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Suboptimality in perception

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ABSTRACT

Human perception is increasingly described as optimal. This view reflects recent successes of Bayesian approaches to perception but ignores an extensive literature documenting suboptimal performance in perceptual tasks. Here we review several classes of suboptimal perceptual decisions, including improper placement, maintenance, and adjustment of perceptual criteria, inadequate tradeoff between speed and accuracy, and inappropriate confidence ratings. We examine suboptimalities in the cue combination literature, which has often been taken as evidence for optimality in perception. We further discuss how findings regarding visual illusions, adaptation, and appearance relate to the concept of optimality. We extract a number of principles that account for suboptimal behavior across all of these studies. Finally, we outline an "Objectives-Constraints-Mechanisms" approach for guiding future investigations and discussions of optimal and suboptimal perception. In this approach, findings of optimality or suboptimality are not ends in themselves but serve to characterize the perceptual system's objectives (i.e., what it aims to achieve), constraints (i.e., what costs it incurs), and mechanisms (i.e., what processes it uses to trade off the objectives and constraints). We suggest that this conceptual framework, more than a focus on optimality per se, can advance our understanding of perception.

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1. INTRODUCTION

How do people make perceptual judgments based on the available sensory information? This fundamental question has been a focus of psychological research from the 19th century on (Fechner 1860; Helmholtz 1856). Many perceptual tasks naturally lend themselves to what has traditionally been called "ideal observer" analysis, whereby the optimal behavior is mathematically determined and human behavior is compared to this standard (Geisler 2011; Green and Swets 1966; Ulehla 1966). The extensive literature on this topic includes many examples of humans performing similar to an ideal observer but also features many examples of suboptimal behavior. Perceptual science has a strong tradition of developing models and theories that attempt to account for the full range of empirical data on how humans perceive (Macmillan and Creelman 2005).

Recent years have seen an impressive surge of Bayesian theories of human cognition and perception (Gershman, Horvitz, and Tenenbaum 2015; Griffiths, Lieder, and Goodman 2015; Tenenbaum et al. 2011). These theories often depict humans as optimal decision-makers, especially in the area of perception. A number of high-profile papers have shown examples of human perceptual behavior that is close to optimal (Ernst and Banks 2002; Körding and Wolpert 2004; Landy et al. 1995), while other papers have attempted to explain apparently suboptimal behaviors as being in fact optimal (Weiss, Simoncelli, and Adelson 2002). Consequently, many statements by researchers in the field leave the impression that humans are essentially optimal in perceptual tasks:

> "... psychophysics is providing a growing body of evidence that human perceptual computations are 'Bayes' optimal'." (Knill & Pouget, 2004, p. 712)

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> "Across a wide range of tasks, people seem to act in a manner consistent with optimal Bayesian models" (Vul, Goodman, Griffiths, & Tenenbaum, 2014, p.1)
>
> "These studies with different approaches have shown that human perception is close to the Bayesian optimal" (Körding & Wolpert, 2006, p. 321)

Despite a number of recent criticisms of such assertions regarding human optimality (Bowers and Davis 2012a, 2012b; Eberhardt and Danks 2011; Jones and Love 2011; Marcus and Davis 2013, 2015), as well as statements from some of the most prominent Bayesian theorists that their goal is not to demonstrate optimality (Griffiths et al. 2012), the quotes above indicate that the view that humans are (close to) optimal in perceptual tasks has taken strong foothold.

To facilitate a more comprehensive view of human perceptual decisions, here we review a number of areas in which humans are suboptimal. We aim to cover as much ground as possible without discussing each finding in detail. Given the vast literature on the topic, our review is necessarily incomplete, but we hope that the numerous examples of suboptimality in perceptual decision making will lead to a more balanced view and facilitate a deeper understanding of human perception. We also discuss how findings of suboptimality can reveal the principles of the human perceptual system. We advocate for an Objectives-Constraints-Mechanisms approach to understanding perception and explore the implications of this approach for the concept of perceptual optimality. Finally, we provide recommendations for discussing optimality in empirical studies of perception and sensory processing.

2. DEFINING OPTIMALITY

Optimality can be a difficult concept to define. Most research to date thus focuses on simple cases in which one of two stimulus categories is presented on each trial and observers make a

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perceptual judgment discriminating between the two possibilities. In such cases, optimal performance is usually defined as the response strategy that produces the highest level of reward. If reward is not explicitly provided, then it is usually assumed that observers attempt to maximize accuracy (which is equivalent to rewarding equally all correct responses and punishing equally all incorrect ones).

What is the strategy that maximizes reward? In the general case, on each trial an observer attempts to estimate the expected value (EV) of deciding on stimulus $\boldsymbol{s}$ based on the internal response $\boldsymbol{r}$ and reward for that stimulus $R(s)$:

$$EV(s) = P(s|r) * R(s) \tag{1}$$

which, by using Bayes' formula, can be further expanded to

$$EV(s) = P(r|s) * P(s) * R(s)/P(r) \tag{2}$$

where $P(s|r)$ is the posterior probability of s, $P(r|s)$ is the likelihood function that depends on the internal response, while $P(s)$ and $P(r)$ are the probability of occurrence for s and r. In the case of two stimulus classes s_1 and s_2, the optimal strategy is to respond s_2 when $EV(s_2) \geq EV(s_1)$, which, using equation 2, is equivalent to responding s_2 when:

$$\frac{P(r|s_2)}{P(r|s_1)} \geq \frac{P(s_1)}{P(s_2)} * \frac{R(s_1)}{R(s_2)} \tag{3}$$

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In formula 3, $\frac{P(r|s_2)}{P(r|s_1)}$ is known as the "likelihood ratio", $\frac{P(s_1)}{P(s_2)}$ is the prior odds, and $\frac{R(s_1)}{R(s_2)}$ is the reward bias. Formulas 2 and 3 assume no penalty for incorrect responses. In cases in which a full reward matrix is provided such that $R_{i,j}$ is the reward for responding "s_i" when stimulus s_j was presented, then a straightforward computation shows that the reward bias term in equation 3 needs to be replaced by $\frac{R_{1,1}-R_{2,1}}{R_{2,2}-R_{1,2}}$.

Theoretically, the likelihood function $P(r|s)$ can take any shape, but in practice, $P(r|s)$ can almost always be approximated with a Gaussian distribution (Macmillan and Creelman 2005; See et al. 1997; Swets 1986). See the first point in "Methodological considerations that can account for suboptimal behavior" below for further discussion of this distributional assumption.

3. EXAMPLES OF SUBOPTIMALITY

3.1 Criterion in simple 2-choice tasks

In the simplest and most ubiquitous case, only two stimuli s_1 and s_2 are presented with equal probability and equal reward (that is often not defined explicitly). From formula 3 we can derive that the location of the optimal criterion is the point where the two likelihood functions $P(r|s_1)$ and $P(r|s_2)$ intersect and the likelihood ratio is 1 (**Figure 1**).

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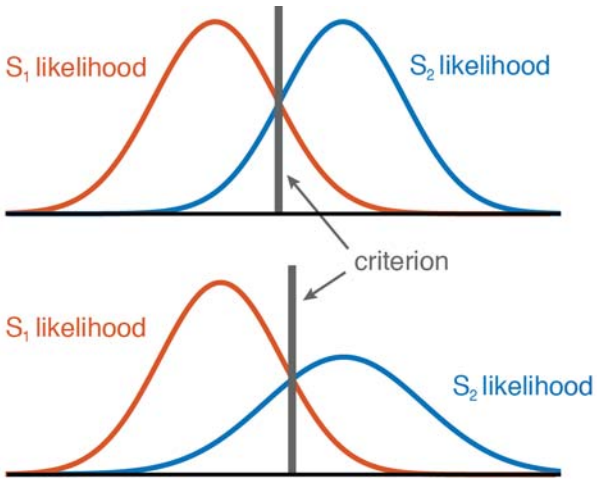

***Figure 1**. Depiction of optimal criteria in simple 2-choice tasks. The upper panel shows the likelihood functions for two stimuli and with equal variability. The gray vertical line represents the location of the optimal criterion such that the region on the left of the criterion is categorized as " " and the region on the right as " ". The lower panel shows the location of the optimal criterion when the variability of the two likelihood functions differs (in the case depicted here, the optimal criterion results in a higher proportion of responses).*

3.1.1 Detection criteria

Many tasks involve the simple distinction between noise ( ) and signal+noise ( ). These are usually referred to as detection tasks. In most cases is found to have smaller variance than (Green and Swets 1966; Macmillan and Creelman 2005; Swets, Tanner, and Birdsall 1961), from where it follows that an optimal observer would choose more often than even when the two stimuli are presented at equal rates (**Figure 1**). Indeed, many detection studies find that observers choose the noise distribution more than half of the time (Gorea & Sagi, 2000; Green & Swets, 1966; Rahnev et al., 2011; Reckless et al., 2014; Solovey, Graney, & Lau, 2015; Swets et al., 1961) thus suggesting that observers do have access to their internal uncertainty. However, most studies do not allow for the estimation of the likelihood functions for

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individual observers and thus it is an open question how optimal observers in those studies actually are. A few studies have reported conditions in which observers choose the noise distribution s_1 *less* than half of the time (Morales et al., 2015; Rahnev et al., 2011; Solovey et al., 2015) thus providing examples of clearly suboptimal strategies.

3.1.2 Discrimination criteria

Detection tasks require observers to distinguish between the noise vs. signal+noise stimuli but other tasks require observers to discriminate between two roughly equivalent stimuli. For example, observers might discriminate left vs. rightward motion or clockwise vs. counterclockwise grating orientation. For these types of stimuli, the likelihood functions for each stimulus category can safely be assumed to have approximately equal variability (Macmillan and Creelman 2005; See et al. 1997). Such studies find that the average criterion location across the whole group of observers is usually close to the optimal (likelihood ratio = 1) but individual observers can still exhibit substantial biases in either direction (e.g., Whiteley & Sahani, 2008). For example, we re-analyzed the data from a recent study in which observers discriminated between a grating tilted 45 degrees clockwise or counterclockwise from vertical (Rahnev et al. 2016). Seventeen observers came for four sessions on different days completing 480 trials each time. Using a binomial test, we found that only 11 of the 68 total sessions did not exhibit significant deviation from unbiased responding of clockwise or counterclockwise. Further, observers tended to have relatively stable biases as demonstrated by a positive criterion correlation across all pairs of sessions (all p's < .003). Thus, even if the performance of the group appears to be close to optimal, individual observers often deviate substantially from optimality.

3.1.3 Two-stimulus tasks

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The biases observed in detection and discrimination experiments led to the development of the popular two-alternative forced-choice (2AFC) task, in which both stimulus categories are presented on each trial, in order to reduce individual bias (Macmillan and Creelman 2005). 2AFC tasks separate the two stimuli either temporally (these tasks are also commonly referred to as 2-interval-forced-choice or 2IFC tasks) or spatially. Note that in recent years many researchers have begun to use the term "2AFC" for tasks in which only one stimulus is presented. To avoid confusion, we adopt the term "2-stimulus tasks" to refer to tasks where two stimuli are presented (the original meaning of 2AFC) and the term "1-stimulus tasks" to refer to the common single-stimulus tasks such as detection and discrimination tasks discussed in 1.1 and 1.2.

Even though 2-stimulus tasks were designed to remove observer bias, significant biases have been observed for them, too. While biases in spatial 2AFC tasks have received less attention, several biases have been documented for 2IFC tasks. For example, early research suggested that if the two stimuli in a trial differ in intensity then the second one is more often selected as the one of higher intensity, a phenomenon called *time-order errors* (Fechner 1860; Osgood 1953). More recently Yeshurun et al. (2008) re-analyzed 2IFC data from seventeen previous experiments and found significant interval biases. The direction of the bias varied between the different experiments, suggesting that the specific experimental design has an influence on observers' bias.

3.1.4 Explaining suboptimality in simple 2-choice tasks

Why do people have trouble setting appropriate criteria in simple 2-choice tasks? One possibility is that they have a tendency to give the same response when uncertain. For example, a given observer may respond that they saw left (rather than right) motion every time they get distracted

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or had very low evidence for either choice. This could be due to a preference for one of the two stimuli or one of the two motor response options. Re-analysis of another previous study (Rahnev, Lau, and De Lange 2011) where we withheld the stimulus-response mapping until after the stimulus presentation still found that 12 of the 21 observers revealed a significant response bias for motion direction. This suggests that a preference in motor behavior cannot fully account for this type of suboptimality. Thus, it appears that many observers automatically develop a preference for one of the stimulus categories, and this preference leads to suboptimal behavior. Nevertheless, it is possible that this bias does not arise from an inappropriate decision strategy but rather from biases in the sensory system. For example, for some observers left vs. rightward motion may generate likelihood functions with different variability, similar to detection tasks in which the noise and noise+signal distributions have different variability (see **Figure 1**).

3.2 Maintaining stable criteria

Above we considered observers' average criterion in simple cases when the optimal criterion should be placed at the point at which the likelihood ratio equals 1. We now turn our attention to whether factors other than the likelihood functions have an impact on the perceptual decision. An optimal observer would be insensitive to any other factors (**Figure 2**).

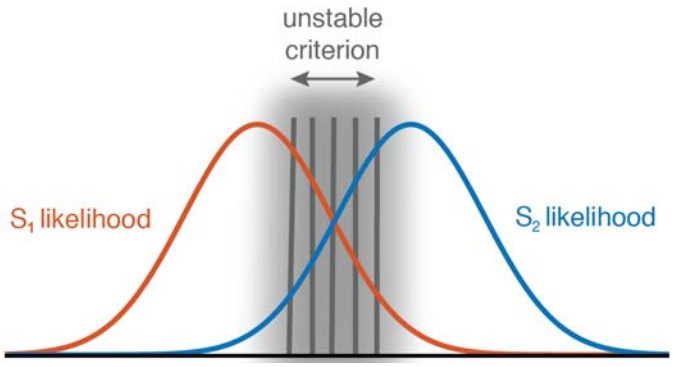

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Figure 2*. Depiction of a failure to maintain a stable criterion. The optimal criterion is shown in Figure 1 but observers often fail to maintain that criterion over the course of the experiment, resulting in a criterion that effectively varies over trials.*

3.2.1 Sequential effects

Optimality requires that judgments are made based on the evidence from the current stimulus independently of previous stimuli. However, sequential effects are ubiquitous in perceptual tasks (Fischer and Whitney 2014; Fründ, Wichmann, and Macke 2014; Kaneko and Sakai 2015; Liberman, Fischer, and Whitney 2014; Tanner, Haller, and Atkinson 1967; Treisman and Faulkner 1984; Ward and Lockhead 1970; Yu and Cohen 2009). The general finding is that observers' responses are positively autocorrelated such that the response on the current trial is likely to be the same as on the previous trial, though in some cases negative autocorrelations have also been reported (Tanner et al. 1967; Ward and Lockhead 1970). Further, observers are able to adjust to new trial-to-trial statistics but this adjustment is only strong in the direction of default biases and weak in the opposite direction (Abrahamyan et al. 2016). Similar biases have been observed in other species such as mice (Busse et al. 2011).

3.2.2 Switching between different criteria

Another factor that has been found to influence optimal criterion placement is interleaving trials that require different criteria. Gorea & Sagi (2000) demonstrated that when high-contrast stimuli (optimally requiring a relatively liberal detection criterion) and low-contrast stimuli (optimally requiring a relatively conservative detection criterion) were presented simultaneously, observers tended to use the same compromised detection criterion that was suboptimal for both the high- and low-contrast stimuli. This happened despite the fact that on each trial observers knew with 100% certainty which contrasts might have been present in each location. Similar effects have

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been shown in a variety of paradigms that involved using stimuli of different contrasts (Gorea, Caetta, and Sagi 2005; Gorea and Sagi 2001, 2002; Zak et al. 2012), attended vs. unattended stimuli (Morales et al. 2015; Rahnev, Maniscalco, et al. 2011), and central vs. peripheral stimuli (Solovey et al. 2015).

3.2.3 Irrelevant reward influencing the criterion

Several lines of research suggest that observers change their criteria based on factors that should optimally be ignored. For example, maximizing one's payoff necessitates using the same (unbiased) criterion whether a large or small reward is offered on a particular trial. However, both monetary rewards and punishments lead observers to adopt a more liberal detection criterion such that more stimuli are identified as targets (Reckless et al. 2013, 2014). Note that in these studies the rewards were given simply for responding correctly and thus should not have changed the criterion. Similar changes to response criterion due to monetary motivation are obtained in a variety of paradigms (Henriques, Glowacki, and Davidson 1994; Taylor et al. 2004). To complicate matters, observers' personality traits interact with the type of monetary reward in altering response criteria (Markman, Baldwin, and Maddox 2005). Thus, even though optimality necessitates that observers only use task-relevant information, it appears that a wide range of irrelevant factors are capable of shifting the response criterion.

3.2.4 Explaining suboptimality in maintaining stable criteria

Why do people shift their response criteria based on factors that should be irrelevant for the criterion placement? Some of the above examples can be explained as memory limitations. For example, maintaining two criteria for two different stimulus classes simultaneously may be too taxing to the system. On the other hand, sequential effects have been explained in terms of an automatic tendency to exploit the continuity in our normal environment, even though such

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continuity is not present in most experimental setups (Fischer and Whitney 2014; Liberman et al. 2014). However, it is harder to explain why personality traits or task features such as the presence of monetary rewards that should be irrelevant to the response criterion change observers' criteria. It appears that in some situations observers may use additional factors besides what is suggested by Bayesian inference to make their perceptual decisions (Mueller and Weidemann 2008). Very little research has focused on identifying such factors and explaining why they influence observers' responses.

3.3 Adjusting choice criteria

One of the most common ways to assess optimality in perceptual decision making is to manipulate the prior probability of the stimulus classes and/or provide unequal payoffs that bias responses towards one of the stimulus categories (Macmillan and Creelman 2005). As can be seen from formula 3, the two manipulations have an equivalent effect on the optimal response strategy: both require observers to shift their criterion in likelihood space by a factor dictated by the specific prior probability and reward structure (**Figure 3**).

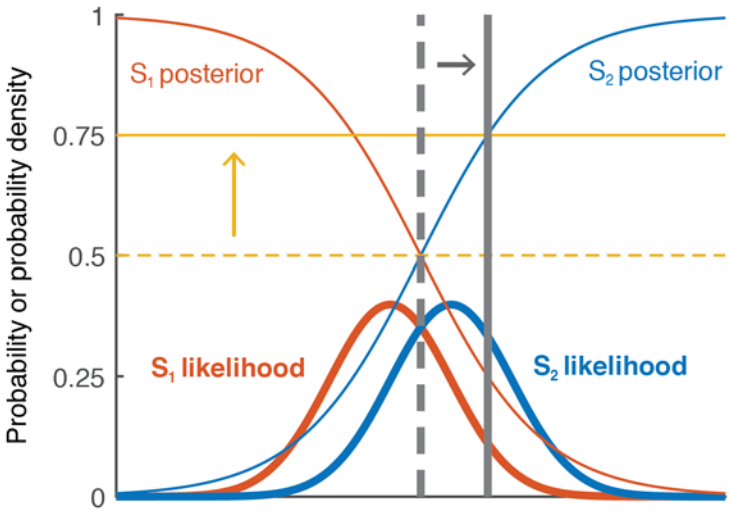

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Figure 3*. Depiction of optimal adjustment of choice criteria. In addition to the S_1 and S_2 likelihood functions (in thick red and blue lines), the figure shows the corresponding posterior functions (in thin red and blue lines). The vertical criteria in likelihood space (thick gray lines) correspond to the horizontal thresholds in posterior probability space (thin yellow lines). Optimal criterion and threshold for equal prior probabilities or payoffs are shown in dashed lines. If unequal prior probability or unequal payoff is provided such that S_1 ought to be chosen three times as often as S_2, then the threshold would optimally be shifted to 0.75, corresponding to a shift in the criterion such that the horizontal threshold and vertical criterion intersect on the S_2 posterior. The y-axis is probability density for the likelihood functions, and probability for the posterior distributions.*

3.3.1 Priors

Two main approaches have been used to determine whether observers can optimally adjust their criterion when one of two stimuli has a higher probability of occurrence. In base-rate manipulations, long blocks of the same occurrence frequency are used, and often observers are not told the probabilities of occurrence in advance (e.g., Maddox, 1995). Most studies find that observers adjust their criterion to account for the unequal base rate, but this adjustment is smaller than what is required for optimal performance, resulting in a conservative criterion placement (Bohil and Maddox 2003a; Green and Swets 1966; Maddox and Bohil 2001, 2003, 2005; Maddox, Bohil, and Dodd 2003; Maddox and Dodd 2001; Tanner 1956; Tanner et al. 1967; Vincent 2011). Some studies have suggested that observers become progressively more suboptimal as the base-rate becomes progressively more extreme (Bohil and Maddox 2003a; Green and Swets 1966). However, a few studies have reported that certain conditions result in extreme criterion placement such that observers rely more on base-rate information than is optimal (Maddox and Bohil 1998b). Finally, other studies found that with extensive training

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approximately half of the observers are able to adjust the criterion to a near optimal level (Maddox 1995).

A second way to manipulate the probability of occurrence is to do it on a trial-by-trial basis and explicitly inform the observers about the stimulus probabilities before each trial. Research that has used this method also finds conservative criterion placement such that observers do not adjust their criterion enough (Ackermann and Landy 2014; de Lange et al. 2013; Rahnev, Lau, et al. 2011; Summerfield and Koechlin 2010; Ulehla 1966).

3.3.2 Payoffs

The decision criterion can also be manipulated by giving different payoffs for different responses. The general finding with this manipulation is that observers, again, do not adjust their criterion enough (Ackermann and Landy 2014; Bohil and Maddox 2001, 2003a, 2003b; Busemeyer and Myung 1992; Maddox and Bohil 1998a, 2000, 2001, 2003, 2005; Maddox et al. 2003; Maddox and Dodd 2001; Markman et al. 2005; Taylor et al. 2004; Ulehla 1966) and, as with base rates, become more suboptimal for more extreme payoffs (Bohil and Maddox 2003a). Nevertheless, one study that involved a very large number of sessions with two monkeys reported extreme criterion changes (Feng et al. 2009).

Criterion adjustments in response to unequal payoffs are usually found to be more suboptimal compared to adjustments in response to unequal base-rates (Ackermann and Landy 2014; Bohil and Maddox 2001, 2003a; Busemeyer and Myung 1992; Healy and Kubovy 1981; Maddox 2002; Maddox and Bohil 1998a; Maddox and Dodd 2001) though the opposite pattern was found by Green and Swets (1966).

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Finally, the exact payoff structure may also influence observers' optimality. For instance, introducing a cost for incorrect answers leads to more suboptimal criterion placement compared to conditions with the same optimal criterion shift but without a cost for incorrect answers (Maddox and Bohil 2000; Maddox et al. 2003; Maddox and Dodd 2001).

3.3.3 Explaining suboptimality in choice criterion

Why are observers unable to adjust their decision criteria optimally in response to priors and rewards? One possibility is that they do not have an accurate internal representation of the relevant probability (Acerbi, Vijayakumar, and Wolpert 2014; Ackermann and Landy 2014; Zhang and Maloney 2012). For example, Zhang & Maloney (2012) proposed that observers represent the logarithm of a given probability rather than the probability itself. A related possibility is that observers set their criteria directly on the likelihood function without properly computing the posterior probability (Dusoir 1975, 1983; Ingham 1970; Macmillan and Creelman 1990; Matthews, Davies, and Holley 1993; See et al. 1997; Swets 1986).

Another possible explanation is the "flat-maxima" hypothesis, according to which the observer adjusts the decision criterion based on the change in reward and has trouble finding its optimal value if other criterion positions result in similar reward rates (Bohil and Maddox 2003b; Busemeyer and Myung 1992; Maddox and Bohil 2001, 2003, 2004, 2005; Maddox et al. 2003; Maddox and Dodd 2001; von Winterfeldt and Edwards 1982).

When the criterion-reward function has flat regions, large deviations in criterion can be seen as only mildly suboptimal because of the relatively small losses in reward. In fact, some authors have argued that observers in their studies should be considered basically optimal since they

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achieved close to maximal reward even though their decision criteria were far from the optimal criteria (Feng et al. 2009; Whiteley and Sahani 2008).

3.4 Adjusting tradeoff between speed and accuracy

In the examples above, the only variable of interest has been observers' choice irrespective of their reaction times (RTs). However, if instructed, observers can provide responses faster at lower accuracy, a phenomenon known as speed-accuracy tradeoff (SAT; Fitts, 1966; Heitz, 2014). An important question here is whether observers can adjust their decision process optimally in response to a particular payoff structure that includes reaction time (**Figure 4**).

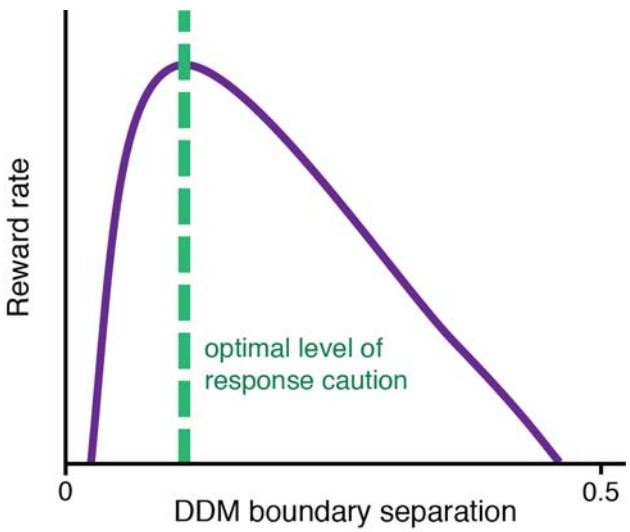

Figure 4*. Depiction of the reward rate curve for different boundary separations in the drift-diffusion model (DDM) framework* (Starns and Ratcliff 2010)*. A larger separation means more evidence is needed to make a response, so responses are more accurate but take longer. The optimal level of response caution - the amount of boundary separation that maximizes the reward per unit time - is depicted with the dashed line.*

3.4.1 Trading off speed and accuracy

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A number of studies that explored this question found that while observers are able to adjust their behavior to account for both accuracy and RT, they cannot do this optimally (Balci et al. 2011; Bogacz et al. 2010; Simen et al. 2009; Starns and Ratcliff 2010, 2012; Tsetsos et al. 2015). In most cases, observers set their decision threshold more conservatively than optimal, leading to slightly higher accuracy but substantially slower responses than optimal (Bogacz et al. 2010; Simen et al. 2009; Starns and Ratcliff 2010, 2012). This is true when observers have a fixed period of time to complete as many trials as possible (Bogacz et al. 2010; Simen et al. 2009; Starns and Ratcliff 2010, 2012), and in the more familiar design with a fixed number of trials per block (Starns and Ratcliff 2010, 2012). Further, observers set more conservative decision criteria for more difficult conditions when optimality demanded that they do the opposite (Starns and Ratcliff 2012). Older adults are even more suboptimal than college-age participants by this measure (Starns and Ratcliff 2010, 2012). Similar to adjusting one's criterion based on rewards, extensive training reduces suboptimal behavior but does not eliminate it (Balci et al. 2011). Finally, recent research shows that when speed is emphasized, observers' usage of sensory information becomes suboptimal (Ho et al. 2012; Rae et al. 2014).

3.4.2 Keeping a low error rate under implicit time pressure

Even though observers tend to overemphasize accuracy, they are also suboptimal in tasks that require an extreme emphasis on accuracy. This conclusion comes from a line of research on visual search in which observers are typically given unlimited amount of time to decide whether a target is present or not (Eckstein 2011). In certain situations, such as airport checkpoints or detecting tumors in mammograms, the goal is to keep a very low miss rate, because misses can have dire consequences (Evans, Birdwell, and Wolfe 2013; Wolfe et al. 2013). However, such situations typically feature very low target prevalence, so an observer may need to adjust both the criterion for target detection and a quitting threshold that dictates when to stop searching (Wolfe and Van Wert 2010). A series of studies by Wolfe and colleagues found that observers,

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even trained doctors and airport checkpoint screeners, are suboptimal in such tasks in that they allow overly high rates of misses (Evans et al. 2011, 2013; Wolfe et al. 2013; Wolfe, Horowitz, and Kenner 2005; Wolfe and Van Wert 2010). Further, this effect is robust and resistant to a variety of methods designed to help observers set more appropriate criteria (Wolfe et al. 2007) or correct motor errors (Van Wert, Horowitz, and Wolfe 2009). An explanation of this suboptimality based on capacity limits is rejected by two studies that found that observers use a relatively more optimal criterion after a block of high prevalence targets accompanied with feedback (Wolfe et al. 2007, 2013).

3.4.3 Explaining suboptimality in speed-accuracy tradeoff

Why are observers unable to trade off speed and accuracy optimally? It is possible to account for overly conservative decision criteria by postulating that, in addition to maximizing their reward rate, observers place a premium on being accurate (Balci et al. 2011; Bogacz et al. 2010). Another possibility is that observers' judgments of elapsed time are noisy, and placing the threshold higher than the optimal value leads to a higher reward rate than setting it lower than optimal by the same amount (Simen et al. 2009; Zacksenhouse, Bogacz, and Holmes 2010). Finally, observers may be unable or unwilling to set extreme criteria, making it impossible to keep a very low error rate (Wolfe et al. 2013). More than one of these factors may play a role in each of the demonstrations of suboptimality discussed above.

3.5 Confidence in one's decision

Up to this point we have considered how observers place, maintain, and adjust their perceptual criteria in order to maximize their task performance. Observers are also able to provide metacognitive confidence ratings of their subjective belief that each of their decisions was

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correct. We now turn our attention to the question of whether these confidence judgments are optimal (**Figure 5**).

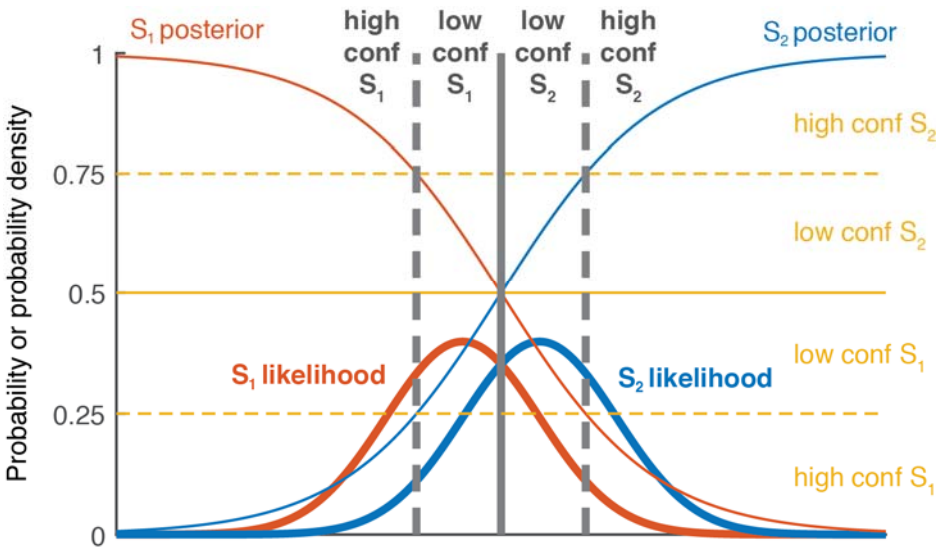

Figure 5*. Depiction of how an observer should give confidence ratings. Similar to Figure 3, both the likelihood and posterior functions are depicted. Confidence thresholds are optimally given on the posterior probability space (depicted as thin yellow lines). These thresholds correspond to criteria in likelihood space (depicted as thick gray lines). This horizontal thresholds and vertical criteria intersect on the S_2 posterior. The y-axis is probability density for the likelihood functions, and probability for the posterior distributions.*

3.5.1 Over- and under-confidence (confidence calibration)

The most straightforward test of the optimality of observers' confidence ratings is to determine whether their stated probability of being correct matches the actual probability of being correct. For example, if we consider all trials in which an optimal observer has 70% confidence of being correct, then we would expect the average accuracy on those trials to be 70%. This logic

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applies when an observer chooses between two equally likely options without a significant bias for one over the other (though does not hold in the presence of unequal prior probabilities or choice biases, Drugowitsch, Moreno-Bote, & Pouget, 2014). This type of relationship between confidence and accuracy is often referred to as "confidence calibration" (Baranski and Petrusic 1994). Studies of confidence have found that for certain tasks observers are overconfident (i.e., they overestimate their accuracy) (Adams 1957; Baranski and Petrusic 1994; Dawes 1980; Harvey 1997; Keren 1988; Koriat 2011) while for other tasks observers are underconfident (i.e., they underestimate their accuracy) (Baranski and Petrusic 1994; Björkman, Juslin, and Winman 1993; Dawes 1980; Harvey 1997; Winman and Juslin 1993). One pattern that emerges consistently is that overconfidence often occurs for difficult tasks while underconfidence appears in easy tasks (Baranski and Petrusic 1994, 1995, 1999), a phenomenon known as the *hard-easy effect* (Gigerenzer, Hoffrage, and Kleinbölting 1991). Similar results are seen for tasks outside of the perceptual domain such as answering general knowledge questions (Griffin and Tversky 1992). Over- and under-confidence are stable over different tasks (Ais et al. 2015; Song et al. 2011) and depend on non-perceptual factors such as one's optimism bias (Ais et al. 2015).

3.5.2 Dissociations of confidence and accuracy

Miscalibration of confidence, as with over- and under-confidence findings, can arise from a variety of causes such as misrepresentation of probabilities or observer-specific factors like shyness about giving high confidence. However, regardless of such factors, one would predict that an observer who performs at a similar level in two different experimental conditions would have similar confidence for those conditions. However, despite the intuitiveness of such a proposition, many studies have demonstrated dissociations between confidence and accuracy. For example, a requirement to provide quick responses decreases accuracy while leaving confidence unchanged (Baranski and Petrusic 1994; Vickers and Packer 1982). A similar effect

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was obtained by Kiani, Corthell, & Shadlen (2014), who showed that a stimulus manipulation that increases RT but does not affect accuracy leads to a decrease in confidence. Dissociations between confidence and accuracy have also been found in conditions that differ in attention (Rahnev, Bahdo, et al. 2012; Rahnev, Maniscalco, et al. 2011; Wilimzig et al. 2008), the variability of the perceptual signal (de Gardelle and Mamassian 2015; Koizumi, Maniscalco, and Lau 2015; Samaha et al. 2016; Song, Koizumi, and Lau 2015; Spence, Dux, and Arnold 2015; Zylberberg, Roelfsema, and Sigman 2014), the stimulus-onset asynchrony in metacontrast masking (Lau and Passingham 2006), the presence of unconscious information (Vlassova, Donkin, and Pearson 2014), and the relative timing of a concurrent saccade (Navajas, Sigman, and Kamienkowski 2014). Further, some of these biases seem to arise from individual differences that are stable across multiple sessions (de Gardelle and Mamassian 2015). Finally, dissociations between confidence and accuracy have been found in studies that applied transcranial magnetic stimulation (TMS) to the visual (Rahnev, Maniscalco, et al. 2012), premotor (Fleming et al. 2015), or frontal cortex (Chiang et al. 2014), possibly indicating disruptions to the normal circuits that compute confidence.

3.5.3 Metacognitive sensitivity (confidence resolution)

The sections above were concerned with the average magnitude of confidence ratings over many trials. Another measure of interest is the degree of correspondence between confidence and accuracy on individual trials (Fleming and Lau 2014; Metcalfe and Shimamura 1994). The degree to which the two go together on a trial-by-trial basis is often taken as a measure of metacognitive sensitivity (Fleming and Lau 2014) and has also been referred to as “confidence resolution” (Baranski and Petrusic 1994).

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Research in this domain finds that metacognitive sensitivity is often imperfect (Metcalfe and Shimamura 1994; Nelson and Narens 1990). Recently, Maniscalco & Lau (2012) developed a method that reveals how optimal an observer's metacognitive sensitivity is. Their method computes *meta-d'*, a measure of how much information is available for metacognition, which can then be compared with the actual *d'* value. An optimal observer would have a *meta-'d*/*d'* ratio of 1. Maniscalco & Lau obtained a ratio of .77, suggesting a 23% loss of information for confidence judgments. Even though some studies that used the same measure but different perceptual paradigms found values close to 1 (Fleming et al. 2014), many others arrived at values substantially lower than 1 (Maniscalco and Lau 2015; Maniscalco, Peters, and Lau 2016; Massoni 2014; McCurdy et al. 2013; Schurger, Kim, and Cohen 2015; Sherman et al. 2015; Vlassova et al. 2014). Interestingly, at least one study has reported values significantly above 1, suggesting that in certain cases the metacognitive system has more information than was used for the primary decision (Charles et al. 2013).

3.5.4 Other factors influencing confidence ratings

Confidence judgments are influenced by a host of factors unrelated to the perceptual signal at hand. Maniscalco and Lau demonstrated that working memory load (Maniscalco and Lau 2015) affects the relationship between confidence and accuracy. Emotional states also have an influence; for instance, worry increases metacognitive performance (Massoni 2014). Other factors, such as eye gaze stability (Schurger et al. 2015) and age (Weil et al. 2013) also correlate with the relationship between confidence and accuracy.

Sequential effects have also been reported for confidence judgments such that a high confidence rating is more likely to follow a high, rather than low, confidence rating (Mueller and Weidemann 2008). Confidence dependencies exist even between different tasks, such as letter

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and color discrimination, that depend on different neural populations in the visual cortex (Rahnev et al. 2015). This effect was dubbed "confidence leak" and was shown to be negatively correlated with observers' metacognitive sensitivity.

Confidence has also been shown to exhibit a "positive evidence" bias (Maniscalco et al. 2016; Zylberberg, Barttfeld, and Sigman 2012). In 2-choice tasks, one can distinguish between sensory evidence in a trial that is congruent with the observer's response on that trial (positive evidence) and sensory evidence that is incongruent with the response (negative evidence). Even though the perceptual decisions usually follow the optimal strategy of weighting equally both of these sources of evidence, confidence ratings are suboptimal in depending more heavily on the positive evidence (Koizumi et al. 2015; Maniscalco et al. 2016; Samaha et al. 2016; Song et al. 2015; Zylberberg et al. 2012).

3.5.5 Explaining suboptimality in confidence ratings

Why do observers give inappropriate confidence ratings? Most modeling approaches - such as signal detection theory (Green and Swets 1966; Macmillan and Creelman 2005), drift diffusion models (Ratcliff and Starns 2009; Vickers 1979), and Bayesian theories of population coding (Drugowitsch and Pouget 2012) - assume that subjective reports of confidence are based directly on the perceptual signal. However, even though these approaches can sometimes account for particular types of suboptimal behavior (King and Dehaene 2014; Rahnev, Maniscalco, et al. 2011), it is hard to see how they can account for the full array of suboptimalities reviewed above. Instead, other theories postulate the existence of either different processing streams that contribute differentially to the perceptual decision and the subjective confidence judgment (Del Cul et al. 2009; Jolij and Lamme 2005; Weiskrantz 1996) or a second processing stage that determines the confidence judgment, and which builds upon

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the information in an earlier processing stage responsible for the perceptual decision (Lau and Rosenthal 2011; Maniscalco and Lau 2010, 2016; Pleskac and Busemeyer 2010). Both types of models could be used to explain the various findings of suboptimal behavior.

3.6 Comparing sensitivity in different tasks

The above sections discussed observers' performance on a single task. Another way of examining optimality is to compare the performance on two related tasks. If the two tasks have a formal relationship, then an optimal observers' sensitivity on the two tasks would follow that relationship.

3.6.1 Comparing performance in 1-stimulus and 2-stimulus tasks

Visual sensitivity has traditionally been measured by employing either (1) a 1-stimulus (detection or discrimination) task in which a single stimulus from one of two stimulus classes is presented on each trial, or (2) a 2-stimulus task in which both stimulus classes are presented on each trial (see 1.3 above). Intuitively, 2-stimulus tasks are easier because the final decision is based on two, rather than one, stimulus. In fact, assuming independent processing of each stimulus, the relationship between sensitivity on these two types of tasks can be mathematically defined: the sensitivity on the 2-stimulus task should be $\sqrt{2}$ times higher than on the detection task (Macmillan & Creelman, 2005; **Figure 6**). Nevertheless, empirical studies have often contradicted this predicted relationship, suggesting that one or both tasks were performed in a suboptimal manner. Some studies found sensitivity ratios larger than $\sqrt{2}$ (Creelman and Macmillan 1979; Jesteadt 1974; Leshowitz 1969; Markowitz and Swets 1967; Pynn 1972; Schulman and Mitchell 1966; Swets and Green 1961; Viemeister 1970; Watson et al. 1973; Yeshurun, Carrasco, and Maloney 2008), while others have found ratios smaller than $\sqrt{2}$ (Leshowitz 1969; Markowitz and Swets 1967; Swets and Green 1961).

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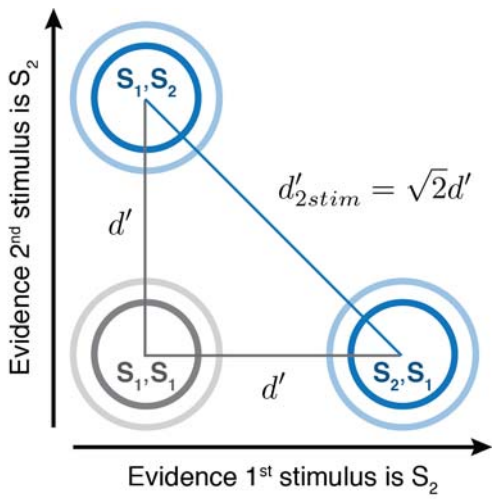

***Figure 6**. Depiction of the relationship between 1-stimulus and 2-stimulus tasks. Each axis corresponds to a 1-stimulus signal detection task (e.g. Figure 1). The three sets of concentric circles represent 2D circular Gaussian distributions corresponding to presenting two stimuli in a row (e.g., S_2,S_1 means that S_2 was presented first and S_1 was presented second). If the discriminability between S_1 and S_2 is d' (1-stimulus task; gray lines in triangle), then the Pythagorean theorem gives us the expected discriminability between S_1,S_2 and S_2,S_1 (2-stimulus task; blue line in triangle).*

3.6.2 Comparing performance in other tasks

Many other comparisons between tasks have been performed in the literature. In temporal 2IFC tasks observers often have different sensitivity to the two stimulus intervals (García-Pérez and Alcalá-Quintana 2010, 2011; Yeshurun et al. 2008), suggesting an inability to distribute resources optimally. Other studies find that longer inter-stimulus intervals in 2IFC tasks lead to decreases in sensitivity (Berliner and Durlach 1973; Kinchla and Smyzer 1967; Tanner 1961), presumably due to memory limitations. Creelman and Macmillan (1979) compared the

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sensitivity on 9 different psychophysical tasks and found a complex pattern of dependencies, many of which were at odds with optimal performance. Finally, Olzak (1985) demonstrated deviations from the expected relationship between detection and discrimination tasks.

An alternative approach to comparing an observer's performance on different tasks is allowing observers to choose which tasks they prefer to complete and analyzing the optimality of these decisions. In particular, one can test for the presence of transitivity: if an observer prefers task A to task B and task B to task C, then the observer should choose task A to task C. Human observers, however, have been shown to violate the transitivity principle (Zhang, Morvan, and Maloney 2010).

3.6.3 Explaining suboptimality in between-task comparisons

Why does perceptual sensitivity violate the expected relationship between different tasks? One possibility is that observers face certain capacity limits in one but not the other task. For example, compared to 1-stimulus tasks, the more complex 2-stimulus task requires the simultaneous processing of two stimuli. If limited resources hamper the processing of the two stimuli, then sensitivity in that task would fall short of what is predicted based on the single-stimulus task. One possible limitation on the processing of the two stimuli comes from the idea of "correlated noise" (Klein 1985), according to which neural processing of each stimulus in 2-stimulus tasks is correlated and thus not independent. Nevertheless, in some experiments observers performed worse than expected on the detection, rather than on the 2-stimulus task. A possible explanation of this effect is the presence of a larger "criterion jitter" in the detection task, i.e., a larger variability in the decision criterion from trial to trial (see **Figure 2**). Since 2-stimulus tasks involve the comparison of two stimuli on each trial, these tasks are less susceptible to criterion jitter. Such criterion variability, which could stem from sequential

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dependencies or even random criterion fluctuations, decreases the estimated sensitivity (Mueller and Weidemann 2008).

3.7 Cue combination

Studies of cue combination have been fundamental to the view that sensory perception is optimal (Trommershäuser, Körding, and Landy 2011). Cue combination (also called cue integration) is needed whenever different sensory features provide separate pieces of information about a single physical quantity. For example, auditory and visual signals can separately inform about the location of an object. Each cue provides imperfect information about the physical world, but different cues have different sources of variability. As a result, integrating them can provide a more accurate and reliable estimate of the physical quantity of interest.

Cue combination lends itself to ideal observer modeling and tests of optimality by focusing on the question of whether a perceptual estimate formed from two cues is the best estimate one could make given the estimates available from each cue individually. The optimal estimate is typically taken to be the one that maximizes precision (minimizes variability) across trials, which is called the minimum-variance unbiased estimator. When the variability for each cue is Gaussian and independent of the other cues, the minimum-variance unbiased estimator is a linear combination of the estimates from each cue, weighted by their individual reliabilities (Landy, Banks, and Knill 2011). This is equivalent to maximum likelihood estimation (MLE) and is optimal in the sense that it is the maximum a posteriori estimate when the prior over estimates is uniform. Whether this weighted sum formula holds for a given cue integration task can be readily tested psychophysically, and a large number of studies have done exactly this for different types of cues and tasks (see Ma, 2010 and Trommershauser et al., 2011 for reviews).

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In particular, the optimal mean perceptual estimate (x) after observing cue 1 (with feature estimate x_1 and variance of σ_1^2) and cue 2 (with feature estimate x_2 and variance of σ_2^2) is:

$$x = \frac{x_1}{\sigma_1^2} / (\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}) + \frac{x_2}{\sigma_2^2} / (\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2})$$

and the optimal variance of the perceptual estimate (σ^2) is:

$$\sigma^2 = \frac{\sigma_1^2 * \sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

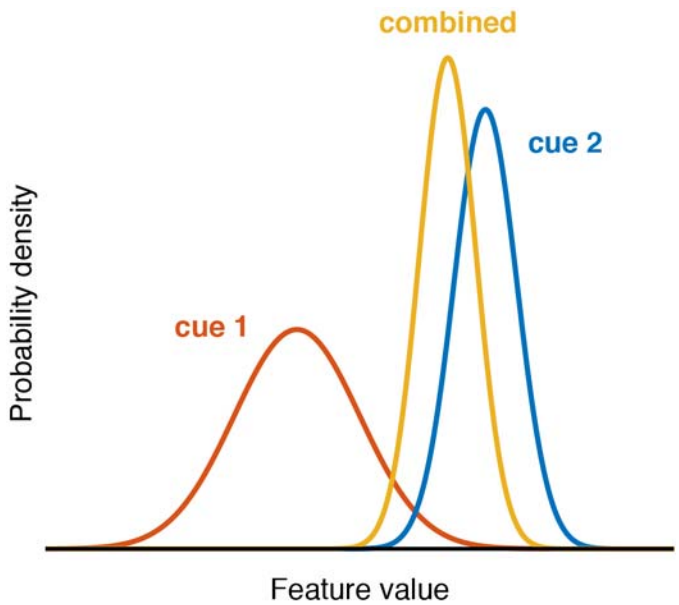

Figure 7*. Optimal cue integration. Two cues that give independent information about the value of a sensory feature (red and blue curves) are combined to form a single estimate of the feature value (green curve). The combined cue distribution is narrower than both individual cue distributions, and its mean is closer to the more certain cue (cue 2).*

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3.7.1 Examples of optimality in cue combination

A classic example of this approach is a study of visual-haptic cue combination by Ernst & Banks (2002). In this study, observers estimated the height of a rectangle that they could 1) only see, 2) only touch, or 3) both see and touch. Performance in the visual-haptic condition was well described by the MLE formula: the single cue measurements predicted both the reliability of visual-haptic estimates and the weights given to the two cue types for these estimates. Many studies have observed similar optimal cue combination behavior in a range of tasks estimating different physical quantities (Trommershäuser et al. 2011). These studies have investigated integration across two modalities (including vision, touch, audition, the vestibular sense, and proprioception; e.g., Alais & Burr, 2004; Ernst & Banks, 2002; Gu, Angelaki, & DeAngelis, 2008; van Beers, Sittig, & Denier van der Gon, 1996) and across two features in the same modality, such as various visual cues to depth (e.g., (Jacobs 1999; Landy et al. 1995). Common among these experiments is that trained observers complete many trials of a psychophysical task, and the two cues provide similar estimates of the quantity of interest. Optimal cue combination has also been observed during sensory-motor integration (Maloney and Zhang 2010; Trommershäuser 2009; Wei and Körding 2011; Yeshurun et al. 2008).

3.7.2 Examples of suboptimality in cue combination

Because optimality is often the hypothesized outcome in cue integration studies, findings of suboptimality may be underreported or underemphasized in the literature (Rosas and Wichmann 2011). Still, a number of studies have demonstrated suboptimal cue combination behavior that violates some part of the MLE formula. These violations fall into two categories: 1) those in which the cues are integrated but are not weighted according to their independently measured reliabilities, and 2) those in which the reliability of estimates from two cues is no better than those from a single cue.

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In the first category are findings from a wide range of combined modalities: visual-auditory (Battaglia, Jacobs, and Aslin 2003; Burr, Banks, and Morrone 2009; Maiworm and Röder 2011), visual-vestibular (Fetsch et al. 2012; Prsa, Gale, and Blanke 2012), visual-haptic (Battaglia, Kersten, and Schrater 2011; Rosas et al. 2005), and visual-visual (Knill and Saunders 2003; Rosas, Wichmann, and Wagemans 2007). For example, auditory and visual cues were not integrated according to the MLE rule in a localization task; instead, observers treated the visual cue as though it were more reliable than it really was (Battaglia et al. 2003). Similarly, visual and haptic depth cues were integrated according to their reliabilities, but observers underweighted the visual texture cue (Rosas et al. 2005). In some of these studies, cue misweighting was restricted to low-reliability cues: In a visual-vestibular heading task, observers overweighted vestibular cues when visual reliability was low (Fetsch et al. 2012), and in a visual-auditory temporal order judgment task, observers overweighted auditory cues when auditory reliability was low (Maiworm and Röder 2011). However, overweighting does not only occur within a limited range of reliabilities (e.g., Battaglia et al., 2003; Prsa et al., 2012).

Several studies have failed to find optimal cue integration in the temporal domain. In an audio-visual rate combination task, observers only partially integrated the auditory and visual cues, and they did not integrate them at all when the rates were very different (Roach, Heron, and McGraw 2006). Observers also overweight auditory cues in temporal order judgment tasks (Maiworm and Röder 2011) and temporal bisection tasks (Burr et al. 2009). It is well-established that when two cues give very different estimates, observers tend to discount one of them (Gepshtein et al. 2005; Jack and Thurlow 1973; Körding et al. 2007; Roach et al. 2006; Warren and Cleaves 1971), an effect which has been called “robust fusion” (Maloney and Landy 1989) and may arise from inferring that the two cues come from separate sources (Körding et al.

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2007). However, in most of these studies, suboptimal cue combination was observed even when the cues gave similar estimates.

In the second category of suboptimal cue combination findings, two cues are no better than one (Chen and Tyler 2015; Drugowitsch, DeAngelis, et al. 2014; Landy and Kojima 2001; Oruç, Maloney, and Landy 2003; Rosas et al. 2005, 2007). (Note that some of these studies found a mix of optimal and suboptimal observers.) Picking the best cue is known as a "veto" type of cue combination (Bülthoff and Mallot 1988) and is considered a case of "strong fusion" (Clark and Yullie 1990; Landy et al. 1995). This is an even more serious violation of optimal cue combination, since it is as though no integration has taken place at all -- the system either picks the best cue or in some cases does *worse* with two cues than with one. Cues may also be mandatorily combined even when doing so is not suitable for the observer's task. For example, texture and disparity information about slant is subsumed in a combined estimate, rendering the single cue estimates unrecoverable (Hillis 2002). Interestingly, the single cue estimates are not lost in children, allowing them to outperform adults when the cues disagree (Nardini, Bedford, and Mareschal 2010). In a related finding, observers used multiple visual features to identify a letter even when the optimal strategy was to use only a single, relevant feature (Saarela and Landy 2015).

3.7.3 Combining stimuli of the same type

So far we have only considered cue combination studies in which the two cues come from different sensory modalities or dimensions. Suboptimal behavior has also been observed when combining cues from the same dimension. For example, Summerfield and colleagues have recently shown that observers do not weight every sample stimulus equally in a decision (Summerfield and Tsetsos 2015). For simultaneous samples, observers underweight "outlier"

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stimuli lying far from the mean of the sample (de Gardelle and Summerfield 2011; Michael, de Gardelle, and Summerfield 2014). For sequential samples, observers overweight stimuli toward the end of the sequence (a recency effect) as well as stimuli that are similar to recently presented items (Cheadle et al. 2014). Observers also use only a subset of a sample of orientations to estimate its mean (Dakin 2001).

3.7.4 Combining sensory and motor cues

Suboptimal cue integration has also been found in sensory-motor tasks. For example, when integrating the path of a pointing movement with online visual feedback, although observers took uncertainty into account, they tended to underestimate the uncertainty indicated by the feedback (Körding and Wolpert 2004). In a pointing task in which observers were rewarded for hitting the correct visual target, observers tended to underweight the difficulty of the motor task (the size of the target), attempting to hit the target consistent with their perceptual decision even when that decision was based on uncertain sensory information (Fleming, Maloney, and Daw 2013). Within the action domain (and so beyond our focus on perception), Maloney and Zhang (2010) have reviewed studies showing both optimal and suboptimal behavior.

3.7.5 Cue combination in children

Optimal cue integration takes time to develop. Children are suboptimal until around 10 years of age when combining multisensory (Gori et al. 2008; Nardini et al. 2008; Petrini et al. 2014) or visual (Dekker et al. 2015; Nardini et al. 2010) cues.

3.7.6 Explaining suboptimal cue combination

Why do observers sometimes combine cues suboptimally? One possible explanation is that people do not have accurate representations of the reliability of the cues. (Knill and Saunders

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2003; Rosas et al. 2005). This might be a particular problem in unnatural tasks. For example, in one task for which cue combination was suboptimal, observers haptically explored a surface with a single finger to estimate its slant. However, observers may have little experience with single-finger slant estimation, since multiple fingers or the whole hand might ordinarily be used for such a task (Rosas et al. 2005). Reliability estimation might also be difficult when the reliability is very low. This possibility may apply to studies in which observers were optimal within a range of sensory reliability, but not outside it (Fetsch et al. 2012; Maiworm and Röder 2011).

Some authors suggest that another reason for over- or under-weighting a certain cue could be prior knowledge about how cues ought to be combined. This could include a prior assumption about how likely a cue is to be related to the desired physical property (Battaglia et al. 2011), how likely two cue types are to correspond to one another (and thus be beneficial to integrate) (Roach et al. 2006), or a general preference to rely on a particular modality, such as audition in a timing task (Maiworm and Röder 2011).

For certain tasks, some researchers question the assumptions of the MLE model, such as Gaussian noise (Burr et al. 2009) or the independence of the neural representations of the two cues (Rosas et al. 2007). Nevertheless, worse performance with two cues than with one is difficult to reconcile with any normative cue integration model in the absence of other constraints.

Sometimes suboptimality in cue combination can depend on task requirements and observers' strategies for performing the task. The importance of these factors was revealed in the finding that observers' accuracy was suboptimal unless reaction time was taken into account

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(Drugowitsch, DeAngelis, et al. 2014) and the demonstration that observers can be optimal in one task but suboptimal in another with the same stimuli (Saarela and Landy 2015). These examples show how assumptions about task demands and the particular task tested can influence conclusions drawn about optimality.

"Robust averaging," or down-weighting of outliers, has been observed when observers must combine multiple pieces of information that give very different perceptual estimates. It has been suggested that these suboptimal behaviors can be explained by adaptive gain changes that result in highest sensitivity to stimuli close to the mean of the sample (or in the sequential case, the subset of the sample that has been presented so far; Summerfield & Tsetsos, 2015). This adaptive gain mechanism is similar to models of sensory adaptation (Barlow 1990; Carandini and Heeger 2012; Wark, Lundstrom, and Fairhall 2007). By following principles of efficient coding that place the largest dynamic range at the center of the sample (Barlow 1961; Brenner, Bialek, and de Ruyter van Steveninck 2000; Wainwright 1999), different stimuli receive unequal weightings, leading to suboptimality. Note that psychophysical studies in which stimulus variability is low would not be expected to show this kind of suboptimality (Cheadle et al. 2014).

It is debated whether suboptimal cue combination in children reflects immature neural mechanisms for integrating cues, or whether the developing brain is optimized for a different task, such as multisensory calibration or conflict detection (Gori et al. 2008; Nardini et al. 2010).

3.8 Other examples of suboptimality

Thus far we have specifically focused on tasks where the optimal behavior can be specified mathematically in a relatively uncontroversial manner. However, the issue of optimality has been discussed in a variety of other contexts. Often these discussions center on cases in which

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perception is not veridical. From the perspective that the visual system should represent the true state of the world, such perception is suboptimal. While veridicality and optimality need not go hand in hand, we still review the relevant research and later use it to discuss some of the difficulties in defining optimality.

3.8.1 Perceptual biases, illusions, and improbabilities

A number of basic visual biases have been documented. Some examples include repulsion of orientation or motion direction estimates away from cardinal directions (**Figure 8A**; Jastrow, 1892; Rauber & Treue, 1998), a bias to perceive speeds as slower than they are when stimuli are low contrast (Stone & Thompson, 1992; Thompson, 1982; but see Thompson, Brooks, & Hammett, 2006), a bias to perceive surfaces as convex (Langer and Bülthoff 2001; Sun and Perona 1997), and a bias to perceive visual stimuli closer to fixation than they are (whereas the opposite is true for auditory stimuli; Odegaard, Wozny, & Shams, 2015).

When biases, context, or other factors lead to something looking dramatically different from its physical reality, we might call it a visual illusion. A classic example is the brightness illusion (**Figure 8B**) in which two squares on a checkerboard appear to be different shades of gray even though they actually have the same luminance (Adelson 1993). Perceptual illusions persist even when the observer knows about the illusion and even after thousands of trials of exposure (Gold et al. 2000).

Some illusions are difficult to reconcile with existing theories of optimal perception. Anderson, O'Vari, & Barth (2011), for example, reported strong percepts of illusory surfaces that were improbable according to optimal frameworks for contour synthesis. In the size-weight illusion,

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smaller objects are perceived as heavier than larger objects of the same weight, even though the prior expectation is that smaller objects are lighter (Brayanov and Smith 2010).

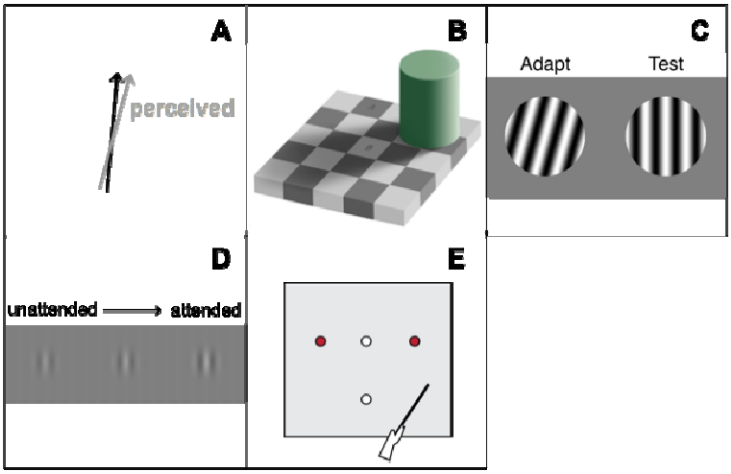

Figure 8*. Examples of adaptation, illusions, and biases. A) Cardinal repulsion. A nearly vertical (or horizontal) line looks more tilted away from the cardinal axis than it is. B) Adelson's checkerboard brightness illusion. Square 2 appears to be brighter than square 1, even though the two squares have the same luminance. Figure courtesy of Michael Bach (http://www.michaelbach.de/ot/lum-adelsonCheckShadow/index.html). C) Tilt aftereffect. After viewing a tilted adapting grating, observers perceive a vertical test grating to be tilted away from the adaptor. D) Effects of spatial attention on contrast appearance* (Carrasco, Ling, and Read 2004)*. An attended grating appears to have higher contrast than the same grating when it is unattended. E) Effects of action affordances on perceptual judgments* (Witt 2011)*. Observers judge an object to be closer (far white circle compared to near white circle) relative to the distance between two landmark objects (red circles) when they are holding a tool that allows them to reach that object than when they have no tool.*

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3.8.2 Adaptation

Adaptation is a widespread phenomenon in sensory systems in which responsiveness to prolonged or repeated stimuli is reduced (Webster 2015). Adaptation can be seen as suboptimal from a Bayesian perspective because subsequent perceptual estimates tend to diverge from rather than conform to the prior stimulus. For example, after prolonged viewing of a line tilted slightly away from vertical, a vertical line looks tilted in the opposite direction (the "tilt aftereffect", **Figure 8C**; Gibson & Radner, 1937). Or, after viewing motion in a certain direction, a stationary stimulus appears to drift in the opposite direction (Wohlgemuth 1911). After adapting to a certain color, perception is biased toward the complementary color (Sabra 1989; Turnbull 1961), and after adapting to a specific face, another face appears more different from that face than it would have otherwise (Webster et al. 2004; Webster and MacLeod 2011). In all of these examples, perception is repelled away from the prior stimulus.

3.8.3 Appearance changes due to visual attention

The same physical stimulus can also be perceived in different ways depending on the state of visual attention. Directing spatial attention to a stimulus can make it appear higher contrast (**Figure 8D**; (Carrasco et al. 2004; Liu, Abrams, and Carrasco 2009; Störmer, Mcdonald, and Hillyard 2009), larger (Anton-Erxleben, Henrich, and Treue 2007), faster (Anton-Erxleben, Herrmann, and Carrasco 2013; Fuller, Park, and Carrasco 2009; Turatto, Vescovi, and Valsecchi 2007), brighter (Tse 2005), and higher spatial frequency (Abrams, Barbot, and Carrasco 2010; Gobell and Carrasco 2005) than it would otherwise. Often attentional effects improve performance on a visual task, but sometimes they make performance worse (Ling and Carrasco 2006; Yeshurun and Carrasco 1998).

3.8.4 Cognition-based biases

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Other studies have documented visual biases associated with more cognitive factors, including action affordances (Witt 2011), motivation (Balcetis 2015), and language (Lupyan 2012). For example, when people will be reaching for an object with a tool that allows them to reach further, they report the object as looking closer than when they will be reaching without the tool (**Figure 8E**; Witt, Proffitt, & Epstein, 2005). In the linguistic domain, calling an object a "triangle" leads observers to report the object as having more equal sides than when the object is called "three-sided" (Lupyan 2016). How much these more cognitive factors affect perception *per se*, as opposed to post-perceptual judgments, and to what extent the observed visual biases are mediated by attention, remain controversial questions (Firestone and Scholl 2015).

3.8.5 Explaining these other examples of suboptimality

How can one explain these findings that are often seen as examples of suboptimality? Some biases and illusions have been explained as arising from priors in the visual system. Misperceptions of motion direction (Weiss et al. 2002) and biases in reporting the speed of low contrast stimuli (Stocker and Simoncelli 2006a; Thompson 1982; Vintch and Gardner 2014) have been explained as optimal percepts for a visual system with a prior for slow motion (Stocker and Simoncelli 2006a; Weiss et al. 2002), because natural objects tend to be still or move slowly. The suggestion is that these priors have been made stable over a lifetime and influence perception even when they do not apply (i.e., in a laboratory task).

Optimal decoding of sensory representations in one task can be accompanied by suboptimal biases in another task using the same stimuli. For example, optimal computation in a motion direction discrimination task should lead to suboptimal perceptual biases in an estimation task -- which is exactly what was observed when these two tasks were interleaved in a single experiment (Jazayeri and Movshon 2007). Another interpretation of these results is that when

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observers make a discrimination decision, they throw away sensory information related to the rejected decision outcome, which biases subsequent estimation (Stocker and Simoncelli 2008).

Various efforts have been made to reconcile adaptation with Bayesian optimality (Grzywacz and Balboa 2002; Hohwy, Roepstorff, and Friston 2008; Schwiedrzik et al. 2014; Snyder et al. 2015). One of the most well-developed lines of work explains the repulsive effects of adaptation as a consequence of efficient coding (Stocker and Simoncelli 2006b). In this framework, a sensory system adapts to maximize its dynamic range around the value of previous input. This change in coding does not affect the prior (as might be expected in a Bayesian treatment of adaptation) but rather affects the likelihood function that relates the system's measurement of the stimulus to the true stimulus value. Specifically, it skews likelihoods away from the adapted stimulus, giving rise to repulsive aftereffects. A similar principle has been suggested to explain why perceptual estimates are repelled from long-term priors, such as those determined by the statistics of natural images (Wei and Stocker 2012, 2015).

4. COMMON CAUSES OF SUBOPTIMALITY

We have so far described a number of examples of suboptimal behavior. Now we discuss several common causes of suboptimality that emerge from the findings we reviewed. These include general perceptual principles as well as methodological considerations that could account for some findings of suboptimal behavior.

4.1 Perceptual principles associated with suboptimality

4.1.1 Probability transformation

In order to optimally incorporate prior information into their perceptual decisions, observers need to be able to accurately represent this information. However, it appears that they can't always

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do this. Researchers commonly find S-shaped distortions of probability and frequency such that small values are overestimated while large values are underestimated (Brooke and MacRae 1977; Varey, Mellers, and Birnbaum 1990). Zhang and Maloney (2012) argued for the presence of "ubiquitous log odds" that systematically distort people's judgments.

4.1.2 Wrong priors

Observers have many priors that help them behave in their natural environment. If these priors are not flexibly changed in a new environment, such as a laboratory experiment, the behavior in that environment will be suboptimal (Brainard et al. 2006; Girshick, Landy, and Simoncelli 2011; Glennerster et al. 2006). Priors that are wrong for a given task can lead to visual illusions, as discussed in Section 3.8.1. Priors acquired during an experiment can also lead to suboptimal behavior when observers base their priors only on the last few trials, rather than the full stimulus history (Raviv, Ahissar, and Loewenstein 2012). The question of how different kinds of priors are learned is still relatively open (Seriès and Seitz 2013).

4.1.3 Hard-to-learn priors

Some priors are inherently complex. Even though most research uses or assumes Gaussian priors, some studies have asked observers to use priors from more complex distributions, such as bimodal functions. Observers are usually suboptimal with such priors (Acerbi, Wolpert, and Vijayakumar 2012; Gekas et al. 2013). Acerbi et al. (2014) tested observers' optimality when the priors are explicitly represented on each trial and found no difference in the degree of suboptimality for Gaussian and bimodal distributions. Thus, suboptimal implementation of complex priors appears to be due to inability to acquire and maintain the prior rather than in the ability to use it correctly. In some cases the priors are hard or impossible to calculate even for the experimenter let alone for the observer (Beck et al. 2012; Geisler and Najemnik 2013).

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4.1.4 Wrong likelihoods

If an observer does not have an accurate representation of his or her own likelihood function, it can lead to under- or over-weighting the sensory evidence relative to the prior. Inaccurate likelihoods could explain suboptimal behavior in some cue combination tasks (as discussed in Section 3.7.2). Observers can also fail to optimally use explicitly presented likelihood information (Körding and Wolpert 2004), as described in Section 3.7.4.

4.1.5 Non-random guesses

It is possible that when the difference in the posterior probability between two options is small, observers "guess" between the two. If such guesses were not random (e.g., an observer is more likely to choose one stimulus category over another when uncertain), then this would lead to suboptimal performance. As discussed in Section 3.1, such guesses are one possible reason for biased criteria in 2-choice tasks. Many descriptive models of behavior include a separate parameter to account for guessing (Acerbi et al. 2014; García-Pérez and Alcalá-Quintana 2010; Whiteley and Sahani 2008; Wichmann and Hill 2001).

4.1.6 Memory limitations

Most perceptual tasks do not have high memory demands and thus memory limitations are not regularly considered. However, many tasks involve non-trivial memory requirements. For example, in temporal 2-stimulus tasks (that is, 2IFC tasks), the first target may not be remembered perfectly while processing the second one. Similarly, in cases when observers should maintain separate criteria for different classes of stimuli, these criteria may attract each other (Gorea and Sagi 2000, 2001, 2002; Morales et al. 2015; Rahnev, Maniscalco, et al. 2011; Solovey et al. 2015). Finally, in tasks that involve confidence ratings, observers need to

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maintain the location of several confidence criteria, which may again create larger memory demands. Phenomena like the "contraction bias", in which size comparisons of two successive stimuli depend on how large they are relative to the sample, have been explained as a stronger influence of a prior on the first of two successive stimuli due to a noisier representation of that stimulus in memory (Ashourian and Loewenstein 2011).

4.1.7 Suboptimal confidence ratings

As discussed in Section 3.5, it is likely that subjective confidence ratings are based on partially different information than the perceptual decisions themselves (Del Cul et al. 2009; Jolij and Lamme 2005; Lau and Rosenthal 2011; Maniscalco and Lau 2010; Pleskac and Busemeyer 2010; Weiskrantz 1996), leading to an imperfect relationship between these two quantities.

4.1.8 Suboptimal biases

The above principles postulate various hard limitations that could lead to suboptimal behavior. In other words, these limitations may be (nearly) insurmountable for observers despite their best efforts. However, it is likely that in some situations observers are suboptimal despite a potentially intact ability for optimality. For example, Stocker & Simoncelli (2008) suggest the presence of *self-consistency bias*: once observers decide on a model for how sensory input was generated, they no longer consider alternative models. Stocker and Simoncelli show how this bias can account for suboptimality in a second perceptual decision since observers may only consider scenarios consistent with their first decision. Similarly, Fleming, Maloney, & Daw (2013) propose that in certain cases observers may make a categorical commitment to one or other state of the world and discard probabilistic information about the likelihood of each state. A similar selection of evidence is seen in the "positive evidence bias" discussed in Section 3.5.4.

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4.1.9 Efficient coding

Influential theories of brain function, such as the *efficient coding hypothesis* (Barlow 1961; Simoncelli 2003; Summerfield and Tsetsos 2015), posit that the brain is optimized for efficient rather than optimal coding. Efficient coding is a strategy to use a limited number of neurons to encode a large stimulus space. Suboptimal behavior arising from adaptation, a pervasive feature of sensory processing, has begun to be explained in this framework, as discussed in Section 3.8.2. A related mechanism - neural normalization (Reynolds and Heeger 2009) - also postulates that sensory information is transformed in a way that leads to partial loss of information, such as magnitude information. All strategies that involve partial information loss necessarily lead to suboptimal behavior on tasks that require the lost original information. If the processing constraints that lead to efficient coding are taken into account, optimal neural coding and behavioral performance can be specified, conditional on these constraints (Ganguli and Simoncelli 2014).

4.2 Methodological considerations that can account for suboptimal behavior

4.2.1 Experimenters making wrong distributional assumptions

Many of the examples of suboptimality above have relied on the assumption that the likelihood function $P(r|s)$ has a Gaussian distribution. This is a reasonable assumption in many common cases (Macmillan and Creelman 2005; See et al. 1997; Swets 1986) but, if incorrect, can make otherwise optimal behavior appear suboptimal (Healy and Kubovy 1981; Kubovy 1977; Maloney and Thomas 1991). Nevertheless, inferences about suboptimality related to, among others, sequential effects, speed-accuracy tradeoffs, confidence-accuracy dissociations, perceptual illusions, and perception for action do not rely on distributional assumptions. On the flip side, most demonstrations of optimal behavior also rely on assumptions of Gaussian variability.

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4.2.2 Lack of motivation to be optimal

One reason noted by many researchers as to why observers may not adjust their criterion in studies with unequal rewards is a desire to be accurate that competes with the desire for maximal reward (the optimal strategy). For example, Maddox & Bohil (1998a) posited the COmpetition Between Reward and Accuracy maximization (COBRA) hypothesis according to which observers attempt to maximize reward but also place a premium on accuracy (Maddox and Bohil 2004, 2005). Placing a premium on accuracy in addition to reward can also be an issue in studies on the speed-accuracy tradeoff. Similarly, the overly conservative criterion in some detection studies could be due to observers not wanting to make false alarms, which can be seen as lying.

4.2.3 Insufficient training with the task

A potential explanation for suboptimal behavior is that observers did not receive sufficient training with the task. Indeed, trial-by-trial feedback does make behavior more optimal in the majority of cases (e.g., Baranski & Petrusic, 1994; Maddox & Bohil, 2005), so it is tempting to blame insufficient training for suboptimal behavior. Nevertheless, a large number of studies have shown that trial-by-trial feedback does not make observers more optimal in a number of specific tasks (Björkman et al. 1993; Keren 1988; Maddox and Bohil 2001; Winman and Juslin 1993). Further, some studies featured continuous trial-by-trial feedback for months and still found suboptimal behavior (e.g., Feng et al., 2009). Thus, even though meaningful feedback appears necessary for proper learning, in many cases it is not sufficient for observers to achieve optimality.

4.2.4 Task specification

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An otherwise optimal observer may appear suboptimal if her assumptions about the task do not match the assumptions of the experimenter or the analysis. For instance, many experiments use a range of different stimulus intensities and then analyze each intensity separately as if it were the only one presented. A potential problem in this approach is that observers develop a prior over all intensities presented. Drugowitsch et al. (2014) showed how this design flaw may exacerbate the hard-easy effect in which observers are overconfident in hard tasks and underconfident in easy tasks. Similarly, observers may change their beliefs about a task and corresponding response criteria due to random variability during the experiment -- for example, inferring that the reliability of a predictive cue has decreased after several invalid trials. These scenarios highlight the notion that great care needs to be taken that observers' assumptions in performing a task match exactly the assumptions implicit in the analysis.

5. OPTIMALITY AND APPROACHES TO UNDERSTANDING THE PERCEPTUAL SYSTEM

5.1 Overarching theory vs. bag of tricks

One attraction of the idea that human perception is optimal is the promise that this approach can lead to an overarching theory of brain processing (Griffiths et al. 2012). Such a theory would allow us to describe diverse perceptual phenomena under the same theoretical framework. Nevertheless, whether an overarching theory of brain function can be constructed is a purely empirical question. The long list of examples of suboptimality in this review should be a cautionary tale about the plausibility of an overarching theory based on optimality alone.

Another extreme view is to see the brain as a "bag of tricks" (Ramachandran 1990). In this view, the brain has adopted a new trick to deal with each new environmental demand over the course of evolution, thus turning itself into a collection of heuristic solutions. Compiling a list of these solutions, no matter how extensive, will likely never add up to a coherent theory.

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5.2 The Objectives-Constraints-Mechanisms (OCM) approach

We suggest that the way forward is to find middle ground between these extreme positions. In particular, we can retain an overarching theoretical *framework*, without ignoring the messiness of human behavior.

We advocate for an Objectives-Constraints-Mechanisms (OCM) approach to understanding the perceptual systems of the brain. In our view, the perceptual system is trying to solve a loosely-defined optimization problem: how to maximize the value of its representations, while minimizing the cost of the resources expended. Therefore, it is critical to specify what types of representations are beneficial (i.e., define the objectives of the system) and what kinds of costs are associated with them (i.e., define the constraints of the system). Critically, these benefits and costs need to be optimized on several, possibly competing, timescales: from the immediate situation to the lifetime of the organism and even the species. Specific examples of each component are listed in Box 1.

5.2.1 Objectives

The first step in the OCM approach is to specify explicitly the objectives of the perceptual system. It is natural to assume that the objective of the system is simply to represent the world as it is. However, it is much more likely that the perceptual system's objective is to produce "useful" rather than veridical representations of the world (Gigerenzer and Brighton 2009; Juslin, Nilsson, and Winman 2009; Simon 1956). Specifying what "useful" means is one of the great challenges that need to be addressed in order to understand brain function.

5.2.2 Constraints

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The flip side of the positive value achieved by satisfying its objectives is that the perceptual system necessarily incurs costs. These costs need to be specified and quantified in order to understand the optimization problem that the brain needs to solve. A mechanism that falls short of fully achieving the brain's objectives but saves considerably on costs is likely to be selected over a mechanism that produces optimal representations at a great cost (Beck et al. 2012; Bowers and Davis 2012a).

5.2.3 Mechanisms

Finally, there could be a number of ways for the perceptual system to maximize its value (i.e., satisfy its objectives) while minimizing its costs (i.e., remain within its constraints). Thus the final step in the OCM approach is to specify the mechanisms that the perceptual system has selected for its computations. Specifying the computational mechanisms used by the brain is at the heart of understanding perception.

BOX 1

Here we compile a short (non-exhaustive) list of objectives, constraints, and well-established mechanisms used by the visual system, to illustrate these concepts.

Objectives

1. Faithful representations

The perceptual system needs to be able to represent the outside world faithfully. Expected objects may be given extra weight but completely new and unexpected objects should still be represented with fidelity.

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2. Relevant representations

Objects that are irrelevant to the current goals of the organism do not need to be represented with high fidelity. On the other hand, highly relevant objects (e.g., faces) should receive more resources.

3. Fast representations

In a dynamic world, accurate representations are of little value unless they are also fast. It is likely that the perceptual system is wired for quick decision even in situations when more time is available.

Constraints

1. Metabolic energy

One cost incurred by the perceptual system is the use of metabolic energy (Carrasco 2011; Lennie 2003). Mechanisms that save metabolic energy may be preferred even if they are suboptimal.

2. Cortical territory

The number of cortical neurons, the number of connections between them, the number of dendrites and axonal length of each neuron, etc. are structurally constrained (Franconeri, Alvarez, and Cavanagh 2013).

3. Physical variability (“noise”)

Several physical and biological factors lead to variability in the properties of neurons, circuits, and their responses. In addition, neurons die, neurotransmitter levels vary, and nutrients

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fluctuate. The perceptual system needs to build representations that are robust to such factors but is inherently limited by the physical hardware available.

4. Computing with neurons

Brain computations are carried by neurons and thus inherit all of neurons' limitations such as relatively slow information transfer, limits on firing rate and frequency, etc.

Mechanisms

The brain implements a number of mechanisms that balance objectives and constraints by prioritizing some information and discarding other information.

1. Higher-level mechanisms

At this relatively more conceptual level, the brain employs mechanisms like spatial and feature-based attention, memory and learning, predictive coding, and efficient coding to prioritize, process efficiently, and retain information.

2. Lower-level mechanisms

At this relatively more computational level, the brain uses mechanisms like normalization (Reynolds and Heeger 2009), adaptation, summary statistics (Alvarez 2011), and biased competition (Desimone and Duncan 1995) to implement the mechanisms from the higher, more conceptual level.

6. IMPLICATIONS OF THE OBJECTIVES-CONSTRAINTS-MECHANISMS APPROACH FOR OPTIMALITY

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The Objectives-Constraints-Mechanisms approach clarifies that optimality is only meaningfully defined *conditional* on objectives and constraints. On the other hand, most research in the field, including the studies reviewed above, tacitly assume a single objective (performing the task at hand) and do not generally incorporate constraints, with the exception of some kind of noise. Thus, while the brain is optimizing performance in a multidimensional space of objectives and constraints, most research attempts to evaluate the performance based on a thin slice of this space. Any specific finding of optimality or suboptimality therefore provides little to no information about the optimality of the system *as a whole*.

Consider the visual system's strategy of creating relative rather than absolute representations of luminance. Such representations can be achieved through normalization (Reynolds and Heeger 2009), efficient coding (Barlow 1961; Simoncelli 2003; Summerfield and Tsetsos 2015), etc. This is a reasonable strategy: the absolute luminance is almost never needed, so the brain discards it, presumably to conserve resources. However, throwing away absolute luminance leads to poor performance whenever this information is actually needed, as in the checkerboard illusion. Indeed, illusions are often framed as a suboptimality that arises because of a more general optimality in the perceptual system. Some have criticized such explanations as "just so stories", able to claim, post-hoc, that any suboptimality is in fact optimal (Bowers and Davis 2012a). In our view, there is a powerful duality between suboptimality and optimality in a Bayesian framework: any prior is, in effect, a bias, which under some circumstances must be suboptimal.

These considerations should lead us away from making any statements that could imply that the perceptual system as a whole is optimal ("the brain is optimal", "people are optimal", "perception

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is optimal", etc.) or suboptimal. Instead, we should emphasize the objectives, constraints, and mechanisms implied by experimental findings of optimality or suboptimality in specific tasks.

The Objectives-Constraints-Mechanisms approach is like previous approaches such as "bounded rationality" (Gigerenzer and Selten 2002; Simon 1957) and "computational rationality" (Gershman et al. 2015) in that it aims to uncover the constraints within the system. However, our approach differs in asserting that an observer's objectives may differ from the experimenter's and that a goal of perceptual science should be to uncover these objectives. The OCM approach is also deliberately agnostic about whether the computational mechanisms underlying behavior are "rational," that is, compatible with Bayesian principles.

7. RECOMMENDATIONS FOR FUTURE STUDIES

7.1 Optimal models as starting points

It is reasonable to assume that people want to perform as well as possible on any task. Thus, an optimal model that incorporates relevant constraints will often be a good starting point in modeling behavior. Conversely, assuming a strong heuristic will often fail to describe human behavior across a variety of tasks. For example, Ma, Shen, Dziugaite, & van den Berg (2015) recently demonstrated that optimal models are as good or better than the max rule (a rule in which the observer only considers the maximum of N inputs) across a range of different tasks. Nevertheless, the countless examples of suboptimality in human perception should serve as a warning that pure optimal models, especially ones that do not take into consideration the objectives, constraints, and mechanisms of the visual system, will often be most useful as starting points rather than as the ultimate description of human behavior.

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According to some Bayesian theorists, models that assume optimal behavior are intrinsically preferable to models that do not (e.g., Drugowitsch & Pouget, 2012). The argument is usually that because people can approximate optimal behavior on some tasks, they must possess the machinery for fully optimal decisions. However, close-to-optimal behavior can also be produced by non-optimal models (Bowers and Davis 2012a; Shen and Ma 2016). On the other hand, the ubiquity of suboptimal perceptual decisions argues against such a strong Bayesian view. It is generally agreed that evolutionary pressures likely produced heuristic but useful, rather than normative, behavior (Gigerenzer and Brighton 2009; Juslin et al. 2009; Simon 1956). Thus, when comparing specific implementational models, optimal ones should not be given more or less weight *a priori*.

7.2 How (not) to demonstrate optimality

Many phenomena in visual perception have been explained as optimal inference. Often these explanations necessitate postulating additional parameters for attentional lapses (Whiteley and Sahani 2008), observers' prior beliefs (Stocker and Simoncelli 2006a), etc. While these extra parameters are usually well justified and are needed to incorporate relevant constraints, the fact that they are treated as free parameters, rather than being estimated separately, can easily lead to overfitting such that an optimal model appears to fit the data better than it should (Bowers and Davis 2012a). The ability of an optimal model containing free parameters to fit the data should not be taken as a *demonstration* that human behavior is indeed optimal; independent tests of the new model should be performed.

Box 2

Recommendations for discussing optimality in experimental research

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1. Describe assumptions about what information the observer is using to perform the task (e.g., stimulus properties, training, experimenter's instructions, feedback, explicit vs. implicit rewards, response time pressure, etc.).
2. Specify the objectives, constraints, and/or mechanisms (OCM) being tested.
3. State predictions for optimal performance conditioned on the task assumptions and hypothesized OCM.
4. Interpret data with respect to the OCM hypotheses, not optimality per se.
5. Discuss any task assumptions that may be incorrect and how different assumptions might affect the interpretation of the results.

8. CONCLUSION

As this review demonstrates, suboptimal behavior is the rule rather than the exception in perception. Suboptimal behavior appears in practically every task in which optimality can be mathematically defined.

In our view, this widespread suboptimality, together with the difficulties in even defining what optimal behavior is outside of constrained laboratory tasks, should steer the field away from arguing that observers are optimal. To be clear, we are not against testing the optimality of performance in well-defined tasks -- such studies are critical in uncovering the principles that govern the perceptual system. Rather, we advocate a shift of emphasis in the framing and discussion of such studies. Psychology has a long history of using ideal observer models to investigate such questions, which we believe has been somewhat obscured in recent years by an increased emphasis on demonstrating optimality. In the framework of our Objectives-Constraints-Mechanisms (OCM) approach, findings should be interpreted as advancing our

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understanding of one or more of these three components, rather than the presence of optimality per se.

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