

# A community-maintained standard library of population genetic models

Jeffrey R. Adrion<sup>1,\*</sup>, Christopher B. Cole<sup>2,\*</sup>, Noah Dukler<sup>3,\*</sup>, Jared G. Galloway<sup>1,\*</sup>, Ariella L. Gladstein<sup>4,\*</sup>, Graham Gower<sup>5,\*</sup>, Christopher C. Kyriazis<sup>6,\*</sup>, Aaron P. Ragsdale<sup>7,\*</sup>, Georgia Tsambos<sup>8,\*</sup>, Franz Baumdicker<sup>9</sup>, Jedidiah Carlson<sup>10</sup>, Reed A. Cartwright<sup>11</sup>, Arun Durvasula<sup>12</sup>, Ilan Gronau<sup>13</sup>, Bernard Y. Kim<sup>14</sup>, Patrick McKenzie<sup>15</sup>, Philipp W. Messer<sup>16</sup>, Ekaterina Noskova<sup>17</sup>, Diego Ortega-Del Vecchyo<sup>18</sup>, Fernando Racimo<sup>5</sup>, Travis J. Struck<sup>19</sup>, Simon Gravel<sup>7,†</sup>, Ryan N. Gutenkunst<sup>19,†</sup>, Kirk E. Lohmueller<sup>6,†</sup>, Peter L. Ralph<sup>1,20,†</sup>, Daniel R. Schrider<sup>4,†</sup>, Adam Siepel<sup>3,†</sup>, Jerome Kelleher<sup>21,†,Ⓝ</sup>, and Andrew D. Kern<sup>1,†,Ⓝ</sup>

<sup>1</sup>Department of Biology and Institute of Ecology and Evolution, University of Oregon,

<sup>2</sup>Wellcome Trust Centre for Human Genetics, University of Oxford, <sup>3</sup>Simons Center for Quantitative Biology, Cold Spring Harbor Laboratory, <sup>4</sup>Department of Genetics, University of North Carolina at Chapel Hill, <sup>5</sup>Lundbeck GeoGenetics Centre, Globe Institute, University of Copenhagen, <sup>6</sup>Department of Ecology and Evolutionary Biology, University of California, Los Angeles, <sup>7</sup>Department of Human Genetics, McGill University, <sup>8</sup>Melbourne Integrative Genomics, School of Mathematics and Statistics, University of Melbourne, <sup>9</sup>Department of Mathematical Stochastics, University of Freiburg, <sup>10</sup>Department of Genome Sciences, University of Washington, <sup>11</sup>The Biodesign Institute and The School of Life Sciences, Arizona State University, <sup>12</sup>Department of Human Genetics, David Geffen School of Medicine, University of California, Los Angeles, <sup>13</sup>The Efi Arazi School of Computer Science, The Herzliya Interdisciplinary Center, Israel, <sup>14</sup>Department of Biology, Stanford University, <sup>15</sup>Department of Ecology, Evolution, and Environmental Biology, Columbia University, <sup>16</sup>Department of Computational Biology, Cornell University, <sup>17</sup>Computer Technologies Laboratory, ITMO University, <sup>18</sup>International Laboratory for Human Genome Research, National Autonomous University of Mexico, <sup>19</sup>Department of Molecular and Cellular Biology, University of Arizona, <sup>20</sup>Department of Mathematics, University of Oregon, <sup>21</sup>Big Data Institute, Li Ka Shing Centre for Health Information and Discovery, University of Oxford, \*Denotes shared first authorship, listed alphabetically, †Denotes shared senior authorship, listed alphabetically, ⓃDenotes corresponding authors, listed alphabetically

## Abstract

The explosion in population genomic data demands ever more complex modes of analysis, and increasingly these analyses depend on sophisticated simulations. Recent advances in population genetic simulation have made it possible to simulate large and complex models, but specifying such models for a particular simulation engine remains a difficult and error-prone task. Computational genetics researchers currently re-implement simulation models independently, leading to inconsistency and duplication of effort. This situation presents a major barrier to empirical researchers seeking to use simulations for power analyses of upcoming studies or sanity checks on existing genomic data. Population genetics, as a field, also lacks standard benchmarks by which new tools for inference might be measured. Here we describe a new resource, `stdpopsim`, that attempts to rectify this situation. `Stdpopsim` is a community-driven open source project, which provides easy access to a growing catalog of published simulation models from a range of organisms and supports multiple simulation engine backends. This resource is available as a well-documented python library with a simple command-line interface. We share some examples demonstrating how `stdpopsim` can be used to systematically compare demographic inference methods, and we encourage a broader community of developers to contribute to this growing resource.

**Keywords:** Population genetics, Simulation, Inference, Reproducibility

## Introduction

While population genetics has always used statistical methods to make inferences from data, the degree of sophistication of the questions, models, data, and computational approaches used have all increased over the past two decades. Currently there exist a myriad of computational methods that can infer the histories of populations (Gutenkunst et al., 2009; Li and Durbin, 2011; Excoffier et al., 2013; Schiffels and Durbin, 2014; Terhorst et al., 2017; Ragsdale and Gravel, 2019), the distribution of fitness effects (Boyko et al., 2008; Kim et al., 2017; Tataru et al., 2017; Fortier et al., 2019; Huang and Siepel, 2019; Ortega-Del Vecchyo et al., 2019), recombination rates (McVean et al., 2004; Chan et al., 2012; Lin et al., 2013; Adrion et al., 2020; Barroso et al., 2019), and the extent of positive selection in genome sequence data (Kim and Stephan, 2002; Eyre-Walker and Keightley, 2009; Alachiotis et al., 2012; Garud et al., 2015; DeGiorgio et al., 2016; Kern and Schrider, 2018; Sugden et al., 2018). While these methods have undoubtedly increased our understanding of genetic and evolutionary processes, very little has been done to systematically benchmark the quality of these inferences or their robustness to deviations from their underlying assumptions. As large databases of population genetic variation begin to be used to inform public health procedures, the accuracy and quality of these inferences is becoming ever more important.

72 Assessing the accuracy of inference methods for population genetics is challenging in  
73 large part because the “ground-truth” in question generally comes not from direct empir-  
74 ical observations, as the relevant historical processes can rarely be observed, but instead  
75 from simulations. Population genetic simulations are therefore critically important to  
76 the field, yet there has been no systematic attempt to establish community standards  
77 or best practices for executing them. Instead, the general *modus operandi* to date has  
78 been for individual groups to validate their own methods using simulations coded from  
79 scratch. Often these simulations are more useful to showcase a novel method than to rig-  
80 orously compare it with competing methods. Moreover, this situation results in a great  
81 deal of duplicated effort, and contributes to decreased reproducibility and transparency  
82 across the entire field. It is also a barrier to entry to the field, because new researchers  
83 can struggle with the many steps involved in implementing a state-of-the-art population  
84 genetics simulation, including identifying appropriate demographic models from the lit-  
85 erature, translating them into input for a simulator, and choosing appropriate values for  
86 key population genetic parameters, such as the mutation and recombination rates.

87 A related issue is that it has been challenging to assess the degree to which model-  
88 ing assumptions and choices of data summaries can affect population genetic inferences.  
89 Standardized simulations would enable these questions to be systematically examined.  
90 Importantly, there are clear examples of different methods yielding fundamentally differ-  
91 ent conclusions. For example, Markovian coalescent methods applied to human genomes  
92 have suggested large ancient ( $> 100,000$  years ago) ancestral population sizes and bot-  
93 tlenecks that have not been detected by other methods based on allele frequency spectra  
94 (see Beichman et al., 2017). These distinct methods differ in how they model, summarize,  
95 and optimize fit to genetic variation data, suggesting that such design choices can greatly  
96 affect the performance of the inference. Furthermore, some methods are likely to perform  
97 better than others under certain scenarios, but researchers lack principled guidelines for  
98 selecting the best method for addressing their particular questions. The need for guidance  
99 from simulated data will only increase as researchers seek to apply population genetic  
100 methods to a growing collection of non-model taxa.

101 For these reasons, we have generated a standardized, community-driven resource for  
102 simulating published demographic models from a number of popular study systems. This  
103 resource, which we call `stdpopsim`, makes running realistic simulations for population  
104 genetic analysis a simple matter of choosing pre-implemented models from a community-  
105 maintained catalog. The `stdpopsim` catalog currently contains six species: humans,  
106 *Pongo abelii*, *Canis familiaris*, *Drosophila melanogaster*, *Arabidopsis thaliana*, and *Es-*  
107 *cherichia coli*. For each species, the catalog contains curated information on our cur-  
108 rent understanding of the physical organization of its genome, inferred genetic maps,  
109 population-level parameters (e.g., mutation rate and generation time estimates), and  
110 published demographic models. These models and parameters are meant to represent

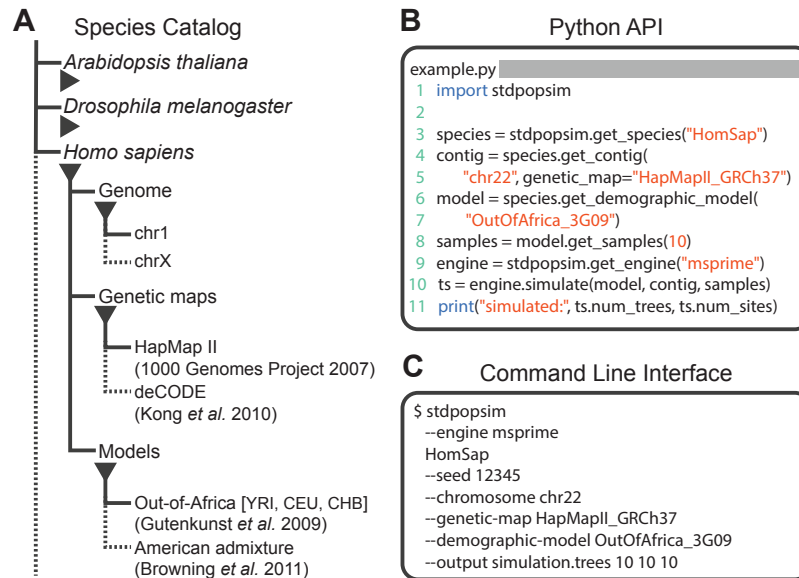


Figure 1: **Structure of stdpopsim.** (A) The hierarchical organization of the stdpopsim catalog contains all model simulation information within individual species (expanded information shown here for *H. sapiens* only). Each species is associated with a representation of the physical genome, and one or more genetic maps and demographic models. Dotted lines indicate that only a subset of these categories is shown. At right we show example code to specify and simulate models using (B) the python API or (C) the command line interface.

111 the field's current understanding, and we intend for this resource to evolve as new results  
112 become available, and other existing models are added to **stdpopsim** by the community.  
113 We have implemented both a command line interface and a simple Python API that can  
114 be used to simulate genomic data from a choice of organism, genetic map, chromosome,  
115 and demographic history. In this way, **stdpopsim** will lower the barrier to high-quality  
116 simulation for exploratory analyses, enable rigorous evaluation of population genetic soft-  
117 ware, and contribute to increased reliability of population genetic inferences.

118 The **stdpopsim** library has been developed by the PopSim Consortium using a dis-  
119 tributed open source model, with strong procedures in place to continue its growth and  
120 maintain quality. Importantly, we developed rigorous quality control methods to ensure  
121 that we have correctly implemented the models as described in their original publication  
122 and provided documented methods for others to contribute new models. We invite new  
123 collaborators to join our community: those interested should visit our developer docu-  
124 mentation at <https://stdpopsim.readthedocs.io/en/latest/development.html>. Below  
125 we describe the resource and give examples of how it can be used to benchmark demo-  
126 graphic inference methods.

## 127 Results

128 The `stdpopsim` library is a community-maintained collection of empirical genome data  
129 and population genetics simulation models, illustrated in Figure 1. The package cen-  
130 ters on a catalog of genomic information and demographic models for a growing list of  
131 species (Fig. 1A), and software resources to facilitate efficient simulations (Fig. 1B-C).  
132 Given the genome data and simulation model descriptions defined within the library, it is  
133 straightforward to run standardized simulations across a range of organisms. `Stdpopsim`  
134 has a Python API and a user-friendly command line interface, allowing users with min-  
135 imal experience direct access to state-of-the-art simulations. Simulations are output in  
136 the “succinct tree sequence” format (Kelleher et al., 2016, 2018, 2019), which contains  
137 complete genealogical information about the simulated samples, is extremely compact,  
138 and can be processed efficiently using the `tskit` library (Kelleher et al., 2016, 2018). The  
139 tree sequence format could also be converted to other formats (e.g., VCF) by the user if  
140 desired.

## 141 The species catalog

142 The central feature of `stdpopsim` is the species catalog, a systematic organization of the  
143 key quantitative data needed to simulate a given species. Data are currently available  
144 for humans, *P. abelii*, *C. familiaris*, *D. melanogaster*, *A. thaliana*, and *E. coli*. A species  
145 definition consists of two key elements. Firstly, the library defines some basic information  
146 about our current understanding of each species’ genome, including information about  
147 chromosome lengths, average mutation rate estimates, and generation times. We also  
148 provide access to detailed empirical information such as inferred genetic maps, which  
149 model observed heterogeneity in recombination rate along chromosomes. Such maps are  
150 often large, so we do not distribute them directly with the software, but make them  
151 available for download in a standard format. When a simulation using such a map is  
152 requested by the user, `stdpopsim` will transparently download the map data into a local  
153 cache, where it can be quickly retrieved for subsequent simulations. In the initial version  
154 of `stdpopsim` we support the HapMapII (International HapMap Consortium et al., 2007)  
155 and deCODE (Kong et al., 2010) genetic maps for humans; the Nater et al. (2017)  
156 maps for *P. abelii*; the Campbell et al. (2016) map for *C. familiaris*; the Salomé et al.  
157 (2011) map for *A. thaliana*; and the Comeron et al. (2012) map for *D. melanogaster*.  
158 Adding further maps to the library is straightforward. The second key element of a  
159 species description within `stdpopsim` is a set of carefully curated population genetic  
160 model descriptions from the literature, which allow simulation under specific historical  
161 scenarios that have been fit to present-day patterns of genetic variation (See the Methods  
162 for a description of the community development and quality-control process for these  
163 models.)

Model ID	Citation	CPU(s)	RAM(MB)	File(MB)
<i>HomSap (Homo sapiens)</i>				
Africa_1T12	Tennessen et al. (2012)	10.4	193.3	23.3
Zigzag_1S14	Schiffels and Durbin (2014)	3.4	105.0	7.9
AshkSub_7G19	Gladstein and Hammer (2019)	15.7	215.3	26.4
OutOfAfrica_3G09	Gutenkunst et al. (2009)	10.9	181.3	21.1
OutOfAfrica_2T12	Tennessen et al. (2012)	11.3	198.0	24.1
AncientEurasia_9K19	Kamm et al. (2019)	69.4	304.1	41.2
AmericanAdmixture_4B11	Browning et al. (2018)	11.1	187.3	22.3
PapuansOutOfAfrica_10J19	Jacobs et al. (2019)	234.7	526.3	77.8
OutOfAfricaArchaicAdmixture_5R19	Ragsdale and Gravel (2019)	9.6	184.5	21.7
<i>DroMel (Drosophila melanogaster)</i>				
OutOfAfrica_2L06	Li and Stephan (2006)	0.6	68.7	1.6
African3Epoch_1S16	Sheehan and Song (2016)	0.5	60.9	0.2
<i>AraTha (Arabidopsis thaliana)</i>				
African2Epoch_1H18	Huber et al. (2018)	434.1	359.2	50.7
African3Epoch_1H18	Huber et al. (2018)	208.6	400.6	58.0
SouthMiddleAtlas_1D17	Durvasula et al. (2017)	159.6	315.4	43.1
<i>PonAbe (Pongo abelii)</i>				
TwoSpecies_2L11	Locke et al. (2011)	7.4	170.5	14.7

Table 1: Initial set of demographic models in the catalog and summary of computing resources needed for simulation. For each model, we report the CPU time, maximum memory usage and the size of the output `tskit` file, as simulated using the `msprime` simulation engine (version 0.7.4). In each case, we simulate 100 samples drawn from the first population, for the shortest chromosome of that species and a constant chromosome-specific recombination rate. The times reported are for a single run on an Intel i5-7600K CPU. Computing resources required will vary widely depending on sample sizes, chromosome length, recombination rates and other factors.

164 The current demographic models in the `stdpopsim` catalog are shown in Table 1.  
165 *Homo sapiens* currently has the richest selection of population models. These include: a  
166 simplified version of the Tennessen et al. (2012) model with only the African population  
167 specified (expansion from the ancestral population and recent growth; Africa\_1T12); the  
168 three-population model of Gutenkunst et al. (2009), which specifies the out-of-Africa  
169 bottleneck as well as the subsequent divergence of the European and Asian popula-  
170 tions (OutOfAfrica\_3G09); the Tennessen et al. (2012) two-population variant of the  
171 Gutenkunst et al. model, which does not include Asian populations but more explic-  
172 itly models recent rapid human population growth in Europe (OutOfAfrica\_2T12); the  
173 Browning et al. (2018) admixture model for American populations, which specifies ances-  
174 tral African, European, and Asian population components (AmericanAdmixture\_4B11);  
175 a three-population out-of-Africa model from Ragsdale and Gravel (2019), which includes  
176 archaic admixture (OutOfAfricaArchaicAdmixture\_5R19); a complex model of ancient

177 Eurasian admixture from Kamm et al. (2019) (AncientEurasia\_9K19); and a synthetic  
178 model of oscillating population size from Schiffels and Durbin (2014) (Zigzag\_1S14).

179 For *D. melanogaster*, we have implemented the three-epoch model estimated by Shee-  
180 han and Song (2016) from an African sample (African3Epoch\_1S16), as well as the out-  
181 of-Africa divergence and associated bottleneck model of Li and Stephan (2006), which  
182 jointly models African and European populations (OutOfAfrica\_2L06). For *A. thaliana*,  
183 we implemented the model in Durvasula et al. (2017) inferred using MSMC. This model in-  
184 cludes a continuous change in population size over time, rather than pre-specified epochs  
185 of different population sizes (SouthMiddleAtlas\_1D17). We have also implemented a two-  
186 epoch and a three-epoch model estimated from African samples of *A. thaliana* in Huber  
187 et al. (2018) (African2Epoch\_1H18 and African3Epoch\_1H18).

188 In addition to organism-specific models, `stdpopsim` also includes a generic piecewise  
189 constant size model and isolation with migration (IM) model which can be used with any  
190 genome and genetic map. Together these models contain many features believed to affect  
191 observed patterns of polymorphism (e.g., bottlenecks, population growth, admixture) and  
192 therefore provide useful benchmarks for method development.

193 To guarantee reproducibility, we have standardized naming conventions for species,  
194 genetic maps, and demographic models that will enable long-term stability of unique  
195 identifiers used throughout `stdpopsim`, as described in our documentation ([https://  
196 stdpopsim.readthedocs.io/en/latest/development.html#naming-conventions](https://stdpopsim.readthedocs.io/en/latest/development.html#naming-conventions)).

## 197 Simulation engines

198 Currently, `stdpopsim` uses the `msprime` coalescent simulator (Kelleher et al., 2016) as  
199 the default simulation engine. Coalescent simulations, while highly efficient, are limited  
200 in their ability to model continuous geography or complex selection scenarios, such as  
201 recurrent sweeps and background selection. For these reasons, we have also implemented  
202 the forward-time simulator, `SLiM` (Haller et al., 2019; Haller and Messer, 2019), as an  
203 alternative backend engine to `stdpopsim`, allowing for the simulation of processes that  
204 cannot be modeled under the coalescent. However, as forward-time simulators explicitly  
205 model all individuals in a population, simulating large population sizes can be highly  
206 demanding of computational resources. One common practice used to address this chal-  
207 lenge is to simulate a *smaller* population, but to rescale resulting times, mutation rates,  
208 recombination rates, and selection coefficients so that the intensity of mutation, recom-  
209 bination, and allele frequency change due to selection per unit time remains the same  
210 (see the `SLiM` manual and Uricchio and Hernandez, 2014). Our implementation of the  
211 `SLiM` backend allows easy use of this *rescaling* through a single “scaling factor” argument.  
212 Such down-scaled simulations are not completely equivalent to simulating all individuals  
213 in the population, and may lead to subtle differences, especially in the presence of selec-

214 tion. However, since many sequence-based measures of population diversity remain nearly  
215 unchanged when rescaling in this fashion, this practice is effective for many purposes and  
216 widely employed.

217 We validated our implementation of the SLiM engine by comparing estimates of several  
218 population genetic summary statistics for neutral simulations generated by both SLiM and  
219 `msprime`. Examples of this validation for the AncientEurasia\_9K19 model (Kamm et al.,  
220 2019) are shown in Figures S1 and S2. For this model, down-scaling factors of up to 10  
221 produce patterns of both diversity and linkage disequilibrium that are indistinguishable  
222 from those observed under the coalescent (i.e., `msprime`). Scaling down by a factor of  
223 50 does appear to modify the distribution of these sequence statistics. Interestingly, the  
224 apparent difference between distributions is somewhat larger when simulating using a  
225 uniform recombination rate (Figure S2), likely due to the lower variation in the values  
226 of these statistics. Importantly, both comparisons validate the equivalence of SLiM and  
227 `msprime` when no down-scaling is applied. The results are also optimistic about the  
228 rescaling strategy to reduce computational burden, but the possible effects are not well-  
229 understood, so results relying on rescaled simulations should be carefully validated.

## 230 **Documentation and reproducibility**

231 The `stdpopsim` command-line interface, by default, outputs citation information for the  
232 models, genetic maps, and simulation engines used in any particular run. We hope that  
233 this feature will encourage users to appropriately acknowledge the resources used in pub-  
234 lished work, and encourage authors publishing demographic models to contribute to our  
235 ongoing community-driven development process. Together with the `stdpopsim` version  
236 number and the long-term stable identifiers for population models and genetic maps, this  
237 citation information will result in well-documented and reproducible simulation work-  
238 flows. The individual tree sequence files produced by `stdpopsim` also contain complete  
239 provenance information including the command line arguments, operating system envi-  
240 ronment and versions of key libraries used.

## 241 **Use case: comparing methods of demographic inference**

242 As an example of the utility of `stdpopsim`, we demonstrate how it can be easily used  
243 to perform a fair comparison of popular demographic inference methods. Although we  
244 present comparison of results from several methods, our aim at this stage is not to provide  
245 an exhaustive evaluation or ranking of these methods. Our hope is instead to demon-  
246 strate how `stdpopsim` will facilitate more detailed future explorations of the strengths  
247 and weaknesses of the numerous inference methods that are available to the population  
248 genetics community (see Discussion).

249 We start by comparing popular methods for estimating population size histories of



250 single populations and subsequently show simple examples of multi-population infer-  
251 ence. To reproducibly evaluate and compare the performance of inference methods,  
252 we developed workflows using `snakemake` (Köster and Rahmann, 2012), available from  
253 <https://github.com/popsim-consortium/analysis>, that allow efficient computing in  
254 multicore or cluster environments. Our workflow generates  $R$  replicates of  $C$  chromo-  
255 somes, producing  $n$  population samples in each of a total of  $R \times C$  simulations for each  
256 demographic model. After simulation, the workflow prepares input files for each inference  
257 method by grouping all  $n \times R \times C$  simulated chromosomes into a single file. Each file  
258 is then converted into an input file appropriate for each inference method (such that all  
259 inference methods run on the same simulation replicates). Each of the inference pro-  
260 grams are then run in parallel, and finally, estimates of population size history from each  
261 program are plotted.

### 262 **Single-population demographic models.**

263 For single-population demographic models, we compared `MSMC` (Schiffels and Durbin,  
264 2014), `smc++` (Terhorst et al., 2017), and `stairway plot` (Liu and Fu, 2015) on sim-  
265 ulated genomes sampled from a single population, under several of the demographic  
266 models described above. However, these experiments raise the question of what to use  
267 as the “true” population sizes in the case of multi-population models with migration.  
268 In particular, a simple single-population model that is fit to data simulated under a  
269 multi-population model, is not expected to recover the actual simulated population sizes  
270 because of model misspecification. Instead, we argue that the best one may expect in such  
271 a scenario is to infer a model that accurately reflects the coalescence time distribution of  
272 the simulated model. Under a multi-population model, the coalescence time distribution  
273 is influenced by migration between the target population and populations not analyzed  
274 in inference, as well as by the ancestral effective population sizes. The inverse coalescence  
275 rate is commonly interpreted as the effective population size, since these are equal in a  
276 single-population model with random mating. We thus analytically computed inverse  
277 coalescence rates in `msprime` for each simulated model, and used them as benchmarks  
278 for the “true” effective population sizes. See the Appendix for a precise definition and  
279 description of the inverse coalescence rate computation.

280 Figure 2 presents the results from simulations under `OutOfAfricaArchaicAdmixture_5R19`,  
281 a model of human migration out of Africa that includes archaic admixture (Ragsdale and  
282 Gravel, 2019), along with an empirical genetic map. In each column of this figure we  
283 show the inferred population size history (denoted  $N(t)$ ) from samples taken from each  
284 of the three extant populations in the model. In each row we show comparisons among  
285 the methods (including two sample sizes for `MSMC`). Blue lines show estimates from each  
286 of three replicate whole genome simulations, and black lines indicate the “true” values

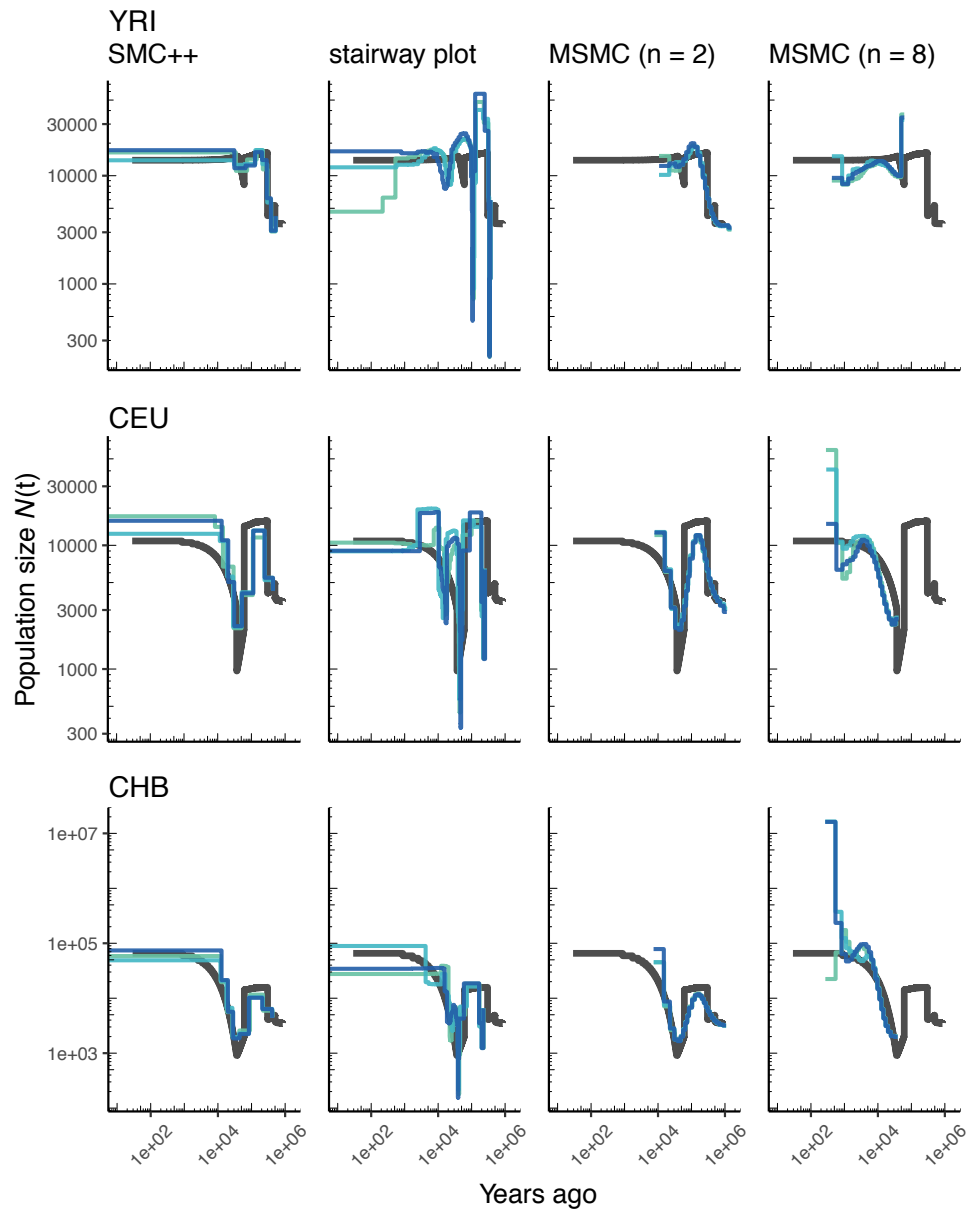


Figure 2: **Comparing estimates of  $N(t)$  in humans.** Here we show estimates of population size over time ( $N(t)$ ) inferred using 4 different methods: `smc++`, `stairway plot`, and `MSMC` with  $n = 2$  and  $n = 8$  samples. Data were generated by simulating replicate human genomes under the `OutOfAfricaArchaicAdmixture_5R19` model (Ragsdale and Gravel, 2019) and using the `HapMapII_GRCh37` genetic map (International HapMap Consortium et al., 2007). From top to bottom we show estimates for each of the three populations in the model (YRI, CEU, and CHB). In shades of blue we show the estimated  $N(t)$  trajectories for each of three replicates. As a proxy for the “truth”, in black we show inverse coalescence rates as calculated from the demographic model used for simulation (see text).

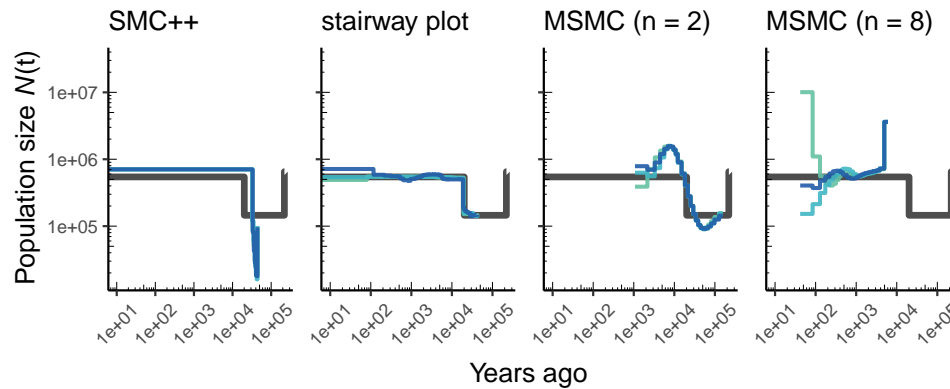


Figure 3: **Comparing estimates of  $N(t)$  in *Drosophila*.** Population size over time ( $N(t)$ ) estimated from an African population sample. Data were generated by simulating replicate *D. melanogaster* genomes under the African3Epoch.1S16 model (Sheehan and Song, 2016) with the genetic map of Comeron et al. (2012). In shades of blue we show the estimated  $N(t)$  trajectories for each replicate. As a proxy for the “truth”, in black we show inverse coalescence rates as calculated from the demographic model used for simulation (see text).

287 depicted by the inverse coalescence rates (although in this specific model the inverse  
288 coalescence rates are very close to the simulated population sizes; Figure S3). While  
289 there is variation in accuracy among methods, populations, and individual replicates, the  
290 methods generally produce a good estimate of the true effective population sizes of the  
291 simulations, with inferred values mostly within a factor of two of the truth, and most  
292 methods inferring a bottleneck at approximately the correct time.

293 Using `stdpopsim`, we can readily compare performance on this benchmark to that  
294 based on a different model of human history. In Figure S4 we show estimates of  $N(t)$  from  
295 simulations using the same physical and genetic maps, but from the OutOfAfrica.3G09  
296 demographic model that does not include archaic admixture. Again we see that each of  
297 the methods is capturing relevant parts of the population history, although the accuracy  
298 varies across time. In comparing inferences between the models it is interesting to note  
299 that  $N(t)$  estimates for the CHB and CEU simulated populations are generally better  
300 across methods than estimates from the YRI simulated population.

301 We can also see how well methods might do at recovering the population history  
302 of a constant-sized population, with human genome architecture and genetic map. We  
303 show results of such an experiment in Figure S5. All methods recover population size  
304 within a factor of two of the simulated values, however SMC-based methods tend to infer  
305 sinusoidal patterns of population size even though no such change is present.

306 As most method development for population genetics has been focused on human  
307 data, it is important to ask how such methods might perform in non-human genomes.  
308 Figure 3 shows parameter estimates from the African3Epoch.1S16 model, originally es-

309 timated from an African sample of *D. melanogaster* (Sheehan and Song, 2016), and Fig-  
310 ure S6 shows estimates from simulations of *A. thaliana* under the African2Epoch\_1H18  
311 model originally inferred by Huber et al. (2018). In both cases, as with humans, we use  
312 **stdpopsim** to simulate replicate genomes using an empirically-derived genetic map, and  
313 try to infer back parameters of the simulation model. Accuracy is mixed among meth-  
314 ods when doing inference on simulated data from these *D. melanogaster* and *A. thaliana*  
315 models, and generally worse than what we observe for simulations of the human genome.

### 316 **Multi-population demographic models.**

317 As **stdpopsim** implements multi-population demographic models, we also explored pa-  
318 rameter estimation of population divergence parameters. In particular, we simulated data  
319 under multi-population models for humans and *D. melanogaster* and then inferred pa-  
320 rameters using *dad*, *fastsimcoal2*, and *smc++*. For simplicity, we conducted inference in  
321 *dad* and *fastsimcoal2* by fitting an isolation with migration (IM) model with constant  
322 population sizes and bi-directional migration (Hey and Nielsen, 2004). Our motivation  
323 for fitting this simple IM model was to mimic the typical approach of two population  
324 inference on empirical data, where the user is not aware of the ‘true’ underlying demog-  
325 raphy and the inference model is often misspecified. For human models with more than  
326 two populations (e.g., Gutenkunst et al., 2009) this limitation means that users are in-  
327 ferring parameters for a model that does not match the model from which the data were  
328 generated (Figures 4A and B). However, since the model used for inference also allows  
329 gene flow between populations, we directly compare estimated effective population sizes  
330 to the values used in simulations (black line in Figure 4C) and not the inverse coalescence  
331 rates.

332 In Figure 4C we show estimates of population sizes and divergence time, for each of  
333 the inference methods, using samples drawn from African and European populations sim-  
334 ulated under the OutOfAfrica\_3G09 model. Our results highlight many of the strengths  
335 and weaknesses of the different methods. For instance, the SFS-based approaches with  
336 simple IM models do not capture recent exponential growth in the CEU population, but  
337 do consistently recover the simulated YRI population size history. Moreover, these ap-  
338 proaches allow migration rates to be estimated (Figure S7), and lead to more accurate  
339 inferences of divergence times. However, these migration rate estimates are somewhat  
340 biased. In contrast, *smc++* is much better at capturing the recent exponential growth in  
341 the CEU population, though it consistently underestimates divergence times because it  
342 assumes no migration between populations (Figure 4C).

343 Again, we can extend this analysis to other taxa and examine the performance of  
344 these methods for a two-population model of *D. melanogaster*. Figure S8 shows inference  
345 results using data simulated under the OutOfAfrica\_2L06 model. This model includes

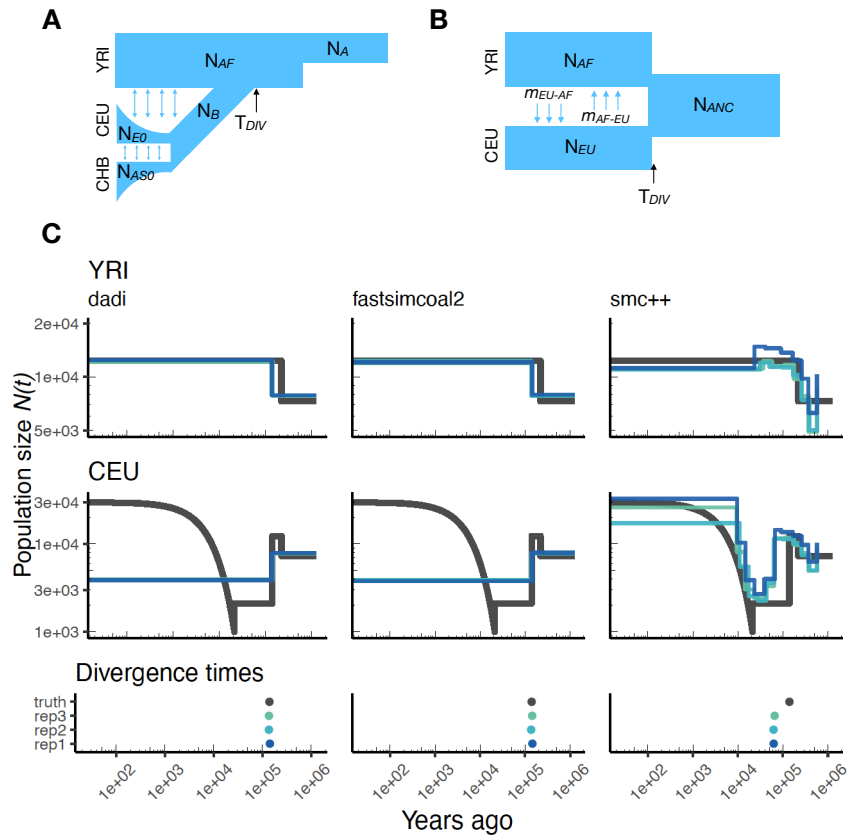


Figure 4: **Parameters estimated using a multi-population human model.** Here we show estimates of  $N(t)$  inferred using *dadi*, *fastsimcoal2*, and *smc++*. **(A)** Data were generated by simulating replicate human genomes under the OutOfAfrica\_3G09 model and using the HapMapII\_GRCh37 genetic map inferred in International HapMap Consortium et al. (2007). **(B)** For *dadi* and *fastsimcoal2* we show parameters inferred by fitting the depicted IM model, which includes population sizes, migration rates, and a split time between CEU and YRI samples. **(C)** Population size estimates for each population (rows) from *dadi*, *fastsimcoal2*, and *smc++* (columns). In shades of blue we show  $N(t)$  trajectories estimated from each simulation, and in black simulated population sizes for the respective population. The population split time,  $T_{DIV}$ , is shown at the bottom (simulated value in black and inferred values in blue), with a common  $x$ -axis to the population size panels.

346 an ancestral population in Africa from which a European population splits off follow-  
347 ing a bottleneck, with no post-divergence gene flow between the African and European  
348 population (Figure S8A). Here again, we find that *∂a∂i* and *fastsimcoal2* infer more  
349 consistent histories, but they do not detect the brief bottleneck in Europe, due to the  
350 inference model not allowing for population size changes after the population split. In  
351 addition, *∂a∂i* and *fastsimcoal2* both do reasonably well at correctly inferring the ab-  
352 sence of migration (Figure S9). In contrast, the inferred demographic parameters from  
353 *smc++* are more noisy, though in some cases better capture the short bottleneck in the  
354 European population.

355 Although these results do not represent an exhaustive benchmarking, we have begun  
356 to highlight some of the strengths and weaknesses of these methods. Future work should  
357 build on these results and undertake more in-depth comparisons under a wider range of  
358 simulated demographic models.

## 359 Discussion

360 Here we have described the first major product from the PopSim Consortium: the  
361 *stdpopsim* library. We have founded the Consortium with a number of specific goals in  
362 mind: standardization of simulation within the population genetics community, increased  
363 reproducibility and ease of use of complex simulations, community-based development  
364 and decision making guiding best practices in population genetics, and benchmarking of  
365 inference methods.

366 The *stdpopsim* library allows for rigorous standardization of complex population  
367 genetic simulations. Population genetics, as a field, has yet to coalesce around a set of  
368 standards for the crucial task of method evaluation, which in our discipline hinges on  
369 simulation. In contrast, other fields such as structural biology (Moult et al., 1995) and  
370 machine learning (Russakovsky et al., 2015) have a long track record of standardized  
371 method testing. We hope that our efforts represent the beginning of what will prove to  
372 be an equally longstanding and valuable tradition in population genetics.

373 Besides being a resource for developers of computational methods, we aim for *stdpopsim*  
374 to be a resource for empirical researchers using genomic data. For instance, *stdpopsim*  
375 could be used in power analyses to determine adequate sample sizes, or in sanity checks  
376 to see if observed data (e.g., levels of divergence or the allele frequency spectrum) are  
377 roughly consistent with the hypothesized scenario. Currently, many studies would benefit  
378 from such simulation-based checks. However, there are major barriers to implementation,  
379 since individual research groups must reimplement complex, previously published demo-  
380 graphic models, a task made especially daunting by additional layers of realism (e.g.,  
381 recombination maps).

382 **Benchmarking population size inference.** We have illustrated in this paper how  
383 `stdpopsim` can be used for direct comparisons of inferential methods on a common set  
384 of simulations. Our benchmarking comparisons have been limited, but nevertheless re-  
385 veal some informative features. For example, at the task of estimating population size  
386 histories for simulated human populations, we find that the sequence-based methods  
387 (`MSMC` and `smc++`) perform somewhat better overall—at least for moderate times in the  
388 past—than the site frequency spectrum-based method (`stairway plot`), which tends to  
389 over-estimate the sizes of oscillations (Figures 2 and S4). In contrast, `stairway plot` out-  
390 performs the sequence-based methods on simulations of *D. melanogaster* or *A. thaliana*  
391 populations, in which linkage disequilibrium is reduced (Figures 3 and S6). In simula-  
392 tions of two human populations (Figure 4), `∂a∂i` and `fastsimcoal2` do reasonably well at  
393 reconstructing the simulated YRI history and estimating divergence times, but struggle  
394 with the more complex simulated CEU history, in large part because the methods assume  
395 constant population sizes. On the other hand, `smc++` does not have the same restrictions  
396 on its inferred history, and as a result does much better with the CEU history but tends  
397 to underestimate divergence times due to the assumption of no migration. The results  
398 for the two-population *D. melanogaster* model (Figure S8) are generally similar. In these  
399 comparisons, `fastsimcoal2` and `∂a∂i` perform almost identically, which is expected be-  
400 cause they fit the same models to the same summaries of the data, differing only in how  
401 they calculate model expectations and optimize parameters.

402 All methods for inferring demographic history have strengths and weaknesses (as  
403 recently reviewed by Beichman et al., 2018). We compared inferences from simulated  
404 whole genome data, but many factors affect choice of methodology. Markovian coales-  
405 cent methods (`MSMC` and `smc++`) require long contiguous stretches of sequence data. In  
406 contrast, frequency spectrum methods (`stairway plot`, `∂a∂i`, and `fastsimcoal2`) can  
407 use reduced-representation sequencing data, such as RADseq (Andrews et al., 2016).  
408 `∂a∂i` and `fastsimcoal2` require a pre-specified parametric model, unlike `MSMC`, `smc++`,  
409 and `stairway plot`. Using a parametric approach yields less noisy results, but a model  
410 that is too simple may not capture important demographic events (Figures 4 and S8),  
411 and other forms of model misspecification may also produce undesirable behavior. From  
412 a software engineering perspective, methods also differ in their ease of installation and  
413 use. We hope our workflows will assist in the application of all the methods we have  
414 considered.

415 Altogether, these preliminary experiments highlight the utility of `stdpopsim` for com-  
416 paring a variety of inference methods on the same footing, under a variety of different  
417 demographic models. In addition, the ability of `stdpopsim` to generate data with and  
418 without significant features, such as a genetic map or population-size changes (e.g., Figure  
419 S5), allows investigation of the failure modes of popular methods. Moreover the compari-  
420 son of methods across the various genome organizations, genetic maps, and demographic

421 histories of different organisms, provides valuable information about how methods might  
422 perform on non-human systems. Finally, comparison of results across methods or simu-  
423 lation runs provides an estimate of inference uncertainty, analogous to parametric boot-  
424 strapping, especially when different methods are vulnerable to model misspecification in  
425 different ways.

426 **Next steps.** `Stdpopsim` is intended to be a fully open, community-developed project.  
427 Our implementations of genome representations and genetic maps for the some of the  
428 most common study systems in computational genetics—humans, *Drosophila*, and *Ara-*  
429 *bidopsis* (among others)—are only intended to be a starting point for future development.  
430 Researchers are invited to contribute to the resource by adding their organisms and mod-  
431 els of choice. The `stdpopsim` resource is accompanied by clearly documented standard  
432 operating procedures that are intended to minimize barriers to entry for new developers.  
433 In this way, we expect the resource to expand and adapt to meet the evolving needs of  
434 the population genomics community.

435 One of our goals is to engage research communities studying other taxa, so as to  
436 expand the resource to many more species. Although we have included demographic  
437 models and recombination maps, there are many biological processes that we do not  
438 model. Some of the additions that we are enthusiastic to add are: selection (including  
439 distributions of fitness effects, maps of functional elements, both single and recurrent  
440 hitchhiking events, and selection on polygenic traits), gene conversion, mutation models  
441 (rate heterogeneity), more realistic demography (overlapping generations, separate sexes,  
442 mortality/fecundity schedules), geographic population structure, and downstream aspects  
443 of data quality (genotyping and mapping error). Moreover, an in-depth investigation into  
444 the effects of population-size rescaling under many of the above scenarios is warranted,  
445 given our preliminary findings using neutral simulations (Figures S1 and S2). Some other  
446 important processes are more challenging to model with current simulation software, such  
447 as structural variation, changing recombination maps over time, transposable elements,  
448 and context-dependent mutation.

449 We wish to emphasize that although the included demographic histories are some of  
450 the most widely used models for our current set of species, we anticipate the set of avail-  
451 able models to expand as new methods and new modeling frameworks are developed. For  
452 instance, the current models all describe a small set of discrete, randomly mating popula-  
453 tions, which are likely good approximations for deep-time population history, but may be  
454 less useful for methods describing dynamics of contemporary populations. `Stdpopsim`'s  
455 framework is sufficiently general that more realistic population models will be easily in-  
456 corporated, as they are published. Additional aspects of the framework, such as genome  
457 builds, will also continue to change as improvements are made to our understanding of  
458 genome structure.



## 459 Methods

### 460 Model quality control

461 As a consortium we have agreed to a standardized procedure for model inclusion into  
462 `stdpopsim` that allows for rigorous quality control. Imagine Developer A wants to in-  
463 troduce a new model into `stdpopsim`. Developer A implements the demographic model  
464 for the relevant organism along with clear documentation of the model parameters and  
465 populations. This model is submitted as a “pull request”, where it is evaluated by a  
466 reviewer and then included as ‘preliminary’, but is not linked to the online documen-  
467 tation nor the command line interface. Developer A submits a quality control (QC)  
468 issue, after which a second developer, Developer B (perhaps found by requesting review  
469 from the broader Consortium), then independently reimplements the model from the  
470 relevant primary sources and adds an automatic unit test for equality between the QC  
471 implementation and the preliminary production model. If the two implementations are  
472 equivalent, the original model is included in `stdpopsim`. If not, we move to an arbitration  
473 process whereby A and B first try to work out the details of what went wrong. If that  
474 fails, the original authors of the published model must be contacted to resolve ambigu-  
475 ities. Further details of our QC process can be found in our developer documentation  
476 (<https://stdpopsim.readthedocs.io/en/latest/development.html>).

477 The possibility for error and the importance of careful quality control was illustrated  
478 very clearly during our own development process: while carrying out the final revisions of  
479 this paper, we noticed that the `OutOfAfrica_3G09` model (Gutenkunst et al., 2009) had  
480 not gone through our QC process. The subsequent QC revealed that our implementation  
481 was in fact slightly wrong—migration rates had not been set to zero to the European  
482 population in the most ancient time period when there should have only been a single  
483 population. This error was propagated from the `msprime` documentation, where the  
484 model was presented as an illustrative example. A number of studies have been published  
485 using copies of this erroneous example code.

### 486 Workflow for analysis of simulated data

487 To demonstrate the utility of `stdpopsim` we created `Snakemake` workflows (Köster and  
488 Rahmann, 2012) that perform demographic inference on tree sequence output from our  
489 package using a few common software packages (see Figure S10 for an example workflow).  
490 Our choice of `Snakemake` allows complete reproducibility of the analyses shown, and all  
491 code is available from <https://github.com/popsim-consortium/analysis>.

492 We performed two types of demographic inference. Our first task was to infer effective  
493 population size over time (denoted  $N(t)$ ). This was done using three software packages:  
494 `stairway plot`, which uses site frequency spectrum information only (Liu and Fu, 2015);

495 MSMC (Schiffels and Durbin, 2014), which is based on the sequentially Markovian coales-  
496 cent (SMC), run with two different sample sizes ( $n = 2, 8$ ); and `smc++` (Terhorst et al.,  
497 2017), which combines information from the site frequency spectrum with recombina-  
498 tion information as in SMC-based methods. No attempt was made at trying to optimize  
499 the analysis from any particular software package, as our goal was not to benchmark  
500 performance of methods but instead show how such benchmarking could be easily done  
501 using the `stdpopsim` resource. In this spirit we ran each software package as near to de-  
502 fault parameters as possible. For `stairway plot` we set the parameters `numRuns=1` and  
503 `dimFactor=5000`. For `smc++` we used the “estimate” run mode to infer  $N(t)$  with all other  
504 parameters set to their default values. For MSMC we used the `--fixedRecombination` op-  
505 tion and used the default number of iterations.

506 For the single-population task we ran human (HomSap) simulations using a variety  
507 of models (see Table 1): `OutOfAfricaArchaicAdmixture_5R19`, `OutOfAfrica_3G09`, and  
508 a constant-sized generic model. Each simulation used the `HapmapII_GRCh37` genetic  
509 map. For *D. melanogaster* we estimated  $N(t)$  from an African sample simulated under  
510 the `DroMel`, `African3Epoch_1S16` model using the `Comeron2012_dm6` map. Finally, we  
511 ran simulations of *A. thaliana* genomes using the `AraTha African2Epoch_1H18` model  
512 under the `Salome2012_TAIR7` map. For each model, three replicate whole genomes were  
513 simulated and the population size estimated from those data. In all cases we set the  
514 sample size of the focal population to  $N = 50$  chromosomes.

515 Following simulation, low-recombination portions of chromosomes were masked from  
516 the analysis in a manner that reflects the “accessible” subset of sites used in empirical  
517 population genomic studies (e.g., Danecek et al., 2011; Langley et al., 2012). Specifically  
518 we masked all regions of 1 cM or greater in the lowest 5th percentile of the empirical  
519 distribution of recombination, regions which are nearly uniformly absent for empirical  
520 analysis. This approach to masking was chosen to prevent marginal trees with low or no  
521 recombination from biasing the comparisons of demographic inference methods. It should  
522 be noted that masking is not implemented within `stdpopsim` proper; tree sequences  
523 generated by `stdpopsim` are always raw and unmasked. This allows users the flexibility  
524 to implement masking approaches that are specific to their needs for downstream analysis.

525 Our second task was to explore inference with two-population models using some of  
526 the multi-population demographic models implemented in `stdpopsim`. For HomSap we  
527 used the `OutOfAfrica_3G09` model with the `HapmapII_GRCh37` genetic map, and for  
528 DroMel we used the `OutOfAfrica_2L06` model with the `Comeron2012_dm6` map. The  
529 HomSap model is a three population model (Africa, Europe, and Asia) including post-  
530 divergence migration and exponential growth (Figure 4C), whereas the DroMel model  
531 is a two population model (Africa and Europe) with no post-divergence migration and  
532 constant population sizes (Figure S8).

533 To conduct inference on these models, we applied three commonly used methods:

534 *∂a∂i* (Gutenkunst et al., 2009), *fastsimcoal2* (Excoffier et al., 2013), and *smc++* (Ter-  
535 horst et al., 2017). As above, these methods were used generally with default settings  
536 and we did not attempt to optimize their performance or fit parameter-rich demographic  
537 models.

538 For both *∂a∂i* and *fastsimcoal2*, we fit a two population isolation-with-migration  
539 (IM) model with constant population sizes. This IM model contains six parameters: the  
540 ancestral population size, the sizes of each population after the split, the divergence time,  
541 and two migration rate parameters. Importantly, this meant that for both species, the  
542 fitted model did not match the simulated model (Figures 4 and S8). In the HomSap case,  
543 we therefore performed inference solely on the Africa and Europe populations, meaning  
544 that the Asia population functioned as a “ghost” population that was ignored by our  
545 inference. To validate our inference approach, we also conducted inference on a generic  
546 IM model that was identical to the model used for inference (Figure S11).

547 From HomSap simulations we took 20 whole genome samples each from the Europe  
548 and Africa populations from each replicate. Runtimes of DroMel simulations were pro-  
549 hibitively slow when simulating whole genomes with the Comeron2012\_dm6 map due to  
550 large effective population sizes leading to high effective recombination rates. For this  
551 reason, we present only data from 50 samples of a 3 MB region of chromosome 2R from  
552 simulations under OutOfAfrica\_2L06. For the generic IM simulations, we used the Hom-  
553 Sap genome along with the HapmapII\_GRCh37 genetic map and sampled 20 individuals  
554 from each population.

555 Following simulation, we output tree sequences and masked low-recombination re-  
556 gions using the same approach described for the single population workflow above. We  
557 converted tree sequences into a two-dimensional site frequency spectrum for all chro-  
558 mosomes in the appropriate format for *∂a∂i* and *fastsimcoal2*. For each simulation  
559 replicate, we performed 10 runs of *∂a∂i* and *fastsimcoal2*, checking to ensure that each  
560 method reached convergence.

561 Detailed settings for *∂a∂i* and *fastsimcoal2* can be found in the Snakefile on our git  
562 repository (<https://github.com/popsim-consortium/analysis>). Estimates from the  
563 highest log-likelihood (out of 10 runs) for each simulation replicate are shown in Figures  
564 4C and S8C.

565 For *smc++*, we converted the tree sequences into VCF format and performed inference  
566 with default settings. Importantly, *smc++* assumes no migration post-divergence, deviat-  
567 ing from the simulated model. However, because *smc++* allows for continuous population  
568 size changes, it is better equipped to capture many of the more complex aspects of the  
569 simulated demographic models (e.g., exponential growth).

570 To visualize our results, we plotted the inferred population size trajectories for each  
571 simulation replicate alongside the simulated population sizes (Figures 4C and S8C). Here,  
572 unlike the single-population workflow, we compare our inferred population sizes only to

573 the simulated population sizes and not the inverse coalescence rates.

## 574 Resource availability

575 The stdpopsim package is available for download on the Python Package Index: <https://pypi.org/project/stdpopsim/>. Documentation for the project can be found here:  
576 <https://stdpopsim.readthedocs.io/en/latest/>.  
577

## 578 Acknowledgments

579 We thank the Probabilistic Modeling in Genomics conference organizers for making this  
580 collaboration possible, and the Simons Center for Quantitative Biology at Cold Spring  
581 Harbor Laboratory for sponsoring the first workshop. Early on in the project we were en-  
582 couraged by many people including Patrick Phillips, Richard Durbin, Dmitri Petrov, and  
583 Sohini Ramachandran. In addition we would like to thank NESCENT and Matt Hahn,  
584 Victoria Sork, and Michael Whitlock for organizing a 2014 catalysis meeting in which  
585 many of the goals of this effort were first laid out. CCK and KEL were funded under NIH  
586 Award R35GM119856. JRA and ADK were funded under NIH Award R01GM117241.  
587 TJS and RNG were funded under NIH Award R01GM127348. ALG and DRS were funded  
588 under NIH award R00HG008696. ND and AS were supported in part by NIH Awards  
589 R01HG010346 and R35GM127070. FR and GG were supported by a Villum Young In-  
590 vestigator award (project no. 00025300). DODV is funded by a UC MEXUS-CONACYT  
591 Collaborative Grant and a DGAPA-PAPIIT grant (PAPIIT-IA200620). JK is supported  
592 by the Robertson Foundation.

## 593 References

- 594 Jeffrey R Adrion, Jared G Galloway, and Andrew D Kern. Predicting the landscape of  
595 recombination using deep learning. *Molecular Biology and Evolution*, 02 2020. ISSN  
596 0737-4038. doi: 10.1093/molbev/msaa038. URL [https://doi.org/10.1093/molbev/  
597 msaa038](https://doi.org/10.1093/molbev/msaa038). msaa038.
- 598 Nikolaos Alachiotis, Alexandros Stamatakis, and Pavlos Pavlidis. OmegaPlus: a scalable  
599 tool for rapid detection of selective sweeps in whole-genome datasets. *Bioinformatics*,  
600 28(17):2274–2275, 2012.
- 601 Kimberly R Andrews, Jeffrey M Good, Michael R Miller, Gordon Luikart, and Paul A  
602 Hohenlohe. Harnessing the power of RADseq for ecological and evolutionary genomics.

- 603 *Nat. Rev. Genet.*, 17(2):81–92, 2016. ISSN 1471-0064. doi: 10.1038/nrg.2015.28. URL  
604 <https://doi.org/10.1038/nrg.2015.28>.
- 605 Gustavo V. Barroso, Nataša Puzović, and Julien Y. Dutheil. Inference of recombination  
606 maps from a single pair of genomes and its application to ancient samples. *PLoS*  
607 *Genetics*, 15(11):e1008449, 2019. doi: 10.1371/journal.pgen.1008449. URL <https://dx.plos.org/10.1371/journal.pgen.1008449>.  
608
- 609 Annabel C Beichman, Tanya N Phung, and Kirk E Lohmueller. Comparison of single  
610 genome and allele frequency data reveals discordant demographic histories. *G3: Genes,*  
611 *Genomes, Genetics*, 7(11):3605–3620, 2017.
- 612 Annabel C. Beichman, Emilia Huerta-Sanchez, and Kirk E. Lohmueller. Using genomic  
613 data to infer historic population dynamics of nonmodel organisms. *Annual Review*  
614 *of Ecology, Evolution, and Systematics*, 49(1):433–456, 2018. doi: 10.1146/annurev-  
615 ecolsys-110617-062431. URL [https://doi.org/10.1146/annurev-ecolsys-110617-](https://doi.org/10.1146/annurev-ecolsys-110617-062431)  
616 [062431](https://doi.org/10.1146/annurev-ecolsys-110617-062431).
- 617 Adam R. Boyko, Scott H. Williamson, Amit R. Indap, Jeremiah D. Degenhardt, Ryan D.  
618 Hernandez, Kirk E. Lohmueller, Mark D. Adams, Steffen Schmidt, John J. Sninsky,  
619 Shamil R. Sunyaev, Thomas J. White, Rasmus Nielsen, Andrew G. Clark, and Car-  
620 los D. Bustamante. Assessing the evolutionary impact of amino acid mutations in  
621 the human genome. *PLoS Genetics*, 4(5):e1000083, may 2008. ISSN 15537390. doi:  
622 DOI 10.1371/journal.pgen.1000083.
- 623 Sharon R Browning, Brian L Browning, Martha L Daviglus, Ramon A Durazo-Arvizu,  
624 Neil Schneiderman, Robert C Kaplan, and Cathy C Laurie. Ancestry-specific recent  
625 effective population size in the Americas. *PLoS Genetics*, 14(5):e1007385, 2018.
- 626 Christopher L. Campbell, Claude Bhérier, Bernice E. Morrow, Adam R. Boyko, and  
627 Adam Auton. A pedigree-based map of recombination in the domestic dog genome.  
628 *G3: Genes, Genomes, Genetics*, 6(11):3517–3524, 2016. doi: 10.1534/g3.116.034678.  
629 URL <https://www.g3journal.org/content/6/11/3517>.
- 630 Andrew H Chan, Paul A Jenkins, and Yun S Song. Genome-wide fine-scale recombination  
631 rate variation in *Drosophila melanogaster*. *PLoS Genetics*, 8(12):e1003090, 2012.
- 632 Josep M Comeron, Ramesh Ratnappan, and Samuel Bailin. The many landscapes of  
633 recombination in *Drosophila melanogaster*. *PLoS Genetics*, 8(10):e1002905, 2012.
- 634 James F. Crow and Carter Denniston. Inbreeding and variance effective population num-  
635 bers. *Evolution*, 42(3):482–495, 1988. URL <http://www.jstor.org/stable/2409033>.

- 636 Petr Danecek, Adam Auton, Goncalo Abecasis, Cornelis A Albers, Eric Banks, Mark A  
637 DePristo, Robert E Handsaker, Gerton Lunter, Gabor T Marth, Stephen T Sherry,  
638 et al. The variant call format and VCFtools. *Bioinformatics*, 27(15):2156–2158, 2011.
- 639 Michael DeGiorgio, Christian D Huber, Melissa J Hubisz, Ines Hellmann, and Rasmus  
640 Nielsen. Sweepfinder2: increased sensitivity, robustness and flexibility. *Bioinformatics*,  
641 32(12):1895–1897, 2016.
- 642 Arun Durvasula, Andrea Fulgione, Rafal M Gutaker, Selen Irez Alacakaptan, Pádraic J  
643 Flood, Célia Neto, Takashi Tsuchimatsu, Hernán A Burbano, F Xavier Picó, Carlos  
644 Alonso-Blanco, et al. African genomes illuminate the early history and transition to  
645 selfing in *Arabidopsis thaliana*. *Proceedings of the National Academy of Sciences*, 114  
646 (20):5213–5218, 2017.
- 647 Laurent Excoffier, Isabelle Dupanloup, Emilia Huerta-Sánchez, Vitor C Sousa, and  
648 Matthieu Foll. Robust demographic inference from genomic and SNP data. *PLoS*  
649 *Genetics*, 9:e1003905, 2013.
- 650 Adam Eyre-Walker and Peter D Keightley. Estimating the rate of adaptive molecular  
651 evolution in the presence of slightly deleterious mutations and population size change.  
652 *Molecular Biology and Evolution*, 26(9):2097–2108, 2009.
- 653 Alyssa Lyn Fortier, Alec J. Coffman, Travis J. Struck, Megan N. Irby, Jose E. L. Burguete,  
654 Aaron P. Ragsdale, and Ryan N. Gutenkunst. DFEnitely different: Genome-wide  
655 characterization of differences in mutation fitness effects between populations. *bioRxiv*,  
656 2019. doi: 10.1101/703918. URL [https://www.biorxiv.org/content/early/2019/  
657 07/16/703918](https://www.biorxiv.org/content/early/2019/07/16/703918).
- 658 Nandita R. Garud, Philipp W. Messer, Erkan O. Buzbas, and Dmitri A. Petrov. Recent  
659 selective sweeps in North American *Drosophila melanogaster* show signatures of soft  
660 sweeps. *PLoS Genetics*, 11(2):1–32, 02 2015. doi: 10.1371/journal.pgen.1005004. URL  
661 <https://doi.org/10.1371/journal.pgen.1005004>.
- 662 Ariella L Gladstein and Michael F Hammer. Substructured population growth in the  
663 Ashkenazi Jews inferred with Approximate Bayesian Computation. *Molecular Biology*  
664 *and Evolution*, 36(6):1162–1171, Mar 2019. ISSN 1537-1719. doi: 10.1093/molbev/  
665 msz047. URL <http://dx.doi.org/10.1093/molbev/msz047>.
- 666 Ryan N Gutenkunst, Ryan D Hernandez, Scott H Williamson, and Carlos D Bustamante.  
667 Inferring the joint demographic history of multiple populations from multidimensional  
668 SNP frequency data. *PLoS Genetics*, 5(10):e1000695, 2009.
- 669 Benjamin C Haller and Philipp W Messer. SLiM 3: forward genetic simulations beyond  
670 the Wright–Fisher model. *Molecular Biology and Evolution*, 36(3):632–637, 2019.

- 671 Benjamin C Haller, Jared Galloway, Jerome Kelleher, Philipp W Messer, and Peter L  
672 Ralph. Tree-sequence recording in SLiM opens new horizons for forward-time simula-  
673 tion of whole genomes. *Molecular Ecology Resources*, 19(2):552–566, 2019.
- 674 Jody Hey and Rasmus Nielsen. Multilocus methods for estimating population sizes,  
675 migration rates and divergence time, with applications to the divergence of *Drosophila*  
676 *pseudoobscura* and *D. persimilis*. *Genetics*, 167(2):747–760, 2004. ISSN 0016-6731. doi:  
677 10.1534/genetics.103.024182. URL <https://www.genetics.org/content/167/2/747>.
- 678 Yi-Fei Huang and Adam Siepel. Estimation of allele-specific fitness effects across hu-  
679 man protein-coding sequences and implications for disease. *Genome Research*, page  
680 gr.245522.118, 2019. ISSN 1088-9051. doi: 10.1101/gr.245522.118.
- 681 Christian D. Huber, Arun Durvasula, Angela M. Hancock, and Kirk E. Lohmueller. Gene  
682 expression drives the evolution of dominance. *Nature Communications*, 9(1), Jul 2018.  
683 ISSN 2041-1723. doi: 10.1038/s41467-018-05281-7. URL [http://dx.doi.org/10.](http://dx.doi.org/10.1038/s41467-018-05281-7)  
684 [1038/s41467-018-05281-7](http://dx.doi.org/10.1038/s41467-018-05281-7).
- 685 International HapMap Consortium et al. A second generation human haplotype map of  
686 over 3.1 million SNPs. *Nature*, 449(7164):851, 2007.
- 687 Guy S Jacobs, Georgi Hudjashov, Lauri Saag, Pradiptajati Kusuma, Chelzie C Darusal-  
688 lam, Daniel J Lawson, Mayukh Mondal, Luca Pagani, François-Xavier Ricaut, Mark  
689 Stoneking, et al. Multiple deeply divergent Denisovan ancestries in Papuans. *Cell*, 177  
690 (4):1010–1021, 2019.
- 691 Jack Kamm, Jonathan Terhorst, Richard Durbin, and Yun S. Song. Efficiently in-  
692 ferring the demographic history of many populations with allele count data. *Jour-*  
693 *nal of the American Statistical Association*, page 1–16, Jul 2019. ISSN 1537-274X.  
694 doi: 10.1080/01621459.2019.1635482. URL [http://dx.doi.org/10.1080/01621459.](http://dx.doi.org/10.1080/01621459.2019.1635482)  
695 [2019.1635482](http://dx.doi.org/10.1080/01621459.2019.1635482).
- 696 Jerome Kelleher, Alison M Etheridge, and Gilean McVean. Efficient coalescent simulation  
697 and genealogical analysis for large sample sizes. *PLoS Computational Biology*, 12(5):  
698 e1004842, 2016.
- 699 Jerome Kelleher, Kevin R. Thornton, Jaime Ashander, and Peter L. Ralph. Ef-  
700 ficient pedigree recording for fast population genetics simulation. *PLoS Compu-*  
701 *tational Biology*, 14(11):1–21, 11 2018. doi: 10.1371/journal.pcbi.1006581. URL  
702 <https://doi.org/10.1371/journal.pcbi.1006581>.
- 703 Jerome Kelleher, Yan Wong, Anthony W. Wohns, Chaimaa Fadil, Patrick K. Albers, and  
704 Gil McVean. Inferring whole-genome histories in large population datasets. *Nature*

- 705 *Genetics*, 51(9):1330–1338, 2019. ISSN 15461718. doi: 10.1038/s41588-019-0483-y.  
706 URL <https://doi.org/10.1038/s41588-019-0483-y>.
- 707 John G Kemeny, J Laurie Snell, and Anthony W Knapp. *Denumerable Markov chains*,  
708 volume 40. Springer Science & Business Media, 2012.
- 709 Andrew D Kern and Daniel R Schrider. diploS/HIC: an updated approach to classifying  
710 selective sweeps. *G3: Genes, Genomes, Genetics*, 8(6):1959–1970, 2018.
- 711 Bernard Y Kim, Christian D Huber, and Kirk E Lohmueller. Inference of the distribu-  
712 tion of selection coefficients for new nonsynonymous mutations using large samples.  
713 *Genetics*, 206(1):345–361, 2017.
- 714 Yuseob Kim and Wolfgang Stephan. Detecting a local signature of genetic hitchhiking  
715 along a recombining chromosome. *Genetics*, 160(2):765–777, 2002. ISSN 0016-6731.  
716 URL <https://www.genetics.org/content/160/2/765>.
- 717 Augustine Kong, Gudmar Thorleifsson, Daniel F Gudbjartsson, Gisli Masson, Asgeir  
718 Sigurdsson, Aslaug Jonasdottir, G Bragi Walters, Adalbjorg Jonasdottir, Arnaldur  
719 Gylfason, Kari Th Kristinsson, et al. Fine-scale recombination rate differences between  
720 sexes, populations and individuals. *Nature*, 467(7319):1099, 2010.
- 721 Johannes Köster and Sven Rahmann. Snakemake—a scalable bioinformatics workflow  
722 engine. *Bioinformatics*, 28(19):2520–2522, 2012.
- 723 Charles H Langley, Kristian Stevens, Charis Cardeno, Yuh Chwen G Lee, Daniel R  
724 Schrider, John E Pool, Sasha A Langley, Charlyn Suarez, Russell B Corbett-Detig,  
725 Bryan Kolaczkowski, et al. Genomic variation in natural populations of *Drosophila*  
726 *melanogaster*. *Genetics*, 192(2):533–598, 2012.
- 727 Haipeng Li and Wolfgang Stephan. Inferring the demographic history and rate of adaptive  
728 substitution in *Drosophila*. *PLoS Genetics*, 2(10):e166, 2006.
- 729 Heng Li and Richard Durbin. Inference of human population history from individual  
730 whole-genome sequences. *Nature*, 475(7357):493, 2011.
- 731 Kao Lin, Andreas Futschik, and Haipeng Li. A fast estimate for the population recom-  
732 bination rate based on regression. *Genetics*, 194(2):473–484, 2013.
- 733 Xiaoming Liu and Yun-Xin Fu. Exploring population size changes using SNP frequency  
734 spectra. *Nature Genetics*, 47(5):555, 2015.
- 735 Devin P. Locke, LaDeana W. Hillier, Wesley C. Warren, Kim C. Worley, Lynne V.  
736 Nazareth, Donna M. Muzny, Shiaw-Pyng Yang, Zhengyuan Wang, Asif T. Chinwalla,



- 737 Pat Minx, and et al. Comparative and demographic analysis of orang-utan genomes.  
738 *Nature*, 469(7331):529–533, Jan 2011. ISSN 1476-4687. doi: 10.1038/nature09687.  
739 URL <http://dx.doi.org/10.1038/nature09687>.
- 740 Gilean A. T. McVean, Simon R. Myers, Sarah Hunt, Panos Deloukas, David R. Bentley,  
741 and Peter Donnelly. The fine-scale structure of recombination rate variation in the  
742 human genome. *Science*, 304(5670):581–584, 2004. ISSN 0036-8075. doi: 10.1126/  
743 science.1092500. URL <https://science.sciencemag.org/content/304/5670/581>.
- 744 John Moulton, Jan T Pedersen, Richard Judson, and Krzysztof Fidelis. A large-scale exper-  
745 iment to assess protein structure prediction methods. *Proteins: Structure, Function,*  
746 *and Bioinformatics*, 23(3):ii–iv, 1995.
- 747 Alexander Nater, Maja P Mattle-Greminger, Anton Nurcahyo, Matthew G Nowak, Marc  
748 De Manuel, Tariq Desai, Colin Groves, Marc Pybus, Tugce Bilgin Sonay, Christian  
749 Roos, et al. Morphometric, behavioral, and genomic evidence for a new orangutan  
750 species. *Current Biology*, 27(22):3487–3498, 2017.
- 751 Diego Ortega-Del Vecchyo, Kirk E. Lohmueller, and John Novembre. Haplotype-based  
752 inference of the distribution of fitness effects. *bioRxiv*, 2019. doi: 10.1101/770966. URL  
753 <https://www.biorxiv.org/content/early/2019/09/16/770966>.
- 754 Aaron P Ragsdale and Simon Gravel. Models of archaic admixture and recent history  
755 from two-locus statistics. *PLoS Genetics*, 15(6):e1008204, 2019.
- 756 Olga Russakovsky, Jia Deng, Hao Su, Jonathan Krause, Sanjeev Satheesh, Sean Ma,  
757 Zhiheng Huang, Andrej Karpathy, Aditya Khosla, Michael Bernstein, et al. Imagenet  
758 large scale visual recognition challenge. *International Journal of Computer Vision*, 115  
759 (3):211–252, 2015.
- 760 P A Salomé, K Bomblies, J Fitz, R A E Laitinen, N Warthmann, L Yant, and D Weigel.  
761 The recombination landscape in *Arabidopsis thaliana* F2 populations. *Heredity*, 108  
762 (4):447–455, Nov 2011. ISSN 1365-2540. doi: 10.1038/hdy.2011.95. URL [http://dx.](http://dx.doi.org/10.1038/hdy.2011.95)  
763 [doi.org/10.1038/hdy.2011.95](http://dx.doi.org/10.1038/hdy.2011.95).
- 764 Stephan Schiffels and Richard Durbin. Inferring human population size and separation  
765 history from multiple genome sequences. *Nature Genetics*, 46(8):919, 2014.
- 766 Sara Sheehan and Yun S Song. Deep learning for population genetic inference. *PLoS*  
767 *Computational Biology*, 12(3):e1004845, 2016.
- 768 Lauren Alpert Sugden, Elizabeth G Atkinson, Annie P Fischer, Stephen Rong, Brenna M  
769 Henn, and Sohini Ramachandran. Localization of adaptive variants in human genomes  
770 using averaged one-dependence estimation. *Nature Communications*, 9(1):703, 2018.

- 771 Paula Tataru, Maéva Mollion, Sylvain Glémin, and Thomas Bataillon. Inference of dis-  
772 tribution of fitness effects and proportion of adaptive substitutions from polymorphism  
773 data. *Genetics*, 207(3):1103–1119, 2017.
- 774 Jacob A Tennessen, Abigail W Bigham, Timothy D O'Connor, Wenqing Fu, Eimear E  
775 Kenny, Simon Gravel, Sean McGee, Ron Do, Xiaoming Liu, Goo Jun, et al. Evolution  
776 and functional impact of rare coding variation from deep sequencing of human exomes.  
777 *Science*, 337(6090):64–69, 2012.
- 778 Jonathan Terhorst, John A Kamm, and Yun S Song. Robust and scalable inference of  
779 population history from hundreds of unphased whole genomes. *Nature Genetics*, 49(2):  
780 303, 2017.
- 781 Lawrence H. Uricchio and Ryan D. Hernandez. Robust forward simulations of recurrent  
782 hitchhiking. *Genetics*, 197(1):221–236, 2014. ISSN 0016-6731. doi: 10.1534/genetics.  
783 113.156935. URL <https://www.genetics.org/content/197/1/221>.
- 784 John Wakeley. *Coalescent Theory, an Introduction*. Roberts and Company, Greenwood  
785 Village, CO, 2005. URL <http://www.coalescenttheory.com/>.

786 **Supplemental Figures**

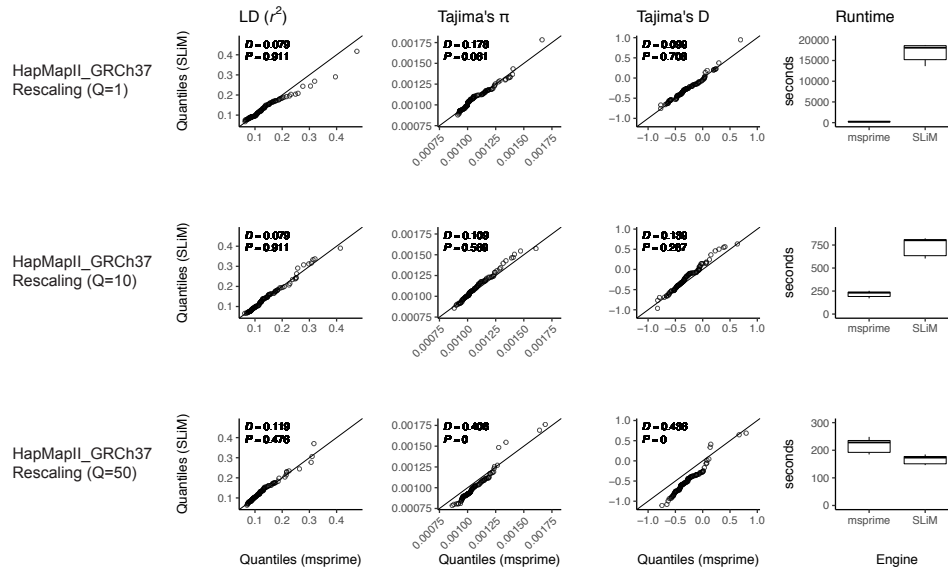
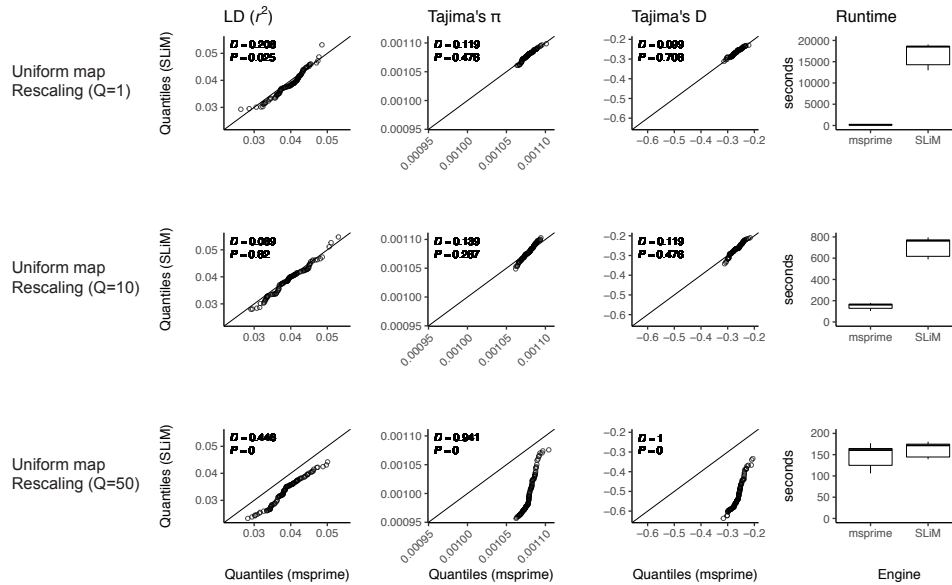


Figure S1: **Validating the SLiM engine backend under a genetic map.** Here we validate our integration of the SLiM (Haller et al., 2019; Haller and Messer, 2019) engine backend. We show quantile-quantile plots between SLiM and msprime engines for three population genetic summary statistics:  $r^2$ , Tajima's  $\pi$ , and Tajima's D. Additionally, we show runtimes for generating each simulation replicate. Data were generated by simulating 100 replicates of human chromosome 22 under the AncientEurasia\_9K19 model (Kamm et al., 2019) using the HapMapII\_GRCh37 genetic map (International HapMap Consortium et al., 2007). 12 samples were drawn from each population (excluding basal Eurasians). From top to bottom we show results using three scaling factors for the population sizes: Q=1, Q=10, and Q=50. Kolmogorov-Smirnov 2-sample test statistics ( $D$ ) and p-values are shown, testing the null hypothesis that the quantiles were drawn from the same continuous distribution.



**Figure S2: Validating the SLiM engine backend under uniform recombination.** Here we validate our integration of the SLiM (Haller et al., 2019; Haller and Messer, 2019) engine backend. We show quantile-quantile plots between SLiM and msprime engines for three population genetic summary statistics:  $r^2$ , Tajima's  $\pi$ , and Tajima's D. Additionally, we show runtimes for generating each simulation replicate. Data were generated by simulating 100 replicates of human chromosome 22 under the AncientEurasia\_9K19 model (Kamm et al., 2019) using a uniform rate of recombination across the chromosome. 12 samples were drawn from each population (excluding basal Eurasians). From top to bottom we show results using three scaling factors for the population sizes:  $Q=1$ ,  $Q=10$ , and  $Q=50$ . Kolmogorov-Smirnov 2-sample test statistics ( $D$ ) and p-values are shown, testing the null hypothesis that the quantiles were drawn from the same continuous distribution.

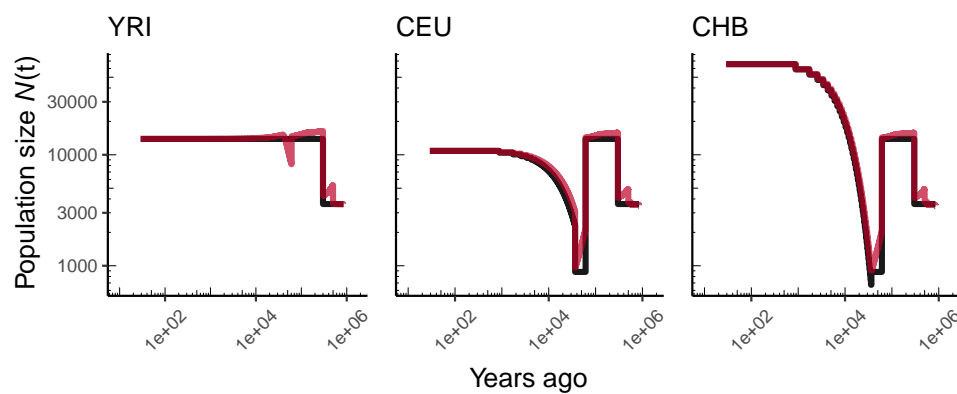


Figure S3: **Comparing simulated population sizes and inverse coalescence rates in humans.** Data are shown from human genomes under the OutOfAfricaArchaicAdmixture\_5R19 model (Ragsdale and Gravel, 2019) and using the HapMapII\_GRCh37 genetic map (International HapMap Consortium et al., 2007). From left to right we show sizes for each of the three populations in the model: YRI, CEU, and CHB. We plot the simulated sizes for each population in black, and in red we plot inverse coalescence rates as calculated from the demographic model used for simulation (see text). In this specific model, these two measures are near identical, but in other models with higher migration rates we expect to see a larger departure between the two.

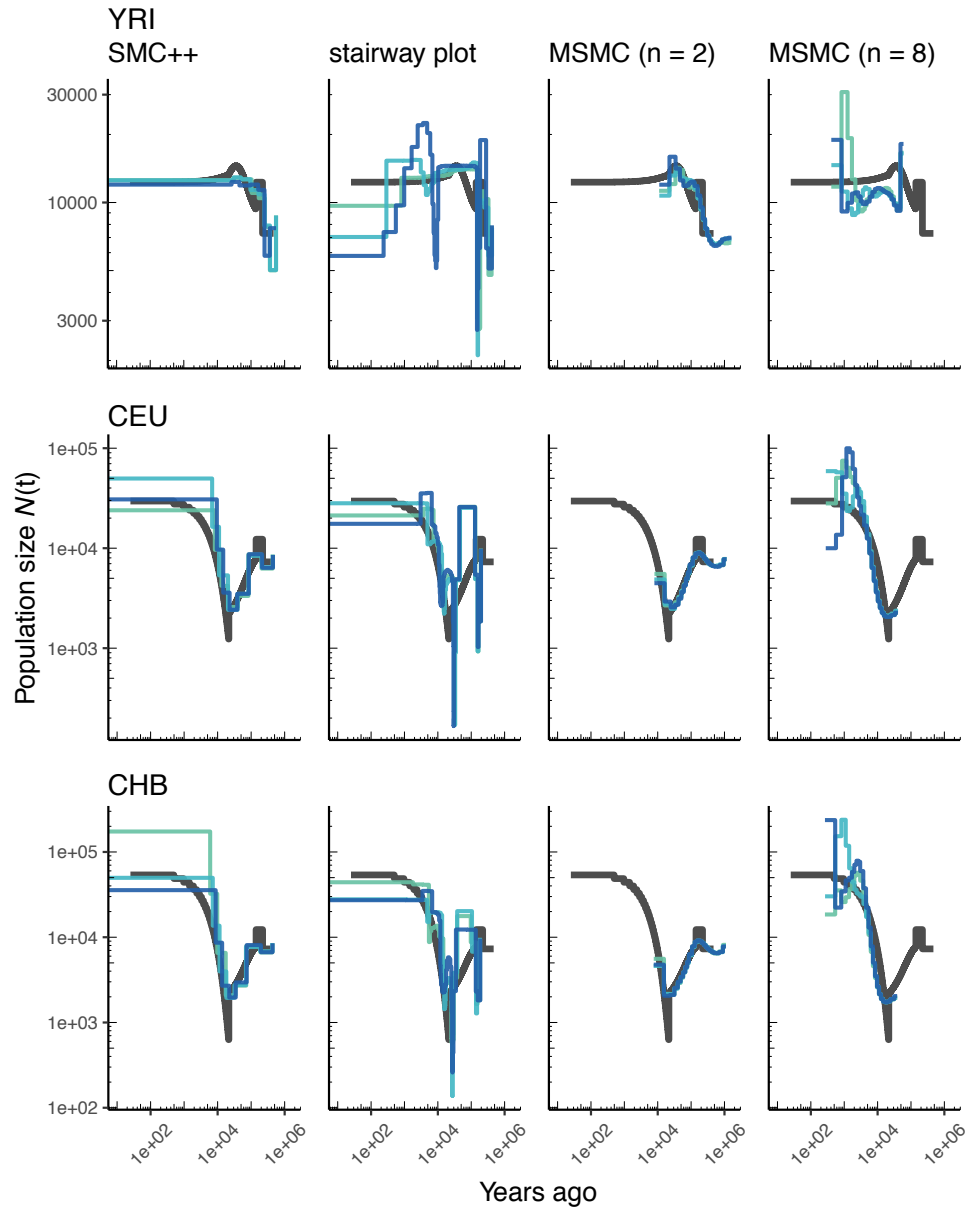


Figure S4: **Comparing estimates of  $N(t)$  in humans.** Estimates of population size over time ( $N(t)$ ) inferred using 4 different methods, `smc++`, `stairway plot`, and `MSMC` with  $n = 2$  and  $n = 8$ . Data were generated by simulating replicate human genomes under the `OutOfAfrica_3G09` model (Gutenkunst et al., 2009) and using the `HapMapII.GRCh37` genetic map (International HapMap Consortium et al., 2007). From top to bottom we show estimates for each of the three populations in the model: YRI, CEU, and CHB. In shades of blue we show the estimated  $N(t)$  trajectories for each replicate. As a proxy for the “truth”, in black we show inverse coalescence rates as calculated from the demographic model used for simulation (see text).

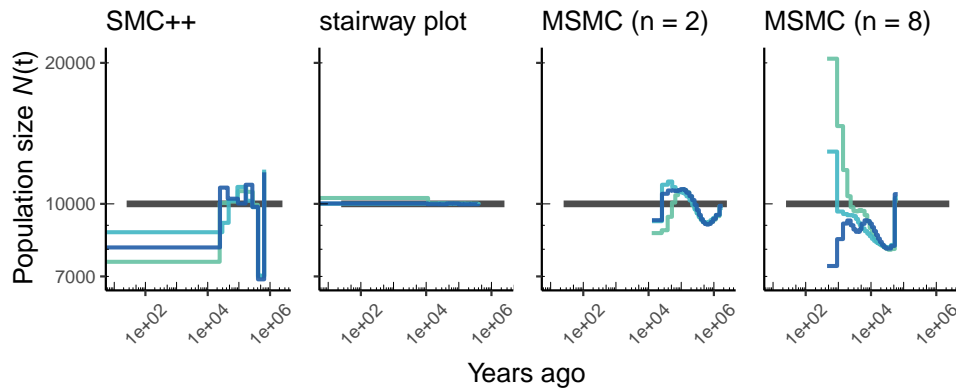


Figure S5: **Comparing estimates of  $N(t)$  in humans.** Here we show estimates of population size over time ( $N(t)$ ) inferred using 4 different methods, `smc++`, and `stairway plot`, and MSMC with  $n = 2$  and  $n = 8$ . Data were generated by simulating replicate human genomes under a constant sized population model with  $N = 10^4$  and using the HapMapII\_GRCh37 genetic map (International HapMap Consortium et al., 2007). As a proxy for the “truth”, in black we show inverse coalescence rates as calculated from the demographic model used for simulation (see text).

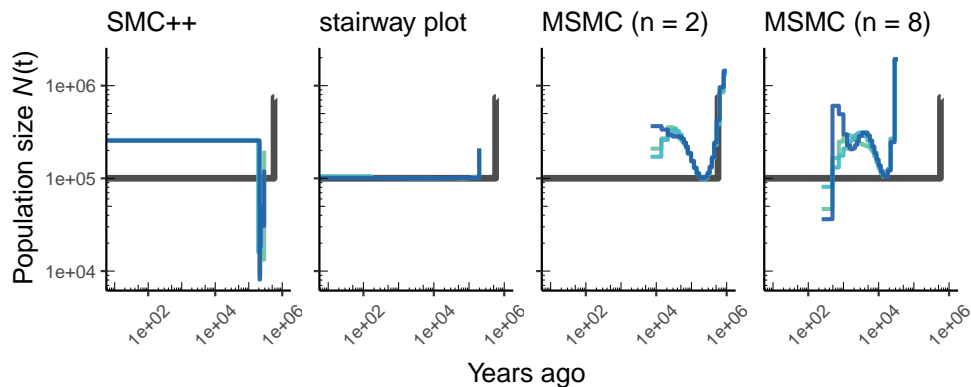


Figure S6: **Comparing estimates of  $N(t)$  in *A. thaliana*.** Here we show estimates of population size over time ( $N(t)$ ) inferred using 4 different methods, `smc++`, and `stairway plot`, and MSMC with  $n = 2$  and  $n = 8$ . Data were generated by simulating replicate *A. thaliana* genomes under the African2Epoch\_1H18 model (Durvasula et al., 2017) and using the SalomeAveraged\_TAIR7 genetic map (Salomé et al., 2011). As a proxy for the “truth”, in black we show inverse coalescence rates as calculated from the demographic model used for simulation (see text).



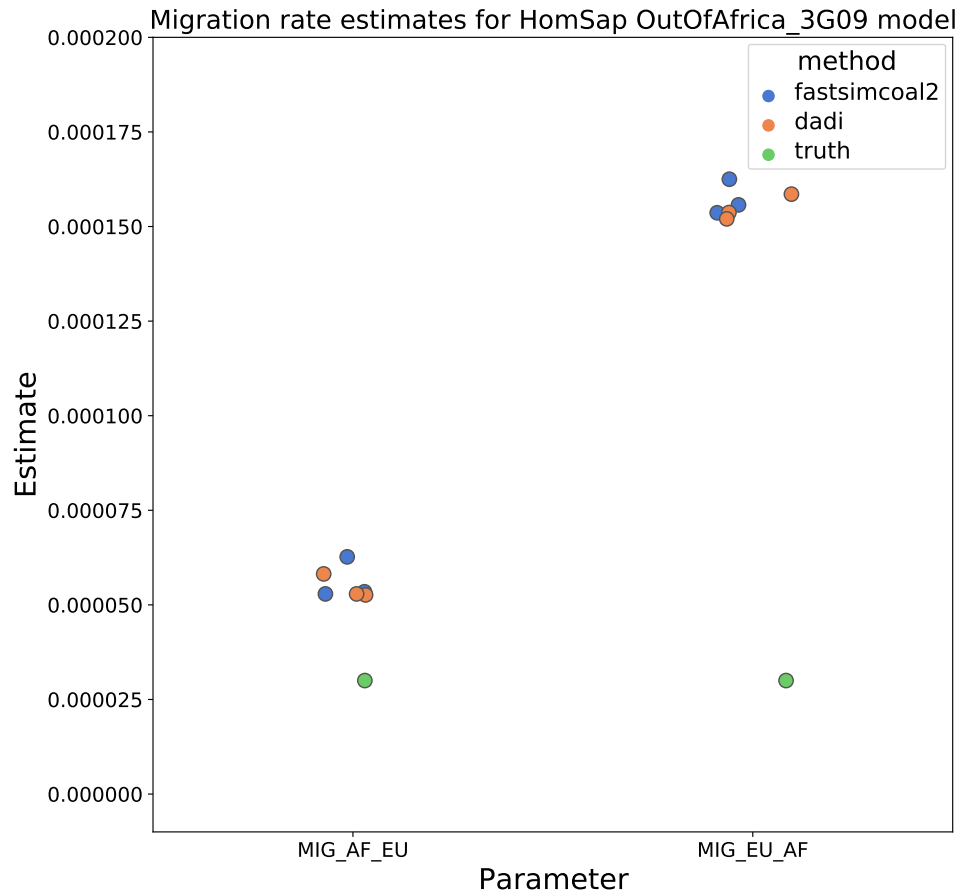


Figure S7: **Migration rate estimates for the human Gutenkunst model.** Here we show inferred migration rates from *dadi* and *fastsimcoal2*. Data were generated by simulating replicate human genomes under the Gutenkunst et al. (2009) model and using the genetic map inferred in International HapMap Consortium et al. (2007). Directional migration from Europe to Africa is represented as *MIG\_AF\_EU* and migration from Africa to Europe is represented as *MIG\_EU\_AF*. Note that the *x*-axis coordinates are arbitrary.

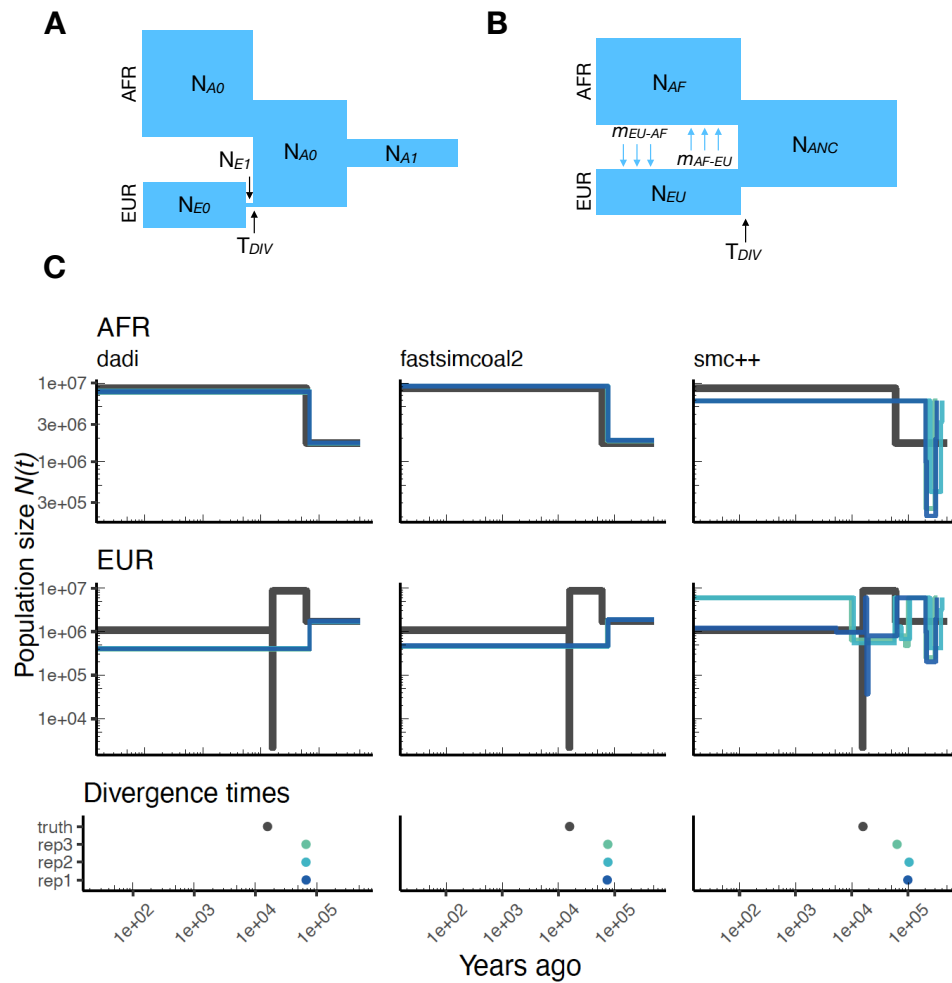


Figure S8: **Parameters estimated using a two-population *Drosophila* model.** Here we show estimates of  $N(t)$  inferred using *dadi*, *fastsimcoal2*, and *smc++*. Data were generated by simulating replicate *Drosophila* genomes under the Li and Stephan (2006) model and using the genetic map inferred in Comeron et al. (2012). See legend of Figure 4 for details. In shades of blue we show the estimated  $N(t)$  trajectories for each replicate. In black we show the simulated population sizes.

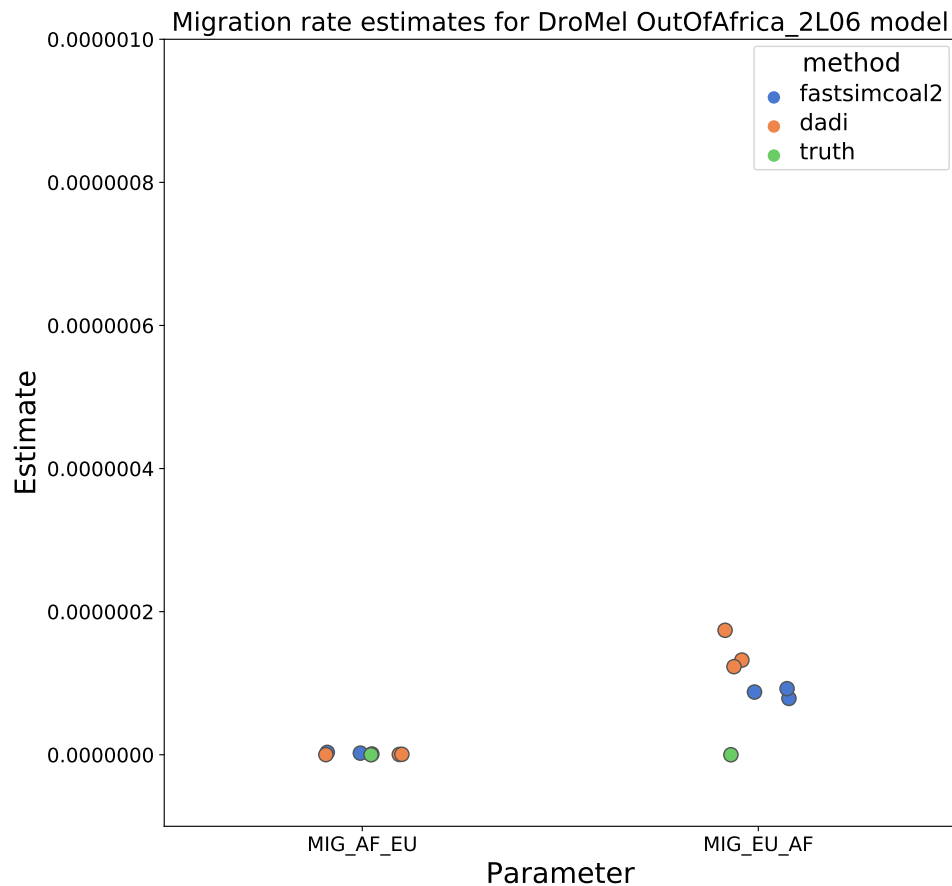


Figure S9: **Migration rate parameters estimated under a two-population *Drosophila* model.** Here we show inferred migration rates from *dadi* and *fastsimcoal2*. Data were generated by simulating replicate *Drosophila* genomes under the Li and Stephan (2006) model and using the genetic map inferred in Comeron et al. (2012). Directional migration from Europe to Africa is represented as *MIG\_AF\_EU* and migration from Africa to Europe is represented as *MIG\_EU\_AF*. Note that the *x*-axis coordinates are arbitrary.



Figure S10: **Workflow for our  $N(t)$  inference methods comparison.** Here we show single replicate for two chromosomes, chr22 and chrX, simulated under the HomSap OutOfAfrica\_3G09 demographic model, with a HapmapII\_GRCh37 genetic map. Note that the data used as input by all inference methods `smc++`, `MSMC`, and `stairway plot`, come from the same set of simulations.

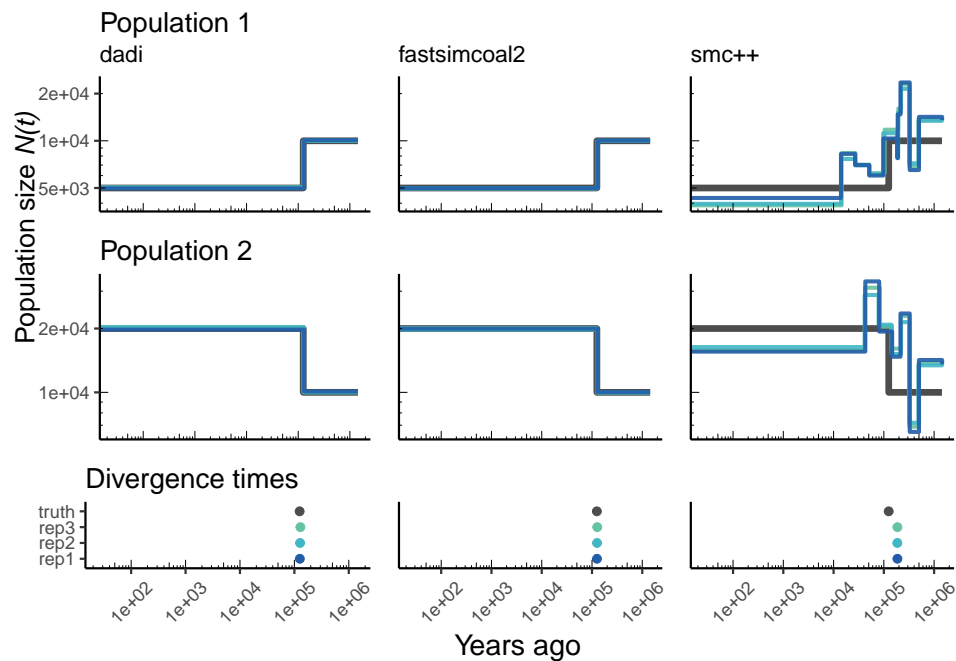


Figure S11: **Parameters estimated from a generic IM model** Here we show estimates of  $N(t)$  inferred using *dadi*, *fastsimcoal2*, and *smc++*. Data were generated by simulating under a generic IM model with a human genome and International HapMap Consortium et al. (2007) genetic map. In shades of blue we show the estimated  $N(t)$  trajectories for each replicate. In black we show the simulated population sizes.

## 787 Appendix: Calculating coalescence rates

788 In population genetics, the “effective population size” of a population model with constant  
789 (census) size is often defined to be the number of diploids in a Wright-Fisher population  
790 that would have the same coalescence rate (or, equivalently, genetic drift) as the popu-  
791 lation in question (reviewed in Crow and Denniston, 1988). One reason the concept is  
792 useful is because theory predicts that genetic data from distinct populations with the  
793 same effective population size will look similar in many ways: for instance, their mean  
794 coalescence times will be the same. Conversely, this implies that effective population size  
795 should be easier to infer from genomic data than aspects of population demography that  
796 do not affect effective population size. An analogous observation holds for populations  
797 of changing size, if we define the “coalescence rate” of a given demographic model at a  
798 particular point back in time to be the rate of coalescence of remaining lineages and de-  
799 fine the “coalescence effective size” at that time, denoted  $N_e(t)$ , so that the coalescence  
800 rate at time  $t$  in the past is  $1/(2N_e(t))$ . With these definitions, any two models with  
801 the same effective population size trajectory ( $N_e(t)$ ) will have the same *distribution* of  
802 coalescence times. For this reason, we might guess that if we apply an inference method  
803 that assumes a Wright-Fisher population with changing size through time to a different  
804 population model, the inferred demographic history will match the “effective population  
805 size history” defined in this way. These observations and the following calculations are  
806 standard in coalescent theory (see e.g., Wakeley, 2005), but they are provided here for  
807 completeness.

We compute the coalescence rate of a collection of samples in a given demographic model at a particular point back in time as the expected number of coalescences happening at that time per unit of time and per pair of as-yet-uncoalesced lineages. More concretely, let  $p(t)$  denote the probability that the lineages of a randomly chosen pair of samples have not yet coalesced  $t$  units of time ago, let  $p(z, t)$  denote the probability that those lineages have not yet coalesced and are furthermore both in location  $z$ , and let  $N_e(z, t)$  be the (effective) diploid population size in location  $z$  at the time, so that  $1/(2N_e(z, t))$  is the rate of coalescence there. Then, we compute the mean coalescence rate as

$$r(t) = \frac{1}{p(t)} \sum_z \frac{p(z, t)}{2N_e(z, t)}.$$

This follows because if we have  $m$  diploid samples, and hence  $\binom{2m}{2}$  lineages, the expected number of coalescences in location  $z$  between times  $t$  and  $t + dt$  ago is

$$\binom{2m}{2} p(z, t) \frac{dt}{2N_e(z, t)},$$

and the expected number of pairs of uncoalesced lineages at that time is

$$\binom{2m}{2} p(t).$$

808 The expression for  $r(t)$  is a ratio of these two quantities; to obtain it we need to compute  
809  $p(t)$  and  $p(z, t)$ . This is relatively straightforward using the general theory of Markov  
810 chains (e.g., Kemeny et al., 2012), and is implemented in `msprime`.

811 Note that since these quantities are *per pair of lineages*, this definition depends on the  
812 locations of the samples. The coalescence rate also has the intuitive interpretation that it  
813 is the average between-lineage coalescence rate, averaged over where uncoalesced lineages  
814 might be. Since the local coalescence rate is the inverse of the population size,  $1/r(t)$  (as  
815 shown for instance in Figure 2) is a weighted harmonic mean of the census sizes of the  
816 different populations present at that time. This is as expected: suppose that we have two  
817 populations, one big and one small, connected by migration. If all our samples are from  
818 the big population, the number of recent coalescences should be small, reflecting the large  
819 population size, while in the long run, the coalescence rate approaches an intermediate  
820 rate. On the other hand, more recent coalescences are expected if all samples are from  
821 the small population, A method that fits a single, time-varying population size to the  
822 data might be expected to find a population size trajectory to match these time-varying  
823 rates of coalescence.

We use the same computations to analytically compute *mean coalescence times*: since  
for any nonnegative random variable  $T$ , the mean value is  $\mathbb{E}[T] = \int_0^\infty \mathbb{P}\{T > t\} dt$ , we  
can obtain the mean coalescence time as

$$\int_0^\infty p(t) dt,$$

824 where  $p(t)$  is defined above.

825 The coalescence rate trajectories can be computed from a model in `msprime` using  
826 the `coalescence_rate_trajectory` method of the `Demography Debugger` class, which  
827 can be obtained from a `stdpopsim` model using the `model.get_demography_debugger()`  
828 method.