# Distinguishing within- from between-individual effects:

# How to use the within-individual centering method for quadratic patterns

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**Abstract**

1. Any average pattern observed at the population level may confound two different types of processes: some processes that occur among individuals and others that occur within individuals. Separating within- from among-individual processes is critical for our understanding of ecological and evolutionary dynamics.

2. The within-individual centering method allows distinguishing within- from among-individual processes and this method has been largely used in ecology to investigate both linear and quadratic patterns. Here we show that two alternative equations could be used for the investigation of quadratic within-individual patterns. We explain under which hypotheses each is valid. Reviewing the literature, we found that mainly one of these two equations has been used by the studies investigating quadratic patterns. Yet this equation could be inappropriate in many cases.

3. We show that these two alternative equations make different assumptions about the shape of the within-individual pattern. The choice of using one equation instead of the other should depend upon the biological process investigated. One equation assumes that all individuals show the same quadratic pattern over the whole range of the explanatory variable whereas the other assumes that the quadratic patterns depend on the average explanatory variable of each individual. We give examples of biological processes corresponding to each equation.

4. Using simulations, we showed that a mismatch between the assumptions made by the equation used to analyze the data and the biological process investigated led to flawed inference affecting both output of model selection and accuracy of estimates. We stress that the equation used should be chosen carefully to ensure that the assumption made about the shape of the within-individual

42  pattern matches the biological process investigated. We hope that this manuscript will encourage

43  the use of the within-individual centering method, promoting its correct application for non-linear

44  relationships.

46  Keywords: age pattern, individual trajectory, linear mixed models, selective appearance, selective

47  disappearance, senescence.

**Introduction**

The task of distinguishing trait variation occurring within individuals from that occurring

among individuals is a recurrent challenge in ecology (Vaupel et al. 1979; Vaupel and Yashin

1985; Bolnick et al. 2011; Vindenes and Langangen 2015; Hamel et al. 2018). As both within-

and among-individual effects drive trait variation, the patterns observed at the population level do

not necessarily reflect the changes occurring at the individual level, and vice versa. A famous

example is the investigation of the relationships between a response variable and individual age

(van de Pol and Verhulst 2006; Nussey et al. 2008). Variation in the response variable may arise

from within-individual variation, e.g., aging effect due to accumulating experience in early-life

or/and due to senescence in late life, as well as from among-individual variation, by the selective

disappearance of frail individuals (Aubry et al. 2009; Bouwhuis et al. 2009; Hämäläinen et al.

2014). Indeed, in heterogeneous populations, frail individuals are expected to die first, leaving

robust individuals to be overrepresented at old ages (Pigeon et al. 2017; Vedder and Bouwhuis

2018). This change in the composition of the population over time generates positive age-related

relationships at the population level. Thus, the average age-related trajectory observed at the

population level could differ from the average trajectory observed at the individual scale, because

independent processes, arising within- and among-individuals, co-occur (Vaupel and Yashin

1985).

This problem of distinguishing between within-individual and among-individual patterns

is not specific to age-related patterns. It may concern the relationships between any response

variable and any type of explanatory traits, i.e., behavioral, morphometric, physiological,

environmental, each time several observations per individual are obtained and aggregated (e.g.,

Pick et al. 2016; Dammhahn et al. 2017; Morrongiello et al. 2019; Siracusa et al. 2019).

Analyzing such aggregated data, we need to distinguish the within-individual effect, i.e., how

variation in the explanatory variable X within an individual affects the response variable Y, from

the among-individual effect, i.e., how the average value of X for a given individual is related to

the average value of Y. This problem could even be extended beyond the individual level since

similar problems could be encountered every time data are aggregated at different scales, a

common data structure in ecology and evolution. For the rest of the manuscript, we will mainly

focus on the individual level, but similar distinctions could be relevant when comparing patterns

within and among clusters of data, e.g. groups of individuals, populations, species and so on (e.g.,

Dakin et al. 2018; Lundin et al. 2019; Lemoine et al. 2020).

From an eco-evolutionary perspective, it is crucial to separate within- from among-

individual variation in traits (Bolnick et al. 2011). For example, the investigation of life-history

trade-offs, which are within-individual effects, requires accounting for among-individual

variation in resource acquisition, which is an among-individual effect (van Noordwijk and de

Jong 1986). Similarly, our understanding of the age-related change in demographic rates has been

limited for a long time because most studies considered only the population-level pattern

(Forslund and Pärt 1995; Nussey et al. 2008). Patterns within and among individuals provide

different information which are of interest to understand the behavior, physiology or demography

of biological systems. A within-individual effect may provide information on processes involving

individual plasticity such as phenotypic change, learning or the occurrence of trade-off. On the

other hand, among-individual effects depict consistent inter-individual differences and thus may

suggest among-individual differences in fitness, personality or pace of life, for instance (Wilson

and Nussey 2010; Reid et al. 2010; Dingemanse and Dochtermann 2013). Ignoring the distinction

between within- and among-individual effects may thus lead to flawed inferences confounding

processes occurring at different levels (Vaupel and Yashin 1985; van de Pol and Verhulst 2006;

Kendall et al. 2011; Stover et al. 2012).

A statistical method to distinguish within- from among-individual effects is the within-

individual centering method (Kreft et al. 1995; Hofmann and Gavin 1998; Snijders and Bosker

1999). This powerful method is simple to use and gained popularity rapidly following its

introduction to ecologists (van de Pol and Wright 2009). To date, this work has been cited 479

times and the approach has been used in 346 published studies (research among the papers citing

van de Pol and Wright 2009 using Web of Science in January 2021). While this method has been

presented in the context of behavioral studies for linear patterns (van de Pol and Wright 2009), it

has been used in various ecological fields including demography, aging, eco-physiology, and

parasitology, and has been extended to investigate quadratic patterns (Fig. 1).

In this paper we clarify the use of the within-individual centering method for quadratic

patterns providing guidelines for its application. After briefly presenting the within-individual

centering approach in case of linear patterns, we present the two alternative equations which can

be used for the investigation of quadratic within-individual effects. We explain the different

assumptions made by each equation about the shape of the within-individual pattern and suggest

in which circumstances each equation should be used. We stress that previous studies

investigating quadratic patterns failed to directly recognize these assumptions and, in some cases,

may have used an inappropriate equation. Based on simulations, we assess the consequences of

using the inappropriate equation on model selection and quality of estimates (bias and precision).

We also illustrate the consequences of using the inappropriate equation for the specific case of

quadratic-age patterns, which correspond to the most frequent individual-centered variable used

when investigating quadratic individual patterns. We hope that this manuscript will encourage the

use of the within-individual centering method promoting its correct application for non-linear

relationships.

Figure 1: Number of articles citing the van de Pol & Wright (2009)'s work and using the within-individual centering

method including all explanatory variables (upper panel) and age in particular with linear (light grey) and quadratic

(dark grey) models. The articles have been classified according to the type of response variable(s) used (life-history,

behavioral, physiological, morphological, and other traits).

**1. The within-individual centering method**

In ecology, linear mixed models are nowadays standard tools to analyze data aggregated

at different scales. However, such models may fail to estimate the true within-individual changes

because the individual random effect may not capture the total among-individual effects (van de

Pol & Wright 2009). The within-individual centering method is an extension of a simple mixed

model that allows distinguishing explicitly within- from among-individual variation using

additional fixed effects (Kreft et al. 1995; Hofmann and Gavin 1998; Snijders and Bosker 1999,

van de Pol & Wright 2009). We present only briefly this method and refer to van de Pol &

Wright (2009) for further information.

The standard random effect model can be described by the following regression equation:

$$y_{ij} = \beta_0 + \beta_1 x_{ij} + u_j + e_{ij} \qquad (1)$$

where $\beta_0$ is the intercept, $\beta_1$ is the slope, $x_{ij}$ is the value of a trait $x$ at measurement $i$ from

individual $j$, $u_j$ is the deviation from the intercept for individual $j$ assumed to be drawn from a

normal distribution with a mean of zero and among-subject variance $\sigma_u^2$, and $e_{ij}$ is the residual

error for each measurement assumed to be drawn from a normal distribution with a mean of zero

and variance $\sigma_e^2$.

Starting from this standard mixed model, the within-individual centering approach decomposes

the effect of x in two terms: the within-individual and the among-individual effects. The among-

individual effect is fitted as the average x value for each individual ($\overline{x_j}$). The within-individual

effect is fitted as the deviation from the individual mean for a given observation ($x_{ij} - \overline{x_j}$). The

previous equation becomes:

$$y_{ij} = \beta_0 + \beta_W(x_{ij} - \overline{x_j}) + \beta_B \overline{x_j} + u_j + e_{ij} \qquad (2)$$

where $\beta_W$ is the slope of the within-individual effect and $\beta_B$ is the slope of the among-individual

effect. Thus, equation 1 corresponds to a specific case of equation 2 when $\beta_W = \beta_B$ and assumes

that among- and within-individual effects are identical. van de Pol & Wright (2009) showed that

a standard mixed model does not allow estimation of within-individual effects as reliably as the

within-individual centering model.

## 2. The within-individual centering method to investigate quadratic patterns

The within-individual centering method has been regularly used in ecology to investigate

quadratic changes in traits (11.5% of all the articles citing van de Pol & Wright (2009) and

applying this method, Fig. 1). Quadratic terms could be added to both the within- and the among-

individual effects independently since they can arise from different processes. Among the

published articles using a quadratic relationship, all introduced it for the within-individual effect

but only some of those used it simultaneously for the among-individual effect. The relevance of

including quadratic terms to the within-individual effect, the among-individual effect, or both,

depends on the system studied and the question investigated. Here, for the sake of clarity, we first

present how quadratic terms can be added to the within- and among-individual effects separately.

Then we give the general equation including a quadratic term for both within- and among-

individual effects.

### 2.1 Quadratic within-individual effect

177 To use an individual centering approach including a quadratic within-individual effect,

178 the individual centering should be applied on both linear and quadratic terms. We suggest the

179 following equation to estimate within-individual quadratic effect (see Appendix S1 for full

180 derivation and explanation of the equation):

181
$$y_{ij} = \beta_0 + \beta_{W1}(x_{ij} - \overline{x}_j) + \beta_B \overline{x}_j + \beta_{W2}(x_{ij}^2 - \overline{x}_j^2) + u_j + e_{ij} \qquad (3)$$

182 An alternative equation which has been used by most of the studies investigating quadratic

183 individual trajectories is:

184
$$y_{ij} = \beta_0 + \beta_{W1}(x_{ij} - \overline{x}_j) + \beta_B\overline{x}_j + \beta_{W2}(x_{ij} - \overline{x}_j)^2 + u_j + e_{ij} \qquad (4)$$

In equation 3, the quadratic within-individual effect ($\beta_{W2}$) is included as the deviation of the

squared $x$ value from the squared average $x$ value of an individual, i.e., $x_{ij}^2 - \overline{x}_j^2$, whereas in

equation (4), the quadratic within-individual effect ($\beta_{W2}$) is included as the square of the centered

variable, i.e., $(x_{ij} - \overline{x}_j)^2$. It should be noted that equations (3) and (4) are not equivalent and

generate very different individual patterns (Fig. 3).

To better understand the difference between these two equations, we can develop equation

(4) as:

$$y_{ij} = \beta_0 + \beta_{W1}(x_{ij} - \overline{x}_j) + \beta_B\overline{x}_j + \beta_{W2}(x_{ij}^2 - \overline{x}_j^2) + 2\beta_{W2}x_{ij}\overline{x}_j + u_j + e_{ij} \qquad (4i)$$

From equation (4i), we can see that the difference between the two equations is the presence of

the term $2\beta_{W2}x_{ij}\overline{x}_j$ in equation (4) only. This term introduces an interaction between $x_{ij}$, the value

of a trait $x$ at measurement $i$ from individual $j$, and $\overline{x}_j$, the average x value sampled for individual

$j$. By doing so, equation (4) assumes that the quadratic within-individual effect ($\beta_{W2}$) is not

related to the absolute value of X but depends on the average $\overline{x}_j$ value sampled for each

individual. Thus, equation (4) assumes that within-individual changes depends on the range of the

explanatory variable sampled for each individual (Fig. 2B). In contrast, because there is no

interaction between $x_{ij}$ and $\overline{x}_j$ in equation (3), this equation assumes that the within-individual

effect of X on Y depends on the absolute value of X. In other words, equation 3 assumes that

there is a general quadratic pattern over the entire range of X which is shared by all the

individuals. Ignoring individual variation due to the among-individual effect and random

variation in the intercept, the effect of X on Y is the same for all individuals (Fig. 2A).

As a consequence, the two equations differ in where the quadratic pattern is expected to

reach its maximum/minimum within each individual. Equation 3 assumes that the

maximum/minimum is identical for all individuals on the absolute scale of X values,

independently of the ranges of each individual trajectory. Equation 4 assumes that the

maximum/minimum is different among individuals but is reached at the same relative place

within the range of the explanatory variable observed for each individual (mean average

measured for each individual, for instance). See appendix S2 for a mathematical demonstration of

this difference.

Figure 2. Individual trajectories illustrating quadratic within-individual effects simulated with equation (3) (A) and

(4) (B). In both graphs, the four same individual life histories have been simulated using the equations (3) and (4),

respectively, with the following parameters: $\beta_0 = 0$, $\beta_{W1} = -0.2$, $\beta_{W2} = 0.12$, $\beta_B = 1$, $\sigma_u^2 = 1$ and $\sigma_e^2 = 0$. Black

circles show $\overline{x}_j$, the average X values for each individual. The equation (3) (A) assumes that all individuals show the

same pattern over the range of the explanatory variable. In contrast, equation (4) (B) assumes that the intra-individual

changes of a given individual depend on the range of the explanatory variable sampled for this individual.

## 2.2 Quadratic among-individual effect

To use an individual centering approach including a quadratic among-individual effect, equation (2) becomes:

$$y_{ij} = \beta_0 + \beta_W(x_{ij} - \bar{x}_j) + \beta_{B1}\bar{x}_j + \beta_{B2}\bar{x}_j^{\,2} + u_j + e_{ij} \quad (5)$$

The quadratic among-individual effect ($\beta_{B2}$) is fitted by the square of the average $x$ value for each individual ($\bar{x}_j^{\,2}$) (Fig. 3). Contrary to the within-individual effect, we found no alternative equation in studies investigating quadratic individual trajectories. Because including a quadratic among-individual effect is not a source of confusion, the rest of the article focuses on the quadratic within-individual effect.

Figure 3. Individual trajectories illustrating a quadratic among-individual effect. Here, Y decreases linearly in relation to X within each individual but Y increases quadratically among individuals in relation to the average $\bar{x}_j$ values. Individual trajectories are simulated with equation (5) with the following parameters: $\beta_0 = 0$, $\beta_W = $ -0.5, $\beta_{B1} = 1.2$, $\beta_{B2} = $ -0.08, $\sigma_u^2 = 1$ and $\sigma_e^2 = 0$. Black circles show $\bar{x}_j$.

**2.3 Quadratic within-and among-individual effects**

To use an individual centering approach including both a quadratic within-individual

effect and a quadratic among-individual effect, the two alternative equations are:

$$y_{ij} = \beta_0 + \beta_{W1}(x_{ij} - \bar{x}_j) + \beta_{B1}\bar{x}_j + \beta_{W2}(x_{ij}^2 - \bar{x}_j^2) + \beta_{B2}\bar{x}_j^2 + u_j + e_{ij} \text{ (3')}$$

and

$$y_{ij} = \beta_0 + \beta_{W1}(x_{ij} - \bar{x}_j) + \beta_{B1}\bar{x}_j + \beta_{W2}(x_{ij} - \bar{x}_j)^2 + \beta_{B2}\bar{x}_j^2 + u_j + e_{ij} \qquad \text{(4')}$$

It should be noted that a key difference between the two approaches equations is that when

within- and among-individual effects are similar, i.e., $\beta_{W1} = \beta_{B1}$ and $\beta_{W2} = \beta_{B2}$, only equation

(3') is equivalent to a model not using within-individual centering. Indeed, assuming identical

within- and among-individual effect, we may simplify equation (3') and (4') as follows:

$$y_{ij} = \beta_0 + \beta_{W1}x_{ij} + \beta_{W2}x_{ij}^2 + u_j + e_{ij} \text{ (3'')}$$

and

$$y_{ij} = \beta_0 + \beta_{W1}x_{ij} + \beta_{W2}x_{ij}^2 + 2\beta_{W2}x_{ij}\bar{x}_j + u_j + e_{ij} \qquad \text{(4'')}$$

**3. When should each equation be used? Examples of biological processes fitting each**

**equation**

Equation (3) assumes that all individuals show the same quadratic pattern over the range

of the explanatory variable. Equation (3) can thus be used when researchers expect a functional

relationship shared by all the individuals regardless of the mean value of the explanatory variable

sampled for each individual (Figs. 3A & 6A). Taking the example of the flying speed

performance in an insect, it is reasonable to expect consistent among-individual responses to

variation in temperature. Because insects are ectotherms, flying performance of all individuals is

expected to increase with temperature until a certain point where extreme temperature may have

a negative effect. Here, there is a general quadratic pattern over the entire range of temperature

shared by all the individuals (Fig. 4A). The effect of temperature on flying speed depends on the

absolute value of the temperature, not on the deviation from the average temperature sampled for

each individual. In this case, using equation (3) allows correct estimation of the quadratic within-

individual effect. More generally, equation (3) could be used any time the within-individual

response depends on the absolute value of the explanatory variable. This is expected to be the

case in most studies investigating the response of individuals to biotic and abiotic environmental

factors.

In contrast, equation (4) assumes that each individual has a quadratic pattern that depends

on its individual mean. Continuing with the example of flying performance in an insect, an

example of biological process that could fit equation 4 is the effect of body mass on flying

performance. In this case, absolute body mass might not be critical to determine flying

performance, but rather the deviation from the optimum individual body mass (Fig. 4B). As a

consequence, patterns are centered on individual means in this example. The same change in

body mass may be associated with higher or lower flying speed according to the size of the

individuals. More generally, equation (4) assumes that the effect of the explanatory variable

varies among individuals according to an individual optimum. This may correspond to any

variables that have strong individual optima dependent on sampled individual $x_j$ values. It could

for instance correspond to an acclimation process when individuals have optimal performance

based on the environment they experienced in the past.

Figure 4. Theoretical effect of temperature (A) and body mass (B) on flying speed in a butterfly. These examples illustrate relationships following assumptions made by equation (3) and (4), respectively. Variation in flying speed is due to both a within-individual effect, i.e., effect of variation in temperature or body mass on individual performance, and an among-individual effect generated by variation in cohort (A) and size (B), for instance. In accordance with assumptions made by equation 3, all individuals show the same relationship between flying speed and temperature. In other words, the effect of the temperature on flying speed is independent of the range of temperature encountered by each individual. In contrast, variation in flying speed according to body mass shows a strong individually-specific response. Each quadratic pattern is centered on the mean body mass of each individual. In this case it is the mass deviation related to the individual optimum, not the absolute mass value, which is critical.

## 4. The case of quadratic age trajectories with selective disappearance: which equation should be used?

The importance in distinguishing within-individual from among-individual effects in age-related pattern has been recognized for a long-time (Forslund and Pärt 1995; Reid et al. 2003; Nussey et al. 2008). Unsurprisingly, the within-individual centering method has been commonly

used in ecology to investigate age-related changes in behavioral, physiological, morphological or

life-history traits (Fig. 1). When individual life-trajectories are considered in their entirety, the

observed within-individual age patterns generally display a dome shape with increasing

performance in early life and decreasing performance in late life due to senescence (Forslund and

Pärt 1995; Aubry et al. 2009; Bouwhuis et al. 2009; Hämäläinen et al. 2014). Consequently,

quadratic relationships have often been used to investigate age-related patterns. We found that

27% of the studies using the within-individual centering method to investigate age-related

patterns included a quadratic within-individual effect. Among them, all studies describing the

method with enough detail used equation (4), but none used equation (3). Although equation 4

has been applied systematically, this equation might be inappropriate for age-related patterns.

Using the within-individual centering method, the age effect is split into two terms: the

average individual age whose associated slope estimates the among-individual effect (i.e., the

effect due to selective disappearance of certain phenotypes), and the age-related changes (i.e., the

difference between the actual and the average age of a given individual), whose slope estimates

the within-individual effect. When we compare the individual-trajectories predicted by the

equations (3) and (4), it appears that the equation 4 is not consistent with empirically observed

age patterns (Loison et al. 1999; Reid et al. 2003; Zhang et al. 2015) (Fig. 5). For instance,

equation 4 assumes that the age trajectory of an individual dying at 5 years old ($\bar{x}_j = \overline{age}_j = 3$) is

similar to the age pattern between 6 and 10 years old of an individual dying at 15 years old ($\bar{x}_j = $

$\overline{age}_j = 8$). In both cases the age-related changes are the same, ranging from -2 to 2 (Fig. 4B).

With equation (3), an individual dying young (low average X) is expected to display a trajectory

that differs from that of an individual dying old (high average X). The former would show only

the classical early-life increase, whereas the latter would show a more general pattern including

an increase, a plateau and a decrease when older due to senescence (Fig. 5A). This pattern

matches what is generally observed in longitudinal studies; that is, a decrease in individual

performance occurring in later life after picking up in middle age (Loison et al. 1999; Reid et al.

2003; Zhang et al. 2015). In contrast, equation (4) does not allow modelling only the early-life

increase for individuals dying young, but constrains all individuals to display the full quadratic

pattern regardless of their age of death (Fig. 5B). This is generally inappropriate since in

longitudinal studies, most of the individuals die before reaching the age of onset of senescence

(Clutton-Brock 1988; Newton 1998). Finally, equation (4) makes the assumption of a fixed

ageing pattern with a varying age of onset of senescence whereas equation (3) assumes a fixed

age of onset of senescence estimating a general senescence pattern. It should be noted that

extending equation 3 with random slopes for the linear and quadratic components of the within-

individual effect would allow estimation of both varying rate and onset of senescence. However,

those models, although theoretically sound, require much more data (van de Pol 2012; Westneat

et al. 2020).

In the case of age patterns, the among-individual effect may be generated by both

selective appearance and disappearance effects (van de Pol and Verhulst 2006), making

estimation of the among-individual effect more complicated. In addition, several studies strongly

suggest moving away from polynomial models in favour of threshold models when studying

aging patterns (Berman et al. 2009; Froy et al. 2017; Murgatroyd et al. 2018; Rodríguez-Muñoz

et al. 2019). In this context, an interesting alternative method closely related to the within-

individual centering is the use of age at first and last reproduction as fixed covariates (van de Pol

and Verhulst 2006). This method allows disentangling of the selective appearance and

disappearance, and facilitates the implementation of threshold models. For these reasons, we

encourage the use of the method described by van de Pol and Verhulst (2006) instead of the more

generic within-individual centering method in the case of age patterns.

Figure 5: Age-related individual trajectories simulated according to equations 3 (A) and 4 (B). The five

individual life histories are similar between the graphs in terms of the intercept and age range. Equation 4

assumes that the age pattern of an individual dying at 5 years old is similar to the age pattern between 6

and 10 years old of an individual dying at 15 years old (black dot with associated age-related changes).

**5. Consequences of mismatches between generating and analytical equations**

Above we showed that two alternative equations could be used for the investigation of

quadratic within-individual effects. Here we illustrate the consequences of a mismatch between

the equation generating the quadratic within-individual patterns and the equation used to analyze

the data.

**5.1. Methods**

We simulated 1,000 data sets consisting of 50 individuals with 15 measurements each.

First, for each individual, we used a normal distribution with a mean $\overline{X}_j$, taken randomly between

-7 and 7, and a variance 1.5 to sample 15 measurements. Then, we simulated the response

variable Y using either equation (3) or (4). In both cases, we used the same parameters: $\beta_0 = 0$,

$\beta_{W1} = 0.4$, $\beta_{W2} = -0.11$, $\beta_{B1} = 1$, $\beta_{B2} = 0$, $\sigma_u = 2$ and $\sigma_e^2 = 0.5$. Finally, for each simulated dataset, we

fitted three models: the matching and mismatching models (equation (3) and (4)) and a simpler

linear model (equation 2).

Based on these sets of simulations, we assessed the consequences for model selection and

quality of estimates of analyzing the data with the mismatching equation. We used the Akaike

information criterion (AIC) to compare matching, mismatching and linear models. This

comparison allows us to check if a mismatch between the generating and analytical equations

affects our ability to detect quadratic within-individual effects. When differences of AIC between

the two models were below 2, we followed the principle of parsimony, selecting the model with

the lower number of parameters. To assess the effect of mismatches between generating and

analytical equations on the quality of the estimates, we computed bias by subtracting the

estimated parameters from the true parameter values used to generate the data. Results were

summarized with violin plots to visualize both mean and dispersion of bias. Finally, we also

computed the mean square error (MSE) which gives the accuracy of the estimates, i.e.,

combination of bias and precision to define the performance of an estimator.

**5.2. Results**

Quadratic models matching the equation used to simulate the data were always selected as

the best of the three models (lower AIC). However, simpler linear models were selected against a

mismatching quadratic model in 3% of the data simulated with equation (3) and analyzed with

equation (4) and in 38% of the data simulated with equation (4) and analyzed with equation (3).

This demonstrates that using the inappropriate equation may lead to flawed inferences.

383                        Regarding the performance of the estimators, no bias was observed when the

equation used to generate the data matched the one use to analyze the data (equation (3) bias

$\beta_{w1}$= -0.0005[-0.027:0.024] and $\beta_{w2} = -0.0001[-0.003:0.003]$, equation (4) bias $\beta_{w1}$= -0.0009[-

0.027:0.025] and $\beta_{w2} = -0.0001[-0.012:0.013]$) (Fig. 6). However, it should be noted that the

performance of equation (4) depends on how accurately the average explanatory variable

sampled for each individual corresponds to the true/biological average explanatory variable.

When these quantities differ, for instance due to sampling bias, estimates of the quadratic within-

individual pattern could be biased (Appendix S3).

391           Mismatch between the generating and analytical equations affects both bias and precision

of the estimates. When data were simulated with equation 4 but analyzed with equation 3, the

intercept was negatively biased and the quadratic within-individual effect was positively biased.

When data were simulated with equation 3 but analyzed with equation 4, estimates were unbiased

but very imprecise. In particular, the linear and quadratic within-individual estimates showed

poor performance in this mismatching case (Figure 6 & S1). Whatever the mismatch, using the

inappropriate equation may lead to strong misestimation of the within-individual pattern (Fig. 7).

Nevertheless, the among-individual effect was estimated without bias and with a consistent

precision whichever generating and analytical equations were used (Fig. 6).

Figure 6: Bias in the estimates of the within- and among-individual effects from 1000 datasets simulated

and analyzed by the two alternative quadratic equations of the within-centering method (data analyzed

with equation 3 in black and equation 4 in grey). Parameters include the intercept (beta0), the linear (W1)

and the quadratic (W2) within-individual effect of X, and the linear among-individual effect (B1).

Figure 7: Estimation of the quadratic within-individual effect for data simulated with equation (3) (A) and

(4) (B). Each dot represents one observation per individual and the gray lines depict observed within-

individual trajectories. For illustration, we show the row observations for only 10 individuals. The green

line shows the simulated within-individual effect, the dashed black line shows predictions of the naïve

model that corresponds to a simple quadratic linear model, the red line shows predictions of the within-

individual effect estimated using a standard quadratic mixed model (quadratic extension of equation 1),

and the blue and orange lines show prediction of the within-individual effect estimated using the within-

individual centering method according to equation 3 and 4 respectively.

## 6. Simulation of quadratic age trajectories with selective disappearance

### 6.1. Methods

427    We ran further simulations for age-related patterns corresponding to the most frequent

428    individual-centered variable used for quadratic patterns. Instead of using equations (3) and (4),

429    we generated age-related data with selective disappearance.

430    First, we simulated individual age-related trajectories using the following equation:

431
$$y_{ij} = \beta_0 + \beta_{w1} age_{ij} + \beta_{w2} age_{ij}^2 + u_j + e_{0ij},$$

where $\beta_0 = 0$ is the intercept, $\beta_{w1} = 0.8$ and $\beta_{w2} = $ -0.06 are the linear and quadratic within-

individual effects respectively, $u_j$ is deviation from the intercept for individual $j$ assumed to be

drawn from a normal distribution with zero mean and among-subject variance $\sigma_u^2 = 2$, and $e_{ij}$ is

the residual error assumed to be drawn from a normal distribution with zero mean and variance

$\sigma_e^2 = 1$. To simulate an among-individual effect, we generated the selective disappearance of frail

individuals simulating age of death as a function of the individual-specific deviation in the

response variable ($u_j$). This differential survival among individuals in relation to their age-

independent performance corresponds to what has been reported by empirical studies as

individual quality (McCleery et al. 2008; Pigeon et al. 2017; Vedder and Bouwhuis 2018). We

simulated individual age at death using an annual individual survival probability of

ilogit($u_j$/2+1.5). Individuals with negative deviation, that is those with lower performances, also

have lower survival prospects (see appendix S4 for the code used). Using the procedure described

above, we simulated 1000 datasets with perfect detection, each consisting of 50 individuals. We

analyzed these datasets assessing the relationships between the response variable and age with

both equations (3) and (4) and computed bias and MSE as explained in section 5.1. Finally,

because detection is often imperfect, we performed the same simulation analysis but with

individual recapture probability set to 0.5 and to 0.2 instead of 1.

**6.2. Results**

Results confirmed that equation (3), but not equation (4), is suitable for the investigation

of quadratic age patterns when selective disappearance occurs. Estimates of the within-individual

age effect from equation (4) were strongly biased (bias $\beta_{w1}$= -0.818[-0.860:-0.763] and $\beta_{w2} =$

$-0.0002[-0.008 : 0.008]$). Bias was higher in $\beta_{w1}$ than in $\beta_{w2}$ because $\beta_{w1}$ allows

counterbalancing of the supplemental component $2\beta_{w2}x_{ij}\bar{x}_j$ included in equation (4). Only

equation (3) provided unbiased estimates (bias $\beta_{w1} = -0.002[-0.093 : 0.097]$ and $\beta_{w2} =$

$-0.0001[-0.006: 0.006]$) (Figure S2). Using equation (4) instead of equation (3) for describing

age-related individual trajectory led to flawed inferences with a strong overestimation of

senescence (earlier onset and higher rate, Figure S3). Since real data often show incomplete

histories of individual trajectories, i.e., detection probability is typically lower than 1, we

reassessed the performance of these models considering incomplete individual capture histories.

Results show that missing information within the lifetime of an individual does not change the

respective performance of the models previously considered, but increases uncertainty for all

parameters. Even when the detection probability is very low i.e., 0.2, leading to very patchy

capture histories for each individual, the within-individual centering estimates from equation (3)

are accurate whereas those from equation (4) are consistently biased (Figure S2 & S4).

**Conclusion**

The within-individual centering method should be used with caution for non-linear

relationships. We showed that two different equations could be used to estimate within-individual

quadratic relationships and that each equation makes different assumptions about the shape of the

individual pattern. The choice of using one equation instead of the other should depend upon the

biological process investigated. We believe that equation (3), which assumes that the individual

response depends on the absolute value of the explanatory variable, better fits many biological

processes than equation (4), which assume that the individual response depends on relative value

of the explanatory variable to the subject mean. Typically, poor and good conditions in climatic

factors, food abundance or parasite load is expected to be uniform among-individuals as assumed

by equation 3 (Fig. 3A). Equation (4) assumes that the average explanatory variable sampled for

each individual matches the average condition driving the quadratic within-individual pattern. It

is questionable whether this assumption is frequently met in practice. In many ecological studies,

the range of environmental conditions sampled for a given individual is at least in part randomly

determined. For instance, if we investigate the individual response to a climatic variable such as

precipitation, the range of values considered for each individual will depend on the conditions

occurring during the period of observation of those individuals in the field. As a consequence,

even if the process generating the response variable corresponds to equation 4, the average

explanatory variable sampled for each individual would not necessarily match the average

condition driving the quadratic within-individual pattern.

Using the within-individual centering approach with polynomial terms requires larger

sample sizes at the individual level to be able to properly assess the linear and quadratic patterns

both at the within- and among-individual levels. Even if the simulations showed no biases when

matching equations were used, it should be noted that we used data sets with numerous repeated

measures (50 individuals with 15 observations each). The biases and reliability of within-

individual centering with quadratic patterns have not yet been properly assessed and we urge

researchers to consider their sample size and data structure before using this type of approach.

In any case, when using the within-individual centering for non-linear relationships, the

equation used should be chosen carefully in order to run meaningful analyses. In line with

Westneat et al (2020), we stress that it is critical to have a clear understanding of the equation

used to ensure that the assumptions made by a statistical model (using within-individual centering

or not) match the biological process investigated. We hope that this manuscript will encourage

the use of the within-individual centering method and promote its correct application for non-

linear relationships.

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