Using visual encounter data to improve capture-recapture abundance estimates

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Abstract

Capture-recapture studies are widely used in ecology to estimate population sizes and demographic rates. In some capture-recapture studies, individuals may be visually encountered but not identified. For example, if individual identification is only possible upon capture and individuals escape capture, visual encounters can result in failed captures where individual identities are unknown. In such cases, the data consist of capture histories with known individual identities, and counts of failed captures for individuals with unknown identities. These failed captures are ignored in traditional capture-recapture analyses that require known individual identities. Here we show that if animals can be encountered at most once per sampling occasion, failed captures provide lower bounds on population size that can increase the precision of abundance estimates. Analytical results and simulations indicate that visual encounter data improve abundance estimates when capture probabilities are low, and when there are few repeat surveys. We present a hierarchical Bayesian approach for integrating failed captures and auxiliary encounter data in statistical capture-recapture models. This approach can be integrated with existing capture-recapture models, and may prove particularly useful for hard to capture species in data-limited settings.

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¹ Introduction

Capture-recapture studies are widely used for estimating abundance and demographic 2 rates, using information about the identities of captured individuals (Jolly 1965). This 3 paper examines the case where individual identification requires capture, and identities 4 of animals that are visually encountered but not captured are unknown. In such cases, 5 capture" is synonymous with "identification". This often applies in capture-recapture studies 6 of amphibians, where individual identification is only possible with the animal in hand. 7 We also assume that if an individual escapes capture, it is not encountered again in the 8 sampling occasion because it is hiding or otherwise inaccessible (Bailey and Nichols 2010, 9 Joseph and Knapp 2018). Under these conditions, encounters leading to failed captures 10 provide information about abundance, but this information is not readily used in traditional 11 capture-recapture models. 12

When individuals can be encountered at most once on a survey, encounter and capture data 13 both provide lower bounds on the total number of individuals in the population. Total 14 abundance must be greater than or equal to the number of animals encountered in a survey. 15 Similarly, total abundance must be greater than or equal to the number of unique individuals 16 identified in the capture data. Capture data differ however, in that information accumulates 17 over multiple surveys (Pollock 1982). For example, if two surveys occur on consecutive days 18 in a closed population, then the total number of unique individuals captured across both 19 surveys provides a lower bound on abundance. 20

Here, we show how visual encounter data can improve abundance estimates in capture-recapture studies for study designs where 1) individual identification requires capture, 2) a subset of encountered individuals are captured, and 3) individuals can be encountered at most once per sampling occasion. We develop a modified capture-recapture observation model, and investigate conditions under which encounter data improve abundance estimates. The methods presented here accommodate both failed captures and counts of animals ²⁷ collected separately from capture-recapture surveys, and can be integrated with existing
²⁸ capture-recapture models.

$_{29}$ Methods

³⁰ Model description

We adopt a hierarchical Bayesian approach in which an observation model depends on a state model, and both depend on some parameters (Berliner 1996). The state model describes the presence or absence of individuals in a population, and the observation model describes the visual encounter and capture process. The parameter model represents prior distributions for all remaining unknowns.

36 State model

Abundance can be estimated from capture-recapture data in a Bayesian framework using parameter-expanded data augmentation (Royle and Dorazio 2012). Here, N^* unique individuals are observed, but $M > N^*$ individuals are modeled, augmenting the observed data with $M - N^*$ additional capture histories of animals that were never captured. The assumption is that the true abundance N is less than M, but greater than the number of observed individuals N^* .

Individuals i = 1, ..., M are either "in the population" $(z_i = 1)$ or not $(z_i = 0)$, where the parameters $z_1, ..., z_M$ are state parameters to be estimated, and abundance is $N = \sum_i z_i$. These states can be modeled as conditionally independent Bernoulli random variables with probability parameter ω , where ω is the probability of an individual being in the population:

$$z_i \sim \text{Bernoulli}(\omega).$$

47 Observation model

⁴⁸ On surveys k = 1, ..., K an observer searches for individuals, encountering each individual ⁴⁹ with probability η . We assume individuals can only be encountered once at most. If an animal ⁵⁰ is encountered, it is captured with probability κ . Because individuals must be captured to be ⁵¹ identified, encounter data are observed for captured individuals, but not for failed captures. ⁵² Thus, encounter histories are partly observed. We assume the number of failed captures on ⁵³ each survey is observed.

Let $y_{i,k}^*$ represent the categorical outcome for individual *i* on survey *k*. There are three possibilities (Figure 1):

- 56 1. The individual was not encountered $(y_{i,k}^* = 1)$, with probability $z_i(1-\eta) + 1 z_i$.
- 57 2. The individual was encountered but not captured $(y_{i,k}^* = 2)$, with probability $z_i \eta (1-\kappa)$.
- 58 3. The individual was captured $(y_{i,k}^* = 3)$, with probability $z_i \eta \kappa$.

The first two outcomes are not observed. We observe a binary record of whether individual iwas captured on survey k: $y_{i,k}$, so that $y_{i,k} = I(y_{i,k}^* = 3)$, where I is an indicator function that is equal to one if the condition inside the parentheses is satisfied, otherwise it equals zero. In other words, $y_{i,k}^*$ is observed only if $y_{i,k}^* = 3$. Additionally, the observed number of failed captures f_k corresponds to the sum $f_k = \sum_i I(y_{i,k}^* = 2)$. The observation model consists of two parts: one for the capture data:

$$[y_{i,k} \mid y_{i,k}^*] = \text{Bernoulli}(I(y_{i,k}^* = 3)),$$

⁶⁵ and another for the failed capture counts:

$$[f_k \mid y_{1,k}^*, ..., y_{M,k}^*] = I(f_k = \sum_i I(y_{i,k}^* = 2)),$$

⁶⁶ where square brackets denote a probability function. Alternatively, a "soft" constraint can be ⁶⁷ imposed as an approximation or to account for uncertainty in the number of failed captures:

$$[f_k \mid y_{1,k}^*, ..., y_{M,k}^*] = \text{Normal}(f_k \mid \sum_i I(y_{i,k}^* = 2), \sigma),$$

with σ set to some small fixed value.

69 Parameter model

The hierarchical model specification is completed by specifying prior distributions for remaining unknowns. Here, we use independent Uniform(0, 1) priors for all probabilities. This prior over the inclusion probability ω implies a discrete uniform prior for the true abundance from 0 to M.

74 Posterior distribution

The parameters consist of states $z_1, ..., z_M$, outcomes $y_{1,1}^*, ..., y_{M,K}^*$, and the probabilities of inclusion (ω), encounter (η), and capture (κ). The data consist of the capture histories $y_{1,1}, ..., y_{M,K}$, and counts of failed captures from each survey $f_1, ..., f_K$. The posterior distribution of parameters given data, $[z_1, ..., z_M, y_{1,1}^*, ..., y_{M,K}^*, \omega, \eta, \kappa | y_{1,1}, ..., y_{M,K}, f_1, ..., f_K]$, is proportional to:

$$\underbrace{\prod_{i} \prod_{k} [y_{i,k} \mid y_{i,k}^*]}_{i} \times \underbrace{\prod_{k} [f_k \mid y_{1,k}^*, ..., y_{M,k}^*]}_{i} \times \underbrace{\prod_{i} [y_{i,k}^* \mid z_i, \eta, \kappa]}_{i} \times \underbrace{\prod_{i} [z_i \mid \omega]}_{i} \times \underbrace{[\omega][\eta][\kappa]}_{i}.$$

This model can be implemented in the popular BUGS language (Lunn et al. 2009), and we provide example specifications in Appendix S1.

⁸² Auxiliary encounter data

In some cases, auxiliary encounter data are collected, such as during visual encounter surveys where individuals are counted on surveys, but no captures are attempted (Crump and Scott Jr 1994). Let a_k represent the number of unique individuals that were encountered on survey k. If encounters of individuals are conditionally independent, a binomial observation model provides a reasonable choice, where the number of trials is the population abundance $N = \sum_i z_i$ and the probability of success is the encounter probability η :

$$[a_k \mid \eta, z_1, ..., z_M] = \text{Binomial}(\eta, \sum_i z_i),$$

for surveys k = 1, ..., K. This is the observation model used in N-mixture models (Royle 2004). In addition to potentially providing a higher lower bound on abundance, auxiliary encounters also provide additional information about encounter probabilities, because the encounter data are conditionally independent from encounters leading to captures. When combined with the capture-recapture model outlined above, the joint model of captures, failed captures, and auxiliary encounters comprise an integrated population model (Besbeas et al. 2002, Abadi et al. 2010). The posterior distribution is proportional to:

$$\overbrace{\prod_{i}\prod_{k}[y_{i,k} \mid y_{i,k}^{*}]}^{\text{Captures}} \times \overbrace{\prod_{k}[f_{k} \mid y_{1,k}^{*}, ..., y_{M,k}^{*}]}^{\text{Failed captures}} \times \overbrace{\prod_{k}[a_{k} \mid \sum_{i}z_{i}, \eta]}^{\text{Auxiliary encounters}} \times \overbrace{\bigcup_{k}[a_{k} \mid \sum_{i}z_{i}, \eta]}^{\text{Auxiliary encounters}} \times$$

⁹⁶ Abundance lower bounds from encounter and capture data

The total population size is bounded from below by the number animals encountered on any one survey, assuming each animal can be encountered once at most (i.e., individuals are not double-counted). If n_k is the number of unique animals encountered on survey k,

the lower bound on abundance from encounter data $n_{\min} = \max(n_1, ..., n_K)$ is the maximum of K independent binomial random variables with sample size N and probability η . The probability mass function of this lower bound is thus given by:

$$\Pr(n_{\min} = n) = F(n)^K - F(n-1)^K,$$

where F(n) is the cumulative distribution function of a binomial random variable.

Population size must also be greater than or equal to the number of unique captured individuals. A probability mass function for the lower bound on abundance from capture data (c_{\min}) : the number of unique captured individuals) can be derived with a binomial distribution. The binomial sample size is the true population size (N), and the probability of success is the probability of being captured one or more times: $1 - (1 - \eta \kappa)^K$. The probability mass function for c_{\min} , the abundance lower bound derived from the capture data, is:

$$\Pr(c_{\min} = n) = \operatorname{Binomial}(n \mid 1 - (1 - \eta \kappa)^K, N).$$

When the expected lower bound from encounter data exceeds the expected lower bound from capture data ($\mathbb{E}(n_{\min}) > \mathbb{E}(c_{\min})$), encounter data are expected to increase the precision of abundance estimates.

113 Simulations

¹¹⁴ We empirically verified our theoretical results about the expected lower bounds on abundance ¹¹⁵ provided by encounter and capture data using Monte Carlo simulation. We generated ¹¹⁶ five replicate encounter-capture-recapture data sets for each parameter combination of ¹¹⁷ N = 10, 50, 100, K = 3, 6, 9, and η and κ ranging from 0.01 to 0.99 in increments of 0.01, ¹¹⁸ resulting in 441,045 unique data sets. For each parameter combination, we computed the

empirical mean lower bounds from encounter and capture data, averaging over the five replicate iterations, and compared the results to the theoretical expectations generated from the probability mass functions for n_{\min} and c_{\min} .

To understand the implications of bounding abundance for other parameters, we used a 122 simulation study with known parameters across a range of repeat surveys (K = 3, K = 6, 123 and K = 9). We visualized the joint posterior distribution of abundance and the probability 124 of being encountered and captured, and compared these results to a simpler model: M_0 125 a capture-recapture model of a closed population with identical detection probabilities 126 $p_1 = \dots = p_K = p$, which ignores encounters that do not lead to captures (Royle and Dorazio 127 2008). This model can only estimate the marginal probability of capture $p = \eta \kappa$. The 128 observation model is $y_{i,k} \sim \text{Bernoulli}(z_i p)$ for individuals i = 1, ..., M on survey k = 1, ..., K. 120 The state model for z is unchanged, and uniform priors over (0, 1) were assigned to ω and p. 130

¹³¹ Draws from the posterior distributions of all models were simulated using JAGS, with ¹³² six parallel Markov chain Monte Carlo chains, and 400,000 iterations per chain with an ¹³³ adaptation period of 200,000, a burn-in period of 40,000, and posterior thinning by 400 to ¹³⁴ reduce memory usage (Plummer and others 2003). Convergence was assessed using visual ¹³⁵ inspection of traceplots, and the potential scale reduction factor (\hat{R}) statistic (Gelman et ¹³⁶ al. 1992). All code to replicate the analyses is available in a research compendium at ¹³⁷ https://github.com/mbjoseph/secmr.

138 **Results**

Across a range of abundances, encounter data are expected to provide a higher lower bound on abundance when capture probabilities are low, and when there are few repeat surveys (Figure 2). The boundary in the bivariate encounter-capture parameter space delineating the region where $\mathbb{E}(c_{\min} - n_{\min}) < 0$ shifts toward lower capture probabilities as the number

of repeat surveys increases, and as abundance increases. Empirical average bounds from simulated capture-recapture data were in agreement with the theoretical expectations derived from the probability mass functions of n_{\min} and c_{\min} (Figure 3).

When the lower bound on abundance is greater for encounter data than capture data, the joint 146 model of encounters and captures produces a more precise estimate of abundance because 147 there is zero probability mass below the lower bound on abundance. Furthermore, if there is 148 posterior correlation between abundance and another parameter, encounter data can also 149 increase the precision of the correlated parameter. For example, the marginal probability of 150 capture and abundance are correlated in the posterior, so that increased posterior precision 151 for abundance implies increased posterior precision of marginal capture probability (Figure 152 4). 153

154 Discussion

Failed captures and auxiliary encounters are likely to be most useful in capture-recapture studies when animals are hard to capture and the number of surveys is small. This expectation holds across a range of population abundance and encounter probabilities. In such cases, encounter data increase the precision of population abundance estimates by increasing the lower bound on abundance. Such data are essentially "free" in encounter-capture-recapture study designs, and can be included by modifying the likelihood function (and not the underlying state model) of capture-recapture models.

In addition to increasing the precision of abundance estimates, encounter data can increase the precision of parameter estimates for parameters that are correlated with abundance in the posterior distribution. For the simple model presented here, this includes the detection and inclusion probabilities. For more complex models that allow state evolution through time, this might include survival and recruitment probabilities. Thus, we expect that encounter data in general might provide information about abundance, and parameters relating to
abundance and the measurement process.

These results are consistent with related findings for mark-resight studies where marked 169 individuals are subject to incomplete identification. In such studies, accounting for failed 170 identifications of marked individuals – analogous to failed captures – is most advantageous 171 when identification probabilities are low – analogous to capture probabilities being low 172 (McClintock et al. 2014b). However, in the encounter-capture-recapture scenario considered 173 here, whether an animal is marked or not is unknown until it is captured, as would be the case 174 for subdermal passive integrated transponder tags in an amphibian (Gibbons and Andrews 175 2004). 176

Here we assumed that individuals could be encountered at most once per sampling occasion. 177 In other words, there is no double counting: each failed capture corresponds to one unique 178 individual. The observation model developed here is not robust to violations of this assumption, 179 because the total number of encounters sets a lower bound on the true population size. As a 180 consequence, if individuals escape capture multiple times in the sampling sampling occasion, 181 it is possible that the posterior for abundance might be misleadingly precise (i.e., the lower 182 bound on population size would be too high). Therefore, we do not recommend this approach 183 if individuals might be encountered multiple times on the same sampling occasion. This 184 assumption is likely to hold for example in capture-recapture studies of amphibians in high 185 elevation lakes, where individual animals that escape capture hide afterwards, e.g., underwater 186 where they cannot be seen or captured (Joseph and Knapp 2018). If individuals are captured 187 multiple times in the same sampling occasion, then it will be clear that the assumption of at 188 most one encounter has been violated. In such cases, alternative encounter models may be 189 necessary, e.g., a Poisson model that allows repeat encounters on a sampling occasion (Royle 190 et al. 2009). 191

¹⁹² The model developed here relates to other approaches for handling imperfect individual

identification in capture-recapture studies including misidentification (Link et al. 2010, 193 McClintock et al. 2014a, Schofield and Bonner 2015) and partial identification (Augustine et 194 al. 2018), and also to approaches that account for latent encounter histories with observed 195 summary counts (Chandler et al. 2013). In particular, this approach might be viewed as an 196 aspatial analog of the spatial model presented in Chandler et al. (2013) in which a subset of 197 individual identities are available, with a Bernoulli (instead of a Poisson) encounter model, 198 and a stochastic second stage "identification upon encounter" component. This approach 199 can also be seen as a degenerate case of a partial identification model, in which there are no 200 spatial (Augustine et al. 2018) or genetic data (Wright et al. 2009) available to inform the 201 identities of individuals that have escaped capture. 202

Looking ahead, there are opportunities to build upon this approach. First, in terms of 203 implementation, marginalization over the discrete latent variables might allow more efficient 204 sampling from the posterior distribution. Second, because this model includes separate 205 parameters for encounter and capture probabilities, covariates can be included separately 206 for each of these components. This could be useful for example to account for predator 207 avoidance behavior that might influence capture probabilities, and weather conditions that 208 might influence encounter probabilities. Observer effects provide an additional use case: some 200 observers might be better than others at finding or capturing individual animals. 210

In this paper, we presented a motivation for including encounter data in capture-recapture studies based on abundance lower bounds from encounter and capture data. Given that encounter data are included via a modified likelihood and not a modified state model, this approach can be readily integrated with a variety of capture-recapture models, and may be useful for hard to capture species in data-limited settings.

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²⁷⁹ Figure legends

²⁸⁰ Figure 1

Conceptual diagram to represent the data model for individuals i = 1, ..., M and sampling 281 occasions or surveys k = 1, ..., K. Each individual is either in the population $(z_i = 1)$ or not 282 $(z_i = 0)$. Those that are not are never encountered. Those that are may be encountered on 283 occasion k or not, and encountered individuals may or may not be captured and identified (we 284 assume that identification requires capture, so that capture and identification are synonymous). 285 Each path leads to a value of the partly observed quantity $y_{i,k}^*$, where $y_{i,k}^* = 1$ when animals 286 are not encountered, $y_{i,k}^* = 2$ when animals are encountered but not identified, and $y_{i,k}^* = 3$ 287 when animals are encountered and identified. 288

²⁸⁹ Figure 2

Expectations for the difference in abundance lower bounds provided by capture and encounter data as a function of the number of surveys K, abundance N, the encounter probability η , and the probability of capture conditional on an encounter κ . When the surface is red, encounter data are expected to increase the precision of abundance estimates by increasing the lower bound on true abundance. The heavy black line marks the null isocline where the expected difference is zero. Lighter lines represent contours spaced by 2 individuals.

²⁹⁶ Figure 3

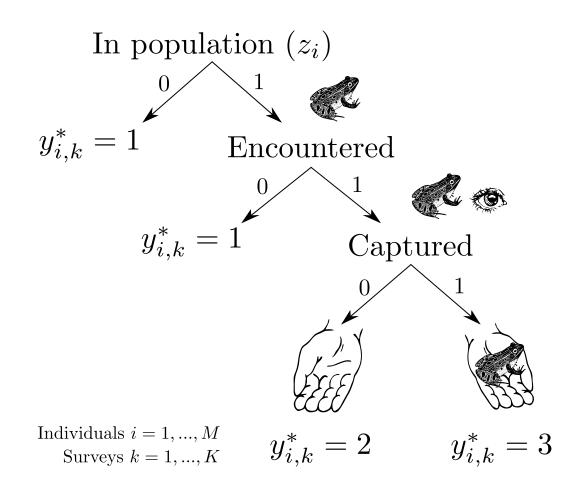
Empirical verification of theoretical expectations for lower bounds on abundance provided by encounter and capture data. Each point represents the empirical average of five replicate simulations across a range of parameter values, with panels separated by population size (N)and whether the lower bounds are derived from capture data (c_{\min}) or encounter data (n_{\min}) .

³⁰¹ Figure 4

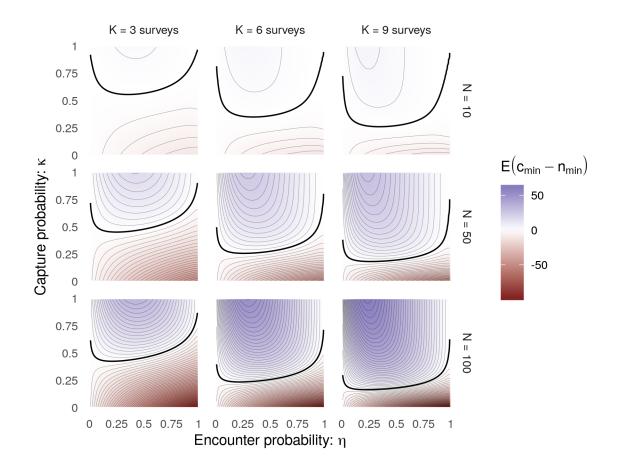
Samples from the posterior distribution of abundance (N, x-axis) and marginal capture probability $p = \eta \kappa$ for an individual in the population (y-axis). Each point is a sample from the posterior. Black points correspond to the baseline capture-recapture model M_0 , which does not include encounter data. Blue points correspond to an encounter-capture-recapture model that uses encounter data to bound abundance. Vertical dashed lines are shown for the lower bounds on abundance derived from encounter (n_{\min}) and capture (c_{\min}) data.

308 Figures

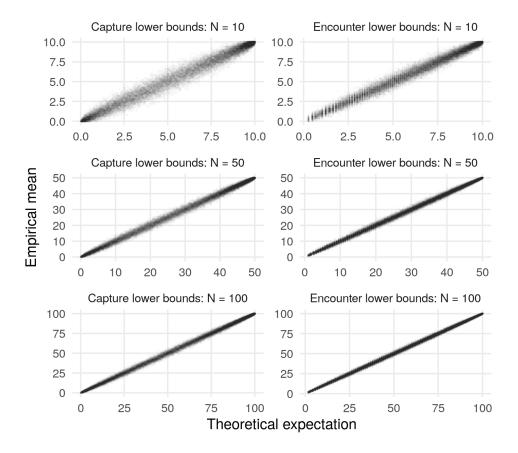
³⁰⁹ Figure 1



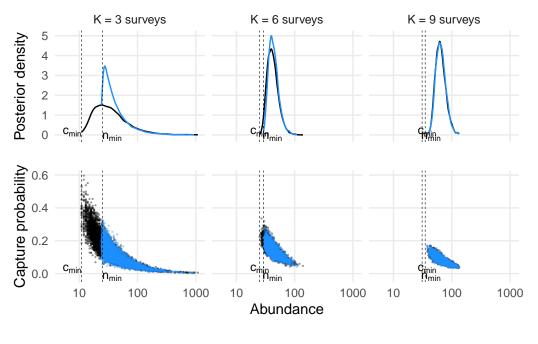
³¹⁰ Figure 2



³¹¹ Figure 3



³¹² Figure 4



Model:
• Capture-recapture • Encounter-capture-recapture

313 Appendix S1

This appendix includes model specifications and BUGS/JAGS code for failed capture and auxiliary encounter models. These models differ in the observation process, but the model of presence/absence states is identical. Each individual i = 1, ..., M is either in the population $(z_i = 1)$ or not $(z_i = 0)$, where $z_i \sim \text{Bernoulli}(\omega)$, and ω is an inclusion probability parameter. Abundance is the sum of these values $N = \sum_i z_i$.

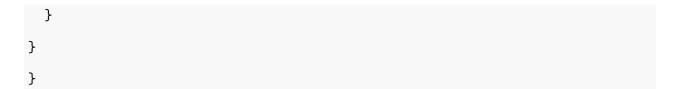
319 Baseline model

The baseline model is M_0 and assumes equal detection probabilities of individuals i = 1, ..., Mon surveys k = 1, ..., K, where $y_{i,k} = 1$ represents a capture of individual i on survey k, and $y_{i,k} = 0$ indicates no capture. If p is the capture probability, then $y_{i,k} \sim \text{Bernoulli}(z_i p)$. The posterior distribution is:

$$\underbrace{\begin{bmatrix} z_1, ..., z_M, \omega, p \mid y_{1,1}, ..., y_{M,K} \end{bmatrix} \propto}_{i} \underbrace{\prod_{i=1}^{\text{Captures}} [y_{i,k} \mid z_i, p]}_{k} \times \underbrace{\prod_{i=1}^{\text{State model}} \sum_{i=1}^{\text{Priors}} [\omega][p]}_{i}.$$

JAGS code for this model with uniform priors over ω and p is:

```
model {
  omega ~ dbeta(1, 1)
  p ~ dbeta(1, 1)
  for (i in 1:M) {
    z[i] ~ dbern(omega)
    for (k in 1:K) {
        y[i, k] ~ dbern(z[i] * p)
```



325 Baseline model with failed captures

We add failed captures to the baseline model by introducing a categorical parameter $y_{i,k}^*$, which represents "not encountered" $(y_{i,k}^* = 1)$, "failed capture" $(y_{i,k}^* = 2)$, or "capture" $(y_{i,k}^* = 3)$. If η is the encounter probability, κ is the probability of capture conditional on encounter, and f_k is the number of failed captures, then as described in the main text the posterior is:

$$\begin{bmatrix} z_1, \dots, z_M, y_{1,1}^*, \dots, y_{M,K}^*, \omega, \eta, \kappa \mid y_{1,1}, \dots, y_{M,K}, f_1, \dots, f_K \end{bmatrix} \propto \underbrace{\prod_{i} \prod_{k} [y_{i,k} \mid y_{i,k}^*]}_{i} \times \underbrace{\prod_{k} [f_k \mid y_{1,k}^*, \dots, y_{M,k}^*]}_{i} \times \underbrace{\prod_{i} \prod_{k} [y_{i,k}^* \mid z_i, \eta, \kappa]}_{i} \times \underbrace{\prod_{i} [z_i \mid \omega]}_{i} \times \underbrace{[\omega][\eta][\kappa]}_{i}.$$

JAGS code for this model with uniform priors over ω , η , and κ is:

model {

```
omega ~ dbeta(1, 1)
p_encounter ~ dbeta(1, 1)
p_capture ~ dbeta(1, 1)
p_encounter_and_capture = p_encounter * p_capture
p_failed_capture = p_encounter * (1 - p_capture)
p_not_encountered = 1 - p_encounter
```

```
for (i in 1:M) {
    z[i] ~ dbern(omega)
    pi[i, 1] = z[i] * p_not_encountered + 1 - z[i]
    pi[i, 2] = z[i] * p_failed_capture
    pi[i, 3] = z[i] * p_encounter_and_capture
    for (k in 1:K) {
        y_star[i, k] ~ dcat(pi[i, ])
        y[i, k] ~ dbern(y_star[i, k] == 3)
    }
    # number of failed captures (soft constraint)
    for (k in 1:K) {
        failures[k] ~ dnorm(sum(y_star[, k] == 2), 4)
    }
```

Note that the normal distribution for failed captures imposes a "soft" constraint on the sum $\sum_{i} y_{i,k}^{*}$. By adjusting the normal precision parameter, this constraint can be relaxed or made more strict. In the strictest case a hard constraint can be imposed via the "ones trick", where a vector of ones is passed as data, and the likelihood for failed captures is represented as ones[k] ~ dbern(sum(y_star[, k]) == failures[k]).

³³⁷ Baseline model with auxiliary encounters

}

To add auxiliary encounter data to the baseline model, we use the following likelihood for the number of auxiliary encounters a_k in survey k conditional on the true abundance N and the encounter probability η :

$a_k \sim \text{Binomial}(\eta, N),$

where $N = \sum_{i} z_{i}$. Note that there need not be an equal number (K) capture-recapture surveys and auxiliary encounter surveys, but here we assume this is the case to simplify notation. This model permits decomposing the parameter p, which represents the marginal probability of capture, into the product of the encounter probability η and the probability of capture conditional on encounter κ .

³⁴⁶ Then, the posterior is:

$$\begin{bmatrix} z_1, ..., z_M, \omega, \eta, \kappa \mid y_{1,1}, ..., y_{M,K}, a_1, ..., a_K \end{bmatrix} \propto \underbrace{\prod_{i=1}^{\text{Captures}} [y_{i,k} \mid z_i, \eta, \kappa]}_{k} \times \underbrace{\prod_{i=1}^{\text{Auxiliary encounters}} [z_i, \eta]}_{k} \times \underbrace{\prod_{i=1}^{\text{State model}} [z_i \mid \omega]}_{i} \times \underbrace{[\omega][\eta][\kappa]}_{i}.$$

JAGS code for this model with uniform priors over ω , η , and κ is:

```
model {
  omega ~ dbeta(1, 1)
  p_encounter ~ dbeta(1, 1)
  p_capture ~ dbeta(1, 1)
  for (i in 1:M) {
    z[i] ~ dbern(omega)
    for (k in 1:K) {
        y[i, k] ~ dbern(z[i] * p_encounter * p_capture)
    }
}
```

```
N <- sum(z[])
for (k in 1:K) {
    a[k] ~ dbinom(p_encounter, N)
}
</pre>
```

³⁴⁸ Baseline model with failed captures and auxiliary encounters

Combining failed captures and auxiliary encounters includes the N-mixture model likelihood for auxiliary encounters in the failed capture model. As described in the main text, the posterior is:

$$\underbrace{ \begin{bmatrix} z_1, \dots, z_M, y_{1,1}^*, \dots, y_{M,K}^*, \omega, \eta, \kappa \mid y_{1,1}, \dots, y_{M,K}, f_1, \dots, f_K, a_1, \dots, a_K \end{bmatrix} \propto }_{\substack{\text{Captures} \\ \prod_i \prod_k [y_{i,k} \mid y_{i,k}^*] \\ } \times \underbrace{\prod_k [f_k \mid y_{1,k}^*, \dots, y_{M,k}^*]}_{\substack{\text{Failed captures} \\ \text{Failed captures} \\ } \times \underbrace{\prod_k [g_{i,k} \mid y_{1,k}^*, \dots, y_{M,k}^*]}_{\substack{\text{Failed captures} \\ \text{Failed captures} \\ \text{Failed$$

JAGS code for this model with uniform priors over ω , η , and κ is:

model {

```
omega ~ dbeta(1, 1)
p_encounter ~ dbeta(1, 1)
p_capture ~ dbeta(1, 1)
p_encounter_and_capture = p_encounter * p_capture
p_failed_capture = p_encounter * (1 - p_capture)
p_not_encountered = 1 - p_encounter
```

```
for (i in 1:M) {
  z[i] ~ dbern(omega)
  pi[i, 1] = z[i] * p_not_encountered + 1 - z[i]
 pi[i, 2] = z[i] * p_failed_capture
 pi[i, 3] = z[i] * p_encounter_and_capture
  for (k in 1:K) {
    y_star[i, k] ~ dcat(pi[i, ])
    y[i, k] ~ dbern(y star[i, k] == 3)
  }
}
# number of failed captures (soft constraint)
for (k in 1:K) {
 failures[k] ~ dnorm(sum(y_star[, k] == 2), 4)
}
N <- sum(z[])
for (k in 1:K) {
  a[k] ~ dbinom(p_encounter, N)
}
```

}