

Gene-culture co-inheritance of a behavioral trait

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Abstract

Human behavioral traits are complex phenotypes that result from both genetic and cultural transmission. But different inheritance systems need not favor the same phenotypic outcome. What happens when there are conflicting selection forces in the two domains? To address this question, we derive a Price equation that incorporates both cultural and genetic inheritance of a phenotype where the effects of genes and culture are additive. We then use this equation to investigate whether a genetically maladaptive phenotype can evolve under dual transmission. We examine the special case of altruism using an illustrative model, and show that cultural selection can overcome genetic selection when the variance in culture is sufficiently high with respect to genes. Finally, we show how our basic result can be extended to non-additive effects models. We discuss the implications of our results for understanding the evolution of maladaptive behaviors.

1 Introduction

Behavioral traits are among the most complex phenotypes under study in evolutionary biology. At the heart of that complexity is the interaction between genetic inheritance and the environment (Turkheimer, 2000). In organisms with social learning, a significant component of the environment can be conspecific individuals who will serve as models for socially learned behaviors, leading to the emergence of cultural transmission. Thus, in organisms with significant social learning, some behaviors will be determined by both genetic and nongenetic inheritance systems. Examples of behaviors that are influenced by both genetic and cultural transmission span a wide range, such as antisocial behavior (Maes et al., 2007), parental behavior (Pérusse et al., 1994), handedness (Laland, 2008), fertil-

12 ity (Alvergne et al., 2011; Colleran and Mace, 2015; Colleran, 2016; Kosova et al.,
13 2010), and Alzheimer's disease biomarkers (Levy et al., 2016) in humans, as well
14 as song form in passerine birds Feher et al. (2009); Freeberg (2000), mate choice
15 in Trinidadian guppies (Dugatkin, 1992), and foraging behavior in bottlenose dol-
16 phins (Krützen et al., 2005). In each of these cases, an important fitness-related
17 trait or behavior is determined not only by genetic inheritance, but also cultural
18 transmission from individuals that may not have contributed any genetic material.
19 Future investigations are likely to reveal even more examples across behavioral do-
20 mains and taxa. Given the likely importance of genetic and nongenetic inheritance
21 in determining so many behavioral traits, it is imperative to develop a better theo-
22 retical understanding of how such co-inheritance affects the evolution of behavioral
23 traits.

24 In recent years, evolutionary theorists have begun to investigate the consequences
25 of multiple inheritance systems (Otto et al., 1995; Day and Bonduriansky, 2011; Bon-
26 duriansky and Day, 2009). In a pair of papers, Day & Bondurianski used the Price
27 equation to construct a general framework for modeling genetic and nongenetic
28 traits that jointly determined phenotype, though their method only kept track of
29 the reproductive fitness consequences of both systems of inheritance. While this
30 approach gives a mathematically valid description of evolutionary change in a trait,
31 it obscures the separate roles of genetic and cultural inheritance and selection in
32 causing that change. To account for these separate causal roles, one needs to con-
33 sider fitness measures in both systems of inheritance. A fitness measure implies
34 a mapping from ancestral to descendant individuals; ancestors who map to more

35 descendants have higher fitness. Multiple inheritance systems mean the possibility
36 of multiple mappings, as illustrated with the following example. Imagine a pop-
37 ulation of asexual organisms (as in Figure 1) with a phenotype p determined by
38 genetic and cultural inheritance. Let p_a be the phenotype of an ancestor and p_d the
39 phenotype of her genetic descendant. If both genes and culture are inherited from
40 the same ancestor, and we assume no flaws in transmission, then $p_a = p_d$. How-
41 ever, if one's genetic parent and cultural role model are not the same individual,
42 then it is possible that $p_a \neq p_d$. If we consider the mapping solely from genetic
43 parents to offspring, this discrepancy will appear simply to be an unexplained de-
44 viation between parents and offspring. However, we might also keep track of the
45 mapping between cultural role models and pupils. Under this mapping, we might
46 find that certain individuals map to more cultural descendants as a result of their
47 phenotype due to selection in the cultural domain. Thus what appears under one
48 mapping to be an unexplained deviation between parent and offspring is revealed
49 under another mapping to be a force of selection in its own right.

50 The argument above underscores the importance of considering fitness in each
51 domain of inheritance when multiple forms of inheritance are present. We are
52 not the first authors to highlight this point. Nearly forty years ago, Richerson &
53 Boyd (1978) remarked that when both genes and culture determine a single phe-
54 notype, the value of the phenotype that maximizes genetic fitness may differ from
55 the value that maximizes cultural fitness, leading to conflicts between the two in-
56 heritance systems. Using a weak selection model they showed that under certain
57 conditions, the cultural fitness optimum could be reached at the expense of genetic

58 fitness. In the ensuing decades, cultural evolution theory has largely focused on the
59 case when the genetic trait encodes a learning rule that determines how a cultural
60 trait is acquired (Boyd and Richerson, 1988; Cavalli-Sforza and Feldman, 1981; Boyd
61 et al., 2003; Guzmán et al., 2007; Lehmann et al., 2008). By contrast, the problem of
62 conflict between inheritance systems that affect the same trait has received surpris-
63 ingly little attention, with the notable exception of the model of Findlay (Findlay,
64 1992), which only treated vertical cultural transmission. Indeed, some recent pa-
65 pers (El Mouden et al., 2014; Morin, 2014) claim that such conflicts will always be
66 resolved in favor of reproductive fitness.

67 In this paper, we present a general approach to the evolution of a co-inherited
68 trait by deriving a Price equation that explicitly incorporates both forms of inheri-
69 tance. The Price equation is an exact description of an evolutionary process under a
70 certain set of minimal assumptions (Price et al., 1970; Frank, 1998; Rice, 2004). Soon
71 after its introduction, Hamilton (1975) pointed out that the Price equation can apply
72 equally well to cultural transmission, and recent authors have developed it exclu-
73 sively for that purpose (Henrich, 2004a; El Mouden et al., 2014). Others have also
74 extended the Price equation to include multiple forms of inheritance (Day and Bon-
75 duriansky, 2011; Helanterä and Uller, 2010), though with the limitation of a single
76 fitness measure. Here, we use a simple additive model to derive a Price equation
77 that incorporates both domains of inheritance and their relevant fitness measures
78 directly. We then analyze the condition for the evolution of a phenotype when
79 selection in the two domains is in conflict. We take altruistic behaviors as a spe-
80 cial case and present a series of illustrative models to explore the implications of

81 our results, including assortative interactions and gene-culture correlations. Our
82 model elucidates the conditions under which selection in one domain can over-
83 come counter-selection in the other domain. We then extend our Price equation
84 framework to more complicated models. We end with a discussion of the implica-
85 tions of our results for understanding the evolution of reproductively maladaptive
86 behaviors.

87 **2 Gene-Culture Price equation**

88 We model the evolution of a trait that results from both genetic and cultural inheri-
89 tance. Evolution here means the change in the phenotypes in a population, not only
90 the change in the genetic or culturally inherited information that underlies them.
91 An individual's phenotype is represented by a continuous variable, p . We can take
92 this to represent a behavioral trait, such as one of the big five personality traits (e.g.
93 extraversion, agreeableness, conscientiousness, etc.) (Goldberg, 1993). We assume
94 that the effects of genetic and cultural inheritance are additive, i.e., we express an
95 individual's phenotype as the following:

$$p_j = c_j + g_j + e. \quad (1)$$

96 The final term, e , is the effect of the environment that does not include cultural
97 transmission (i.e. is not heritable). The two terms, c_j and g_j will be referred to as
98 the culture-type and genotype, respectively. These terms only describe the state
99 of the continuous variables, and are not meant to imply any particular mode of

100 inheritance (e.g. haploidy, diploidy, etc.). Equation (1) is similar to the quantitative
101 genetic formulation in Otto et al. (Otto et al., 1995). The culture- and geno-types
102 are determined by the corresponding values in j 's genetic and cultural ancestors.
103 We assume that a descendant's culture-type and genotype are linear functions of
104 her ancestors' values given by

$$g_j = \sum_{i=1}^N \nu_{ij} g_i + \Delta g_j \quad (2a)$$

$$c_j = \sum_{i=1}^N \gamma_{ij} c_i + \Delta c_j, \quad (2b)$$

105 where $\nu_{ij}, \gamma_{ij} \in [0, 1]$ and $\sum_{i=1}^N \nu_{ij} = \sum_{i=1}^N \gamma_{ij} = 1$; these values are the weights that
106 describe the degree of influence an ancestor i has on descendant j in the genetic or
107 cultural domain. When i is not a genetic ancestor to j , then $\nu_{ij} = 0$; when i is not a
108 cultural ancestor, $\gamma_{ij} = 0$. By normalizing these weights we have assumed that all
109 individuals have at least one genetic and cultural ancestor. While this assumption
110 is perfectly natural for genetic reproduction, one can imagine traits for which some
111 individuals might receive no cultural input, or more cultural input than genetic.
112 The delta terms, Δg_j and Δc_j , represent departures in j from the inherited genetic
113 and cultural values. As an example, Δg_j may be nonzero in the event of mutation
114 or recombination, while Δc_j may be nonzero due to individual learning or expe-
115 rience. This model generalizes that presented by El Mouden et al. (2014), though
116 our analysis and conclusions differ.

117 Fitness captures the contribution of an ancestor to the next generation. In this
118 model, that contribution, whether genetic or cultural, is determined by the weights

119 given to an ancestor by her descendants (again, as in El Mouden et al. (2014)). Thus,
120 the fitness of an individual in either domain of inheritance is simply the sum of the
121 weights given to an ancestor by all descendants. Specifically, we define the genetic
122 fitness of an ancestor i as $w_i = \sum_{j=1}^{N'} \nu_{ij}$ and the cultural fitness, $s_i = \sum_{j=1}^{N'} \gamma_{ij}$,
123 where the sums are taken over the descendant generation and N' is the number
124 of descendants. For example, for a haploid organism, all ν_{ij} are either 1 or 0, and
125 w_i is simply equal to the number of offspring; in the diploid, sexually reproducing
126 case, $\nu_{ij} = \{0, 1/2\}$. For s_i , the values will range from 0 to a maximum of N' , which
127 occurs when i is the sole cultural ancestor of all descendants in the population. In
128 the cultural domain, the definition of s_i shows that the total amount of influence an
129 ancestor i has on descendant phenotypes is what matters most, not just the number
130 of individuals over which i has had some non-zero influence.

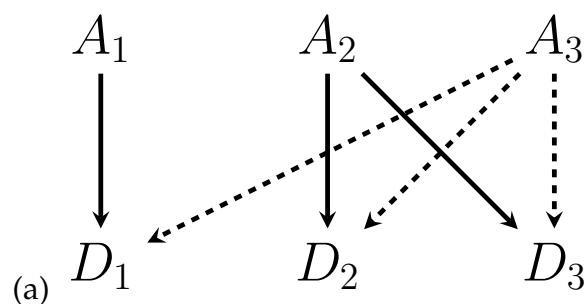
131 Using these definitions and equation 1, we can derive the following Price equa-
132 tion to describe the evolutionary change in the mean value of the phenotype (see
133 SI-1),

$$\Delta \bar{p} = \frac{1}{\bar{w}} \text{cov}(w_i, g_i) + \frac{1}{\bar{s}} \text{cov}(s_i, c_i) + \langle \Delta g_j \rangle + \langle \Delta c_j \rangle. \quad (3)$$

134 Note that angled brackets indicate averages over the descendant population, in-
135 dexed by j . Just as in the standard Price equation, the covariance terms represent
136 the effects of selection and drift (Rice, 2004) on evolutionary change. Importantly,
137 we can separate the effects of differential reproduction, $\frac{1}{\bar{w}} \text{cov}(w_i, g_i)$, and differen-
138 tial influence in cultural transmission, $\frac{1}{\bar{s}} \text{cov}(s_i, c_i)$. Note also that the mean fitness,
139 $\bar{w} = \frac{N'}{N} = \bar{s}$, which is a direct result of the normalization conditions on γ_{ij} and ν_{ij} ,
140 and again implies that everyone receives the same amount of cultural input as ge-

141 netic input, and that cultural descendants must equal the mean number of genetic
142 offspring. The remaining terms are the effects due to spontaneous departure from
143 one's inherited information, such as mutation or recombination in genes, or indi-
144 vidual trial-and-error learning in culture. These terms differ somewhat from the
145 transmission term in the standard Price equation, which is the fitness-weighted av-
146 erage departure of mean offspring phenotype from parental phenotype ($E[w\Delta p]$).
147 This difference results from the fact that we are measuring the differences ($\Delta g_j, \Delta c_j$)
148 between an individual offspring's type and its ancestral contribution.

149 The standard Price equation uses a single fitness measure and provides a math-
150 ematically valid description of evolutionary change. Why then do we need equa-
151 tion (3) that keeps track of two kinds of fitness? To see why, consider the popu-
152 lation in Figure 1, which depicts an asexually reproducing population where de-
153 scendants receive both genes and culture from ancestors, though not necessarily
154 the same ancestors. The solid arrows indicate parent-offspring relationships, and
155 the dashed arrows social learning relationships. Using only reproductive fitness
156 (solid arrows) we could capture the evolutionary change with the standard Price
157 equation: $\Delta \bar{p} = cov(w_i, p_i) + E[w\Delta p] = -1/3 + 1 = 2/3$. This expression allows
158 us to see the effect of natural selection acting against the phenotype, but leads us
159 to conclude that the transmission term, for reasons that are obscure, more than
160 compensates for natural selection. Thus it appears that natural selection has been
161 overtaken by a faulty inheritance system. However, computing the terms in equa-
162 tion (3), we have $cov(w_i, g_i) = 0$, $cov(s_i, c_i) = 2/3$, $E[\Delta g] = 0$, $E[\Delta c] = 0$, and so
163 $\Delta \bar{p} = 0 + 2/3 + 0 + 0 = 2/3$. Considering both genetic and cultural mappings,



	g	c	Phenotypes	Reproductive fitness	Cultural fitness
A_1	1	0	$p_{a_1} = 1$	$w_1 = 1$	$s_1 = 0$
A_2	1	0	$p_{a_2} = 1$	$w_2 = 2$	$s_2 = 0$
A_3	1	1	$p_{a_3} = 2$	$w_3 = 0$	$s_3 = 3$
D_1	1	1	$p_{d_1} = 2$		
D_2	1	1	$p_{d_2} = 2$		
D_3	1	1	$p_{d_3} = 2$		

Figure 1: (a) The diagram shows the hereditary relationships between ancestors (A_1, A_2, A_3) and descendants (D_1, D_2, D_3). The solid line indicates reproductive relationships, while the dashed lines show cultural learning. While A_3 sired no offspring, he is the cultural learning model for all descendants. (b) Genotype, culture-type, phenotypic and fitness values for each ancestor and descendant (excepting fitness values). Each descendant has only one genetic and cultural ancestor, thus each solid edge corresponds to $\nu_{ij} = 1$ and each dashed edge, $\gamma_{ij} = 1$.

164 we see that there is in fact no natural selection on the phenotype in the genetic do-
 165 main, and no flaws in either inheritance system; however, there is positive selection
 166 in the cultural domain that produces evolutionary change. This is a distinctly differ-
 167 ent cause than was revealed by considering only the reproductive fitness mapping.
 168 In summary, if the two modes of inheritance were not explicitly described as in
 169 equation (1), then a departure in phenotype from one's genetic ancestors would in-
 170 clude the effect of cultural inheritance, while a departure in phenotype from one's

171 cultural ancestors would include genetic inheritance. By explicitly accounting for
172 both inheritance mechanisms, our approach avoids confounding their evolutionary
173 effects.

174 We can use equation (3) to examine evolutionary change when there are con-
175 flicts between cultural and genetic selection forces. Is it possible for a trait that is
176 favored by social learning but detrimental to reproductive fitness to evolve? For
177 example, consider a socially acquired preference that leads to decreased reproduc-
178 tion, as in some cultural evolution models of the demographic transition (Ihara and
179 Feldman, 2004; Kolk et al., 2014). Let higher values of p reduce fitness, that is to say,
180 $cov(w_i, p_i) < 0$. Then we have the following condition,

$$cov(s_i, c_i) > -cov(w_i, g_i) - \bar{w} \langle \Delta c_j \rangle, \quad (4)$$

181 where we have ignored the genetic transmission term $\langle \Delta g_j \rangle$ under the assumption
182 that mutation and recombination effects are unbiased with respect to genotypic
183 value. Putting aside for the moment the cultural transmission term, this condi-
184 tion states that the mean value of p can increase—despite reducing reproductive
185 fitness—so long as the covariance between cultural value and influence on descen-
186 dants exceeds the absolute value of the covariance between genotype and repro-
187 ductive fitness. In essence, a loss in reproductive fitness can be compensated for
188 by increased importance as a learning model. However, the cultural transmission
189 term means that this condition will be harder to meet if social learning biases indi-
190 viduals toward lower cultural values than their learning models, for example, as a
191 result of biased learning error (Henrich, 2004b).

192 Intuitively, whether individuals give higher or lower weights to ancestors with
193 higher cultural values should determine the direction of evolution of p . This can be
194 seen by observing that the cultural covariance term can be rewritten as (*see* SI-1)

$$\text{cov}(s_i, c_i) = N' \langle \text{cov}(\gamma_{ij}, c_i) \rangle = N' \langle \beta_{\gamma c}^j \rangle \text{var}(c), \quad (5)$$

195 where again, brackets indicate the mean over the descendant population (index j)
196 and N' is the descendant population size. The regression coefficient, $\beta_{\gamma c}^j$ character-
197 izes the learning rule employed by a descendant j ; formally, it is the regression of
198 the weight that descendant j ascribes to an ancestor on an ancestor's cultural type.
199 When this term is positive, it means that, on average, greater weight is given to an-
200 cestors with higher values of c . We can now rewrite eq. (4) as a new inequality that
201 shows explicitly how strong the bias in favor of higher c must be in order for there
202 to be positive evolutionary change,

$$\langle \beta_{\gamma c}^j \rangle > -\frac{1}{N} \left[\frac{\beta_{wg} \text{var}(g)}{\bar{w} \text{var}(c)} + \frac{\langle \Delta c_j \rangle}{\text{var}(c)} \right] \quad (6)$$

203 Condition (4) gives us the criterion for maladaptive phenotypes with respect to
204 how ancestors' c values translate into cultural fitness. The condition in (6) allows
205 us to see the same condition from the 'descendant's point of view'. The population
206 average of the learning rule employed by descendants determines the direction of
207 evolutionary change. Importantly, we also see that the strength of the genetic se-
208 lection term (first term inside the brackets) is modified by the relative variance in
209 genotypes and culture-types. This is a result of having multiple selection terms in

210 our Price equation. In fact, Hamilton (1975) pointed out a similar effect in his multi-
211 level selection version of the Price equation, where the variances corresponded to
212 individual and group level characters. One important difference is that while group
213 and individual level variances are just different ways of partitioning the population
214 variance (and hence have to add up to the total variance), here we have variances
215 of two different variables whose values are unconstrained by one another.

216 **2.1 Cultural Evolution of Altruism**

217 Hamilton's rule Hamilton (1964a,b) states that an altruistic allele will spread in the
218 population when $rB > C$, where B is the fitness benefit to a recipient of altruism,
219 C is the fitness cost to an altruist, and r measures the assortment between altruists
220 (often interpreted as a relatedness coefficient). Cultural evolution theorists have
221 claimed that altruism is more likely to evolve under cultural evolution, as this re-
222 latedness parameter for culture is likely to be higher than for genes (Fehr and Fis-
223 chbacher, 2003; Boyd and Richerson, 2010; Henrich, 2004a). This claim implies that
224 cultural evolution makes the spread of altruism possible even when the classical
225 form of Hamilton's rule does not hold (El Mouden et al., 2014), i.e. when genetic
226 selection is opposed to altruism. To investigate this claim, the effect of evolutionary
227 forces in the cultural and genetic domains must be compared directly, which has
228 not been done before. In this section, we use our framework to derive the precise
229 conditions under which cultural selection can favor altruism despite being opposed
230 by genetic selection.

231 By altruism we mean here a behavior that reduces the fitness (genetic and/or

232 cultural) of a focal individual while increasing the fitness of others, when the fit-
 233 ness effects of others on the focal individual are ignored (Hamilton, 1964a; Rousset,
 234 2013). We assume that the fitness cost is both genetic and cultural. Let p now rep-
 235 resent the level of altruistic behavior and the cultural and genetic fitnesses be given
 236 by the following equations:

$$s_i = s_0 + \beta_{sp}p_i + \beta_{s\tilde{p}}\tilde{p}_i \quad (7)$$

$$w_i = w_0 + \beta_{wp}p_i + \beta_{w\tilde{p}}\tilde{p}_i \quad (8)$$

237 The tilde over a variable indicates the mean value of that variable across i 's neigh-
 238 bors. We have assumed both kinds of fitness are linear functions of an individu-
 239 als own phenotype and the phenotypes of her neighbors, where s_0 and w_0 are the
 240 baseline fitnesses. As in the standard derivation of Hamilton's rule using the Price
 241 equation, it is customary to identify β_{wp} and $\beta_{w\tilde{p}}$ as the cost (C) to an altruist and
 242 benefit (B) to recipients of altruism, respectively (Frank, 1998; Rice, 2004; McElreath
 243 and Boyd, 2008). We will use the same convention, but add subscripts to indicate
 244 costs and benefits to genetic *and* cultural fitnesses: $\beta_{wp} = -C_g$; $\beta_{cp} = -C_c$; $\beta_{w\tilde{p}} = B_g$;
 245 $\beta_{c\tilde{p}} = B_c$. By labeling these terms, we'll be able to more clearly interpret our key
 246 results. We can derive the following condition (*see* SI-2),

$$B_c(\beta_{\tilde{c}c} + \beta_{\tilde{g}c}) - C_c(1 + \beta_{gc}) > - \left[B_g(\beta_{\tilde{g}g} + \beta_{\tilde{c}g}) - C_g(1 + \beta_{cg}) \right] \frac{\text{var}(g)}{\text{var}(c)}, \quad (9)$$

247 where we've ignored the transmission terms. Written this way, we can see that
 248 the left-hand side is the cultural selection coefficient, and the term in the brackets

249 on the right-hand side is the genetic selection coefficient. One thing that is clear
250 immediately is that the genetic selection coefficient is different than it would be
251 under purely genetic transmission of the phenotype (i.e. $B_g\beta_{\bar{g}g} - C_g$, as follows
252 from the canonical form of Hamilton's rule). This is due to the presence of the
253 additional regression coefficients $\beta_{\bar{c}g}$ and β_{cg} . The same can be said for the cultural
254 selection coefficient, which would be $B_c\beta_{\bar{c}c} - C_c$ under purely cultural transmission
255 (El Mouden et al., 2014).

256 Taking first the LHS, we see three regression coefficients. The first, $\beta_{\bar{c}c}$, is the cul-
257 tural relatedness term, and describes how likely actors are to behave altruistically
258 toward individuals with similar culture-types. The second, $\beta_{\bar{g}c}$, is one of the gene-
259 culture relatedness terms and captures the correlation between an actor's culture-
260 type and neighbor's genotype. Thus, if individuals with higher culture-type val-
261 ues are more likely to direct their altruism towards those with higher genotypic
262 values, the cultural fitness benefit is greater. The final regression coefficient, β_{gc} ,
263 captures the correlation between an actor's culture-type and its genotype. If altru-
264 ism is costly both to cultural and genetic fitness, then a higher correlation between
265 culture-type and genotype will make it even more difficult for altruism to evolve.

266 Now we turn to the genetic selection coefficient, given by the term in brackets
267 on the RHS. The regression coefficient, $\beta_{\bar{c}g}$, is the regression of neighbor culture-
268 type on focal genotype. The term β_{cg} is the regression of focal culture-type and
269 focal genotype. Both of these terms mean that the presence of cultural transmis-
270 sion changes genetic selection on altruism (i.e. genetic selection is no longer given
271 by $B_g\beta_{\bar{g}g} - C_g$), since there are now gene-culture relatedness terms to be taken into

272 account. For example, if individuals with an altruistic genotype are likelier to
273 direct their altruism towards those with an altruistic culture-type then genetic se-
274 lection can favor altruism even with low genetic relatedness (β_{gg}).

275 The inequality states that the cultural selection coefficient must exceed the ge-
276 netic selection coefficient scaled by the ratio of the variance in genotypes to cultural
277 types. Thus, even relatively weak cultural selection can overcome genetic selection
278 if the variance in culture-types is sufficiently high compared to the variance in geno-
279 types. Below we will explore the consequences of (9) using three simple illustrative
280 models.

281 **3 Illustrative models**

282 We imagine a population of haploid individuals interacting assortatively in each
283 generation. These interactions determine the reproductive output of each individ-
284 ual and, potentially, their cultural influence on the next generation. Each individual
285 possesses two loci with a single ‘allele’ at each locus. At the first locus, alleles are
286 transmitted genetically, from a single parent to her offspring; at the other locus, a
287 ‘cultural allele’ is acquired from a single cultural parent. An individual’s pheno-
288 type is determined by the combined additive effect of the alleles at the two loci in
289 the following way: when two individuals interact they play a prisoner’s dilemma;
290 each individual employs a mixed strategy where the phenotype, p , is the proba-
291 bility of playing ‘cooperate’. Those with both the genetic and cultural alleles for
292 altruism play a pure strategy of cooperate; those with only the genetic or cultural

293 allele, play cooperate half of the time; finally, an individual that lacks both the ge-
294 netic and cultural alleles will play a pure strategy of defect. Thus we have four
295 types of individuals in the population $\{0, 0\}, \{0, 1\}, \{1, 0\}, \{1, 1\}$, with phenotypes
296 $p_{00} = 1, p_{01} = p_{10} = 1/2, p_{11} = 1$.

297 An individual of type ψ has an expected reproductive fitness of

$$w_\psi = w_0 + B_g \tilde{p}_\psi - C_g p_\psi \quad (10)$$

298 where w_0 is the baseline fitness, \tilde{p}_ψ is the expected phenotype of a type ψ individ-
299 ual's opponent in the game, and p_ψ is the phenotype of a type ψ individual.

300 Players in the model interact assortatively with respect to both genes and cul-
301 ture. The probability that a player encounters an opponent of the same genotype is
302 f_g , while the probability for culture-types is f_c . If individuals were interacting with
303 kin, f_g would be the probability of being identical-by-descent, and f_c would be the
304 analogous value computed for a cultural genealogy (Aguilar and Ghirlanda, 2015).
305 We allow for these assortment parameters to vary by domain because population
306 structure may vary with respect to the two systems of inheritance. For example, in a
307 genetically panmictic population in which cultural learning only takes place within
308 demes, f_g is 0 while f_c is nonzero. These two values allow us to consider a broad
309 range of population structure models. We assume that with some probability, f_g ,
310 an individual chooses a partner of identical genotype, and otherwise selects her
311 partner at random (with an analogous situation for culture-type). We then com-
312 pute the probability of having a player of a certain type will be conditional on one's

313 own type. Thus, the probability that a type $\{1, 1\}$ interacts with another $\{1, 1\}$ is,

$$P(1, 1|1, 1) = f_g f_c + f_c(1 - f_g)q_g + f_g(1 - f_c)q_c + (1 - f_c)(1 - f_g)q_g q_c \quad (11)$$

314 where q_g and q_c are the population frequencies of the genetic and cultural altruistic
315 alleles. The first term is the probability that two $\{1, 1\}$ individuals are identical due
316 to assortment; the second is the probability of being identical due to assortment for
317 culture but not genes; the third is the probability of being identical due to assort-
318 ment for genes and not culture; and the final term is the probability of not being
319 identical due to assortment either genetically or culturally. These conditional prob-
320 abilities then determine the expected phenotype of an individual's opponent in the
321 game, \tilde{p}_ψ . Further details on the model are provided in SI-3.

322 Offspring inherit their parent's genetic allele. They must then choose a cultural
323 model whose allele they will inherit at the cultural locus. In the next two subsec-
324 tions, we consider two models differing in how cultural parents are chosen. In both,
325 cultural models are independent of the genotype of the offspring. In section 4, we
326 consider the more general case where there is genetically biased cultural transmis-
327 sion.

328 **3.1 Model 1: Neutral cultural trait**

329 First, we assume that the cultural propensity for altruism is neutral with respect to
330 cultural fitness. In other words, ancestors are chosen as cultural parents without
331 regard to their cultural traits, so the probability of acquiring the cultural propensity

332 for altruism is q_c , the population frequency of the cultural allele in the parental gen-
333 eration. We can use (9) to determine the condition for the increase in the altruistic
334 phenotype by multiplying both sides of the inequality by $var(c)$ and computing the
335 covariances directly from the model. We have no cultural selection, so $B_c = C_c = 0$.
336 Since culture is chosen at random, genetic and cultural type are uncorrelated, so
337 that $cov(c, g) = cov(\tilde{c}, g) = cov(\tilde{g}, g) = 0$. Thus, (9) reduces to

$$B_g f_g > C_g,$$

338 the canonical form of Hamilton's rule. This result follows directly from the cul-
339 tural allele being chosen at random. Under random copying the expected change
340 in the frequency of the cultural allele is zero and the only change in mean pheno-
341 type will be due to changes in the frequency of the genetic allele. Further, with
342 no correlations between the genetic and cultural allele, the only forces affecting the
343 evolution of the genetic allele will be the reproductive fitness effects. However, it
344 should be noted that due to the dual inheritance of altruism, the value of the phe-
345 notype may be maintained at significant levels in the population if the frequency of
346 the cultural allele is high. Take the extreme case where $q_c = 1$. Even if the inequal-
347 ity above is not met and the genetic allele is driven to extinction, the cultural allele
348 will be unaffected and the mean value of the phenotype in the population will be
349 $\bar{p} = q_c/2 = 1/2$. In other words, there will be no perfect altruists, but everyone will
350 be a 'half' altruist. As the mean reproductive fitness, \bar{w} , depends on the mean phe-
351 notype, this could have important implications for population growth, including
352 eventual extinction.

353 3.2 Model 2: Cultural prisoner's dilemma

354 Next, we consider a case where offspring no longer choose their cultural parent
355 at random. In particular, we assume that individuals meet to play the prisoner's
356 dilemma, this time with respect to both reproduction and cultural propagation.
357 For simplicity, we'll imagine individuals producing cultural 'gametes' or behavioral
358 tokens that can then be acquired by offspring. The number of tokens an individual
359 produces will affect her probability of being copied. This can be interpreted as her
360 visibility, or salience as a cultural model. The expected number of cultural gametes,
361 z , that an individual of type ψ produces is,

$$z_\psi = z_0 + B_c^z \tilde{p}_\psi - C_c^z p_\psi \quad (12)$$

362 The terms B_c^z and C_c^z are the gametic fitness benefit and cost (i.e. the regression of
363 the number of cultural gametes produced on neighbor phenotype and focal pheno-
364 type, respectively), with $B_c = B_c^z / \bar{z}$, $C_c = C_c^z / \bar{z}$ (see SI-4). Recall that in the previ-
365 ous section cultural fitness was defined in terms of the total influence ($s_i = \sum_{j=1}^{N'} \gamma_{ij}$)
366 an ancestor has on the descendant population. In this model, offspring have a single
367 cultural ancestor (i.e. $\gamma_{ij} = 1$), and s_i is just the total number of descendant indi-
368 viduals who count i as an ancestor. The number of offspring available as cultural
369 descendants is determined by the reproductive output of the population, thus,

$$s_i = \frac{z_i}{\bar{z}} \bar{w}. \quad (13)$$

370 Substituting (12) and (13) into the gene-culture Price equation and making simpli-
371 fications (see SI-4 for details), we obtain:

$$B_c^z f_c - C_c^z > - [B_g f_g - C_g] \frac{q_g(1 - q_g)}{q_c(1 - q_c)} \frac{\bar{z}}{\bar{w}}. \quad (14)$$

372 We now have an additional scaling term, $\bar{w}/\bar{z} = s_i/z_i$, the number of cultural de-
373 scendants per cultural gamete produced. This term converts the payoff in the cul-
374 tural Prisoner's Dilemma game (z_i) to the true cultural fitness (s_i), which in this
375 case is the number of individuals that count i as an ancestor.

376 Above, we have written out the variance terms explicitly. For given cultural and
377 genetic selection coefficients, the ratio of the variances in (14) means that the effect
378 of cultural selection will be maximized when the genetic allele is at very high or
379 very low frequency (q_g close to 0 or 1) and $q_c = .5$.

380 In Figure 2, we plot the the values of the total selection coefficient on the altru-
381 istic phenotype (i.e. LHS minus RHS in (14)) against the frequency of the genetic
382 allele, q_g . The frequency of the cultural altruistic allele is set to .5 to maximize $var(c)$,
383 which allows us to restrict the values of $var(g)/var(c)$ between 0 and 1. As $var(g)$
384 is minimized at $q_g = 0, 1$, selection is convex in q_g . This is generally true so long as
385 the cultural selection coefficient is positive and the genetic selection coefficient is
386 negative. Thus, when $q_c = .5$, cultural selection will have the strongest effect when
387 the genetic allele is extinct or at fixation.

388 We defined altruism with respect to both cultural and genetic fitnesses. In model
389 1, cultural transmission was neutral with respect to the altruistic phenotype, while
390 in model 2 there was also a direct cultural fitness cost to the phenotype. It is possi-

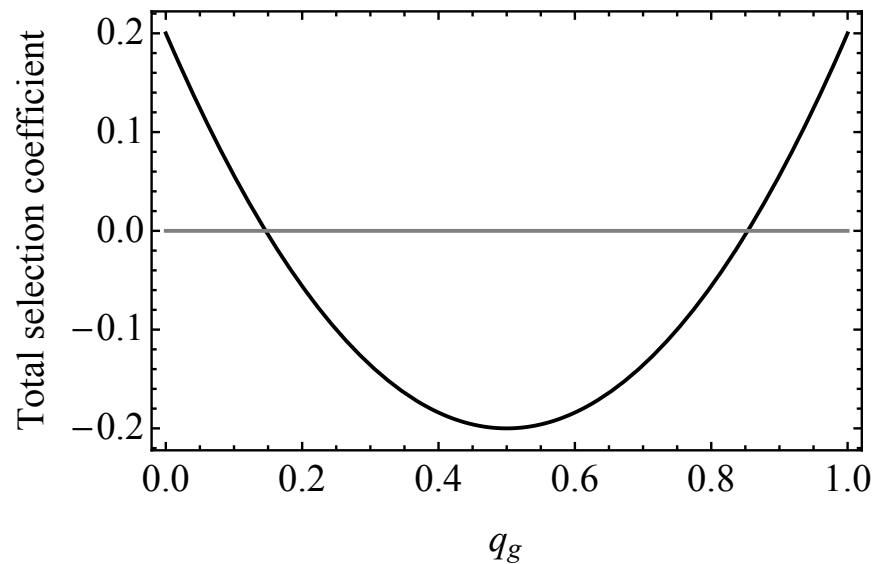


Figure 2: Total selection coefficient (LHS-RHS in (14)) for $B_g = B_c^z = 2, C_g = C_c^z = 1, r_g = .3, r_c = .6,$ and $q_c = .5,$ to maximize $var(c)$. Total selection coefficient is maximized when q_g is at its minimum or maximum values. Parameter changes adjust the range, but not the shape of the function, so long as $B_c^z f_c - C_c^z > 0, B_g f_g - C_g < 0$ (i.e. there are some parameter combinations for which all values are negative/positive but the curve remains convex).

391 ble that a phenotype may be beneficial in the cultural domain while detrimental to
392 reproduction. In this case, we simply change the sign of the cost term on the LHS
393 of (14) and see that it makes the condition easier to meet. Conversely, we could
394 imagine a behavior that is costly with respect to cultural fitness and entirely bene-
395 ficial in the genetic domain, which would again make condition (14) more difficult
396 to meet. This highlights the importance of specifying the fitness consequences of a
397 trait in each domain of inheritance.

398 **4 Gene-Culture covariance**

399 In the two illustrative models above, we assumed that genetic and cultural trans-
400 mission were independent, i.e. having a particular genetic type had no effect on
401 the probability of acquiring a particular cultural type. This allowed us to ignore
402 the covariance between genetic and cultural types. Now we allow for genes and
403 culture to be nonindependent. In particular, we assume that with probability k a
404 newborn individual will acquire the cultural allele that mirrors her genetic allele.
405 For example, an individual born with the genetic allele for altruism ($g = 1$), will
406 acquire the cultural allele for altruism with probability k . With probability $1 - k$,
407 she acquires her cultural allele randomly from the cultural gamete pool. This kind
408 of non-random learning can result, for example, from a genetic predisposition to-
409 wards particular cultural models, such as when the locus that determines cultural
410 learning is linked to the locus that determines the altruistic genotype. The effect of
411 this construction is to introduce a correlation between genotype and cultural type.

412 The full recursion equations are given in a Mathematica notebook in the SI. In
 413 SI-5 we show that the altruistic phenotype increases when:

$$B^z(\beta_{\bar{g},c} + \beta_{\bar{c}c}) - C_z(1 + \beta_{gc}) > - [B_g(\beta_{\bar{g}g} + \beta_{\bar{c}g}) - C_g(1 + \beta_{cg})] \frac{(1+k) \bar{z} \text{var}(g_i)}{(1-k) \bar{w} \text{var}(c_i)} - \frac{k(q_g - q_c) \bar{z}}{\text{var}(c_i)},$$

414 which evaluates to:

$$B_c \left(f_c + \frac{\text{var}(g)}{\text{var}(c)} k f_g (1 + k(1 - f_c)) \right) - C_c \left(1 + \frac{\text{var}(g)}{\text{var}(c)} k \right) > \\
 - [B_g (k(f_c + f_g - f_c f_g) + f_g) - C_g(1 + k)] \frac{\text{var}(g) (1+k) \bar{z}}{\text{var}(c) (1-k) \bar{w}} - \frac{k(q_g - q_c) \bar{z}}{\text{var}(c)}. \quad (15)$$

415 When $k = 0$ the above expression reduces to (14). When $k = 1$, the RHS goes
 416 to positive or negative infinity, depending on the sign of genetic selection; when
 417 genetic selection is negative, the condition becomes impossible to meet. Thus, the
 418 effect of k , the correlation between genotype and cultural type, is to make it more
 419 difficult for cultural selection to overcome natural selection when they are in con-
 420 flict. This is intuitive as higher k makes the cultural inheritance system more cou-
 421 pled to the genetic inheritance system.

422 As in the general condition, (9), the genetic selection term (i.e. the term on the
 423 RHS in brackets) is different from what it would be under purely genetic transmis-
 424 sion of altruism. One way to see this effect is to fix B_g and C_g and see above what
 425 value of f_g the genetic selection coefficient is positive, which we'll call f_g^{crit} . For
 426 example, under purely genetic transmission of altruism, for $B_g = 2$, $C_g = 1$, then
 427 $f_g^{crit} = 0.5$; for all $f_g > 0.5$ genetic selection favors altruism. In figure 3 we plot f_g^{crit}
 428 for the same values of B_g and C_g and varying r_c and k . For all constellations of f_c

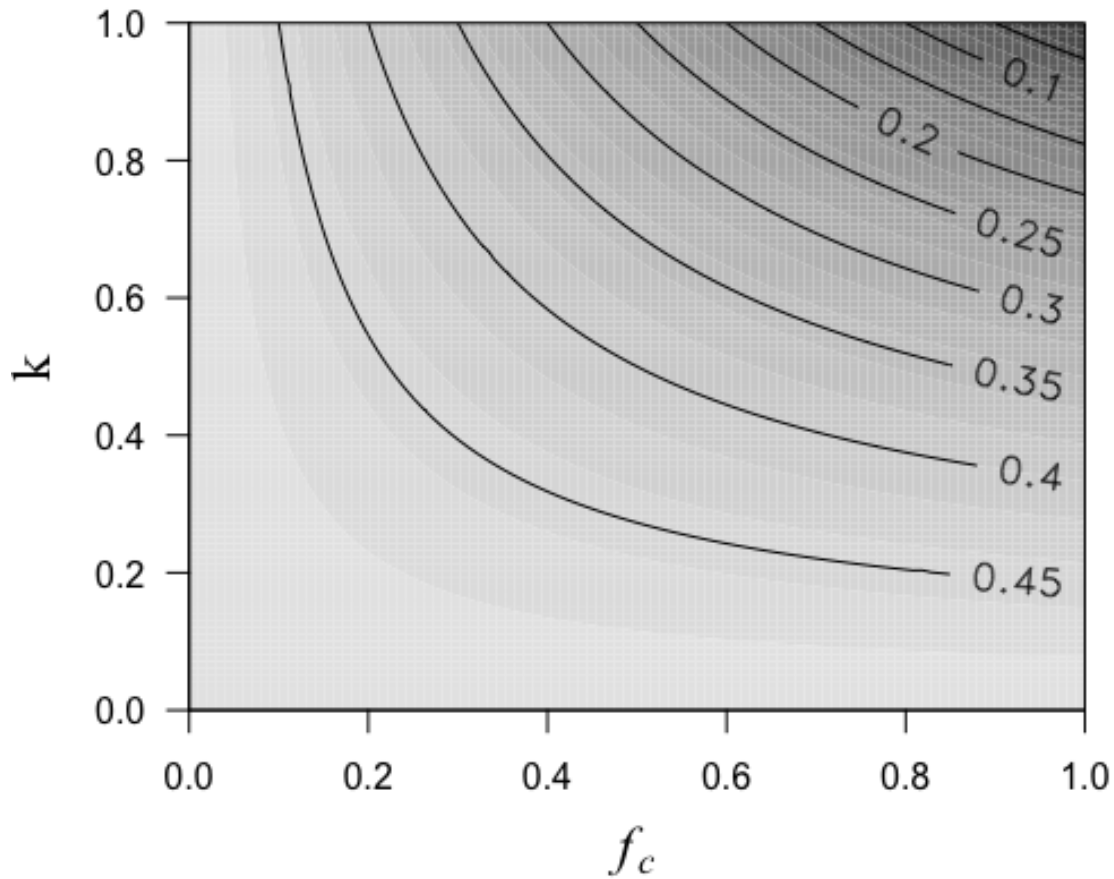


Figure 3: Critical values of f_g^{crit} for $B_g = 2, C_g = 1$ and varying r_c and k . For all combinations of r_c and k , f_g^{crit} is lower than for what it would be under purely genetic transmission of altruism ($f_g^{crit} = .5$). Values $f_g > f_g^{crit}$ mean positive genetic selection.

429 and k we see that f_g^{crit} under gene-culture transmission is always lower than f_g^{crit}
430 under purely genetic transmission (in fact, it can be shown that for $B_g > C_g$, the
431 critical value will always be lower). Thus, cultural transmission affects the evolu-
432 tion of altruism in two important ways: (1) by introducing a cultural selection force
433 that may overcome genetic selection; (2) by changing the nature of genetic selection
434 itself. This latter effect means that for given values of B_g and C_g , genetic selection
435 may be positive in the presence of joint cultural and genetic transmission when it
436 would have been negative under purely genetic transmission.

437 5 Non-additive phenotypes

438 The results described above all assumed an additive phenotype function, which is a
439 standard starting point in social evolution and population genetics theory (Van Cleve,
440 2015). However, biological reality may be much more complicated, particularly
441 when trying to incorporate the effects of multiple inheritance systems. One way to
442 deal with this problem in evolutionary theory has been to observe that most genetic
443 variants have small effects on phenotypes and genetic variation in the population
444 is small, in which case, an additive approximation gives satisfactory results (Tay-
445 lor and Frank, 1996; Akçay and Van Cleve, 2012). In this section, we translate this
446 approach to phenotypes that are jointly determined by genes and culture.

447 We begin by assuming that an individual descendant j 's phenotype is given
448 by a function $p_j(c_j, g_j)$, where the arguments are a descendant's heritable cultural
449 and genetic information. This information in turn is a function of the heritable

450 cultural and genetic information of that individual's ancestors, which implies that
 451 we can instead write the phenotype mapping function as $p_j(c_1, \dots, c_N, g_1, \dots, g_N)$,
 452 a direct function of the ancestral culture-types and genotypes. Assuming that all
 453 p_j are differentiable with respect to ancestral values, and that the variances in c
 454 and g are small, we can make a first-order Taylor approximation of p_j around the
 455 point $(\bar{c}, \bar{g}) = (\bar{c}_1, \dots, \bar{c}_N, \bar{g}_1, \dots, \bar{g}_N)$. We then substitute this expansion into $\Delta\bar{p} =$
 456 $\frac{1}{N} \sum_{i=1}^N p_j - \bar{p}$ to arrive at a Price equation for the non-additive case (see 5),

$$\Delta\bar{p} = \frac{N}{N'} \text{cov}(\mathcal{S}_i, c_i) + \frac{N}{N'} \text{cov}(W_i, g_i) + \langle p_j(\bar{c}, \bar{g}) \rangle - \bar{p}, \quad (16)$$

457 where $\mathcal{S}_i = \sum_{j=1}^{N'} \frac{\partial p_j}{\partial c_i} \Big|_{(\bar{c}, \bar{g})}$ and $W_i = \sum_{j=1}^{N'} \frac{\partial p_j}{\partial g_i} \Big|_{(\bar{c}, \bar{g})}$, refer to *generalized fitnesses* in
 458 the sense that we are measuring not only the number of descendant individuals an
 459 ancestor has, but also the combined effect of that ancestor on her descendants' phe-
 460 notypes. For example, in a haploid genetic model in the absence of mutation, where
 461 the 'phenotype' of interest is just the genotype, then $\frac{\partial p_j}{\partial g_i} = 1$ when i is a genetic an-
 462 cestor of j , while $\frac{\partial p_j}{\partial g_k} = 0$ for all individuals k that are not genetic ancestors to j . In
 463 this case, the generalized fitness just reduces to the number of descendant individu-
 464 als who count i as an ancestor. Similarly, in the model presented in the first section,
 465 the partial derivative of the phenotype function p_j with respect to c_i will yield γ_{ij} ,
 466 and $\mathcal{S}_i = s_i$. The advantage of this formulation is that more complicated phenotype
 467 mapping functions can be incorporated into the idea of a generalized fitness. The
 468 formulation also allows for the generalized fitness to be negative, as might happen
 469 in a case where 'offspring' react by contrasting themselves with parental individ-

470 uals (Findlay, 1992). This generalized fitness concept captures the causal effect of
471 an ancestor on a descendant individual's phenotype, where it induces a positive
472 or negative correlation in their phenotypes, a feature missing from more restrictive
473 fitness concepts.

474 Equation (16) looks similar to equation (3); first, we have two covariance terms
475 that account for the effect of selection (now with respect to generalized fitness).
476 We've replaced the inverse of the mean fitness with a more direct measure of pop-
477 ulation growth, N/N' ; this is because generalized fitness refers to the effect of an
478 ancestor on the phenotypes in the next generation, and is no longer synonymous
479 merely with her contribution to the growth of the population. The remaining term,
480 $\langle p_j(\bar{c}, \bar{g}) \rangle - \bar{p}$, is the difference between (1) the average phenotype that would occur
481 if every individual inherited the mean values of c and g , and (2) the mean pheno-
482 type among ancestors (\bar{p}). This isolates the effect of the phenotype functions among
483 descendants, p_j , on evolutionary change and can be seen as analogous to the tra-
484 ditional transmission term in that it captures the effect of constructing phenotypes
485 from inherited information on evolutionary change.

486 From eq. 16 we can simply derive a condition for the evolution of a maladaptive
487 trait. When $\Delta\bar{p} > 0$, we have,

$$\beta_{S_i, c_i} > - \left[\beta_{W_i, g_i} \frac{\text{var}(g)}{\text{var}(c)} + \frac{(\langle p_j(\bar{c}, \bar{g}) \rangle - \bar{p})}{\text{var}(c)} \right] \quad (17)$$

488 This result is exactly analogous to (4) in the first section and can be summarized
489 similarly: a loss in generalized reproductive fitness can be compensated for by a
490 gain in generalized cultural fitness. Again, we have assumed that the rules of trans-

491 mission remain constant over the timescale being considered in the Price equation.

492 This approach could of course be extended to higher order expansions of the
493 phenotype function: in Section 5 we show that the infinite expansion of the phe-
494 notype function leads to a more precise definition of generalized fitness than the
495 approximation given in this example. Most importantly, without making assump-
496 tions about either the phenotype mapping function or the fitness function, we have
497 shown an important relationship between these two fundamental concepts in evo-
498 lutionary theory.

499 This approach bears some resemblance to the model presented by Richerson
500 and Boyd in (Richerson and Boyd, 1978), where generic phenotype functions de-
501 pended on genotype, cultural-type, and a third argument called the penetrance,
502 which determined the relative influence of either inheritance system on phenotype.
503 In the foregoing presentation this penetrance value is implicit in the phenotype
504 function.

505 **6 Discussion**

506 **6.1 Co-inherited Behaviors**

507 Our model was inspired by the fact that many complex behavioral traits, from psy-
508 chological traits, to disease risk, are likely affected by both genetic and cultural
509 transmission. For instance Levy et al. (2016) found that culturally transmitted neg-
510 ative stereotypes about aging predicted later biomarkers of Alzheimer's disease
511 (reduced hippocampal volume and increased amyloid plaque development). Our

512 model indicates that if these negative stereotypes are culturally selected for, per-
513 haps because a rapidly changing environment makes older individuals seem less
514 valuable as cultural role models, then cultural selection can increase the incidence
515 and severity of Alzheimer's biomarkers, especially as natural selection is likely to be
516 very weak in this case. In another interesting example, Kong et al. recently showed
517 evidence for selection against genetic variants associated with educational attain-
518 ment, a complex trait which is clearly influenced by social learning (Kong et al.,
519 2017). If more highly educated individuals exercise greater influence on the edu-
520 cational attainment of others then cultural and genetic selection will be in direct
521 conflict. Our model suggests that measuring the relative variance in culture and
522 genes for this co-inherited trait (educational attainment) is an important next step
523 in understanding its evolution.

524 Research into the evolutionary basis of human behavior has long puzzled over
525 the existence of maladaptive behaviors (Glanville, 1987; Logan and Qirko, 1996).
526 These are behaviors that persist via cultural transmission despite detrimental re-
527 productive fitness effects, such as clubbing pregnant women to induce birth in
528 Colombia (Reichel-Dolmatoff and Reichel-Dolmatoff, 2013), unhygienic neonatal care
529 practices in Bangladesh (McConville, 1988), and folk medical practices like ingest-
530 ing rhino horn (Ayling, 2013) or bloodletting (Wootton, 2007). While these practices
531 are likely spread almost exclusively by cultural transmission, they may be influ-
532 enced by genetic inheritance via broader behavioral traits with a significant genetic
533 component, such as risk-taking, which also shows cross-cultural variation (Weber
534 and Hsee, 1998; Hsee and Weber, 1999). Fertility behavior provides another po-

535 tential example of a dually inherited trait. The reduction in fertility seen across
536 the globe, known as the demographic transition, may be a result of cultural se-
537 lection overcoming genetic selection for higher fertility. Our model demonstrates
538 more broadly the possibility that maladaptive behavioral traits may evolve under
539 dual transmission, despite their reproductive fitness costs. In fact, using our Tay-
540 lor expansion approach for non-additive phenotypes, the Kolk et al. model can be
541 expressed in terms of our Price equation.

542 **6.2 Relative strength of genetic and cultural selection**

543 Explicitly modeling genetic and cultural inheritance for a co-inherited trait did not
544 only reveal the effect of each inheritance system on evolutionary change; it also re-
545 vealed the effect of each inheritance system on the other. As we saw in sections
546 2.1 and 4, the presence of cultural transmission changed the genetic selection coef-
547 ficient from what it would be under purely genetic transmission (the same is true
548 with respect to purely cultural transmission). The addition of another inheritance
549 system for a single trait meant that there was both an additional selection force to
550 be considered, and that genetic selection itself took a different form that could favor
551 the evolution of a trait when purely genetic transmission would not.

552 Our results show the importance of the ratio of genetic to cultural variance in
553 scaling the effect of genetic selection. It is interesting to consider empirical esti-
554 mates of cultural and genetic diversity to gauge the expected relative strength of
555 genetic selection. Bell et al. compared F_{st} values for culture and genes in pop-
556 ulations using the World Values Survey (Bell et al., 2009). Their results suggested

557 greater-between population variation in culture than in genes. Unfortunately, these
558 results say little about the within-group variance in culture relative to genes. Other
559 studies have shown parallels in the patterns of linguistic and genetic diversity (Per-
560 reault and Mathew, 2012; Longobardi et al., 2015; Creanza et al., 2015; Hunley et al.,
561 2008), but again provided no information about the ratio of genetic to cultural vari-
562 ance. However, this question is well-suited to empirical study; given our results,
563 empirical estimates of the ratio can shed light on qualitative expectations about the
564 evolution of behavioral traits.

565 The ratio of genetic to cultural variance also has an important relationship to the
566 narrow-sense heritability (h^2), which measures the proportion of phenotypic vari-
567 ance attributable to the ‘heritable’ component of phenotype (Falconer and Mackay,
568 1996). In a series of papers, Danchin and co-authors (Danchin and Wagner, 2010;
569 Danchin et al., 2011, 2013) introduced the idea of ‘inclusive heritability’, which par-
570 titions the variance in the heritable component of phenotype into the contributions
571 from each system of inheritance. This allows for narrow-sense heritability to be ex-
572 pressed as the sum of the heritabilities in each domain (assuming no interactions
573 between the inheritance systems). In our model, this means $h^2 = h_g^2 + h_c^2$ (where
574 h_g^2 and h_c^2 are the genetic and cultural heritabilities). The ratio of these heritabili-
575 ties is exactly the term that appears in our results as the scaling factor of genetic
576 selection, further demonstrating the importance of inclusive heritability when con-
577 sidering evolutionary outcomes.

578 **6.3 Related results**

579 Nearly forty years ago Richerson & Boyd showed that under certain conditions, the
580 equilibrium value of a trait that is both genetically and culturally inherited could be
581 that which optimizes cultural fitness (Richerson and Boyd, 1978). But surprisingly,
582 given the intense attention gene-culture coevolution received, very little theoretical
583 work has been done to follow up on the evolution of phenotypes that are directly
584 co-inherited, as opposed to culturally inherited behavioral phenotypes and genet-
585 ically inherited learning rules (e.g., as in Lehmann and Feldman, 2008). Findlay
586 (Findlay, 1992) modeled gene-culture transmission of a phenotype in a structured
587 population. He found that when individual level selection is weak and migration
588 low, gene-culture transmission was more favorable to the evolution of altruism than
589 genetic transmission alone. This is because the effect of migration is to erode be-
590 tween group variance. In our paper the total population variance in culture was
591 shown to have an effect on making the evolution of an altruistic phenotype easier,
592 and we implicitly included the effect of population structure in our illustrative mod-
593 els. Another paper close to our model is that of Lehmann et al. (2008), who model
594 the evolution of a purely culturally inherited altruistic behavior in a subdivided
595 population. In their model, Lehmann et al. consider the phenotype to be affecting
596 either only cultural or reproductive fitness, and assume no genetic contribution to
597 the phenotype. As such, their model can be recovered by modifying our Price equa-
598 tion (9) (by setting $p_i = c_i$, which replaces the right-hand side of the condition with
599 zero). Importantly, biological offspring in Lehmann et al's model serve as "vectors"
600 of the cultural types of their parents, which means that even when their cultural

601 trait only affects reproductive fitness, our corresponding B_c and C_c terms would be
602 non-zero. That means that the transmission rate of different cultural types are not
603 the same over the entire lifecycle, and there is cultural (but no genetic) selection.

604 More recently, El Mouden et al. used a Price equation to describe cultural evolu-
605 tion and made some claims that at face value differ from our results. In particular,
606 they claim that cultural selection can only increase genetic fitness (e.g., through al-
607 truism that benefits others) if cultural and genetic fitness are tightly coupled. We
608 have instead shown that such co-inherited altruism can increase through cultural
609 selection even when opposed by genetic selection. The source of the discrepancy
610 lies in the fact that El Mouden et al. base their statements on the *partial change in*
611 *genetic fitness due to selection* (cf. their equation S12), which by Fisher's fundamen-
612 tal theorem (Fisher, 1930; Bijma, 2010) is always positive, even though total mean
613 fitness can be increasing or decreasing. To give the simplest possible example, con-
614 sider our equation (14), and an altruistic trait with $B_g > C_g$ but $f_g B_g < C_g$. This trait
615 would be opposed by genetic selection, but if it spreads through cultural selection
616 (which happens when $f_c B_c^z - C_c^z$ is sufficiently large) it would increase the mean ge-
617 netic fitness, despite the fact cultural and genetic fitness are negatively correlated.
618 This is a direct consequence of the well-known fact the total change in fitness in-
619 cludes changes in the population composition (e.g., social effects) that are ignored
620 in the Fisherian partial change. This bears directly on the claim of El Mouden et al.
621 (on p. 235) that cultural selection opposed by genetic selection cannot explain the
622 success and spread of human populations through increased reproductive fitness,
623 because the latter is clearly an argument about realized (or total) fitness, and not

624 about partial changes. That claim is false as the above argument illustrates: cultural
625 altruism, even if it is opposed by genetic selection *can* increase the mean fitness of
626 populations. Furthermore, because El Mouden et al. do not consider co-inheritance
627 of a phenotype underlying fitness along with potential gene-culture covariances,
628 they overlook the possibility that cultural transmission and assortment might alter
629 the direction of genetic selection. Thus, contrary to the arguments of El Mouden et
630 al. , the sovereignty of genetic selection over cultural selection is far from absolute,
631 and a careful accounting of the operation of both modes of transmission is needed.

632 **6.4 Limitations and extensions**

633 The additive model we used in this paper is both the simplest model and a nat-
634 ural extension of the standard assumption in quantitative genetics (Falconer and
635 Mackay, 1996). However, even under this simple model we observed some non-
636 trivial results. In the general model we assumed that mean cultural fitness was
637 equal to the mean reproductive fitness (i.e. $\bar{s} = \bar{w}$). This was convenient and sim-
638 ply resulted in every individual receiving some cultural input, though there exist
639 traits for which some individuals may never receive cultural input. For example,
640 though underlying genetic variation may determine one's reading ability, one may
641 never be taught to read. In these cases, the equality of \bar{s} and \bar{w} will not necessarily
642 hold. Our general framework could easily be extended to compass this possibility.
643 Indeed, we discarded this assumption in our second illustrative model; when the
644 mean number of replications for culture and reproduction were not the same—in
645 this case \bar{z} and \bar{w} —the conversion factor \bar{z}/\bar{w} scaled the effect of genetic selection. In

646 the event that cultural replication might affect fewer individuals than are actually
647 born, $\bar{z}/\bar{w} < 1$, and the effect genetic selection is further reduced.

648 In the course of deriving our results on the effects of selection, we often ignored
649 the transmission terms, $\langle \Delta c \rangle$ and $\langle \Delta g \rangle$. In relatively simple genetic systems, it may
650 be safe to assume that the expected difference between parents and offspring is
651 zero. However, culture very often makes this assumption untenable, as the cultural
652 transmission system allows for biased or directed ‘mutation’ in the form of individ-
653 ual learning and other factors. For example, individuals may systematically differ
654 from their parents because they learn more appropriate responses to their environ-
655 ment through their own trial-and-error learning. El Mouden et al. (2014) offered an
656 interpretation of the transmission term as evolved biases in favor of reproductive-
657 fitness-maximizing behaviors. However, biases (or individual learning) may not al-
658 ways produce reproductive fitness-maximizing biases, especially under frequency
659 dependence. Meanwhile, Henrich (2004) took the transmission term to represent
660 systematic error in cultural learning that biased individuals to trait values lower
661 than their cultural parents. These examples hint at the diverse interpretations that
662 can be ascribed to the transmission term, particularly in lieu of empirical evidence
663 on how a specific trait is passed on. These effects also present important future
664 directions for a more complete framework of gene-culture co-evolution.

665 In our section on non-additivity, we took an unusual approach to deriving the
666 Price equation. Most models of social evolution make an explicit assumption about
667 the fitness function (e.g. linearity, as in our derivation of the gene-culture Hamil-
668 ton’s rule) and an implicit assumption about the phenotype function (e.g. $p = g$, as

669 in the phenotypic gambit). By contrast, we made no assumptions about the form
670 of the phenotype function, with the exception of infinite differentiability, and were
671 able to derive a definition of fitness that similarly relied on no previous assumptions
672 about the fitness function. This approach demonstrates the relationship between
673 how phenotypes are actually constructed from inherited information and fitness it-
674 self. Also, our notion of generalized fitness captures both the effect of reproduction
675 (the number of individuals who receive any heritable information from an ances-
676 tor) and the effect of transmission (how much heritable information flows from an
677 ancestor to a descendant). The relationship between generalized fitness and other
678 important fitness concepts, such as inclusive fitness, is worth exploring, but beyond
679 the scope of the present paper.

680 Lastly, we took the transmission rules for both genes and culture to be stable over
681 the timescale assumed in the Price equation. While this is the standard assumption
682 for genetic systems, the long-term evolution of culture will be determined by the
683 ways in which individuals acquire cultural information, a trait that may itself be
684 culturally or genetically transmitted. Exploring the evolution of transmission rules
685 in the context of a trait that is co-inherited is an important future direction for this
686 work.

687 **6.5 Conclusion**

688 The Price equation offers a general statement of how evolutionary change can be
689 partitioned among different evolutionary factors (Frank, 2012). We derived a Price
690 equation for the evolution of a trait that is transmitted via both modes of inheri-

691 tance. In using the Price equation, we have offered a general framework for par-
692 titioning the evolution of a co-inherited trait across the distinct causes (selection,
693 transmission effects, etc.) in each domain of inheritance. Given the evidence for
694 the long evolutionary history of cultural transmission in the human lineage (Lind
695 et al., 2013), it is likely many behavioral traits evolved under the combined influence
696 of genetic and cultural transmission. As the importance of non-genetic inheritance
697 systems becomes clearer, we propose that accounting for multiple inheritance sys-
698 tems explicitly, as we do in our framework, will contribute to a better theoretical
699 understanding of the evolution of these traits.

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862 Supplementary Information

863 SI-1 Derivation of Gene Culture Price equation.

864 The phenotype of individual j is given by,

$$p_j = \sum_{i=1}^N \nu_{ij} g_i + \Delta g_j + \sum_{i=1}^N \gamma_{ij} c_i + \Delta c_j + e \quad (\text{SI-1})$$

865 where the coefficients ν_{ij} and γ_{ij} represent the influence an ancestor i has on de-
866 scendant j in the genetic and cultural domains, respectively (Note: $\sum_{i=1}^N \nu_{ij} =$
867 $\sum_{i=1}^N \gamma_{ij} = 1$)¹. The mean value of p in the descendant generation is,

$$\bar{p}' = \frac{1}{N'} \sum_{j=1}^{N'} \sum_{i=1}^N \nu_{ij} g_i + \frac{1}{N'} \sum_{j=1}^{N'} \Delta g_j + \frac{1}{N'} \sum_{j=1}^{N'} \sum_{i=1}^N \gamma_{ij} c_i + \frac{1}{N'} \sum_{j=1}^{N'} \Delta c_j \quad (\text{SI-2})$$

868 where e is assumed to have mean zero. Reversing the orders of the double sum
869 terms and noting that $w_i = \sum_{j=1}^{N'} \nu_{ij}$, and $s_i = \sum_{j=1}^{N'} \gamma_{ij}$, we can rewrite eq. SI-2 as,

$$\bar{p}' = \frac{1}{N'} \sum_{i=1}^N g_i w_i + \frac{1}{N'} \sum_{i=1}^N c_i s_i + \frac{1}{N'} \sum_{j=1}^{N'} \Delta g_j + \frac{1}{N'} \sum_{j=1}^{N'} \Delta c_j \quad (\text{SI-3})$$

¹In this derivation we assume that for every descendant j there exists some ancestor i for whom $\gamma_{ij} > 0$.

870 Using the definition of covariance ($cov(x, y) = E[xy] - E[x]E[y]$) we can replace the
 871 first two terms on the RHS,

$$\bar{p}' = \frac{N}{N'} cov(w_i, g_i) + \frac{N}{N'} cov(s_i, c_i) + \langle \Delta g_j \rangle + \langle \Delta c_j \rangle + \frac{N}{N'} (\bar{w}\bar{g} + \bar{s}\bar{c}) \quad (\text{SI-4})$$

872 The angle brackets here mean averages over the descendant population. Noting
 873 $N\bar{w} = N\bar{s} = N'$ we can rewrite the final term on the RHS as $\bar{g} + \bar{c}$. Subtracting the
 874 mean phenotype in the ancestral population, $\bar{p} = \bar{g} + \bar{c}$, we have (3).

875 The cultural covariance term in (3) takes the ‘ancestral’ point of view, in that it in-
 876 cludes ancestral cultural values and their fitnesses. However, we can re-express
 877 this term from the descendant point of view with the following quick restatement
 878 (note: overbars are ancestor averages and brackets are descendant averages),

$$\begin{aligned} cov(s_i, c_i) &= \overline{c_i s_i} - \bar{c}\bar{s} \\ &= \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^{N'} c_i \gamma_{ij} - \bar{c}\bar{s} = \sum_{j=1}^{N'} \overline{c_i \gamma_{ij}} - \bar{c}\bar{s} \\ &= \sum_{j=1}^{N'} cov(c_i, \gamma_{ij}) + \sum_{j=1}^{N'} \bar{c} \bar{\gamma}_j - \bar{c}\bar{s} \\ &= \sum_{j=1}^{N'} cov(\gamma_{ij}, c_i) + \bar{c} \frac{N'}{N} - \bar{c}\bar{s} \\ &= N' \langle cov(\gamma_{ij}, c_i) \rangle. \end{aligned} \quad (\text{SI-5})$$

879 For clarity, the final mean of the covariance term is taken over the descendant pop-
 880 ulation.

881 **SI-2 Derivation of Gene-culture Hamilton's rule**

882 We begin with the following cultural and genetic fitness functions:

$$s_i = s_0 + \beta_{sp}p_i + \beta_{s\bar{p}}\bar{p} = s_0 + \beta_{sp}c_i + \beta_{sp}g_i + \beta_{s\bar{p}}\tilde{c}_i + \beta_{s\bar{p}}\tilde{g}_i \quad (\text{SI-6})$$

$$w_i = w_0 + \beta_{wp}p_i + \beta_{w\bar{p}}\bar{p} = w_0 + \beta_{wp}c_i + \beta_{wp}g_i + \beta_{w\bar{p}}\tilde{c}_i + \beta_{w\bar{p}}\tilde{g}_i \quad (\text{SI-7})$$

883 The tilde over a variable indicates the mean value of that variable across i 's neigh-
 884 bors. We have assumed both kinds of fitness are linear functions of an individuals
 885 own phenotype and the phenotypes of her neighbors. As in the standard derivation
 886 of Hamilton's rule using the Price equation, it is customary to identify β_{wp} and $\beta_{w\bar{p}}$
 887 as the cost (C) to an altruist and benefit (B) to recipients of altruism, respectively.
 888 We will use the same convention, but add subscripts to indicate costs and benefits
 889 to genetic *and* cultural fitnesses,

$$\beta_{wp} \Rightarrow C_g$$

$$\beta_{sp} \Rightarrow C_c$$

$$\beta_{w\bar{p}} \Rightarrow B_g$$

$$\beta_{s\bar{p}} \Rightarrow B_c$$

Substituting SI-7 into our Price equation in 3, and ignoring the transmission terms,
 we have,

$$\begin{aligned} \bar{w}\Delta\bar{p} = & B_c [\text{cov}(\tilde{c}, c) + \text{cov}(\tilde{g}, c)] + C_c [\text{var}(c) + \text{cov}(g, c)] \\ & + B_g [\text{cov}(\tilde{c}, g) + \text{cov}(\tilde{g}, g)] + C_g [\text{cov}(c, g) + \text{var}(g)] \quad (\text{SI-8}) \end{aligned}$$

891 The equation above allows us to derive a condition for the evolution of the altru-
892 istic trait p in the population. Using $cov(x, y) = \beta_{xy}var(y)$, where β_{xy} is the linear
893 regression coefficient of x on y , and dividing through by $var(c)$, we can rearrange
894 the above expression to find,

$$B_c(\beta_{\bar{c}c} + \beta_{\bar{g}c}) - C_c(1 + \beta_{gc}) > -\{C_g(1 + \beta_{cg}) + B_g(\beta_{\bar{c}g} + \beta_{\bar{g}g})\} \frac{var(g)}{var(c)}. \quad (\text{SI-9})$$

895 **SI-3 Model 1**

896 We imagine a population of haploid individuals who, once born, select a cultural
897 parent to copy. Each individual has two loci with a single allele present at each.
898 The allele at the first locus is genetically transmitted while the allele at the second is
899 received from a cultural parent. Individuals interact assortatively, with some prob-
900 ability of being genetically identical due to assortment, (f_g) , and culturally identical
901 due to assortment, (f_c) . At discrete time steps individuals meet a random kin mem-
902 ber and play a prisoner's dilemma according to a mixed strategy. The phenotype, p ,
903 is the probability of playing cooperate. The two loci mean four types of individuals
904 $\{0, 0\}$, $\{0, 1\}$, $\{1, 0\}$, $\{1, 1\}$, with phenotypes, $p_{00} = 0$, $p_{01} = 1/2$, $p_{10} = 1/2$, $p_{11} = 1$.

The expected reproductive fitnesses for each type are

$$w_{00} = w_0 + B_g(P(11|00) + P(01|00)/2 + P(10|00)/2) \quad (\text{SI-10a})$$

$$w_{01} = w_0 + B_g(P(11|01) + P(01|01)/2 + P(10|01)/2) - C_g/2 \quad (\text{SI-10b})$$

$$w_{10} = w_0 + B_g(P(11|10) + P(01|10)/2 + P(10|10)/2) - C_g/2 \quad (\text{SI-10c})$$

$$w_{11} = w_0 + B_g(P(11|11) + P(01|11)/2 + P(10|11)/2) - C_g. \quad (\text{SI-10d})$$

905 The conditional probabilities are probability of encountering a certain type given
 906 one's own type. For example, $P(10|00)$ denotes the "probability of encountering a
 907 $\{1, 0\}$ given that the player is a $\{0, 0\}$." Rather than enumerate all of these condi-
 908 tional probabilities we take advantage of the following identity:

$$P(g_o, c_o|g_p, c_p) = P(g_o|g_p)P(c_o|c_p), \quad (\text{SI-11})$$

where the o subscript indicates the opponent and p the player. We need only specify the following conditional probabilities,

$$P(g_o = 1|g_p = 1) = (1 - f_g)q_g + f_g \quad (\text{SI-12a})$$

$$P(g_o = 1|g_p = 0) = (1 - f_g)q_g \quad (\text{SI-12b})$$

$$P(c_o = 1|c_p = 1) = (1 - f_c)q_c + f_c \quad (\text{SI-12c})$$

$$P(c_o = 1|c_p = 0) = (1 - f_c)q_c. \quad (\text{SI-12d})$$

909 Note that the remaining marginal conditional probabilities are given by

$$P(g_o = 0|g_p = 0) = 1 - P(g_o = 1|g_p = 0) \quad (\text{SI-13a})$$

$$P(g_o = 0|g_p = 1) = 1 - P(g_o = 1|g_p = 1) \quad (\text{SI-13b})$$

$$P(c_o = 0|c_p = 0) = 1 - P(c_o = 1|c_p = 0) \quad (\text{SI-13c})$$

$$P(c_o = 0|c_p = 1) = 1 - P(c_o = 1|c_p = 1). \quad (\text{SI-13d})$$

910 Using (SI-11) we can calculate all the conditional probabilities of encounters be-
 911 tween types.

912 For each type we can write the following recursions for the frequency at the
 913 successive time step by simply multiplying the frequency of each phenotype after
 914 selection by the frequency of the cultural allele (which will not change between
 915 generations),

$$\phi'_{00} = \frac{1}{\bar{w}}(w_{00}\phi_{00} + w_{01}\phi_{01})(1 - q_c) \quad (\text{SI-14})$$

$$\phi'_{01} = \frac{1}{\bar{w}}(w_{00}\phi_{00} + w_{01}\phi_{01})q_c \quad (\text{SI-15})$$

$$\phi'_{10} = \frac{1}{\bar{w}}(w_{11}\phi_{11} + w_{10}\phi_{10})(1 - q_c) \quad (\text{SI-16})$$

$$\phi'_{11} = \frac{1}{\bar{w}}(w_{11}\phi_{11} + w_{10}\phi_{10})q_c. \quad (\text{SI-17})$$

916 We wish to know when the mean population phenotype increases,

$$\bar{p}' > \bar{p} \Rightarrow \phi'_{00} + \frac{1}{2}(\phi'_{01} + \phi'_{10}) > \phi_{00} + \frac{1}{2}(\phi_{01} + \phi_{10}). \quad (\text{SI-18})$$

917 Substituting (SI-10a)–(SI-10d) and (SI-14)–(SI-17) into (SI-18), gives,

$$\text{var}(g)(B_g f_g - C_g) > 0, \quad (\text{SI-19})$$

918 and multiplying both sides by $1/\text{var}(g)$ leads to (12).

919 **SI-4 Model 2**

In this model, individuals encounter one another and play a prisoner’s dilemma. This time, the game determines both the reproductive fitness and cultural fitness of the players. We imagine individuals producing ‘cultural gametes’, or behavioral tokens. The probability of acquiring a given cultural allele will be determined by the proportion that allele constitutes of all the available cultural gametes. The expected number of cultural gametes produced by individuals of each type are:

$$z_{00} = z_0 + B_c^z(P(11|00) + P(01|00)/2 + P(10|00)/2) \quad (\text{SI-20a})$$

$$z_{01} = z_0 + B_c^z(P(11|01) + P(01|01)/2 + P(10|01)/2) - C_c^z/2 \quad (\text{SI-20b})$$

$$z_{10} = z_0 + B_c^z(P(11|10) + P(01|10)/2 + P(10|10)/2) - C_c^z/2 \quad (\text{SI-20c})$$

$$z_{11} = z_0 + B_c^z(P(11|11) + P(01|11)/2 + P(10|11)/2) - C_c^z. \quad (\text{SI-20d})$$

920 It is important to note that the terms B_c^z and C_c^z are the gametic fitness benefit and
 921 cost, as opposed to B_c and C_c that appear in (9). The actual cultural fitness of an
 922 individual i is $s_i = z_i \frac{\bar{w}}{\bar{z}}$. Writing the cultural fitnesses in terms of the z_i , we can
 923 write the following recursions for the frequencies of types in this model:

$$\phi'_{00} = (1 - q'_g)(1 - q'_c) \quad (\text{SI-21a})$$

$$\phi'_{01} = (1 - q'_g)q'_c \quad (\text{SI-21b})$$

$$\phi'_{10} = q'_g(1 - q'_c) \quad (\text{SI-21c})$$

$$\phi'_{11} = q'_gq'_c, \quad (\text{SI-21d})$$

924 where,

$$q'_g = \frac{1}{\bar{w}}(w_{11}\phi_{11} + w_{10}\phi_{10}) \quad (\text{SI-22})$$

$$q'_c = \frac{1}{\bar{z}}(z_{11}\phi_{11} + z_{01}\phi_{01}). \quad (\text{SI-23})$$

925 We wish to solve for the condition when the mean phenotype increases in the $\bar{p}' > \bar{p}$,

$$\phi'_{11} + \frac{1}{2}(\phi'_{01} + \phi'_{10}) > \phi_{11} + \frac{1}{2}(\phi_{01} + \phi_{10}) \quad (\text{SI-24a})$$

⇓

$$q'_g + q'_c > q_g + q_c$$

⇓

$$\frac{1}{\bar{w}}(w_{11}\phi_{11} + w_{10}\phi_{10}) + \frac{1}{\bar{z}}(z_{11}\phi_{11} + z_{01}\phi_{01}) > q_g + q_c$$

(SI-24b)

926 Substituting $q_g = \phi_{11} + \phi_{10}$, $q_c = \phi_{11} + \phi_{01}$, we can rewrite SI-24a as,

$$\phi_{11}(z_{11} - \bar{z}) + \phi_{01}(z_{01} - \bar{z}) > -[\phi_{11}(w_{11} - \bar{w}) + \phi_{10}(w_{10} - \bar{w})] \frac{\bar{z}}{\bar{w}}$$

↓

$$q_g q_c (z_{11} - \bar{z}) + (1 - q_g) q_c (z_{01} - \bar{z}) > -[q_g q_c (w_{11} - \bar{w}) + q_g (1 - q_c) (w_{10} - \bar{w})] \frac{\bar{z}}{\bar{w}}.$$

927 Finally, substituting the fitness expressions (SI-10a)-(SI-10d) into the RHS, and af-
928 ter considerable algebra, we arrive at,

$$-[B_g f_g - C_g] q_g (1 - q_g) \frac{\bar{z}}{\bar{w}}. \quad (\text{SI-26})$$

929 Similarly, substituting the (SI-20a) into the LHS, and after some further algebra, we
930 have,

$$[B_c^z f_c - C_c^z] q_c (1 - q_c) \quad (\text{SI-27})$$

931 Combining the two sides of the inequality and dividing both sides by $q_c(1 - q_c)$, we
932 arrive at (14).

933 Alternatively, we could have arrived at (14) more directly by using (9). Substi-
934 tuting for the cultural fitness, $s_i = z_i \frac{\bar{w}}{\bar{z}}$, we can then rewrite (9) as,

$$B_c^z (\beta_{\bar{c}c} + \beta_{\bar{g}c}) - C_c^z (1 + \beta_{gc}) > -[B_g (\beta_{\bar{c}g} + \beta_{\bar{g}g}) - C_g (1 + \beta_{cg})] \frac{\text{var}(g)}{\text{var}(c)} \frac{\bar{z}}{\bar{w}} \quad (\text{SI-28})$$

935 Again, we compute the relevant terms:

$$\beta_{\tilde{c}c} = f_c \quad (\text{SI-29})$$

$$\beta_{\tilde{g}c} = 0 \quad (\text{SI-30})$$

$$\beta_{\tilde{c}g} = 0 \quad (\text{SI-31})$$

$$\beta_{\tilde{g}g} = f_g \quad (\text{SI-32})$$

$$\beta_{cg} = 0 \quad (\text{SI-33})$$

$$\text{var}(c) = \frac{1}{4}q_c(1 - q_c) \quad (\text{SI-34})$$

$$\text{var}(g) = \frac{1}{4}q_g(1 - q_g) . \quad (\text{SI-35})$$

936 Substituting these terms into (9) gives us (14).

937 **SI-5 Covariance between genetic and cultural trans-** 938 **mission**

939 The previous models all assumed that offspring acquired their genotype and culture-
940 type independently. Here we introduce a correlation between both types of inheri-
941 tance. Let k be the probability that an offspring individual with certainty acquires
942 the cultural allele that corresponds to their genetic allele (e.g. both genetic and cul-
943 tural altruism allele). In this case we have the following encounter probabilities:

$$P(11|11) = (f_g + (1 - f_g)q_g) \left[f_c + (1 - f_c) \frac{\phi_{11}}{\phi_{11} + \phi_{10}} \right] \quad (\text{SI-36a})$$

$$P(01|11) = (1 - f_g)(1 - q_g) \left[f_c + (1 - f_c) \frac{\phi_{01}}{\phi_{00} + \phi_{01}} \right] \quad (\text{SI-36b})$$

$$P(10|11) = (f_g + (1 - f_g)q_g)(1 - f_c) \frac{\phi_{10}}{\phi_{10} + \phi_{11}} \quad (\text{SI-36c})$$

$$P(00|11) = (1 - f_g)(1 - q_g)(1 - f_c) \frac{\phi_{00}}{\phi_{00} + \phi_{01}} \quad (\text{SI-36d})$$

$$P(11|10) = f_g(1 - f_c) \left(\frac{\phi_{11}}{\phi_{11} + \phi_{10}} + (1 - f_g)q_g(1 - f_c) \frac{\phi_{11}}{\phi_{11} + \phi_{10}} \right) \quad (\text{SI-36e})$$

$$P(01|10) = (1 - f_g)(1 - q_g)(1 - f_c) \frac{\phi_{01}}{\phi_{01} + \phi_{00}} \quad (\text{SI-36f})$$

$$P(10|10) = f_g \left[f_c + (1 - f_c) \frac{\phi_{10}}{\phi_{10} + \phi_{11}} \right] + (1 - f_g)q_g \left[f_c + (1 - f_c) \frac{\phi_{10}}{\phi_{10} + \phi_{11}} \right] \quad (\text{SI-36g})$$

$$P(00|10) = (1 - f_g)(1 - q_g) \left[f_c + (1 - f_c) \frac{\phi_{00}}{\phi_{00} + \phi_{01}} \right] \quad (\text{SI-36h})$$

$$P(11|01) = (1 - f_g)q_g \left[f_c + (1 - f_c) \frac{\phi_{11}}{\phi_{11} + \phi_{10}} \right] \quad (\text{SI-36i})$$

$$P(01|01) = (f_g + (1 - f_g)(1 - q_g)) \left[f_c + (1 - f_c) \frac{\phi_{01}}{\phi_{01} + \phi_{00}} \right] \quad (\text{SI-36j})$$

$$P(10|01) = (1 - f_g)q_g(1 - f_c) \frac{\phi_{10}}{\phi_{10} + \phi_{11}} \quad (\text{SI-36k})$$

$$P(00|01) = (f_g + (1 - f_g)(1 - q_g))(1 - f_c) \frac{\phi_{00}}{\phi_{00} + \phi_{01}} \quad (\text{SI-36l})$$

$$P(11|00) = (1 - f_g)q_g(1 - f_c) \frac{\phi_{11}}{\phi_{11} + \phi_{10}} \quad (\text{SI-36m})$$

$$P(01|00) = (f_g + (1 - f_g)(1 - q_g))(1 - f_c) \frac{\phi_{01}}{\phi_{01} + \phi_{00}} \quad (\text{SI-36n})$$

$$P(10|00) = (1 - f_g)q_g \left[f_c + (1 - f_c) \frac{\phi_{10}}{\phi_{10} + \phi_{11}} \right] \quad (\text{SI-36o})$$

$$P(00|00) = (f_g + (1 - f_g)(1 - q_g)) \left[f_c + (1 - f_c) \frac{\phi_{00}}{\phi_{00} + \phi_{01}} \right] \quad (\text{SI-36p})$$

$$(\text{SI-36q})$$

945 The recursions for the frequencies of types are:

$$\phi'_{11} = q'_g(k + (1 - k)q'_c) \quad (\text{SI-37})$$

$$\phi'_{10} = q'_g(1 - k)(1 - q'_c) \quad (\text{SI-38})$$

$$\phi'_{01} = (1 - q'_g)(1 - k)q'_c \quad (\text{SI-39})$$

$$\phi'_{00} = (1 - q'_g)(k + (1 - k)(1 - q'_c)) \quad (\text{SI-40})$$

946 Our model occurs in two stages. First, each individual produces a number of ge-
 947 netic and cultural gametes according to selection in both domains. Then those ga-
 948 metes are paired, either according to gene-culture assortment, or the pure gamete
 949 frequencies. We will call the frequency of the altruistic alleles amongst gametes, q_g
 950 and q_c , and the frequency of the alleles amongst actual individuals, q_g^\bullet and q_c^\bullet .

951 Again, we consider the condition $\bar{p}' > \bar{p}$, which leads to,

$$q'_g + q'_c + k(q'_g - q'_c) > q_g + q_c + k(q_g - q_c) \quad (\text{SI-41a})$$

⇓

$$(1 - k)(q'_c - q_c) > -(1 + k)(q'_g - q_g) \quad (\text{SI-41b})$$

Here, we note that $q'_c - q_c = (q'_c - q_g^\bullet) + (q_g^\bullet - q_c)$. The first term is the change in frequency from ancestral individuals to descendant gametes, while the second term is the change in frequency from ancestral gametes to ancestral individuals.

We have,

$$q'_c - q_g^\bullet = \phi_{11}(z_{11} - \bar{z}) + \phi_{01}(z_{01} - \bar{z}) = \frac{1}{\bar{z}} \text{cov}(z_i, c_i), \quad (\text{SI-41c})$$

and,

$$q_c^\bullet - q_c = q_g(k + (1 - k)q_c) + (1 - q_g)(1 - k)q_c - q_c = k(q_g - q_c). \quad (\text{SI-41d})$$

952 Noting that $q'_g - q_g = \frac{1}{\bar{w}} \text{cov}(w_i, g_i)$, we have,

$$(1 - k)\frac{1}{\bar{z}} (\text{cov}(z_i, c_i) + k(q_g - q_c)) > -(1 + k)\frac{1}{\bar{w}} \text{cov}(w_i, g_i) \quad (\text{SI-41e})$$

953 Expanding the covariance terms in the fitness functions and rearranging terms,

954 gives,

$$B^z(\beta_{\bar{g},c} + \beta_{\bar{c}c}) - C_z(1 + \beta_{gc}) > -[B_g(\beta_{\bar{g}g} + \beta_{\bar{c}g}) - C_g(1 + \beta_{cg})] \frac{(1+k)\bar{z}}{(1-k)\bar{w}} \frac{\text{var}(g_i)}{\text{var}(c_i)} - \frac{k(q_g - q_c)\bar{z}}{\text{var}(c_i)}$$

955 Computing all of the above regression coefficients we arrive at,

$$B_c \left(f_c + \frac{\text{var}(g)}{\text{var}(c)} k f_g (1 + k(1 - f_c)) \right) - C_c \left(1 + \frac{\text{var}(g)}{\text{var}(c)} k \right) > \\ - [B_g (k(f_c + f_g - f_c f_g) + f_g) - C_g (1 + k)] \frac{\text{var}(g)}{\text{var}(c)} \frac{(1+k)\bar{z}}{(1-k)\bar{w}} - \frac{k(q_g - q_c)\bar{z}}{\text{var}(c)}.$$

956 We can see that when $k = 1$ —meaning genotype and culture-type are perfectly

957 correlated—the condition is impossible to meet if genetic selection is opposed to

958 altruism.

959 **SI-6 Non-additive phenotypes**

We assume that all descendant individuals have a (potentially) unique function for mapping from heritable inputs to phenotype, $p_j(f_j(c_1, \dots, c_N), h_j(g_1, \dots, g_N))$. Assuming that the change in phenotype is small over small fluctuations in heritable inputs (e.g. because we are considering small evolutionary time scales), we can take a first order Taylor approximation of a phenotype function around the point $(\bar{c}, \dots, \bar{c}, \bar{g}, \dots, \bar{g}) = (\bar{\mathbf{c}}, \bar{\mathbf{g}})$,

$$p_j \approx p_j(\bar{\mathbf{c}}, \bar{\mathbf{g}}) + \sum_{i=1}^N \frac{\partial p_j}{\partial c_i} \Big|_{(\bar{\mathbf{c}}, \bar{\mathbf{g}})} (c_i - \bar{c}) + \sum_{i=1}^N \frac{\partial p_j}{\partial g_i} \Big|_{(\bar{\mathbf{c}}, \bar{\mathbf{g}})} (g_i - \bar{g}).$$

960 To obtain the Price equation, we can substitute the above expression into $\Delta \bar{p} = \bar{p}' - \bar{p}$,

$$\Delta \bar{p} \approx \frac{1}{N'} \sum_{j=1}^{N'} p_j(\bar{\mathbf{c}}, \bar{\mathbf{g}}) + \frac{1}{N'} \sum_{j=1}^{N'} \sum_{i=1}^N c_i \frac{\partial p_j}{\partial c_i} \Big|_{(\bar{\mathbf{c}}, \bar{\mathbf{g}})} + \frac{1}{N'} \sum_{j=1}^{N'} \sum_{i=1}^N g_i \frac{\partial p_j}{\partial g_i} \Big|_{(\bar{\mathbf{c}}, \bar{\mathbf{g}})} \quad (\text{SI-42})$$

$$- \frac{\bar{c}}{N'} \sum_{j=1}^{N'} \sum_{i=1}^N \frac{\partial p_j}{\partial c_i} \Big|_{(\bar{\mathbf{c}}, \bar{\mathbf{g}})} - \frac{\bar{g}}{N'} \sum_{j=1}^{N'} \sum_{i=1}^N \frac{\partial p_j}{\partial g_i} \Big|_{(\bar{\mathbf{c}}, \bar{\mathbf{g}})} - \bar{p} \quad (\text{SI-43})$$

961 Switching the order of all the summations, and defining the quantities, $\mathcal{S}_i = \sum_{j=1}^{N'} \frac{\partial p_j}{\partial c_i} \Big|_{(\bar{\mathbf{c}}, \bar{\mathbf{g}})}$,

962 and $W_i = \sum_{j=1}^{N'} \frac{\partial p_j}{\partial g_i} \Big|_{(\bar{\mathbf{c}}, \bar{\mathbf{g}})}$, we can write,

$$\Delta \bar{p} = \frac{N}{N'} [\text{cov}(\mathcal{S}_i, c_i) + \text{cov}(W_i, g_i) + \bar{c}\bar{\mathcal{S}} + \bar{g}\bar{W}] - \frac{N}{N'} \bar{c}\bar{\mathcal{S}} - \frac{N}{N'} \bar{g}\bar{W} \quad (\text{SI-44})$$

$$+ \frac{1}{N'} \sum_{j=1}^{N'} p_j(\bar{\mathbf{c}}, \bar{\mathbf{g}}) - \bar{p} \quad (\text{SI-45})$$

963 Canceling terms we arrive at Eq. (16).

964 If we continue our expansion of the phenotype function, we arrive at the follow-
 965 ing result,

$$\Delta\bar{p} = \frac{N}{N'}\text{cov}(\mathcal{S}_i, c_i) + \frac{N}{N'}\text{cov}(\mathcal{W}_i, g_i) + \frac{N}{N'}\text{cov}(\mathcal{I}_i, g_i) + \overline{p_j(\bar{\mathbf{c}}, \bar{\mathbf{g}})} - \bar{p} \quad (\text{SI-46})$$

966 where,

$$\begin{aligned} \mathcal{S}_i &= \sum_{j=1}^{N'} \left(\frac{\partial p_j}{\partial c_i} + \frac{1}{2} \sum_{k=1}^N \frac{\partial^2 p_j}{\partial c_i \partial c_k} (c_k - \bar{c}) + \frac{1}{3!} \sum_{k=1}^N \sum_{l=1}^N \frac{\partial^3 p_j}{\partial c_i \partial c_k \partial c_l} (c_k - \bar{c})(c_l - \bar{c}) + \dots \right) \\ \mathcal{W}_i &= \sum_{j=1}^{N'} \left(\frac{\partial p_j}{\partial g_i} + \frac{1}{2} \sum_{k=1}^N \frac{\partial^2 p_j}{\partial g_i \partial g_k} (g_k - \bar{g}) + \frac{1}{3!} \sum_{k=1}^N \sum_{l=1}^N \frac{\partial^3 p_j}{\partial g_i \partial g_k \partial g_l} (g_k - \bar{g})(g_l - \bar{g}) + \dots \right) \\ \mathcal{I}_i &= \sum_{j=1}^{N'} \left(\frac{1}{2} \sum_{k=1}^N \frac{\partial^2 p_j}{\partial g_i \partial c_k} (c_k - \bar{c}) + \frac{1}{3!} \sum_{k=1}^N \sum_{l=1}^N \frac{\partial^3 p_j}{\partial g_i \partial c_k \partial c_l} (c_k - \bar{c})(c_l - \bar{c}) \right. \\ &\quad \left. + \frac{1}{3!} \sum_{k=1}^N \sum_{l=1}^N \frac{\partial^3 p_j}{\partial g_i \partial c_k \partial g_l} (c_k - \bar{c})(g_l - \bar{g}) + \dots \right). \end{aligned}$$

967 The dots represent higher order terms in the expansion. The \mathcal{S}_i and \mathcal{W}_i terms are ex-
 968 clusive to the cultural and genetic domains, while the \mathcal{I}_i term captures interactions
 969 between the two forms of inheritance. The additional covariance term captures the
 970 effect of interactions between genes and culture. In expanding these phenotype
 971 functions in a Taylor series, we've been able to directly relate the concepts of fitness
 972 to phenotype while making only minimal assumptions about either.