

1 Spatial Correlation as an Early Warning Signal of
2 Regime Shifts in a Multiplex Disease-Behaviour
3 Network

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7 **Abstract**

Early warning signals of sudden regime shifts are a widely studied phenomenon for their ability to quantify a system's proximity to a tipping point to a new and contrasting dynamical regime. However, this effect has been little studied in the context of the complex interactions between disease dynamics and vaccinating behaviour. Our objective was to determine whether critical slowing down (CSD) occurs in a multiplex network that captures opinion propagation on one network layer and disease spread on a second network layer. We parameterized a network simulation model to represent a hypothetical self-limiting, acute, vaccine-preventable infection with short-lived natural immunity. We tested five different network types: random, lattice, small-world, scale-free, and an empirically derived network. For the first four network types, the model exhibits a regime shift as perceived vaccine risk moves beyond a tipping point from full vaccine acceptance and disease elimination to full vaccine refusal and disease endemicity. This regime shift is preceded by an increase in the spatial correlation in non-vaccinator opinions beginning well before the bifurcation point, indicating CSD. The early warning signals occur across a wide range of parameter values. However, the more gradual transition exhibited in the empirically-derived network underscores the need for further research before it can be determined whether trends in spatial correlation in real-world social networks represent critical slowing down. The potential upside of having this monitoring ability suggests that this is a worthwhile area for further research.

8 *Keywords:* adaptive networks, multiplex networks, behavioral modelling,
9 coupled behavior-disease models, regime shifts, early warning signal

10 **1. Introduction**

11 Vaccine-preventable infectious diseases continue to impose significant bur-
12 dens on populations around the world [1]. Access to vaccines remains a sig-
13 nificant barrier to providing more widespread protection against infectious
14 diseases. However, a growing obstacle to infection control is vaccine refusal,
15 which can have a large effect on disease prevalence. For instance, the drop
16 in vaccine coverage after Andrew Wakefield’s fraudulent 1998 paper about
17 the mumps-measles-rubella vaccine reduced MMR coverage to as low as 61
18 % in some areas of the United Kingdom [2]. There were numerous measles
19 outbreaks in the years following the publication of the Wakefield paper [3].
20 Elimination of polio in Africa was similarly interrupted when a rumor that
21 the vaccine could cause infertility or HIV infection began spreading in 2003,
22 when leaders of three states in north-central Nigeria boycotted the vaccine
23 until it could be tested independently. The impasse was not resolved until
24 the following year, a time period during which these states accounted for over
25 50% of polio cases worldwide [4, 5]. Vaccine refusal and hesitancy are also
26 common for influenza vaccine, with non-vaccinators citing concern for side
27 effects, lack of perception of infection risk, and doubts about vaccine efficacy
28 as reasons to not become vaccinated [6].

29 Simple differential equation models such as the Kermack-McKendrick SIR
30 (susceptible-infected-recovered) model published in 1927 (originally formu-
31 lated as an integro-differential equation) [7], allow us to characterize useful
32 measures such as the expected number of new infections caused by each in-
33 fection, and are readily fitted to epidemiological data. Classical infection
34 transmission models such as the Kermack-McKendrick model assume that
35 members of the population mix homogeneously. However, in many situa-
36 tions, infection transmission through a network—where individuals are nodes
37 and contacts through which infection may pass are edges—are a more accu-
38 rate description of infection dynamics [8]. Networks tend to be analytically
39 intractable and therefore agent-based models are often used to simulate net-
40 works. Agent-based simulations on networks allow us to specify complex in-
41 dividual node behavior in a natural way. One of the most ambitious examples
42 of these is the Global-Scale Agent Model, which models the daily behavior
43 and relationships of 6.5 billion people using worldwide GIS data[9]. How-
44 ever, agent-based network simulations have also been studied in the context

45 of nonlinear interactions between disease dynamics and individual behaviour
46 concerning vaccines and contact avoidance [10, 11, 12, 13, 14].

47 The trajectory that an infection takes as it moves through a population is
48 heavily influenced by the spread of health information between individuals, so
49 more sophisticated models of disease spread often combine disease dynam-
50 ics and social dynamics. The coupled interactions between individual be-
51 haviour and disease dynamics have been modelled under various frameworks
52 and placed under various rubrics including: epidemic games [15], coupled
53 behaviour-disease models [11, 16, 17], socio-epidemiology, economic epidemi-
54 ology and behavioural modeling [18]. A more recent trend in epidemiological
55 modeling is to abstract these two subsystems into (1) an information trans-
56 fer network through which information flows between individuals, and (2) a
57 separate physical disease transmission network. A system where each node
58 is part of two or more different networks is called a multiplex network, and
59 is a natural way to implement a coupled disease-behaviour system [19, 17].
60 For instance, the simultaneous spread of disease and disease awareness over
61 adaptive multiplex networks with scale-free degree distributions has been
62 studied [20]. Similarly, a three layer network to model the diffusion of infec-
63 tion, awareness, and preventative measures along different contact networks
64 was found to reasonably approximate empirical influenza data[21].

65 The nonlinear coupling between disease and social processes creates feed-
66 back loops between infection prevention mechanisms and disease spread.
67 Nonlinear feedback in other complex systems such as from solid state physics
68 and theoretical ecology has often been shown to yield critical transitions
69 [22, 23]. A critical transition is defined as an abrupt shift from an existing
70 dynamical regime to a strongly contrasting (and sometimes unfavourable)
71 dynamical regime as some external parameter is pushed past a bifurcation
72 point [24]. Fortunately, critical transitions (and other regime shifts associ-
73 ated with a bifurcation where the eigenvalue of the Jacobean matrix around
74 the equilibrium approaches zero) often exhibit characteristic early warning
75 signals beforehand that allow these shifts to be predicted. Critical slowing-
76 down (CSD) based indicators were one of the first early warning signals to
77 be studied. CSD occurs because the speed with which a system responds
78 to perturbations slows as it approaches bifurcations where the magnitude
79 of the eigenvalue of the Jacobean approaches zero at the bifurcation point.
80 Since nearly all systems in the real world are subject to perturbations, the
81 lag-1 autocorrelation of a time series can be used as a relatively universal (or
82 at least potentially common) indicator of CSD. Lag-1 autocorrelation ap-

83 pears to be a robust statistic and has been shown to be present in predicting
84 catastrophic bifurcations in complex real world systems such as the global
85 climate[25], human nervous systems[26], and stock markets[27].

86 The discrete fourier transform (DFT) of a network is another example
87 of a CSD-based early warning signal. Under some assumptions, the Weiner-
88 Kinchin Theorem shows that we can use the discrete Fourier transform (DFT)
89 to measure spatial correlation in system state, and this has been shown to
90 work in some ecological applications [28] [29]. Lag-1 spatial correlation can in
91 some cases provide a better early warning signal than time-domain methods,
92 because "a spatial pattern contains much more information than does a single
93 point in a time series, in principle allowing shorter lead times" before the
94 critical transition occurs [30, 31]. This observation has been corroborated in
95 three ecological dynamical systems[31].

96 Early warning signals of regime shifts in coupled behaviour-disease net-
97 works have received relatively little attention in the literature on modelling
98 interactions between disease dynamics and human behaviour. This appears
99 to be a significant knowledge gap because early warning signals for vaccine
100 scares could help public health anticipate widespread vaccine refusal and
101 prepare for outbreak response in advance, as well as build efforts to improve
102 trust between the public and the health authorities. In this paper we use an
103 agent-based model on a two-layer multiplex network to simulate the coupled
104 disease dynamics of a vaccine-preventable infection and social dynamics of
105 vaccination in a population. We show that spatial correlation can be used as
106 an early warning signal for regime shifts in this system on most (but not all)
107 network topologies. In the next section we discuss the model structure and
108 methods of analysis, followed by a section on results and finally a discussion
109 section.

110 **2. Methods**

111 *2.1. Simulation*

112 Our agent-based model simulated a population of 10,000 individuals (nodes),
113 where every node belongs to two different connectivity networks: a transmis-
114 sion network and a social network. In the transmission network, each node
115 is connected to other nodes from which they can contract infection. Two
116 nodes are linked in the social network if they can be influenced by one an-
117 other's opinions on vaccination. These networks were simulated as fixed

118 graphs upon which stochastic processes occurred, with a variety of degree
 119 distributions and average path lengths.

120 We modelled a hypothetical acute, self-limiting infection with rapidly
 121 waning natural immunity. Each node on the physical layer is in one of four
 122 possible states: susceptible (S), infected (I), recovered (R), or vaccinated
 123 (V). A node on the social layer also has an opinion on the vaccine: they are
 124 either a non-vaccinator (proportion η), or a vaccinator (proportion ν). We
 125 will denote the biological state of a node v by $B(v)$, and the opinion of a
 126 node v by $\Theta(v)$. The transmission network is a graph denoted by $T(V, E_T)$,
 127 and the social network is a graph denoted by $O(V, E_O)$. We assume that they
 128 share the same set of vertices V although this assumption could be relaxed
 129 in future work. The set of nodes in the neighbourhood of v is $adj_T(v)$ or
 130 $adj_O(v)$ for the transmission and the social network respectively.

131 The algorithm used to simulate the social and transmission processes used
 132 discrete timesteps. At each time step, for each $v \in V$:

- 133 • If $B(v) = I$, then for all $u \in adj_T(v)$ such that $B(u) = S$ and $\Theta(u) \neq \nu$,
 134 set $B(u) = I$ with probability p (infection event)
- 135 • If $B(v) = I$, let $B(v) = R$ with probability r (natural recovery event)
- 136 • If $B(v) = R$, set $B(v) = S$ with probability γ (loss of immunity event)
- 137 • If $B(v) = S$, set $B(v) = I$ with probability $\sigma \ll 1$ (case importation
 138 event)
- 139 • Choose some node $u \in adj_O(v)$ uniformly at random. If $\Theta(v) \neq \Theta(u)$,
 140 then $P(\eta \rightarrow \nu) = \Phi(E_V - E_N)$, where

$$E_V = -c_v + c_n, \tag{1}$$

$$E_N = -c_I \mathfrak{J}(v), \tag{2}$$

142 where Φ is a sigmoid function such that $\Phi(\infty) = 1$, $\Phi(-\infty) = 0$, $\Phi(0) =$
 143 0.5 as described in previous models (opinion change event) [32]. In our
 144 implementation, $\Phi(x) = \frac{1}{1+e^{-\beta x}}$, c_v is the perceived cost of vaccination
 145 (due to infection risks), c_I is the perceived cost of infection (due to
 146 infection risks), β controls the steepness of the sigmoid function, and
 147 $\mathfrak{J}(v) = \{u \in adj_T(v) : B(u) = I\}$. c_n represents some outside incentive
 148 that a person might have for vaccinating, such as peer approval, school

149 admission requirements, or tax incentives. Normalizing both payoff
150 equations by c_I yields

$$151 \quad E_V = -c + \xi \quad (3)$$

$$E_N = -\mathcal{J}(v) \quad (4)$$

152 where c is the ratio of perceived vaccine risk to perceived disease risk,
153 and $\xi = \frac{c_v}{c_I}$ is the ratio of the vaccination incentive to the perceived
154 disease risk. Since changes in perceived vaccine risk are controlled
155 through changes in c , we will vary c in our analysis. We assume the
156 vaccine is perfectly efficacious.

- 157 • With probability ϵ , v changes opinions (random opinion change event).
158 That is, if $\Theta(v) = \nu$, set $\Theta(v) = \eta$ and vice-versa.

159 We applied synchronized updating to the network: the change in state re-
160 sulting from each rule is stored and applied after every rule is checked, so
161 the order of the above steps does not matter.

162 The result of these rules is a feedback loop where, depending on the rel-
163 ative costs of vaccination and infection, the population tends not to exhibit
164 a mixture of strategies except near the critical values of c . When $c < \xi$, the
165 payoff to vaccinate E_V is positive and thus exceeds the payoff not to vacci-
166 nate E_N which always obeys $E_N \leq 0$. In this case, in the limit as $\beta \rightarrow \infty$, all
167 nodes are therefore vaccinators and the infection dies out. However, when
168 $c > \xi$ and thus $E_V < 0$, the disease-free equilibrium destabilizes since $E_N \approx 0$
169 in the absence of sustained transmission. In general, since the vast major-
170 ity of nodes do not have infected neighbours at the disease-free equilibrium,
171 there is a rapid shift in the population to non-vaccinator opinions as well
172 as epidemic outbreaks. For larger values of β , the function controlling the
173 opinion-switching as a function of the payoff difference between vaccinator
174 and non-vaccinator strategies is steeper, and the population transition from
175 non-vaccinator to vaccinator strategies is therefore sharper, yielding a crit-
176 ical transition. However, we will use the more general term ‘regime shift’
177 throughout this paper, since the transition can be made more or less abrupt
178 by changing the value of β .

179 2.2. Early Warning Signal Analysis

180 As the system approaches a regime shift, the dominant eigenvalue of
181 the underlying dynamical system will approach zero. Therefore, it will take

182 longer to recover from perturbations to the steady state. In a spatially ex-
183 tended population, this should cause each node to become more correlated
184 to its immediate neighbours, on average. This correlation can be reflected in
185 a statistic called the lag-1 spatial correlation (lag-1 SC). We used Moran's
186 I to measure the lag-1 SC of non-vaccinators as described in [33]. Moran's
187 I is widely used to calculate the spatial correlation for early warning signals
188 [34, 35, 36].

189 Let $G = (V, E)$ be a graph with n nodes, $adj(v)$ be the set of vertices
190 adjacent to v , and $f(v)$ be a binary function such that $f(v) = 1$ if v is a
191 vaccinator, and $f(v) = 0$ otherwise. We define Moran's I at lag-1 in the
192 following way:

$$I = \frac{\sum_{v \in V} I_v}{|E|} \quad (5)$$

$$I_v = \frac{n(f(v) - \bar{x}) \sum_{w \in adj(v)} (f(w) - \bar{x})}{\sum_{w \in V} (w - \bar{x})^2} \quad (6)$$

193 where $\bar{x} = \frac{1}{n} \sum_{v \in V} f(v)$ is the fraction of vaccinators in the network.

194 The simulation was run long enough for the spatial correlation to stabilize
195 (3500 timesteps), and the equilibrium value was calculated as the average of
196 the next 500 measurements. This procedure was followed 100 times for each
197 value of c , and these values were averaged to obtain a data point for every
198 value of c . The social network and the transmission network are always both
199 the same type of network, but independently generated.

200 2.3. Parameter Values

201 Baseline parameter values appear in Table 1. The parameter values were
202 chosen to qualitatively represent a hypothetical acute-self limiting infection
203 with waning natural immunity, such as the case of meningococcal infection,
204 influenza or pertussis [37, 38, 39, 40]. The value for r corresponds to a mean
205 duration of infection of 14 days, the value for γ corresponds to losing nat-
206 ural immunity after two years, and the value for σ corresponds to a case
207 importation event in the network once every two months. We conduct uni-
208 variate sensitivity analysis with respect to r and σ , since they are important
209 parameters governing the natural history of the infection. For the baseline
210 parameter values, ξ is set to zero without loss of generality. The value of
211 c will be varied in the analysis of early warning signals. $\epsilon > 0$ is required

Parameter	Value	Definition
p	0.5	Probability that an infected node infects a given susceptible neighbour
r	0.07143	Probability that an infected node recovers
γ	0.001369	Probability that a recovered node becomes susceptible
ϵ	0.001369	Probability that a node randomly switches their opinion on vaccination
σ	0.016666	Probability of disease reintroduction
ξ	0	Parameter governing incentive to become vaccinated
c	0.1	Ratio of perceived risk of vaccine to perceived risk of disease
β	1	Parameter controlling the steepness of Φ

Table 1: Parameter definitions and baseline parameter values in probability per timestep (unless otherwise stated). One timestep was interpreted to correspond to one day.

212 to prevent the population from fixating on one of the two strategies. To
 213 initialize each stochastic realization, one randomly chosen node is infected,
 214 and each node is a vaccinator with probability 0.5.

215 2.4. Networks

216 We ran our model on five different networks: Erdos-Renyi [41], Barabasi-
 217 Albert [42], square lattice (or grid), Kleinberg small world[43], and ten sub-
 218 sets of a network constructed by the Network Dynamics and Simulation and
 219 Science Laboratory (NDSSL), based on GIS data from the city of Portland
 220 [44].

221 An Erdos-Renyi network is simply given a set of nodes V and $v, w \in V$,
 222 v is connected to w with some probability p . In our Erdos-Renyi network
 223 model, we used a connection probability of 0.001, so each node has degree
 224 10 on average.

225 The Barabasi-Albert model yields networks with a scale-free (or power-
 226 law) node degree distribution. Starting with a small initial connected network
 227 (V, E) , new nodes are added to V one at a time. Where the probability that
 228 the new node is connected to an existing node $v \in V$ is $p_v = \frac{deg(v)}{\sum_{w \in V} deg(w)}$. To
 229 ensure that the network is always connected, new nodes are also connected

230 to m existing vertices, chosen uniformly at random. The Barabasi-Albert
231 networks we used had $m = 1$.

232 Our lattice with $n = 10,000$ nodes was built as follows: if the nodes
233 are arranged on the integer points of a square \sqrt{n} units wide, each node is
234 connected to the nodes within a unit distance up or down (but not both).
235 Because lattice networks are not random, there is no difference between the
236 social and transmission networks and therefore this is effectively not a mul-
237 tiplex network.

238 The Kleinberg small world network is defined as a square lattice, where
239 additional edges are added between some nodes v and w with a probability
240 proportional to $1/d(v, w)$. The result of this process is a network with a very
241 short average path length. In our implementation, nodes only gain extra
242 edges with 0.1 probability.

243 The empirically-derived networks from the NDSSL dataset are designed to
244 have some of the properties of a real contact network, being derived from the
245 population of Portland, Oregon. We used a set of ten subnetworks sampled
246 from the NDSSL dataset and constructed in such a way to share the same
247 properties as the original dataset (see Ref. [32] and supplementary appendix
248 for details). The subnetworks had an average path length of 4.020 ± 0.126 ,
249 and an average clustering coefficient of 0.747 ± 0.006 . For each run, two
250 networks were chosen from the 10 networks uniformly at random and one
251 was set as the social network, with the other as the transmission network.

252 3. Results

253 3.1. Model dynamics

254 We generated time series of the percentage of vaccinators and percent-
255 age of infected persons for each of the networks, in order to illustrate the
256 basic dynamics exhibited by the model. We used baseline parameter values
257 everywhere (Table 1) except that $c = 0.3$. For all networks we initialized
258 the population to have a low initial number of vaccinators and a large initial
259 number of susceptible persons. These initial conditions caused the incidence
260 of infection to skyrocket at the beginning of the simulation for all network
261 types (Figure 1). Immediately after this initial outbreak, susceptible neigh-
262 bours of infected persons get vaccinated, thereby reducing prevalence.

263 After this initial spike, the dynamics settle down into pseudo-stable pat-
264 terns that vary widely depending on network type. More frequent outbreaks

265 appear to occur on networks with higher degree, which is consistent with intu-
266 tion (Figure 1). The random network exhibits relatively regular outbreaks
267 (Figure 1a), while the square lattice, Barabasi-Albert network, and small
268 world network exhibit more irregular dynamics consisting of large outbreaks
269 interspersed with periods of very low vaccine coverage and infection preva-
270 lence (Figure 1b-d). However, during certain phases in the time series, the
271 small-world network appears to transition to a regime of sustained endemic
272 infection similar to that observed for the random network (Figure 1d). The
273 empirically-derived network exhibits small stochastic fluctuations around an
274 equilibrium, and the percentage of vaccinators is significantly higher in the
275 empirically-derived network than in the other four networks (Figure 1e).

276 *3.2. Regime shifts*

277 We carried out this simulation experiment for a range of values of c to
278 understand how dynamics respond to changes in the perceived vaccine risk
279 c . We computed the long-term average prevalence of infected persons (I)
280 and vaccinators (ν) for each value of c tested. As c approaches zero from
281 below (for $\xi = 0$), a transition from a regime of high vaccine coverage and
282 low infection prevalence to a regime of low vaccine coverage and endemic
283 infection should be observed, since for $c > 0$, the payoff to vaccinate becomes
284 less than the payoff not to vaccinate.

285 In the simulations we observe a transition in the percentage of non-
286 vaccinators as a function of the perceived vaccine risk c in most of the network
287 types (Figure 2). As c approaches zero, the prevalence of vaccinators declines
288 dramatically in the first four networks. The transition appears gradual (non-
289 critical) in the empirically-derived network (Figure 2e). We speculate this
290 is due to the greater heterogeneity exhibited by the empirically-derived net-
291 work than the other four idealized network types. The percentage of infected
292 persons in each network shows similar transitions, even in the latter network
293 (Figure 2e). We also note that the transition is sharper when the sigmoid
294 function used in decision-making is steeper (higher β ; results not shown).

295 *3.3. Early warning signals*

296 Indicators such as spatial correlation can signal an impending critical
297 transition in spatially structured ecological systems [31]. Although theo-
298 retical results are not available for coupled behaviour-disease dynamics on
299 multiplex networks, the universality of dynamics near local bifurcations of

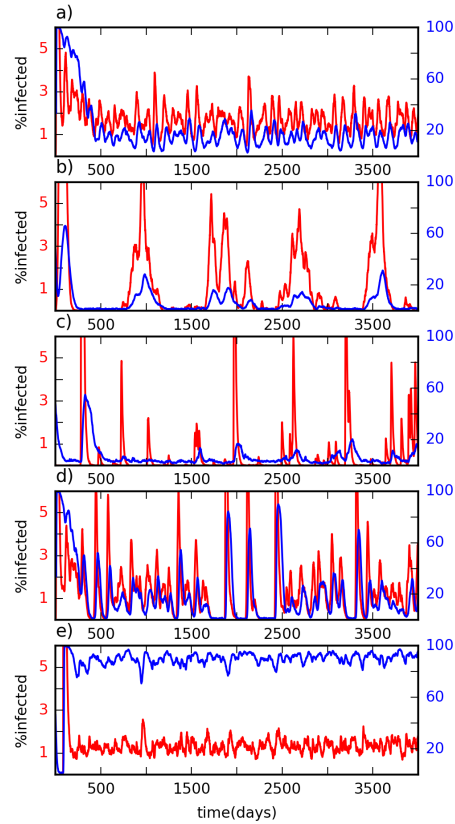


Figure 1: Time series for a typical simulation on each network type: a) random network, b) square lattice, c) Barabasi-Albert network, d) Small world network, e) empirically-derived networks. Red line is percentage of infected individuals (I) in the population; blue line is percentage of vaccinators (ν) in the population. Parameter values are as in Table 1 except $c = 0.3$.

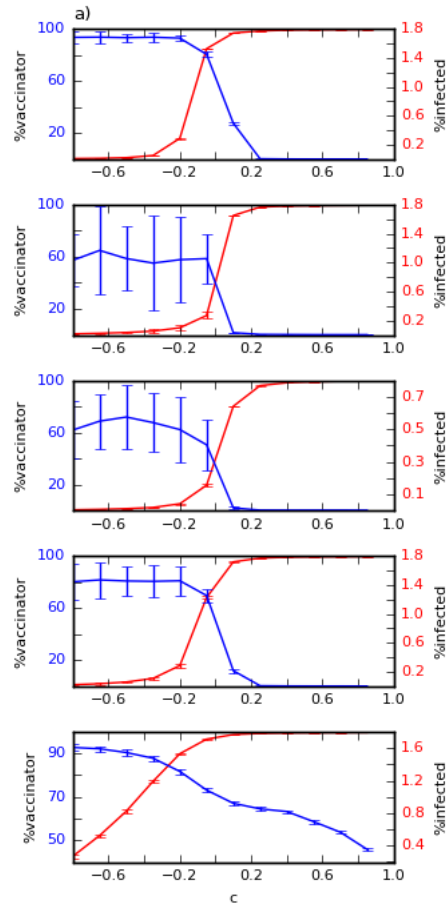


Figure 2: The time-averaged percentage of infected persons and vaccinators as a function of relative vaccine cost c , showing a critical transition near $c = 0$ on the a) random network, b) square lattice, c) Barabasi-Albert network, d) Small world network, and a more gradual transition on the e) empirically-derived networks. All parameters are as in Table 1 except for c , which is being varied. The blue line represents the percentage of vaccinators, and the red line percentage of infected. Error bars represent the standard deviation over the 100 realizations.

300 dynamical systems [45] suggests that similar early warning signals should be
301 observed in our system.

302 In spatially extended critical phenomena, the plot of spatial correlation
303 versus a bifurcation parameter such as c is linear on a log-linear plot [46].
304 Hence, we computed the average lag-1 spatial correlation (SC) across the
305 entire time series. We repeated this for many values of c and plotted lag-1
306 AC versus c on a log-linear scale. As noted previously, we expect near the
307 threshold $c = 0$ where the costs and benefits of the vaccine become balanced,
308 that critical slowing down should emerge in the network, and that this should
309 manifest as increased spatial correlation. As we increase c from negative to
310 positive, small clusters of non-vaccinators begin to appear. Each day every
311 node samples a random neighbour, and the only other way for that node to
312 switch opinions is if the randomly sampled neighbour has a different opinion
313 that they do (see Methods). As a result, we expect to see clusters of non-
314 vaccinators emerge, which causes the lag-1 SC to increase before the critical
315 transition (and after which almost everyone because a non-vaccinator) (figure
316 3).

317 This pattern is observed in simulations for all network types. As the
318 regime shift at $c = 0$ is approached from negative values of c (corresponding
319 to a rise in perceived vaccine risks), we observe a clear and linear increase
320 in the time-averaged lag-1 SC, in plots of the natural logarithm of lag-1 SC
321 versus c (Figure 4). This is robust to values of the disease transmission
322 probability, p (Figure 4).

323 However, there is a notable difference in y-axis scales for the random and
324 small-world networks (Figure 4a,d). Overall these networks show a smaller
325 increase in spatial correlation, possibly due to the smaller average path length
326 in these networks. Furthermore, lag-1 SC in the empirically-derived network
327 has a nonlinear and more gradual response to changes in c , which matches
328 the lack of a sharp critical transition in that network. Sensitivity analyses
329 over r and σ confirm the same patterns, except in the extreme case of $r = 0$
330 where infected individuals never recover (Figure 5).

331 We observe that the rise in the natural logarithm of lag-1 SC begins well
332 before the number of non-vaccinators begins to increase appreciably (com-
333 pare $c \in [-0.8, -0.2]$ in Figure 4 versus Figure 2). Therefore, tracking lag-1
334 SC can provide an early warning signals of potential shifts in population vac-
335 cinating behaviour that would not be accessible simply by extrapolating the
336 number of non-vaccinators using a linear regression, for instance. Moreover,
337 this rise in lag-1 SC is highly robust to network type and parameter value,

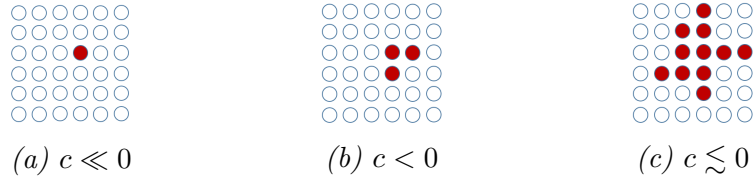


Figure 3: Visualization of non-vaccinator spatial correlation on a square lattice. As c approaches the critical transition at $c = 0$, clusters of non-vaccinators (red) begin to appear, increasing the spatial correlation of non-vaccinators.

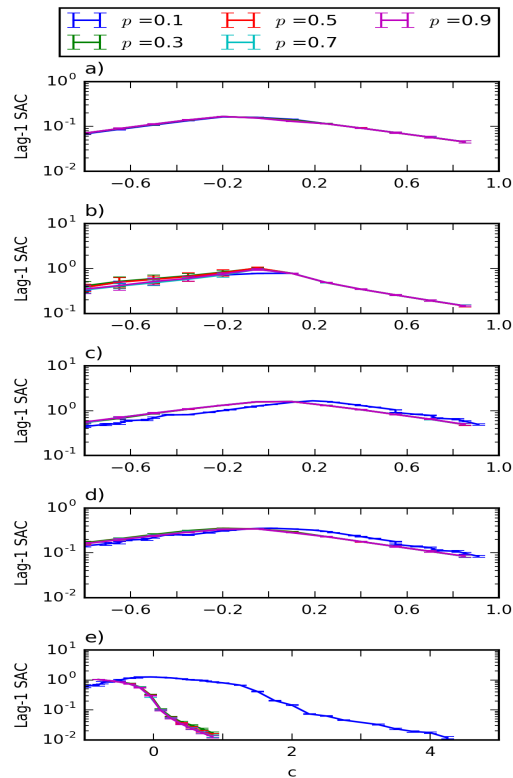


Figure 4: The natural logarithm of the time-averaged lag-1 SC of nonvaccinators for a range of values of c , showing a linear increase in lag-1 SC in a log-linear plot as the critical transition is approached on a) random network, b) square lattice, c) Barabasi-Albert network, d) Small world network, e) empirically-derived networks. All other parameter values are as in Table 1.

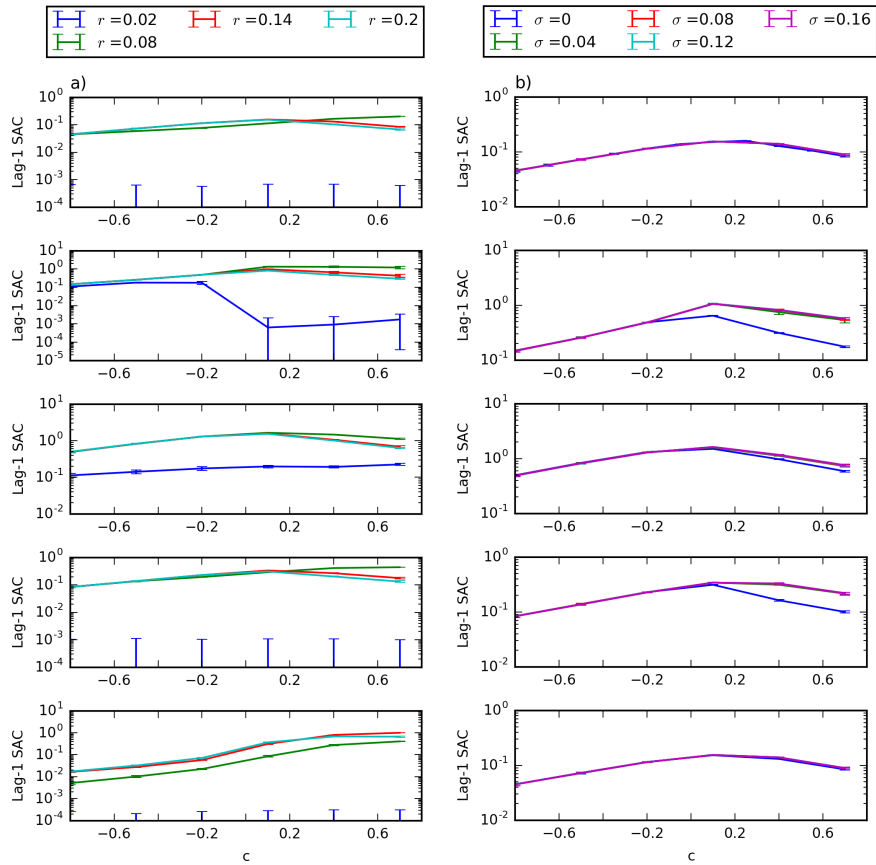


Figure 5: The natural logarithm of the time-averaged lag-1 SC of nonvaccinators for a range of values of c at selected values of a) r and b) σ , showing a linear increase in lag-1 SC in a log-linear plot as the critical transition is approached. Networks types from top row to bottom row are: random network, square lattice, Barabasi-Albert network, small world network, and empirically-derived networks. All parameters besides r , σ and c are the same as Table 1.

338 due to the fundamental assumption that a node's vaccination status is influ-
339 enced by the opinions of the nodes in their social neighbourhood. However,
340 the location of the regime shift in c is related to the average node degree:
341 with an average node degree of 100, the regime shift occurs at approximately
342 $c = 2.4$.

343 4. Discussion

344 Here we studied regime shifts in coupled behaviour-disease dynamics on
345 a multiplex network where an infectious disease is transmitted through the
346 physical network layer, and the social layer describes a population where
347 everyone has either a pro-vaccine or an anti-vaccine opinion. These simu-
348 lation results show the presence of critical slowing down near a bifurcation
349 in the multiplex network corresponding to a switch from predominant vac-
350 cinating behaviour and disease elimination, to predominant non-vaccinating
351 behaviour and disease endemicity. Critical slowing down was clearly man-
352 ifested in all network types and across a broad range of parameter values,
353 with the exception of the empirically derived network. This exception may
354 have been on account of the greater heterogeneity of the network causing
355 lack of a sharp transition to non-vaccinating behaviour.

356 Hence, the results suggest that it may be possible to use lag-1 spatial
357 correlation in social networks as an early warning signal of widespread vaccine
358 refusal in a population. However, the lack of a clear transition in the case of
359 the network that was empirically derived (from NDSSL data) suggests that
360 further research must be conducted in order to determine how and whether
361 it would be possible to detect such early warning signals in real-world social
362 networks, and what the trends in correlation indicators might signify. Our
363 model also assumed that networks are static and that the two layers are
364 perfectly correlated. Neither condition holds in real populations, and these
365 simplifying assumptions could be relaxed in future work.

366 It is also possible to tailor this model to specific infectious diseases such as
367 measles or influenza by modifying the model to include relevant vital dynam-
368 ics, disease natural history, and vaccine characteristics. This is particularly
369 important since disease natural history can have a significant impact on dis-
370 ease dynamics [37, 47], and vaccine coverage can vary widely between both
371 vaccines and populations [48, 49]. Finally, future research could seek early
372 warnings signals in lag-1 SC measurements from social networks derived from
373 social media data sources such as Twitter. Lag-1 SC is readily calculated if

374 the sentiment of Twitter users toward vaccines can be assessed as pro- or
375 anti-vaccine. However, the Twitter follower network is a directed graph that
376 changes in time, therefore additional theoretical refinements are necessary.

377 Lag-1 spatial correlation appears to be a robust early warning signal for
378 predicting regime shifts in vaccine uptake under the conditions we studied,
379 indicating potential for worthwhile additional study in the context of coupled
380 behaviour-disease interactions.

381 5. References

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