

1 **Polar Gini Curve: a Technique to Discover Single-cell Biomarker**
2 **Using 2D Visual Information**

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9 **Running title:** *Nguyen et al / Polar Gini Curve single cell*

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16

17 Total word counts: 3446

18 Total figures: 10

19 Total tables: 0

20 Total supplementary figures: 0

21 Total supplementary tables: 0

22 Total supplementary files: 3

23

24 **Abstract**

25 In this work, we design the Polar Gini Curve (PGC) technique, which combines the gene
26 expression and the 2D embedded visual information to detect biomarkers from single-cell data.
27 Theoretically, a Polar Gini Curve characterizes the shape and ‘evenness’ of cell-point distribution
28 of cell-point set. To quantify whether a gene could be a marker in a cell cluster, we can combine
29 two Polar Gini Curves: one drawn upon the cell-points expressing the gene, and the other drawn
30 upon all cell-points in the cluster. We hypothesize that the closers these two curves are, the more
31 likely the gene would be cluster markers. We demonstrate the framework in several simulation
32 case-studies. Applying our framework in analyzing neonatal mouse heart single-cell data, the
33 detected biomarkers may characterize novel subtypes of cardiac muscle cells. The source code and
34 data for PGC could be found at https://figshare.com/projects/Polar_Gini_Curve/76749.

35

36 **KEYWORDS:** Single-cell gene expression; Gini coefficient; Polar Gini Curve; Biomarker

37

38 **Introduction**

39 Discovering biomarkers from the single-cell gene expression data is an interesting yet
40 challenging problem [1]. Compared to the well-established bulk gene expression data, the
41 expression distribution in single-cell is significantly more heterogeneous [2-4]. Therefore, as
42 shown in [5, 6], the bulk-analysis strategies [7, 8] achieve low sensitivity in detecting markers. In
43 addition, as embedding [9-11] and clustering [12-14] are the essential components in many single-
44 cell expression analytical pipelines [15, 16], the biomarker detection techniques would need to
45 tackle the challenges and errors from embedding and clustering [17, 18].

46

47 From the statistical point of view, there are two different directions among the current state-
48 of-the-art methods in solving the single-cell biomarker discovery problem. The first direction is
49 using non-parametric approaches [19]. Non-parametric approaches do not attempt to construct the
50 model characterizing the gene expression distribution [20]. They do not require too many prior
51 assumptions about the expression data. Therefore, in theory, they could be applied in most of the
52 heterogeneous scenarios in single-cell expression. For example, Seurat [16] and the SINCERA
53 [21] pipelines use the Mann–Whitney test [22]. The disadvantages of non-parametric approaches
54 include lacking the point-estimator (for example, we could not tell how much of fold-change when

55 comparing the expressions of the same gene in two populations) and the lower true positive rate
56 [5, 6]. On the other hand, the parametric approaches model the underlying expression distribution.
57 For example, [23] applies Bayesian statistics, Monocle2 [11, 24] and MAST [2] apply different
58 linear models, and [25] applies the Poisson models to single-cell differential expression analysis.
59 The parametric approaches, compared to the non-parametric ones, are significantly more sensitive
60 [5, 6], especially in detecting markers in small cell-cluster since they may require less number of
61 cell-samples. However, these approaches assume that the gene expression distribution has specific
62 shapes; therefore, these approaches tend to have higher false-positive rates.

63

64 In this work, we developed a new framework based on the novel idea of integrating expression
65 and the embedded visual information of single-cell data into one metric to identify biomarkers.
66 This idea has been successfully implemented in spatial single-cell data, in which the visualization
67 space reflects the relative position of the cells in a tissue image [26, 27]. In this framework, we
68 decided to take advantage of cluster shape and cell-point distribution from the 2D visual space.
69 Our strategy was to project the single-cell 2D cluster onto multiple angle-axes to explore all
70 viewing angles of the cluster. On each ‘viewing angle’, we captured the visual distribution using
71 the Gini coefficient [28]. Together, for each set of points in 2D, we constructed a Polar Gini Curve
72 (PGC) from the correspondent between viewing angle and Gini coefficient. We hypothesized that
73 for the marker gene, its expressing cell set should have its PGC close to the PGC computed from
74 the whole cluster cell-set. We demonstrated the framework in several simulation case-studies.
75 Applying our framework in analyzing neonatal mouse heart single-cell data [29], the detected
76 biomarkers may characterize novel subtypes of cardiac muscle cells. We named the framework
77 PGC-RSMD (Polar Gini Curve – Root Mean Square Deviation). The source code and dataset,
78 including supplemental data, used in this manuscript could be found in
79 https://figshare.com/projects/Polar_Gini_Curve/76749.

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81

82 **Material and Method**

83 **Computing PGC-RSMD for one gene in one cluster**

84 **Figure 1** demonstrates the workflow to compute PGC-RSMD for one gene in a cell cluster
85 from the single-cell expression data. Our approach used the 2D embedding [9] and clustering

86 results from single-cell expression data as the input. Starting from the 2D x - y embedding space,
87 for an arbitrary angle θ , the pipeline projects the x - y coordinate [30] *for every cell-point* onto the
88 θ -axis (z score)

$$89 \quad z = x\cos(\theta) + y\sin(\theta) \quad (1)$$

90 We subtracted the scores from (1) with the smallest z to ensure that all z scores are non-negative,
91 which is the requirement for computing the Gini coefficient. Then, it computed two Gini
92 coefficients g_{sub} and g_{whole} to measure the inequality among the z scores. The g_{sub} coefficient only
93 used the distribution of z scores obtained from cells expressing the gene. The g_{whole} coefficient
94 would use the distribution of all z scores. The Gini coefficient formula is as in [28]

$$95 \quad g = \frac{1}{2n^2\bar{z}} \sum_{i=1}^n \sum_{j=1}^n |z_i - z_j| \quad (2)$$

96 Here, i and j are arbitrary indices in the list of z scores being used in the computation, \bar{z} is the
97 average of these z scores, and n is the size of the z score list. Repeating (1) and (2) for multiple
98 angles θ spanning from 0 to 2π would yield the corresponding lists between g and θ , as shown in
99 the bottom-right table in **Figure 1**. This would lead to two polar curves for Gini coefficients, one
100 for the cell-points expressing the gene in the cluster, and one for all cell-points in the cluster. We
101 hypothesize that *the two curves would be closer in the marker-gene scenario than in the non-*
102 *marker gene scenario*. Therefore, we used the root-mean-square deviation (RSMD) metric, which
103 is popular in computing fitness in Bioinformatics [31], to determine whether a gene is a marker in
104 the cluster.

$$105 \quad RSMD = \frac{\sum_{\forall\theta} (g_{\text{sub}}(\theta) - g_{\text{whole}}(\theta))^2}{n_{\theta}} \quad (3)$$

106 Here, n_{θ} , also called resolution, is the number of angles θ for which we repeat (1) and (2). In this
107 work, we chose $n_{\theta} = 1000$, which makes the angle list $\theta = 0, \pi/500, 2\pi/500, \dots, 999\pi/500, 2\pi$.

108
109 To compute the RSMD statistical p-value for each gene in each cluster, first, we linearly
110 normalized (scaled) the RSMD computed in (3) such that the normalized RSMD is between 0 and
111 1. This could be done by dividing (3) by the largest RSMD among all genes in each cluster. Then,
112 we applied the estimated p-value calculation in [32] to assign a p-value for each gene in each
113 cluster. Briefly, from the RSMD scores in (3), we verified that the RSMD scores followed a bell-

114 shaped distribution. Then, we computed the mean μ and standard deviation σ of the normalized
115 RSMD. Then, the p-value for each gene in the cluster is

116
$$p - value(i) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^U e^{-\frac{(u-\mu)^2}{2\sigma^2}} du \quad (4)$$

117 In (4), U stands for the normalized RSMD.

118

119

120 **Setting up simulation**

121 In this work, to demonstrate how the PGC-RSMD functions, we setup two simulations. In the
122 first simulation, the cell cluster in the x - y embedding space had 5000 points, which were uniformly
123 generated in the unit circle $x^2 + y^2 \leq 1$. In the second simulation, the 2D visualization of cell clusters
124 had the shape identical to the real-world cluster obtained from visualizing the mouse fetal lung
125 single-cell data [29] using tSNE [33]. We applied the sampling-by-rejection technique [34] to
126 generate these cluster points as follows. In the first simulation, we randomly generated a point
127 whose coordinates are between -1 and 1 using uniform sampling, then accepted the point if it had
128 $x^2 + y^2 \leq 1$. In the second simulation, the random point coordinates were within the cluster
129 coordinate range. We extracted the cluster boundary points, compute the polygon from these
130 boundary points, which allowed deciding whether a point was inside the polygon using Matlab
131 [35, 36]. In each simulation, we randomly chose m percentage of points ($m = 5, 10, 15, \dots, 95$) and
132 assumed that they represent the cells expressing gene. For each m percentage, we repeat the
133 simulation 1000 times.

134 In addition, to evaluate how the performance of PGC-RSMD would change in drop-out
135 scenario, we modified the single-cell data simulator in [37] as follow. First, we use [37] default
136 parameters to synthesize 2 clusters such that each cluster has 6000 cells, 250 markers (total 500
137 cluster markers) and other 4500 genes. For each cluster marker, the average expression fol-change
138 when comparing two clusters was between 4 and 1000. We assigned the drop-out probability for
139 each gene from 0, 5, 10, ..., to 45% such that there were 25 markers for each drop-out probability.
140 Then, in each cell, we randomly change the marker expression to 0 according to the markers' drop-
141 out probability. For each of the 4500 non-cluster markers, the expression in each cell was randomly
142 between 0 and 500. We assigned the sparisity – defined as the percentage of non-expressing cells
143 (0 expression) – for each non-cluster marker from 0, 5, 10, to 95%. In each cell, we randomly

144 changed the non-cluster marker to 0 according to its sparsity. We used the AUC metric to evaluate
145 whether the PGC-RSMD score could differentiate the 500 cluster-markers: whether each marker
146 is specific for the first or the second clusters.

147

148

149 **Identifying cardiac muscle cell clusters and marker genes from the neonatal mouse heart** 150 **single-cell data**

151 We obtained the neonatal mouse heart single-cell case-study from the Mouse Cell Atlas [29].
152 We processed the data as specified in [29]. After preprocessing, the dataset covered 19,494 genes
153 expression in 5075 cells. We use tSNE [33] (without dimensional reduction) to embed the dataset
154 into the 2D space. We used the density-based clustering algorithm [38] implemented in Matlab
155 [39] to identify 9 cell clusters. In the implementation [39], we chose the clustering parameters
156 $\epsilon = 4$, $\text{minpts} = 40$. There were 788, 397, 2966, 156, 288, 123, 76, 125, 87 cell-points in
157 cluster 1, 2, ..., 9, correspondingly. There were 69 cell-points for which the algorithm is unable to
158 assign to any clusters (Supplemental Data 3).

159

160 We computed the percentage of expressing cells (the naïve approach) and PGC-RSMD for all
161 genes in all clusters. We removed genes expressing in less than 10% of the cluster cells. For
162 comparison, in the naïve approach, in each cluster, we selected the top genes sorted by the highest
163 percentage of expressing cells as the cluster markers. In the PGC-RSMD approach, we selected
164 the smallest-RSMD genes with its p-value < 0.05 as the cluster marker. In this work, we focused
165 on identifying the heart muscle cell clusters and their markers. We manually examine the
166 distribution of cells expressing the well-known heart muscle cell markers: *Myh7*, *Actc1*, and *Tnnt2*
167 [40-47].

168

169

170 **Setting up the re-identifying cluster ID problem**

171 To compare the robustness of our PGC-RSMD markers with other approaches, we setup the
172 re-identifying cluster ID as follow. From the visual coordinates and 9 clusters of 5075 cells in [29],
173 we randomly divided the dataset into the training set (4060 cells – 80%) and the test set (1015 cells
174 – 20%) such that set has samples of all 9 clusters. Using the training set and markers' expression

175 found by PGC-RSMD, in comparison with other approaches, we applied the neural network
176 algorithm [48] to train models that identify cluster ID. We evaluated these models in the test set
177 and recorded the classification accuracy and area-under-receiving characteristic curve (AUC).
178 Here, we hypothesized that the ‘better’ markers would yield higher classification accuracy and
179 AUC. The other approaches being compared with PGC-RSMD are:

180 - The baseline approach: in this approach, we would train the classification models using all
181 genes expression.

182 - The differential expression approach: in this approach, we use Fisher’s exact test [49], which
183 computes the likelihood of a gene being expressed (raw expression > 0) in a cluster, compare to
184 the likelihood of the gene being expressed outside the cluster. In this work, we select the DEG
185 markers in each cluster according to the following criteria: odd ratio > 5 and the percentage of
186 expressing cell (m) $> 50\%$.

187 - The SpatialDE [26] approach: SpatialDE finds the gene with high variance regarding the
188 distribution of ‘point’ on the spatial 2D space. The ‘null’ hypothesis in this approach is the gene
189 distribution in the ‘spatial space’ follows a multivariate normal distribution. The marker is selected
190 if the gene expression distribution is significantly different from the null distribution, recorded in
191 the p-value. In this work, we select the SpatialDE marker according to the following criteria: q-
192 value (adjusted p-value) < 0.05 and percentage of expressing cell (m) $> 50\%$.

193 In both the DEG and the SpatialDE approach, we sort the markers according to the decreasing
194 order of m . To make a fair comparison, we use the same number of markers, ranging from 5 to
195 100, found by PGC-RSMD, DEG and SpatialDE to train the classification models.

196

197

198 **Results**

199 **PGC-RSMD strongly correlates to the percentage of expressing cell in a cluster**

200 In **Figure 2**, we show that the fitness between the cluster PGC and the sub-cluster PGC strongly
201 correlates to the percentage of expressing cell-points as the ‘sub-cluster’ m in the circle-shaped
202 simulation. In addition, as m increases, the RSMD variance decreases. We represented the fitness
203 by the **root-mean-square deviation (RSMD)** as showed in the method section. In this figure, for
204 each m (from 5 to 95), we randomly generate 1000 sub-clusters and their PGCs. The detailed result
205 of this simulation could be found at the Supplemental Data 1.

206 In addition, we observed a similar correlation when experimenting with the mouse fetal lung
207 single-cell data [29]. **Figure 3a** shows the dataset clusters visualization using tSNE [33] and the
208 chosen cluster. To synthesize a 3000-point cluster with the same shape to the chosen cluster, we
209 still applied the random-by-rejection [34] as presented in the Material and Method section. **Figure**
210 **3b** still shows a strong correlation between m and RSMD. The detailed result of this simulation
211 could be found at the Supplemental Data 2.

212
213 On the other hand, the PGC approach has the potential to answer whether the marker could
214 identify subpopulations of cells in a cluster. **Figure 4a** demonstrates the 30000-point cluster with
215 ring-shape $0.25 \leq x^2 + y^2 \leq 1$, which appears to be a sub-cluster marker. In this case, $m = 0.75$. In
216 this example, RSMD = 0.033 (**Figure 4b**), which is greater than the RSMD distribution computed
217 from the random and uniformly-distributed cluster with the same m (**Figure 4c**).

218
219 **Figure 5** shows a decrease of PGC-RSMD performance in the drop-out scenario. Briefly, the
220 synthetic data has 2 clusters, 250 distinct markers for each cluster. Each gene has a specific drop-
221 out rate as presented in the Material and Method section. Using the PGC-RSMD scores in each
222 cluster to differentiate these 500 the cluster-specific markers, we observed that PGC-RSMD
223 achieves very high area-under-receiver-characteristic curve (AUC) (>0.95) when the drop-out
224 probability is small ($\leq 5\%$). However, AUC decreases significantly with the probability of drop-
225 out (**Figure 5a**). This phenomenon further demonstrates the strong association between RSMD
226 and the percentage of expressing-cell. When the drop-out rate increases, the percentage of
227 expressing-cell decreases; therefore, RSMD may mischaracterize a high-dropout marker as non-
228 marker.

229
230

231 **Case-study: PGC identifies heart muscle cell in neonatal mouse heart single-cell**

232 *PGC-RSMD detects markers to support cell-type identification in single-cell mouse*
233 *neonatal heart data*

234 **Figure 6** summarizes the neonatal mouse heart single-cell data [29] and its 9-cluster markers.
235 **Figure 6a** visualizes these 9 clusters with tSNE. The PGC-RSMD founds 258 genes, which are
236 the union of the smallest 100-PCG-RSMD genes found in each cluster, marking these clusters

237 (Supplemental Data 3). **Figures 6b** and **6c** showed that the gene-cluster marker-association reflects
238 the underlying gene expression in the single-cell data. In these heatmap figures, each row
239 corresponds to one gene.

240

241 We identified the muscle-cell clusters 1, 4 and 9 by the expression of *Myh7*, *Actc1*, and *Tnnt2*,
242 which strongly express in muscle cell type [40-47] (**Figure 7**). Compared to the naïve method
243 using the percentage of expressing cell, our PGC-RSMD is significantly better by detecting *Actc1*,
244 which are missed by the naïve approach (**Figure 8**). Furthermore, our approach identified *Mgrn1*
245 [50, 51], *Ifitm3* [52], *Myl6b* [53] marking cluster 1, which could play important roles in cardiac
246 muscle functionality, heart failure, and heart development. These genes are not identified using
247 the naïve approach (**Figure 8**). On the other hand, among genes having a high percentage of
248 expressing cell, our PGC-RSMD suggests that *Ndufa4l2*, *Mdh2*, and *Atp5g1* may not be heart
249 muscle cell markers. However, they could suggest a subtype of heart muscle cells (**Figure 9**). The
250 percentage of expressing cells, PGC-RSMD, statistical p-value and ranks for all genes could be
251 found in Supplemental Data 3.

252

253 *Re-identifying the cells' cluster ID from markers*

254 We observe that the markers found by the PGC-RSMD approach achieve better performance
255 than the similar SpatialDE [26] markers, and similar performance to the differentially-expressed-
256 gene (DEG) when being used to re-identify cell's cluster ID. Briefly, after computing the visual
257 coordinate and cluster ID of all cells, we randomly split the dataset [29] into the training (80%)
258 and test (20%) sets. We only applied the baseline PGC-RSMD, SpatialDE and DEG approaches
259 to find the markers and built machine learning models to predict the cells' cluster ID from these
260 markers in the training set. In this experiment, we used all genes to train the predictor in the
261 baseline approach. The detailed description of this experiment could be found in the method
262 section. Evaluating the prediction models in the test set, the PGC-RSMD approach performs
263 closely to the DEG; both have cluster ID prediction accuracy above 0.9 and AUC above 0.95 on
264 average (**Figure 10**). These two approaches significantly outperform SpatialDE, whose accuracy
265 is just above the baseline.

266

267

268 Discussion

269 In this work, we show that integrating the embedded information, which does not often have a
270 deterministic relationship with gene expression and is primarily for clustering a visualization,
271 could lead to new insights to biomarkers in single-cell data. In the mouse neonatal heart case-
272 study, our PGC-RSMD approach could recall *Actc1* as the marker characterizing heart muscle cell.
273 Meanwhile, the approach using the ratio of expressing cell may fail to recall because a large
274 percentage of cells does not capture *Actc1* transcript. Therefore, our proposed technique has the
275 potential to handle analytical issues due to single-cell data quality, such as short-read and low
276 sequencing depth [54-56]. On the other hand, for genes having high percentage of expressing cell,
277 the PGC approach could further show that these genes may characterize novel cardiac muscle cell
278 sub-types for future studies, such as in *Mdh2* and *Myl6b*. Therefore, we suggest that the biomarker
279 discovery problem could be divided into two sub-problems: the ‘global markers’ specify cell types
280 and the ‘local markers’ specify subtypes. We could solve these two sub-problems by the right
281 integration of gene expression and visual information.

282
283 In this work, we primarily demonstrate how PGC detects markers for single cluster, which
284 does not need the gene expression from other clusters in the dataset. The approach could be
285 extended to incorporate the ‘global’ expression as follow. First, a PGC analysis can be performed
286 with marker cells as the foreground and all cells (regardless of their cluster assignments) as the
287 background. Second, a PGC analysis can be performed for each cluster in the dataset
288 independently and compare among the clusters’ marker lists. In the neonatal mouse heart case-
289 study, this approach shows two types of marker: one expressing globally in all clusters, which are
290 likely heart-tissue specific; the other express locally in one or some specific cluster, which are
291 likely cell-type specific.

292
293 In addition to our proposed PGC approach, we could apply several alternative strategies to
294 integrate the gene expression and visual information to solve the single-cell biomarker discovery
295 problem. For example, the fractal dimension analysis strategies [57, 58], which focus on evaluating
296 the uniformity of cell-point distribution, could be applied to identify markers in which the
297 expressing cells distribute more densely than they are in the overall cluster. In addition, we could
298 also customize the statistical texture analysis in image processing, such as homogeneity and

299 integrity [59, 60], to analyze the difference between the overall cluster cell-point and cell-
300 expressing gene point as the metric to determine markers. On the other hand, choosing the
301 appropriate visual approach depends on the nature of the data and the problem. Our experiment
302 with the re-identifying cluster ID shows that the well-established SpatialDE [26] does not
303 outperform our approach and the DEG approach. One explanation is that in our problem, a good
304 marker for identifying cell type usually follows a good ‘default’ distribution over the visual space;
305 meanwhile, the SpatialDE aims to find markers that express significantly different from a default
306 distribution.

307

308 The major limitation of our proposed PGC-RSMD approach is the long computational time,
309 especially when comparing to the DEG approaches. This is similar to SpatialDE, which also used
310 visual information to detect marker genes. The DEG approaches may only need to compute one
311 statistical test to determine whether a gene is a marker for all clusters. Meanwhile, to draw the
312 curves, PGC-RSMD would need to compute hundreds to thousands, which depends on the desired
313 curve resolution, to characterize one gene in one cluster. Due to the long computational time, we
314 were not able to create multiple simulations, which is the ideal approach, run to compute the
315 statistical [32] p-value for the RSMD score. Therefore, we decided to reapply the estimation
316 presented to compute the p-value. This approach is computationally more efficient but may not
317 well-reflect the statistical characteristic of the single-cell data. In addition, we have not fully
318 tackled the problem of choosing the right threshold to determine whether a gene expresses in a
319 cell. Because of the strong association between PGC-RSMD and the percentage of expressing-
320 cell, we expect that the result would significantly different when choosing a different threshold to
321 determine whether a gene expresses in a cell. In this work, choosing 0 as the threshold still yields
322 good performance because of the high sparsity in the real dataset.

323

324 **Conclusions**

325 In this work, we have presented Polar Gini Curve, a novel technique to detect markers from
326 the single-cell RNA expression data using visual information. In principle, our technique could
327 complement the state-of-the-art approach: the PGC technique finds markers such that the
328 expressing cells are evenly distributed throughout the cluster space; meanwhile, the state-of-the-
329 art approach finds markers assuming a multivariate normal distribution of gene expression in the

330 visual space. We have demonstrated that the PGC technique performs better in some tasks in
331 single-cell analysis.

332

333

334 **Authors' contribution**

335 TN designed and implemented the core polar Gini algorithm, designed the sampling strategies
336 in the simulation, performed the neonatal heart single-cell case-study, and primarily prepared the
337 manuscript. JJ prepared the software package, preprocessed the single-cell data, and executed the
338 simulation designs. NX designed different simulation scenarios, interpreted the statistical
339 outcomes, and prepared the literature review. JC originated the idea of using Gini coefficient
340 curves to integrate gene expression, cell-point distribution, and cluster shape to solve the
341 biomarker discovery problem, designed the performance evaluation, and supervised the overall
342 technical development. All authors reviewed/ revised and approved the manuscript.

343

344

345 **Competing interests**

346 The authors have declared that no competing interests exist.

347

348

349 **Acknowledgments**

350 This work was supported by the 'startup budget' granted to Jake Chen from the University of
351 Alabama at Birmingham.

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482

483

484 **Figure legends**

485 Figure 1. Overall workflow to compute the PGC-RSMD metric for one gene in one cluster of cells.
486 Here, the data points, histogram, and PGC for cells expressing the gene are cyan. The ones for the
487 whole cells in the cluster are red.

488

489 Figure 2. Boxplot showing a strong correlation between ‘subcluster’ percentage (m) and cluster-
490 subcluster PGC fitness (RSMD) in uniformly-distributed and a circular cluster.

491

492 Figure 3. a) The selected cluster for the experiment in [29]. b) Correlation between ‘subcluster’
493 percentage (m) and cluster-subcluster PGC fitness (RSMD) in the selected cluster.

494

495 Figure 4. The ring-shape simulation study: a) Visualization of the cluster and ring-shape sub-
496 cluster ($m = 0.75$); b) PGC yield RSMD = 0.033; c) Distribution of RSMD, extracted from Figure
497 2 with $m = 75\%$, when the sub-cluster uniformly distributed on the cluster area.

498

499 Figure 5. PGC-RSMD performance in recalling cluster marker in drop-out simulation. a) heatmap
500 showing the simulation design of 500 markers and 4500 neutral genes, with drop out / percentage
501 of cell expressing between 5 and 100%; b) The simulation data 2D visualization; c) the AUC drops
502 when drop-out increases.

503

504 Figure 6. The result from mouse neonatal heart single-cell [29] analysis. a) the tSNE plot shows 9
505 clusters. b) gene-cluster marker relationship (from 258 genes) found by PGC-RSMD; ■ gene is
506 found as marker, ■ gene is found as non-marker. c) expression heatmap for these genes.

507

508 Figure 7: Heart muscle cell clusters, identified by *Myh7*, *Actc1*, and *Tnnt2*

509

510 Figure 8. PGC-RSMD highlight makers that do not have high percentage of expressing cells: PGCs
511 of *Actc1*, *Mgrn1*, *Ifitm3*, *Myl6b* in cluster 1. The numbers in the parenthesis are ranks of these
512 genes in each metric

513

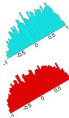
514 Figure 9. PGC-RSMD shows that gene has high percentage of expressing cells: *Ndufa4l2*, *Mdh2*,
515 and *Atp5g1*, may not be markers in cluster 1. These genes appear to highlight a local subcluster.
516 The numbers in the parenthesis are ranks of these genes in each metric.

517

518 Figure 10. Performance of the PGC-RSMD, SpatialDE, and DEG in re-identifying the cell's
519 cluster ID problem using dataset [26]. The x-axis shows the number of top-significant markers
520 being selected to train the prediction models. a) accuracy; b) AUC over 9 clusters.

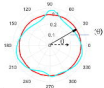
521

2D visualization



$$Gini(g)$$

$$\frac{1}{2n^2} \sum_{i=1}^n \sum_{j=1}^n |x_i - x_j|$$

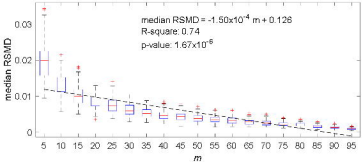


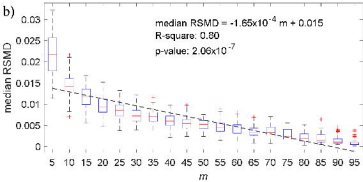
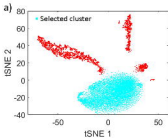
Angle θ	S_{sub}	S_{whole}	$f_{\theta} = S_{sub} - S_{whole} $
...
30°	0.26	0.29	0.03
...

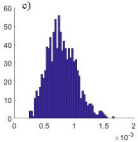
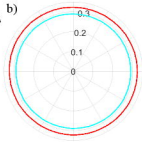
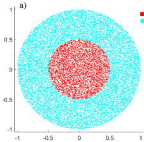
$$RSMD = \sum f_{\theta} / n_{\theta}$$

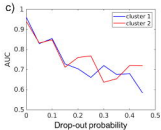
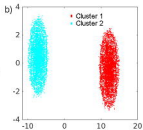
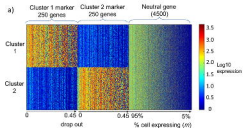
 Cell expressing genes (sub cluster)

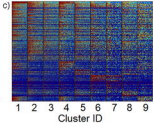
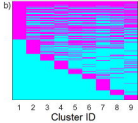
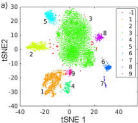
 All cells (whole cluster)

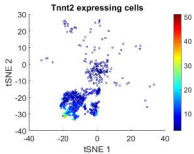
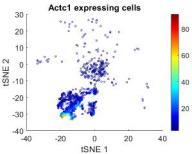
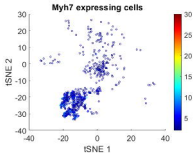
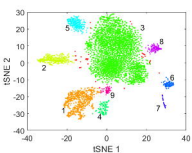


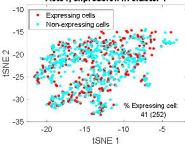
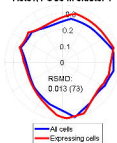
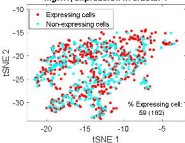
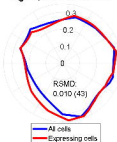
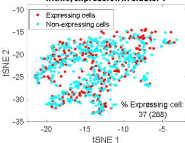
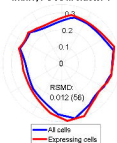
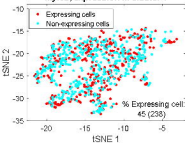
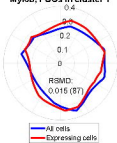


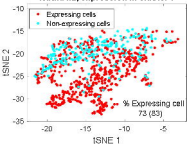
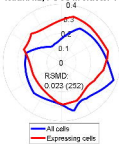
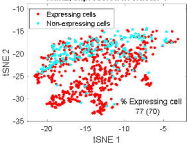
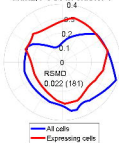
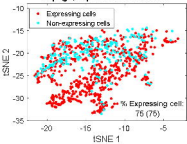








Actc1, expression in cluster 1**Actc1, PGCs in cluster 1****Mgrn1, expression in cluster 1****Mgrn1, PGCs in cluster 1****Ifitm3, expression in cluster 1****Ifitm3, PGCs in cluster 1****Myf6b, expression in cluster 1****Myf6b, PGCs in cluster 1**

Ndufa4l2, expression in cluster 1**Ndufa4l2, PGCs in cluster 1****Mdh2, expression in cluster 1****Mdh2, PGCs in cluster 1****Atp5g1, expression in cluster 1****Atp5g1, PGCs in cluster 1**