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Antisocial rewarding in structured populations

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Abstract

Cooperation in collective action dilemmas usually breaks down in the absence of additional incentive mechanisms. This tragedy can be escaped if individuals have the possibility to invest in prosocial reward funds that are shared exclusively among cooperators. Yet, the presence of defectors who do not contribute to the public good, but do reward themselves deters cooperation in the absence of additional countermeasures. Although a recent simulation study suggests that interactions in spatially structured populations are sufficient to prevent such antisocial rewarding from deterring cooperation, the exact role of spatial structure on the evolution of public goods cooperation with pool-rewarding remains unclear. Here, we develop an analytical model of public goods with prosocial and antisocial pool rewarding in spatially structured populations. We show that increasing reward funds facilitates the maintenance of prosocial rewarding but prevents its evolution from rare in spatially structured populations, and that spatial structure can sometimes hinder, rather than promote, the evolution of prosocial rewarding. This is due to the fact that, counterintuitively, antisocial rewarding can be a more cooperative alternative than prosocial rewarding in some instances. Our results suggest that, even in spatially structured populations, additional mechanisms are required to prevent antisocial rewarding from deterring cooperation in public goods dilemmas.

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1 Introduction

Explaining the evolution of cooperation has been a long-standing challenge in evolutionary biology and the social sciences (Axelrod, 1984; Sugden, 1986; Frank, 1998; Lehmann and Keller, 2006; West et al., 2007; Sigmund, 2010). The problem is to explain how cooperators, whose contributions to the common good benefit everybody in a group, can prevent defectors from outcompeting them, leading to a tragedy of the commons where nobody contributes and no common good is created or maintained (Hardin, 1968).

A solution to the problem of cooperation is to provide individuals with additional incentives to contribute, thus making defection less profitable (Oliver, 1980; Hilbe and Sigmund, 2010). Incentives can be either negative (punishment) or positive (rewards). Punishment occurs when individuals are willing to spend resources in order for defectors to lose even more resources (Sigmund, 2007). Punishment can be stable against defection, since rare defectors are effectively punished (Boyd and Richerson, 1992). However, to evolve from rare and resist invasion by individuals who cooperate but restrain from investing into incentives, i.e., second-order defectors, punishers must gain from punishing, for example, through reputational benefits in future interactions (Hilbe and Sigmund, 2010; dos Santos et al., 2011; Hilbe and Traulsen, 2012).

Defection in collective action problems can also be prevented via positive incentives. Using rewards, cooperators can pay to increase the payoff of other cooperators. While the emergence of such behavior is usually favored, as there are very few cooperators to reward when cooperators are rare, it becomes increasingly costly as cooperators become more abundant in the population (Hauert, 2010). Hence, to resist second-order defectors, non-rewarding players must benefit less from rewards (Sasaki and Unemi, 2011). Alternatively, when both rewards and punishment are present, rewards can foster the emergence of punishment, which in turn can be stable provided second-order punishment is available (Sasaki et al., 2015).

Individuals can either decide to impose incentives unilaterally, or they can pool their effort to impose incentives collectively. When acting collectively, individuals invest into a fund used to either punish defectors or reward cooperators; in the latter scenario one speaks of "prosocial rewarding". These collective mechanisms can be viewed as primitive institutions, as group members both design and enforce the rules to administer incentives to overcome social dilemmas (Ostrom, 1990). Pool rewards (Sasaki and Unemi, 2011; Sasaki and Uchida, 2014) are particularly interesting because they involve the creation of resources, as opposed to their destruction (as in punishment). Prosocial rewarding can favor cooperation only if non-rewarding players can be sufficiently prevented from accessing reward funds so that second-order defectors benefit less from rewards than do rewarders (Sasaki and Unemi, 2011). However, the presence of "antisocial rewarders", i.e., individuals who do not contribute to the public good but reward themselves, destroys cooperation unless additional mechanisms, such as better rewarding abilities for prosocials, work in combination with exclusion (dos Santos, 2015).

Pool rewards can also be viewed as a consecutive collective action dilemma played exclusively among those who made a similar choice in the first public goods game, i.e., cooperators with each other, and defectors with each other. Situations where individuals are involved in different levels of social dilemmas are particularly likely to happen in bacterial communities (Hibbing et al., 2010). Indeed, many species of bacteria secrete public

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good molecules (e.g., iron-binding siderophores and other signaling molecules), which are susceptible to exploitation from both their own and other strains (Griffin et al., 2004; Khan et al., 2006; Hughes and d'Ettorre, 2008). In addition, bacteria are also involved in within-species public goods games, as some of those public good molecules can also be strain-specific (Hohnadel and Meyer, 1988; Hughes and d'Ettorre, 2008). We might then argue that the relevance of pool-reward mechanisms extends to non-human species.

A recent theoretical study challenged the view that additional mechanisms are required to prevent antisocial rewarding from deterring cooperation in public goods games [Szolnoki and Perc (2015); hereafter, SP15]. Instead, their authors showed that prosocial rewarding outcompetes antisocial rewarding if individuals interact preferentially with neighbors in a spatially structured population. Additionally, SP15 showed that increasing the amount of rewards is beneficial for prosocial rewarding. However, SP15 focused on one particular type of population structure (a lattice with overlapping groups), so it remains unclear whether their results generalize to a broader range of spatial models.

Here, we formulate a mathematical model of public goods with reward funds in spatially structured populations. Our model clarifies the role of spatial structure for cooperation to be favored through pool rewarding. The conditions under which cooperation is favored are derived analytically, using the "scaled relatedness coefficient" (Lehmann and Rousset, 2010; Van Cleve, 2015) as a natural measure of spatial structure. Scaled relatedness depends on demographic assumptions (e.g., whether or not generations are overlapping), and it has been determined for many models of spatial structure (Lehmann and Rousset, 2010; Van Cleve and Lehmann, 2013; Peña et al., 2015). To simplify our analysis, however, we treat scaled relatedness as an exogenous parameter. This allows us to make general predictions about the effect of spatial structure on cooperation, and to make connections between our results and the vast literature on inclusive fitness theory.

2 Model

2.1 Public goods game with prosocial and antisocial reward funds

Following SP15, let us assume that individuals interact in groups of size n and play a public goods game (PGG) followed by a rewarding stage. There are two types of actions available to individuals: "rewarding cooperation" (RC, or "prosocial rewarding"), whereby a benefit r_1/n is provided to each of the other $n-1$ group members at a cost γ, and "rewarding defection" (RD, or "antisocial rewarding"), whereby no benefit is provided and no cost is payed. The parameter r_1 is the synergy factor of the PGG, and it is such that $1 < r_1 < n$ holds.

Individuals choosing RC or RD also invest in their own reward funds. Each reward fund yields a per capita net reward $r_2 - \gamma$ (reward benefit r_2 minus cost of contributing to the reward pool γ) provided there is at least another individual playing the same action among the $n-1$ other group members, and zero otherwise, i.e., self-rewarding is not allowed and the cost γ is payed only if the rewarding institution is created. For example, a focal individual playing RC will pay the cost and receive the reward only if there is at

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least another RC among its $n-1$ partners. This reflects a situation where reward funds yield non-linear returns reminiscent of those of a volunteer's dilemma (Diekmann, 1985). Since the net reward $r_2 - \gamma$ does not depend on the group size n, r_2 can in principle take any value greater or equal than γ. Note that individuals choosing the most common action are more likely to get the reward, even under random group formation. Hence, RC can prevail as long as its frequency in the global population is above one half and rewards outweigh the net cost of contributing to the PGG. However, if self-rewarding is allowed, cooperation is never favored even when all individuals play RC (dos Santos, 2015).

With the previous assumptions, and letting without loss of generality $\gamma = 1$, the payoffs for a focal individual choosing either RC or RD when k co-players choose RC (and $n-1-k$ co-players choose RD) are respectively given by (cf. Equations 3.1, 3.2, and 3.3 in SP15):

$$c_k = \begin{cases} \frac{r_1}{n} - 1, & \text{if } k = 0 \\ r_1 \frac{k+1}{n} + r_2 - 2, & \text{if } 1 \leq k \leq n-1 \end{cases} \tag{1}$$

and

$$d_k = \begin{cases} r_1 \frac{k}{n} + r_2 - 1, & \text{if } 0 \leq k \leq n-2 \\ r_1 \frac{n-1}{n}, & \text{if } k = n-1. \end{cases} \tag{2}$$

Note that if everybody plays RC, everybody gets a payoff $c_{n-1} = r_1 + r_2 - 2$. Instead, if everybody plays RD, everybody gets $d_0 = r_2 - 1$. Since $r_2 > 1$ holds, $c_{n-1} > d_0$ holds for all values of r_1 and r_2, which means that prosocial rewarding Pareto dominates antisocial rewarding. If players stick to pure strategies, they are collectively better being prosocial rewarders than being antisocial rewarders.

2.2 Evolutionary dynamics

We assume that individuals implement mixed strategies, i.e., they play RC with probability z and RD with probability $1-z$. We are interested in the long-term evolution of the phenotype z in an homogenous spatially structured population; more precisely, we look into convergence stable strategies (Geritz et al., 1998) under trait substitution dynamics (Rousset, 2004). This requires us to investigate the sign of the selection gradient on z, which under standard assumptions (Van Cleve and Lehmann, 2013; Van Cleve, 2015) can be shown to be proportional to the "inclusive gain function" (Peña et al., 2015)

$$\mathcal{G}(z) = -\mathcal{C}(z) + \kappa \mathcal{B}(z), \tag{3}$$

where

$$-\mathcal{C}(z) = \sum_{k=0}^{n-1} \binom{n-1}{k} z^k (1-z)^{n-1-k} \Delta_k \tag{4}$$

is the effect of the individual's behavior on itself (i.e., the "direct effect"),

$$\mathcal{B}(z) = \sum_{k=0}^{n-1} \binom{n-1}{k} z^k (1-z)^{n-1-k} \Theta_k \tag{5}$$

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is the effect of co-players' behavior on the focal individual (i.e., the "indirect effect"), and κ is the "scaled relatedness coefficient", which accounts for both increased genetic relatedness and increased local competition brought about by spatial structure (Lehmann and Rousset, 2010; Van Cleve and Lehmann, 2013; Van Cleve, 2015). Here,

$$\Delta_k = c_k - d_k \tag{6}$$

are the "direct gains from switching" recording the changes in payoff experienced by a focal if it unilaterally switches its action from RD to RC when k co-players stick to RC (and $n-1-k$ stick to RD), and

$$\Theta_k = k\,(c_k - c_{k-1}) + (n-1-k)\,(d_{k+1} - d_k) \tag{7}$$

are the "indirect gains from switching" recording the changes in the total payoff accrued by co-players when the focal unilaterally switches its action from RD to RC (Peña et al., 2015). Eq. (4) and (5) express the direct effect $-\mathcal{C}(z)$ and the indirect effect $\mathcal{B}(z)$ as expected values of the direct and indirect gains from switching when the number of other individuals playing RC is distributed according to a binomial distribution with parameters n and z.

A necessary and sufficient condition for a mutant with phenotype $z+\delta$ to invade a resident population of phenotype z when δ is vanishingly small is that the inclusive gain function evaluated at z is greater than zero, i.e., that $-\mathcal{C}(z) + \kappa\mathcal{B}(z) > 0$ holds. This corresponds to a scaled form of the marginal version of Hamilton's rule (Lehmann and Rousset, 2010; Van Cleve, 2015). More importantly for our purposes, the inclusive gain function allows to identify "convergence stable" evolutionary equilibria (Eshel, 1983, 1996; Taylor, 1989); these are given either by singular strategies z^* (i.e., the zeros of the inclusive gain function) satisfying $\mathcal{G}'(z^*) < 0$, or by the extreme points $z = 0$ (if $\mathcal{G}(0) < 0$) and $z = 1$ (if $\mathcal{G}(1) > 0$). Convergence stability is a standard way of characterizing long-term evolutionary attractors; a phenotype z^* is convergence stable if for resident phenotypes close to z^* mutants can invade only if mutants are closer to z^* than the resident (Geritz et al., 1998).

3 Results

Calculating the gains from switching by first replacing Eq. (1) and (2) into Eq. (6) and (7), then replacing the resulting expressions into Eq. (4) and (5), and simplifying, we obtain that the inclusive gain function for the PGG with reward funds can be written as (see Appendix A for a derivation)

$$\begin{aligned}\mathcal{G}(z) = {} & \underbrace{\frac{r_1}{n} - 1 + (r_2 - 1)\big[z^{n-1} - (1-z)^{n-1}\big]}_{-\mathcal{C}(z)} \\ & + \kappa\,\underbrace{(n-1)\Big\{\frac{r_1}{n} + (r_2-1)\big[z(1-z)^{n-2} - z^{n-2}(1-z)\big]\Big\}}_{\mathcal{B}(z)}.\end{aligned} \tag{8}$$

In the following, we identify convergence stable equilibria and characterize the evolutionary dynamics, first for well-mixed and then for spatially structured populations.

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3.1 Well-mixed populations

For well-mixed populations, the scaled relatedness coefficient is equal to zero. In this case, the inclusive gain function reduces to $\mathcal{G}(z) = -\mathcal{C}(z)$ and we obtain the following characterization of the evolutionary dynamics (see Appendix B for a derivation). If $r_1/n + r_2 \leq 2$, $z = 0$ is the only stable equilibrium, and RD dominates RC. Otherwise, if $r_1/n + r_2 > 2$, both $z = 0$ and $z = 1$ are stable, and there is a unique $z^* > 1/2$ that is unstable. In this case, the evolutionary dynamics are characterized by bistability or positive frequency dependence, with the basin of attraction of full RD ($z = 0$) being always larger than the basin of attraction of full RC ($z = 1$). Moreover, z^* (and hence the basin of attraction of $z = 0$) decreases with increasing r_1 and r_2. In particular, higher reward funds lead to less stringent conditions for RC to evolve. In any case, RC has to be initially common ($z > 1/2$) in order for full RC to be the final evolutionary outcome.

3.2 Spatially structured populations

Interactions in spatially structured populations (for which κ is not necessarily equal to zero) can dramatically alter the evolutionary dynamics of public goods with prosocial and antisocial rewards. In particular, we find that whether or not the extreme points $z = 0$ and $z = 1$ are stable depends on how the scaled relatedness coefficient κ compares to the critical values

$$\kappa_* = \frac{n(2 - r_2) - r_1}{(n-1)r_1}, \tag{9}$$

and

$$\kappa^* = \frac{nr_2 - r_1}{(n-1)r_1}, \tag{10}$$

which satisfy $\kappa_* \leq \kappa^*$, in the following way (Fig. 1). For low values of κ ($\kappa < \kappa_*$), full RD ($z = 0$) is stable and full RC ($z = 1$) is unstable. For intermediate values of κ ($\kappa_* < \kappa < \kappa^*$), both full RD and full RC are stable. For large values of κ ($\kappa > \kappa^*$), full RC is stable and full RD is unstable. Note that, for a given group size n and PGG synergy factor r_1, $\kappa_* = \kappa^*$ if and only if $r_2 = 1$, i.e., if rewards are absent. In this case, full RD and full RC cannot be both stable.

Rewards have contrasting effects on κ_* (the critical scaled relatedness value below which full RC is unstable) and κ^* (the critical scaled relatedness value above which full RD is unstable). On the one hand, κ_* is decreasing in the reward benefit r_2, so larger rewards increase the parameter space where full RC is stable. If spatial structure is maximal, i.e., $\kappa = 1$, the condition for full RC to be stable is $r_1 + r_2 > 2$, which always holds. On the other hand, κ^* is an increasing function of r_2. Hence, larger rewards make it harder for spatial structure to destabilize the full RD equilibrium, and hence for RC to increase when rare. For $\kappa = 1$, full RD is still stable whenever $r_1 < r_2$. Contrastingly, full defection can never be stable if $\kappa = 1$ in the absence of rewards (i.e., $r_2 = 1$) since, by definition, $r_1 > 1$. From this analysis we can already conclude that even maximal spatial structure does not necessarily allow RC to invade and increase when rare. In addition, a minimum critical value of scaled relatedness is required for prosocial rewarding to be stable once it is fully adopted by the entire population.

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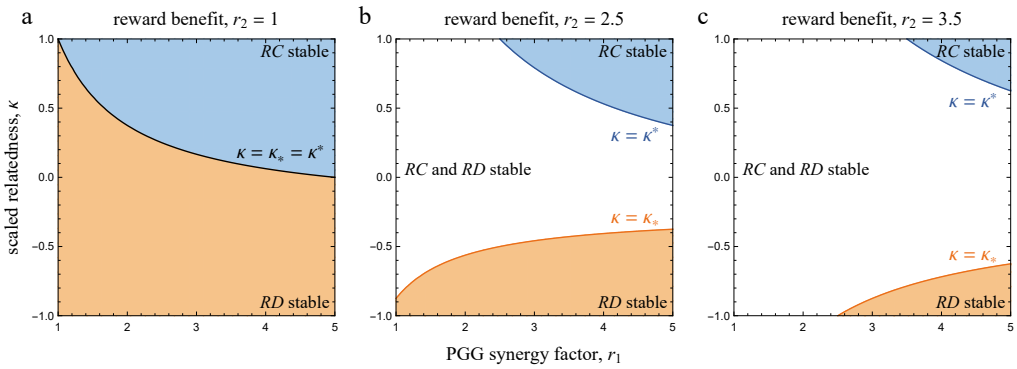

Figure 1: Phase diagrams illustrating the possible dynamical regimes of public goods games with prosocial and antisocial reward funds. Prosocial rewarding (RC) is stable if $\kappa > \kappa_*$, while antisocial rewarding (RD) is stable if $\kappa < \kappa^*$. The critical values κ_* and κ^* are functions of the public goods game synergy factor r_1, the reward benefit r_2, and the group size n, as given by Eqs. (9) and (10). Increasing the reward benefit r_2 makes it more difficult for both prosocial and antisocial rewarding to increase from rare. Parameters: $n = 5$.

Let us now investigate singular strategies. Depending on the parameter values, there can be either zero, one, or three interior points at which the inclusive gain function (and hence the selection gradient) vanishes (see Appendix C for a proof). If there is a unique singular point, then it is unstable while $z = 0$ and $z = 1$ are stable, and the evolutionary dynamics is characterized by bistability. If there are three singular points (probabilities z_{L}, z_{M}, and z_{R}, satisfying $0 < z_{\mathrm{L}} < z_{\mathrm{M}} < z_{\mathrm{R}} < 1$), then $z = 0$, z_{M}, and $z = 1$ are stable, while z_{L} and z_{R} are unstable. In this case the evolutionary dynamics allows for the coexistence of both RD and RC at the stable mixed strategy z_{M}; a necessary condition for this dynamical outcome is both relatively large reward benefits and relatively large scaled relatedness.

We calculated the singular strategies numerically, as the equation $\mathcal{G}(z) = 0$ cannot be solved algebraically in the general case (Fig. 2 and Fig. 3). Increasing scaled relatedness generally increases the parameter space where RC is favored. Yet, there are cases where increasing scaled relatedness can hinder the evolution of RC. Specifically, when the reward benefit is considerably larger than the public goods share, increasing scaled relatedness can increase the basin of attraction of the full RD equilibrium (Fig. 2*c*). Also, increasing rewards can be detrimental to RC in spatially structured populations by increasing the basin of attraction of full RD (Fig. 3*c*, *f*, *h*, *i*); this is never the case when there is no spatial structure (Fig. 3*a*, *d*, *g*; cf. section 3.1). Finally, the best case scenario from the point of view of a rare mutant playing $z = \delta$ (where δ is vanishingly small) is in the absence of rewards (i.e., $r_2 = 1$), because that is the case where the required threshold value of scaled relatedness to favor prosocial rewarding is the lowest (i.e., where κ^* attains its minimum value in Eq. (10)).

In order to understand why, contrary to naive expectations, increasing spatial structure might sometimes select against RC, note first that the derivative of the inclusive gain function with respect to κ is equal to the indirect effect $\mathcal{B}(z)$. This is nonnegative if

$$\frac{r_1}{n} + (r_2 - 1)\big[\underbrace{z(1-z)^{n-2} - z^{n-2}(1-z)}_{q(z)}\big] \geq 0. \tag{11}$$

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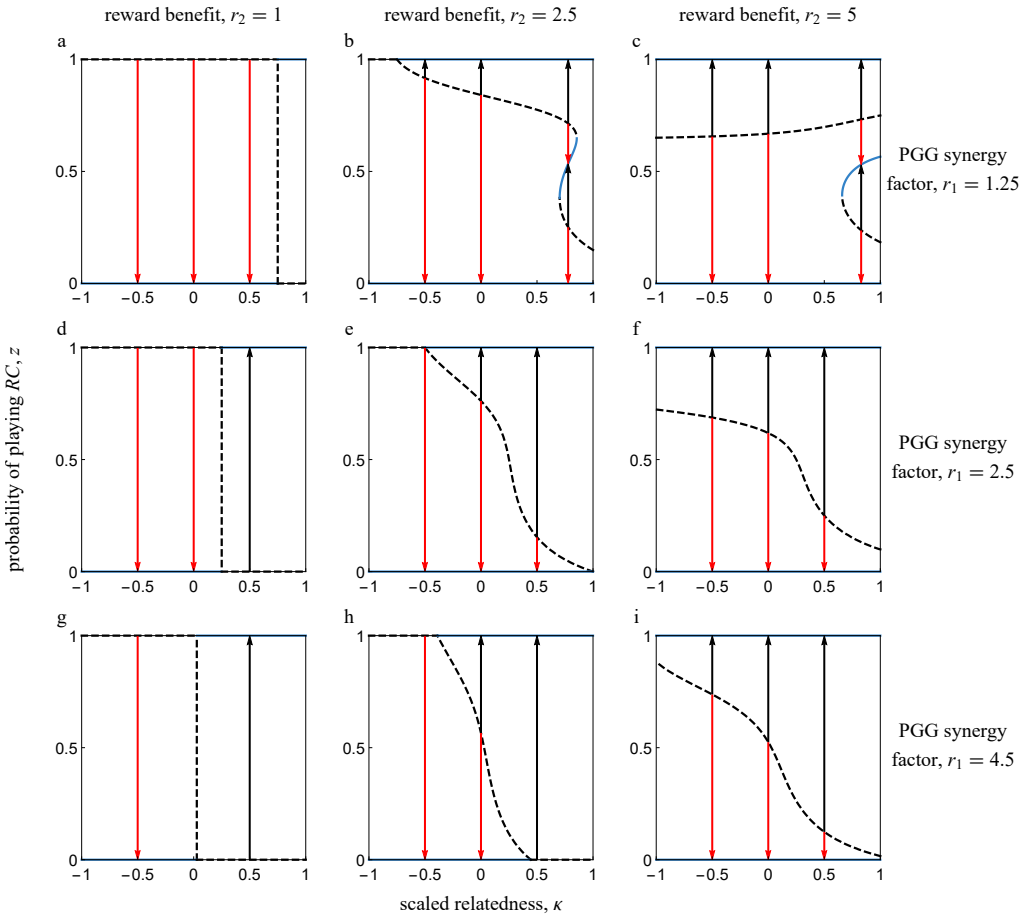

Figure 2: Bifurcation plots illustrating the evolutionary dynamics of pool rewarding in spatially structured populations. The scaled relatedness coefficient serves as a control parameter. Arrows show the direction of evolution for the probability of playing prosocial rewarding. Solid (dashed) lines correspond to convergence stable (unstable) equilibria. In the left column panels (a, d, g), rewards are absent (i.e., $r_2 = 1$). In the middle column panels (b, e, h), $r_2 = 2.5$. In the right column panels (c, f, i), $r_2 = 4.5$. In the top row panels (a, b, c), $r_1 = 1.25$. In the middle row panels (d, e, f), $r_1 = 2.5$. In the bottom row panels (g, h, i), $r_1 = 4.5$. In all panels, $n = 5$.

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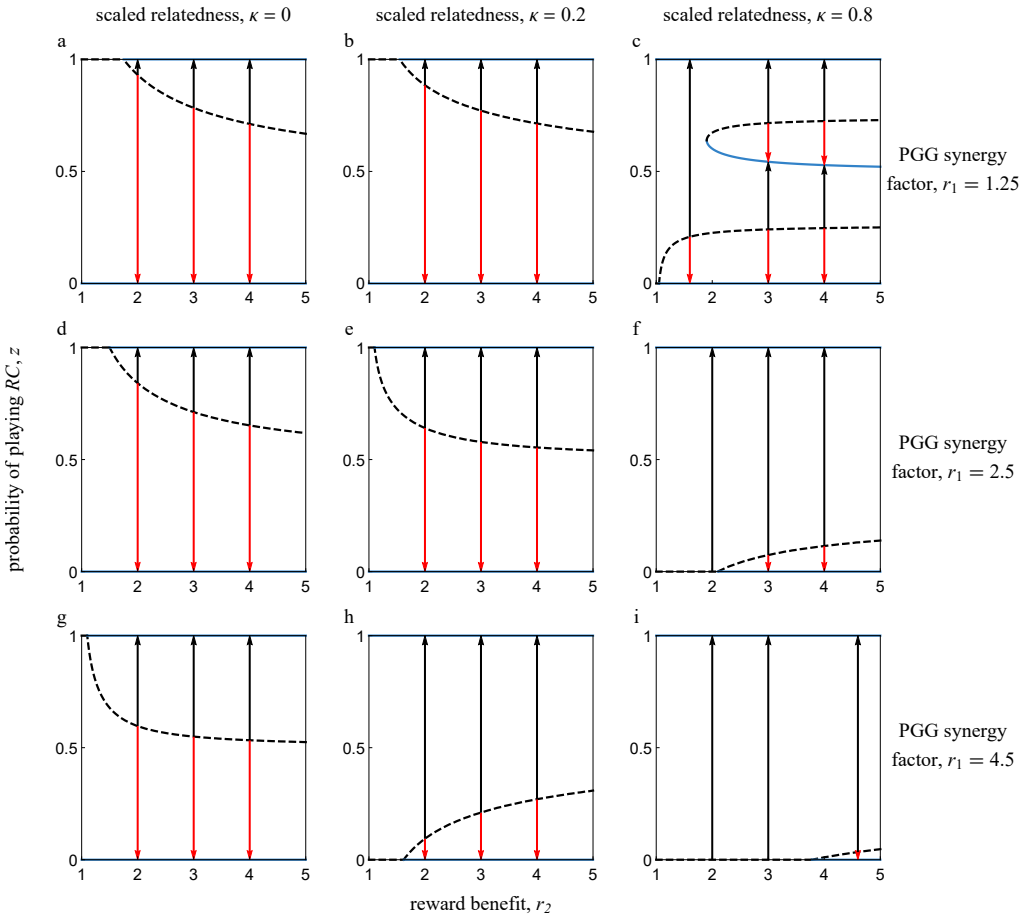

Figure 3: Bifurcation plots illustrating the evolutionary dynamics of of pool rewarding in spatially structured populations. The reward benefit serves as a control parameter. Arrows show the direction of evolution for the probability of playing prosocial rewarding. Solid (dashed) lines correspond to convergence stable (unstable) equilibria. In the left column panels (a, d, g), there is no spatial structure (i.e., $\kappa = 0$). In the middle column panels (b, e, h), $\kappa = 0.2$. In the right column panels (c, f, i), $\kappa = 0.8$. In the top row panels (a, b, c), $r_1 = 1.25$. In the middle row panels (d, e, f), $r_1 = 2.5$. In the bottom row panels (g, h, i), $r_1 = 4.5$. In all panels, $n = 5$.

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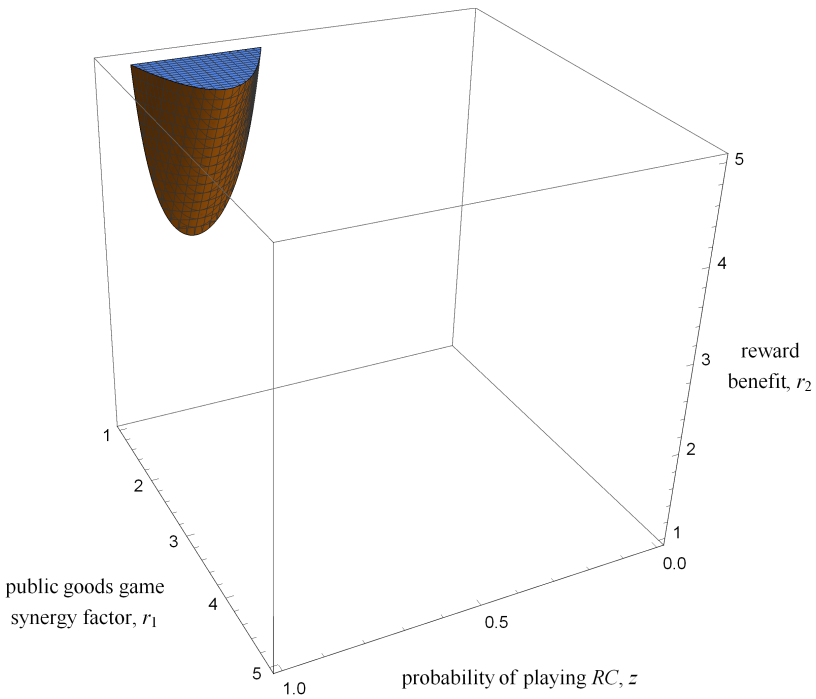

Figure 4: Parameter space where condition (11) does not hold and increasing spatial structure is detrimental to prosocial rewarding for some values of the probability of playing prosocial rewarding, z. Parameters: $n = 5$.

In the absence of rewards (i.e., $r_2 = 1$), condition (11) always holds. That is, increasing scaled relatedness always promotes cooperation when there are no rewards. In addition, when $0 \leq z \leq 1/2$, the function $q(z)$ is nonnegative, so that condition (11) holds and $\mathcal{B}(z)$ is positive. Hence, increasing scaled relatedness is always beneficial for RC when such behavior is expressed less often than RD. However, increasing scaled relatedness might not always favor RC when such behavior is already common in the population, i.e., if $z > 1/2$. Indeed, when the synergy factor of the PGG is relatively small and rewards are relatively large, condition (11) is not fulfilled for some z and $\mathcal{B}(z)$ is negative for some probability of playing RC (Fig. 4).

A closer look at the indirect gains from switching Θ_k (Eq. 7) reveals why $\mathcal{B}(z)$, and hence the effect of scaled relatedness on the selection gradient, can be negative for some z. The indirect gains from switching are nonnegative for all $k \neq n-2$. For $k = n-2$ and $n \geq 4$ we have $\Theta_{n-2} = (n-1)r_1/n - r_2 + 1$, which can be negative if

$$r_2 - 1 > (n-1)r_1/n \tag{12}$$

holds. Inequality (12) is hence a necessary condition for $\mathcal{B}(z)$ to be negative for some z and for prosocial rewarding to fail to qualify as payoff cooperative or payoff altruistic [*sensu* Peña et al. (2015)]. Indeed, when condition (12) holds and hence $\Theta_{n-2} < 0$, prosocial rewarding cannot be said to be altruistic according to the "focal-complement" interpretation of altruism (Matessi and Karlin, 1984; Kerr et al., 2004). This is because the sum of the payoffs of the $n-1$ co-players of a given focal individual, out of which $n-2$ play RC and one plays RD, is larger if the focal plays RD than if the focal plays RC. We also point out that RC is not altruistic according to an "individual-centered" interpretation (Uyenoyama and Feldman, 1980; Kerr et al., 2004) or "cooperative" [*sensu* Peña

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et al. (2016b)] if

$$r_2 - 1 > r_1/n, \tag{13}$$

since in this case the payoff to a focal individual playing RD as a function of the number of other players choosing RC in the group, d_k (see Eq. (2)), is decreasing (and not increasing) with k at $k = n - 2$. Indeed, if condition (13) holds, players do not necessarily prefer other group members to play RC irrespective of their own strategy: a focal RD player would prefer one of its $n - 1$ co-players to play RD rather than play RC. In the light of this, it is perhaps less surprising that for some parameters increasing spatial structure can be detrimental to the evolution of prosocial rewarding, even if prosocial rewarding Pareto dominates antisocial rewarding.

4 Discussion

We have investigated the effect of spatial structure on the evolution of public goods cooperation with reward funds. Measuring spatial structure by means of the scaled relatedness coefficient allowed us to capture both the effects of increased assortment and increased local competition that characterize evolution in spatially structured populations. We have found that (i) prosocial rewarding cannot invade full antisocial rewarding unless the scaled relatedness coefficient is sufficiently large, but (ii) increasing scaled relatedness can be detrimental to prosocial rewarding in cases where rewards are considerably larger than the public goods share. We have also demonstrated the contrasting effects of increasing rewards, which (iii) only benefits prosocial rewarding in well-mixed populations, but (iv) can also benefit antisocial rewarding in spatially structured populations. These results illustrate the complex nature of the evolutionary dynamics of multi-player games in the presence of spatial structure.

Our study revealed a perhaps counterintuitive result, namely, that increasing spatial structure can be detrimental to prosocial rewarding. While spatial structure generally favors the evolution of cooperation (Ohtsuki et al., 2006; Lehmann et al., 2007; Débarre et al., 2014; Peña et al., 2016b), it can sometimes oppose it (Hauert and Doebeli, 2004; Van Cleve and Lehmann, 2013; Peña et al., 2016a). In our case, spatial structure can oppose selection because the indirect gains from switching from antisocial to prosocial rewarding can be negative. In particular, if $n \geq 4$ and exactly $n - 2$ co-players choose prosocial rewarding and one co-player chooses antisocial rewarding, choosing antisocial rewarding (RD) rather than prosocial rewarding (RC) might increase (rather than decrease) the payoffs of co-players. The reason is that, by choosing RD, the focal player helps its RD co-player getting the reward fund, while allowing also its RC co-players to keep theirs, as the focal contribution is not critical to the creation of the prosocial reward fund. If the reward benefit is so large that Eq. (12) holds, the benefit to the single RD co-player is greater than what everybody loses by the focal not contributing to the public goods game, and the sum of payoffs to co-players is greater if the focal plays RD than if it plays RC. This implies that, although prosocial rewarding Pareto dominates antisocial rewarding, it does not strictly qualify as being payoff altruistic or payoff cooperative (Peña et al., 2015), hence the mixed effects of increasing spatial structure.

Our results also revealed the fact that higher values of the reward benefit r_2 make it more difficult for prosocial rewarding to invade from rare in spatially structured populations.

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Indeed, the critical value of scaled relatedness required for prosocial rewarding to be favored over antisocial rewarding is greater in the presence of rewards than in their absence. This is striking because rewards are meant to be mechanisms incentivizing provision in public goods games (Sasaki and Unemi, 2011; Sasaki and Uchida, 2014), rather than making collective action more difficult to emerge. Obviously, our result hinges on the assumption that prosocial rewarders and antisocial rewarders are both equally effective in rewarding themselves, i.e., r_2 is the same for both prosocial and antisocial rewarders. Challenging this assumption by making investments in rewards contingent on the production of the public good, or by increasing the ability of prosocials to reward each other relative to that of antisocials (dos Santos, 2015), will necessarily change this picture and promote prosocial rewarding in larger regions of the parameter space.

Although higher rewards prevent the invasion of prosocial rewarding from rare, we have also shown that, once prosocial rewarding is common, higher rewards can further enhance the evolution of prosocial rewarding. These results are in line with the findings of SP15, who showed that when both population structure is strong (their spatial model supports cooperation even in the absence of rewards) and the initial frequency of prosocial rewarding is relatively high (i.e., 1/4 in all their simulations), larger rewards promote prosocial rewarding. However, SP15 focused on one particular type of population structure (a square lattice with overlapping groups of size $n = 5$). Instead, our model considered in an abstract way a wide array of spatially structured populations, each characterized by a particular value of the scaled relatedness coefficient. This allowed us to identify regions of the parameter space where increasing spatial structure can be detrimental to prosocial rewarding. In addition, our analytical treatment allowed us to consider the entire range of different initial conditions (including extreme rareness of prosocial rewarding). However, such straightforward analysis required us to assume mixed strategies and δ-weak selection (Peña et al., 2015). An analytical description of the evolutionary dynamics of multi-player games with discrete strategies in spatially structured populations (as in SP15) would require tracking higher-order genetic associations (Roze and Rousset, 2008; Ohtsuki, 2010, 2014) or structure coefficients (Wu et al., 2013; Peña et al., 2016b) which cannot be in general captured by a simple scaled relatedness coefficient, as the one used in the present study. Therefore, the invasion and equilibrium conditions of our model and that of SP15 might not necessarily be identical. In fact, such conditions differ even between discrete- and continuous-strategy two-player games (Grafen, 1979; Wild and Traulsen, 2007).

Both our model and that of SP15 have not considered the presence of individuals who are able to benefit from reward funds without contributing to them. In other words, second-order defection is avoided by design. Allowing for second-order defection makes cooperation through pool rewarding vulnerable to defection, even in the absence of antisocial rewarding (Sasaki and Unemi, 2011). Therefore, even though the conclusions of SP15 contradict the findings of dos Santos 2015, namely that antisocial rewarding deters cooperation except in certain conditions (e.g., better rewarding abilities for prosocials), SP15 did not investigate standard pool-rewarding (Sasaki and Unemi, 2011; Sasaki and Uchida, 2014; dos Santos, 2015). Hence, their claim that spatial structure prevents antisocial rewarding from deterring cooperation, while not always true as we have shown here, does not apply to the more general case of pool-reward funds where second-order defection is allowed. Exploring the effects of spatial structure in these more realistic cases remains an interesting line of research.

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To conclude, we find that antisocial rewarding deters the evolution of cooperation from rare unless scaled relatedness is sufficiently high and rewards are relatively low, or ideally absent. We argue that additional countermeasures, such as exclusion and better rewarding abilities for prosocials (dos Santos, 2015), are still required to (i) prevent antisocial rewarding from deterring cooperation between unrelated social partners, and (ii) allow prosocial rewarding to invade either when relatedness is low or when rewards are too large.

Competing interests

The authors declare no competing interests.

Authors' contributions

M.D.S. and J.P. conceived and designed the study, performed the analysis, and wrote the manuscript. All authors gave final approval for submission.

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A Inclusive gain function

To derive the inclusive gain function $\mathcal{G}(z)$, we first calculate the direct and indirect gains from switching (Eq. (6) and (7)) associated to the payoffs of the game. We find that the gains from switching depend on the group size n in the following way.

1. For $n = 2$: $(\Delta_0, \Delta_1) = (r_1/2 - r_2, r_1/2 + r_2 - 2)$ and $(\Theta_0, \Theta_1) = (r_1/2 - r_2 + 1, r_1/2 + r_2 - 1)$.
2. For $n = 3$: $(\Delta_0, \Delta_1, \Delta_2) = (r_1/3 - r_2, r_1/3 - 1, r_1/3 + r_2 - 2)$ and $(\Theta_0, \Theta_1, \Theta_2) = (2r_1/3, 2r_1/3, 2r_1/3)$.
3. For $n = 4$: $(\Delta_0, \Delta_1, \Delta_2, \Delta_3) = (r_1/4 - r_2, r_1/4 - 1, r_1/4 - 1, r_1/4 + r_2 - 2)$ and $(\Theta_0, \Theta_1, \Theta_2, \Theta_3) = (3r_1/4, 3r_1/4 + r_2 - 1, 3r_1/4 - r_2 + 1, 3r_1/4)$.
4. For $n \geq 5$: $(\Delta_0, \Delta_1, \ldots, \Delta_{n-2}, \Delta_{n-1}) = (r_1/n - r_2, r_1/n - 1, \ldots, r_1/n - 1, r_1/n + r_2 - 2)$ and $(\Theta_0, \Theta_1, \Theta_2, \ldots, \Theta_{n-3}, \Theta_{n-2}, \Theta_{n-1}) = ((n-1)r_1/n, (n-1)r_1/n + r_2 - 1, (n-1)r_1/n, \ldots, (n-1)r_1/n, (n-1)r_1/n - r_2 + 1, (n-1)r_1/n)$.

Replacing the direct gains from switching Δ_k into the expression for the direct effect $-\mathcal{C}(z)$ (Eq. (4)) and the indirect gains from switching Θ_k into the expression for the indirect effect $\mathcal{B}(z)$ (Eq. (5)), and then both of these functions into the expression for the inclusive gain function $\mathcal{G}(z)$ (Eq. (3)), and simplifying, we obtain the formula given in Eq. (8), which is valid for all $n \geq 2$.

B Evolutionary dynamics for $\kappa = 0$

For $\kappa = 0$, the inclusive gain function $\mathcal{G}(z)$ reduces to $-\mathcal{C}(z)$. This function is increasing and its end-points are given by $-\mathcal{C}(0) = r_1/n - r_2$ and $-\mathcal{C}(1) = r_1/n + r_2 - 2$. Since $1 < r_1 < n$ and $r_2 > 1$, $-\mathcal{C}(0) < 0$ always hold, and $z = 0$ is always stable. If $r_1/n + r_2 \geq 2$, $-\mathcal{C}(z)$ is nonpositive for all z and $z = 0$ is the only stable equilibrium. If $r_1/n + r_2 < 2$, $-\mathcal{C}(1) > 0$ and $z = 1$ is also stable. In this case, and since $-\mathcal{C}(z)$ is increasing, $-\mathcal{C}(z)$ has a single zero z^* in $(0, 1)$ giving rise to an unstable equilibrium. Such zero is given by the unique solution to

$$\underbrace{z^{n-1} - (1-z)^{n-1}}_{p(z)} = \underbrace{\frac{n - r_1}{n(r_2 - 1)}}_{\alpha}.$$

Since $p(z)$ is increasing in z, $p(1/2) = 0$, and $\alpha > 0$ always holds, $z^* > 1/2$ holds true. Additionally, since α is decreasing in both r_1 and r_2, z^* is increasing in both r_1 and r_2.

C Evolutionary dynamics for $\kappa \neq 0$

Rearranging terms, the inclusive gain function $\mathcal{G}(z)$ given by Eq. (8) can be alternatively written as

$$\mathcal{G}(z) = \frac{r_1}{n}\left[1 + \kappa(n-1)\right] - 1 + (r_2 - 1)\mathcal{P}(z)$$

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where

$$\mathcal{P}(z) = \sum_{k=0}^{n-1} \binom{n-1}{k} z^k (1-z)^{n-1-k} \zeta_k$$

is a polynomial in Bernstein form (Peña et al., 2014) of degree $n-1$ with coefficients given by

1. $(\zeta_0, \zeta_1) = (-(1+\kappa), 1+\kappa)$ if $n = 2$.
2. $(\zeta_0, \zeta_1, \zeta_2) = (-1, 0, 1)$ if $n = 3$.
3. $(\zeta_0, \zeta_1, \zeta_2, \zeta_3) = (-1, \kappa, -\kappa, 1)$ if $n = 4$.
4. $(\zeta_0, \zeta_1, \zeta_2, \ldots, \zeta_{n-3}, \zeta_{n-2}, \zeta_{n-1}) = (-1, \kappa, 0, \ldots, 0, -\kappa, 1)$ if $n \geq 5$.

The number of sign changes (and hence of singular points) of $\mathcal{G}(z)$ is bounded from above by the number of sign changes of $\mathcal{P}(z)$. Moreover, and by the variation-diminishing property of polynomials in Bernstein form (Peña et al., 2014), the number of sign changes of $\mathcal{P}(z)$ is equal to the number of sign changes of the sequence of coefficients $(\zeta_0, \ldots, \zeta_{n-1})$ minus an even integer. It then follows that the number of singular points is at most one if $n \leq 3$ or if scaled relatedness is nonpositive, $\kappa \leq 0$. In this case, the unique singular point z^* is convergence unstable. However, if $n \geq 4$ and $\kappa > 0$, there could be up to three singular points z_{L}, z_{M}, and z_{R} satisfying $0 < z_{\mathrm{L}} < z_{\mathrm{M}} < z_{\mathrm{R}} < 1$ such that z_{L} and z_{R} are convergence unstable and z_{M} is convergence stable.

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