, we confronted our

compiled data sets with a number of the scalar measures used to examine the size,

shape and structure of **G** (Table 1). We observed that many of these measures have

considerable shared information (Figures 4 & 5), though in general this pattern of

relationships would recommend the use of at least two measures; one to express the

magnitude of **G** and a second relating to evenness/variance of the eigenvalues. While

there may be particular instances where these measures result in widely divergent

estimates, at least with respect to the empirical estimates we have collated, the

marginal benefits of using all of them are an illustration of diminishing returns. We

speculate that one potential use (which would require considerable additional research)

would be in a fashion analogous to the molecular population geneticists' use of the

parameter Tajima's D, which is simply a scaled measure of two different estimates of

the population mutation rate, $4N_e\mu$. It is possible that subtle differences among scalar **G**

measures may ultimately provide important insights into the structure of **G**. One

surprising observation that emerges from our results, is that the number of traits used

to estimate **G** is not well correlated with any of the scalar measures we used. One

explanation for this becomes clear when considering that the magnitudes of the

principal eigenvalue of **G** is so highly correlated with total genetic variation (the trace of

G). This suggests that an overwhelming proportion of all of the variation is found along

this principal vector (which would differ for each **G**). However, it is well known that

estimating G can be difficult and insufficient sampling at the level of families can inflate

the magnitude of the principal eigenvalue [55,56].

Reviews based on published estimates of evolutionary rates [30,31] have provided a

number of important insights into the evolutionary process. Hendry & Kinnison [30]

provided the theoretical foundations for measuring evolutionary rates and used a small

sample of published estimates to propose that rapid evolution should be viewed as the

norm rather than the exception. In a much larger study, however, Kinnison & Hendry

[31] showed that the frequency distribution of evolutionary rates measured in haldanes

is log-normal (i.e. many slow rates and few fast rates, median haldanes = 5.8×10^{-3}) and

that life-history and morphological traits appear to evolve equally as fast when

measured in haldanes. In agreement with these reviews, we found that the frequency

distribution of evolutionary rates in our study was also log-normal and that the median

rate across trait types and taxa was similar (median haldanes = 7.6×10^{-3}) to that

reported in Kinninson & Hendry [31]. Moreover, we found little evidence to suggest that

the evolutionary rates of life-history and morphological traits differed in either plants or

animals. A novel outcome of our analysis, however, is the finding that sexual traits

evolve faster than life-history and morphological traits in animals. Our findings provide

evidence for a general pattern of faster evolution in sexual traits in animals to add to the

highly cited individual examples of very rapid evolution of sexual traits [57,58] and their

role in speciation [59,60]. More work is needed to determine whether this pattern also

exists in plants.

Reviews synthesizing estimates of selection gradients are far more extensive in the

literature [33-39]. In their seminal review, Kingsolver *et al*. [33] found that the frequency

distributions of absolute linear and quadratic selection gradients were exponential and

generally symmetrical around zero, suggesting that stabilizing and disruptive selection

occur equally frequently and with similar strength in nature. Kingsolver *et al.* [33] also

found that linear selection was stronger on morphological than life-history traits. The

most recent review [38] containing an updated data set and using formal Bayesian

meta-analysis to control for potential biases [34,35,37] confirmed many of the main

findings of Kingsolver *et al*. [33], with the notable exception that linear selection appears

stronger in plants than animals.

In agreement with this most recent review [38], we found that the distribution of

absolute linear and quadratic selection gradients were exponential (see Supplement).

However, our estimates for absolute linear selection gradients were higher than

reported by Kingsolver *et al.* [38] (0.24 (0.17, 0.26) versus 0.14 (0.13, 0.16)). This raises

the obvious question of why this difference occurs. There has been much discussion on

the general limitations of using selection gradients in synthetic reviews (e.g.

[33,35,37,38] and these arguments undoubtedly also apply to our study. However, as

most of these limitations are inherent to both studies, it is unlikely that they explain the

observed differences. Furthermore, given that we used the same Bayesian framework as

Kingsolver et al. [38] it is also unlikely that our analytical approach has generated the

observed differences. The most likely reason for the observed differences is the way

that traits and taxa were categorized across these studies. Kingsolver et al. [38] used

four different trait categories (size, morphological (not including size), phenology and

life-history (not including phenology)) and categorized taxa as invertebrates, vertebrates

or plants in their analysis. In contrast, we only distinguished between animals and plants

and used three different trait categories (morphological, life-history and sexual) in our

analysis, the latter of which includes a mixture of morphological and behavioural traits.

Thus, there are likely to be some differences in how selection gradients are distributed

amongst categories in our analyses compared to those in Kingsolver et al. [38] and this

may account for the observed differences between studies. Irrespective of the

underlying reasons for these differences, our main finding that there is little difference

in selection gradients across trait types and taxa suggests that selection alone is unlikely

to explain the higher rate of evolutionary response we observe for sexual traits in

animals.

After decades of quantitative genetic research it is now widely accepted that the

additive genetic variance-covariance matrix (**G**) plays a major role in

facilitating/constraining phenotypic evolution [16,19,20]. The way in which **G** shapes

phenotypic evolution can be envisaged using the concept of genetic degrees of freedom

(Figure 1; [9,15]). Whenever there is genetic covariation between the individual traits

contained in **G**, there is the potential for fewer axes of genetic variation than observed

traits [9,15,61,62] (but see [63]), which can influence evolutionary rates [64]. Where the

majority of the genetic variance is concentrated in a few direction – known as "lines of

least evolutionary resistance" [15] – these have been shown to play an important role in

directing the short-term evolutionary trajectory of a population [15,65-69]. Quantifying

these properties of **G** is an essential step if we are to explore these ideas empirically.

Our comparison of a number of the measures that have been proposed to fit this role

indicates that, however informative it may or may not prove to be, we are able to

reliably quantify the magnitude of **G**, since we find broad agreement among magnitude

measures. Perhaps unsurprisingly, it seems that the magnitude of a matrix is somewhat

more straightforward to describe with a scalar measure than the eigenvalue

evenness/eccentricity/dimensionality. The measures available for quantification of the

shape of **G** in multiple dimensions are much less tightly inter-correlated than those

dealing with matrix magnitude when compared using empirical data. What this

ultimately means for our understanding of evolvability is unclear, but it is important to

acknowledge the gaps in out current understanding if we are to progress.

The finding that genetic variance for sexual traits may be spread less evenly across

dimensions in animals runs counter to our hypothesis, and suggests that the potential

for genetic constraints does not explain the higher rate of evolution we observe for

these traits. We found at best, only weak evidence for differences in the measures to

capture the size and shape of **G** with respect to our trait groupings. There has been

debate over the importance of sexual selection in plants [70], but there is theoretical

[48] and empirical [71] evidence suggesting that floral morphology is indeed subject to

sexual selection. Unfortunately though, there are currently no data on evolutionary

rates for sexual traits in plants, making it difficult to understand the implications of this

increased dimensionality. Our findings indicate that the subject warrants greater

attention. Additionally, when considering these issues, researchers also need to keep in

mind that their decisions about measurement scaling issues are likely to be important

when measuring selection [35] and genetic variability [6]. This is especially important

when addressing the question of evolvability, where both these measures must be

brought together [19].

Collectively, our results suggest that the higher rate of evolution observed for sexual

traits in animals is only weakly associated with these scalar measures summarizing **G** for

these traits rather than stronger selection. However, as our data set is based on derived

estimates of evolutionary rates, standardized selection gradients and **G**, there are a

number of inevitable limitations that apply to our findings. First, there are limitations

with using the matrix structure measures (n_D, $E\lambda$, $Var(\lambda)$ or $Var_{rel}(\lambda)$) to capture the

dimensionality of **G** [20]. Although these measure are calculable from published

estimates of **G**, they do not explicitly test how many of the dimensions of **G** actually

exist (i.e. have statistical support). A number of approaches [63,72] have been taken to

directly estimate the dimensionality of **G** [72,73] though such studies have found both

populations that have evolutionary access to all dimensions of **G** [63] and others that

are constrained by LLER's [72,74]. Second, our analysis does not consider the alignment

between the vectors of selection and **G**. LLER's only constrain the response to selection

when they are poorly aligned with vectors of selection [26,28,75]. These limitations can

only be resolved by further analysis of the raw data sets contained in the original studies

we review. This is particularly true for better estimation of **G** itself, as well as its actual

dimensionality, which can only be performed with the raw data [56,61,63,76-79]. Future

studies would therefore greatly benefit from researchers publishing their raw datasets

in open repositories [80] and we strongly encourage researchers to do so. Our database

can be found at DRYAD DOI:xxxxxxxx.

Acknowledgements

We thank S.F. McDaniel and W.U. Blanckenhorn for donating unpublished matrices and

to R. Snook, S. Chenoweth, T. Chapman, R. Firman, and M. Reuter for data on

experimental evolution. I. Stott, F. Ingleby and two anonymous reviewers provided

comments on an earlier draft of this manuscript. JH, TT and JBW were funded by NERC,

JH and JBW by the BBSRC and JH by a University Royal Society Fellowship. WP was

supported by a NERC studentship (awarded to JH and TT) and ID & WP were funded by

NIH grant no. 1R01GM094424.

References

1.   Arnold, S., Pfrender, M. & Jones, A. 2001 The adaptive landscape as a
483   conceptual bridge between micro- and macroevolution. *Genetica* **112**, 9–32.

2.   Merilä, J., Kruuk, L. & Sheldon, B. 2001 Natural selection on the genetical

component of variance in body condition in a wild bird population. *Journal*
of Evolutionary Biology **14**, 918–929.

3. Bégin, M. & Roff, D. A. 2003 The constancy of the G matrix through species
divergence and the effects of quantitative genetic constraints on
phenotypic evolution: A case study in crickets. *Evolution* **57**, 1107–1120.

4. Walsh, B. & Blows, M. W. 2009 Abundant genetic variation+ strong
selection= multivariate genetic constraints: a geometric view of adaptation.
Annual Review of Ecology

5. Lande, R. 1979 Quantitative Genetic-Analysis of Multivariate Evolution,
Applied to Brain - Body Size Allometry. *Evolution* **33**, 402–416.

6. Houle, D. 1992 Comparing evolvability and variability of quantitative traits.
Genetics **130**, 195–204.

7. Roff, D. A. 1997 Evolutionary quantitative genetics.

8. Roff, D. A. & Mousseau, T. A. 1987 Quantitative genetics and fitness: lessons
from Drosophila. *Heredity* **58 (Pt 1)**, 103–118.

9. Kirkpatrick, M. & Lofsvold, D. 1992 Measuring selection and constraint in
the evolution of growth. *Evolution* **46**, 954–971.

10. Bell, G. 1997 *Selection: the mechanism of evolution*. Springer.

11. Schluter, D. 2000 *The Ecology of Adaptive Radiation (Oxford Series in
Ecology & Evolution)*. 1st edn. Oxford University Press, USA.

12. Schluter, D. 2001 Ecology and the origin of species. *Trends Ecol Evol* **16**,
372–380.

13. Moose, S. P., Dudley, J. W. & Rocheford, T. R. 2004 Maize selection passes
the century mark: a unique resource for 21st century genomics. *Trends in
Plant Science* **9**, 358–364. (doi:10.1016/j.tplants.2004.05.005)

14. Powell, R. L. & Norman, H. D. 2006 Major Advances in Genetic Evaluation
Techniques. *J Dairy Sci* **89**, 1337–1348. (doi:10.3168/jds.S0022-
0302(06)72201-9)

15. Schluter, D. 1996 Adaptive radiation along genetic lines of least resistance.
Evolution **50**, 1766–1774.

16. Blows, M. & Hoffmann, A. 2005 A reassessment of genetic limits to
evolutionary change. *Ecology* **86**, 1371–1384.

17. Blows, M. W. 2007 A tale of two matrices: multivariate approaches in
evolutionary biology. *Journal of Evolutionary Biology* **20**, 1–8.
(doi:10.1111/j.1420-9101.2006.01164.x)

18. Blows, M. & Walsh, B. 2009 Spherical cows grazing in flatland: constraints to
selection and adaptation. *Adaptation and Fitness in Animal Populations*, 83–
101.

19. Hansen, T. F. & Houle, D. 2008 Measuring and comparing evolvability and
constraint in multivariate characters. *Journal of Evolutionary Biology* **21**,
1201–1219. (doi:10.1111/j.1420-9101.2008.01573.x)

20. Kirkpatrick, M. 2008 Patterns of quantitative genetic variation in multiple
dimensions. *Genetica* **136**, 271–284. (doi:10.1007/s10709-008-9302-6)

21. Lande, R. & Arnold, S. 1983 The Measurement of Selection on Correlated
Characters. *Evolution* **37**, 1210–1226.

22. Hansen, T. F., Pélabon, C. & Houle, D. 2011 Heritability is not Evolvability.
Evol Biol **38**, 258–277. (doi:10.1007/s11692-011-9127-6)

23. Agrawal, A. F. & Stinchcombe, J. R. 2008 How much do genetic covariances
alter the rate of adaptation? *P R Soc B* **276**, 1183–1191.
(doi:10.1098/rspb.2008.1671)

24. Hunt, J., Wolf, J. & Moore, A. 2007 The biology of multivariate evolution.
Journal of Evolutionary Biology **20**, 24–27.

25. Dickerson, G. E. 1955 Genetic Slippage in Response to Selection for Multiple
Objectives. *Cold Spring Harbor Symposia on Quantitative Biology* **20**, 213–
224. (doi:10.1101/SQB.1955.020.01.020)

26. Blows, M. W., Chenoweth, S. F. & Hine, E. 2004 Orientation of the Genetic
Variance-Covariance Matrix and the Fitness Surface for Multiple Male
Sexually Selected Traits. *Am Nat* **163**, 329–340. (doi:10.1086/381941)

27. Hine, E., Chenoweth, S. F. & Blows, M. W. 2004 Multivariate quantitative
genetics and the lek paradox: Genetic variance in male sexually selected
traits of Drosophila serrata under field conditions. *Evolution* **58**, 2754–2762.

28. van Homrigh, A., Higgie, M., McGuigan, K. & Blows, M. W. 2007 The
depletion of genetic variance by sexual selection. *Curr Biol* **17**, 528–532.
(doi:10.1016/j.cub.2007.01.055)

29. Simonsen, A. K. & Stinchcombe, J. R. 2010 Quantifying Evolutionary Genetic
Constraints in the Ivyleaf Morning Glory, Ipomoea hederacea. *Int J Plant Sci*

171, 972–986. (doi:10.1086/656512)

30. Hendry, A. & Kinnison, M. 1999 Perspective: the pace of modern life:
measuring rates of contemporary microevolution. *Evolution* **53**, 1637–1653.

31. Kinnison, M. & Hendry, A. 2001 The pace of modern life II: from rates of
contemporary microevolution to pattern and process. *Genetica* **112**, 145–
164.

32. Mousseau, T. & Roff, D. 1987 Natural-selection and the heritability of fitness
components. *Heredity* **59**, 181–197.

33. Kingsolver, J., Hoekstra, H., Hoekstra, J., Berrigan, D., Vignieri, S., Hill, C.,
Hoang, A., Gibert, P. & Beerli, P. 2001 The strength of phenotypic selection
in natural populations. *Am Nat* **157**, 245–261.

34. Hoekstra, H., Hoekstra, J., Berrigan, D., Vignieri, S., Hoang, A., Hill, C., Beerli,
P. & Kingsolver, J. 2001 Strength and tempo of directional selection in the
wild. *P Natl Acad Sci Usa* **98**, 9157–9160.

35. Hereford, J., Hansen, T. F. & Houle, D. 2004 Comparing strengths of
directional selection: how strong is strong? *Evolution* **58**, 2133–2143.

36. Siepielski, A. M., DiBattista, J. D. & Carlson, S. M. 2009 It's about time: the
temporal dynamics of phenotypic selection in the wild. *Ecol Lett* **12**, 1261–
1276. (doi:10.1111/j.1461-0248.2009.01381.x)

37. Kingsolver, J. G. & Diamond, S. E. 2011 Phenotypic Selection in Natural
Populations: What Limits Directional Selection? *Am Nat* **177**, 346–357.
(doi:10.1086/658341)

38. Kingsolver, J. G., Diamond, S. E., Siepielski, A. M. & Carlson, S. M. 2012
Synthetic analyses of phenotypic selection in natural populations: lessons,
limitations and future directions. *Evol Ecol* **26**, 1101–1118.
(doi:10.1007/s10682-012-9563-5)

39. Siepielski, A. M., DiBattista, J. D., Evans, J. A. & Carlson, S. M. 2011
Differences in the temporal dynamics of phenotypic selection among fitness
components in the wild. *P R Soc B* **278**, 1572–1580.
(doi:10.1098/rspb.2010.1973)

40. Morrissey, M. B. & Hadfield, J. D. 2012 Directional selection in temporally
replicated studies is remarkably consistent. *Evolution* **66**, 435–442.
(doi:10.1111/j.1558-5646.2011.01444.x)

41. Houle, D., Pélabon, C., Wagner, G. P. & Hansen, T. F. 2011 Measurement

and meaning in biology. *Q Rev Biol* **86**, 3–34.

42.   Pomiankowski, A. & Moller, A. 1995 A resolution of the lek paradox. *P Roy*
Soc Lond B Bio **260**, 21–29.

43.   Haldane, J. B. S. 1949 Suggestions as to quantitative measurement of rates
of evolution. *Evolution* **3**, 51–56.

44.   Gingerich, P. D. 1983 Rates of evolution: effects of time and temporal
scaling. *Science* **222**, 159–161. (doi:10.1126/science.222.4620.159)

45.   Gingerich, P. 1993 Quantification and comparison of evolutionary rates.
American Journal of Science **293-A**, 453–478.

46.   Uyeda, J. C., Hansen, T. F., Arnold, S. J. & Pienaar, J. 2011 The million-year
wait for macroevolutionary bursts. *Proceedings of the National Academy of*
Sciences **108**, 15908–15913. (doi:10.1073/pnas.1014503108)

47.   Hendry, A. P., Farrugia, T. J. & Kinnison, M. T. 2008 Human influences on
rates of phenotypic change in wild animal populations. *Molecular Ecology*
17, 20–29. (doi:10.1111/j.1365-294X.2007.03428.x)

48.   Moore, J. C. & Pannell, J. R. 2011 Sexual selection in plants. *Curr Biol* **21**,
R176–82. (doi:10.1016/j.cub.2010.12.035)

49.   Pavlicev, M., Cheverud, J. M. & Wagner, G. P. 2009 Measuring
Morphological Integration Using Eigenvalue Variance. *Evol Biol* **36**, 157–170.
(doi:10.1007/s11692-008-9042-7)

50.   Team, R. C. D. In press. R: A language and environment for statistical
computing.

51.   Hadfield, J. D. 2010 MCMC Methods for Multi-Response Generalized Linear
Mixed Models: The MCMCglmm R Package. *Journal of Statistical Software*
33, 1–22.

52.   Spiegelhalter, D. J., Best, N. G., Carlin, B. P. & Van Der Linde, A. 2002
Bayesian measures of model complexity and fit. *Journal of the Royal*
Statistical Society: Series B (Statistical Methodology) **64**, 583–639.

53.   Bates, D., Maechler, M. & Bolker, B. In press. lme4: Linear mixed-effects
models using S4 classes.

54.   Hill, W. G. & Zhang, X. S. 2012 On the Pleiotropic Structure of the Genotype-
Phenotype Map and the Evolvability of Complex Organisms. *Genetics* **190**,
1131–1137. (doi:10.1534/genetics.111.135681)

55. Hill, W. G. & Thompson, R. 1978 Probabilities of Non-Positive Definite
Between-Group or Genetic Covariance Matrices. *Biometrics* **34**, 429–439.

56. Meyer, K. & Kirkpatrick, M. 2010 Better Estimates of Genetic Covariance
Matrices by 'Bending' Using Penalized Maximum Likelihood. *Genetics* **185**,
1097–1110. (doi:10.1534/genetics.109.113381)

57. Mendelson, T. C. & Shaw, K. L. 2005 Sexual behaviour: rapid speciation in an
arthropod. *Nature* **433**, 375–376. (doi:10.1038/433375a)

58. Zuk, M., Rotenberry, J. T. & Tinghitella, R. M. 2006 Silent night: adaptive
disappearance of a sexual signal in a parasitized population of field crickets.
Biol Lett-Uk **2**, 521–524. (doi:10.1098/rsbl.2006.0539)

59. Panhuis, T. M., Butlin, R., Zuk, M. & Tregenza, T. 2001 Sexual selection and
speciation. *Trends Ecol Evol* **16**, 364–371. (doi:10.1016/S0169-
5347(01)02160-7)

60. Ritchie, M. G. 2007 Sexual Selection and Speciation. *Annu Rev Ecol Evol S* **38**,
79–102. (doi:10.1146/annurev.ecolsys.38.091206.095733)

61. Hine, E. & Blows, M. W. 2006 Determining the effective dimensionality of
the genetic variance-covariance matrix. *Genetics* **173**, 1135–1144.
(doi:10.1534/genetics.105.054627)

62. McGuigan, K. & Blows, M. W. 2007 The phenotypic and genetic covariance
structure of drosphilid wings. *Evolution* **61**, 902–911. (doi:10.1111/j.1558-
5646.2007.00078.x)

63. Mezey, J. & Houle, D. 2005 The dimensionality of genetic variation for wing
shape in Drosophila melanogaster. *Evolution* **59**, 1027–1038.

64. McGuigan, K. 2006 Studying phenotypic evolution using multivariate
quantitative genetics. *Molecular Ecology* **15**, 883–896. (doi:10.1111/j.1365-
294X.2006.02809.x)

65. Badyaev, A. V. & Foresman, K. R. 2000 Extreme environmental change and
evolution: stress-induced morphological variation is strongly concordant
with patterns of evolutionary divergence in shrew mandibles. *Proc. Biol. Sci.*
267, 371–377. (doi:10.1098/rspb.2000.1011)

66. McGuigan, K., Chenoweth, S. F. & Blows, M. W. 2005 Phenotypic divergence
along lines of genetic variance. *Am Nat* **165**, 32–43.

67. Renaud, S., Auffray, J.-C. & Michaux, J. 2006 Conserved phenotypic variation
patterns, evolution along lines of least resistance, and departure due to

652   selection in fossil rodents. *Evolution* **60**, 1701–1717.

653   68.   Chenoweth, S. F. & McGuigan, K. 2010 The Genetic Basis of Sexually
654   Selected Variation - Annual Review of Ecology, Evolution, and Systematics,
655   41(1):81. *Annual Review of Ecology*

656   69.   Boell, L. 2013 Lines of least resistance and genetic architecture of house
657   mouse (Mus musculus) mandible shape. *Evol Dev* **15**, 197–204.
658   (doi:10.1111/ede.12033)

659   70.   Skogsmyr, I. & Lankinen, S. 2002 Sexual selection: an evolutionary force in
660   plants? *Biol Rev* **77**, 537–562. (doi:10.1017/S1464793102005973)

661   71.   Carlson, J. E. 2008 Hummingbird responses to gender-biased nectar
662   production: are nectar biases maintained by natural or sexual selection? *P R*
663   *Soc B* **275**, 1717–1726. (doi:10.1098/rspb.2008.0017)

664   72.   Hine, E. & Blows, M. 2006 Determining the effective dimensionality of the
665   genetic variance-covariance matrix. *Genetics* **173**, 1135–1144.
666   (doi:10.1534/genetics.105.054627)

667   73.   Mezey, J. G. 2005 Naturally Segregating Quantitative Trait Loci Affecting
668   Wing Shape of Drosophila melanogaster. *Genetics* **169**, 2101–2113.
669   (doi:10.1534/genetics.104.036988)

670   74.   Hunt, J., Blows, M. W., Zajitschek, F., Jennions, M. D. & Brooks, R. 2007
671   Reconciling strong stabilizing selection with the maintenance of genetic
672   variation in a natural population of black field crickets (*Teleogryllus*
673   *commodus*). *Genetics* **177**, 875–880. (doi:10.1534/genetics.107.077057)

674   75.   McGuigan, K. 2006 Studying phenotypic evolution using multivariate
675   quantitative genetics. *Molecular Ecology* **15**, 883–896. (doi:10.1111/j.1365-
676   294X.2006.02809.x)

677   76.   Meyer, K. 2007 WOMBAT—A tool for mixed model analyses in quantitative
678   genetics by restricted maximum likelihood (REML). *J. Zhejiang Univ. - Sci. B*
679   **8**, 815–821. (doi:10.1631/jzus.2007.B0815)

680   77.   Kirkpatrick, M. 2004 Direct Estimation of Genetic Principal Components:
681   Simplified Analysis of Complex Phenotypes. *Genetics* **168**, 2295–2306.
682   (doi:10.1534/genetics.104.029181)

683   78.   de los Campos, G. & Gianola, D. 2007 Factor analysis models for structuring
684   covariance matrices of additive genetic effects: a Bayesian implementation.
685   *Genet Sel Evol* **39**, 481–494. (doi:10.1186/1297-9686-39-5-481)

79.   Runcie, D. E. & Mukherjee, S. 2013 Dissecting High-Dimensional Phenotypes
with Bayesian Sparse Factor Analysis of Genetic Covariance Matrices.
Genetics **194**, 753–767. (doi:10.1534/genetics.113.151217)

80.   Whitlock, M. C., McPeek, M. A., Rausher, M. D., Rieseberg, L. & Moore, A. J.
2010 Data Archiving. *Am Nat* **175**, 145–146. (doi:10.1086/650340)

Figure Legends

Figure 1. The effect of $\mathbf{g}_{max}$ on the response to selection where traits genetically covary.

The axes represent the breeding values for 2 hypothetical traits. The population mean is

at the solid point and the surrounding ellipse is the 95% confidence region for the

distribution of trait values about the mean. That these traits covary is evident as the

ellipse is at an angle relative to the trait axes. The axes of the ellipse represent the 2

orthogonal directions (eigenvectors) of variance present – there is more standing

genetic variance along the major axis ($\mathbf{g}_{max}$) than the minor axis. They grey lines are

'contours' on a fitness landscape, with an adaptive peak at 'S'. Rather than evolving

directly toward the peak (dashed arrow), the influence of $\mathbf{g}_{max}$ may cause the population

to evolve along an indirect course (bold arrow). In some cases this may even, result in

the population evolving toward an alternate fitness peak (e.g. at 'A', modified contours

not shown) in line with $\mathbf{g}_{max}$, even though it is more distant from the current mean.

Figure 2. Posterior means and 95% credible intervals for estimates of absolute rate of

evolution (haldanes). Open points are for plants and filled points for animals. Trait types

are life-history (LH), morphology (M) and sexually selected (S) and filled points are for

animals and open points for plants (no data available for sexual traits in plants).

Figure 3. Posterior means and 95% credible intervals for estimates of standardized

selection gradients (β) by trait type. Trait labels and taxon symbols are as in Figure 2.

Figure 4. Pairs plot to illustrate the relationships between measures used to describe

the structure of G expressed as covariance matrices. Measures are 'total genetic

variance' (tgv), 'maximum evolvability' (e_{max}) & 'effective number of dimensions' (n_D)

[20], the first eigenvalue of **G** (g_{max}), 'average evolvability' ($\bar{e}$) [19], 'eigenvalue evenness'

(E_λ – originally intended for use with correlation matrices [23]) and the number of traits

included in the matrix (k). Figures in the lower off-diagonal are pairwise correlations

between the measures.

Figure 5. Pairs plot to illustrate the relationships between measures used to describe

the structure of G expressed as correlation matrices. Measures are 'eigenvalue variance'

($Var(\lambda)$) [49], 'eigenvalue evenness' (E_λ) [23], 'relative eigenvalue variance' ($Var_{rel}(\lambda)$)

[49], the first eigenvalue of **G** (g_{max}) and the number of traits included in the matrix (k).

Figures in the lower off-diagonal are pairwise correlations between the measures.

Figure 6. Posterior means and 95% credible intervals for the four measures used to

characterise **G** matrices expressed as covariances (see methods section); (a) 'maximum

evolvability' (e_{max}), (b) 'total genetic variance' (tgv), (c) 'average evolvability' ($\bar{e}$) and (d)

'effective dimensionality' (n_D). Trait types are life-history (LH), morphology (M) and

sexually selected (S) and filled points are for animals and open points for plants.

Figure 7. Posterior means and 95% credible intervals for the four measures used to

characterise **G** matrices expressed as correlations (see methods section); (a) 'eigenvalue

variance' (Var(λ)), (b) relative eigenvalue variance' (Var$_{rel}$(λ)) and (c) 'eigenvalue

evenness' (E_λ). Trait labels and taxon symbols are as in Figure 6.

Table 1. G-matrix metrics used in this study. Eigenvalue variance, relative eigenvalue

variance and eigenvalue evenness are calculated from correlation matrices, whereas the

other four metrics are calculated from covariance matrices. In all formulae λ are

eigenvalues and *n* is the number of traits in the matrix (rank).* n_D does not measure

dimensionality per se, but eccentricity.

Measure	Cov/cor	Reference	Equation #	Formula
effective number of dimensions* (n_D)	cov	[20]	#2 (pg 273)	$n_D = \sum_{i=1}^{n} \lambda_i / \lambda_1$
maximum evolvability (e_{max})	cov	[20]	#3 (pg 274)	$e_{max} = \sqrt{\lambda_1}$
total genetic variance (v_T)	cov	[20]	#4 (pg 274)	$v_T = \sum_{i=1}^{n} \lambda_i$
average evolvability ($\bar{e}$)	cov	[19]	#4 (pg 1206)	$\bar{e} = \dfrac{\sum_i \lambda_i}{n}$
eigenvalue variance (Var(λ))	cor	[49]	n/a (pg 158)	$Var(\lambda) = \dfrac{\sum_{i=1}^{n} (\lambda_i - 1)^2}{n}$
Relative eigenvalue variance (Var$_{rel}$(λ))	cor	[49]	n/a (pg 159)	$Var_{rel}(\lambda) = \dfrac{Var(\lambda)}{n-1}$
eigenvalue evenness (E_λ)	cor	[23]	#3.2 (pg 1187)	$E_\lambda = -\sum_{i=1}^{n} \dfrac{\tilde{\lambda}_i \ln(\tilde{\lambda}_i)}{\ln(n)}$ where $\tilde{\lambda}_i = |\lambda_i| / \sum_{j=1}^{n} |\lambda_j|$

Table 2. The main effects included in the final models for each analysis. (Effects of 'trait

type' refer to life-history, morphology or sexual and 'taxa' to plant or animal. 'Study

type' refers to field observation or experimental evolution. Random effects of 'study'

and 'species' refer to models where an intercept was fitted to each species and study,

and the random effect of 'trait type:species' indicates where both a species-level

intercept and a species-level trait type effect were fitted.) Full sets of models can be

found in the scripts and data on Dryad.

measure	fixed effects	random effects
Rate (Animals)	trait type + study type	study + trait type:species
Rate (Plants)	trait type	species
|β|	trait type + taxon + trait type x taxon	study + species
(**G**) nD	trait type + taxon + trait no.	study
(**G**) e_{max}	trait type + taxon + trait no.	study + trait type:species
(**G**) tgv	trait type + taxon + trait no.	study + trait type:species
(**G**) $\bar{e}$	trait type + taxon + trait no.	study + trait type:species
(**G**) Var(λ)	trait type + taxon + trait no.	study
(**G**) Var$_{rel}$(λ)	trait type + taxon + trait no.	study
(**G**) E_{λ}	trait type + taxon + trait no.	study

Table 3. Summary statistics for estimates of the rate of evolutionary response, linear

and quadratic selection gradients and measure capturing the size, shape and structure

of **G**. (Statistics are reported by taxa and trait type, together with overall estimates

across trait types and taxa. For each combination of taxa and trait type, the summary

statistics for each measure are provided in the following order: posterior mean,

posterior mode, lower and upper 95% credible intervals (in parenthesis) and sample size

(in italics).)

measure	animals			overall	plants	
	LH	M	SS	all traits	LH	M
Rate (haldanes)	0.12	0.13	0.18	0.13	0.3	0.15
	0.122	0.12	0.193	0.101	0.332	0.181
	(0.02,0.22)	(0.09,0.17)	(0.10,0.26)	(0.08,0.17)	(0.18,0.42)	(0.05,0.25)
	781	*7*	*1667*	*2571*	*26*	*90*
$|\beta|$	0.09	0.22	0.19	0.21	0.31	0.22
	0.157	0.215	0.167	0.242	0.334	0.344
	(0.00,0.19)	(0.16,0.28)	(0.11,0.27)	(0.17,0.26)	(0.22,0.41)	(0.09,0.36)
	65	*342*	*150*	*776*	*156*	*44*
(**G**) nD	1.13	1.20	1.31	1.53	1.23	1.29
	1.40	1.19	2.06	1.50	1.28	0.98
	(0.76,1.5)	(0.82,1.54)	(0.85,1.82)	(1.39,1.67)	(0.77,1.65)	(0.86,1.71)
(**G**) e_{max}	0.43	1.25	0.78	0.61	0.59	0.91
	0.26	0.01	0.86	0.47	0.89	0.57
	(0,1.04)	(0,2.83)	(0,1.88)	(0.26,0.97)	(0,1.39)	(0,2.22)
(**G**) tgv	8.62	25.41	14.32	3.14	9.38	24.09
	17.16	23.57	21.19	7.67	17.37	39.68
	(0.01,18.11)	(0.3,52.47)	(0,30.89)	(0,7.57)	(0.01,19.64)	(0.01,50.48)
(**G**) $\bar{e}$	1.17	3.69	1.95	0.48	1.29	3.40
	0.55	1.74	1.29	0.25	0.80	0.67
	(0,2.94)	(0.01,8.55)	(0,4.75)	(0,1.15)	(0,3.2)	(0,7.83)
n (cov)	*14*	*38*	*10*	*81*	*1*	*3*
(**G**) Var(λ)	0.43	0.60	0.80	1.40	0.63	0.49
	0.39	0.79	1.19	1.11	0.99	0.25
	(0,1.04)	(0,1.32)	(0,1.68)	(0.92,1.86)	(0,1.41)	(0,1.17)
(**G**) Var$_{rel}$(λ)	0.36	0.43	0.50	0.32	0.16	0.23
	0.45	0.46	0.49	0.35	0.07	0.24
	(0.15,0.56)	(0.22,0.63)	(0.25,0.75)	(0.23,0.41)	(0,0.33)	(0,0.42)
(**G**) E_{λ}	0.76	0.70	0.67	0.73	0.86	0.79

	0.78	0.77	0.69	0.76	0.80	0.81
	(0.67,0.84)	(0.61,0.77)	(0.57,0.77)	(0.70,0.77)	(0.77,0.94)	(0.71,0.87)
n (cor)	42	82	27	*221*	14	26

762   Short Title for Page Headings: **Evolutionary rate as a function of selection & the G**

763   **matrix**

(a)   (b)   (c)