Flagellar dynamics reveal fluctuations and kinetic limit in the *Escherichia coli* chemotaxis network

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Abstract

Biochemical signaling networks allow living cells to adapt to a changing environment, but these networks must cope with unavoidable number fluctuations ("noise") in their molecular constituents. *Escherichia coli* chemotaxis, by which bacteria modulate their random run/tumble swimming pattern to navigate their environment, is a paradigm for the role of noise in cell signaling. The key signaling protein, CheY, when activated by (reversible) phosphorylation, causes a switch in the rotational direction of the flagellar motors propelling the cell, leading to tumbling. CheY-P concentration, [CheY-P], is thus a measure of the chemotaxis network's output, and temporal fluctuations in [CheY-P] provide a proxy for network noise. Here we quantify the fluctuations in [CheY-P] from the switching dynamics of individual flagella, observed using time-resolved fluorescence microscopy of optically trapped *E. coli* cells. This approach reveals [CheY-P] fluctuations at steady state, which may play a critical role in driving flagellar switching and cell tumbling. A stochastic theoretical model, inspired by work on gene expression noise, points to CheY activation occurring in bursts, driving the [CheY-P] fluctuations. When the network is stimulated chemically to higher activity, we observe a dramatic decrease in [CheY-P] fluctuations. Our stochastic model shows that an intrinsic kinetic ceiling on network activity places an upper limit on [CheY-P], which when encountered suppresses its fluctuations. This limit may also prevent cells from tumbling unproductively in steep gradients.

Introduction

A common feature of living organisms is their ability to sense environmental signals and respond to these signals by modifying their behavior. This ability is enabled by signal transduction, which converts environmental inputs into behavioral outputs[1]. Studies have shown that signaling networks must operate in the presence of noise not only in their inputs but also in the circuit components themselves[2,3]. Until recently, it was thought that signaling networks had evolved to be robust against undesirable noise[4–7]. More recent studies have shifted to understanding the role of noise in regulating and fine-tuning biological function[8,9]. For example, noise resulting from transcription/translation has been measured precisely and shown to play a critical role in networks for gene expression and regulation[10–12]. In contrast, studying noise resulting from signaling networks at the post-translation level has proven more challenging.

One extensively studied signaling system is the chemotaxis network of *E. coli*, which cells use to navigate their environment[13–17]. *E. coli* cells swim in a random walk consisting of "runs"—during which their flagella rotate counter-clockwise (CCW)—and "tumbles"—during which one or more flagella rotate clockwise (CW)[18–21]. Changing environmental conditions are sensed through a two-component signaling system comprising receptors that bind extracellular ligands and a kinase, CheA, that transfers its phosphoryl group onto downstream effectors[22]. The response regulator, CheY, when phosphorylated by the receptor kinase complex to CheY-P, binds to the flagellar motor and increases the probability of CW motor rotation[23–25]. The phosphatase CheZ carries out the opposite reaction, maintaining a dynamic equilibrium between CheY and CheY-P (**Figure 1a**)[26,27]. Additionally, the methyltransferase CheR and the methylesterase CheB covalently modify receptors, and the resulting modulation of CheA activity affects CheY-P levels over the timescales of chemotactic adaptation[28–30]. While the population averaged relationships between these signaling protein concentrations and cellular motility in *E. coli* are now well understood[13–16,24], the role of fluctuations in protein concentrations in modulating cell behavior remains comparatively unexplored.

The intracellular CheY-P concentration, [CheY-P], is a dynamic indicator of the cell's environment and determines its trajectory of motion. Thus, [CheY-P] fluctuations are the primary indicator of noise upstream in the signaling network and affect downstream behavior. Cluzel et

al. [25] showed that the motor CW bias, the probability of CW rotation of a single flagellar motor,

has an ultrasensitive switch-like dependence on the intracellular [CheY-P]. Thus, the cell's

swimming behavior is sensitive to small temporal fluctuations in [CheY-P] (**Figure 1b, i-iii**). Noise

due to CheY-P fluctuations was first inferred by Korobkova et al.[31] from long-term measurements

of individual rotating flagellar motors, where larger-than-expected noise in the motor CW bias

was attributed to methylation/demethylation dynamics of CheR/CheB over long timescales (10s

of seconds). Recently, single-cell measurements probing intracellular [CheY-P] using fluorescent

reporters revealed a potential new source of noise arising from the dynamics of the receptor

kinase complex itself[32,33]. Noise in the *E. coli* chemotaxis signaling network is thought to have

functional consequences for sensing and navigation in complex natural microenvironments. It

has been proposed to allow cells to sample 3D space more effectively[31,34], enhance chemotactic

drift up a gradient[35–37], and synchronize flagellar switching to mitigate differences in swimming

behavior among cells with different numbers of flagella[38,39].

Here, we use optical trapping coupled with time-resolved fluorescence microscopy[39,40] to

infer temporal [CheY-P] fluctuations from the statistics of flagellar switching in individual multi-

flagellated *E. coli* cells. In contrast to fluorescence-based approaches that provide a relative

measure of noise[32,33], here it is measured directly in terms of CheY-P concentration fluctuations.

Analyzing cells at steady state, we determine the single-cell distribution of [CheY-P], which

reveals that the network is poised at a low CW bias, favoring CCW flagellar rotation/running, and

that transient fluctuations in [CheY-P] drive CW flagellar rotation/tumbling. Unlike previous

studies where fluctuations were studied over long timescales (>10 s)[31–33,41], our method

measures fluctuations at the timescales of individual runs and tumbles (0.5-5 s), which allows us

to track the time evolution of signaling noise in cells undergoing adaptation. We find that

fluctuations are significantly reduced upon applying a stimulus that increases CheA activity. A

simple stochastic model in which noise in signaling arises from large, burst-like fluctuations in

CheA activity recapitulates all of our experimental results and suggests that the decrease in CheY-

P fluctuations upon stimulation is due to a kinetic ceiling on the CheA activity.

Results

Flagellar switching statistics reveal temporal [CheY-P] fluctuations

We inferred the temporal fluctuations in the intracellular CheY-P concentration from the dynamics of flagellar switching in individual cells. We imaged fluorescently labeled flagella on individual, swimming cells to determine their CCW/CW rotation state, using an optical trapping assay described previously[39,40,42] (see **Materials and Methods**). Briefly, *E. coli* cells were injected into a laminar flow chamber containing two parallel channels. A single cell was captured using a dual beam optical trap from one channel containing many cells (labeled "cells" in **Figure 1c**) and moved for observation to a second channel containing no other cells (labeled "blank" in **Figure 1c**). The light scattered by the optically trapped cell was used to monitor the run/tumble swimming behavior as described previously (data not shown, see Ref. [40]). Simultaneously, high-speed, epi-fluorescent stroboscopic imaging[39] was used to monitor the rotation state of all the individual flagella on the cell at high temporal resolution (0.1 s) and long-time duration (typically ~15-40 s, until the flagella photobleached) (**Figure 1d,** top). Following Darnton et al.[21] and our previous work[39], the CCW/CW rotation state of each flagellum was visually identified by its different helical waveforms (see **Materials and Methods**).

We next inferred the distribution of [CheY-P], making use of the statistics of switching in multi-flagellated cells. For a cell with N flagellar motors, the probability that n_{cw} rotate in a CW direction, $p_N(n_{cw})$, is equal to (see **Materials and Methods**):

$$p_N(n_{cw}) = \int_0^1 dc \, p_N(n_{cw} \mid c) p(c), \tag{1}$$

where c is the instantaneous motor CW bias, or probability that any one motor is CW, $p_N(n_{cw} \mid c)$ is the conditional probability that n_{cw} out of N motors are in the CW state given a certain c, and $p(c)$ is the distribution of CW biases (**Figure 1b, iii**). Given the finding that flagellar motors switch independently of each other[39], $p_N(n_{cw} \mid c)$ is given by the binomial distribution:

$$p_N(n_{cw} \mid c) = \binom{N}{n_{cw}} c^{n_{cw}} (1-c)^{N-n_{cw}}. \tag{2}$$

Transitions between CW and CCW flagellar rotation are engendered by a switch complex at the base of each motor that binds CheY-P with high cooperativity. The motor CW bias c depends

112   on the intracellular CheY-P concentration in a switch-like manner, customarily modeled by a

113   sigmoidal Hill equation (**Figure 1b, ii**)[25]:

$$c(y_P) = \frac{y_P^H}{K^H + y_P^H} , \tag{3}$$

115   where $y_P \equiv$ [CheY-P], K is the CheY-P concentration at 0.5 CW bias, and H is the Hill coefficient.

116   (Analytical forms for $c(y_P)$ other than Eq. (3) are also commonly used[43], and are indistinguishable

117   in shape; see **Appendix**). For simplicity, we adopted the Hill equation, Eq. (3), in our analysis).

118   Thus, Eq. (1) for $p_N(n_{cw})$ can be written in terms of [CheY-P]:

$$p_N(n_{cw}) = \int_0^{y_{tot}} dy_P \, p_N(n_{cw} | y_P) p(y_P) , \tag{4}$$

120   where y_{tot} is the total intracellular CheY concentration, reported to be ~10 μM for cells under

121   growth conditions similar to ours[44] (see **Materials and Methods**). $p(y_P)$ is the distribution of

122   [CheY-P] (**Figure 1b, i**), and the conditional probability $p_N(n_{cw} | y_P)$ is obtained by combining Eq.

123   (2) and (3) to yield:

$$p_N(n_{cw} | y_P) = \binom{N}{n_{cw}} \frac{y_P^{n_{cw} H}}{(K^H + y_P^H)^N} . \tag{5}$$

Since the conditional probability is known, Eq. (4) in principle allows one to connect the

probability $p_N(n_{cw})$—which can be determined from measurements of the CCW/CW rotational

state of each flagellum of a cell with N flagellar motors—to the distribution of [CheY-P], $p(y_P)$.

(Eq. (1) can similarly be used to connect to the distribution of CW biases $p(c)$). In practice,

solutions to Eqs. (1) and (4) are not unique, but one can show that an increasing number of

moments of the distribution $p(c)$ are determined as N increases (see **Materials and Methods**).

Thus, one expects that the distribution $p(c)$, and thus the distribution $p(y_P)$ which captures

fluctuations in [CheY-P], can be estimated over a given time window with increasing accuracy as

N increases. Essentially, since each flagellar motor independently measures the intracellular level

of CheY-P, the more motors a cell possesses, the better our estimate of the mean [CheY-P] and

its fluctuations.

We note that while $p(c)$ is determined exactly from Eq. (1) with no additional input

parameters, $p(y_P)$ requires values for the parameters H, K, and y_{tot} in Eqs. (4) and (5). In particular,

$p(y_P)$ depends non-negligibly on K and H (see **Appendix** for a detailed discussion on the sensitivity

to K and H). Initial single-cell measurements of the motor c vs. y_P response function[25] yielded

values of $H = 10.3$ and $K = 3.1$ μM. However, the discovery of remodeling in the flagellar motor[45–

47]—which causes the number of FliM, the component of the motor switch complex that binds

CheY-P, to vary according to intracellular CheY-P levels—has led to revised estimates of the Hill

coefficient, $H \sim 20$, nearly twice as large[47]. Furthermore, remodeling causes the motor response

function to shift depending on CheY-P level, such that K (and to a lesser extent H) depend on the

steady-state CW bias of the motor[43,45] (see **Appendix** for a discussion on remodeling). As detailed

below, we account for the effect of motor remodeling in determining the appropriate values of

K and H in Eqs. (4) and (5) given the CW bias of the cell. We also note that, since y_P is determined

only relative to K, we plot and report values for the ratio y_P/K, which is the functionally important

parameter for *E. coli* chemotaxis. In contrast to K and H, $p(y_P)$ is insensitive to the exact value of

y_{tot}, because the CW bias $c \approx 1$ when $y_P = y_{tot} > K$ according to Eq. (3). (The upper limit for the

integral in Eq. (4) can be replaced with ∞ with error <0.01%). Lastly we note that writing Eqs. (4)

and (5) implicitly assumes that the motor response function, $c(y_P)$, is identical for all motors. An

analysis of the CW bias across motors and cells indicates that this variability is negligible, and

justifies our use of a single value for K and for H across all cells (see **Appendix** for further

discussion).

Figure 1 shows the workflow to determine $p(y_P)$ experimentally. Fluorescence tracking of the

CCW/CW state of the individual flagella (**Figure 1d,** middle) is represented as a time trace of the

number of CW flagella (**Figure 1d,** bottom). For a cell with N flagella, a histogram of this time

trace was used to construct the probability distribution $p_N(n_{cw})$ that n_{cw} out of N flagella rotate

CW (**Figure 1e**, bottom**,** histogram). We determined the probability distributions $p(c)$ (**Figure 1e**,

middle) and $p_{fit}(y_P)$ (**Figure 1e**, top) via the integral equations Eqs. (1) and (4), respectively, using

a trial distribution function and finding the parameters that generated the probability

distribution $p_{N,fit}(n_{cw})$ (**Figure 1e**, bottom, solid line) that best fitted the experimental distribution

(see **Materials and Methods**). Below we show the results of this analysis for one choice of trial

distribution function (see **Materials and Methods** for a discussion on alternate choices).

The chemotaxis network exhibits [CheY-P] fluctuations at steady-state

We first applied the above approach to free-swimming (i.e. unstimulated) cells for which the

chemotaxis network is in a steady state. Fluorescence tracking data of CCW/CW flagellar states

were obtained for cells of a strain wild-type for chemotaxis (HCB1660, henceforth referred to as

"wild-type"; see **Materials and Methods**) with N = 2 to 5 flagella. Data traces tracking n_{cw} were

pooled together from multiple cells with the same total number of flagella N to generate $p_N(n_{cw})$

(**Figure 2a**, colored histograms). As we show below, pooling over the population of cells did not

affect our results appreciably.

Since we did not expect the number of flagella to have an effect on fluctuations of the

underlying network, we fitted the data globally, determining a single CW bias and [CheY-P]

distribution for all values of N. The best global motor bias distribution $p_{glofit}(c)$ is shown in **Figure**

2b, determined using the beta distribution as a trial function (see **Materials and Methods**; Eq.

(15)). The corresponding distributions of number of CW flagella, $p_{N,glofit}(n_{cw})$, are shown in **Figure**

2a (gray solid lines). The mean and standard deviation in CW bias from this global fit were $\mu_{c,glo}$ =

0.14 ± 0.02 and $\sigma_{c,glo}$ = 0.22 ± 0.02, respectively. Given the above mean CW bias value, we used a

quantitative model by Zhang et al.[43] for FliM remodeling dynamics in the motor switch complex

to determine that K = 2.4 μM and H = 18 for the motor c vs y_P response function, Eq. (3) (see

Appendix). Given these values of K and H, we determined from $p_{glofit}(c)$ and Eq. (10) the

corresponding best fit global [CheY-P] distribution $p_{glofit}(y_P)$, shown in **Figure 2c** (gray solid curve;

see **Materials and Methods**). The mean and standard deviation of [CheY-P] relative to the

parameter K from this global fit were $\mu_{y_p,glo}/K$ = 0.80 ± 0.03 and $\sigma_{y_p,glo}/K$ = 0.18 ± 0.03, respectively.

Figure 2 – figure supplement 1 shows the estimated error in the best fit [CheY-P] distributions,

as determined by bootstrap resampling (see **Materials and Methods**). Alternate choices of trial

functions for the fitting distributions did not change the mean and standard deviation

appreciably, although some functions fit the data better than others (see **Materials and**

Methods, **Figure 2 – figure supplement 2**, and **Table 3**). Furthermore, we inquired whether

fluctuations in the instantaneous CW bias, as captured by $p(c)$, could couple to FliM remodeling

dynamics and affect the estimates of mean and standard deviation of [CheY-P]. We found that

remodeling had no effect on the mean [CheY-P] but led to a small (<25%) increase in the standard

deviation (see **Appendix** and **Appendix – figure 1a**), which we ignored in further analysis.

As a comparison, we also determined the CW bias distribution $p_{fit}(c)$ and [CheY-P] distribution

$p_{fit}(y_P)$ separately for each set of cells with different number of flagella N (**Figure 2b** and **2c**,

colored solid curves), along with the best fit distribution of the number of CW flagella, $p_{N,fit}(n_{cw})$

(**Figure 2a**, colored solid lines). Although the distributions $p_{N,fit}(n_{cw})$ were better fit for each

individual N, the corresponding $p_{fit}(c)$ and $p_{fit}(y_P)$ for each N were comparable to those obtained

by a global fit, as shown in **Figure 2b** and **c** (see **Table 4**). The mean and standard deviation in CW

bias and in [CheY-P]/K averaged over the separate fits, $<\mu_c> = 0.15 \pm 0.03$ and $<\sigma_c> = 0.20 \pm 0.02$,

$<\mu_{y_p}>/K = 0.80 \pm 0.04$ and $<\sigma_{y_p}>/K = 0.17 \pm 0.02$, respectively, were almost identical to those

obtained by a global fit, indicating that a single global distribution describes the data across N

and that the number of flagella does not affect network fluctuations measurably. The above

analyses thus show that [CheY-P] fluctuations in free-swimming wild-type cells results in a

coefficient of variation $\sigma_{y_p}/\mu_{y_p} = 0.22$.

We carried out several tests to confirm that the width of the estimated [CheY-P] distribution

resulted from noise in signal transduction. First, we applied our method of analysis to a strain

that expresses the constitutively active mutant protein CheY[D13K], a mimic for CheY-P[24] (strain

PM87, henceforth referred to as "CheY*"; see **Materials and Methods**). Since CheY in this strain

is decoupled from the upstream network components and the concentration of CheY[D13K] is

determined from gene expression levels, which should not fluctuate on the timescales of our

experiments in nutrient-limited conditions[48], we expected to observe smaller fluctuations in

[CheY[D13K]] as compared to the wild-type strain. Repeating the above analysis, **Figures 2d-f** show

the experimental distributions $p_N(n_{cw})$ along with all fit distributions. Here, the CW bias and FliM

dynamics model yielded parameter values $K = 2.5$ μM and $H = 18$, similar to those for wild-type

cells, which were used to determine the [CheY[D13K]] distribution. Values for the mean [CheY[D13K]]

and standard deviation relative to K from fits for each N and from a global fit are found in **Table**

5. The mean [CheY-P] relative to K is similar for both strains, and, as expected, fluctuations in the CheY* strain are much reduced compared to the wild-type strain, yielding a coefficient of variation of 0.1.

Second, we tested whether the width of the estimated [CheY-P] distribution could be due to cell-to-cell differences in mean [CheY-P] rather than fluctuations arising in signal transduction. To test the contribution of cell-to-cell variation, we determined the best fit [CheY-P] distributions for individual cell traces (**Figure 3 – figure supplement 1** and **figure supplement 2**) and compared the wild-type and CheY* strains. As shown in **Figure 3a** and **3b**, while the mean [CheY-P] relative to K, μ_{y_p}/K, from individual cell distributions was similar for both strains, the coefficient of variation, σ_{y_p}/μ_{y_p}, was significantly smaller for the CheY* strain as compared to the wild-type strain ($p \sim 10^{-4}$), as expected. Importantly, the coefficient of variation for individual wild-type cells averaged over N, $\langle\sigma_{y_p}/\mu_{y_p}\rangle \approx 0.24$, was close to that determined over the population. This indicates that cell-to-cell variation over the wild-type cell population does not contribute greatly to the observed width in the [CheY-P] distribution. In contrast, $\langle\sigma_{y^*}/\mu_{y^*}\rangle$ was significantly smaller for individual cells of the CheY* strain, ~0.05, as compared to the population, 0.1. We suspect cell-to-cell CheY^{D13K} expression differences may contribute to the estimated distributions in **Figure 2d-f**. Nevertheless, our results show that we can attribute the width in the [CheY-P] distribution for wild-type cells to fluctuations in [CheY-P] due to signaling activity. Our estimate of CheY-P fluctuations in wild-type cells is consistent with a theoretically predicted lower bound of 20% of the mean[49].

Our method for determining [CheY-P] fluctuations in terms of the functional [CheY-P] concentration allows us to make one interesting observation. Comparing our estimated distribution to the Hill curve relating CW bias to [CheY-P], Eq. (3) (**Figure 2c**), we notice that the peak of the distribution resides at low CW biases, $c < 0.05$, corresponding to CCW flagellar rotation and running a majority of the time. CW rotation and tumbling thus result from fluctuations away from the peak, in the tail of the distribution at higher [CheY-P]. This finding contrasts with the conventional view that the network is poised near the midpoint of the Hill curve[25,36], i.e. with $p(y_P)$ sharply peaked near [CheY-P] $\approx K$, and suggests that fluctuations are a critical determinant of the tumbling frequency.

[CheY-P] fluctuations decrease upon stimulating the network to increase CW bias

Having characterized fluctuations in the chemotactic network in the steady state, we next investigated them in a network stimulated by a perturbation. In response to a stepwise change in chemical environment, the intracellular [CheY-P] changes rapidly, and then reverts to its steady-state value through adaptation[50–52]. Applying a large stepwise decrease in attractant leads to a transient increase in [CheY-P] and flagellar motor CW bias followed by a decrease to the steady state[42,53], allowing us to measure network fluctuations across the sensitive range of the motor CW bias vs. [CheY-P] curve, Eq. (3).

We measured the response of individual cells to a decrease in concentration of α-methyl-DL-aspartate (MeAsp), a non-metabolizable analog of L-aspartate[54] that acts as a chemoattractant[54,55]. We used a three-channel flow chamber (**Figure 4a**) to observe the effect of a step-down from 1 to 0 mM MeAsp on individual optically trapped wild-type cells. The step-down was applied by quickly moving a trapped cell from the middle channel of the flow chamber containing 1 mM MeAsp (**Figure 4a**; labeled "blank + attractant") to the top channel containing no MeAsp (labeled "blank"; see **Materials and Methods**). The run-tumble behavior of the cell was monitored pre- and post-stimulus via the optical traps (see **Materials and Methods**), whereas the flagellar rotation state was measured by fluorescence imaging only post-stimulus.

Figure 4b shows a representative measurement of a cell with 3 flagella undergoing a step-down stimulus at $t = 0$. In response, the cell exhibits the expected increase in tumbling immediately post-stimulus, followed by relaxation to the pre-stimulus steady state as the cell adapts (**Figure 4 – figure supplement 1**). As the probability of tumbling is directly related to the probability of CW flagellar rotation[39], n_{cw} measured by fluorescence imaging post-stimulus also relaxes to lower values as the cell adapts (**Figure 4b**, blue trace). Since the [CheY-P] distribution is time-dependent in cells responding to a step-down, the time traces of n_{cw} for each cell were divided into time windows corresponding to 20% of the cell's adaptation time (**Figure 4c**, colors corresponding to time window), over which the average tumble bias was constant to within ~10-20%. The best fit [CheY-P] distribution $p_{fit}(y_P)$ was estimated separately for each window (see **Materials and Methods** for more details on windowing). All data from cells with the same total

number of flagella (in **Figure 4c**, three representative cells with N = 3 flagella) were pooled in

similarly constructed time windows and used to create histograms $p_N(n_{cw})$ at each time (**Figure**

4d). $p_{fit}(y_P)$ corresponding to each time window was then found as described above (**Figure 4e**,

colored curves). Relative to unstimulated cells (**Figure 4e**, gray curve for unstimulated wild-type

cells with N = 3 flagella), the peak of the [CheY-P] distribution shifted to a higher value

immediately after the stimulus, as expected for a step down. While we expected the standard

deviation to increase as the mean [CheY-P] increased, the width of the [CheY-P] distribution

instead decreased (**Figure 4e**). We verified that the reduction in noise was due to a change in

chemotaxis network dynamics after stimulation rather than due to windowing of the data; the

[CheY-P] distribution widths in adapting cells compared to those in steady-state cells over

similarly sized 3-s time windows were significantly different (p = 0.05).

The time-dependent, post-stimulus mean relative to the unstimulated global mean $\mu_{y_p}/\mu^0_{y_p,\text{glo}}$

and coefficient of variation σ_{y_p}/μ_{y_p} of [CheY-P] for cells with N = 2, 3, 4 flagella are shown in **Figure**

5a and **5b** (colored filled circles), respectively. The corresponding normalized steady-state mean

and coefficient of variation obtained from **Figure 2** are also plotted for reference (for $t < 0$,

colored open circles). As expected, the trends are similar for cells with different numbers of

flagella. Thus, we estimated these parameters from a fit to a global [CheY-P] distribution across

cells with different N for each time window (**Figure 5a** and **5b**; gray filled circles; see **Materials**

and Methods). Consistent with the distributions in **Figure 4d**, the mean [CheY-P] increases

immediately in response to the step-down stimulus and relaxes toward its steady-state, pre-

stimulus value as the cells adapt. At the same time, the coefficient of variation in [CheY-P] falls

post-stimulus, then gradually increases toward the pre-stimulus value.

These results point to two interesting features. First, the mean [CheY-P] shows a 20% increase

compared to the pre-stimulus steady state immediately after the step down (**Figure 5a**). While

the increase in [CheY-P] and the resulting increase in CW bias is expected for a step-down

stimulus, it is smaller in magnitude than predicted by existing models[44,56]. Given the strength of

the stimulus (from 1 to 0 mM [MeAsp]), these models predict a CW bias close to 1, corresponding

to at least a 75% increase in mean [CheY-P]immediately post stimulus; instead, [CheY-P] remains

near the midpoint of the CW bias vs. [CheY-P] curve, $\mu_{y_p} \approx K$ in Eq. (3). Second, fluctuations are significantly reduced immediately after the stimulus is applied, decreasing from $\sigma_{y_p}/\mu_{y_p} = 0.2$ pre-stimulus to as low as $\sigma_{y_p}/\mu_{y_p} = 0.02$ post-stimulus (**Figure 5b**). This value is as low as that measured in individual CheY* cells (**Figure 3b**), suggesting that fluctuations in the network are largely suppressed when driven to high CW bias.

In the above analysis, we assumed the same values for K and H as for steady-state cells (2.4 µM and 18, respectively). However, we expect FliM remodeling to occur in response to the stimulus, leading to a gradual shift in the motor c vs y_P response curve post-stimulus. To determine the size of this effect on the estimates of μ_{y_p} and σ_{y_p}/μ_{y_p}, we carried out simulations of motor remodeling based on the model by Zhang et al.[43] (see **Materials and Methods**). As shown in **Appendix – figure 1**, the estimated μ_{y_p} increases slightly as a consequence of the reported adaptive effect of remodeling[45]. Nevertheless, the qualitative trend of μ_{y_p} vs. time post-stimulus remains. In contrast, σ_{y_p}/μ_{y_p} exhibits only a moderate increase (<15%) when accounting for motor remodeling (**Appendix – figure 1**).

A stochastic model incorporating CheA activity fluctuations reproduces the experimental results

We next sought a stochastic model to describe our experimental results. The intracellular CheY-P is controlled by fast phosphorylation-dephosphorylation reactions,

$$\text{CheY} \underset{k_z}{\overset{k_A a}{\rightleftharpoons}} \text{CheY-P} , \qquad (6)$$

in which the rate of CheY phosphorylation by the CheA-receptor complex is written as $k_A a$, where a is the probability that CheA is in its active state and k_A is the maximum phosphorylation rate[57,58], and in which k_Z is the rate of CheY-P dephosphorylation by the phosphatase CheZ. These rates are expected to be on the order of 10 s^{-1} [56–58]. We first examined if this set of reactions alone could generate the number fluctuations in CheY-P to reproduce the observed noise. Using the chemical master equation (CME), the following analytical solution for the coefficient of variation in [CheY-P] (see **Materials and Methods**) is obtained[59]:

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$$\frac{\sigma_{y_P}}{\mu_{y_P}} = \sqrt{\frac{1}{\mu_{y_P} V_C}\left(1 - \frac{\mu_{y_P}}{y_{tot}}\right)}, \tag{7}$$

where the mean [CheY-P] is given by $\mu_{y_p} = y_{tot} k_A a / (k_A a + k_z)$ and V_C is the *E. coli* cell volume,

assumed to be 1.4 fL[60] ($V_c^{-1} \approx 1.2$ nM).

To compare this model to the experimental data, we plotted in **Figure 5c** the coefficient of

variation vs. the mean [CheY-P] for cells at steady state (open circles) and for stimulated cells

(filled circles). All of the experimental data fall on one continuous curve. (The same pattern is

observed when accounting for motor remodeling on the estimates of μ_{y_p} and σ_{y_p}/μ_{y_p}; **Appendix**

– figure 1d). The curve traced out by the data differs from the model Eq. (7) in two significant

ways. First, for low mean [CheY-P], Eq. (7) (dotted line in **Figure 5c**) predicts a coefficient of

variation σ_{y_p}/μ_{y_p} an order of magnitude smaller than our experimentally determined value.

Fundamentally, the low number fluctuations predicted from Eq. (7) are due to the large total

number (~8,000) of CheY molecules in the cell. Matching our experimental data would require

decreasing the number of CheY by over two orders of magnitude (equivalent to replacing V_C with

a cell volume two orders of magnitude smaller in Eq. (7)). Although a recent study proposed that

CheY sequestration at the cell poles where many signaling proteins reside may limit the number

of CheY-P participating in signaling[61], such a mechanism is unlikely to account for this discrepancy

between Eq. (7) and our data.

Second, the dramatic decrease in coefficient of variation near $\mu_{y_p}/\mu^0_{y_p,glo} \approx 1.2$, which results

from the observed narrowing of the [CheY-P] distributions in stimulated cells (**Figure 4e**), is not

recapitulated in the model. Eq. (7) predicts a drop to zero at $\mu_{y_p} \approx 10$ μM or $\mu_{y_p}/\mu^0_{y_p,glo} \approx 5$, when

all of the CheY becomes phosphorylated. One possibility to account for these observations is a

ceiling of on the amount of CheY available for phosphorylation, leading to a maximum relative

[CheY-P] ≈ 1.2 and a reduction in the width of the [CheY-P] distribution at this concentration. To

test this mechanism, we constructed a mutant *ΔcheZ* strain ("RB03"; see **Materials and Methods**)

lacking the phosphatase CheZ, in which all the available CheY is expected to be phosphorylated.

We replicated our flagellar imaging analysis and calculated the [CheY-P] distribution for this

strain, assuming the same values for K and H as for wild type cells (2.4 μM and 18, respectively).

This yielded a mean [CheY-P]/K of 1.6 ± 0.1, corresponding to a CW bias close to 1 according to

Eq. (3) (data not shown), higher than that observed following a step-down stimulus, ruling out

such a mechanism. This result holds when we account for the effect of motor remodeling in this

strain, which manifests as an increase in K compared to the wild type strain, since the mean

[CheY-P] scales with K.

An alternate, more plausible mechanism is that additional sources of noise contribute to the

two-species model in Eq. (6). One such source is fluctuations in phosphorylation-

dephosphorylation kinetics. Although Eq. (6) assumes constant rates, the activity of the CheA-

receptor complex is thought to fluctuate over time[31,32]. To model CheA fluctuations, we modified

Eq. (6) to allow CheA to interconvert between active and inactive states, leading to a fluctuating

activity $a(t)$. Adapting the approach of Paulsson et al.[62,63], the following analytical solution for the

coefficient of variation in [CheY-P] in the presence of CheA fluctuations is obtained (see **Materials**

and Methods):

374
$$\frac{\sigma_{y_P}}{\mu_{y_P}} = \sqrt{\frac{1}{\mu_{y_P} V_C}\left(1 - \frac{\mu_{y_P}}{y_{tot}}\right)\left[1 + \langle b \rangle\left(1 - \frac{\mu_{y_P}}{\alpha y_{tot}}\right)\right]},$$
(8)

where the factor in brackets represents the contribution of CheA activity fluctuations to CheY-P

noise. Here $\langle b \rangle$ denotes the number of new CheY-P generated during the time periods when

CheA is active. The factor $\alpha \equiv k_A / (k_A + k_Z)$ is the fraction of CheY that is phosphorylated when all

CheA are active, i.e. when the activity a = 1.

This model recapitulates the features of the data well; **Figure 5c** shows a fit to Eq. (8) (dashed

line) with parameters $\langle b \rangle$ = 280 ± 80 and α = 0.240 ± 0.003 (R^2 = 0.6; errors represent 95%

confidence interval). (Fits of the data in **Appendix – figure 1**, which account for motor remodeling

yield an almost identical value of α and a ~70% higher $\langle b \rangle$). Analogous to "burst size" in

stochastic gene expression[64,65], $\langle b \rangle$ increases number fluctuations in CheY-P, matching the data.

The parameter α represents the fact that the contribution of CheA fluctuations to CheY-P noise

must go to zero (as in in **Figure 5c**) when a = 1, at a mean [CheY-P] of $\mu_{y_P} = \alpha y_{tot} \approx$ 2 µM. The value

of α points to a kinetic ceiling on CheA-receptor complexes of k_A = 0.32k_Z, or ~10 s^{-1} assuming a

constant CheY-P dephosphorylation rate of $k_Z = 30\,\text{s}^{-1}$, within the reported range[56–58]. A maximum

rate $k_A = 10\,\text{s}^{-1}$ is consistent with reported values of the CheA auto-phosphorylation rate, the rate-

limiting step for CheY phosphorylation[35,56–58]. (We note that in the $\Delta cheZ$ strain, the kinetic ceiling

on k_A would not cap [CheY-P] since $k_Z = 0$.)

Discussion

In this work we demonstrate a new approach for inferring fluctuations in [CheY-P] based on

the statistics of flagellar switching in multi-flagellated *E. coli* cells. One advantage of this approach

is that its accuracy in quantifying network noise does not depend on temporal averaging, which

limited previous measurements to long time scales (>10 s)[31,33], but instead relies on multiple

flagellar motors making independent simultaneous "measurements" of intracellular CheY-P

levels. This feature allows us to measure [CheY-P] noise at short timescales (0.5-5 s) and to track

the time evolution of signaling noise in cells as they undergo adaptation.

Another advantage of this method is that noise parameters are determined in terms of CheY-

P concentration (relative to K), which allows for direct comparison to theoretical models. In

contrast, prior fluorescence-based studies determine changes in CheA activity relative to a

baseline[32,33]. However, one limit is that our estimate of $p(y_P)$ depends on the relation between

CW bias and [CheY-P], parametrized by K and H in the Hill curve, Eq. (3). As discussed above, we

consider uncertainties and variations in these parameters in our analysis. It is important to

highlight that the relative differences between various [CheY-P] distributions—such as between

different time points in adapting cells—still remain significant (see **Appendix – figure 1**). Thus,

the trends in our results remain valid independent of parameter values.

Previous single-cell measurements of flagellar rotation by bead tracking[31,41] and of CheA

activity using fluorescent reporters[32,33] have reported long-term noise in the chemotaxis network

attributed to CheR-CheB methylation-demethylation dynamics[31] and, recently, new sources such

as receptor clustering and other dynamics[32,33]. A direct comparison between these results and

ours is difficult because of the differences in timescales between the measurements (0.1-40 s vs.

10-1000 s). While our approach extends to fast timescales, photobleaching of the dyes limits the

duration of our traces and limits the overlap between the measurement timescales.

Nevertheless, to attempt such a comparison, we determined how our estimate of CheY-P noise

varies with the measurement time window. As shown in **Figure 5 – figure supplement 1**, the

noise increases with longer windows. This trend is consistent with our short-time measurements

capturing the tail of a longer-timescale noise source. We are unable to determine the timescale

precisely, but estimate that it must be >10 s, consistent with reported values of ~13 s[32,33]. Thus,

it is plausible that these different measurements are consistent and that source of CheY-P noise

are the same. Nevertheless, we have deliberately made our model for CheA activity fluctuations

in Eq. (8) agnostic to the source of noise to allow for alternatives.

The noise characteristics of the chemotaxis network revealed in our measurements may have

functional consequences. Our findings suggest that [CheY-P] fluctuations are critical drivers for

CW flagellar rotation/tumbling in *E. coli*. Our experimentally determined mean [CheY-P]/K of 0.8

restricts the cell to a CW bias <0.05, corresponding to running the majority of the time.

Previously, Terasawa et al.[38] proposed that waves of CheA activity could cause transient increases

in [CheY-P]. Our data and model appear to be consistent with this mechanism, with the "burst

size" $\langle b \rangle$ in Eq. (8) representing such waves in activity. Our fit parameters indicate that ~3.5% of

all CheY are phosphorylated during such events. We envision that transient increases in [CheY-

P], driven by waves of phosphorylation by the receptor kinase complex[38], increases the CW bias

to generate tumbles. In previous work, it was proposed that [CheY-P] fluctuations act to

synchronize flagellar switching[38,39], which was shown to make swimming behavior robust to

variation in flagella number[39]. *E. coli* has also been proposed to exploit long-timescale [CheY-P]

fluctuations to explore larger volumes of space[31,34] and to increase drift speeds, enhancing

migration along increasing attractant gradients[35–37]. Lastly, the kinetic ceiling in receptor kinase

activity which restricts CheY-P fluctuations when the mean [CheY-P] increases could prevent

CheY-P concentrations from saturating and leading to continuous and unproductive CW flagellar

rotation/tumbling. Such behavior could result in increased *E. coli* swimming efficiency down

steep decreases in attractant concentration, as likely found in natural environments.

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Materials and Methods

Microbiology

Cell preparation

Experiments were performed on the HCB1660 *E. coli* strain, a gift from the Berg lab[66]. This strain is considered wild-type for chemotaxis but is *ΔfliC* expressing FliC^{S219C} under the control of the P$_{araBAD}$ promoter. The mutant FliC^{S219C} protein was specifically labelled with a cysteine reactive fluorescent dye, Alexa Fluor 532 C$_5$ Maleimide (A10255, ThermoFisher Scientific) using the method first described by Turner et al.[66] and later used by Mears et al.[39]. The strain referred to as CheY* is PM87, *ΔfliC ΔcheBYZ* expressing FliC^{S219C} and constitutively active CheY^{D13K}, constructed by Mears et al.[39]. The strain referred to as RB03 (constructed for this work) is *ΔfliC ΔcheZ* expressing FliC^{S219C}.

As described by Mears et al.[39], for each experiment, a single colony from an LB, lysogeny broth, agar plate was inoculated into 1 mL tryptone broth (TB: 0.8% [wt/vol] NaCl, 1% Bacto Tryptone), and grown to saturation overnight (14-18 hours), shaking at 265 RPM at 30°C with the appropriate antibiotics. The overnight culture was diluted 100-fold into 12 mL TB, and grown to OD$_{600}$ ~0.4 to 0.5, with 0.01% [wt/vol] or 666 µM L-arabinose, shaking at 265 RPM at 30°C for ~4.5 hours. The over-day culture was washed twice by slow centrifugation (1300 × g) and gently resuspended in 1 mL motility buffer (MB: 10 mM Phosphate Buffer [pH 7.0], 70 mM NaCl and 0.1 mM EDTA)[21].

To label flagella fluorescently, we followed the protocol reported by Mears et al.[39]. An appropriate volume of the Alexa Fluor 532 C$_5$ Maleimide dye was added to resuspended cells in 0.5 mL MB to a concentration of ~0.01 mg/mL and incubated with slow rotation (~10 RPM) at room temperature in the dark for 90 min. The labeled cells were gently resuspended in 1 mL MB. For trapping, cells were diluted 20-fold in trap motility buffer (TMB: 70 mM NaCl, 100 mM Tris-Cl, 2% [wt/vol] glucose, with 0.1 mM methionine and an oxygen-scavenging system [290 µg ml^{-1} pyranose oxidase and 65 µg ml^{-1} catalase][40]) and injected into the flow chamber for the trapping experiment. When handling resuspended cells, at all times, we avoided pipetting, or we used wide orifice pipette tips to avoid shearing flagella.

1 mg of the dry Alexa Fluor 532 C_5 Maleimide dye was dissolved in 250 µL water by vortexing

and stored in aliquots at -20°C. The stock concentration for each aliquot was determined using

UV-Visible Absorption Spectroscopy. Since this dye does not dissolve readily in water, we found

it important to determine the concentration for each aliquot separately for optimal labeling.

Construction of strains

The bacterial strains and plasmids used for this work are listed in **Table 1**. Oligonucleotides for

generating mutations and creating plasmids are listed in **Table 2.** All primers were purchased

from Integrated DNA Technologies (Coralville, Iowa). The *CheZ* chromosomal deletion was carried

out using the standard λ Red recombinase system developed by Datsenko and Wanner[67]. The

strain referred to as RB03 was created from HCB1613 (AW405, chemotaxis wild-type with a

functional FliC deletion). To create the strain RB01, *CheZ* was replaced by a chloramphenicol

resistance cassette with flanking FRT (Flp recombinase-recombination target) sites using the

primers CheZ-pKDF and CheZ-pKDR with pKD3 as a template. This replacement was accomplished

via the λ Red recombinase enzymes expressed from pKD46. The chloramphenicol cassette was

eliminated via Flp recombinase expressed from pCP20 to obtain strain RB02. Finally, the strain

RB03 was created by transforming strain RB02 with the plasmid pBAD33 that expresses FliC[S219C].

Standard molecular cloning techniques were used to purify plasmids and perform PCRs.

Data Acquisition

Optical trapping of individual *E. coli* with simultaneous fluorescence imaging

Experiments were performed with a dual-trap optical tweezers instrument combined with epi-

fluorescence slim-field imaging, developed and described in detail in previous work by Mears et

al.[39]. Briefly, the optical traps were created from a single 5-W, 1064-nm laser (YLR-5-1064-LP, IPG

Photonics). Two trapping beams were generated by timesharing, intermittently deflecting the

input beam between two positions with an Acousto-Optic Deflector, AOD (DTSXY-250-1064, AA

Opto-Electronic). The two resulting beams were then tightly focused using a 60X, water

immersion (1.2 NA) objective (Nikon, Tokyo, Japan) to create two optical traps. An identical

objective was used to collect the transmitted trap light for position detection and for bright-field

imaging of the cells using Koehler illumination from a blue LED. A bacterial cell could be captured

and aligned between the two optical traps without impeding its rotatory motion. The light

scattered by the trapped cell was collected onto a position sensitive photodetector and used to

monitor cell motion, which was analyzed to determine runs and tumbles as described below and

previously by Min et al.[40].

Fluorescently labeled flagella of trapped cells were imaged using a 532-nm laser (TECGL-30,

World Star Tech) aligned for slim-field illumination[68], with the excitation beam diameter at the

sample plane ~10-15 μm. We strobed the fluorescence excitation and trap laser illumination out

of phase with each other to minimize photobleaching[39]. The fluorescence illumination was

strobed by using an Acousto-Optic Modulator, AOM (802AF1, IntraAction) to deflect the

excitation light intermittently into the sample plane for a duration of 16-20 μs each cycle while

the traps were off. An EMCCD camera (iXon3 860 EMCCD, Andor) recorded images in synchrony

with the excitation pulse at the rate of 100 frames per second to capture focused images of the

flagella as they rotated at a rate of ~100 Hz. In this manner, movies of flagellar motion were

recorded, typically for ~30-40 s until the flagella photobleached. Movies were saved using the

Solis software (Andor) and processed using custom written code in MATLAB. Each movie was

analyzed manually to count flagella and their transitions between CW and CCW rotating states.

Motility assays combined with fluorescence imaging in trapped cells

Custom-built microfluidic chambers shown in **Figure 1c** and **4a** (preparation described in detail

in Mears et al.[39] and Min et al.[42]) were used to perform motility assays coupled with fluorescence

imaging of labeled flagella on individual trapped cells. To prepare chambers, glass coverslips were

sonicated in acetone, then rinsed in deionized water. This step was followed by rinsing in

methanol and drying under nitrogen flow. The flow channels in the chamber were cut out from

Nescofilm (Alfresa Pharma Corporation, Osaka, Japan) and bonded to glass coverslips as

described previously[39,42].

For measurements of free-swimming, or unstimulated, cells, the chamber contained two

channels (**Figure 1c**). The upper channel contained trap motility buffer (TMB) only, while the

lower channel contained *E. coli* cells in TMB. Both channels were continuously injected with the

appropriate solutions using a syringe pump (PHD2000, Harvard Apparatus) with a linear flow rate

of ~30 μm/s. Cells were trapped in the lower channel and moved to the upper channel by

displacing the chamber with respect to the traps using a motorized three-axis translational stage

(ESP301; Newport, Irvine, California). Once the trapped cell was aligned between the two traps

in the upper channel, the bright-field illumination was turned off and the 532-nm excitation

illumination was turned on. Cell motion was observed by simultaneously recording the optical

trap signal sensed by position sensitive photodetectors and the fluorescence signal from labelled

flagella sensed by the EMCCD camera.

For measurements of stimulated cells undergoing chemotactic adaptation, the chamber

contained three channels (**Figure 4a**). For a step-down stimulus, the top ("blank") channel

contained TMB only, the middle ("blank + attractant") channel contained TMB, 1 mM α-methyl-

DL-aspartate (MeAsp) and Rhodamine B (100 nM), and the bottom ("cell + attractant") channel

contained TMB, 1 mM MeAsp and *E. coli* cells. Fluorescence imaging of the Rhodamine B diffusion

profile was used to characterize the gradient formed across the top and the middle channel, as

described by Min et al.[42]. All three channels of the flow chamber were injected with their

solutions at a linear flow rate of ~50 μm/s. In a typical experiment, a swimming cell was first

captured from the "cells + attractant" channel. The trapped cell was moved to the "blank +

attractant" channel, aligned between the two traps, and oriented along the direction of the flow.

Pre-stimulus optical trap signal of the swimming cell was recorded for ~200 s (without any

fluorescence imaging of flagella). The trapped cell was then moved at the speed 100 μm/s to the

"blank" channel so that it experienced a step down in MeAsp concentration over a ~4-s duration

(as shown in Figure S1 in Min et al.[42]). During this move, the bright field illumination was turned

off and the 532-nm excitation illumination was turned on. The chemotactic response of the

captured cell was recorded in the "blank" channel by simultaneously recording the optical trap

signal and the fluorescence signal from labelled flagella. While the trap signal post-stimulus was

recorded for ~400 s, the EMCCD signal was recorded only for ~30-40 s until the flagella

photobleached.

Raw Data Analysis

Run-tumble analysis for optical trap data

Cell motion was detected from imaging the optical trap light scattered by the cell body onto position-sensitive photodetectors, as described previously by Min et al.[40]. This optical trap signal was analyzed for runs and tumbles. Briefly, runs corresponded to oscillatory signals due to cell body roll at a frequency ~10 Hz, while tumbles corresponded to erratic trap signals with a frequency <1 Hz. We used custom-written MATLAB code incorporating a wavelet analysis to identify the peak body roll frequency at every time point of each data trace and to assign runs and tumbles by specifying an appropriate frequency threshold[40]. An example trace of the trap signal and the corresponding run-tumble assignment is shown in **Figure 1c** and **d** in earlier work by Mears et al.[39].

Image analysis for fluorescence data

Images acquired using the high-speed fluorescence imaging described above were adjusted for contrast to best visualize the flagella. Each movie was played in slow motion and Clockwise (CW)/Counter Clockwise (CCW) states were assigned manually to each flagellum over a time window of 100 ms (i.e. 10 frames). As described previously[21,39], the CCW/CW rotation state of the flagella can be visually identified from their different helical waveforms, commonly referred to as 'normal', 'semi-coiled', 'curly-1' and 'curly-2'. CCW-rotating flagella exclusively adopt the 'normal' waveform and CW-rotating flagella adopt the 'semi-coiled' and 'curly-1' waveforms the majority of the time. Waveform transitions take less than ~0.5 s to propagate through the flagella.

Modeling and Analysis

Extracting [CheY-P] distributions from flagellar rotation data

In flagellar imaging movies of individual cells, we tracked the CW/CCW state of each flagellum—hence the number of CW flagella in time—and determined $p_N(n_{cw})$, the probability that n_{cw} out of a total of N flagella rotate CW, where $n_{cw} = 0, 1... N$. From first principles[69], we can relate $p_N(n_{cw})$ to the instantaneous CW bias of the motor, c through Eq. (1), where $p(c)$ is the distribution in

motor bias c, the probability that a motor rotates CW. The motor bias takes values between 0

and 1. $p_N(n_{cw}|c)$, the conditional probability that n_{cw} out of N flagella are CW given a certain bias

c, is given by the binomial distribution in Eq. (2), since as shown in previous work by Mears et

al.[39], the flagellar motors switch independently.

We can thus use Eq. (1) and (2) to estimate the motor bias distribution $p(c)$ from

measurements of $p_N(n_{cw})$. In general, this integral equation cannot be solved by a unique solution

$p(c)$; instead, $p(c)$ becomes more tightly determined as the number of flagella N increases. This

can be seen by noting that $p_N(n_{cw}|c)$ in Eq. (2) is a polynomial in c of order N. Therefore, the

integral Eq. (1) for $p_N(n_{cw})$ yields a sum over the first N moments of the distribution $p(c)$,

$$p_N(n_{cw}) = \sum_{r=n_{cw}}^{N} \frac{N!}{n_{cw}!(r-n_{cw})!(N-r)!}(-1)^{r-n_{cw}} \int_0^1 c^r p(c)dc$$

$$= A_{n_{cw},n_{cw}}^{(N)}\left\langle c^n \right\rangle + A_{n_{cw},n_{cw}+1}^{(N)}\left\langle c^{n+1} \right\rangle + \cdots A_{n_{cw},N}^{(N)}\left\langle c^N \right\rangle = \sum_{r=0}^{N} A_{n_{cw},r}^{(N)}\left\langle c^r \right\rangle$$

(9)

where each moment is given by $\left\langle c^r \right\rangle = \int_0^1 dc\, c^r p(c)$ and the $(N+1) \times (N+1)$ upper triangular matrix

$A_{n_{cw},r}^{(N)}$ is defined in the second line. This matrix equation can be inverted to give $\left\langle c^r \right\rangle$ in terms of

a sum of $p_N(n_{cw})$ over n_{cw}. For a cell with N flagellar motors, inversion of Eq. (9) yields only the

first N moments of the distribution $p(c)$, $\left\langle c \right\rangle$, $\left\langle c^2 \right\rangle$, … $\left\langle c^N \right\rangle$. Short of knowing every moment, the

distribution $p(c)$ can only be approximated using the finite number of moments. Nevertheless,

the first two moments are generally sufficient to assess the overall shape of the distribution,

which is adequate for our purposes, and we analyze cells with $N \geq 2$.

An alternate formulation is to express the integral equation in terms of the concentration of

phosphorylated CheY, [CheY-P] is Eq. (4) where y_P is the cellular [CheY-P] and $p(y_P)$ is the

distribution of [CheY-P]. y_P can take values between 0 to y_{tot}, the total cellular [CheY]. The

conditional probability $p_N(n_{cw}|y_P)$ can be obtained from $p_N(n_{cw}|c)$ in Eq. (2) by making use of

the relation between the motor bias c and y_P determined by Cluzel et al., Eq. (3) [25]. The

conditional probability $p_N(n_{cw}|y_P)$ is obtained by combining Eq. (2) and Eq. (3) to yield Eq. (5).

We can transform between $p(c)$ and $p(y_P)$ using the following relationship:

617
$$p(y_P) = p(c)\frac{dc}{dy_P} \, ,$$

618   which, using Eq. (3) gives

619
$$p(y_P) = p(c)\frac{HK^H y_P^{H-1}}{\left(K^H + y_P^H\right)^2} \, .$$
(10)

Local and global fitting with trial distributions and error analysis

Since the form of the distribution $p_N(n_{cw})$ depends on the number of flagella, we constructed

separate distributions for cells with different numbers of flagella N, as shown in **Figure 2a** and **d**.

[CheY-P] distributions were estimated for each data set by assuming a trial distribution function

p_{fit} (of either motor bias, c, or [CheY-P], y_P) with a small number of shape parameters (two),

evaluating $p_{N,fit}(n_{cw})$ using the integral in Eq. (1) or (4), and finding the shape parameters that

best fit the experimentally observed distribution $p_{N,obs}(n_{cw})$. Without a priori knowledge of the

appropriate form for the trial distribution function p_{fit}, we tested three different functions. One

was a beta distribution in c,

630
$$p_{fit}(c) = \frac{(1-c)^{\beta-1} c^{\alpha-1}}{B(\alpha,\beta)} \, ,$$
(11)

where $B(\alpha,\beta) = \int_0^1 (1-x)^{\beta-1} x^{\alpha-1} dx$ is the beta function and the shape parameters $\alpha, \beta > 0$. The

beta distribution was chosen because it is defined over the range 0 to 1, the same as for the

motor bias c. α and β were the fitting parameters and used to determine the mean and variance

of the motor bias, $\langle c \rangle = \dfrac{\alpha}{\alpha+\beta}$ and $\langle c^2 \rangle - \langle c \rangle^2 = \dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$, respectively. To determine

the corresponding mean, μ_{y_P}, and standard deviation, σ_{y_P}, of the [CheY-P] distribution, we first

calculated the probability density function $p(y_P)$ from $p(c)$ according to Eq. (10) and then

evaluated the mean and standard deviation numerically from their definitions.

For comparison, we also tested Gaussian and gamma trial distributions for $p_{fit}(y_P)$, using the

alternate formulation described in Eq. (4). For the Gaussian distribution,

$$p_{fit}(y_P) = \frac{1}{\sqrt{2\pi\sigma_{y_P}^2}} \exp - \frac{(y_P - \mu_{y_P})^2}{2\sigma_{y_P}^2}. \tag{12}$$

641 The fitting parameters were the mean and standard deviation, μ_{y_P} and σ_{y_P}. For the gamma

642 distribution,

$$p_{fit}(y_P) = \frac{1}{\Gamma(k)\theta^k} y_P^{k-1} \theta^{-\frac{y_P}{\theta}}, \tag{13}$$

644 the fitting parameters were the shape parameters k and θ, with the mean and the standard

645 deviation in [CheY-P] corresponding to $\mu_{y_P} = k\theta$ and $\sigma_{y_P} = \sqrt{k\theta^2}$, respectively.

646 We found the shape parameters that best fit the experimentally observed distribution

647 $p_{N,obs}(n_{cw})$ using a χ^2 minimization routine, where the figure of merit χ_N^2 was defined as:

$$\chi_N^2 = \sum_{n=0}^{N} \frac{N_{data}(p_{N,fit}(n_{cw}) - p_{N,obs}(n_{cw}))^2}{p_{N,fit}(n_{cw})}, \tag{14}$$

where N_{data} is the number points used to construct each distribution $p_{N,obs}(n_{cw})$. We report the

reduced χ^2 in **Table 3**, calculated by dividing χ_N^2 in Eq. (14) by the degrees of freedom, which in

our case is the difference between the number of independent data points in each distribution

$p_{N,obs}(n_{cw})$ and the number of independent parameters used to determine $p_{N,fit}(n_{cw})$. For

example, for cells with $N = 3$ flagella, the number of independent data points in $p_{N,obs}(n_{cw})$ is 3,

and the number of independent fitting parameters used to estimate $p_{N,fit}(n_{cw})$ is 2, hence the

degrees of freedom is 1. We note that for $N = 2$, the degrees of freedom is 0 and thus a reduced

χ^2 is undefined. This reflects the fact that cells with two flagella determine exactly two moments

of the fitting distribution function, and thus the two shape parameters. For a population of cells

with the same number of flagella N, we estimated the error in the calculated [CheY-P] distribution

and hence its mean and standard deviation by bootstrapping, where individual cell time traces

were resampled with replacement and input into the χ^2 minimization routine.

In addition to calculating [CheY-P] distributions separately for each number of flagella N, we

estimated a single global [CheY-P] distribution that fit all datasets across all N. Corresponding to

this distribution, we found a single set of parameters that best fit $p_{N,obs}(n_{cw})$ separately for each N

by minimizing a combined χ^2

$$\chi^2_{global} = \frac{\sum_{N=2}^{5}(N+1)\chi^2_N}{\sum_{N=2}^{5}N+1} ,$$ (15)

where χ^2_N is calculated according to Eq. (14). The reduced χ^2 for this global fitting was calculated

as described above, with the degrees of freedom equal to 12 (equal to the total number of

independent data points in $p_{N,obs}(n_{cw})$ over all N, 14, minus the number of independent fitting

parameters used to estimate $p_{N,fit}(n_{cw})$, 2). To estimate the error in the global [CheY-P]

distribution, we applied the global χ^2 minimization routine similar to above, and bootstrap

resampled all datasets across all flagellar numbers N (as opposed to resampling over each N

separately). We then sorted the resampled data by N for separate analysis using Eq. (1) or (4).

Finally, we also calculated CheY-P distributions for each individual cell for each number of

flagella N. We estimated the error in the individual cells' CheY-P distributions using bootstrapping

as described above. 0.5-s long time windows of individual cell n_{cw} time traces were resampled

with replacement to construct traces for each bootstrapping trial, and the corresponding

$p_{N,obs}(n_{cw})$ was input into the χ^2 minimization routine. A 0.5-s window duration was chosen based

on the average autocorrelation time for individual cell n_{cw} time traces, determined to be ~0.3 s

across N. Time windows of duration of 0.5 s, about twice as long as the autocorrelation time, are

sufficiently long to be uncorrelated and suitable for the purposes of bootstrapping.

Figure 2 – figure supplement 2 compares examples of the three trial distribution functions

$p_{fit}(y_P)$ that best fit the same data set, and **Table 3** displays the corresponding best fit values of

the mean and standard deviation μ_{y_P} and σ_{y_P} for the three trial functions. All three yielded

similarly shaped best fit distributions, and similar values for μ_{y_P} and σ_{y_P}, although the beta- and

Gaussian-based trial distributions, Eq. (11) and (12), consistently yielded better fits than the

gamma-based distribution Eq. (13) (compare χ^2 in **Table 3**).

For the results reported in **Figure 2-5** and their supplements, we used the beta trial function to estimate the [CheY-P] distribution. We also estimated [CheY-P] distributions for the mutant *ΔcheZ* strain lacking the phosphatase CheZ ("RB03), in which all the available CheY is expected to be phosphorylated. We found that the beta distribution trial function was not well-behaved for this extreme case, hence the result reported for this strain is for a Gaussian trial function.

Custom statistical test

In order to make comparisons between sets of data points each with an error bar (for example, in **Figure 3**), standard statistical tests such as the t-test and Wilcoxon rank sum test could not be used. Hence a custom statistical test based on the permutation test[70] was written in order to make such comparisons. The null hypothesis for this test was that the observed means of the two data sets are equal. First, surrogate data sets were created under the assumption of this null hypothesis. Surrogate data for each data point in the two data sets being compared was created by random sampling from a gamma distribution with mean equal to the value of the data point and the standard deviation equal to the value of the error bar on the data point. Next, the labels on each point in the surrogate data were randomly permuted to create two separate surrogate data sets with the same number of points as the ones being compared. For each such pair of surrogate data sets, a test statistic under the null hypothesis was calculated as the difference of their means. For 50,000 such surrogate data sets, the probability distribution of the test statistic under the null hypothesis was plotted. The P-value for this custom statistical test was calculated as the probability of finding the value of the test statistic, i.e. difference between means under the null hypothesis, that is greater than the observed difference of means.

Estimating [CheY-P] distributions for stimulated cells

The flagellar imaging data for trapped cells experiencing a step-down in MeAsp were also analyzed to record the number of CW flagella with time. Since the motor bias varies with time during adaptation, each individual cell time trace was divided into non-overlapping windows the duration of which was determined from the cell's adaptation time. For each cell, the collected optical trap signal was analyzed for runs and tumbles, and the tumble bias was calculated from

the fraction of time the cell tumbled over a 10-s sliding time window. The time trace of the tumble

bias was fit to the following expression:

$$TB(t) = TB_\infty + ae^{-bt} + ce^{-dt} \tag{16}$$

where a, b, c and d are fitting parameters, and TB_∞ is the tumble bias of the cell when it has

adapted to the stimulus. The adaptation time was approximated as $t_{adap} = \ln(2)/b$ following a

procedure used by Min et al.[42] An example fit is shown in **Figure 4 – figure supplement 1.**

Windows of duration $0.2t_{adap}$ were selected starting at the time of the stimulus, $t = 0$, until t

$= 1.6t_{adap}$, where t_{adap} for our dataset was found to be ~3s. A final window of variable length

between $0.5t_{adap}$ and $2.5t_{adap}$ was created based on the number of data points left in the trace.

Data for number of CW flagella n_{cw} from multiple cells with the same total number of flagella N

were pooled in each time window. The time-dependent histogram $p_{N,obs}(n_{cw}, t)$ was constructed

and the time-dependent [CheY-P] distribution $p_{fit}(y_P, t)$ was determined for each time window.

The error in $p_{fit}(y_P, t)$ was calculated by bootstrap resampling as before, over each time window.

We calculated a global [CheY-P] distribution for each time window across all N as described

above. For the results from this analysis reported in **Figure 4** and **5**, we used the beta distribution

trial function to estimate $p_{fit}(y_P, t)$.

Analytical model of CheY-P noise with no CheA fluctuations

We used a stochastic chemical kinetics approach to model CheY phosphorylation and CheY-P

dephosphorylation reactions of the *E. coli* chemotaxis network and determine the fluctuations in

CheY-P. While various models[56,57] describe a wider range of reactions in the chemotactic signaling

network, we chose the simplest possible scenario to verify that it could recapitulate the essential

features of our experimental results.

In our model, we considered the following chemical reaction,

$$CheY \underset{k_z}{\overset{k_A a}{\rightleftharpoons}} CheY\text{-}P \tag{17}$$

where $k_A a$ is the rate of phosphorylation of CheY by the CheA-receptor complex, and k_z is the

rate of dephosphorylation of CheY-P by the phosphatase CheZ. As is standard practice[57,58], we

defined the activity a of the CheA-receptor complex as the probability that it is in the active state

($0 \le a \le 1$) and k_A as the maximum phosphorylation rate. We initially assumed that both $k_A a$ and

k_z remain constant.

We used a chemical master equation (CME) approach to determine number fluctuations in

CheY-P for this set of reactions. The individual reactions in terms of the molecular species are:

Phosphorylation of CheY: $N_{Yp} \xrightarrow{k_A a(N_Y^{tot} - N_{Yp})} N_{Yp} + 1$

Dephosphorylation of CheY-P: $N_{Yp} \xrightarrow{k_z N_{Yp}} N_{Yp} - 1$

where N_{Yp} is number of CheY-P molecules, and N_Y^{tot} is the total number of (unphosphorylated

and phosphorylated) CheY molecules, the latter assumed to be fixed. The CME corresponding

to these reactions is given by

753
$$\frac{dP_{N_{Yp}}}{dt} = k_A a (N_Y^{tot} - N_{Yp} + 1) P_{N_{Yp}-1} + k_z (N_{Yp} + 1) P_{N_{Yp}+1} - \left[k_z N_{Y_p} + k_A a (N_Y^{tot} - N_{Yp}) \right] P_{N_{Yp}}, \qquad (18)$$

754   where $P_{N_{Yp}}(t)$ is the probability that the number of CheY-P molecules at time t is N_{Yp}.

755   Following the approach of McQuarrie and Gillespie[59], the CME Eq. (18) can be solved

756   analytically to yield the following steady-state mean and variance in CheY-P number,

757
$$\left\langle N_{Yp} \right\rangle = \frac{k_A a}{k_A a + k_z} N_Y^{tot}, \qquad (19)$$

758
$$\sigma_{N_{Yp}}^2 = \frac{k_z k_A a}{(k_A a + k_z)^2} N_Y^{tot} = \left\langle N_{Yp} \right\rangle \left(1 - \frac{\left\langle N_{Yp} \right\rangle}{N_Y^{tot}} \right). \qquad (20)$$

759   Eqs. (19) and (20) can be combined to obtain the coefficient of variation

760
$$\frac{\sigma_{N_{Yp}}^2}{\left\langle N_{Yp} \right\rangle^2} = \frac{1 - f_{Yp}}{\left\langle N_{Yp} \right\rangle}, \qquad (21)$$

761   where $f_{Yp} \equiv \left\langle N_{Yp} \right\rangle / N_{Yp}^{tot}$ is the fraction of total CheY that is phosphorylated. Eq. (21) is the expected

762   result for a binomial-distributed process. Eq. (19) and (20) can be converted into expressions for

763   the mean and variance in CheY-P concentration by dividing by the *E. coli* cell volume V_c, assumed

764   to be 1.4 fL[60]: $\mu_{Yp} = \left\langle N_{Yp} \right\rangle / V_c$ and $\sigma_{Yp}^2 = \sigma_{N_{Yp}}^2 / V_c^2$.

Analytical model of CheY-P noise with CheA fluctuations

We next considered the same phosphorylation-dephosphorylation chemical reaction, Eq. (17), with temporal fluctuations in CheA activity. We devised a simple model in which CheA interconverts between active (ON) and inactive states (OFF) with rates $\lambda_A^\pm$,

$$\text{CheA}_{\text{OFF}} \underset{\lambda_A^-}{\overset{\lambda_A^+}{\rightleftharpoons}} \text{CheA}_{\text{ON}}. \tag{22}$$

The individual reactions in Eqs. (17) and (22) in terms of each molecular species are now:

Activation of CheA: $\qquad N_A \xrightarrow{\lambda_A^+(N_A^{tot}-N_A)} N_A + 1$

Deactivation of CheA: $\qquad N_A \xrightarrow{\lambda_A^- N_A} N_A - 1$

Phosphorylation of CheY: $\qquad N_{Yp} \xrightarrow{\kappa_A N_A (N_Y^{tot}-N_{Yp})} N_{Yp} + 1$

Dephosphorylation of CheY-P: $\qquad N_{Yp} \xrightarrow{k_z N_{Yp}} N_{Yp} - 1$

where N_A and N_A^{tot} are the number of active CheA molecules and the total number of CheA molecules in the cell, respectively. Here, we made explicit the dependence of the phosphorylation rate on the number of active CheA, N_A, defining κ_A such that $\kappa_A N_A = k_A a$, with the activity $a = N_A / N_A^{tot}$ and the maximum phosphorylation rate $k_A = \kappa_A N_A^{tot}$.

From these reactions, the CME for both molecular species is given by:

$$\begin{aligned}
\frac{dP_{N_A,N_{Yp}}}{dt} &= \lambda_A^+(N_A^{tot} - N_A + 1)P_{N_A-1,N_{Yp}} + \lambda_A^-(N_A + 1)P_{N_A+1,N_{Yp}} \\
&\quad + \kappa_A N_A(N_Y^{tot} - N_{Yp} + 1)P_{N_A,N_{Yp}-1} + k_z(N_{Yp} + 1)P_{N_A,N_{Yp}+1} \\
&\quad - \left[\lambda_A^+(N_A^{tot} - N_A) + \lambda_A^-(N_A) + \kappa_A N_A(N_Y^{tot} - N_{Yp}) + k_z(N_{Yp})\right]P_{N_A,N_{Yp}}
\end{aligned} \tag{23}$$

where $P_{N_A,N_{Yp}}(t)$ is the joint probability that there are N_A active CheA-receptor complexes and N_{Yp} CheY-P molecules at time t.

The reaction scheme in Eqs. (17) and (22) maps naturally into stochastic models of gene expression, where genes can switch from inactive to active (similar to CheA-receptor complexes), and active genes produce mRNA transcripts (similar to CheY phosphorylation). Paulsson[62,63] has developed a systematic formalism for calculating number fluctuations in gene expression, which

788   we adapted for our reaction scheme. The steady-state mean numbers of active CheA and CheY-

789   P are easily shown to be given by

$$\frac{\langle N_A \rangle}{N_A^{tot}} = a = \frac{\lambda_A^+}{\lambda_A^+ + \lambda_A^-},$$ (24)

$$\frac{\langle N_{Yp} \rangle}{N_Y^{tot}} = f_{Yp} = \frac{k_A a}{k_A a + k_z},$$ (25)

792   where Eq. (25) is identical to Eq. (21) above, as expected. For number fluctuations, Paulsson

793   provides the following general solutions[63]:

$$\frac{\sigma_A^2}{\langle N_A \rangle^2} = \frac{1}{\langle N_A \rangle H_{AA}},$$

$$\frac{\sigma_{Yp}^2}{\langle N_{Yp} \rangle^2} = \frac{1}{\langle N_{Yp} \rangle H_{YY}} + \frac{\sigma_A^2}{\langle N_A \rangle^2} \left(\frac{H_{YA}}{H_{YY}} \right)^2 \frac{H_{YY}/T_Y}{H_{YY}/T_Y + H_{AA}/T_A},$$

796   where the logarithmic gains H_{ij} and times T_i are defined in Eq. (S5) and (S6) in Ref. [17]. Evaluating

797   these factors for the CME in Eq. (23), these expressions simplify into

$$\frac{\sigma_A^2}{\langle N_A \rangle^2} = \frac{1 - a}{\langle N_A \rangle},$$ (26)

799   which is expected for a binomial-distributed process, and

$$\frac{\sigma_{Yp}^2}{\langle N_{Yp} \rangle^2} = \frac{1 - f_{Yp}}{\langle N_{Yp} \rangle} \left(1 + \frac{\langle N_{Yp} \rangle}{\langle N_A \rangle} \frac{\tau_A}{\tau_A + \tau_Y} (1 - a)(1 - f_{Yp}) \right),$$ (27)

801   where $\tau_A = (\lambda_A^+ + \lambda_A^-)^{-1}$ is the characteristic relaxation timescale for CheA activation-deactivation,

802   and $\tau_Y = (k_A a + k_z)^{-1}$ is the timescale for CheY phosphorylation-dephosphorylation. Note that in

803   the absence of CheA noise, Eq. (21) is recovered from Eq. (27).

804   From the steady-state solutions for the means of the molecular species, Eq. (24) and (25), we

805   obtained, after some algebra

$$\frac{\sigma_{Yp}^2}{\langle N_{Yp} \rangle^2} = \frac{1 - f_{Yp}}{\langle N_{Yp} \rangle} \left[1 + \langle b \rangle \left(1 - \frac{f_{Yp}}{\alpha} \right) \right],$$ (28)

in which we defined $\alpha \equiv k_A / (k_A + k_Z)$ and $\langle b \rangle \equiv N_Y^{tot} k_A \tau_{eff} / N_A^{tot}$, and where $\tau_{eff} = (\tau_A^{-1} + \tau_Y^{-1})^{-1}$. We

note that the first factor in Eq. (28) is the coefficient of variation in the absence of CheA

fluctuations. The second factor describes how CheA noise increases CheY-P number fluctuations.

In analogy with transcriptional bursting[64,65], the parameter $\langle b \rangle$ represents the "burst size", the

number of new CheY-P molecules generated per CheA within its active lifetime. The other

parameter, α, describes the effect of a kinetic ceiling of CheA on CheY-P number fluctuations.

f_{Y_p} / α is the number of CheY-P molecules when all CheA-receptor complexes are active ($a = 1$),

i.e. when the system phosphorylates at its maximum possible rate. When CheA-receptor

complexes attain this kinetic ceiling, the added CheY-P fluctuations due to CheA noise drops to

zero in Eq. (28), as expected from Eq. (26).

Table 1

Strains and Plasmids used in this study

Strain	Genotype	Comments	Source
HCB1613	*fliC*::Tn5 (Kan^R)	wild type "deleted" for fliC	Gift of Berg lab
HCB1660	*fliC*::Tn5 (Kan^R)	wild type "deleted" for fliC expressing pBAD33- fliC^{S219C}	Gift of Berg lab
RB01	*cheZ*::Cm, *fliC*::Tn5 (Kan^R)	N/A	This work
RB02	*cheZ*::FRT, *fliC*::Tn5 (Kan^R)	N/A	This work
RB03	*cheZ*::FRT, *fliC*::Tn5 (Kan^R)	wild type "deleted" for fliC and cheZ expressing pBAD33- fliC^{S219C}	This work
Plasmids			
pKD3		Template for Cm^R cassette	[67]
pKD46		Plasmid expressing λ Red recombinases	[67]
pCP20		Plasmid expressing Flp recombinase	[67]
pBAD33	*fliC^{S219C}* under P$_{araBAD}$ promoter, Cm^R, p15a origin	Expresses mutant version of FliC for fluorescence labelling	[66]gift of Berg lab

820 **Table 2**

821 Primers used in this work

Primer	Sequence
CheZ-pKDF	GGAAAAACTCAACAAAATCTTTGAGAAACTGGGCATGTGAG

	GATGCGACTGTGTAGGCTGGAGCTGCTTC
CheZ-pKDR	TTATCAGACCGCCTGATATGACGTGGTCACGCCACATCAGGC AATACAAACATATGAATATCCTCCTTA

Table 3

Distribution and fitting parameters for the [CheY] distribution $p_{fit}(y_P/K)$ for the wild type strain with different trial functions

Distri-bution	Parameter	$N = 2$	$N = 3$	$N = 4$	$N = 5$	Global
Beta	μ_{y_p}/K	0.84 ± 0.02	0.80 ± 0.04	0.70 ± 0.06	0.86 ± 0.03	0.80 ± 0.02
	σ_{y_p}/K	0.16 ± 0.02	0.17 ± 0.02	0.20 ± 0.03	0.12 ± 0.02	0.17 ± 0.02
	Reduced χ^2	Undefined	1.9	7.4	5.4	6.7
Gauss	μ_{y_p}/K	0.86 ± 0.02	0.83 ± 0.02	0.77 ± 0.04	0.88 ± 0.03	0.83 ± 0.02
	σ_{y_p}/K	0.13 ± 0.01	0.12 ± 0.01	0.13 ± 0.02	0.11 ± 0.03	0.13 ± 0.02
	Reduced χ^2	Undefined	0.63	12.6	5.1	6.7
Gamma	μ_{y_p}/K	0.83	0.81	0.73	0.85	0.78
	σ_{y_p}/K	0.19	0.18	0.18	0.19	0.18
	Reduced χ^2	Undefined	39.5	44.1	56	9.9

Table 4

Distribution parameters for the [CheY] distribution $p_{fit}(y_P)$ for the wild type strain

N	Mean [CheY-P], μ_{y_p}/K	Standard deviation in [CheY-P], σ_{y_p}/K	Coefficient of Variation, σ_{y_p}/μ_{y_p}
2	0.84 ± 0.02	0.16 ± 0.02	0.19 ± 0.02

3	0.80 ± 0.04	0.17 ± 0.02	0.21 ± 0.04
4	0.70 ± 0.06	0.20 ± 0.03	0.29 ± 0.07
5	0.86 ± 0.03	0.12 ± 0.02	0.14 ± 0.03
Global	0.80 ± 0.02	0.17 ± 0.02	0.22 ± 0.03

830 **Table 5**

831 Distribution parameters for the [CheY] distribution $p_{fit}(y^*)$ for the mutant CheY* strain

N	Mean [CheY-P], μ_{y^*}/K	Standard deviation in [CheY-P], σ_{y^*}/K	Coefficient of Variation, σ_{y^*}/μ_{y^*}
2	0.94 ± 0.04	0.13 ± 0.04	0.14 ± 0.04
3	0.93 ± 0.02	0.06 ± 0.01	0.06 ± 0.02
4	0.91 ± 0.02	0.06 ± 0.02	0.06 ± 0.02
5	0.84 ± 0.04	0.09 ± 0.03	0.11 ± 0.04
Global	0.90 ± 0.02	0.10 ± 0.02	0.11 ± 0.02

Appendix

Sensitivity of [CheY-P] distribution to experimental parameters

We note that while $p(c)$ in Eq. (1) is determined exactly from the experimental data $p_N(n_{cw})$ with

no additional input parameters, $p(y_P)$ in Eq. (4) depends on the values of the experimental

parameters y_{tot}, K, and H. While y_{tot} is well known[44], values of K and H may vary due to flagellar

motor remodeling as well as differences between individual motors or cells (described in detail

in the following sections). In this section, we describe the sensitivity of the inferred [CheY-P]

distribution on y_{tot}, K, and H.

As mentioned in the text, the integral Eq. (4) is insensitive to the exact value of y_{tot}. Because

the CW bias $c \approx 1$ for $y_P > K$ due to the sigmoidicity of Eq. (3), the integral depends very weakly

on the precise value of the upper limit, and y_{tot} can be replaced with ∞ with an error on the

integral value <0.01%. In contrast, the dependence of Eq. (4) on K and H is more significant and

may affect our inference of [CheY-P] fluctuations. We analyzed the sensitivity of the mean [CheY-

P], μ_{y_p}, and coefficient of variation, σ_{y_p}/μ_{y_p}, on these two experimental parameters.

Making use of Eq. (3) and (4), the moments of the distribution $p(y_P)$ may be written as:

848
$$\left\langle y_P^m \right\rangle = \int_0^\infty dy_P \, y_P^m p(y_P) = \int_0^1 dc \, K^m \left(\frac{c}{1-c} \right)^{\frac{m}{H}} p(c) \qquad (29)$$

849 To estimate the sensitivity of the moments to K and H, we determined their logarithmic gains.

850 For K, it is easy to show that

851
$$\frac{d\ln\left\langle y_P^m \right\rangle}{d\ln K} = m \, ,$$

852 which indicates that $\mu_{y_p} = \left\langle y_P \right\rangle$ scales as K, $\sigma_{y_p}^2 = \left\langle y_P^2 \right\rangle - \left\langle y_P \right\rangle^2$ scales as K^2, and σ_{y_p}/μ_{y_p} is

853 independent of K. For H,

854
$$\frac{d\left\langle y_P^m \right\rangle}{d\ln H} = -\frac{m}{H} K^m \int_0^1 dc \left(\frac{c}{1-c} \right)^{\frac{m}{H}} \ln\left(\frac{c}{1-c} \right) p(c) = -m \int_0^\infty dy_P \, y_P^m \ln\left(\frac{y_P}{K} \right) p(y_P) \, ,$$

855 which cannot be solved analytically. However, since the distributions $p(y_P)$ are peaked functions

856 (see **Figures 2 and supplements**, **Figure 4**), the right integral is well approximated by a Taylor

857   expansion about the mean [CheY-P], $\langle y_P \rangle = \mu_{y_p}$. Defining the function $f_m(y_P) \equiv -my_P^m \ln(y_P / K)$

858   in the integrand, it follows that, to lowest two orders:

859
$$\frac{d\langle y_P^m \rangle}{d\ln H} = \langle f_m(y_P) \rangle \approx f_m(\mu_{y_p}) + \frac{1}{2} f_m''(\mu_{y_p}) \sigma_{y_p}^2 ,$$

860   where '' denotes the second derivative with respect to y_P. After some algebra, the sensitivity of

861   μ_{y_p} to H can be shown to be approximately

862
$$\frac{d\ln \mu_{y_p}}{d\ln H} \approx -\ln\frac{\mu_{y_p}}{K} , \tag{30}$$

863   which ranges between 0.4 and 0 for the range of values observed experimentally for μ_{y_p}/K, 0.7

864   to 1 (**Figure 5c**). The sensitivity of σ_{y_p} to H is given by

865
$$\frac{d\ln \sigma_{y_p}}{d\ln H} \approx -\left(1 + \ln\frac{\mu_{y_p}}{K}\right), \tag{31}$$

which ranges between -0.65 to -1 for the same range of values of μ_{y_p}. Subtracting Eq. (30) from

Eq. (31) gives an expression for the coefficient of variation and shows that σ_{y_p}/μ_{y_p} approximately

scales as H^{-1}. These dependences match those obtained by directly integrating Eq. (4) numerically

while varying H. In summary, our sensitivity analysis indicates that μ_{y_p} scales linearly with K but

does not depend very strongly on H, whereas σ_{y_p}/μ_{y_p} is independent of K but scales inversely with

H. Neither parameter depends appreciably on y_{tot}.

Effect of flagellar motor remodeling on CheY-P distribution

Studies have shown that the flagellar motor exhibits adaptive remodeling[45–47]. FliM, a

component of the switch complex at the base of the flagellar motor that binds CheY-P, is dynamic

and able to bind to or dissociate from the motor. Furthermore, the mean number of FliM

molecules bound to the motor depends on its CW bias. As a result, changes in motor bias—either

from temporal fluctuations or in response to a chemotactic stimulus—lead to FliM exchange and

motor remodeling. This in turn affects the binding of CheY-P to the motor, which shifts the

motor's CW bias, leading to a negative feedback loop and an adaptive response.

Zhang et al. recently developed a quantitative model for this dynamic process[43], using the

Monod-Wyman-Changeux (MWC) model to describe the dependence of the motor's CW bias c

on the number of bound FliM molecules, N_F, and intracellular [CheY-P], y_P:

884
$$c(y_P, N_F) = \frac{1}{1 + e^{(N_F g(y_P) + \varepsilon)/k_B T}},$$
(32)

885   where ε = 33.65 $k_B T$ and

886
$$g(y_P) = k_B T \ln\left(\frac{1 + y_P / K_2}{1 + y_P / K_1}\right),$$
(33)

and where K_2 = 7.5 μM and K_1 = 1.28 μM. Eqs. (32)-(33) describe a function that is nearly

indistinguishable in shape to the Hill curve $c(y_P)$, Eq. (3), first determined by Cluzel et al.[25], despite

their different analytical forms. For a value of N_F = 43.5 (corresponding to our experimentally

determined $c(y_P)$ as detailed later in this section), Eqs. (32)-(33) correspond to a Hill curve with

parameters H = 18 and K = 2.4 μM. (Note also that $\sqrt{K_1 K_2} = K$ = 3.1 μM [43]).

In Zhang et al., motor remodeling is described by the differential equation[43]:

893
$$\frac{dN_F}{dt} = k_{on}(N_{F,max} - N_f) - k_{off}(N_F - N_{F,NE}(c)).$$
(34)

The FliM association and dissociation dynamics are slow, and given by rate constants k_{on} and k_{off}

with similar values of ~0.02 s^{-1} (Lele et al., 2012). FliM bound to the switch complex can adopt

two different states, an unstable state that freely exchanges with cytoplasmic FliM, and a stable

non-exchanging state whose number is denoted by $N_{F,NE}$ in Eq. (34)[43]. $N_{F,NE}$ is a linear function of

the CW bias, adopting a value of 34 for a motor rotating CCW and a value of 12 for one rotating

CW[43], i.e. $N_{F,NE}(c) = 34 - 22c$. $N_{F,max}$ = 56 represents the maximum number of FliM binding sites

on the motor.

We used the above model to determine the effect of motor remodeling on our estimates of

noise in the chemotaxis network. We can predict qualitatively the effect of the negative feedback

loop generated by remodeling. Given a small increase in CheY-P level above the steady state, the

CW bias c increases from Eqs. (32) and (33), and the bound FliM number N_F in turn decreases

from Eq. (34). A decrease in N_F leads to a shift in the curve $c(y_P, N_F)$ to the right according to Eq.

(32), which decreases c. Thus, for a fixed increase in CheY-P, the net effect of the adaptive

remodeling is to suppress corresponding changes in *c*. In turn, to reproduce the observed

fluctuations in *c* in our data, a larger variance in [CheY-P] must be assumed in the presence of

motor remodeling compared to in its absence. We thus expect that ignoring remodeling leads to

an underestimate of the CheY-P fluctuations.

The magnitude of this underestimate will depend on the timescale of CheY-P fluctuations

relative to that of remodeling. Due to the slow remodeling dynamics (of timescale $1/k_{off} \approx 50$ s),

the remodeling feedback loop will not respond to fast CheY-P fluctuations. To model the effect

of remodeling on CheY-P noise estimates, we carried out numerical simulations of CheY-P

fluctuations using a discretized Langevin equation

916
$$y_P(t+dt) = y_P(t) - \frac{1}{\tau_A}(y_P(t) - \mu_{y_p})dt + \sigma_{Y_p}\sqrt{\frac{2dt}{\tau_A}}\xi(t),$$
(35)

where τ_A is the characteristic timescale for fluctuations in [CheY-P] due to CheA activation-

deactivation, ξ are Gaussian-distributed random numbers with unit variance, and μ_{y_p} and σ_{y_p} are

the mean and standard deviation of the [CheY-P], respectively. [Note that by construction Eq.

(35) generates a Gaussian [CheY-P] distribution. However, in the main text, we utilize a beta

distribution as a trial distribution to estimate [CheY-P] fluctuations. For the sake of consistency,

in the following analysis we compare the simulation results to [CheY-P] fluctuations in the

absence of remodeling assuming a Gaussian [CheY-P] distribution (see **Figure 2 – figure**

supplement 2), a trial distribution that also works well to estimate [CheY-P] fluctuations (**see**

Table 3).]

Using $y_p(t)$ generated by Eq. (35) as input to Eqs. (32)-(34), we simulated remodeling motors

at steady state, with the time-dependent CW bias $c(t)$ and number of bound FliM $N_F(t)$ as outputs.

We used $N_F(t = 0) = N_{F,NE}(c(\mu_{y_p}))$ as the initial condition for Eq. (35), although simulation results

were insensitive to this value at long times, and ran multiple (N = 200) repeats of the simulation.

μ_{y_p} and σ_{y_p} were adjusted so that the mean and standard deviation in CW bias matched the values

we determined experimentally for cells at steady state, $\mu_c = 0.14$ and $\sigma_c = 0.22$. **Appendix – figure**

1a plots the coefficient of variation, σ_{y_p}/μ_{y_p}, required to reproduce the experimental CW bias

parameters in the presence of remodeling, as a function of the CheY-P fluctuation timescale τ_A.

As expected, when $\tau_A << 1/k_{off}$, the effect of remodeling on CheY-P fluctuation estimates is

minimal. In contrast, when $\tau_A \approx 1/k_{off}$, CheY-P fluctuations must increase to match the

experimental data; however, the underestimate in CheY-P noise is moderate, <25% for $\tau_A \approx 10$ s,

which is thought to be the timescale of CheY-P fluctuations due CheR-CheB dynamics[32,33].

Additionally, we found $N_f(t \to \infty) = 43.5$ to correspond to experimentally determined CW bias

mean and standard deviation values across all τ_A.

We also used this framework to simulate cells undergoing chemotactic adaptation in

response to a step down in MeAsp from 1 to 0 mM. Here, we did not model adaptation kinetics

explicitly. Instead we used the experimentally measured mean and standard deviation in CW bias

in ~2.5-s time windows post-stimulus (corresponding to windows of duration $0.2t_{adap}$, see **Figure**

4) to guide piecewise simulations over those time windows. Briefly, the appropriate values for

μ_{y_p} and σ_{y_p} in Eq. (35) were selected to generate a time trace $c(t)$ with μ_c and σ_c matching the

experimental data in each time window. As above, we ran multiple (N = 5000) repeats of the

simulations in each time window. The steady-state value of N_F = 43.5 served as the initial

condition for simulations in the first time window post-stimulus. $N_F(t)$ evolved over each time

window, and its average over the simulation repeats at the end of each time window served as

the initial condition for the next time window.

Appendix – figure 1b and 1c plot the mean μ_{y_p} and coefficient of variation σ_{y_p}/μ_{y_p} vs. time

post-stimulus in the absence and presence of remodeling (for τ_A = 10 s only). Again, a moderate

increase of <15% in CheY-P noise is observed as a result of accounting for motor remodeling. The

larger effect is seen on the mean CheY-P, which increases as a consequence of the adaptive effect

of remodeling, as described above. Briefly, the increase in c due to the stimulus is followed by a

decrease in N_F which shifts the curve $c(y_P, N_F)$ to the right. As a result, a larger value of μ_{y_p} is

necessary to produce the same value for μ_c in the absence of remodeling.

Finally, **Appendix – figure 1d** plots the coefficient of variation σ_{y_p}/μ_{y_p} vs. the mean μ_{y_p} for

steady state and post-stimulus in the absence and presence of remodeling (for τ_A = 10 s only). Eq.

(8) in the main text is used to fit both sets of data (R^2 = 0.7 in absence of remodeling and R^2 = 0.8

in presence of remodeling). While the values of the kinetic ceiling α are identical in absence and

presence of remodeling (α = 0.246 ± 0.008 and α = 0.246 ± 0.002 respectively), the burst size <b>

is somewhat different (161 ± 110 and 271 ± 114 respectively).

Effect of motor-to-motor differences on CheY-P distribution

In Eqs. (4)-(5) of the main text, we implicitly assumed that the CW bias vs. CheY-P relation,

Eq. (3), is the same for all motors, i.e. the parameters K and H are constants. However, there may

be differences between motors that lead to variations in these parameters. Recent work[71] has

shown that the load on the flagellar motors shifts the curve midpoint, K. Thus, differences in load

(e.g. from different flagellar lengths) could lead to variation in CW bias across motors for the

same intracellular [CheY-P].

To estimate the variation in c vs. y_P curves across motors of the same cell, we analyzed time

traces from cells of the CheY* strain. Since we could identify each flagellum during our

fluorescence movies, we determined the time-averaged CW bias of each motor individually and

quantified CW bias differences across motors on the same cell. As CheY* expression is controlled,

its concentration is assumed to be constant over the time scale of the measurements (30-40 s)

for each individual cell. Variations in CW bias thus represent either statistical fluctuations or

differences in K between motors (Differences in H are expected to have a negligible effect on

average CW bias). From Eq. (3), the CW bias c_i of the i-th motor gives the ratio K_i/y^*, where $y^* \equiv$

[CheY*] is constant. Normalizing this ratio by its average over all motors on the same cell yields

$K_i / \langle K \rangle$, the distribution of which over all cells provides an upper bound for the coefficient of

variation in K of 3%.

We next determined the effect of a variation in K on the estimated CheY-P fluctuations. If

there exist motor-to-motor differences, then Eq. (2) connecting the CheY-P distribution to the

probability $p_N(n_{cw})$ that n_{CW} flagella rotate CW must be modified by replacing the conditional

probability $p_N(n_{cw} \mid c)$ with the corresponding equation for non-identical motors:

987
$$p_N(n_{cw} \mid \{c_i\}) = \sum_\sigma \prod_{i=1}^{n_{cw}} c_{\sigma(i)} \prod_{j=n_{cw}+1}^{N} (1 - c_{\sigma(j)}),$$
(36)

988   where the sum is carried out over all non-identical permutations σ of the motor index and where

$$c_i = \frac{y_P^H}{K_i^H + y_P^H} \ .$$ (37)

We carried out numerical simulations of cells with N flagellar motors each with a different K_i

drawn from a Gaussian distribution with mean 2.4 μM and coefficient of variation 5% (larger than

our upper bound estimate of 3% above). For each simulated cell, we generated a probability

$p_{N,sim}(n_{cw})$ using Eq. (4) with Eqs. (36) and (37), assuming a common CheY-P distribution $p_{in}(y_P)$

with mean μ_{y_p} = 1.97 μM and coefficient of variation σ_{y_p}/μ_{y_p} = 0.15 as input (these values

correspond to the CheY-P Gaussian distribution parameters determined for wild type HCB1660

cells). We then fitted the $p_{N,sim}(n_{cw})$ with a trial distribution function $p_{fit}(y_P)$, according to the

procedure outlined in the main text, assuming identical motors, i.e. Eqs. (4)-(5). Comparing the

input and fit distributions, $p_{in}(y_P)$ and $p_{fit}(y_P)$, allowed us to quantify the effect of assuming

identical motors in cells with non-identical motors. For a motor-to-motor variation of 5% in K, we

found that μ_{y_p} was 2.1 μM and the coefficient of variation σ_{y_p}/μ_{y_p} decreased by ~25%.

1001

Effect of cell-to-cell variation on CheY-P distribution

We also explored the effect of variation in the c vs. y_P curves between cells. Similarly to above,

we estimated variations in the parameter K across a wild-type cell population by determining the

time-averaged CW bias of individual cells and quantifying cell-to-cell differences CW bias. This

analysis provides an upper bound for the population coefficient of variation in K of <5%. We next

carried out numerical simulations of cells each with a different K_i drawn from a Gaussian

distribution with mean 2.4 μM and coefficient of variation 5%. We then quantified the effect of

assuming an identical value of K for a population of cells in which this parameter varied. Assuming

a common CheY-P distribution $p_{in}(y_P)$ with mean μ_{y_p} = 1.97 μM and coefficient of variation σ_{y_p}/μ_{y_p}

= 0.15 as input, for a cell-to-cell variation of 5% in K, we found that the mean [CheY-P] was 1.97

μM and the coefficient of variation remained unchanged or increased marginally by ~5%. The

increase in estimated [CheY-P] fluctuations is similar to that observed with CheY* cells, when

comparing population and single-cell level distributions (**Figure 2** and **3**). Cell-to-cell variation in

CheY^{D13K} expression in the CheY* strain, equivalent to variation in K, broadens the observed

[CheY-P] distribution.

References

1. Lim WA, Mayer B., P. T. *Cell Signaling*. (Garland Science, 2014).

2. Mcadams, H. H. & Arkin, A. Stochastic mechanisms in gene expression. *Proc. Natl. Acad. Sci. U. S. A.* **94**, 814–819 (1997).

3. Allen, C. & Stevens, C. F. An evaluation of causes for unreliability of synaptic transmission. *Proc. Natl. Acad. Sci. U. S. A.* **91**, 10380–10383 (1994).

4. Barkai, N. & Leibler, S. Robustness in simple biochemical networks to transfer and process information. *Nature* **387**, 913–917 (1997).

5. Silva-Rocha, R. & de Lorenzo, V. Noise and Robustness in Prokaryotic Regulatory Networks. *Annu. Rev. Microbiol.* **64**, 257–275 (2010).

6. Stelling, J., Sauer, U., Szallasi, Z., Doyle, F. J. & Doyle, J. Robustness of Cellular Functions. *Cell* **118**, 675–685 (2004).

7. Eldar, A. *et al.* Robustness of the BMP morphogen gradient in Drosophila embryonic patterning. *Nature* **419**, 304–308 (2002).

8. Rao, C. V, Wolf, D. M. & Arkin, A. P. Control, exploitation and tolerance of intracellular noise. *Nature* **420**, 231–237 (2002).

9. Tsimring, L. S. Noise in biology. *Reports Prog. Phys.* **77**, (2014).

10. Raj, A. & van Oudenaarden, A. Nature, nurture, or chance: stochastic gene expression and its consequences. *Cell* **135**, 216–26 (2008).

11. Eldar, A. & Elowitz, M. B. Functional roles for noise in genetic circuits. *Nature* **467**, 167–173 (2010).

12. Balázsi, G., Van Oudenaarden, A. & Collins, J. J. Cellular decision making and biological noise: From microbes to mammals. *Cell* **144**, 910–925 (2011).

13. Wadhams, G. H. & Armitage, J. P. Making sense of it all: bacterial chemotaxis.

Nat.Rev.Mol.Cell Biol. **5**, 1024–1037 (2004).

14.   Shimizu, T. S., Tu, Y. & Berg, H. C. A modular gradient-sensing network for chemotaxis in
Escherichia coli revealed by responses to time-varying stimuli. *Mol. Syst. Biol.* **6**, 1–14
(2010).

15.   Lan, G., Schulmeister, S., Sourjik, V. & Tu, Y. Adapt locally and act globally: Strategy to
maintain high chemoreceptor sensitivity in complex environments. *Mol. Syst. Biol.* **7**, 1–
16 (2011).

16.   Sourjik, V. & Wingreen, N. S. Responding to chemical gradients: bacterial chemotaxis.
Curr. Opin. Cell Biol. **24**, 262–8 (2012).

17.   Micali, G. & Endres, R. G. Bacterial chemotaxis: Information processing, thermodynamics,
and behavior. *Curr. Opin. Microbiol.* **30**, 8–15 (2016).

18.   Larsen, S. H., Reader, R. W., Kort, E. N., Tso, W. W. & Adler, J. Change in direction of
flagellar rotation is the basis of the chemotactic response in Escherichia coli. *Nature* **249**,
74–77 (1974).

19.   Macnab, R. M. & Ornston, M. K. Normal-to-curly flagellar transitions and their role in
bacterial tumbling. Stabilization of an alternative quaternary structure by mechanical
force. *J. Mol. Biol.* **112**, 1–30 (1977).

20.   Turner, L., Ryu, W. S. & Berg, H. C. Real-time imaging of fluorescent flagellar filaments. *J.*
Bacteriol. **182**, 2793–2801 (2000).

21.   Darnton, N. C., Turner, L., Rojevsky, S. & Berg, H. C. On torque and tumbling in swimming
Escherichia coli. *J. Bacteriol.* **189**, 1756–1764 (2007).

22.   Parkinson, J. S., Hazelbauer, G. L. & Falke, J. J. Signaling and sensory adaptation in
Escherichia coli chemoreceptors: 2015 update. *Trends Microbiol.* **23**, 257–266 (2015).

23.   Scharf, B. E., Fahrner, K. A., Turner, L. & Berg, H. C. Control of direction of flagellar
rotation in bacterial chemotaxis. *Proc. Natl. Acad. Sci.* **95**, 201–206 (1998).

24.   Alon, U. *et al.* Response regulator output in bacterial chemotaxis. *EMBO J.* **17**, 4238–4248
(1998).

25.   Cluzel, P., Surette, M. & Leibler, S. An Ultrasensitive Bacterial Motor Revealed by
Monitoring Signaling Proteins in Single Cells. *Science (80-.).* **287**, 1652–1655 (2000).

26.   Hess, J. F., Oosawa, K., Kaplan, N. & Simon, M. I. Phosphorylation of three proteins in the
signaling pathway of bacterial chemotaxis. *Cell* **53**, 79–87 (1988).

27.   Blat, Y. & Eisenbach, M. Phosphorylation-Dependent Binding of the Chemotaxis Signal
Molecule CheY to Its Phosphatase, CheZ. *Biochemistry* **33**, 902–906 (1994).

28.   Springer, M. S., Goy, M. F. & Adler, J. Protein methylation in behavioural control
mechanisms and in signal transduction. *Nature* **280**, 279–284 (1979).

29.   Kehry, M. R., Bond, M. W., Hunkapiller, M. W. & Dahlquist, F. W. Enzymatic deamidation
of methyl-accepting chemotaxis proteins in Escherichia coli catalyzed by the cheB gene
product. *Proc. Natl. Acad. Sci. U. S. A.* **80**, 3599–3603 (1983).

30.   Borkovich, K. A., Alex, L. A. & Simon, M. I. Attenuation of sensory receptor signaling by
covalent modification. *Proc. Natl. Acad. Sci.* **89**, 6756–6760 (1992).

31.   Korobkova, E., Emonet, T., Vilar, J. M. G., Shimizu, T. S. & Cluzel, P. From molecular noise
to behavioural variability in a single bacterium. *Nature* **428**, 574–578 (2004).

32.   Colin, R., Rosazza, C., Vaknin, A. & Sourjik, V. Multiple sources of slow activity
fluctuations in a bacterial chemosensory network. *Elife* **6**, 1–32 (2017).

33.   Keegstra, J. M. *et al.* Phenotypic diversity and temporal variability in a bacterial signaling
network revealed by single-cell FRET. *Elife* **6**, 1–33 (2017).

34.   Matthäus, F., Jagodič, M. & Dobnikar, J. E. coli superdiffusion and chemotaxis-search
strategy, precision, and motility. *Biophys. J.* **97**, 946–957 (2009).

35.   Emonet, T. & Cluzel, P. Relationship between cellular response and behavioral variability
in bacterial chemotaxis. *Proc. Natl. Acad. Sci. U. S. A.* **105**, 3304–3309 (2008).

36. He, R., Zhang, R. & Yuan, J. Noise-Induced Increase of Sensitivity in Bacterial Chemotaxis.
Biophys. J. **111**, 430–437 (2016).

37. Flores, M., Shimizu, T. S., ten Wolde, P. R. & Tostevin, F. Signaling Noise Enhances
Chemotactic Drift of E. coli. *Phys. Rev. Lett.* **109**, 148101 (2012).

38. Terasawa, S. *et al.* Coordinated reversal of flagellar motors on a single Escherichia coli
cell. *Biophys. J.* **100**, 2193–2200 (2011).

39. Mears, P. J., Koirala, S., Rao, C. V., Golding, I. & Chemla, Y. R. Escherichia coli swimming is
robust against variations in flagellar number. *Elife* **3**, 1–18 (2014).

40. Min, T. L. *et al.* High-resolution, long-term characterization of bacterial motility using
optical tweezers. *Nat. Methods* **6**, 831–835 (2009).

41. Park, H. *et al.* Interdependence of behavioural variability and response to small stimuli in
bacteria. *Nature* **468**, 819–821 (2010).

42. Min, T. L., Mears, P. J., Golding, I. & Chemla, Y. R. Chemotactic adaptation kinetics of
individual Escherichia coli cells. *Proc. Natl. Acad. Sci.* **109**, 9869–9874 (2012).

43. Zhang, C., He, R., Zhang, R. & Yuan, J. Motor Adaptive Remodeling Speeds Up Bacterial
Chemotactic Adaptation. *Biophys. J.* **114**, 1225–1231 (2018).

44. Li, M. & Hazelbauer, G. L. Cellular stoichiometries of the components of the chemotaxis
signaling complex. *J. Bacteriol* **186**, 3687–3694 (2004).

45. Yuan, J., Branch, R. W., Hosu, B. G. & Berg, H. C. Adaptation at the output of the
chemotaxis signalling pathway. *Nature* **484**, 233–236 (2012).

46. Lele, P. P., Branch, R. W., Nathan, V. S. J. & Berg, H. C. Mechanism for adaptive
remodeling of the bacterial flagellar switch. *Proc. Natl. Acad. Sci. U. S. A.* **109**, 20018–
20022 (2012).

47. Yuan, J. & Berg, H. C. Ultrasensitivity of an adaptive bacterial motor. *J. Mol. Biol.* **425**,
1760–1764 (2013).

48.   Shamir, M., Bar-On, Y., Phillips, R. & Milo, R. SnapShot: Timescales in Cell Biology. *Cell*
164, 1302-1302.e1 (2016).

49.   Tu, Y. & Grinstein, G. How White Noise Generates Power-Law Switching in Bacterial
Flagellar Motors. **208101**, 1–4 (2005).

50.   Alon, U., Surette, M. G., Barkai, N. & Leibler, S. Robustness in bacterial chemotaxis.
Nature **397**, 168–171 (1999).

51.   Sourjik, V. & Berg, H. C. Receptor sensitivity in bacterial chemotaxis. *Proc. Natl. Acad. Sci.*
U. S. A. **99**, 123–127 (2002).

52.   Segall, J. E., Block, S. M. & Berg, H. C. Temporal comparisons in bacterial chemotaxis.
Proc. Natl. Acad. Sci. U. S. A. **83**, 8987–8991 (1986).

53.   Block, S. M., Segall, J. E. & Berg, H. C. Adaptation kinetics in bacterial chemotaxis. *J.*
Bacteriol. **154**, 312–323 (1983).

54.   Mesibov, R. & Adler, J. Chemotaxis toward amino-acids in Escherichia coli. *J. Bacteriol.*
112, 315–326 (1972).

55.   Mesibov, R., Ordal, G. W. & Adler, J. The range of attractant concentrations for bacterial
chemotaxis and the threshold and size of response over this range. Weber law and
related phenomena. *J. Gen. Physiol.* **62**, 203–223 (1973).

56.   Vladimirov, N., Løvdok, L., Lebiedz, D. & Sourjik, V. Dependence of bacterial chemotaxis
on gradient shape and adaptation rate. *PLoS Comput. Biol.* **4**, (2008).

57.   Bitbol, A. F. & Wingreen, N. S. Fundamental constraints on the abundances of chemotaxis
proteins. *Biophys. J.* **108**, 1293–1305 (2015).

58.   Clausznitzer, D., Oleksiuk, O., Løvdok, L., Sourjik, V. & Endres, R. G. Chemotactic response
and adaptation dynamics in Escherichia coli. *PLoS Comput. Biol.* **6**, 1–11 (2010).

59.   Mcquarrie, D. A. & Gillespie, D. T. Stochastic Theory and Simulations of Chemical Kinetics.
J. Appl. Probab. **478**, 413–478 (1967).

60.   Sourjik, V. & Berg, H. C. Binding of the Escherichia coli response regulator CheY to its

target measured in vivo by fluorescence resonance energy transfer. *Proc. Natl. Acad. Sci.*

U. S. A. **99**, 12669–12674 (2002).

61.   Tindall, M. J., Porter, S. L., Wadhams, G. H., Maini, P. K. & Armitage, J. P. Spatiotemporal

modelling of CheY complexes in Escherichia coli chemotaxis. *Prog. Biophys. Mol. Biol.*

100, 40–46 (2009).

62.   Paulsson, J. Summing up the noise in gene networks. *Nature* **427**, 415–8 (2004).

63.   Paulsson, J. Models of stochastic gene expression. *Phys. Life Rev.* **2**, 157–175 (2005).

64.   Golding, I., Paulsson, J., Zawilski, S. M. & Cox, E. C. Real-time kinetics of gene activity in

individual bacteria. *Cell* **123**, 1025–1036 (2005).

65.   Thattai, M. & van Oudenaarden, A. Intrinsic noise in gene regulatory networks. *Proc.*

Natl. Acad. Sci. **98**, 8614–8619 (2001).

66.   Turner, L., Zhang, R., Darnton, N. C. & Berg, H. C. Visualization of flagella during bacterial

swarming. *J. Bacteriol.* **192**, 3259–3267 (2010).

67.   Datsenko, K. A. One-step inactivation of chromosomal genes in Escherichia coli K-12

using PCR products. *Proc. Natl. Acad. Sci.* **97**, 6640–6645 (2000).

68.   Plank, M., Wadhams, G. H. & Leake, M. C. Millisecond timescale slimfield imaging and

automated quantification of single fluorescent protein molecules for use in probing

complex biological processes. *Integr. Biol. (Camb).* **1**, 602–612 (2009).

69.   Ross, S. M. *A first course in probability*. (Pearson, 2013).

70.   Wilcox, R. R. *Comparing Two Independent Groups*. *Applyinf Contemporary Statistical*

Techniques (2003). doi:10.4324/9781410613707-10

71.   Wang, F. *et al.* Non-equilibrium effect in the allosteric regulation of the bacterial flagellar

switch. *Nat. Phys.* **13**, 710–714 (2017).

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1167 **Figures**

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**Figure 1. Estimating the temporal [CheY-P] distribution from flagellar dynamics of single *E. coli*
cells. a.** Schematic of a multi-flagellated *E. coli* cell showing key reactions of the chemotaxis
signaling network. CheY is phosphorylated to CheY-P by the receptor kinase complex (CheA; dark
blue); CheY-P is dephosphorylated by the phosphatase CheZ (gray). CheY-P binding to flagellar
motors (mustard) causes a switch from CCW to CW rotation, which leads to cell tumbling. **b.**
Temporal fluctuations in [CheY-P] due to chemotaxis network dynamics and their effect on
flagellar switching statistics. i) [CheY-P] temporal fluctuations and corresponding distribution
function $p(y_p)$. ii) Switch-like response of motor CW bias, c, to [CheY-P]. iii) Resulting temporal
fluctuations in motor bias $c(t)$ and distribution of motor bias $p(c)$. iv) CCW/CW rotation state of
three flagellar motors corresponding to the instantaneous motor bias c. The [CheY-P] distribution
function $p(y_P)$ is directly related to the probability distribution $p_N(n_{CW})$ that n_{CW} out of N flagella
rotate CW through the integral Eq. (4) (large gray arrow). **c.** Schematic of two-channel laminar
flow chamber for optical trap assay. A single cell with fluorescently labelled flagella (gray stars) is
captured from the bottom channel ("Cells"), aligned along the flow between two optical traps
(red cones), moved to the upper channel ("Blank"), and its fluorescent flagella (yellow stars)
imaged by stroboscopic slim-field microscopy (green). **d.** Fluorescence data trace from
representative trapped cell with $N = 3$ flagella. Top, still images of fluorescent flagella, with

position of the unlabeled cell body approximately indicated by the dashed yellow line. CW and

CCW rotating flagella are labelled where clearly visible. Middle, CCW/CW rotation state of each

flagellum vs. time (mustard). Bottom, corresponding number n_{CW} of CW flagella vs. time (light

blue). **e**. Bottom, probability distribution $p_N(n_{CW})$, determined experimentally from d (light blue

bars) and fit (solid blue line) based on the model in Eq. (4). Middle, best fit estimate of the motor

bias probability distribution $p_{fit}(c)$. Top, best fit estimate of the [CheY] probability distribution

$p_{fit}(y_P)$.

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Figure 2. [CheY-P] fluctuations in the chemotaxis network at steady state. Measurement of flagellar dynamics and estimation of [CheY-P] fluctuations in wild-type *E. coli* strain HCB1660 (a-c) and in "CheY*" mutant strain PM87 expressing constitutively active CheY^{D13K} (d-f). **a.** Experimentally determined probability distribution $p_N(n_{CW})$ that n_{CW} out of N flagella rotate CW, grouped by cells with the same total number of flagella N = 2, 3, 4, 5 (number of cells = 13, 15, 14, 9 respectively). Best fit distributions $p_{N,fit}(n_{CW})$ determined separately for each N (colored solid lines) as well as globally (gray solid lines) are overlaid with the observed distributions (colored filled bars). **b.** Best fit probability distributions of the motor bias, $p_{fit}(c)$, determined separately for each N (colored solid curves) and globally (gray solid curve). **c.** Best fit probability distributions of [CheY-P], $p_{fit}(y_P)$, determined separately for each N (colored solid curves) and globally (gray solid curves) overlaid with Eq. (3), the relationship between motor bias c and y_P. **d-f.** Same as a-c for CheY* strain, showing experimental (open bars) and best fit (dashed lines and curves) distributions, grouped by cells with the same total number of flagella N = 2, 3, 4, 5 (number of cells = 13, 13, 9, 9 respectively).

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Figure 2 – figure supplement 1. Error in the estimated [CheY-P] probability distribution at steady state. a. Best fit [CheY-P] distribution $p_{fit}(y_P)$ for wild-type *E. coli* strain HCB1660 (thick colored curves) overlaid with each bootstrap resampling trial (thin colored curves) used to estimate the error and with the average over all bootstrap trials (thick dotted curve). Distributions determined for cells with the same total number of flagella N = 2, 3, 4, 5 as well as globally. **b.** Same as a for the "CheY*" mutant strain PM87.

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1216 **Figure 2 – figure supplement 2. Comparison of different trial functions for estimating the best**

1217 **fit [CheY-P] distribution. a.** Best fit [CheY-P] distribution $p_{fit}(y_P)$ for wild-type *E. coli* with beta

1218 (solid curve), Gaussian (dotted curve) and gamma (dashed curve) distribution trial functions.

1219 Distributions determined for cells with the same total number of flagella N = 2, 3, 4, 5 as well as

1220 globally. **b.** Same as a for the CheY* mutant strain.

Figure 3. Estimate of the mean and coefficient of variation of [CheY-P] in individual cells. a.
Comparison of mean relative [CheY-P], μ_{y_p}/K, determined for individual wild-type (colored circles)
and CheY* cells (open circles), for different number of flagella N. **b.** Comparison of the coefficient
of variation in [CheY-P], σ_{y_p}/μ_{y_p}, determined for individual wild-type (colored circles) and CheY*
cells (open circles), for different N. Averages over each cell population are overlaid (black solid
circles for wild-type, black open circles for CheY*); error bars represent standard deviation.

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Figure 3 – figure supplement 1. [CheY-P] fluctuations in individual wild type *E. coli* cells. **a.** Top, experimentally determined probability distribution $p_N(n_{CW})$ (filled bars) and best fit $p_{N,fit}(n)$ (solid lines) for 5 individual representative cells with N = 2 flagella. Bottom, corresponding best fit [CheY-P] distribution $p_{fit}(y_P)$ (solid curves). **b-d.** Same as a, for N = 3, 4, and 5, respectively.

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Figure 3 – figure supplement 2. [CheY-P] fluctuations in individual mutant *E. coli* CheY* cells. a.

Top, experimentally determined probability distribution $p_N(n_{CW})$ (open bars) and best fit $p_{N,fit}(n)$

1236   (dashed lines) for 5 individual representative cells with N = 2 flagella. Bottom, corresponding best

1237   fit [CheY-P] distribution $p_{fit}(y^*)$ (dashed curves). **b-d.** Same as a, for N = 3, 4, and 5, respectively.

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Figure 4. [CheY-P] fluctuations in the chemotaxis network upon stimulation. a. Schematic of

three-channel laminar flow chamber for flagellar imaging of a cell experiencing a step down in

attractant concentration. A single cell with fluorescently labelled flagella (gray stars) is captured

from the bottom channel containing 1 mM MeAsp ("Cells + Attractant") and aligned along the

flow between two optical traps (red cones) in the middle channel ("Blank + attractant"). The

trapped cell is then moved to the upper channel lacking MeAsp ("Blank") and its fluorescent

flagella (yellow stars) are imaged (green). **b.** Representative trace showing the response of "Cell

1" to a step-down stimulus. Top, schematic of MeAsp concentration over time, showing the step

down from 1 to 0 mM at $t = 0$. Data are not collected during the move from the middle to the top

channel. Bottom, number n_{CW} of CW flagella over time after $t = 0$. **c-e.** Estimation of the [CheY-P]

distribution in cells responding to a step-down stimulus. **c.** Traces from three representative

"Cells 1, 2, 3" showing the number n_{CW} of CW flagella vs. time (normalized by each cell's

adaptation time t_{adap}). Each trace is subdivided into windows of duration $0.2t_{adap}$, and data from

each window is pooled across cells (for $N = 3$ flagella shown here, number of cells = 9). **d.**

Distributions of the number of CW flagella $p_N(n_{CW})$ for each window (colored bars) with the

calculated best fits (black solid lines). **e.** Best fit [CheY-P] distributions, $p_{fit}(y_P)$, (colored curves)

for each window. The [CheY-P] distribution for unstimulated cells is overlaid (gray dotted curve).

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Figure 4 – figure supplement 1. Run-tumble response of a cell to a step-down stimulus. a.

Schematic of MeAsp concentration over time, showing the step down from 1 to 0 mM at $t = 0$. **b.**

Run-tumble binary time trace of a representative cell. Data are not collected during the move

from the middle to the top channel. **c.** Tumble bias of the cell (green), determined at each time

point from the fraction of time tumbling over a 10-s moving window, and fit to bi-exponential

function (red) Eq. (16).

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Figure 5. Reduction in [CheY-P] fluctuations post stimulus due to kinase phosphorylation dynamics. a. Mean [CheY-P], μ_{y_p}, vs. normalized time t/t_{adap} after step down stimulus (top schematic, red) determined separately for each number of flagella N (colored circles) and globally (gray circles). μ_{y_p} is normalized by the global best fit mean [CheY-P] for unstimulated cells, $\mu^0_{y_p,glo}$. **b.** Coefficient of variation in [CheY-P], σ_{y_p}/μ_{y_p}, after step down. Same color code as a. Data are pooled by cells with the same total number of flagella $N = 2, 3, 4$ (number of cells = 9, 9, 9). The pre-stimulus mean and coefficient of variation in [CheY-P] were obtained from the analysis of the unstimulated cells in Fig. 2. **c.** Coefficient of variation in [CheY-P], σ_{y_p}/μ_{y_p}, plotted against the normalized mean [CheY-P], $\mu_{y_p}/\mu^0_{y_p,glo}$, for data from unstimulated and stimulated cells from a and b**.** Fits to Eq. (7) and Eq. (8) are also shown. **d.** Simple model of [CheY-P] fluctuations in the *E. coli* cell. (Left) In unstimulated cells, the fluctuations in CheA activity at the steady state lead

1275   to CheY-P production in bursts. (Right) In stimulated cells, CheA activity is higher on average than

1276   in unstimulated cells and shows reduced fluctuations as it approaches the kinetic ceiling. This

1277   also causes a reduction in [CheY-P] fluctuations at high mean [CheY-P].

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Figure 5 – figure supplement 1. Dependence of [CheY-P] noise estimates on data time window duration. Ratio of standard deviation in [CheY-P] estimated over an entire single-cell trace to the standard deviation estimated over 3-s windows of the same trace, $\sigma_{yp,\text{cell}}/\sigma_{yp,3s}$, plotted vs. trace duration (gray filled circles). Data globally fitted from cells with different numbers of flagella. The noise estimates increase with trace duration, a trend replicated by a simple model (black dotted line). The model assumes that the spectral density in [CheY-P] is given by a Lorentzian with characteristic timescale τ_A = 12.6 s (see Ref. (33)). Integrating the spectral density yields an expression for the standard deviation, $\sigma_{y_P} = \dfrac{c\tau_A}{\pi}(\tan^{-1}(2\pi\tau_A f_{high}) - \tan^{-1}(2\pi\tau_A f_{low}))$, in terms of the data bandwidth. f_{high} = 10 Hz is the sampling rate and f_{low} = 1/trace duration. c is a constant.

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Appendix – figure 1. The effect of flagellar motor remodeling on CheY-P fluctuation estimates.

a. Coefficient of variation in [CheY-P], σ_{y_p}/μ_{y_p}, from simulations accounting for flagellar motor remodeling vs. CheY-P fluctuation timescale τ_A. σ_{y_p}/μ_{y_p} is normalized by the coefficient of variation without motor remodeling, and τ_A is normalized by the motor remodeling rate k_{off}. **b.** Mean [CheY-P] μ_{y_p} vs. normalized time t/t_{adap}, taking into account flagellar motor remodeling (black) and ignoring remodeling (light gray). μ_{y_p} is normalized by the global best fit mean [CheY-P] without remodeling, $\mu^0_{y_p,glo}$. **c.** Same as b, for the coefficient of variation in [CheY-P], σ_{y_p}/μ_{y_p}. **d.** Coefficient of variation in [CheY-P] σ_{y_p}/μ_{y_p} vs. normalized mean [CheY-P], $\mu_{y_p}/\mu^0_{y_p,glo}$ from b and c. Data points are normalized as in Fig. 5a-c. Simulations are based on globally fitted experimental data in Fig. 5a-c. Simulations in b-d assume a CheY-P fluctuation timescale τ_A = 10 s (see Appendix).