

1 *Methods in Description and Validation of* 2 *Local Metagenetic Microbial* 3 *Communities*

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8 *Abstract:*

9

- 10 1. We propose minhash (as implemented by MASH) and NMF as alternative methods to estimate similarity
11 between metagenetic samples. We further describe these results with cluster analysis and correlations
12 with independent ecological metadata.
- 13 2. Species and kmer abundance information is used to determine similarities and create clusters to better
14 understand how communities interact, as well as relate to known environmental variables, such as Ph and
15 Soil Conductivity.
- 16 3. We use cluster silhouettes to assess various approaches for clustering metagenetic samples as well as
17 anova to uncover links between metagenetic samples and the known environmental variables.
- 18 4. By analyzing data from the Atacama desert and determining the relationship between ecological factors
19 and group membership, we show the applicability of these methods.

20 *Introduction*

21 How microbiome communities, and in a broader context local communities, are determined, described, and
22 validated is a matter of some debate (Holyoak et al. 2005). While Principal Components Analysis (PCA) is the
23 most common computational approach with the most divisive components considered the most ecologically
24 relevant, PCA is biased towards components that have the most variance (Parsons et al. 2009), and is not
25 necessarily useful for factor analysis, or determining underlying variables where many observed variables may
26 reflect a few unobserved variables (Jolliffe 1986). A common description of community structure beyond the
27 initial group assignment is also often lacking in PCA. Here we present two alternative computational methods to
28 determine the grouping of metagenetic samples: k-mer minhash sketching and Non-Negative Matrix Factorization
29 (NMF). NMF can be paired with k-means to estimate the number of groups present and has the benefit of
30 determining the most important feature driving inferred relationships. Minhash sketches can be used to quickly
31 estimate similarities between whole samples in an alignment-free approach, i.e., OTUs do not need to be
32 generated first. While minhash and NMF are used here to cluster metagenetic samples based on inferred
33 relationships, note that NMF focuses on what is distinct (in a cluster) while a minhash implementation (Ondov et
34 al. 2016) is combined with hierarchical methods to infer clusters based on pairwise similarities. For this reason we
35 use Silhouette plots for cluster assessment, which is a measure of how close each point in a cluster is to other
36 clusters. Finally, we use ANOVA to determine the relationships of known environmental factors to the inferred
37 clusters using our new and existing approaches.

38 One local community can be delineated from another by inferred differences in the species detected within
39 one or more samples (Rusch et al. 2007);(Seshadri et al. 2007). K-mer estimation methods such as MASH or
40 OTU methods such as PCA or NMF should be able to distinguish local communities based on the inference of
41 species. Using these methods, however, does not necessarily provide distinctions between environments or even
42 interrelated communities. Note that the detected abundance of a species in a metagenetic sample may not correlate
43 with its actual abundance in the broader area, making drawing boundaries between local communities using any
44 computational approach difficult. As a result, prior work in community analysis has often relied on additional

45 metadata such as physical barriers and environmental measurements to refine the structure of estimated local
46 communities based on the species observed (Holyoak et al. 2005).

47 Here, we consider novel, data-driven (unsupervised) approaches for defining communities based on clusters,
48 or different inter-related groups, inferred only from NGS sequence data. These approaches allow us to artificially
49 induce computational cutoffs and, as a result, no prior knowledge/metadata are required to infer potential
50 relationships between samples. Because environmental characteristics can change the viability of a microbial
51 species occupying that area (Hultman et al. 2015);(Gibbons & Gilbert 2015), subsequent comparisons of
52 groupings to independent environmental variables provides a biologically motivated assessment of whether these
53 computationally generated results uncover local communities. To define clusters we introduce MinHash (Ondov
54 et al. 2016) based similarity for determining local community structure, which essentially an approximation of
55 the Jaccard similarity based on shared species within samples (see Rusch et al. 2007 and Ondov et al. 2016 for
56 details). We also apply Non-Negative Matrix Factorization (NMF) (Gaujoux & Seoighe, 2010);(Seung & Lee
57 1999);(Paatero & Tapper 1994) using the nsNMF algorithm (Pascual-Montano et al. 2006) to determine non-
58 shared species based on OTU abundances. Log likelihood statistical analysis of an Atacama desert microbial
59 community indicates that among these *de novo* methods, hierarchical clustering using MinHash similarities has
60 more explicative power than NMF on OTU abundance. This data set is a good choice for this analysis because of
61 the data's wide geographic range and inclusion of environmental variables. For the analysis of the Atacama
62 desert, samples taken from the same sampling location (North/Central/South) were more similar according to
63 alpha diversity (Crits-Christoph et al. 2013); however, we show that other environmental variables can have a
64 statistically higher correlation than sampling location, and specifically that PH, air relative humidity (RH) and soil
65 conductivity best explain observed local communities derived computationally. Combined, these indicate data-
66 driven methods can be directly used to estimate community structure from NGS data.

67 *NMF, MASH, Silhouettes, and ANOVA*

68 Here, NMF (Berman & Plemmons, 1994) and Mash (Ondov et al. 2016) are integral to defining local
69 community structure. NMF—or Non-Negative Matrix Factorization—is method by which to split a matrix into a
70 components, based on the factors that are most important in making that split. For example, for RNA-seq
71 expression analysis, suppose there are ‘k’ known clusters. NMF will break a provided expression matrix (genes
72 by cells or cell tissues) into k total clusters while also producing the most important genes for doing so (Yu-Jui,
73 2016). When applied to observed OTU abundances, NMF will ideally return the most important OTUs to generate
74 a fixed number of community-driven clusters. The power in this method is that different factors may be indicators
75 for each cluster, instead of just the presence or absence of a particular expressed gene or observed species. NMF
76 becomes particularly powerful when paired with k-means (Hartigan & Wong, 1979);(Forgey, 1965), which is a
77 clustering method that can be used to measure how many clusters exist (aka, the ‘fit’). Because NMF combines
78 factor discovery with iterative determination of the total number of clusters, NMF can be a more descriptive
79 alternative to simple PCA-based visualization of the data.

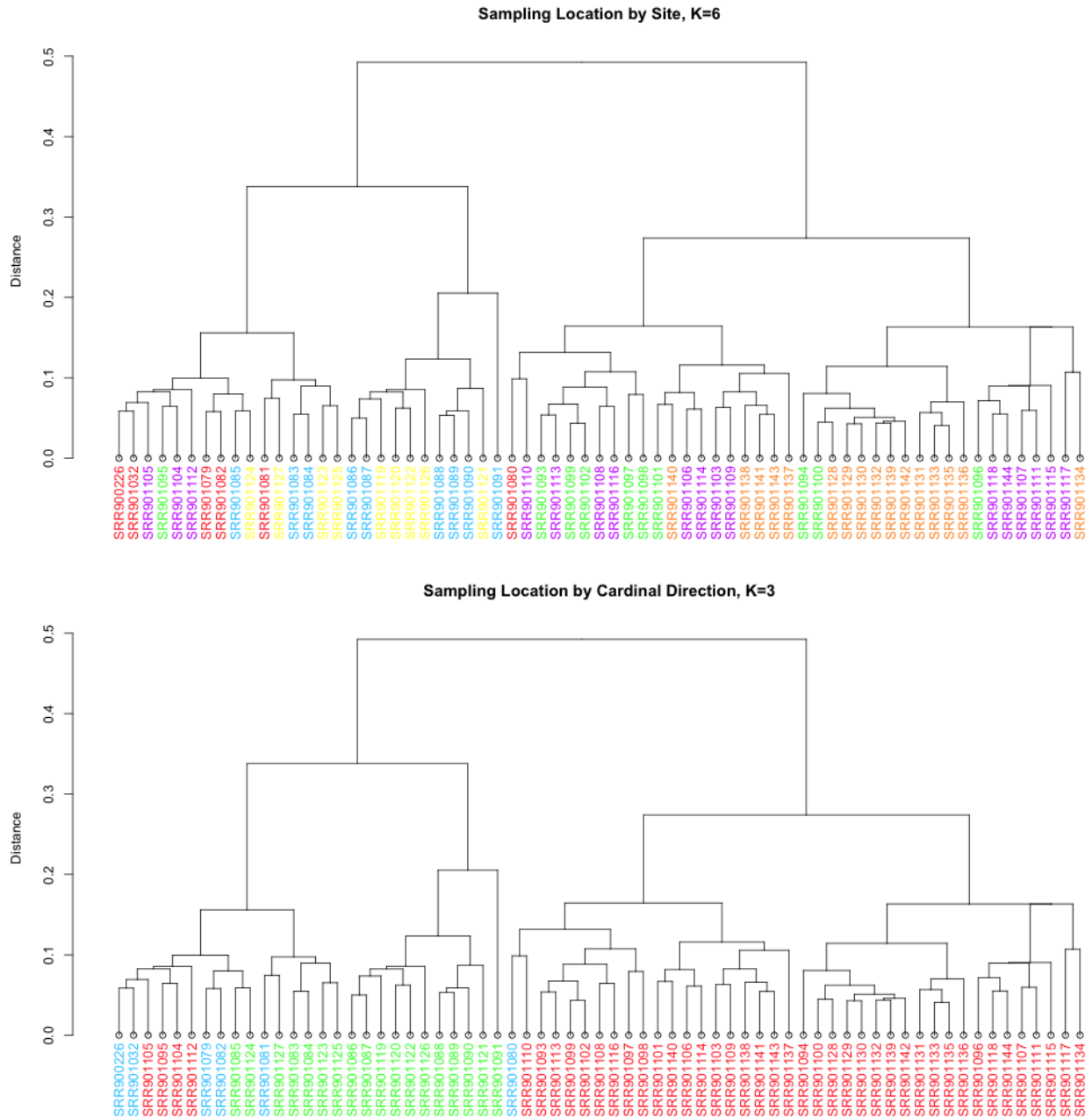
80 Mash, which is based on MinHash sketching (Broder, 1997), is an alignment-free method by which to
81 estimate the distance between two sequences or sets of sequences. Using this computational method a set of
82 samples can be sequenced and then quickly compared to estimate how similar they are. The resulting pairwise
83 similarity matrix can then be clustered hierarchically, which can be visualized in the form of dendrograms and/or
84 heatmaps. Mash can be run on raw samples if desired at the cost of potentially higher inferred distances. Example
85 hierarchical clustering algorithms are Diana (Struyf et al. 1997);(Kaufman & Rousseeuw, 2009) and McQuitty-
86 WPGMA (McQuitty, 1966).

87 Using silhouettes (Rousseeuw, 1987);(Handl et al. 2005) and the clustering information derived from NMF
88 we can further describe structure within a cluster. Specifically, silhouette width highlights the ‘belongingness’ of
89 each data point within a cluster; higher averages indicate cluster points are more tightly correlated with each
90 other.

91 Finally we hypothesize that the local assortment of species is largely determined by the environment in which
92 they live. If so, a change in environment and a corresponding change in observed species should, for the most
93 part, correlate and this correspondence can be tested using both Anova and a mantel test under the right conditions
94 (DeLong, 2013). We also realize that environment itself can correlate with distance, i.e., in the northern
95 hemisphere, northern samples have fewer growing degree days than southern samples. For this reason isolation by
96 distance (IBD) could also manifest as distinct clusters using our computational alternatives just as they would in a
97 traditional PCA analysis.

98 *Results from Atacama Data*

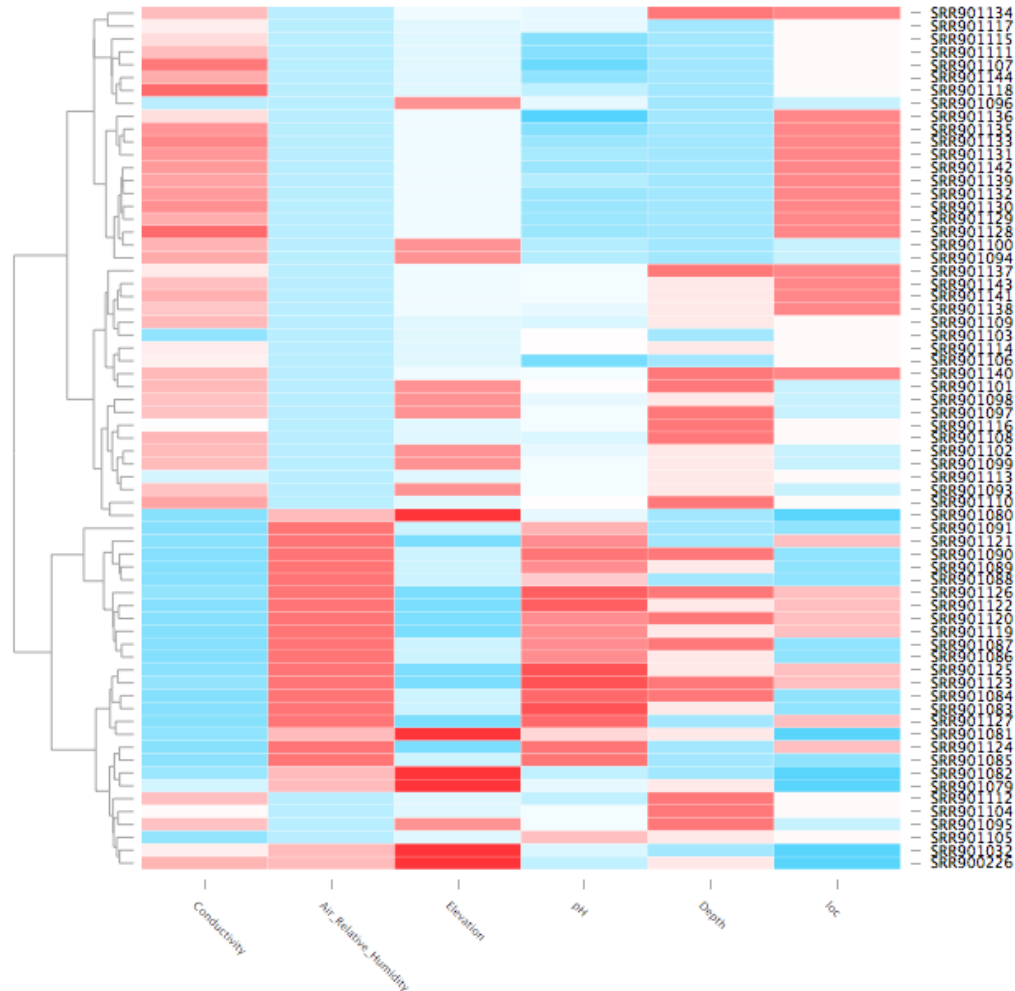
99 Sample clustering based on OTUs was performed using Non-negative matrix factorization (NMF), which
100 determines OTUs that are most informative using linear algebra-based techniques (Ondov et al. 2016);(Seung &
101 Lee 1999);(Paatero & Tapper 1994);(Yu-Jui, 2016). Sample to sample distances were determined based on
102 minhash sketches, which estimate the Jaccard similarity of two samples based on shared subsequences (k-mers).
103 We also determined the OTUs present in these Atacama samples using mothur (see Methods). Given our focus
104 on unsupervised analysis, we processed the mash-based sample distances with multiple clustering methods: K-
105 means (Hartigan & Wong, 1979);(Forgey, 1965), hierarchical (Everitt, 1974);(Hartigan, 1975): Agglomerative
106 and Divisive (Kaufman & Rousseeuw, 2009).



107

108 *Figure 1. Sample clusterings of Crits-Christoph et al. (2013) data using two measures of distance: site*
109 *location (top) and cardinal direction (bottom). Dendrograms were generated with Diana and are colored by*
110 *sampling location (top, 6 total).*

111

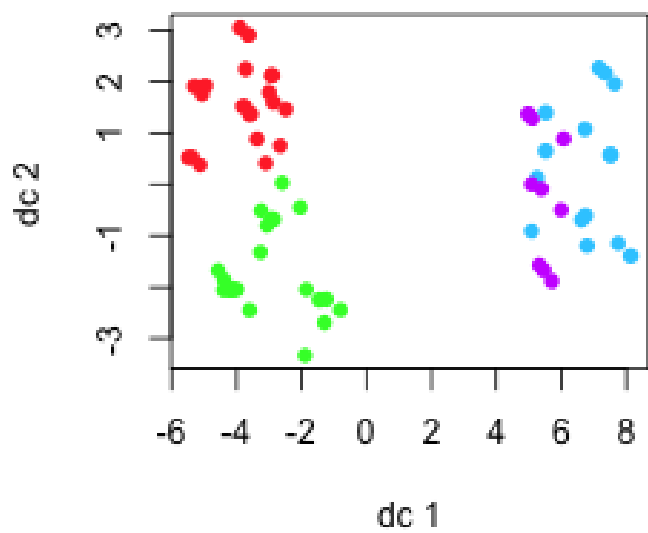
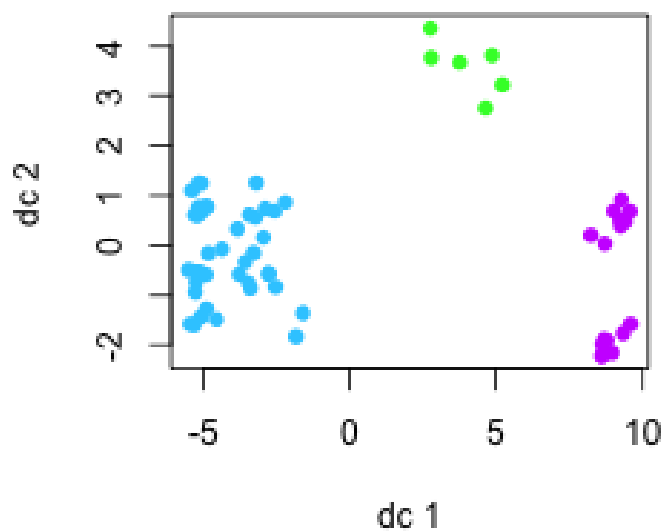


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113 *Figure 2. Heatmap of Various Environmental Variables, red indicating low and blue indicating high values.*

114 *The left hand side is determined by Diana clustering on sample to sample similarities, as in Figure 1.*

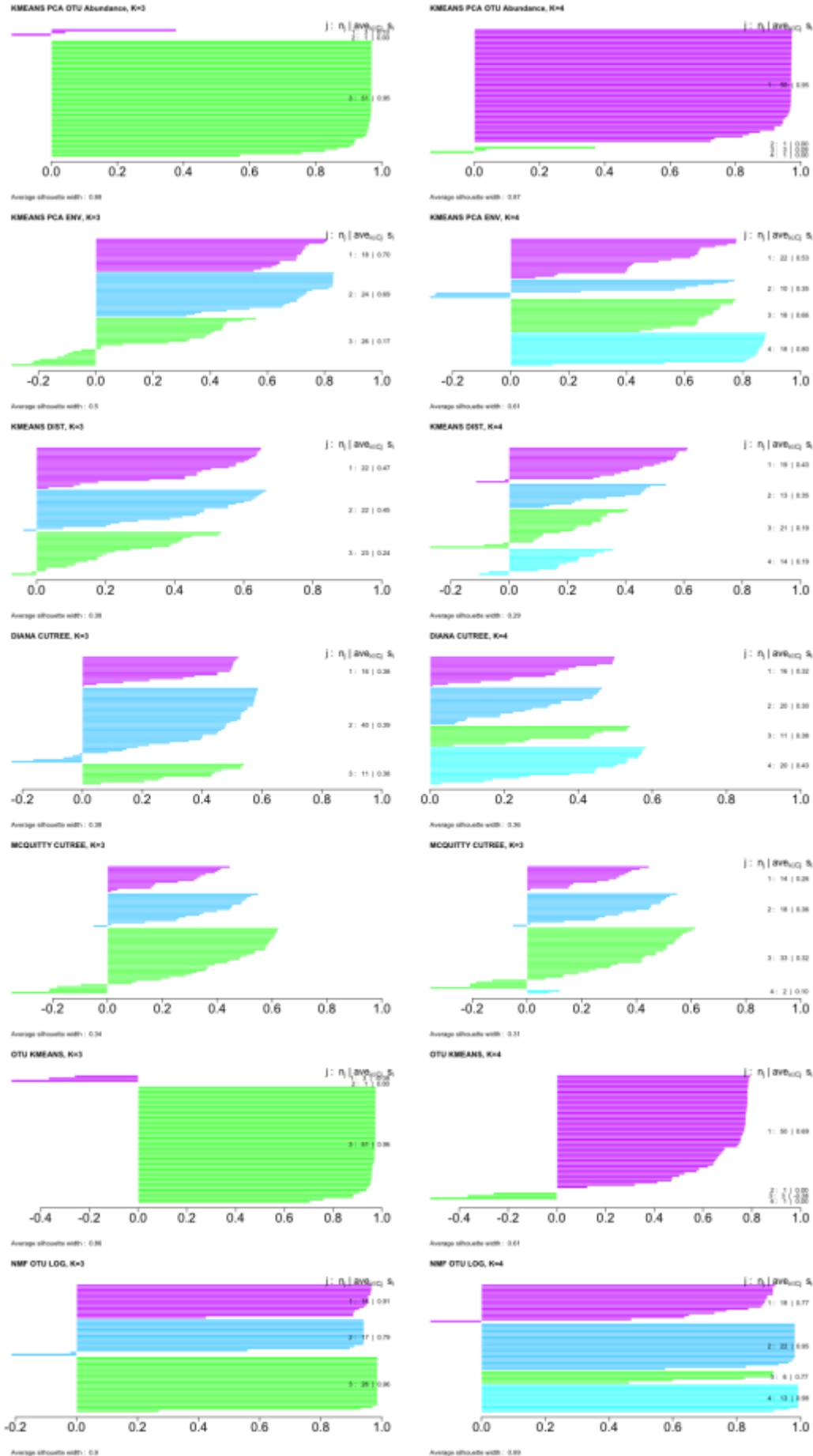
115 Although prior work had shown that alpha diversity relationships among Atacama desert samples were driven
116 by geographic location (Crits-Christoph et al. 2013), our preliminary analysis suggested sample to sample
117 similarities based on mash and NMF were better explained by pH, Relative Air Humidity, and Conductivity as
118 well as the previously reported location variable. Note that this “cluster first” computationally focused approach is
119 a departure from previous techniques that draw local communities using external metadata to overcome species
120 dispersion, although the species’ relationships are often defined by interrelated sequence clusters (OTUs).



121

122 *Figure 3. PCA of environmental variables at (top) K=3, and (bottom) K=4, colored according to assigned*
123 *cluster as determined by K-means (Hartigan & Wong, M 1979).*

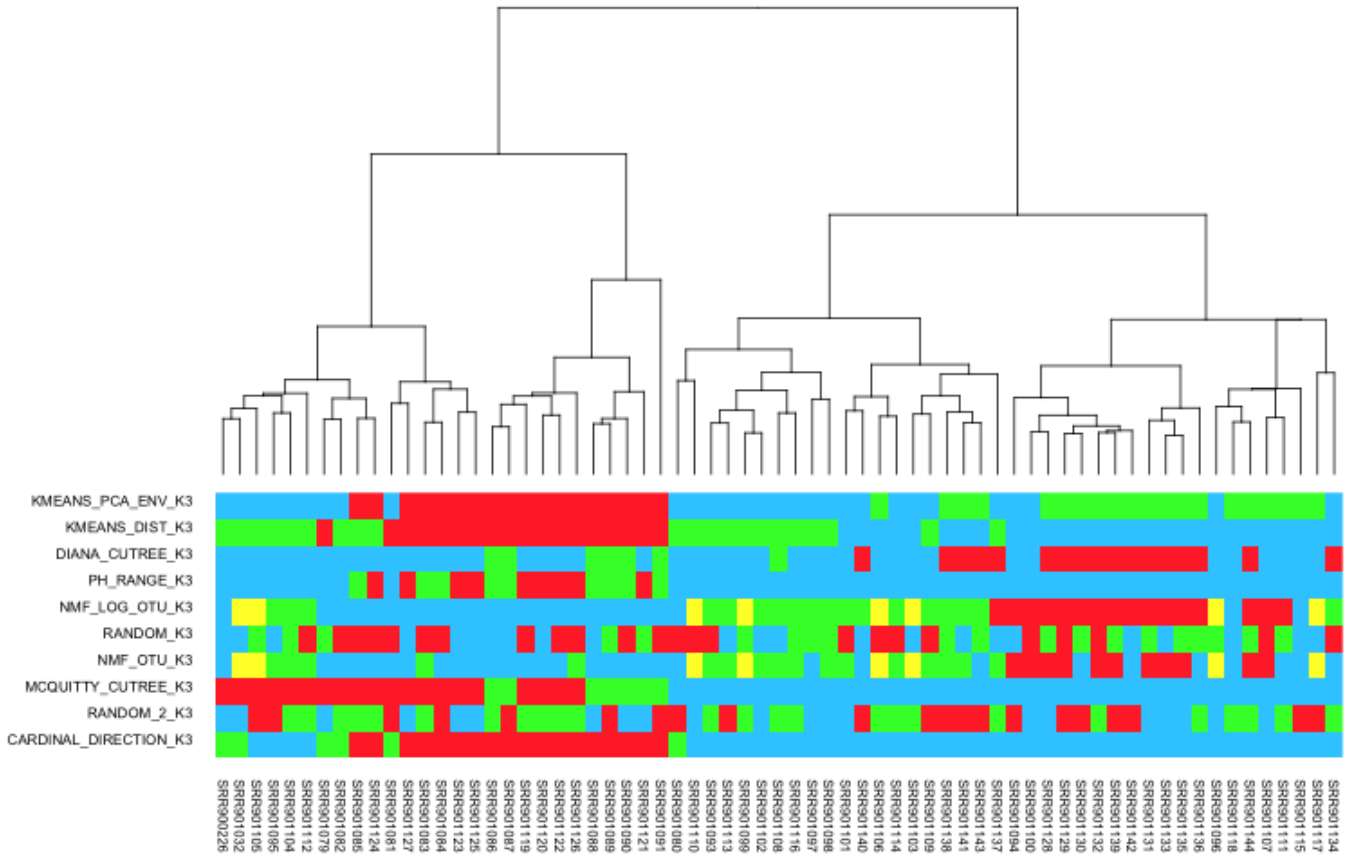
124 To be consistent with current practice, we first applied Principal Component Analysis (PCA) using the
125 samples' environmental variables (Air Humidity, Depth, Elevation, Soil Conductivity, and PH) to assess whether
126 there is a ecological basis for observed clusters (Figure 2). We also used Average Silhouette Width (Rousseeuw,
127 1987), which provides a measure of how dense clusters are, with denser clusters being preferred. Average
128 Silhouette can be used to determine the number of clusters by picking the higher average, in the case of
129 comparing two candidate clusterings.



131 *Figure 4. Left Side, red, k=3, right side, orange, k=4, Top to Bottom: PCA determined clusters on OTU*
132 *abundance, PCA determined environmental clusters, kmeans on mash distances, diana on mash distances,*
133 *kmeans on OTU data (euclidean distance), NMF on OTU data, K=3 and k=4 were found to be viable, on the*
134 *determination that an Average Silhouette Width above .5 was acceptable, while a score above .25 may indicate*
135 *structure (Rousseeuw, 1987). The environmentally driven PCA produced viable clusters at both K=3, and K=4,*
136 *Kmeans on sequence similarity at K=3 weakly indicated structure, Diana clustering weakly indicated structure at*
137 *K=3 and K=4, and NMF on OTUs produced structured clusters at both K=3 and K=4. All cluster silhouettes*
138 *show that clustering at 3 or 4 maybe viable, with the exception of K-means on OTUs themselves, in which most*
139 *samples clustered into a single, large group.*

140 Because silhouette average width for different clustering methods fell at the best values at either K=4, or at
141 K=3, new clusters were generated at both. At K=3, clusterings were generated by Random Assignment, Non-
142 negative Matrix Factorization based on abundance information, as well as log transformed OTU abundance, the
143 three clusters with least within cluster distances from both the Diana, and from Mcquitty-WPGMA hierarchical
144 clustering. Clusters were also made from Sample PH and from a North, South, or Central location. Since
145 environmental variable mixing was previously reported to be the driver of beta diversity at k=3 (cite Atacama
146 paper), we used environmental variable mixing to also generate clusterings. K=4 clusterings were generated with
147 Random Assignment, Non-negative Matrix Factorization based on abundance information, as well as log
148 transformed abundance information, the four clusters with least within cluster distances from both the Diana, and
149 from Mcquitty-WPGMA hierarchical clustering, as well as from PH. Cluster to Cluster correlations show that
150 Mcquitty-WPGMA is more similar to environmental clusterings; however, all non-random clusterings are more
151 similar to each other than to randomly generated clusterings, indicating all detect some elements of community
152 structure present in the data. Although this analysis has indicated that there was an ecological correlation to
153 computationally derived clusters, it has not shown which factors, or how those factors affect clustering. Further,
154 skewed species abundances with a few dominant species could make it more difficult to sample rare species at
155 modest sequencing depth; however, because Mash estimates the similarity between two sets, slight stochastic

156 differences in observed abundances should not significantly affect the results relative to traditional OTU
157 approaches that are also subject to sequence depth to uncover OTUs.

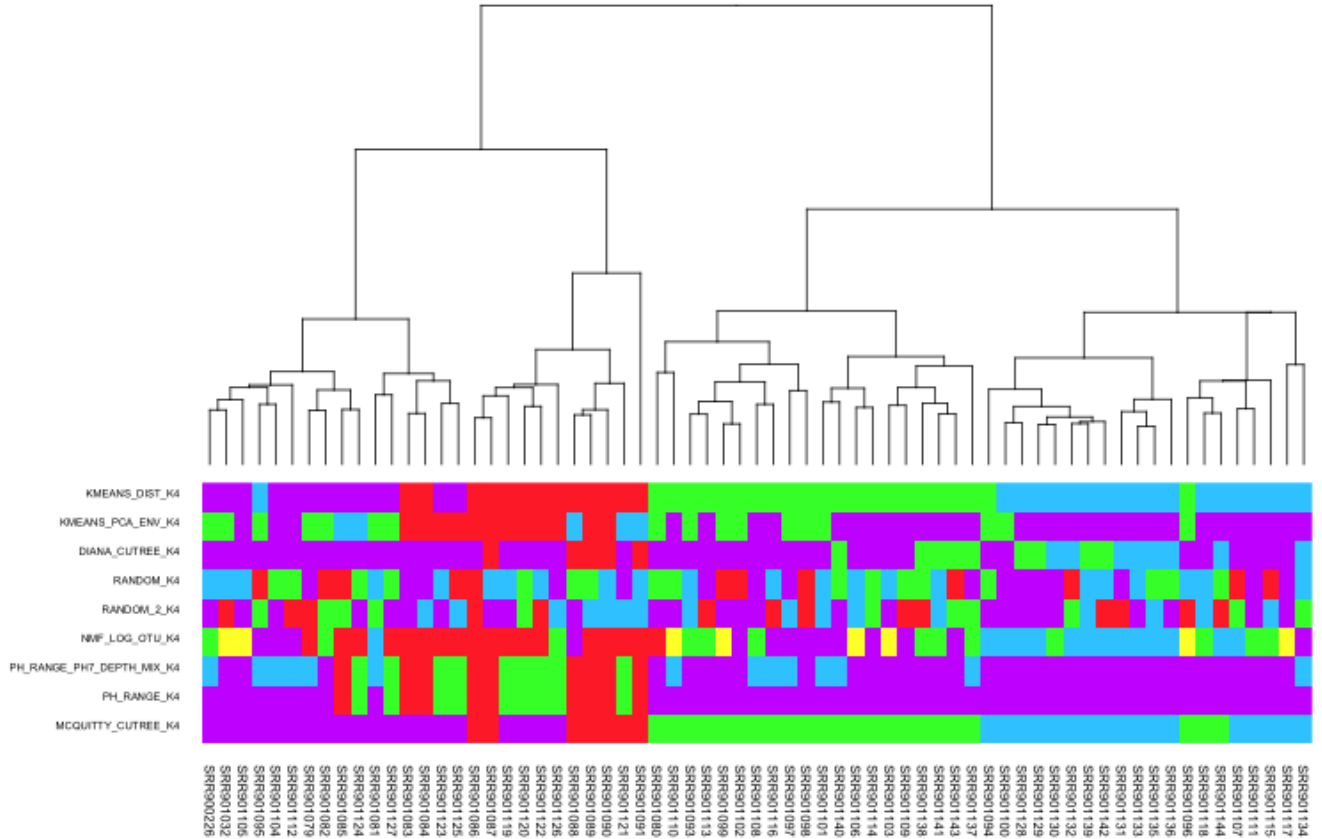


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159

160 *Figure 5. K=3 groupings ordered by a Diana determined dendrogram, yellow means that value was not*
161 *found. It's more important that samples between belong to the same group than to the same color. Each row*
162 *represents a grouping with three total groups. If between two grouping a sample has the same color with another*
163 *sample that means that those two groupings put that sample in the same group. For instance, in K-means on PCA*
164 *of environmental variables there is a large red section, and NMF on log OTU many of those samples that were*

165 *marked red are now marked blue, that is an indication that these two methods have grouped these samples into*
166 *their own clusters.*



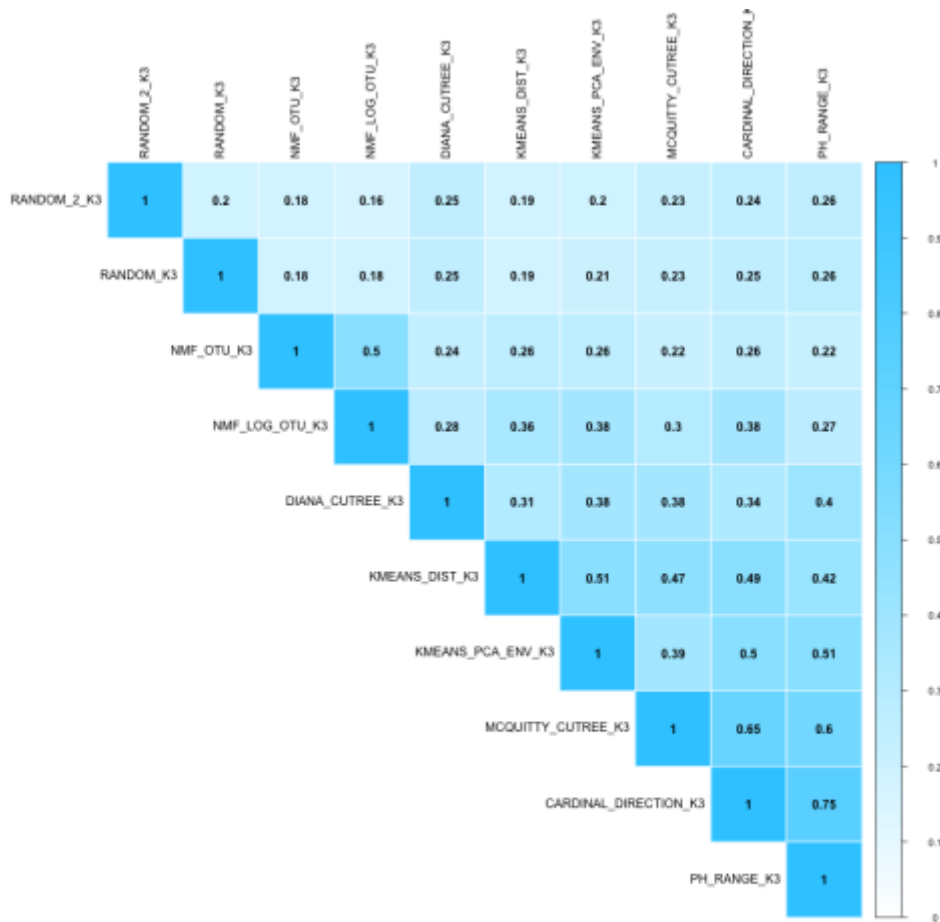
167

168 *Figure 6. K=4 groupings ordered by a Diana determined dendrogram, yellow means that value was not*
169 *found. It's more important that samples between belong to the same group than to the same color. Each row*
170 *represents a grouping with three total groups. If between two grouping a sample has the same color with another*
171 *sample that means that those two groupings put that sample in the same group.*

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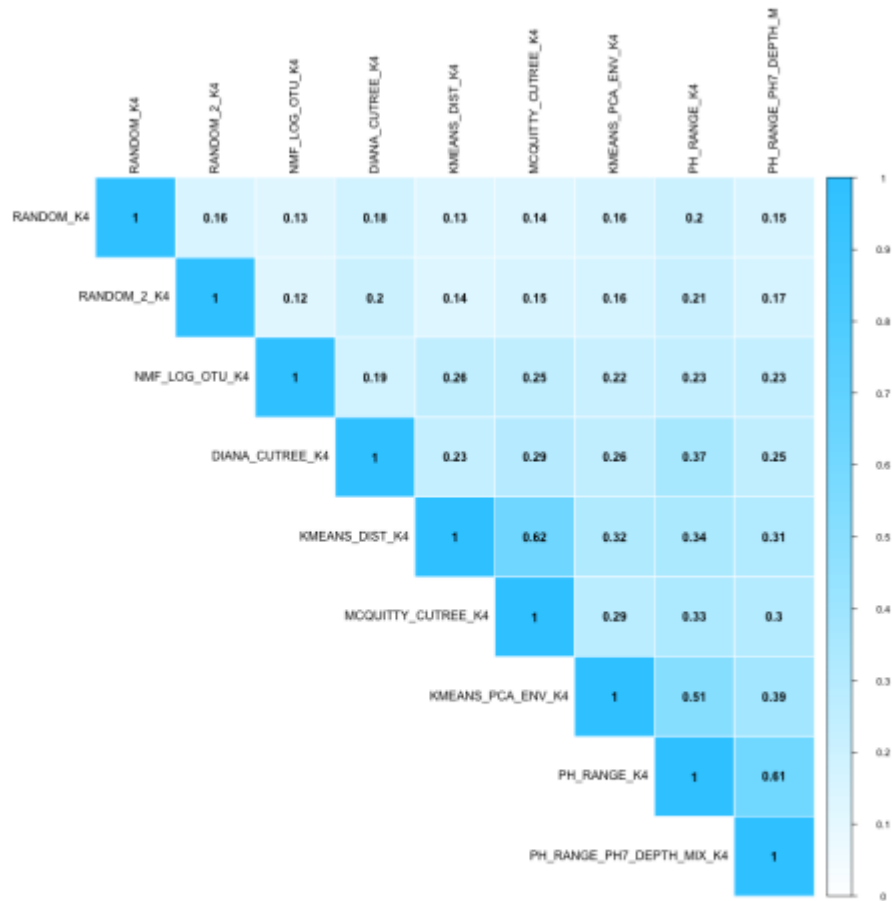
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174



175 *Figure 7. represents cluster similarities between each cluster, cluster similarity jaccard algorithm was used,*

176 *k=3 clusterings are shown.*



177

178 *Figure 8. represents cluster similarities between each cluster, cluster similarity jaccard algorithm was used,*

179 *k=4 clusterings are shown.*

180

ANOVA Results	MCQUITTU_CUTREE_K3		NMF_LOG_OTU_K3		MCQUITTU_CUTREE_K4		NMF_LOG_OTU_K4	
	P-value	Signif.	P-value	Signif.	P-value	Signif.	P-value	Signif.
pH	6.97E-13	***	1.13E-12	***	3.99E-14	***	2.41E-07	***
Elevation	0.14448		0.015637	*	0.000823	***	0.02558	*
Conductivity	0.00573	**	7.32E-05	***	0.398154		0.02377	*
Air_Relative_Humidity	5.03E-07	***	0.000361	***	0.006096	**	0.01114	*

Depth	0.08345		0.310562		0.004934	**	0.02009	*
pH with Conductivity	0.81559		0.026234	*	0.005361	**	0.00579	**
Elevation with Conductivity	0.50011		0.784539		0.14726		0.01553	*
Elevation with Air_Relative_Humidity	0.01035	*	0.375278		0.029011	*	0.69145	
Conductivity with Air_Relative_Humidity	0.00156	**	0.927365		0.022087	*	0.04518	*
pH and Depth	0.1124		0.99117		0.039508	*	0.01784	*
Conductivity with Air_Relative_Humidity with Depth	0.03048	*	0.371664		0.133212		0.91445	
pH with Elevation with Conductivity with Depth	0.22824		0.425284		0.777819		0.03665	*
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1								
8 observations deleted due to missingness in NMF Analyses, 1 observations deleted due to missingness in Mcquitty Analyses								

181

182 The clusterings were modeled by ANOVA, and after calculating a log likelihood test, we found that for both
183 K=3 and K=4 Mcquitty hierarchical clustering, followed by NMF on OTUs, were the most significant and
184 therefore best corresponded to the environmental data. For McQuitty hierarchal clustering, PH and Elevation were
185 found to have the most significance, however, since the elevation was the same for all of the samples of any given
186 sampling site, since elevation is highly correlated with sampling location there maybe some other variable that is
187 being indirectly measured, also highly correlated with sampling location. For NMF on log abundance PH ,
188 Conductivity, and Relative humidity of the air were found to be most significant; however, because relative
189 humidity of each sampling site was the same, it is unknown whether relative humidity of the air was the
190 contributing factor or some other, unknown variable, that also differed from site to site was a factor.

191

192 McQuitty clustering has a .65 similarity with the Cardinal Direction, and similarly High similarities with
193 other environmentally determined groupings. We also see that both McQuitty and NMF have high p-values with
194 some environmental variables in anova, with Ph being particularly significant in both McQuitty and NMF, and to

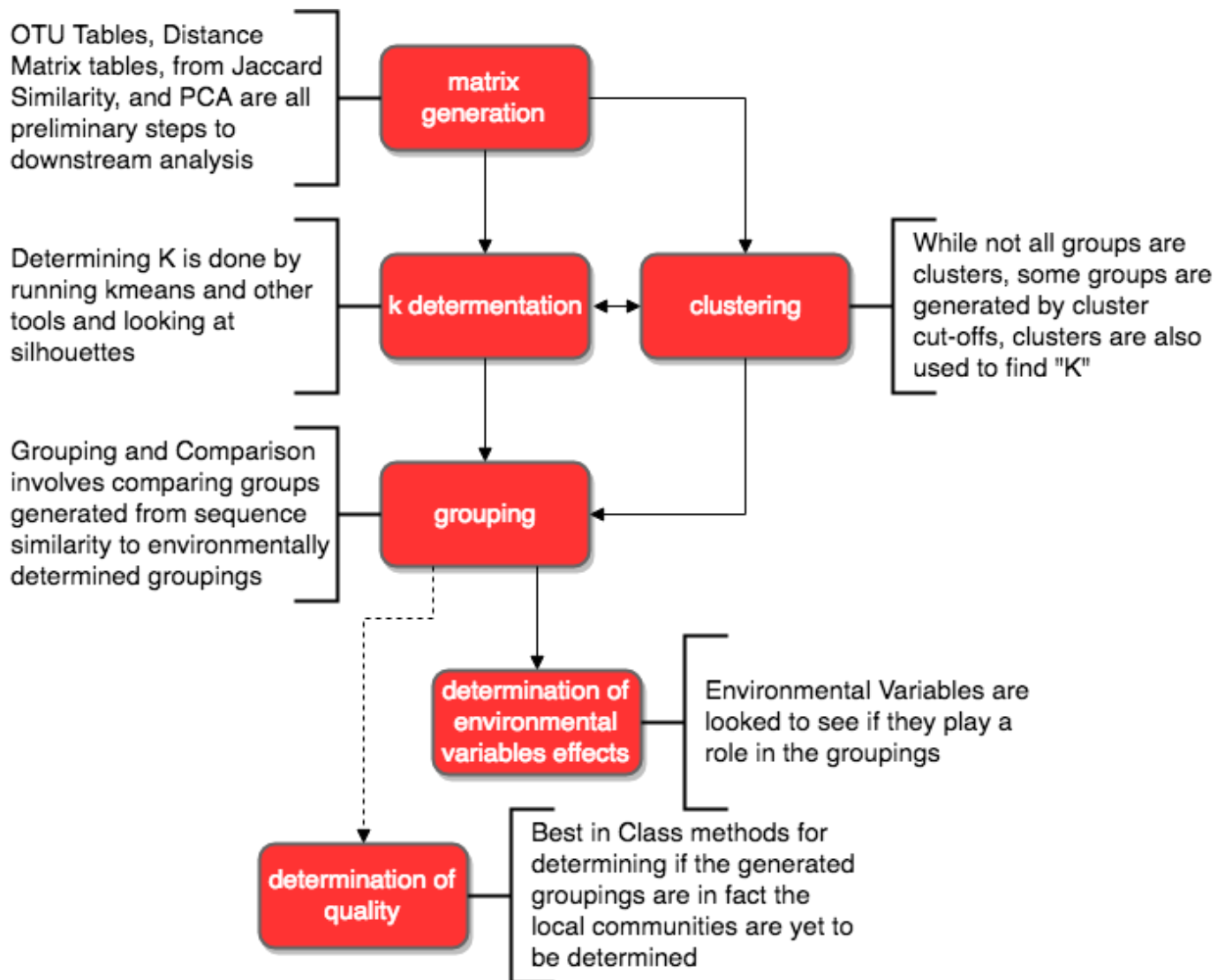
195 a lesser extent Relative Air Humidity being significant as well, and that the sample similarities within these
196 groupings are high. This shows that OTU based methods and distance based methods produce similar results, if
197 driven by slightly different environmental variables, and is getting at the underlying structure of the local
198 communities.

199

200 As per the clustering silhouettes, some of the methods, Diana and NMF, work better at four clusters, while
201 McQuitty and K-means did better at three. The most explicative results, as per ANOVA, NMF on log OTU
202 abundance and McQuitty slightly disagree on which environmental variables have the most importance, but PH
203 and Relative Air Humidity can be seen across all four ANOVAs.

204 *Concluding Remarks and Recommendations*

205



206 Figure 9. Workflow for Determining, Describing, and Validating Atacama data.

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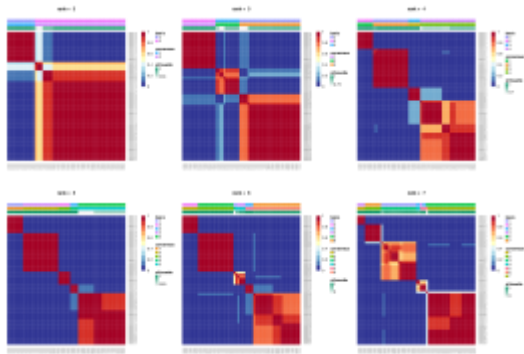
209 As a general workflow (figure 9), after sample collection either OTUs abundances are generated, or sample to
210 sample distances are calculated by comparing their contained trimmed sequences. In the case of the sample to
211 sample distances a distance matrix is generated that can be clustered though hierarchical or other means, and in
212 the case of OTU abundances NMF or K-means is better suited. We calculated pairwise distance on both shared

213 sequences and on OTUs, and then clustered OTUs and shared sequences via K-means, and for shared sequences
214 diana clustering was also utilized, and for NMF was also utilized for OTU abundance. The groupings can then be
215 checked for the influence of independent variables, through a statistical model, in this case anova, which was run
216 on clusters, the 'anova' function from the R 'stats' package was used, and LogLik from the R 'stats' package was
217 used to compare Log-Likelihoods. Clusters were compared to each other using both RAND and Jaccard similarity
218 cluster evaluation methods, as well as a wilcox test (Hollander et al. 2013);(Bauer, 1972).

219 The Atacama data used here is from SRA:SRA091062, Bioproject ID: PRJNA208226, which was thought of
220 as three clusters of data, aligning with sampling site: North, Central, and South. Atacama was chosen for its
221 previous environmental analysis, geographically distinct sampling sites, and curated metadata.

222 Mothur was used to process Raw files for OTU analysis as per non-shhh (Quince et al. 2009) 454 SOP:
223 https://www.mothur.org/wiki/454_SOP. for sequence similarity distance Mothur was used to filter samples
224 based quality scores, as per the shhh and trimming portion of the mothur 454 SOP. Initial NMF analysis (figure
225 10) was done with “sake” (<https://github.com/naikai/sake>), which was originally created to analyze gene
226 expression data, was here utilized to look at OTU abundance data, at k=3 both log transformed and non-log
227 transformed data was utilized, the nsNMF NMF algorithm NMF algorithm was used and the the NMF tool was
228 run at 350 runs, at k=4 only log transformed data was run, with the nsNMF (Pascual-Montano et al. 2006) , NMF
229 algorithm at 350 runs. nsNMF was chosen for its design to deal with perceived sparseness in the data. The R
230 'cluster_similarity' function from the 'clusteval' package was used for Jaccard and RAND similarities, while
231 'wilcox.test' function from the R 'stats' package was used for the wilcox test. wpgma was chosen for
232 Agglomerative clustering because clusters were expected to be of unequal size, as unweighted hierarchical
233 methods can become distorted when large and small groups are compared, and a clear contrast to centroid
234 clustering, as like k-means, was desired. The R 'hclust' function was used from the 'stats' package was used for
235 agglomerative clustering. Diana, from the R 'cluster' package was used for divisive hierarchical clustering, in
236 agglomerative hierarchical clustering samples are combined until all samples are in the same cluster, whereas in
237 divisive hierarchical clustering all samples start in the same cluster and then are partitioned into daughter

238 clusters.. And further analysis and figure analysis was done with the caret ([https://cran.r-](https://cran.r-project.org/package=caret)
239 [project.org/package=caret](https://cran.r-project.org/package=caret)), clusteval (<https://cran.r-project.org/package=clusteval>), cluster ([https://CRAN.R-](https://CRAN.R-project.org/package=cluster)
240 [project.org/package=cluster](https://CRAN.R-project.org/package=cluster)), corrplot (<https://CRAN.R-project.org/package=corrplot>), d3heatmap (<https://CRAN.R-project.org/package=d3heatmap>), fpc (<https://CRAN.R-project.org/package=fpc>), gplots (<https://CRAN.R-project.org/package=gplots>), and NMF (<https://CRAN.R-project.org/package=NMF>) R
241
242
243 packages.



244

245 Figure 10. Correlations of various number of clusters (k) based on K-Means from k=2 to k=7.

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247

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