Effective Biophysical Modeling of Cell Free Transcription and Translation Processes

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Abstract

Transcription and translation are at the heart of metabolism and signal transduction. In this study, we developed an effective biophysical model of transcription and translation processes for the cell free operation of two synthetic circuits. First, we considered a simple circuit in which sigma factor 70 (σ_{70}) induced the expression of green fluorescent protein. This relatively simple case was then followed by a much more complex negative feedback circuit. The negative feedback circuit consisted of two control genes coupled to the expression of a third reporter gene which encoded green fluorescent protein. While plausible values for many of the model parameters were estimated from previous biophysical literature, the remaining unknown model parameters for each circuit were estimated from messenger RNA (mRNA) and protein measurements using multiobjective optimization. In particular, either the literature parameter estimates were used directly in the model simulations, or characteristic literature values were used to establish feasible ranges for the multiobjective parameter search. Next, global sensitivity analysis was used to determine the influence of individual model parameters on the expression dynamics. Taken together, the effective biophysical model captured the expression dynamics, including the transcription dynamics, for two synthetic cell free circuits. While we considered only two circuits here, this approach could potentially be extended to simulate other genetic circuits in both cell free and whole cell biomolecular applications. All model code, parameters, and analysis scripts are available for download under an MIT software license from the Varnerlab GitHub repository.

Introduction

Transcription (TX) and translation (TL), the processes by which information stored in DNA is converted to a working protein, are at the center of metabolism and signal transduction programs important to biotechnology and human heath. For example, evolutionarily conserved developmental programs such as the epithelial to mesenchymal transition (EMT) [1], or retinoic acid induced differentiation [2], rely on multiple rounds of highly coordinated gene expression. From the perspective of biotechnology, even relatively simple industrially important organisms such as *Escherichia coli*, have intricate regulatory networks which control the metabolic state of the cell in response to changing nutrient conditions [3]. Understanding the dynamics of regulatory networks can be greatly facilitated by mathematical models. A majority of these models fall into three categories: logical, continuous, and stochastic models [4]. Logical models such as Boolean networks [5] developed from a variety of data sources [6] represent the state of each entity as a discrete variable, and provide a quick but qualitative description of the network. Linear and non-linear ordinary differential equation (ODE) models fall into the second category, and they generally provide a detailed picture of the network dynamics, although they can be non-physical models, e.g., relying on a gene signal perspective [7]. Lastly, stochastic models describe the interactions between individual molecules, and discrete reaction events [8–11]. Model choice depends on criteria such as speed, the level of detail required and the quantity of experimental data available to estimate the model parameters. While the end goal of the models might be to accurately predict *in vivo* behavior, living systems do not necessarily provide an ideal experimental platform. For example, although there have been significant advancements in metabolomics e.g., [12], the rigorous quantification of intracellular messenger RNA (mRNA) copy number or protein abundance remains challenging. Toward this challenge, cell free systems offer several advantages for the study of transcription and translation processes.

Cell free biology has historically been an important tool to study the fundamental biological mechanisms involved with gene expression. In the 1950s, cell free systems were used to explore the incorporation of amino acids into proteins [13–15], and the role of adenosine triphosphate (ATP) in protein production [16]. Further, *E. coli* extracts were used by Nirenberg and Matthaei in 1961 to demonstrate templated translation [17, 18], leading to a Nobel Prize in 1968 for deciphering the codon code. More recently, as advancements in extract preparation and energy regeneration have extend their durability, the usage of cell free systems has also expanded to both small- and large-scale biotechnology and biomanufacturing applications [19, 20]. Today, cell free systems have been implemented for therapeutic protein and vaccine production [21–23], biosensing [24], genetic part prototyping [25] and minimal cell systems [26]. The versatility of cell free systems offers a tremendous opportunity for the systems-level experimental and computational study of biological mechanism. For example, a number of ordinary differential equation based cell free models have been developed [27–31]. However, despite the obvious advantages offered by a cell free system, experimental determination of the kinetic parameters for these models is often not feasible. For instance, the cell free modeling study of Horvath et al (which included a description of transcription and translation, and the underlying metabolism supplying energy and precursors for transcription and translation), had over 800 unknown model parameters [32]. Moreover, the construction, identification and validation of the Horvath model took well over a year to complete. Thus, constructing, identifying and validating biophysically motivated cell free models, which are simultaneously manageable, is a key challenge. Toward this challenge, effective modeling approaches which coarse grain biological details but remain firmly rooted in a biophysical perspective, could be an important tool.

In this study, we developed an effective biophysical model of transcription and translation processes for the cell free operation of two synthetic circuits. First, we considered a

simple circuit in which sigma factor 70 (σ_{70}) induced the expression of green fluorescent protein. This relatively simple case was then followed by a much more complex negative feedback circuit. The negative feedback circuit consisted of two control genes coupled to the expression of a third reporter gene which encoded green fluorescent protein. While plausible values for many of the model parameters were estimated from previous biophysical literature, the remaining unknown model parameters for each circuit were estimated from messenger RNA (mRNA) and protein measurements using multiobjective optimization. In particular, either the literature parameter estimates were used directly in the model simulations, or characteristic literature values were used to establish feasible ranges for the multiobjective parameter search. Next, global sensitivity analysis was used to determine the influence of individual model parameters on the expression dynamics. Taken together, the effective biophysical model captured the expression dynamics, including the transcription dynamics, for two synthetic cell free circuits. While we considered only two circuits here, this approach could potentially be extended to simulate other genetic circuits in both cell free and whole cell biomolecular applications. All model code, parameters, and analysis scripts are available for download under an MIT software license from the Varnerlab GitHub repository [33].

Materials and Methods

myTX/TL cell-free protein synthesis. The cell-free protein synthesis (CFPS) reactions were carried out using the myTXTL Sigma 70 Master Mix (Arbor Biosciences) in 1.5 mL Eppendorf tubes. The working volume of all the reactions was 12 μL, composed of the Sigma 70 Master Mix (9 μL) and the plasmids (3 μL total): P70a-deGFP (5 nM) for the single-gene system; P70a-deGFP-ssrA (8 nM), P70a-S28 (1.5 nM) and P28a-cI-ssrA (1 nM) for the negative feedback circuit. The plasmids were bought in lyophilized form (Arbor Biosciences) and purified using QIAprep Spin Miniprep Kit (Qiagen) using cell lines DH5-Alpha (for P28a-cI-ssrA) or KL740 (for P70a-deGFP, P70a-deGFP-ssrA, and P70a-S28). The CFPS reactions were incubated at 29°C for the desired time points.

mRNA and protein quantification. Following each CFPS run, the total RNA was extracted from 1 μL of the reaction mixture using PureLink RNA Mini Kit (Thermo Fisher Scientific) and stored at -80°C. The quantitative RT-PCR reactions were done using Applied Biosystems™ TaqMan™ RNA-to-CT™ 1-Step Kit and Custom TaqMan Gene Expression Assays (Thermo Fisher Scientific). At least three technical replicates were performed for each standard. The mRNA standards were prepared as follows: separate CFPS reactions for 5 nM of plasmids (P70a-S28, P70a-deGFP, and P70a-deGFP-ssrA) were carried out for up to 2 hours. Total RNA was extracted using the full reaction volume. This was followed by the removal of 16S and 23S rRNA using the MICROBExpress™ Bacterial mRNA Enrichment Kit (Life Technologies Corporation). Lastly, the MEGAclear™ Kit (Life Technologies Corporation) was used to further purify the mRNA. The concentration of cI-ssrA mRNA was quantified using the deGFP-ssrA mRNA standard. Green fluorescent protein (deGFP) fluorescence was measured using the Varioskan Lux plate reader at 488 nm (excitation) and 535 nm (emission). At the end of the CFPS run, 3 μL of the reaction mixture was diluted in 33 μL phosphate buffered saline (PBS) and stored at -80°C. The fluorescence was measured in triplicate with 10 μL each of this mixture.

Formulation and solution of model equations Consider a gene regulatory motif with genes $\mathcal{G} = 1, 2, \ldots, \mathcal{N}$. Each node in this motif is governed by two differential equations, one for mRNA (m_j) and the corresponding protein (p_j):

$$\dot{m}_j = r_{X,j} u_j - \theta_{m,j} m_j \quad j = 1, 2, \ldots, \mathcal{N} \tag{1}$$

$$\dot{p}_j = r_{L,j} w_j - \theta_{p,j} p_j \tag{2}$$

The term $r_{X,j} u_j$ in the mRNA balance, which denotes the rate of transcription for gene j, is the product of a kinetic limit $r_{X,j}$ (nmol h^{-1}) and a transcription control function $0 \leq u_j(\ldots) \leq 1$ (dimensionless). Similarly, the rate of translation of mRNA j, denoted by $r_{L,j} w_j$, is the also the product of the kinetic limit of translation and a translational control term $0 \leq w_j(\ldots) \leq 1$ (dimensionless). Lastly, $\theta_{\star,j}$ denotes the first-order rate constant (h^{-1}) governing degradation of protein and mRNA. The model equations, automatically generated using the JuGRN tool [34], were encoded in the Julia programming language [35] and solved using the `Rosenbrock23` routine of the `DifferentialEquations.jl` package [36].

Derivation of the transcription and translation kinetic limits. The key idea behind the derivation is that the polymerase (or ribosome) acts as a pseudo-enzyme, it binds a substrate (gene or message), reads the gene (or message), and then dissociates. Thus, we might expect that we could use a strategy similar to enzyme kinetics to derive an expression for $r_{X,j}$ (or $r_{L,j}$). Toward this idea, the kinetic limit of transcription $r_{X,j}$ was developed

by proposing a simple four elementary step model:

$$
\begin{aligned}
\mathcal{G}_j + R_X^{\circ} &\rightleftharpoons (\mathcal{G}_j : \mathrm{R_X})_C \\
(\mathcal{G}_j : \mathrm{R_X})_C &\longrightarrow (\mathcal{G}_j : \mathrm{R_X})_O \\
(\mathcal{G}_j : \mathrm{R_X})_O &\longrightarrow R_X + \mathcal{G}_j \\
(\mathcal{G}_j : \mathrm{R_X})_O &\longrightarrow m_j + R_X + \mathcal{G}_j
\end{aligned}
$$

where $\mathcal{G}_j$, R_X° denote the gene and *free* RNAP concentration, and $(\mathcal{G}_j : \mathrm{R_X})_O$, $(\mathcal{G}_j : \mathrm{R_X})_C$ denote the open and closed complex concentrations, respectively. Let the kinetic limit of transcription be directly proportional to the concentration of the open complex:

$$r_{X,j} = k_{E,j} \left(\mathcal{G}_j : \mathrm{R_X}\right)_O \tag{3}$$

where $k_{E,j}$ is the elongation rate constant for gene j. Then the functional form for $r_{X,j}$ can be derived:

$$r_{X,j} = V_{X,j}^{max} \left(\frac{\mathcal{G}_j}{\tau_{X,j} K_{X,j} + \left(1 + \tau_{X,j}\right) \mathcal{G}_j + \mathcal{O}_{X,j}} \right) \tag{4}$$

where $V_{X,j}^{max}$ denotes the maximum transcription rate (nM/h) of gene j and:

$$\mathcal{O}_{X,j} = \sum_{i=1,j}^{\mathcal{N}} \frac{K_{X,j}\tau_{X,j}}{K_{X,i}\tau_{X,i}} \left(1 + \tau_{X,i}\right) \mathcal{G}_i \tag{5}$$

denotes the coupling of the transcription of gene j with the other genes in the system through competition for RNA polymerase. In a similar way, we developed a model for the translational kinetic limit:

$$r_{L,j} = V_{L,j}^{max} \left(\frac{m_j}{\tau_{L,j} K_{L,j} + \left(1 + \tau_{L,j}\right) m_j + \mathcal{O}_{L,j}} \right) \tag{6}$$

where $V_{L,j}^{max}$ denotes the maximum translation rate (nM/h) and:

$$\mathcal{O}_{L,j} = \sum_{i=1,j}^{\mathcal{N}} \frac{K_{L,j}\tau_{L,j}}{K_{L,i}\tau_{L,i}} \left(1 + \tau_{L,i}\right) m_i \tag{7}$$

describes the coupling of the translation of mRNA j with other messages in the system because of kinetic competition for available ribosomes. The saturation and time constants for each case were defined as $K_{\star,j}^{-1} \equiv k_{+,j}/\left(k_{-,j} + k_{I,j}\right)$ and $\tau_{\star,j}^{-1} \equiv k_{I,j}/\left(k_{A,j} + k_{E,j}\right)$, respectively. In this study, the $\mathcal{O}_{X,j}$ and $\mathcal{O}_{L,j}$ terms were neglected as both circuits had either only one, or a small number of genes.

The maximum transcription rate $V_{X,j}^{max}$ was formulated as:

$$V_{X,j}^{max} \equiv \left[R_{X,T}\left(\frac{\dot{v}_X}{l_{G,j}}\right)\right] \tag{8}$$

The term $R_{X,T}$ denotes the total RNA polymerase concentration (nM), $\dot{v}_X$ denotes the RNA polymerase elongation rate (nt/h), $l_{G,j}$ denotes the length of gene j in nucleotides (nt). Similarly, the maximum translation rate $V_{L,j}^{max}$ was formulated as:

$$V_{L,j}^{max} \equiv \left[K_P R_{L,T}\left(\frac{\dot{v}_L}{l_{P,j}}\right)\right] \tag{9}$$

The term $R_{L,T}$ denotes the total ribosome pool, K_P denotes the polysome amplification constant, $\dot{v}_L$ denotes the ribosome elongation rate (amino acids per hour), and $l_{P,j}$ denotes the length of protein j (aa).

Control functions u and w. Values of the control functions $u\left(\ldots\right)$ and $w\left(\ldots\right)$ describe the regulation of transcription and translation. Ackers et al., borrowed from statistical mechanics to recast the $u\left(\ldots\right)$ function as the probability that a system exists in a configuration which leads to expression [37]. The idea of recasting $u\left(\ldots\right)$ as the probability of expression was also developed (apparently independently) by Bailey and coworkers in a series

of papers modeling the *lac* operon, see [38]. More recently, Moon and Voigt adapted a similar approach when modeling the expression of synthetic circuits in *E. coli* [39]. The $u(\ldots)$ function is formulated as:

$$u(\ldots)_j = \frac{\sum_{i \in \{\chi\}} W_i f_i(\ldots)}{\sum_{j \in \mathcal{C}_j} W_j f_j(\ldots)} \tag{10}$$

where W_i (dimensionless) denotes the weight of configuration i, while $f_i(\cdots)$ (dimensionless) is a binding function which describes the fraction of bound activator/inhibitor for configuration i. The summation in the numerator is over the set of configurations that lead to expression (denoted as χ), while the summation in the numerator is over the set of all possible configurations for gene j (denoted as $\mathcal{C}_j$). Thus, $u(\ldots)_j$ can be thought of as the fraction of all possible configurations that lead to expression. Thus, weights W_i are the Gibbs energy of configuration i given by: W_i = $\exp(-\Delta G_i/RT)$ where ΔG_i denotes the molar Gibbs energy for configuration i (kJ/mol), R denotes the ideal gas constant (kJ mol^{-1} K^{-1}), and T denotes the system temperature (Kelvin).

The translational control function $w(\ldots)$ described the loss of translational activity in the cell free reactions. Loss of translational activity could be a function of many factors, including depletion of metabolic resources. However, in this study we assumed a translational activity factor ϵ of the form:

$$\dot{\epsilon} = -\left(\frac{\ln(2)}{\tau_{L,1/2}}\right)\epsilon \tag{11}$$

where $\tau_{L,1/2}$ denotes the half-life of translation (estimated from the experimental data) and $\epsilon(0)$ = 1. The translational control variable was then given by $w_i = \epsilon$ for all proteins in the circuits.

Estimation of model parameters Model parameters were estimated using the Pareto Optimal Ensemble Technique in the Julia programming language (JuPOETs) [40]. Several of the model parameters were estimated directly from literature, or were set by experimental conditions (Table 1). However, the remaining unknown parameters were estimated from mRNA and protein concentration measurements using JuPOETs. JuPOETs is a multiobjective optimization approach which integrates simulated annealing with Pareto optimality to estimate parameter ensembles on or near the optimal tradeoff surface between competing training objectives. JuPOETs minimized training objectives of the form:

$$\mathcal{E}_j(\mathbf{k}) = \sum_{i=1}^{\mathcal{T}_j} \left(\hat{\mathcal{M}}_{ij} - \hat{y}_{ij}(\mathbf{k}) \right)^2 \tag{12}$$

subject to the model equations, initial conditions, parameter bounds $\mathcal{L} \leq \mathbf{k} \leq \mathcal{U}$ and maximum value constraints for some unmeasured protein species. The objective function(s) $\mathcal{E}_j$ measured the squared difference between the model simulations and experimental measurements for training objective j. The symbol $\hat{\mathcal{M}}_{ij}$ denotes an experimental observation at time index i for training objective j, while the symbol $\hat{y}_{ij}$ denotes the model simulation output at time index i from training objective j. The quantity i denotes the sampled time-index and $\mathcal{T}_j$ denotes the number of time points for experiment j. For the P70-deGFP model, $\mathcal{E}_1$ corresponded to mRNA deGFP, while $\mathcal{E}_2$ corresponded to the deGFP protein concentration. On the other hand, for the full model, $\mathcal{E}_1$ corresponded to mRNA deGFP-ssrA, $\mathcal{E}_2$ to mRNA σ_{28}, $\mathcal{E}_3$ to mRNA C1-ssrA and $\mathcal{E}_4$ to the deGFP-ssrA concentration. Lastly, we penalized non-physical accumulation of the cI-ssrA protein (unmeasured) with a term of the form: $\mathcal{E}_5$ = $C \times \max(0, \hat{y} - U)$ where C denotes a penalty parameter (C = 1×10^5), $\hat{y}$ denotes the maximum cI-ssrA concentration, and U denotes a concentration upper bound (U = 100nM). For the P70-deGFP model, 11 parameters were estimated from data, while 33 parameters were estimated for the full model.

Parameter bounds $\mathcal{L} \leq \mathbf{k} \leq \mathcal{U}$ were established from literature, or from previous experimental or model analysis; parameter values estimated for the P70-deGFP model were also used to establish ranges for the full model. JuPOETs searched over ΔG_i, K_L and $\tau_{L,1/2}$ values directly, while other unknown parameter values took the form of corrections to order of magnitude characteristic literature estimates. For example, we set the mRNA degradation rate constant (θ_m) to a characteristic value taken from literature. Then, the degradation constant for any particular mRNA was represented as: $\theta_{m,i} = \alpha_i \theta_m$, where α_i was an unknown (but bounded) modifier. In this way, we guaranteed the parameter search (and the resulting estimated parameters) were within a specified range of literature values. We used this procedure for all degradation constants (both mRNA and protein) and all time constants (for both transcription and translation). The baseline parameter values are given in Table 1. JuPOETs was run for 20 generations for both models, and all parameters sets with Pareto rank less than or equal to two were collected for each generation. The JuPOETs parameter estimation routine for the models in this study is encoded in the `sa_poets_estimate.jl` script in the model repositories.

Global sensitivity analysis Morris sensitivity analysis was used to understand which model parameters were sensitive [41]. The Morris method is a global method that computes an elementary effect value for each parameter by sampling a model performance function, in this case the area under the curve for each model species, over a range of values for each parameter; the mean of elementary effects measures the direct effect of a particular parameter on the performance function, while the variance of each elementary effect indicates whether the effects are non-linear or the result of interactions with other parameters (large variance suggests connectedness or nonlinearity). The Morris sensitivity coefficients were computed using the `DiffEqSensitivity.jl` package [36]. The parameter ranges were established by calculating the minimum and the maximum value for each parameter in the parameter ensemble generated by JuPOETs. Each range was

then subdivided into 10,000 samples for the sensitivity calculation. The Morris sensitivity coefficients are calculated using the `compute_sensitivity_coefficients.jl` script in the model repositories.

Results

The two cell free genetic circuits used in this study were based upon a regulatory system composed of bacterial sigma factors (Fig. 1). Sigma factor 70 (σ_{70}), endogenously present in the extract, was the primary driver of each circuit. First, σ_{70} induced green fluorescent protein (deGFP) expression was explored in the absence of additional regulators or protein degradation (Fig. 1A). Next, building upon this simple circuit, a more complex case was studied which involved additional regulators and specific protein degradation induced by C-terminal ssrA tags (Fig. 1B). In particular, σ_{70} induced the expression of genes downstream of the P70a promoter: sigma factor 28 (σ_{28}) and deGFP-ssrA. As σ_{28} was synthesized, it then induced the expression of the lambda phage repressor protein cI-ssrA, which was under the σ_{28} responsive P28 promoter. The cI-ssrA protein repressed the P70a promoter via interaction with its OR2 and OR1 operator sites, thereby down-regulating σ_{28} and deGFP-ssrA transcription [42]. Simultaneously, the C-terminal ssrA degradation tag present on the deGFP and cI repressor proteins was recognized by the endogenous ClpXP protease system in the cell free extract, thereby promoting the degradation of these proteins into peptide fragments [43, 44]. In addition, messenger RNAs (mRNAs) were always subject to degradation due to the presence of degradation enzymes in the extract [44, 45]. Taken together, the interactions of the components manifested in an accumulation of deGFP protein for the first circuit, and a pulse signal of deGFP-ssrA in the second. Studying the P70 - deGFP circuit allowed us to estimate parameters governing the interaction of σ_{70} with the P70a promoter. On the other hand, the second circuit allowed us to characterize the interaction of σ_{28} with the P28 promoter, the strength of the transcriptional repression by cI-ssrA, and the kinetics of protein degradation by the endogenous ClpXP protease system. Finally, both circuits allowed us to test our effective biophysical models of transcription and translation rates.

The effective biophysical transcription and translation model captured σ_{70} induced

deGFP expression at the message and protein level (Fig. 2). JuPOETs produced an ensemble (N = 140) of the 11 unknown model parameters which captured the transcription of mRNA (Fig. 2A) and the translation of the deGFP protein (Fig. 2B) to within experimental error. The mean and standard deviation of key parameters is given in Table 2. The deGFP mRNA concentration reached a maximum value of approximately 580 nM within two hours, and stayed at this level for the remainder of the cell free reaction. Thus, the approximate steady state, given that mRNA degradation was continuously occurring over the course of the reaction (the mean lifetime of deGFP mRNA was estimated to be approximately 27 min, which was similar to the 17 - 18 min value reported by Garamella et al [44]), suggested continuous transcriptional activity. On the other hand, the deGFP protein concentration increased more slowly, and began to saturate between 8 to 10 hr at a concentration of approximately 15 μM. Given there was negligible protein degradation (the mean deGFP half-life was estimated to be $\sim$11 days, which was similar to the value of 6 days estimated by Horvath et al, albeit in a different cell free system [32]), the saturating protein concentration suggested that the translational capacity of the cell free system decreased over the course of the reaction. The decrease in translational capacity, which could stem from several sources, was captured in the simulations using a monotonically decreasing translation capacity state variable ϵ, and the translational control variable $w\left(\ldots\right)$. In particular, the mean half-life of translational capacity was estimated to be $\tau_{L,1/2} \sim$ 4 h in the P70-deGFP circuit experiments. Taken together, JuPOETs produced an ensemble of model parameters that captured the experimental training data. Next, we considered which P70 - deGFP model parameters were important to the model performance using global sensitivity analysis.

The importance of the P70 - deGFP model parameters was quantified using Morris sensitivity analysis (Fig. 2B). The Morris method computes an elementary effect value for each parameter by sampling a model performance function, in this case the area under

the curve for each model species, over a range of parameter values; the mean of elementary effects measures the direct effect of a particular parameter on the performance function, while the variance indicates whether the effects are non-linear or the result of interactions with other parameters (large variance suggests connectedness or nonlinearity). Four parameters (the translation saturation coefficient K_L, the translational capacity half-life $\tau_{L,1/2}$, the translation time constant and the protein degradation constant) influence the protein level only. On the other hand, six parameters influenced both mRNA and protein abundance; all six of these parameters were either directly or indirectly associated with transcription. Thus, these parameters influenced the production or stability of mRNA which in turn influenced the protein level. In particular, the mRNA degradation constant and the cooperativity of σ_{70} in the P70a promoter function had the largest direct effect and variance. Surprisingly, the ΔG of σ_{70}/RNAP/promoter configuration was sensitive, but less important than other parameters, and the variance of the elementary effect was small. Taken together, Morris sensitivity analysis of the P70 - deGFP model parameters highlighted the hierarchical structure of the transcriptional and translation model, suggesting experimentally tunable parameters such as mRNA stability were globally important. Next, we used the ensemble of P70a, time constant and degradation parameters estimated for the P70 - deGFP model to constrain the analysis of the more complicated circuit.

The effective biophysical transcription and translation model captured the deGFP-ssrA expression dynamics in the negative feedback circuit (Fig. 3A). JuPOETs produced an ensemble (N = 498) of the 33 unknown model parameters which captured transcription and translation dynamics for σ_{28}, cI-ssrA and deGFP-ssrA. The mean and standard deviation of key parameters is given in Table 3. Compared with the estimated parameters for the P70-deGFP model, the negative feedback circuit model had a higher (approximately two-fold) half life of translation and a lower (approximately two-fold) translation

saturation coefficient. Similarly, there were variations in the values of the transcription and translation time constants for the two systems. Despite the differences, however, the low values of the transcription and translation time constants in both cases qualitatively suggest suggest elongation limited reactions. Unlike the previous P70 - deGFP case, the mRNA expression pattern for σ_{28} and deGFP-ssrA both showed an initial spike, to concentrations similar with the previous pseudo steady state, before the cI-ssrA regulator protein could be expressed. However, after the cI-ssrA protein began to accumulate, the concentrations of the regulated mRNAs dropped by approximately an order of magnitude compared with the unregulated case. Again, as was true previously, the regulated mRNA concentrations reached an approximate steady-state suggesting simultaneous transcription and mRNA degradation. The mean estimated mRNA lifetime for cI-ssrA and deGFP were similar (approximately 16 min), while the degradation of σ_{28} mRNA was predicted to be slower (mean mRNA lifetime was estimated to be approximately 30 min). The secondary effect of cI-ssrA repression was visible in the cI-ssrA mRNA expression pattern. In particular, cI-ssrA expression is induced by σ_{28}, however, σ_{28} expression is repressed by cI-ssrA, thereby completing a negative feedback loop. Initially, before appreciable levels of cI-ssrA had been translated, the cI-ssrA transcription rate was maximum (approximately 200 nM/h). However, the transcription rate decreased and after approximately two hours it remained constant (approximately 12 nM/h) for the remainder of the cell free reaction. Similarly, transcription rates for σ_{28} (approximately 1200 nM/h) and deGFP-ssrA (approximately 750 nM/h) were initially at a maximum due to the presence of endogenous σ_{70}, but they quickly dropped as cI protein accumulated. Protein synthesis followed a similar trend, with the translation rates for σ_{28} and deGFP-ssrA initially present at their maximum values before quickly dropping. After one hour, deGFP levels reached a peak and decayed due to the ClpXP-mediated degradation whereas σ_{28} protein levels continued to slowly rise at a steady rate (approximately 15 nM/h). The negative feedback model also predicted the

expected lag present during the initial phase of cI-ssrA protein synthesis due to the need for σ_{28} protein to reach appreciable levels. Moreover, the high levels of cI-ssrA mRNA (expressed because σ_{28} does not have a degradation tag) led to the accumulation of the cI-ssrA protein. However, the cI-ssrA protein concentration, could not be verified because we did not have experimental measurements of this species. Taken together, the effective biophysical transcription and translation rates, in combination with the control variables u and w, simulated cell free expression dynamics. Next, we considered which model parameters were important using global sensitivity analysis.

Global sensitivity analysis described the influence of the model parameters on the performance of the negative feedback circuit (Fig. 3B). The influence of 33 parameters was computed using the area under the curve of each mRNA and protein species as the performance function. The Morris sensitivity measures (mean and variance) were binned into categories based upon their relative magnitudes, from no influence (white) to high influence (black). Some parameters affected only their respective mRNA or protein target, whereas others had widespread effects. For example, the time constant (tc) modifiers, stability of deGFP-ssrA protein and mRNA, and the binding dissociation constant (K) and cooperativity parameter (n) of cI-ssrA and σ_{70} for deGFP-ssrA affected only the values of deGFP-ssrA protein and mRNA. On the other hand, the tc, stability, K and n parameters for σ_{70}, σ_{28}, or cI-ssrA influenced mRNA and protein expression globally. The σ_{70} and σ_{28} proteins acted as inducers or repressors for more than one gene product: σ_{70} induced both deGFP-ssrA and σ_{28}, and cI-ssrA protein repressesed both of these genes. Degradation constants (denoted as stability) affected the half-lives of the transcribed message or the translated protein in the mixture, while the time constant modifiers changed the time required to form the open gene complex (or translationally active complex). Dissociation and cooperativity constants affected the binding interactions of the inducer (or repressor in the case of cI-ssrA) in the promoter control function. Varying these parameters, there-

fore, had a strong effect on their respective targets. Similarly, the translation saturation and its half-life, which captured the depletion in the translation activity over the course of the reaction, not only affected protein levels but also mRNA levels. This is because these parameters tuned the rate of formation of cI-ssrA, which in turn affected the mRNA levels of its gene targets. Given that cI-ssrA was the main regulator (repressor) of the circuit, the parameters that dictated the levels of cI-ssrA mRNA and protein had a global effect. We also observed high variance (connectedness) for several parameters, in particular involving cI-ssrA. For example, the time constant modifiers for cI-ssrA mRNA and protein had a two-pronged effect. On the one hand, they positively influenced the transcription/translation rates of the gene and mRNA products, directly increasing the cI-ssrA protein. On the other hand, increased cI-ssrA expression could reduce the level of σ_{28}, in turn reducing the cI-ssrA levels over time. Taken together, Morris sensitivity analysis of the negative feedback circuit model parameters suggested that the parameters governing the synthesis rates of the cI-ssrA mRNA and protein were globally important.

Discussion

In this study, we developed an effective biophysical model of transcription and translation processes, and analyzed the influence of different parameters upon model performance. We modeled two constructs, a simple P70-deGFP expression module, and a more complex negative feedback circuit consisting of regulators and ssrA tagged proteins. We first studied the P70 - deGFP module and estimated the time constants, degradation rates, and the parameters governing the interaction of σ_{70} with the P70a promoter, and then used these parameter estimates to constrain the larger negative feedback circuit. We also considered which model parameters were important for both modules using global sensitivity analysis. Our results for the complex circuit showed that the system was most sensitive to the parameters that dictated the levels of cI-ssrA mRNA and protein. In addition, we observed several high variance terms that described connectedness in the system; these terms mainly arose from the nature of the negative feedback circuit. Importantly, for our model formulation, we used parameter values derived from experiments and those having a physiological basis whenever possible. For example, the characteristic value for τ_X, the time constant for transcription, was approximated from a series of *in vitro* experiments using an abortive initiation assay by McClure [46]. Similarly, the weight values appearing in the transcription control function $u(...)$ were physiologically based, in particular upon the Gibbs energies of the respective promoter configurations. The transcriptional control functions themselves, formulated as the fraction of all possible configurations that lead to expression, were based upon a statistical mechanical description of promoter activity, developed from the earlier study of Ackers et al [37]. Thus, values of transcriptional control variables varied continuously between 0 and 1, thus conforming to the smooth (rather than step-like) nature of the regulatory output observed physiologically. Taken together, the proposed effective biophysical model formulation captured the transcription and translation dynamics of synthetic regulatory networks in a cell free system.

The effective TX/TL model described the experimental mRNA and protein training data. However, there were several important questions to address in future studies. First, the model formulation described the data, but did not predict dynamics outside of the training set. If this approach is to be useful to the synthetic biology community, or more broadly as an effective biophysical technique to model gene expression dynamics in *in vivo* applications such as regulatory flux balance analysis, we need to have confidence that the modeling approach is predictive. Thus, while we have established a descriptive model, we have to yet to establish a predictive model. Next, there were several technical or mechanistic questions that should be addressed further. For example, we used a first order approximation of ClpXP mediated protein degradation, while Garamella et al. [44] described this degradation as zero order. Similarly, we did not establish the concentration of ClpXP in the commercially available cell free reaction mixture. The levels of this protein complex could be an important factor controlling protein degradation. Next, we should compare the current modeling approach, and the values estimated for the model parameters, with the study of Marshall and Noireaux [31]. For example, one of the potential limitations of the current study is that we did not consider a separate species for dark GFP. In our previous RNA circuit modeling [47], we did include this term, but failed to do so here. We expect inclusion of a dark versus light GFP species could influence the values for the estimated parameters, particularly the translation time constants. However, previous reports suggested the *in vitro* maturation time of deGFP was approximately 8 min [48], much faster than the typical maturation times for GFP of one hour *in vivo* [49, 50]. Thus, the impact of including a dark and light GFP species may not be worth increased model complexity. Lastly, we should validate the values estimated for the binding function parameters and the promoter configuration free energies. Maeda et al measured the binding affinities of the seven *E. coli* σ factors, which are related to the parameters appearing in the promoter binding functions [51]. There have also been several studies

that have quantified the binding energies of promoter configurations e.g., [37, 52–54]. A perfunctory inspection of the values estimated in this study suggested our estimated free energy values, while the same order of magnitude as previous studies, were off by a factor of up to an order of magnitude compared with other characteristic literature studies (albeit for different promoters). Thus, these other studies could serve as a basis to validate our estimates, and perhaps more importantly constrain the parameter search space for future studies.

The effective TX/TL modeling approach described here has several potential applications. For example, a challenge of *in vivo* constraint based modeling is the description of gene expression [55]. Boolean and probabilisitic approaches [56–58] have largely addressed this challenge. However, the transcriptional state of a boolean gene is either on or off based on the state of its regulators, thus, gene expression is course grained. The current approach could be an interesting biophysical alternative to a boolean description that utilizes a continuous description of transcriptional regulation. In particular, the rules that encode these boolean networks are easily translated into the rational promoter functions described here, however, estimation of the parameters that appear in the promoter functions remains an open question. Another interesting application could be the integration of gene expression with metabolism in cell free synthetic biology applications. For example, synthetic biology models often neglect the integration of the expression of the synthetic circuit with metabolism e.g., [47]. Ultimately, metabolism is centrally important to the operation of any synthetic circuit; gene expression is strongly dependent upon metabolic resources. We have recently started to explore this question by integrating biophysical transcription and translation models with metabolic networks in cell free reactions e.g., [32, 59]. These initial studies provided insights into the potential limitations constraining synthetic circuit operation, and discussed strategies for improving performance metrics such as carbon yield, energy efficiency and productivity of cell free protein production.

A natural extension to these studies would be to use more sophisticated modules such as a genetic circuit like the one used in this study, RNA-based regulation or transcription factor-based biosensors. Taken together, the approach described here can potentially be used as an addition to metabolic models to provide systems-level detail.

Conflict of Interest Statement

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Author Contributions

J.V directed the study. M.V, S.V, H.L and A.A conducted the cell free experimental measurements. J.V, M.V and A.A developed the reduced order models and the parameter ensemble. M.V, A.A and J.V analyzed the model ensemble, and generated figures for the manuscript. The manuscript was prepared and edited for publication by A.A, M.V, S.V and J.V. All authors reviewed this manuscript.

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Data Availability Statement

Model code is available under an MIT software license from the Varnerlab GitHub repository [33]. The mRNA and protein measurements presented in this study are available in the data directory of the model repositories in comma separated value (CSV) and Microsoft Excel format.

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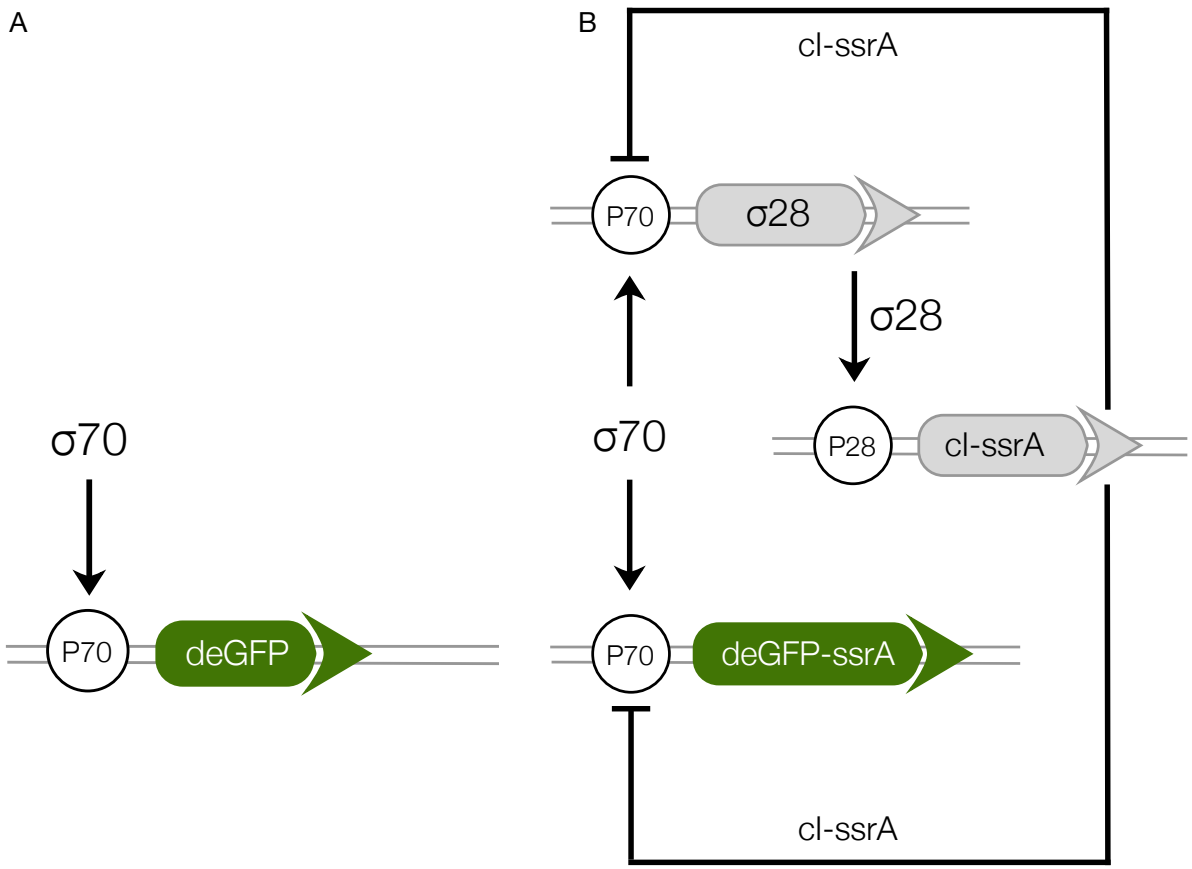

Fig. 1: Schematic of the cell free gene expression circuits used in this study. **A**: Sigma factor 70 (σ_{70}) induced expression of deGFP. **B**: The circuit components encode for a negative feedback loop motif. Sigma factor 28 and deGFP-ssrA genes on the P70a promoters are expressed first because of the endogenous presence of sigma 70 factor in the extract. Sigma factor 28, once expressed, induces the P28a promoter, turning on the expression of the cI-ssrA gene which represses the P70a promoter. The circuit is modified from a previous study [44] by including an ssrA degradation tag on the cI gene.

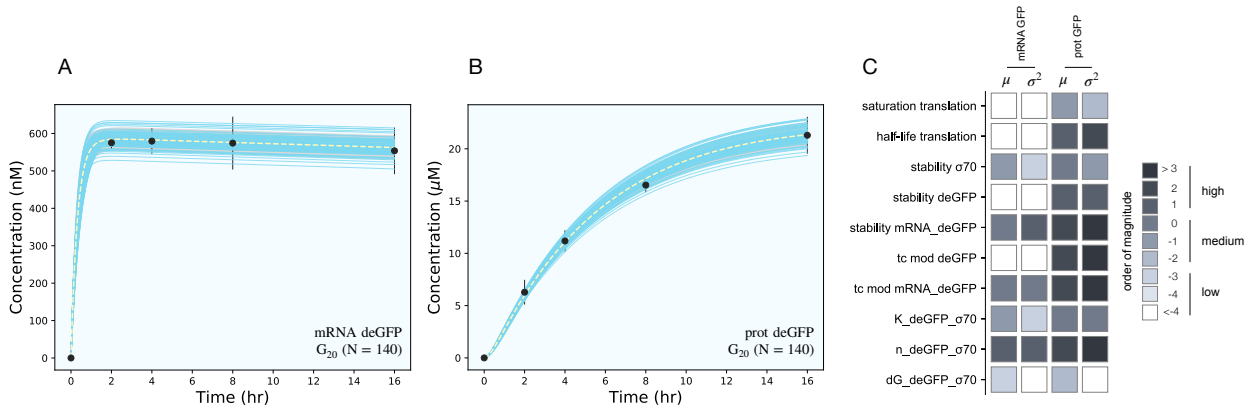

Fig. 2: Model simulations versus experimental measurements for σ_{70} induced deGFP expression. **A**: Simulated and measured deGFP mRNA concentration versus time using the small circuit G_{20} ensemble (N = 140). **B**: Simulated and measured deGFP protein concentration versus time the small circuit G_{20} ensemble (N = 140). **C**: Global sensitivity analysis of the P70-deGFP circuit parameters. Morris sensitivity coefficients were calculated for the unknown model parameters, where the range for each parameter was established from the ensemble. Uncertainty: An ensemble of parameter sets was estimated using multiple generations of the POETs algorithm. Simulations and uncertainty quantification are shown for the generation 20 (G_{20}) which yielded N = 140 parameter sets that were rank two or below. The parameter ensemble was used to calculate the mean (dashed line) and the 95% confidence estimate of the simulation (gray region). Individual parameter set trajectories are shown in blue. Points denote the mean experimental measurement while error bars denote the 95% confidence estimate.

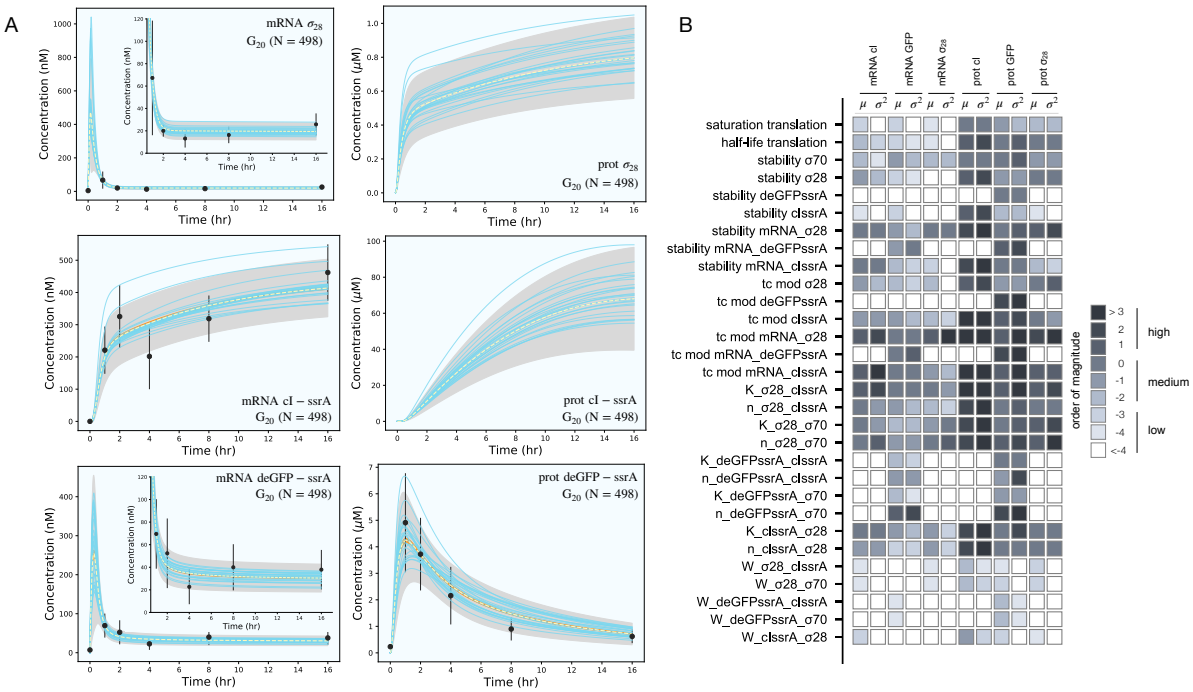

Fig. 3: Model simulations versus experimental measurements for the negative feedback deGFP-ssrA circuit. **A**: Model simulations of the negative feedback deGFP-ssrA circuit using the G_{20} ensemble (N = 498). Uncertainty: Simulations and uncertainty quantification are shown for the generation 20 (G_{20}) which yielded N = 489 parameter sets (rank two or below). The parameter ensemble was used to calculate the mean (dashed line) and the 99% confidence estimate of the simulation (gray region). Individual parameter set trajectories are also shown in blue. Points denote the mean experimental measurement while error bars denote the 95% confidence estimate of the experimental mean. **B**: Global sensitivity analysis of the negative feedback deGFP-ssrA circuit parameters. Morris sensitivity coefficients were calculated for the unknown model parameters, where the range for each parameter was established from the ensemble.

Table 1: Characteristic parameters for TX/TL model equations. Key to references used in the table: (a) [44], (b) [60], (c) set by experiment, (d) [61], (e) estimated in this study, (f) [62], (g) [63], (h) calculated from plasmid sequence, (i) [46], (f) [59]

Description	Parameter	Value	Units	Reference
RNA polymerase concentration	$R_{X,T}$	0.06-0.075	μM	*a*
Ribosome concentration	$R_{L,T}$	$<$ 2.3	μM	*a,b*
σ_{70} concentration	σ_{70}	$<$35	nM	*a*
initial σ_{28} concentration	σ_{28}	$<$20	nM	*a*
Transcription elongation rate	$\dot{v}_X$	12-30	nt/s	*a,d*
Translation elongation rate	$\dot{v}_L$	1-2	aa/s	*a,b*
Transcription saturation coefficient	K_X	0.036	μM	*i*
Polysome amplification constant	K_P	10.0	constant	*e*
Transcription initiation time	k_I^X	22	s	*i*
Translation initiation time	k_I^L	1.5	s	*e*
Default mRNA degradation coefficient	θ_m	3.75	h^{-1}	*a*
Default protein degradation coefficient	θ_p	0.462-1.89	h^{-1}	*f,g*
Gene concentration σ_{28}		1.5	nM	*c*
Gene concentration cI-ssrA		1.0	nM	*c*
Gene concentration deGFP-ssrA		8.0	nM	*c*
Gene length σ_{28}		811	nt	*h*
Gene length cI-ssrA		850	nt	*h*
Gene length deGFP-ssrA		782	nt	*h*
Protein length σ_{28}		240	aa	*h*
Protein length cI-ssrA		248	aa	*h*
Protein length deGFP-ssrA		237	aa	*h*

Table 2: Estimated parameter values for the P70-deGFP model. The mean and standard deviation of each parameter value was calculated over the ensemble of parameter sets meeting the rank selection criteria (N = 139).

Description	**Parameter**	**Value** ($\mu \pm \sigma$)	**Units**
Translation saturation coefficient	K_L	483.13 $\pm$ 10.10	μM
half-life translation	$\tau_{L,1/2}$	4.03 $\pm$ 0.031	h^{-1}
Time constants			
deGFP transcription	$\tau_{\text{X,GFP}}$	0.61 $\pm$ 0.04	dimensionless
deGFP translation	$\tau_{\text{L,GFP}}$	0.16 $\pm$ 0.003	dimensionless
mRNA and protein half-life			
mRNA deGFP	$\ln(2)/\theta_{m,GFP}$	13.5 $\pm$ 2.47	min
protein deGFP	$\ln(2)/\theta_{p,GFP}$	10.86 $\pm$ 0.78	days
protein σ_{70}	$\ln(2)/\theta_{p,\sigma_{70}}$	3.65 $\pm$ 0.17	days
Free energies			
RNAP + deGFP gene	$\Delta G_{\text{GFP,RX}}$	28.82 $\pm$ 1.75	kJ mol^{-1}
RNAP + σ_{70} + deGFP gene	$\Delta G_{\text{GFP},\sigma_{70}}$	-20.38 $\pm$ 1.91	kJ mol^{-1}

Table 3: Estimated parameter values for the negative feedback circuit. The mean and standard deviation for each parameter was calculated over the ensemble of parameter sets (N = 498).

Description	**Parameter**	**Value** ($\mu \pm \sigma$)	**Units**
Translation saturation coefficient	K_L	253.75 $\pm$ 14.12	μM
half-life translation	$\tau_{L,1/2}$	8.86 $\pm$ 0.85	h^{-1}
Time constants			
cI-ssrA transcription	$\tau_{\text{X,cI}}$	$<$ 0.001	dimensionless
deGFP transcription	$\tau_{\text{X,GFP}}$	0.045 $\pm$ 0.003	dimensionless
σ_{28} transcription	$\tau_{\text{X},\sigma_{28}}$	0.0018 $\pm$ 0.0003	dimensionless
cI-ssrA translation	$\tau_{\text{L,cI}}$	0.054 $\pm$ 0.004	dimensionless
deGFP translation	$\tau_{\text{L,GFP}}$	0.058 $\pm$ 0.007	dimensionless
σ_{28} translation	$\tau_{\text{L},\sigma_{28}}$	1.1 $\pm$ 0.13	dimensionless
mRNA and protein half-life			
mRNA cI-ssrA	$\ln(2)/\theta_{m,cI}$	8.1 $\pm$ 0.60	min
mRNA deGFP	$\ln(2)/\theta_{m,GFP}$	7.74 $\pm$ 1.13	min
mRNA σ_{28}	$\ln(2)/\theta_{m,\sigma_{28}}$	14.96 $\pm$ 1.60	min
protein cI-ssrA	$\ln(2)/\theta_{p,cI}$	0.46 $\pm$ 0.043	days
protein deGFP-ssrA	$\ln(2)/\theta_{p,GFP}$	0.051 $\pm$ 0.002	days
protein σ_{28}	$\ln(2)/\theta_{p,\sigma_{28}}$	7.65 $\pm$ 0.91	days
protein σ_{70}	$\ln(2)/\theta_{p,\sigma_{70}}$	14.86 $\pm$ 2.30	days
Free energies			
RNAP + cI gene	$\Delta G_{\text{cI,RX}}$	46.57 $\pm$ 4.28	kJ mol^{-1}
RNAP + σ_{28} + cI gene	$\Delta G_{\text{cI},\sigma_{28}}$	-1.10 $\pm$ 0.04	J mol^{-1}
RNAP + deGFP gene	$\Delta G_{\text{GFP,RX}}$	41.94 $\pm$ 1.80	kJ mol^{-1}
RNAP + σ_{70} + deGFP gene	$\Delta G_{\text{GFP},\sigma_{70}}$	-27.67 $\pm$ 1.79	kJ mol^{-1}
RNAP + cI + deGFP gene	$\Delta G_{\text{GFP,cI}}$	-7.21 $\pm$ 1.14	kJ mol^{-1}
RNAP + σ_{28} gene	$\Delta G_{\sigma_{28},\text{RX}}$	46.67 $\pm$ 3.18	kJ mol^{-1}
RNAP + σ_{70} + σ_{28} gene	$\Delta G_{\sigma_{28},\sigma_{70}}$	-10.46 $\pm$ 1.15	kJ mol^{-1}
RNAP + cI + σ_{28} gene	$\Delta G_{\sigma_{28},\text{cI}}$	-12.89 $\pm$ 1.44	kJ mol^{-1}