FACTORS AFFECTING RESPONSE TO RECURRENT GENOMIC SELECTION IN SOYBEANS

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1 Abstract

2 Herein we report the impacts of applying five selection methods across 40 cycles of recurrent 3 selection and identify interactions with other factors on genetic response using simulated families 4 of recombinant inbred lines derived from 21 homozygous soybean lines used for the Soybean Nested Association Mapping study. The other factors we investigated included the number of 5 6 quantitative trait loci, broad sense heritability on an entry mean basis, selection intensity, and 7 training sets. Both the rates of genetic improvement in the early cycles and limits to genetic 8 improvement in the later cycles are affected by interactions among the factors. All genomic 9 selection methods provided greater rates of genetic improvement (per cycle) than phenotypic selection, but phenotypic selection provided the greatest long term responses. Model updating 10 11 significantly improved prediction accuracy and genetic response for three parametric genomic prediction models. Ridge Regression, if updated with training sets consisting of data from prior 12 cycles, achieved greater rates of response relative to BayesB and Bayes LASSO GP models. A 13 Support Vector Machine method, with a radial basis kernel, resulted in lowest prediction 14 accuracies and the least long term genetic response. Application of genomic selection in a closed 15 breeding population of a self-pollinated crop such as soybean will need to consider the impact of 16 17 these factors on trade-offs between short term gains and conserving useful genetic diversity in the context of goals for the breeding program. 18

20 Background

| 21 | Plant breeding programs consist of 1) recurrent genetic improvement projects, 2) variety |
|----|--|
| 22 | development projects 3) trait introgression projects and 4) product placement projects (Fehr, |
| 23 | 1991). Genetic improvement is assessed using realized genetic gain, which is an estimate of |
| 24 | change of the average genotypic value for traits of interest across cycles of selection and inter- |
| 25 | mating. Perhaps the most relevant trait for assessment of genetic gain is yield per unit land. Yield |
| 26 | gains in many crop species in both developing and some developed countries have stagnated |
| 27 | (Bhatia et al. 2008; Van Ittersum et al. 2013; Liu et al. 2016) |
| 28 | Since 2007, plant breeding programs have investigated genomic prediction (GP) models for |
| 29 | application to genetic improvement (Bernardo & Yu 2007; Heffner et al. 2009). Three aspects of |
| 30 | GP have been investigated: i) estimation of accuracies from prediction models (Habier et al. |
| 31 | 2007; Goddard 2009; Zhong et al. 2009; Jannink 2010; Heffner et al. 2011; Bastiaansen et al. |
| 32 | 2012; Bijma 2012; Wimmer et al. 2013; Lorenz 2013; Hickey et al. 2014, 2017), ii) selection of |
| 33 | experimental lines to include in a crossing nursery (Cochran 1951; Bertan et al. 2007; Bos and |
| 34 | Caligari 2008; Bernardo 2014) and iii) decisions about which lines to cross to create a new cycle |
| 35 | of evaluation and selection (Akdemir & Sánchez 2016; Xu et al. 2017; Goiffon et al. 2017; |
| 36 | Gorjanc et al. 2018). |
| | |

Genetic improvement and cultivar development in crops that are primarily propagated through self-pollination (e.g., barley, canola, maize, oat, rice, soybean, sorghum, wheat, etc.,) involves the derivation of homozygous lines, i.e., replicable genotypes, for phenotypic evaluation across many environments and selection. Creation of replicable lines prior to evaluation and crossing assures that every line selected for initiating a cycle of genetic improvement will produce identical gametes in which all alleles are completely linked. Benefits from creating replicable

lines include: 1) ability to estimate repeatability of line performance across geography and time, 43 2) ability to estimate genotype by environment interactions (GxE) that can be used to distribute 44 lines adapted to specific types of environments and 3) additive genetic variance among 45 homozygous lines will be about twice as large as it would be if replicable lines were not created. 46 As a consequence, genomic selection was not immediately adopted for line development after it 47 48 was introduced in 2001 (Meuwissen et al. 2001). Animal breeders made most of the initial developments and improvements in application of genomic selection (Meuwissen 1997; Li et al 49 50 2008; Dekkers 2010; Dekkers 2012; de los Campos et al. 2013; Henryon et al. 2014). None-the-51 less, several crop breeders have successfully applied GS and demonstrated its short term advantage for genetic gain relative to phenotypic selection in barley, oats, wheat, soybean and 52 53 maize (Bernardo 2008, 2014; Jannink et al. 2010; Asoro et al. 2011; Heslot et al. 2012; Nakaya and Isobe 2012; Hagan et al. 2012; Emily and Bernardo 2013; Crossa et al. 2014; Heslot et al. 54 55 2015; Liu et al. 2015; Beyene et al. 2015; Bassi et al. 2016; Marulanda et al. 2016; Jonas and de Koning 2013, 2016; Hickey et al. 2017; Goiffon et al. 2017). 56 Robertson (Robertson 1960) demonstrated that the rate of genetic gain for selection of traits with 57 additive genetic architectures is greatest in the first few cycles, while later cycles asymptotically 58 59 approach a limit. Hill and Robertson (1966) demonstrated that linkage can affect both the initial rate of genetic gain and selection limits. Felsenstein (1974) termed this the Hill-Robertson (HR) 60 61 effect, where the magnitude of HR effect is determined by selection intensity and initial additive 62 gene frequencies. The HR effect plays a role only in moderate/weak selection, where effective population size (Ne) plays a role in determining the probability of fixation through drift 63 (Comeron et al. 2008). In the absence of epistasis and strong selection pressure, linkage 64

dominates and the effect of linkage disequilibrium (LD) on rate of gain under selection can be

ignored (Felsenstein 1965, 1974; Kimura 1965; Nagylaki 1974). However, the HR effect can
increase negative linkage disequilibrium as the linkage distance between two loci decreases
(Comeron et al. 2008). In addition, greater selection intensities reduce the genetic potential of
founder populations. The drop in genetic potential during the selection process can be prevented
by reintroduction of discarded favorable alleles (Robertson 1960; Hill and Robertson 1966;

71 Maynard and Haigh 1974; Comeron et al. 2008).

72 Previous studies point to differences among models for prediction accuracy and genetic

response, but indicate that specific outcomes depend on conditions of genome organization,

population structure, genetic architecture and selection intensity (Lorenz 2013; Rutkoski et al.

75 2015; Goiffon et al. 2017; Matei et al. 2018; Norman et al. 2018). In addition, training

76 population size affects prediction accuracy and genetic gain. Larger training population sizes

result in more accurate predictions and greater short term genetic improvement (Akdemir et al.

78 2015; Jarquin et al. 2016; Xavier et al. 2017). However, estimates of accuracy approach a limit

79 depending on genome organization and population structure. Prediction accuracy is improved if

80 the training set is selected to represent a larger proportion of a heterogeneous population for a

81 targeted population of environments (Xavier et al. 2016; Jarquin et al. 2018).

Marker density also impacts accuracy and short term genetic gains, with a dense marker set performing better than a sparse set. But the improvement in accuracy and gain reaches a limit within a range of marker density depending on LD associated with the population structure. Empirical and simulation studies on maize, soybean and other crops have identified such a threshold value of marker density above which increasing marker density doesn't result in significant improvements in accuracy and genetic gain (de Roos et al. 2009; Schulz-Streeck et al. 2012; Hickey et al. 2014; Xavier et al, 2016; Norman et al. 2018)

Accuracy and gain are greater for traits with high heritability as a larger fraction of genotypic 89 variance is represented in the phenotypic variance. Genetic architecture also impacts accuracy 90 and gain (Wimmer et al. 2013). One of the important factors that contribute to higher rates of 91 92 short term genetic improvement is shorter time per cycle of GS relative to PS. Phenotyping requires an evaluation phase conducted across multiple years, which is skipped with GS, which 93 94 can reduce the interval between evaluation and crossing to create a new cohort of progeny by two or three years for most crops, depending on the number of generations that can be grown in 95 continuous nurseries (Heffner et al. 2009). 96

97 The type of GP model used affects both prediction accuracy and the magnitudes of residual deviations from predicted values (Long et al. 2011; Heslot et al. 2012; Howard et al. 2014). 98 Comparison of various models for GP have established the important role of genetic architecture 99 100 (de los Campos et al. 2010, 2013; Howard et al. 2014). Howard et al (2014) compared the performance of a set of 14 GP models on F2 and Backcross simulated populations with additive 101 and epistatic genetic architectures. Parametric methods such as Ridge Regression-BLUP and 102 Bayesian regression methods in the mixed effects modeling framework perform well for traits 103 with additive genetic architecture, whereas non-parametric machine-learning methods such as 104 105 Neural Networks and Support Vector Machines provide more accurate predictions for traits with epistatic genetic architectures (Howard et al. 2014, 2016, and 2017). Prediction accuracies are 106 107 essentially the same for all GP models applied to data with additive genetic architectures (Long 108 et al. 2010, 2011; Guo et al. 2012; Howard et al. 2014). This was also the case in a hybrid maize line development program for a set of six traits in recombinant inbred lines derived from bi-109 parental families (Bernardo and Yu 2007). However, a Reproducing Kernel Hilbert Space 110 (RKHS) model (De Los Campos et al. 2009, 2010), a semi-parametric method, showed a higher 111

112 accuracy in F2:3 populations that are comprised of a larger proportion of heterozygotes (Liu et al. 2018). A bagging method that combined RKHS and Bayes-B (BB) demonstrated the best 113 prediction accuracy in SoyNAM population for yield, height and maturity (Xavier et al. 2016). In 114 another study of experimental data, Bayesian methods had similar or better prediction accuracies 115 than SVM and Multi-Layer Perceptron (MLP) (Montesinos-López et al. 2019). 116 117 Given the difficulty of conducting long-term genomic selection in experimental systems, 118 simulations have been used to examine trends across multiple cycles of recurrent genomic 119 selection (Habier et al. 2007; Goddard 2009; Jannink 2010; Bastiaansen et al. 2012; Bijma 2012; 120 de los Campos et al. 2013; Liu et al. 2015; Michel et al. 2016). These studies have shown that selection response, represented as standardized genotypic values, is expected to be faster with 121 122 GS in early cycles and then decrease in later cycles relative to standardized genetic response with phenotypic selection (PS) (Jannink 2010). 123 124 Parametric and non-parametric GP models have different impacts on simulated genetic gains

125 (Bernardo and Yu 2007; Habier et al. 2007). While, Ridge Regression (RR) (Endelman 2011) and Bayesian methods (Pérez and de los Campos 2014) were associated with similar short-term 126 genetic gains, but different long-term gains. In the initial cycles of recurrent selection genetic 127 128 gains and estimates of accuracy were similar between Bayes LASSO (BL) and RR methods, 129 whereas long-term limits to genetic response were better with BL and genetic variance was 130 maintained through later cycles (Liu et al. 2015). In the initial cycles of recurrent selection genetic gains and estimates of accuracy were similar between BL and RR, whereas long-term 131 132 limits to genetic response were better with BL. It was also noted that genetic variance was 133 maintained through later cycles when selection was based on predicted values from BL (Liu et 134 al. 2015).

| 135 | Decreased prediction accuracy of GP models in recurrent GS is often due to decay of LD |
|-----|--|
| 136 | between marker loci and QTL, loss of relationships between lines in early and later cycles of |
| 137 | selection or a combination of both (Habier et al. 2007; Zhong et al. 2009; Hickey et al. 2014; Liu |
| 138 | et al. 2015; Müller and Melchinger 2017, 2018). Shrinkage based methods used for estimation of |
| 139 | marker effects have an impact on relatedness of lines that are selected. The impact of |
| 140 | relationships on prediction accuracies is greater for RR than BL because RR is more dependent |
| 141 | on relationships within a population, whereas accuracy of bayesian models such as BL is more |
| 142 | dependent on LD between marker loci (ML) and QTL, whereas accuracy of BayesB is dependent |
| 143 | on both components (Habier et al. 2007; Zhong et al. 2009; Liu et al. 2015). Closely related |
| 144 | lines tend to be selected as parents with RR, whereas BL maintains a lower rate of inbreeding. |
| 145 | Maintaining a lower rate of inbreeding results in greater long-term genetic gains. (Meuwissen |
| 146 | 1997; Li et al. 2008; Akdemir and Sánchez 2016). |
| 147 | Prediction accuracies can be maintained across cycles of selection by updating GP models with |
| 148 | new genotypic and phenotypic information from each cycle of selection (Jannink 2010; Liu et al. |
| 149 | 2015; Müller et al. 2017, 2018). One approach is to use training data from only the current cycle |
| 150 | of selection. With this approach, predictions do not take into account relationships between the |
| 151 | current population and the founder population or populations from previous cycles of selection. |
| | |

152 At the other extreme, data from all prior cycles of selection can be included with data from the

153 current cycle in the training set. However, there are practical computation limits to the number of

prior cycles of selection that can be included for training GP models. In particular we found that

Bayesian and SVM GP model training time (run time in hours/cycle) increases exponentially

with the size of training populations and requires intensive computing resources that are difficult

to obtain (Figure S1).

154

158 Recurrent cycles of selection could generate populations with little genetic covariance with the founder population, so inclusion of data from early cycles in the training set of later cycles may 159 have limited value. The actual structure of genetic covariance that emerges over cycles of 160 161 recurrent selection will affect number of cycles of data that need to be included in the training set to obtain accurate predictions. However, as noted, the actual impact of including data from prior 162 163 cycles on accuracies of GP models depends on whether the models rely on relationships among lines or on LD between ML and QTL. Moreover, the practice of model updating involves 164 phenotyping, which can adversely affect the relative advantage of GS over PS in terms of gain 165 166 per unit of time as phenotyping takes additional growing season(s) for each cycle (Heffner et al. 2009; Rutkoski et al. 2015; Matei et al. 2018). In practice, animal breeders use training data from 167 168 up to three prior cycles of selection including training data from current cycle (personal 169 communication, Jack Dekkers). Practical guidelines for training sets have not been established 170 for recurrent selection of crop species, in particular soybeans. Recognizing the relatively slow genetic improvement of yield and other polygenic traits of 171 soybeans in the corn-soybean agricultural systems of the primary soybean production region in 172 173 the United States, the North Central Soybean Research Program (NCSRP) supported public

soybean breeders to utilize information from the SoyNAM genome wide association study to

evaluate implementation of GS in soybean breeding populations

176 (https://www.ncsrp.com/NCSRP_research.html#yield). As a precursor to experimental

177 investigations of recurrent GS in Soybean we utilized simulations (Cooper et al. 2002) based on

- the genomic organization and population structure of the SoyNAM founders to evaluate genetic
- 179 responses to five selection methods, three selection intensities, three genetic architectures
- 180 responsible for 0.3 or 0.7 of the total phenotypic variance (H broad sense heritability on an

entry mean basis) and four types of training sets across 40 cycles of recurrent selection. While
the outcomes are specific to soybean genomes adapted to the primary soybean production region,
there are implications for genetic improvement of all line development programs of diploids that
utilize derivation of homozygous lines for evaluation and selection.

185 Methods

186 Simulations and Treatment Design. The impact of number of QTL, selection intensity,

187 heritability, training set and selection methods on response to selection across 40 cycles of

recurrent selection were evaluated using 306 combinations of factors. Explicitly the treatments

189 consisted of three numbers of simulated QTL and three selection intensities, two values for non-

190 genetic variance, five selection methods and four types of training sets used to update four

191 genomic prediction models. In summary the treatment design consists of 18 combinations of

192 factors for phenotypic selection (PS) plus 288 combinations of factors for genomic selection

193 (GS) methods for a total of 306 combinations of factors. Each set of factor combinations was

replicated with ten simulated recurrent selections across 40 cycles resulting in 3060 simulations

195 with 122400 outcomes. Note that different training sets are irrelevant for PS and thus the

196 treatment design is not a complete factorial.

Simulated soybean RILs were generated by crossing *in silico* 20 homozygous SoyNAM founder lines with IA3023 to generate 20 distinct F_1 progeny. The F_1 progeny from each of the 20 crosses were self-pollinated *in silico* for five generations to generate 100 RILs per family. The resulting 200 RILS from 20 families had segregating genotypic information at 4289 genetic loci. Based on NAM genotypic data, we simulated alleles from common founder line with a frequency of 0.9 and alleles from other founder lines with a frequency of 0.1.

203 Subsets of 40, 400, and 4289 SNP marker loci were designated as OTL. The OTL were distributed evenly throughout the genome, and each contributed equal additive effects of 5/-5, 204 0.5/-0.5, or 0.05/-0.05 units respectively to the total genotypic value the simulated RILs. Thus, 205 206 all three genetic architectures had the same potential to create genotypic values ranging from 207 +200 to -200 genotypic units in the initial founder sets of RILs. Phenotypic values were 208 simulated by adding non-genetic variance sampled from an N $(0, \sigma)$ distribution to the simulated genotypic values, where σ was determined by the heritability on an entry mean basis among the 209 210 initial sets of founder sets of RILs. Broad sense heritability on an entry mean basis (H) values of 211 0.7 and 0.3 were simulated for each of the three sets of QTL. After the phenotypic values were simulated in the initial founding sets of RILs, the non-genetic variance was held constant across 212 213 subsequent cycles of selection.

For each cycle of recurrent selection, 1%, 2.5% or 10% of the most positive phenotypic or predicted genotypic values among 2000 simulated RILs were selected as parents to inter-mate for the next cycle (Figure 1). This corresponds to selection intensities of 2.67, 2.34 and 1.75 in terms of standardized selection differential, *i*.

218 Based on previous results from Howard et al (2014), four GS methods were evaluated. Ridge 219 Regression (RR) was selected to represent a frequentist parametric model. Bayes-B (BB) and 220 Bayesian LASSO (BL) were selected to represent parametric bayesian models and Support 221 Vector Machine with Radial Basis Kernel (SVM-RBF) represented a non-parametric method of 222 machine learning. Ridge regression was implemented with a method that employs expectation 223 maximization to obtain Restricted Maximum Likelihood estimates of marker effects (Xavier 224 2019). This computational method is faster than the popular implementation of ridge regression 225 in rrBLUP package (Endelman 2011) and produces values that are highly correlated with the

226 predictions based on the rrBLUP package (Figure S2). The BGLR package (Perez and de los Campos 2014) provided implementations of BB and BL models. The 'Rgtsvm' package in R was 227 used as an implementation of the SVM with RBF kernel method (Wang et al. 2017). 'Rgtsvm' 228 229 implements SVM training on GPUs with computing time several hundred times less than that required for the implementation in 'caret' package on high performance computing clusters, with 230 231 similar prediction accuracies and estimates of mean squared errors (Figure S3). The parameters used to train GP models are provided in Table 3. 232 A preliminary analysis of training sets on genotypic values and prediction accuracies was 233 234 conducted using RR models trained with data from the current cycle as well as 3, 5, 6, 8, 10, 12, and 14 prior cycles. The results were compared with responses from the RR model updated with 235 236 cumulative training set comprised of data from all prior cycles and with no updating using prior 237 cycles. Training sets for each cycle were obtained by randomly sampling 1600 RILs from the set of 2000 simulated RILs in each cycle. The most accurate predictions and maximum genetic 238 response was obtained with training data that is cumulatively added every cycle (Figures S4 and 239 240 S5). The results indicate that including 3-5 prior cycles of training data did not significantly improve prediction accuracies and responses relative to models that were not updated. Also, the 241

standardized genotypic values and prediction accuracies obtained using 10 to 14 prior cycles of

243 data in the training set were not significantly different than results based on training sets

consisting of all prior cycles. Based on the results of this preliminary study, we investigated
responses to recurrent selection using training sets consisting of up to 14 prior cycles of selection
as well as data from the current cycle. After the 14th cycle, training data consisted of only the 14
prior cycles of recurrent selection and before the 14th cycle, training data from all prior cycles

248 were included. For purposes of this manuscript we use the phrase 'model updating' to refer to

retraining GP models with up to 14 previous cycles of training data (Figure 2).

250 Evaluation Metrics. The standardized genotypic value, R_s (1), was estimated every cycle as the

change in genotypic value from the average genotypic value of 2000 RILs derived from the

initial founders and standardized to the maximum genotypic potential (200 units) among the

founders (Meuwissen et al. 2001; Liu et al. 2015).

254
$$R_s = \frac{R_c}{(R_m - R_0)} (1)$$

 255 R_s - Standardized genotypic value R₀ - Average genotypic value of RILs produced by founders
 256 R_c - Average genotypic value in cycle c - R₀ R_m - Maximal possible genotypic value (=200)

257 The maximum genotypic value (M_{gv}) among the RIL's selected in cycle c is a metric used to evaluate the best RIL produced each cycle, while the standardized genotypic variance (SVg) 258 259 defined as the estimated genotypic variance divided by the estimated genotypic variance of the initial population, was used to evaluate the loss of genotypic variability. Note that values for the 260 SV_g range from zero to one. Estimated Linkage disequilibrium (LD) among pairs of marker loci 261 on all 20 chromosomes was evaluated as the deviation of observed gametic frequency of alleles 262 263 at a pair of loci from the product of the individual allele frequencies, assuming independence (Weir 1996). GP models were assessed using the estimated prediction accuracies (r_{ps}) , defined as 264 the estimated linear correlation (Pearson) between predicted and simulated genotypic values and 265 with estimated Mean Squared Error (MSE), defined as the sum of the squared deviations of the 266 267 predicted genotypic values from the simulated values.

268 Modeled response to recurrent selection. The averaged Rs for each cycle, c, of recurrent

selection were modeled with a linear first order recurrence equation:

270
$$f_0(c)y_{(c+1)} + f_1(c)y_{(c)} = g(c)$$

271 Where c is a sequence of integers from 0 to 39 representing each cycle of recurrent selection and

272 f_0, f_1 and g are constant functions of c. By rearranging the equation we note that the response in

273 cycle c+1 can be represented as

274
$$y_{(c+1)} = -\frac{f_1(c)}{f_0(c)} y_{(c)} + \frac{g(c)}{f_0(c)}$$

Since the ratios $f_1(c)/f_0(c)$ and $g(c)/f_0(c)$ are constants, we can represent the response in cycle c+1

276 as

277
$$y_{(c+1)} = \alpha y_{(c)} + \beta$$
 (2)

278 If y_0 specifies the average phenotypic value of the first generation of RILs derived from the

founders, then (2) has a unique solution (Goldberg 1958):

280

$$y_{c} = \alpha^{c} y_{0} + \beta \frac{1 - \alpha^{c}}{1 - \alpha} \text{ if } \alpha \neq 1 \qquad (3)$$

$$y_{c} = \alpha^{c} y_{0} + \beta c \qquad \text{if } \alpha = 1$$

An alternative representation of (3) for the situation of $\alpha \neq 1$ is

282
$$y_{c} = \alpha^{c}(y_{0} - y') + y$$
with $y' = \frac{\beta}{1 - \alpha}$,

283 , where α is less than 1 for genotypic response to recurrent selection and y' represents the 284 asymptotic limit to selection (Goldberg 1958). To illustrate, values of the sequence of c=0 to 39,

,

with $y_0 = 0$, $\alpha = 0.9$ and $\beta = 15$, are plotted in Figure 3. The curve can be interpreted as response to selection as a function of the frequencies of alleles with additive selective advantage, selection intensity, time and effective population size (Robertson 1960).

The parameters, $y_{0,\alpha}$ and β , were estimated with a non-linear least squares method implemented in the 'nls' function of the R base package and the 'nlsList' and 'nlme' functions in the nlshelper

and 'nlme' packages (Pinheiro and Bates 2000; Baty et al. 2015; Pinheiro et al. 2019).

291 Analyses of variance (ANOVA) across 40 cycles of recurrent selection. The purpose of the

ANOVA is to evaluate significant differences in the modeled response pattern of PS and four GP 292 models, based on three genetic architectures, with two levels of non-genetic contributions to the 293 phenotypes, three selection intensities and four training sets for the GP models. The influence of 294 295 multiple factor treatment combinations on estimated non-linear regression models have not been 296 implemented in standard statistical software packages that report the analysis of variance in terms of sums of squares and traditional 'F-tests'. For discussions on the challenges of using 297 standard F-test for non-linear mixed effects models see (Pinheiro et al. 2000; Baty et al. 2015; 298 Pinheiro et al. 2019). Consequently, we analyzed the variance among modeled responses using 299 AIC, BIC and Likelihood metrics that were grouped based on combinations of treatment 300 301 variables consisting of selection methods, training sets, selection intensities, number of simulated 302 QTL and H (Table 1).

We conducted analyses of variance using non-linear least squares on modeled (3) responses grouped by treatment factors (Table 3). In order to provide a balanced data table for analyses by nlme responses from PS were assumed constant for training set levels resulting in responses for 360 combinations of treatment factors. The impact of these factors and their interactions on the modeled response was analyzed using the groupedData function in R to generate data partitions

| 308 C | onditioned on groups | of factors. | Estimates of | of modeled | parameters | were retained | as fixed | effects |
|-------|----------------------|-------------|--------------|------------|------------|---------------|----------|---------|
|-------|----------------------|-------------|--------------|------------|------------|---------------|----------|---------|

- and deviations from estimated means conditioned on grouping variables were modeled as
- random effects using 'nlme' R package. Multiple analyses of 'nlme' objects representing the
- 311 models were used to identify combinations of factors with significant effects on the non-linear
- response model. More information on the analyses can be found in the R package
- 313 'SoyNAMPredictionMethods'
- 314 (http://gfspopgen.agron.iastate.edu/SoyNAM_PredictionMethods.html).

315 Analyses and Data Availability

Simulated data and codes are available as part of R package 'SoyNAMPredictionMethods' (File

S1). All supplemental material including the R package has been uploaded to Figshare and can

be found at <u>https://figshare.com/s/8dba182a46fe1a28c1af</u>. Documentation to use package is

available at http://gfspopgen.agron.iastate.edu/SoyNAM_PredictionMethods.html. SoyNAM

320 genotypic and phenotypic data is available in SoyBase database (Grant et al. 2010).

321

322 **Results**

323 Modeled genotypic values across 40 cycles of recurrent selection. Average genotypic values

324 grouped on each of five factors provide an overview of differences in rate of response and limits

to response among levels within a factor (Figure S6 - S10). The response of averaged Rs (Figure

326 3) were modeled with recurrence equation (3) and the results are consistent with predicted values

and theory by Robertson (1960). There is strong evidence from the analyses of variance (Table

- 328 3) that the response of modeled genotypic values across cycles of selection depend on
- 329 interactions among all simulated factors. The most parsimonious model requires unique

| 330 | estimates of α , and β in equation (3) for each of the 306 combinations of factors (File S2). |
|-----|--|
| 331 | Estimates of α , and β for all factor combinations are provided in File S2. |

Prediction accuracies in the founding sets of RILs: Estimates of prediction accuracies, r_{ps}, of GP

- models trained with the initial set of 2000 F_5 -derived RILs from 20 crosses ranged from 0.75-
- 334 0.82 for H of 0.7 and ranged from 0.38 0.49 for H of 0.3 (Figure 4). The initial r_{ps} for both H
- values was best with BB and poorest with the SVM-RBF. The nQTL had little effect on r_{ps}
- within either value of 0.7 or 0.3 for H. RR and BL had smaller magnitude MSE values than BB
- and SVM RBF for all numbers of simulated QTL and both values for H (Figure 4).
- 338 *Comparisons of Selection Methods*. Both the rates and limits of response from selection in terms
- 339 of R_s are influenced by the five selection methods and their interactions with training sets,

selection intensities, nQTL, and H (Figure 5 and S11). Small nQTL, high values for H and high

341 selection intensities resulted in the greatest initial rates (per cycle) of response while large nQTL,

342 low values of H and relaxed selection intensity showed less rapid initial responses but realized

343 greater R_s values for all PS and GS methods with and without updated training sets. Relative to

344 PS, most GS methods provided greater initial rates of response per cycle, but limits to selection

responses depended on the other factors (Figure 5). In this section, we describe results from top

10% selected fraction for all the GS methods as the differences among GS methods are more

pronounced with relaxed selection intensity. Results from top 1% and 2.5% selected fraction aredescribed in later sections.

When GP models are not updated, BB demonstrated the largest R_s in the early cycles for 40 QTL responsible for 70% of phenotypic variability in the initial population, whereas for 400 and 4289 QTL PS demonstrated greater responses than all GS methods after the 10th cycle (Figure 5 and 6). For H value of 0.3 in the initial populations, BB also had the largest Rs values for 40, 400 and 4289 QTL in the early cycles. Whereas after 10th cycle, PS demonstrated the largest Rs
values for 40, 400 and 4289 simulated QTL (Figure S11 and S12; File S3). In contrast, when the
GP models are updated with training sets consisting of data from up to 14 prior cycles of
recurrent selection, GS using RR models demonstrated largest the Rs values for 40, 400 and
4289 QTL for both heritabilities (Figure 7 and S13; File S3).

358 If the BB models are not updated with data from prior cycles, then the Rs from 40 simulated 359 QTL and 0.7 H in the initial population were 10 to 16% greater than they were with PS in the 360 first five cycles. For the same genetic architecture, recurrent selection with BL and RR models 361 resulted in Rs values that were 4 to 13% greater than Rs values from PS in the first five cycles (Figure 5, 6 and File S3). After five cycles, PS resulted in greater responses than all of the GS 362 363 methods when training sets were not updated. If training sets were not updated after the initial 364 evaluations of RILs and the genetic architectures consisted of 400 and 4289 QTL responsible for 0.7 H in the initial sets of RILs, then RR and Bayesian GP models provided greater genetic 365 366 responses than PS only in the first 2-3 cycles and thereafter PS demonstrated 5-50 % greater standardized genetic responses (File S3). 367

When the RR GP model is updated with data from up to 14 previous cycles of recurrent 368 369 selection, the Rs values for selection on 40 simulated QTL responsible for 70% of phenotypic variability in the initial sets of RILs was 1.5 to 10% greater than PS for the first 10 cycles of 370 recurrent selection, but after the 10th cycle it was similar to PS (Figure 5 and 7; File S3). For the 371 same genetic architecture the Rs values from BB and BL with model updating were 14.4% and 372 373 12% greater than PS respectively for the first five cycles of recurrent selection. After the fifth 374 cycle PS resulted in greater Rs values. If the RR model is updated with data from up to 14 previous cycles of recurrent selection, the responses to selection of RILs with 400 simulated 375

376 QTL responsible for 70% of phenotypic variability in the initial sets of RILs, then the Rs values were 3 to 15% greater than PS across all 40 cycles. The limits of response to selection using BB 377 and BL models were 1 to 14% greater than PS for up to 20 cycles. If the RR model is updated 378 379 with data from 14 previous cycles of recurrent selection, the Rs values with 4289 QTL were 10-380 15% greater than PS for 40 cycles (Figure 5 and 7; File S3). Likewise recurrent selection using 381 BB and BL models resulted in greater responses than PS for 40 cycles (Figure 5 and 7; File S3). Similar, albeit distinctive, comparisons among outcomes from GP models with model updating 382 for genetic architectures responsible for 0.3 of the phenotypic variance in the initial sets of RILs 383 384 are described in File S4.

As noted, if GP models are not updated, PS provides greater responses than GS in the early cycles of recurrent selection, whereas if the parametric GP models are updated then for some combinations of treatment factors, Rs values are greater than PS across many cycles of recurrent selection.

In addition to comparing outcomes of PS with GS, we next consider comparisons of the 389 390 evaluation metrics with and without updated training sets among the GS methods. When GP models are updated, RR models resulted in 10% greater response than RR without updating for 391 392 40 simulated QTL responsible for 70% of phenotypic variability among RILs in the initial cycle. Model updating resulted in 30% and 60% greater responses for 400 and 4289 QTL respectively 393 394 (Figure S14). Recurrent selection with updated BB models resulted in 4% greater Rs values than without updating for 40 QTL and resulted in 22% and 57% greater responses for 400 and 4289 395 396 simulated QTL respectively (Figure S14). Recurrent selection with updated BL models resulted 397 in 3% greater responses than without updating for 40 simulated QTL. If the genetic architecture consisted of 400 QTL and 4289 QTL, updated BL models resulted in 21% and 51% greater 398

399 responses respectively (Figure S14). SVMRBF when updated with training sets demonstrated no significant improvement in Rs values relative to SVMRBF without updating for all genetic 400 architectures (Figure S14 and S15). If the genetic architecture explains only 30% of the 401 phenotypic variability in the initial sets of RILs, the relative improvements in Rs values using 402 updated training sets are greater than simulated QTL that explain 0.7 of the phenotypic variance 403 404 (Figure S15). Percent gain in response in GS with model updating relative to response from GS without updating for forty cycles of selection are provided in File S5. 405 406 When GP models are not updated, Mgv were consistently greater with PS than the four GP 407 models. Among GP models without updating, BB provided the best Mgv, while SVM-RBF had the smallest Mgv (Figure 8 and S16). When GP models are updated, the pattern depends mostly 408 409 on the number of QTL. For initial population H values of both 0.7 and 0.3 and 40 simulated QTL, Mgv are similar for RR, Bayesian GP models and PS, whereas for 400 QTL, RR produces 410 411 greater Mgv than PS and Bayesian GS methods. For 4289 QTL, RR and Bayesian models 412 produce greater Mgv with PS. GS with SVMRBF produced the least desirable Mgv for 40, 400 and 4289 QTL. 413 When GP models are updated, the standardized genotypic variance (Sgv) declines at a rate 414

similar to the rate of decrease when the models are not updated (Figure 9 and S17). Model

416 updating significantly improved estimated prediction accuracies, r_{ps} , for all GP models except

417 SVMRBF (Figure 10 and S18). Among RR and Bayesian GP models, model updating has a

418 slightly larger impact on estimated accuracies with RR than with Bayesian GP models (Figure 10

and S18). For all selection methods pairwise LD among markers on the same chromosome

420 decreases across cycles of recurrent selection (Figure S19-S23). LD decreased slowest with PS

421 (Figure S19). Decay of LD in early and late cycles of selection are similar among parametric GP

models and SVMRBF with relaxed selection intensities. By the 20th cycle of recurrent selection, 422 LD approached zero for all selection methods and there was no evidence that selection methods 423 affected linkage disequilibrium (LD) differentially in the earlier cycles. The rates of LD decay 424 are lower when GP models are updated compared to GP models without updating (Figure S19-425 S23). 426 427 When GP models are not updated, rates and limits of response standardized to change in 428 genotypic variance (RsVar) with RR and Bayesian GS methods are similar to PS for 40, 400 and 429 4289 simulated QTL responsible for both 70% and 30% of phenotypic variability in the initial 430 population. There are no significant differences among GS methods as well as among GS and PS for most treatment combinations. Both rates and limits of RsVar are also comparable across the 431

three nQTL and selection intensity levels. However, with top 10% selected fraction, PS

demonstrated greater limits of RsVar for 400 and 4289 QTL (Figure S24 and S25).

434 However when GP models are updated, the patterns of RsVar are significantly different among

435 GS methods and PS and are also dependent on nQTL, selection intensity and heritability (Figure

436 S26 and S27). With 0.7 heritability, there are no significant difference in RsVar among GS

437 methods for 40 simulated QTL for top 1%, 2.5% and 10% selected fraction. Whereas for 400 and

438 4289 QTL with top 2.5% and 10% selected fraction, RR GS method demonstrated greater limits

439 of RsVar followed by PS and Bayesian GS methods. Gain in RsVar with RR GS is even larger

440 for 0.3 H treatment with relaxed selection intensities (Figure S26 and S27). SVMRBF

demonstrated the least limits of RsVar for treatment combinations with and without modelupdating (Figure S24 -S27).

Selection Intensity. Truncation selection using selection intensities of 2.67 and 2.34 resulted in
 similar rapid responses across the early cycles of recurrent selection (Figure S28 – S31). These

| 445 | selection intensities associated with selection of 1% and 2.5% top RIL's limited responses to the |
|-----|---|
| 446 | early cycles (first 10-20 cycles) for all combinations of selection methods and number of |
| 447 | simulated QTL. In contrast a selection intensity of 1.75, associated with retaining top 10% of the |
| 448 | RILs each cycle, provided continued opportunities for response to selection for additional cycles |
| 449 | depending on the number of simulated QTL and selection method (Figure 5-7 and S11-S15). |
| 450 | Selection intensity also impacted the effectiveness of updating GP models. With relaxed |
| 451 | selection intensity, the proportional gains are greater with model updating than without updating |
| 452 | for the three parametric GP methods, whereas stringent selection intensities with model updating |
| 453 | for these methods resulted in proportionally lower gains (Figure S14, S15 and S32 $-$ S35). |
| 454 | Percent gain in responses from GP model updating for all three selection intensities are described |
| 455 | in File S6. |
| 456 | Mgv's also increased with the relaxed selection intensities and differences of Mgv's among GS |
| 457 | methods due to selection intensities were affected by model updating (Figure 8, S16 and S36- |
| 458 | S39). As expected, the standardized genotypic variance, Sgv, decreased rapidly with increasing |
| 459 | selection intensities (Figure 9, S17, and S40–S43). However, model updating didn't have any |
| 460 | significant effect on rate of decrease in Sgv. Coincident with the relationship between loss of Sgv |
| 461 | and selection intensities, LD also decays with increasing selection intensities (Figure S19). In |
| 462 | contrast to Sgv, the rates of loss of LD among the GS methods are slowed by including training |
| 463 | sets that are updated (Figure S20-S23). |
| 161 | By adjusting selection intensity genetic variance in the population can be maintained for longer |

466 migration. Rate of decrease in genetic variance increases with increasing selection intensity

465

number of cycles without contribution from other sources of variation such as mutation or

467 Relaxed selection intensity of top 10% showed the least rate of decay of genetic variance,

whereas selection with top 1.0% demonstrated the largest rate of decay of genetic variance
(Figure 9, S17, S40 – S43).

The rates at which estimated prediction accuracies decline under relaxed selection intensities
result are less than the loss with stringent selection intensities. As with other metrics, the impact
of selection intensity on estimated predictions accuracies depend on the GS method, model
updating, number of QTL and heritability. Differences among GP models with updating are most
pronounced for high selection intensities, 4289 QTL and high heritability (Figure 10, S18, S44 S47).

476 *Number of simulated QTL*. The number of simulated QTL had the largest consistent impact on differences among the response curves for Rs values and Mgv's. This is most obvious by noting 477 478 that the Rs values (Figure 5, S11, S28-S31) are as high as 80% of the maximum value of 200 479 and reach the limit in less than ten cycles of recurrent selection if there are 40 simulated QTL. In contrast, Rs values are no greater than 40% of the maximum value of 200 and reach the limit in 480 10-15 cycles of recurrent selection for most selection methods if there are 400 simulated QTL 481 while Rs values are no greater than 15% of the maximum value and only begin to approach the 482 limit after 20 cycles if there are 4289 simulated QTL (Figures 5, S11 and S28 – S31). 483 484 The loss of Sgv's across cycles for the simulated number of QTL is consistent with the rate at which limits to response from selection are approached (Figure 9, S17, S41 -43). As genotypic 485

486 variance is eliminated, response to selection approaches a limit. Likewise the estimated

487 prediction accuracies approach zero as the genotypic variance approaches zero (Figure 10, S18

488 ,S44 –S47) although the covariance between the two metrics depend on the other simulated

489 factors.

| 490 | Heritability Most of the differences in response metrics between the two simulated H values |
|-----|---|
| 491 | have been reported above. In summary, Rs values are greater for simulated QTL responsible for |
| 492 | 70% of the initial phenotypic variance than Rs values for all genetic architectures responsible for |
| 493 | 30% of the initial phenotypic variance (Figure 5 and S11). These trends are correlated with the |
| 494 | other response metrics, in particular prediction accuracies of the GP models. |
| 495 | The loss of estimated prediction accuracies are greater with H values of 0.3 than 0.7 with relaxed |
| 496 | selection intensities. Other combinations of selection intensity and heritability require model |
| 497 | updating to provide reasonable GP model prediction accuracies and achieve greater responses |
| 498 | across more cycles of selection. As we would expect, for all combinations of selection intensity |
| 499 | and number of QTL, limits of response and loss of genotypic variance are greater with H values |
| 500 | of 0.7 than 0.3 (Figure 10, S18, and S44 – S47). |

503 Discussion

504 We did not use a coalescent process to establish a set of founders. Rather we used a set of publicly available founders to create a breeding population similar to that found in soybean 505 506 variety development projects in the primary soybean production region of North America. In both academic and commercial soybean development projects it is not unusual to cross multiple 507 508 lines to a single exceptional variety and generate 50 to 150 RILs from each cross. Of the 4289 509 SNP markers with genotypic scores for the SoyNAM population 3818 were polymorphic among 510 the 20 families that were used as founders for the simulations. On average, 773 were 511 polymorphic within a family with a variance among families of 34 polymorphic loci. In the initial founding set of RILs, the average heterozygosity per SNP locus across 20 families was 512 513 0.09. The average estimated G_{st} value across the genome for the initial founding set of RILs from 514 20 families was 0.32. 'Gst' is a measure of sub-population differentiation estimated as ratio of difference between sub population expected heterozygosity and total expected heterozygosity to 515 total expected heterozygosity (Jombart 2008; Ryman and Leimar 2009; Jombart and Ahmed 516 2011). Relative to previous reported founders in self-pollinated crops derived using a coalescent 517 process, our simulations began with a structure more likely to be found in actual soybean 518 519 breeding populations and with much less, albeit more realistic, genetic diversity.

Recurrent selection was conducted in the context of a cultivar development process in which
RILs are created and phenotypically evaluated each cycle. Field evaluations of replicated RILs
provide information on the repeatable performance of RIL's across environments before they are
used for creating a new cycle of genetic improvement. Comparisons among selection methods
assumed equal time required to develop and evaluate RIL's for each cycle of recurrent selection.
In practice, one of the advantages of using recurrent GS relative to recurrent PS is that

response/per year will be greater with GS. Even if both GS and PS require the same amount of
time to develop RILs, selection with GS methods can be conducted without multiple years of
phenotypic evaluations (Heffner et al. 2009).

529 An alternative recurrent selection strategy is to decouple genetic improvement from variety development (Gaynor et al. 2017). By separating the two types of breeding projects, GS can be 530 531 applied every generation using training sets composed of genotypic and phenotypic data obtained 532 on RILs derived in a previous cycle of recurrent selection. The consequence of using GP models 533 to select and cross individual F1 plants instead of RILs should be to create more opportunities for 534 recombination. In such a system training sets are updated with data obtained at regular intervals from annual field trials, although the training sets may be several selection cycles removed from 535 536 the cycle used to create the RILs used in current field evaluations. Implementing such a two part 537 strategy in soybean would require significant changes. In particular, intercrossing soybeans is labor intensive and expensive. Therefore, a two part system that requires intercrossing individual 538 plants, rather than replicable lines, three times per year will require significant investments. 539 Whether such investments can be justified will need to be investigated. The results reported 540 herein provide a basis for comparing alternative breeding strategies with the established 541 542 strategies for genetic improvement and cultivar development.

We simulated only simple additive genetic architectures. Alleles at adjacent QTL were assigned alternating positive genotypic values to alleles from each founder. Also recognize that some marker alleles are identical to the QTL alleles. Thus loss of prediction accuracies across cycles is not due to loss of LD, rather it is entirely due to loss of genotypic variance. Thus, the results indicate best case scenarios for implementation of GS methods. For experimental applications of GS, maintaining relationships among RILs in selected populations and LD between marker-QTL

will help maintain prediction accuracy even when the genotypic variance is reduced in later
cycles of selection (Meuwissen 1997; Zhong et al. 2009; Wimmer et al. 2013; Müller et al. 2017,
2018).

552 Based on estimates of the number of effectively segregating genomic segments among RILs 400 553 and 4289 simulated QTL were associated with only about 40 and 400 effective haplotype blocks 554 respectively (data not shown). The reduced number of effective linkage blocks means that the net 555 magnitude of the allelic effects for each linkage block is less than for the case with 40 simulated 556 QTL. For example by summing +/- alleles across a linkage block consisting of five QTL the 557 simulated genetic effects will be 2a(+-++) or -2a(-+-+). Because the maximum net effect is 200 units regardless of the number of QTL, a large number of linkage blocks with small net 558 559 effects requires more recombination for RILs in the population to realize the genetic potential of 560 the founders. Hence we hypothesize if GS is applied to individuals in every generation such as proposed by (Gaynor et al. 2017) the increased recombination will effectively release useful 561 genetic variability that is locked up in linkage blocks when RIL's are used as the selection units. 562 Our simulations also used constant values of non-genetic variance to produce phenotypic 563 variance every cycle. While it is possible to simulate constant values of H, rather than σ_r , across 564 565 all cycles of selection, the translation of such simulated values to experimental field plots is 566 equivalent to planting (exponentially) increasing numbers of field plots as the genotypic variance 567 decreases. Because closed soybean genetic improvement programs cannot afford to increase the number of plots to offset the loss of genotypic variance, we decided that it would be more 568 569 realistic to assume that the non-genetic variance rather than H will be constant over cycles of 570 recurrent selection.

571 Modeled genotypic responses across 40 cycles of recurrent selection. The use of a first order recurrence equation (3) to model genotypic responses (Rs) to recurrent selection provided a 572 method for comprehensive analyses of variance including interactions among the combinations 573 of factors. Curiously, we have not been able to find previous applications of recurrence 574 equations to model responses from genomic selection methods in the literature. We hope that 575 576 our explanation of how to implement such models in available R packages will encourage others to investigate the dynamics of recurrent selection across multiple cycles in both closed and open 577 genetic populations. 578

579 Prediction accuracies of GP models trained with founder populations are consistent with

prediction accuracies observed in previous studies (Habier et al. 2007; Goddard 2009; Howard et

al. 2014). Prediction accuracies of RR and Bayesian methods with the population structure and

582 genomic architecture created with SoyNAM founders are similar to previously reported

accuracies for additive genetic architectures responsible for 0.7 and 0.3 of the phenotypic

variance while the non-parametric SVM-RBF model produced the least accurate predictions

from the founding sets of RILs (Habier et al. 2007; Howard et al. 2014).

The numbers of QTL also impacted prediction accuracy, with greater prediction accuracies for smaller numbers of QTL and a given training set size. With large numbers of QTL and smaller simulated additive effects, larger training sets are required to maintain similar levels of accuracy as compared to small number of QTL with large additive effects (Goddard 2009; Dekkers 2010).

590 *Comparisons of Selection Methods*. The comparisons of GS methods using all metrics suggest

that RR when updated with training data from prior cycles will provide the best long-term

response to selection in a closed breeding population derived from founders of SoyNAM.

593 Without updating the models using training data from prior cycles, responses to recurrent

594 selection using parametric GS methods are greater than PS in early cycles, but are not better in later cycles, which is of concern for closed population improvement. RR models that use updated 595 training sets achieve greater responses than Bayesian GP models and PS in both the early and 596 late cycles of recurrent selection, whereas Bayesian GP models, when updated, result in greater 597 responses than PS only in the early cycles. With relaxed selection intensities and large number of 598 599 QTL, it is possible to achieve similar or greater limits of response than PS with RR and Bayesian GS methods with updating without compromising on rate of response with GS in early cycles 600 given certain conditions. 601

602 It has been suggested that long-term response in GS will never be greater than PS because PS maintains genotypic variance for a longer number of cycles of recurrent selection (Goddard 603 604 2009; Zhong et al. 2009; Jannink 2010). First, prediction accuracies of GS methods will decrease with every cycle of selection, whereas decay of prediction accuracies is irrelevant for PS. 605 Second, GS results in rapid loss of genetic variance in the initial cycles, which results in 606 607 approaching the asymptotic response limit in early cycles. However, response per cycle in the breeder's equation is also dependent on other factors such as selection intensity and loss of 608 609 standardized genotypic variance in addition to prediction accuracy. Depending on other factors, 610 GS still has the potential to realize greater gains for some treatments.

Model updating with training sets from prior cycles improves the relationship between training sets and validations sets compared to responses to GS without updated training sets resulting in greater prediction accuracies for RR GS method in late cycles of recurrent selection. Model updating also resulted in greater responses standardized to the rate of decrease of genetic variance in selected populations with updated RR GP models. Response standardized to change

in genotypic variance is very similar to efficiency of converting lost genetic diversity into geneticgain discussed by Gorjanc et al (Gorjanc G et al. 2018).

Instead of estimating efficiency as a slope of genetic gain regressed linearly on change in genotypic variance, we evaluated the non-linear pattern of changes. Given similar rates of decrease of genetic variance among RR and Bayes GS methods, the differences in responses among methods is possibly due to greater efficiency of translating loss of genetic diversity to gain with RR GS in later cycles of selection, although for most treatment factors the limits of response with RR and Bayesian GS methods are about the same.

It is not clear from our examination of patterns of relationship among selected lines and LD decay why prediction accuracies are greater in late cycles for RR compared to Bayesian methods when GP models are updated. Even though model updating resulted in significantly different rates of change of genotypic variance in selected populations, average heterozygosity, rate of inbreeding and rate of LD decay relative to GS methods without updating, there are no significant differences among GS methods.

Selection increased linkage disequilibrium (LD) while decreasing genetic variance in early 630 cycles of selection, whereas in late cycles, LD decayed due to recombination. In PS, LD is 631 influenced mostly by selection intensity, whereas the effects of LD and linkage are complicated 632 in GS. Prediction accuracies of GP models are dependent on LD among marker-QTL and affect 633 634 selection of lines and genotypic variance of selected populations, which in turn affect prediction accuracies in subsequent cycles of selection. In addition to the factors investigated in this study 635 636 additional forces can affect LD including selection, drift, epistasis, and GxE effects (Roze and 637 Barton 2006; Hickey et al. 2014). From these data it is not clear which of these forces has the 638 greatest impact on LD, but use of simulations could be used to address this question.

639 Partitioning the contribution of LD and linkage blocks to prediction accuracy could help estimate the contribution of each of these factors to prediction accuracy of GP models. However, it 640 requires a design similar to that employed by (Müller et al. 2017, 2018) for synthetic 641 populations. In their study, populations with unrelated training and prediction sets with LD and 642 SNP based relationship estimates showed low prediction accuracy and low genetic response in 643 644 recurrent GS. This is similar to GS without updating in this study. Whereas populations with relationship between training and prediction sets with LD and SNP based relationship estimates 645 646 is similar to GS with model updating, which showed greater prediction accuracy and greater 647 genetic response in both of their published studies. The genetic covariance among cycles of RILs depends on the number of QTL and influences the 648 649 number of prior cycles that needs to be included in the training data set. If the genetic covariance 650 among cycles of RILs changes rapidly, then data from fewer prior cycles are needed for accurate genomic predictions. Small numbers of QTL with larger effects requires fewer prior cycles of 651

data to achieve good prediction accuracies and responses, whereas large numbers of QTL each
with smaller additive effects produced slower changes to the genetic co-variance structure among
cycles and required training data from more prior cycles to maintain good prediction accuracies
and responses.

The rate at which genetic variance is reduced from cycle to cycle depends on number of QTL, heritability, selection intensity and GS methods (Figures 9 and S16) and we could model the loss of genotypic variance using a first order recurrence equation (3). However, it is likely that the estimated genotypic variance from each cycle is underestimated because the covariance among QTL, due to LD among QTL, was not taken into account (Bulmer 1971, 1976; Lehermeier et al.

2017). None-the-less, the changes of genotypic variance from cycle to cycle mirrored theresponse of Rs and Mgv, decreasing to zero when the Rs and Mgv reached their limits.

We also assumed that the training set size increases with every cycle when we cumulatively add 663 664 data from prior cycles to the training data set. Increasing training population size confounds estimation of effects of maintaining relationship between training and prediction sets on 665 666 accuracy. But, there are some approaches to remedy this situation which still needs to be 667 evaluated. One of the potential approaches is to randomly sample subsets of data from each of 668 the prior cycles to maintain a constant cumulative training population size. It is also possible to 669 assign weights to the samples from prior cycles to place more weight on data from more recent cycles. This essentially involves determining the optimal trade-off for sample sizes and weights 670 671 that will assure maximum prediction accuracy with minimal computational requirements. Some 672 aspects of this optimization problem have been addressed. Akdemir et al (2015) devised a genetic algorithm for selecting optimal training populations to minimize prediction error 673 variance and Xavier et al (2017) developed sampling methods for training bayesian GP models. 674 Another approach is to retrain GP models at regular frequencies instead of updating every cycle, 675 676 while maintaining a constant training population size. Application of sampling and optimization 677 methods for selection of training populations for GP model updating requires further study in the context of recurrent genomic selection. 678

Selection Intensity. Replicated responses to high values of selection intensity quickly reach a
limit in five to ten cycles of recurrent selection. Replicated responses to lower values of selection
intensity consistently result in greater gains over more cycles, indicating that genetic drift is the
most likely mechanism for loss of genotypic variance. These constraints on plant breeding
programs are well characterized (Brisbane and Gibson 1995; Hayes et al. 2009; Jannink 2010;

Hung et al. 2012; Liu et al. 2015; Akdemir and Sánchez 2016; Yabe et al. 2016). The optimal
trade-off between achieving genetic gains and maintaining genetic diversity depend on the
objectives. For example, if the objective is to enter and capture market share in a short time
frame, then maintenance of genetic diversity is not important.
Most simulation studies apply a constant selection intensity across cycles of selection. The

689 effects of applying a dynamic selection strategy is an interesting problem. We hypothesize that a

690 strategy consisting of applying different selection intensities, optimized for each cycle, will

achieve improved long-term genetic response by differentially emphasizing genetic variance and

692 genetic response across multiple cycles of selection.

693 Conclusion

Using simulations we examined the impact of five factors on genetic response through 40 cycles

of recurrent selection in simulated Soybean populations for 306 unique combinations of

treatment factors in a factorial design. Two of these factors, number of QTL and heritability are

697 characteristics that the plant breeder cannot alter in a closed breeding population, whereas the

698 plant breeder can alter selection intensity, GP models and GP model updating.

Interactions among the five factors significantly affect the rate of response and limits to selection response. Responses to selection approached a limit after 10 to 20 cycles of recurrent selection for most combinations of the five factors. If GP models are not updated after an initial training set, then bayesian parametric methods performed better than ridge-regression and machine learning methods in terms of prediction accuracy and limits of response to selection. If GP models are updated by re-training using current and prior cycles of genotypic and phenotypic data, then ridge regression models demonstrated greater prediction accuracies and limits of responses. Relaxed selection intensity resulted in greater limits of responses but at slightly lower
rates of genetic gain in early cycles compared to stringent selection intensities.

708 By utilizing a first order recurrence equation to model response to selection and evaluating 709 factors that could affect the response to selection our results provide an objective framework for 710 further investigations of selection methods for genetic improvement and line development 711 projects in self-pollinated crops. For example, it has been suggested that the occasional 712 emergence of a line with exceptional characteristics that dominate all other genotypes for several 713 cycles of selection is likely due to a unique combination of genetic alleles, i.e., epistasis. If 714 epistasis is simulated, will the first order recurrence equation be sufficiently robust to model response to selection? Second, we employed truncation selection with a crossing strategy that 715 716 uses high contributions from top ranked RILs among the selected population of RILs in each of 717 the simulated cycles of selection. We did not consider relationships among selected RILs nor the trade-offs between genetic gain, genetic variance (inbreeding) when selecting RILs to cross. 718 719 There exist multi-objective optimization breeding methods such as genomic mating and optimal 720 cross selection (Rutten et al. 2002; Woolliams et al. 2015; Akdemir and Sánchez 2016; Gorjanc 721 et al. 2018), that could provide both greater rates and limits of responses across cycles. 722 Importantly, we simulated a closed breeding population in which culled lines were not resampled 723 for discarded favorable alleles, nor did we simulate a more open breeding population with 724 exchange of lines among breeding programs. There are evolutionary algorithms such as island 725 models with parameters that fit these more realistic breeding programs (Yabe et al. 2016). Evaluation of these other types of selection strategies and design of hybrid strategies that 726 727 incorporate elements of two or more distinct algorithmic approaches is a promising direction of exploration. 728

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Figures



Figure 1 Flow Chart for Simulations of Recurrent Genomic Selection. The upper half panel represents the steps involved in generating the base population of 2000 F_5 RILs derived from 20 NAM founder lines crossed, *in silico*, to IA3023. It includes the model training step for genomic prediction models. The lower half panel represents recurrent steps of prediction, sorting, truncation selection, crossing, and generation of F_5 RILs for each cycle as well as the decision steps to check if the training set should be updated and if the recurrent process is to be continued for another cycle.



Figure 2 Flow Chart for Model Updating in Simulations of Recurrent Genomic Selection. Model updating involves combining training data from 'n-1' previous cycles (t-n-1...t-1) with training data from the tth cycle to retrain genomic prediction models. 't' refers to the selection cycle and ranges from 1 to 40 and 'n' refers to number of prior cycles that are included in the training set. The treatment design has four levels for 'n' ranging from 0 - 14. 'n=0' refers to no updating, whereas 'n= 10', 'n=12' and 'n=14' refer to inclusion of training data from 9, 11 and 13 prior cycles along with data from tth cycle of selection.



Figure 3 Theoretical and Simulated Genotypic Values: A) (Top Panel) Theoretical Genotypic Values from 40 cycles of recurrent selection modeled with the recurrence equation,

 $y_c = \alpha^c y_0 + \beta \frac{1 - \alpha^c}{1 - \alpha}$ where *y*_c represents the genotypic value in cycle c, with c= 1, 2... 40 and

values of α and β are 0.9 and 15 respectively. B) (Bottom Panel) Averaged genotypic values fo r 40 cycles of simulated recurrent selection. Genotypic values are averaged across selection m ethods, training sets, selection intensities, number of simulated QTL and simulated heritabilitie s. Predicted curve is modelled with α = 0.82, β = 12.37, that were obtained from 'nls' fit with co mpleted dataset without any grouping.



Figure 4 Estimated prediction accuracies and MSE in Founding Set of RILs: Estimated prediction accuracies (left panel) and mean squared errors (right panel) for four genomic prediction (GP) models: BayesB, BL (Bayes LASSO), RRREML (Ridge Regression with REML) and SVMRBF (Support Vector Machines with Radial Basis Function Kernel) trained with F₅ RILs derived from crosses of 20 homozygous founder lines with IA3023. Phenotypes used to train the GP models consisted of genetic architectures comprised of 40, 400 and 4289 simulated QTL (top, middle and bottom) that were responsible for 70% (blue) and 30% (red) of phenotypic variability in the initial populations.



Figure 5 Standardized Responses for Comparison of GS methods with and without Updating for 0.7 H and Top 10% Selected Fraction Forty cycles of standardized responses to selection of 10% of 2000 soybean RILs per cycle. Standardized responses are plotted by selection methods without (left panels) and with (right panels) model updating using prior cycles as training sets for four genotypic prediction models. Phenotypic selection (PS) is not updated and hence is the same in the left and right panels. The top panels consist of responses for genetic architectures consisting of 40 simulated QTL. Middle panels consist of responses for genetic architectures consisting of 400 simulated QTL and the bottom panels consist of responses for genetic architectures consisting of 4289 simulated QTL. All 40, 400, and 4289 simulated QTL are responsible for 70% of phenotypic variability in the initial population. PS - Phenotypic Selection, RR-REML- Ridge Regression with Restricted Maximum Likelihood, BL - Bayes LASSO, and SVMRBF- Support Vector Machine with Radial Basis Kernel.

| Mathad | ΟΤΙ | horitability | TU5 | TU10 | TU15 | TU20 | ти25 | TU20 | ти25 | TU 40 |
|---------|------|--------------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | | 1115 | 10.51 | 1115 | 11120 | 11125 | 11150 | 11155 | 11140 |
| RR-REML | 40 | 0.7 | -2.36 | -12.71 | -9.63 | -9.61 | -9.61 | -9.61 | -9.61 | -9.61 |
| BayesB | 40 | 0.7 | 10.96 | -4.18 | -5.49 | -5.49 | -5.49 | -5.49 | -5.49 | -5.49 |
| BL | 40 | 0.7 | 3.46 | -9.96 | -8.77 | -8.24 | -8.24 | -8.24 | -8.24 | -8.24 |
| SVMRBF | 40 | 0.7 | -39.67 | -49.09 | -46.90 | -46.67 | -46.67 | -46.67 | -46.67 | -46.67 |
| | | | | | | | | | | |
| RR-REML | 400 | 0.7 | 2.42 | -7.37 | -14.57 | -17.25 | -19.61 | -20.96 | -20.93 | -21.11 |
| BayesB | 400 | 0.7 | 0.10 | -9.53 | -14.89 | -17.58 | -19.94 | -21.26 | -21.05 | -21.11 |
| BL | 400 | 0.7 | -3.35 | -11.95 | -15.15 | -18.26 | -19.62 | -21.21 | -20.80 | -21.37 |
| SVMRBF | 400 | 0.7 | -21.91 | -43.06 | -49.93 | -52.95 | -54.26 | -55.48 | -55.46 | -55.42 |
| | | | | | | | | | | |
| RR-REML | 4289 | 0.7 | -6.26 | -20.12 | -25.33 | -27.60 | -29.16 | -30.98 | -32.60 | -33.57 |
| BayesB | 4289 | 0.7 | -9.07 | -19.79 | -26.28 | -29.14 | -31.30 | -33.37 | -34.90 | -35.84 |
| BL | 4289 | 0.7 | -5.85 | -17.94 | -23.50 | -25.70 | -27.24 | -29.66 | -31.36 | -32.35 |
| SVMRBF | 4289 | 0.7 | -34.49 | -55.74 | -63.38 | -66.01 | -66.85 | -67.79 | -68.60 | -69.15 |

Figure 6 Heat Map for Percent Gain in Rs Relative to PS for 0.7 H for GP Models without Updating: Heat map indicating standardized response relative to PS as percentage gain after 40 cycles of recurrent selection using genomic prediction models without updated training sets for 40, 400, 4289 simulated QTL responsible for 70% of phenotypic variability in the initial population. Blue to red shaded cells represent increasing gain in response relative to PS. RR-REML- Ridge Regression with Restricted Maximum Likelihood, BL - Bayes LASSO, and SVMRBF- Support Vector Machine with Radial Basis Kernel.

| Method | QTL | heritability | No. Prior Cycles | TH5 | TH10 | TH15 | TH20 | TH25 | TH30 | ТН35 | TH40 |
|---------|------|--------------|------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | | | | | | | | | | |
| RR-REML | 40 | 0.7 | 14 | 9.04 | 1.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| BayesB | 40 | 0.7 | 14 | 14.41 | -1.05 | -2.06 | -2.06 | -2.06 | -2.06 | -2.06 | -2.06 |
| BL | 40 | 0.7 | 14 | 11.89 | -4.70 | -5.49 | -5.49 | -5.49 | -5.49 | -5.49 | -5.49 |
| SVMRBF | 40 | 0.7 | 14 | -39.85 | -48.58 | -47.63 | -47.36 | -47.36 | -47.36 | -47.36 | -47.36 |
| | | | | | | | | | | | |
| RR-REML | 400 | 0.7 | 14 | 11.34 | 14.84 | 12.30 | 8.71 | 5.73 | 3.29 | 3.50 | 3.22 |
| BayesB | 400 | 0.7 | 14 | 13.05 | 12.00 | 8.38 | 1.75 | -1.11 | -2.96 | -3.02 | -3.91 |
| BL | 400 | 0.7 | 14 | 13.85 | 10.44 | 6.45 | 1.02 | -1.89 | -4.24 | -3.92 | -4.55 |
| SVMRBF | 400 | 0.7 | 14 | -21.66 | -42.81 | -50.39 | -53.52 | -54.86 | -55.38 | -55.48 | -55.40 |
| | | | | | | | | | | | |
| RR-REML | 4289 | 0.7 | 14 | 10.01 | 10.51 | 11.79 | 13.39 | 14.52 | 13.67 | 11.70 | 10.13 |
| BayesB | 4289 | 0.7 | 14 | 10.10 | 10.41 | 9.09 | 8.31 | 7.20 | 4.56 | 2.06 | 0.60 |
| BL | 4289 | 0.7 | 14 | 10.32 | 9.22 | 8.87 | 8.78 | 8.22 | 6.11 | 3.59 | 2.10 |
| SVMRBF | 4289 | 0.7 | 14 | -36.73 | -56.79 | -64.13 | -66.87 | -68.08 | -68.97 | -69.49 | -69.96 |

Figure 7 Heat Map for Percent Gain in Rs Relative to PS for 0.7 H for GP Models with Updating: Heat map indicating standardized response relative to PS as percentage gain after 40 cycles of recurrent selection using genomic prediction models with updated training sets from up to 14 prior cycles of selection for 40, 400, and 4289 simulated QTL responsible for 70% of phenotypic variability in the initial population. Blue to red shaded cells represent increasing gain in response relative to PS. RR-REML-Ridge Regression with Restricted Maximum Likelihood, BL - Bayes LASSO, and SVMRBF- Support Vector Machine with Radial Basis Kernel.



Figure 8 Attained Genotypic Value for Comparison of GS methods with and without Updating for 0.7 H and Top 10% Selected Fraction: Maximum attained genotypic values (Mgvs) in recurrent genomic selection and phenotypic selection (PS) without updating the training sets in the left panels and with training set updates from up to 14 prior cycles in the right panels. PS has no training sets and hence does not change between the left and right panels. a) 40 QTL (top), b) 400 QTL (middle) and c) 4289 QTL (bottom) responsible for 70% of phenotypic variability in the initial population and selection of 10% of the RILs in each cycle. PS - Phenotypic Selection, RR-REML- Ridge Regression with Restricted Maximum Likelihood, BL - Bayes LASSO, and SVMRBF- Support Vector Machine with Radial Basis Kernel.



Figure 9 Standardized Genotypic Variance (Sgv) for Comparison of GS methods with and without Updating for 0.7 H and Top 10% Selected Fraction Standardized genotypic variance without training set updating (left panels) and with training set updating with prior cycle training data (right panels) for the four GP models. PS has no updating and hence is the same in both left and right panels. A) Training data from up to 14 prior cycles for 40 simulated QTL (top), 400 simulated QTL (middle) and 4289 simulated QTL (bottom) responsible for 70% of phenotypic variability in the initial population and top 10% of RILs with the greatest predicted values. PS - Phenotypic Selection, RR-REML- Ridge Regression with Restricted Maximum Likelihood, BL - Bayes LASSO, and SVMRBF- Support Vector Machine with Radial Basis Kernel.



Figure 10 Estimated Prediction Accuracies for Comparison of GS methods with and without Updating for 0.7 H and Top 10% Selected Fraction: Estimated prediction accuracies with updates to the training sets used in genomic prediction (GP) models. Training data from up to 14 prior selection cycles were used to update all four GP models for 40 QTL (top), 400 QTL (middle) and 4289 QTL (bottom) responsible for 70% of phenotypic variability in the initial population and top 10% of RILs with the greatest predicted values. RR-REML- Ridge Regression with Restricted Maximum Likelihood, BL -Bayes LASSO, and SVMRBF- Support Vector Machine with Radial Basis Kernel.



Figure 11 Average Genotypic Variance in Selected Populations for Comparison of GS methods with and without Updating for 400 QTL, 0.7 H and Top 10% Selected Fraction: Average genotypic variance for 40 cycles of recurrent selection of 10% of RILs using five selection methods without updated training sets (left panel) and with updated training sets (right panel). Training sets consisted of genotypic and phenotypic data from up to14 prior cycles of recurrent selection. Simulated phenotypic values of the RILs consisted of 400 simulated QTL responsible for 70% of phenotypic variability in the initial population. PS - Phenotypic Selection, RR-REML- Ridge Regression with Restricted Maximum Likelihood, BL - Bayes LASSO, and SVMRBF- Support Vector Machine with Radial Basis Kernel.



Figure 12: Number of Lost Favorable Alleles for Comparison of GS methods with and without Updating for 400 QTL, 0.7 H and Top 10% Selected Fraction: Number of favorable alleles that are cumulatively discarded across 40 cycles of recurrent selection of 10% of the RILs created every generation using five selection methods without updated training sets (left panel) and with updated training sets (right panel). Training sets consisted of genotypic and phenotypic data from up to14 prior cycles of recurrent selection. Simulated phenotypic values of the RILs consisted of 400 simulated QTL responsible for 70% of phenotypic variability in the initial population. PS - Phenotypic Selection, RR-REML- Ridge Regression with Restricted Maximum Likelihood, BL - Bayes LASSO, and SVMRBF- Support Vector Machine with Radial Basis Kernel.



Figure 13 Average Expected Heterozygosity for Comparison of GS methods with and without Updating for 400 QTL, 0.7 H and Top 10% Selected Fraction: Average Expected Heterozygosity in recurrent GS without GP model updating (left panel) and GP models updated every cycle with training data from upto14 prior cycles (right panel). All treatment combinations have 400 simulated QTL responsible for 70% of phenotypic variability in the initial population and 10% top selected fraction. PS - Phenotypic Selection, RR-REML- Ridge Regression with Restricted Maximum Likelihood, BL - Bayes LASSO, and SVMRBF- Support Vector Machine with Radial Basis Kernel.



Figure 14 Average Rate of Inbreeding for Comparison of GS methods with and without Updating for 400 QTL, 0.7 H and Top 10% Selected Fraction: Average rate of inbreeding in recurrent GS without GP model updating (left panel) and GP models updated every cycle with training data from upto14 prior cycles. Inbreeding coefficient is estimated as the harmonic mean of all individuals in the population for 40 cycles (top panel). Inset plot (bottom panel) shows magnified region from 3-12 cycles in average rate of inbreeding for PS, RR, BayesB, and BL GS methods. All treatment combinations have 400 simulated QTL responsible for 70% of phenotypic variability in the initial population and 10% top selected fraction. PS - Phenotypic Selection, RR-REML- Ridge Regression with Restricted Maximum Likelihood, BL - Bayes LASSO, and SVMRBF- Support Vector Machine with Radial Basis Kernel.

Tables

TABLE 1: FACTORIAL DESIGN

| Factors | Number of Levels | Values for Levels |
|--|---------------------------------|--|
| Number of QTL | 3 | 40, 400, 4289 |
| Heritability | 2 | 0.7, 0.3 |
| Selection Intensity | 3 | 2.67, 2.34, 1.75 |
| Selection Model | 5 | i) PS- Phenotypic value ii) GS - GP(RR) iii) GS - GP(Bayes B) iv) GS - GP(Bayes LASSO) v) GS - GP(SVM, Radial basis Function Kernel) |
| Model Update: number of prior cycles used in training sets | 4 | i) 0 previous cyclesii) 10 previous cyclesiii) 12 previous cyclesiv) 14 previous cycles |
| Total Number of unique combinations | 360 | |
| Total Number of Simulations | 3600 with 10 reps /condition | |

| TABLE 2 PACKAGES IN R FOR PARAMETRIC AND NON-PARAMETRIC MODELS WITH TUNING PARAMETERS | | | | | | | | |
|--|--|--|--|--|--|--|--|--|
| GS Model | Package (R) | Model Tuning Parameters (BGLR package) | | | | | | |
| Ridge Regression | REML-EM (custom R script Xavier A, 2018) | EM algorithm for estimation of parameters with REML method without using matrix inversion | | | | | | |
| Bayesian LASSO | BGLR (Perez P et al, 2014) | Priors for varE (df=3,S=0.25); varU (df=3,S=0.63); lambda(shape=0.53,rate=5e-5) type='random',value=30), nlter=20000, burnIn=2000, thin=1 | | | | | | |
| Bayes B | BGLR (Perez P et al, 2014) | nlter=41000, burnIn =1000, df0=4, R2=0.7 | | | | | | |
| SVM | Rgtsvm (Wang Zhong et al, 2017) | SVM with Radial basis function kernel on GPU | | | | | | |

| TABLE 3 ANOVA FOR NON-LINEAR MIXED MODELS | | | | | | | | |
|--|----|------------|------------|------------|--|--|--|--|
| Model Description | df | AIC | BIC | logLik | | | | |
| M1: 2 curves, one for each simulated h2 | 6 | 1490883.86 | 1490943.12 | -745435.93 | | | | |
| M2: 3 curves, one for each simulated SI | 6 | 1487947.33 | 1488006.60 | -743967.67 | | | | |
| M3: 3 curves, one for each nQTL level | 6 | 1271056.31 | 1271115.57 | -635522.15 | | | | |
| M4: 4 curves, one for each number of prior cycle TS levels | 6 | 1491707.23 | 1491766.50 | -745847.61 | | | | |
| M5: 5 curves, one for each of SM levels | 6 | 1482602.97 | 1482662.24 | -741295.49 | | | | |
| M6: 6 curves, one for each of h2 and nQTL levels | 9 | 1262672.35 | 1262761.25 | -631327.17 | | | | |
| M7: 6 curves, for each of h2 and SI levels | 9 | 1486685.15 | 1486774.05 | -743333.58 | | | | |
| M8: 8 curves, one for each combination of h2 and TS levels | 9 | 1490439.19 | 1490528.09 | -745210.59 | | | | |
| M9: 9 curves, one for each level of SI and nQTL | 9 | 1243669.76 | 1243758.65 | -621825.88 | | | | |
| M10: 10 curves, one for each level of SM and h2 | 9 | 1481466.69 | 1481555.59 | -740724.34 | | | | |
| M11: 12 curves, one for each level of TS and nQTL | 9 | 1268945.50 | 1269034.40 | -634463.75 | | | | |
| M12: 12 curves, one for each level of TS and SI | 9 | 1487510.42 | 1487599.31 | -743746.21 | | | | |
| M13: 15 curves, one for each combination of SM and nQTL | 9 | 1194913.36 | 1195002.26 | -597447.68 | | | | |
| M14: 15 curves, one for each combination of SM and SI | 9 | 1477442.03 | 1477530.93 | -738712.01 | | | | |
| M15: 20 curves , one for each combination of SM and TS | 9 | 1481835.46 | 1481924.36 | -740908.73 | | | | |
| M16: 18 curves, one for each combination of SI, nQTL and h2 | 12 | 1233180.40 | 1233298.93 | -616578.20 | | | | |
| M17: 24 curves, one for each combination of SI, TS and h2 | 12 | 1486217.33 | 1486335.86 | -743096.66 | | | | |
| M18: 24 curves, one for each combination of TS, nQTL and h2 | 12 | 1260244.61 | 1260363.14 | -630110.31 | | | | |
| M19: 30 curves, one for each combination of SM, h2 and SI | 12 | 1179040.14 | 1179158.67 | -589508.07 | | | | |
| M20: 30 curves, one for each combination of SM, nQTL and h2 | 12 | 1476040.79 | 1476159.32 | -738008.40 | | | | |
| M21: 36 curves, one for each combination of SI, nQTL, and TS | 15 | 1229771.09 | 1229919.25 | -614870.54 | | | | |
| M22: 40 curves, one for each combination of SM, h2 and TS | 12 | 1480446.72 | 1480565.25 | -740211.36 | | | | |
| M23: 45 curves, one for each combination of SI, nQTL and SC | 15 | 1107823.41 | 1107971.57 | -553896.70 | | | | |
| M24: 60 curves, one for each combination of SM, nQTL and TS | 12 | 1188346.20 | 1188464.74 | -594161.10 | | | | |
| M25: 60 curves, one for each combination of SI, TS and SM | 12 | 1476793.27 | 1476911.80 | -738384.64 | | | | |
| M26: 72 curves, one for each combination of TS, h2, nQTL, SI | 15 | 1230064.21 | 1230212.38 | -615017.11 | | | | |
| M27: 90 curves, one for each combination of SM, h2, nQTL, SI | 15 | 1107717.81 | 1107865.98 | -553843.91 | | | | |

| M28: 120 curves, one for each combination of SM, h2, TS, SI | 15 | 1475380.15 | 1475528.31 | -737675.07 |
|---|----|------------|------------|------------|
| M29: 120 curves, one for each combination of SM, nQTL, TS, h2 | 15 | 1171315.89 | 1171464.05 | -585642.94 |
| M30: 180 curves, one for each combination of SM, nQTL, TS, SI | 15 | 1123398.62 | 1123546.78 | -561684.31 |
| M31: 360 curves, one for each combination of SM, nQTL, TS, SI, and h2 | 6 | 1094075.09 | 1094134.35 | -547031.54 |

Table 3: Non-linear Mixed Models fit with increasing number of factors from M1 to M31. Factors include selection method with five levels comprising of PS and four GS models ('SM'), two levels for heritability ('h2'), three levels for number of QTL ('nQTL'), three levels for selection intensity ('SI'), and four levels for prior cycles in training set ('TS')

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