

1 **An empirical test of the role of value certainty in decision making**

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Abstract

Most contemporary models of value-based decisions are built on value estimates that are typically self-reported by the decision maker. Such models have been successful in accounting for choice accuracy and response time, and more recently choice confidence. The fundamental driver of such models is choice difficulty, which is almost always defined as the absolute value difference between the subjective value ratings of the options in a choice set. Yet a decision maker is not necessarily able to provide a value estimate with the same degree of certainty for each option that he encounters. We propose that choice difficulty is determined not only by absolute value distance of choice options, but also by their value certainty. In this study, we first demonstrate the reliability of the concept of an option-specific value certainty using three different experimental measures. We then demonstrate the influence that value certainty has on choice, including accuracy (consistency), choice confidence, response time, and choice-induced preference change (i.e., the degree to which value estimates change from pre- to post-choice evaluation). We conclude with a suggestion of how popular contemporary models of choice (e.g., race model, drift-diffusion model) could be improved by including option-specific value certainty as one of their inputs.

38 INTRODUCTION

39 For many decades, researchers in economics, psychology, and cognitive neuroscience have
40 studied the concept of subjective value, and how implicit subjective value influences explicit
41 choices. In more recent years, decision researchers have frequently relied on self-reported
42 estimates of subjective value (value ratings) to support their theories and models (see Rangel et
43 al, 2008, for a review). Value ratings collected by self-report have served as the fundamental
44 input of most contemporary models of choice. One key component of many models is what is
45 referred to as “choice difficulty”, which is most often defined as the distance between the
46 subjective value ratings of the options in the choice set (where difficulty declines with distance).
47 Choice difficulty has been shown to reliably predict both choice (i.e., the probability of choosing
48 the higher-rated option decreases with difficulty) and reaction time (i.e., deliberation time
49 increases with difficulty). Yet until recently, most models did not explicitly incorporate the
50 possibility that a decision maker (DM) might not be fully certain about the value ratings that he
51 reports. For example, a DM might have an estimate about the (subjective) value of an option,
52 yet simultaneously have a belief about the accuracy of his value estimate. Said another way,
53 sometimes we might feel that we like something a certain amount, but also feel more or less sure
54 about that amount—sometimes we might feel that we know for sure precisely how much we
55 value something, other times we might feel unsure about our value estimates. Consequently,
56 when choosing between items, certainty about the values of the individual options directly
57 impacts choice confidence, which can be defined as a feeling of certainty about which of the
58 options has a higher value. The more uncertain the individual option values are, the more difficult

59 it is to determine which one has the higher value. It would therefore follow that a more complete
60 definition of choice difficulty should account for such a concept as subjective certainty.

61 Some recent studies have indeed included value certainty as an independent variable in their
62 models. DeMartino et al (2013) examined the effects of what they called “bid confidence” on
63 the dependent variables in their model. The authors suggested that bid certainty should
64 moderate the impact of choice difficulty (defined as value rating distance) on choice accuracy,
65 choice confidence, and choice response time (RT). Lee and Daunizeau (2020b) took this a step
66 further, demonstrating in their model how value certainty might be explicitly involved in choice.
67 In that study, the authors suggested that when value ratings are uncertain, a DM will be less
68 accurate and less confident in his choice, but he will also be more inclined to invest mental effort
69 (for which RT could serve as a proxy) in order to decrease the value uncertainty and enable him
70 to confidently choose his preferred option. In fact, in that model, one of the principal drivers of
71 the proposed effort-confidence tradeoff is the desire on the part of the DM to increase the
72 certainty that he has about his value estimates. The model builds upon previous work that
73 demonstrated the same principle in a less formal manner, by showing that both RT and
74 preference change (difference between post- and pre-choice ratings) were decreasing functions
75 of value certainty (for which the authors used the average value certainty of the options being
76 compared), while choice confidence was an increasing function of value certainty (Lee and
77 Daunizeau, 2020b). In their model, the authors explain that lower value certainty impairs the
78 ability of the DM to distinguish the options. This dampens choice confidence, which the DM
79 attempts to boost through mental effort (proxied by RT). In turn, the effort allocation leads to
80 value estimate refinements, and potentially changes of mind (preference reversals).

81 Recent work in other, non-subjective value-based domains also suggests that a measure of
82 certainty about the options is conceptually important when examining choice behavior. Frydman
83 and Jin (2019) invoke the principle of efficient coding to suggest a link between value certainty
84 and choice behavior. In this study, certainty spawns from repeated exposure, which causes
85 greater precision in neural representation (i.e., efficient coding). Higher certainty, thus defined,
86 leads to higher choices accuracy (i.e., consistency with value estimates). Padoa-Schioppa and
87 Rustichini (2014) illustrate a similar concept based on *adaptive* coding, where neural activity is
88 normalized according to the range of option values in the current environment, thus causing
89 choice stochasticity to increase as representation precision decreases. Along similar lines,
90 Woodford (2019) explains how many important aspects of economic choice (e.g., choice
91 stochasticity, risk aversion, decoy effects) could result from noisy neural representations of value.
92 Although the author does not directly refer to a subjective feeling of certainty, it is no far stretch
93 to relate the precision of neural representations to a subjective feeling of certainty. In Polania,
94 Woodford, and Ruff (2019), the authors refer to efficient coding as well as Bayesian decoding
95 principles to explain how choice behavior is influenced by value certainty. Interestingly, the
96 authors do not record any self-reports of value certainty to validate their model. Instead,
97 certainty in this study is captured by rating consistency (i.e., similarity of repeatedly self-reported
98 value estimates). The basic idea here is that greater precision in the neural encoding of value will
99 lead to greater consistency across multiple interrogations of the value-encoding neural
100 population. This precision thus leads to both more consistent ratings and more choices
101 consistent with those ratings.

102 In spite of recent theoretical and empirical evidence that value certainty plays an important role
103 in choice behavior, most popular models of value-based decision making still do not include this
104 variable. In particular, so-called accumulation-to-bound models such as the race model (e.g.,
105 Tajima et al, 2019) and the drift-diffusion model (e.g., Krajbich and Rangel, 2011) do not include
106 option-specific certainty. Tajima et al (2019) model noise in the accumulation process at the
107 system level, where all options have the same degree of uncertainty imposed upon them by the
108 environment, rather than at the option level (although the authors themselves suggest that
109 future studies should explore the various sources of value uncertainty). Other studies have
110 similarly included systemic, but not option-specific, uncertainty (e.g., Louie, Khaw, and Glimcher,
111 2013). Krajbich and Rangel (2011), along with most other published versions of the drift-diffusion
112 model (DDM), fail to include a variable to represent value certainty.

113 Perhaps more researchers would be willing to include value certainty in their models if there was
114 more available evidence demonstrating that certainty could be reliably measured. In this study,
115 we hope to provide some such evidence. In line with Lee and Daunizeau (2020a, 2020b), we
116 explicitly ask decision makers to report their subjective feelings of certainty about the ratings
117 they provide about each of a large set of options. In line with Polania, Woodford, and Ruff (2019),
118 we also implicitly capture value certainty by calculating consistency across multiple ratings of the
119 same options. We then show that the explicit and implicit measurements of value certainty are
120 highly correlated for each individual DM, which suggests that they are both reliably expressing
121 the same internal representation precision. We show that RT during value estimation for each
122 item is also strongly correlated with both measures of certainty, which suggests a link between
123 the representation precision and the cognitive effort required to decode the value signal. Finally,

124 we show that self-reported estimates of certainty generally increase across repeated value
125 estimations. This should be expected if contemplating the value of an option is tantamount to
126 constructing its internal representation, because as the cumulative total of processed
127 information rises (i.e., across multiple rating sessions), the precision of the representation should
128 also increase.

129 In a second study, which is essentially a replication of the rating-choice-rating paradigm of Lee
130 and Daunizeau (2020a, 2020b), we reproduce the previous findings that value estimate certainty
131 positively influences choice consistency and choice confidence, and negatively influences
132 response time and choice-induced preference change. We also show an interesting novel
133 result—we can predict which option a DM will choose based on the relative value estimate
134 certainty of the options being compared, even while ignoring the options' value estimates
135 themselves.

136 In sum, we suggest that any of the considered measures of value estimate certainty (self-reports,
137 rating consistency, rating RT) could be used as a proxy for value certainty in future studies, and
138 that researchers should no longer neglect the concept of value certainty when building their
139 models. In particular, we provide confirmatory and novel evidence that value certainty plays an
140 important role in the cognitive process of decision making.

141 Note: for clarity, we explicitly use different terminology throughout this paper for subjective
142 beliefs about value estimates and about choices: "certainty" refers to the subjective feeling of
143 certainty about a value estimate rating; "confidence" refers to the subjective feeling of
144 confidence that the chosen item is the better one. We never use these terms interchangeably.

145 **METHODS**

146 We conducted a pair of behavioral experiments with the intention of demonstrating the
147 reliability of various measures of value certainty, which include: self-reports, rating consistency,
148 and response time. Participants considered a set of 200 options and provided three separate
149 value ratings for each option, as well as three separate certainty reports about those ratings. We
150 also recorded response time for each evaluation. Participants also made choices between pairs
151 of options, as well as choice confidence reports. Detailed task descriptions can be found below.

152 In Study 1, we asked participants to rate the value of each of a series of items. In addition to the
153 standard subjective value question, we also asked participants to rate their subjective certainty
154 regarding each subjective value judgment. Because we are interested in assessing the
155 consistency of value ratings and how that relates to subjective certainty ratings, we repeated the
156 value and certainty ratings three times during the experiment. This allows us to assess each of
157 our hypotheses, in particular: the correlation between value rating consistency and certainty
158 rating; the increase in certainty across repeated value ratings; the decrease in response time
159 across repeated value ratings.

160 In Study 2, we asked a different group of participants to rate the value of each of a series of items,
161 as well as to make choices between pairs of items. In addition to the standard subjective value
162 question, we also asked participants to rate their subjective certainty regarding each subjective
163 value judgment, and their subjective confidence regarding each choice.

164

165 Materials

166 We built our experiment in Gorilla (gorilla.sc). The experimental stimuli consisted of 200 digital
167 images, each representing a distinct snack item food item. The stimulus set included a wide
168 variety of items. Prior to commencing the experiment, participants received a written description
169 about the tasks and detailed instructions on how to perform them.

170 Ethics statement

171 Our analysis involved de-identified participant data and was approved by the ethics committee
172 of the University of Southern California (USC). In accordance with the Helsinki declaration, all
173 participants gave informed consent prior to commencing the experiment.

174 **Study 1**

175 Participants

176 A total of 37 people participated in this study (22 female; age: mean=21.4, stdev=3.6, min=18,
177 max=35). All participants were recruited from the undergraduate population at USC using
178 SonaSystems. Each participant received course credit as compensation for one hour of time.

179 Experimental Design

180 The experiment was divided into four sections. There was no time limit for the overall
181 experiment, nor for the different sections, nor for the individual trials.

182 In the first section (Exposure), participants merely observed as all individual items were displayed
183 in a random sequence for 750ms each. The purpose of the Exposure section was to familiarize
184 the participants with the full set of items that they would later evaluate, allowing them to form
185 an impression of the range of subjective value for the set. This would diminish the possibility
186 that value ratings would become more accurate across time merely due to a dynamic adaptation
187 of the range as the participants viewed new items.

188 The second through fourth sections (Rating1, Rating2, Rating3) were identical, except for the
189 sequence of trials. In each rating section, all stimuli were displayed on the screen, one at a time,
190 in a random sequence (randomized across participants and across sections for each participant).
191 At the onset of each trial, a fixation cross appeared at the center of the screen for 750ms. Next,
192 a solitary image of a food item appeared at the center of the screen. Participants responded to
193 the question, “How pleased would you be to eat this?” using a horizontal slider scale. The
194 leftmost end of the scale was labeled “Not at all.” and the rightmost end was labeled “Very
195 much!” The scale appeared to participants to be continuous, and the data was captured in
196 increments of 1 (ranging from 1 to 100). Participants could revise their rating as many times as
197 they liked before finalizing it. Participants clicked the “Enter” button to finalize their value rating
198 response and proceed to the next screen. Participants then responded to the question, “How
199 sure are you about that?” by clicking on a horizontal Likert scale (left to right: “not at all”,
200 “slightly”, “somewhat”, “fairly”, “very”, “extremely”) to indicate their level of subjective certainty
201 regarding the preceding value judgment. At that time, the next trial began. Participants were
202 not aware that there were three different rating sections, as the design technically only included
203 one rating section. Within that section, the sequence of trials included a random ordering of all

204 items, followed by another random ordering of all items, followed by another random ordering
205 of all items. From the perspective of the participant, the study consisted of a long series (600) of
206 item evaluations, where repetition could (and indeed, always did) occur. This design, in which
207 participants could not anticipate that each item was going to be rated multiple times, helped to
208 preclude participants from explicitly trying to remember and replicate their evaluations across
209 repetitions. A typical within-trial event sequence is shown in Figure 1.



210

211 **Figure 1:** an example of a within-trial even sequence for the rating tasks (study 1 and study 2)

212

213 **Study 2**

214 **Participants**

215 A total of 50 people participated in this study (18 female; age: mean=30.5, stdev=11.4, min=18,
216 max=64; 8 missing gender info; 2 missing age info). All participants were recruited from the online
217 subject pool using Prolific (prolific.co). Each participant received \$5 as compensation for 45
218 minutes of time.

219 Experimental Design

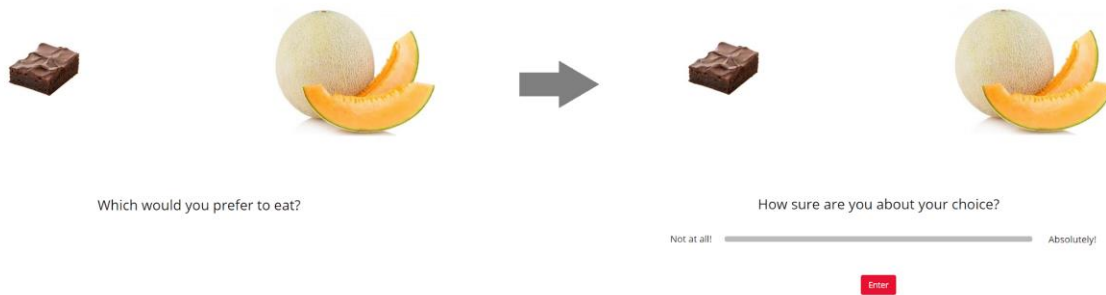
220 The experiment contained three different tasks: exposure, rating, and choice. There was no time
221 limit for the overall experiment, nor for the different tasks, nor for the individual trials.

222 The Exposure task was structurally identical to that described above for Study 1.

223 The Rating1 and Rating2 tasks were identical, except for the sequence of trials. For each rating
224 task, all of the stimuli to which the participant had initially been exposed were again displayed
225 on the screen, one at a time, in a random sequence (randomized across participants and across
226 sections for each participant). The structure of the rating trials was identical to that described
227 above for Study 1. Prior to commencing Rating2, participants were reminded that they should
228 report what they felt at that time and not try to remember what they reported during Rating1.
229 This helped to preclude participants from explicitly trying to remember and replicate their
230 evaluations across repetitions. A typical within-trial event sequence is shown in Figure 1 above.

231 For the Choice task, all stimuli were displayed on the screen, two at a time, in a random sequence
232 (randomized across participants and across sections for each participant). At the onset of each
233 trial, a fixation cross appeared at the center of the screen for 750ms. Next, a pair of images of
234 food items appeared on the screen, one towards the left, one towards the right. Participants
235 responded to the question, "Which would you prefer to eat?" by clicking on the image of their
236 preferred item. Participants then responded to the question, "Are you sure about your choice?"
237 using a horizontal slider scale. The leftmost end of the scale was labeled "Not at all!" and the
238 rightmost end was labeled "Absolutely!" Participants could revise their confidence report as

239 many times as they liked before finalizing it. Participants clicked the “Enter” button to finalize
240 their confidence report and proceed to the next screen. A typical within-trial event sequence is
241 shown in Figure 2.



242

243 **Figure 2:** an example of a within-trial even sequence for the choice task (study 2)

244

245 The pairings of items for each choice trial were created in a deliberate manner. Specifically, we
246 wanted to maximize the number of difficult choices that participants would be faced with. Here
247 we define difficulty as the similarity of the value ratings between choice pair items. Because our
248 simplified online experimental design did not allow for choice pairs to be created dynamically
249 based on each participant’s personal subjective value ratings, we relied on our data from Study
250 1. That data provided us with value ratings for 200 items across 37 participants, which we used
251 to calculate population statistics (median and variance of value estimate ratings) for each item.
252 We first calculated the population value estimate variability, which was the variance of the value
253 estimate ratings for each item across all 37 participants. Because we only wanted 150 items for
254 Study 2, we sorted the original 200 items from lowest to highest population variance and

255 removed the 50 highest-variability items (i.e., the items for which different participants had
256 provided the most variable value estimate ratings) from our set. We thought that this would
257 improve our chances that a new participant would rate the items similarly to the population
258 average ratings. Next, we calculated the population median value for each item. We used the
259 median instead of the mean so as not to be unduly influenced by extreme ratings. Sorting the
260 item set from highest to lowest value, we created triplets of items (i.e., [item1 item2 item3],
261 [item4 item5 item6], ...). We created 50 choice pairs for by selecting the first and second
262 elements from each triplet. We created an additional 50 choice pairs by selecting the first and
263 third elements from each triplet. We thus had a total of 100 choice pairs, all of which should be
264 difficult trials based on population statistics. (The reason why we created two separate sets in
265 the manner described was to allow us to pilot test a hypothesis for a future study, but is irrelevant
266 to this current study.) Obviously, individual ratings deviate from population ratings, which would
267 naturally cause many of the choice pair trials to be more or less difficult for individual
268 participants.

269 The usage of population median ratings in this way was solely to create choice pairs that would
270 *a priori* be likely to be difficult for most participants—it had no impact on the choice data itself,
271 which was based on the individual value estimate ratings of the participants who would actually
272 make the choices. Therefore, although choice pairs were created based on population value
273 estimate ratings from Study 1, the actual choice difficulty analyzed in the data for Study 2 was
274 determined entirely by the personal value ratings provided by each participant in that
275 study. Fortunately, this technique did indeed result in each participant in Study 2 facing a large
276 number of difficult choices (defined by their own personal value ratings). Note: the validity of

277 our analysis would not have been impacted either way, but the effects of interest would likely
278 have diminished.

279 **RESULTS**

280 Before conducting our main analyses, we first validated that our data were reliable (see
281 Supplementary Material for details). We determined that the data were generally reliable,
282 although we decided to exclude 10 participants from Study 1 and six participants from Study 2,
283 for failing to perform the tasks properly for the duration of the experiment (see Supplementary
284 Material for details).

285 **Study 1**

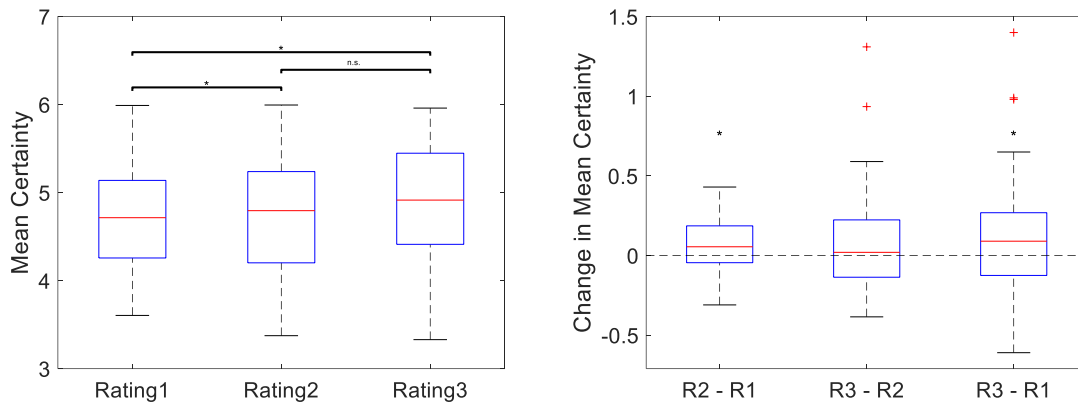
286 *Hypothesis 1: Certainty should negatively correlate with rating inconsistency.*

287 Certainty reports were provided by the participants during the study, but we needed to obtain a
288 measure of rating inconsistency. For each participant, we thus calculated the within-item across-
289 section variance of value ratings (i.e., $V[\text{Rating1}_i \text{Rating2}_i \text{Rating3}_i]$ for $i=1:200$). We deemed
290 that variance is a measure of inconsistency, because perfect consistency would yield a variance
291 of zero and higher degrees of inconsistency would yield higher variance. For each participant,
292 we used the average certainty for each item across the three rating sections as our measure of
293 certainty. The correlation between certainty and inconsistency was negative and significant, as
294 expected (median Spearman's $\rho = -0.239$, $p < 0.001$, two-sided t-test).

295

296 *Hypothesis 2: Certainty should increase with repeated ratings.*

297 We first calculated the within-participant mean of certainty reports separately for Rating1,
298 Rating2, and Rating3. We then calculated the group averages for these values. The across-
299 participant across-item mean certainty for Rating1, Rating2, and Rating3 was 4.78, 4.87, and 5.03,
300 respectively (see Figure 3). The increase in average certainty between Rating1 and Rating2,
301 between Rating2 and Rating3, and between Rating1 and Rating3 were all significant ($p=0.022$,
302 $p=0.041$, $p=0.008$; two-sided t-tests).



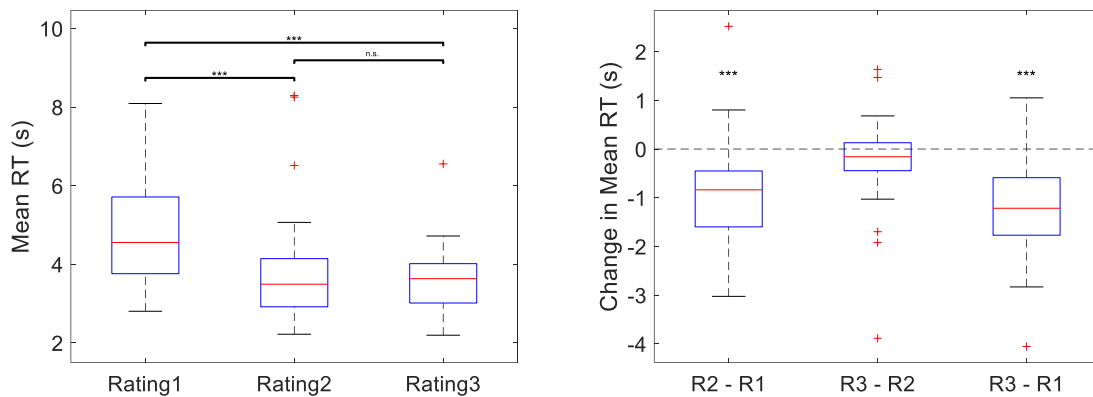
303

304 **Figure 3:** Across participants, the mean value rating certainty (across items) increased with each additional rating.
305 Left figure shows the cross-participant mean of within-participant mean certainty ratings across trials, separate for
306 each rating task. Right figure shows the cross-participant mean of within-participant change in mean certainty
307 across trials, between rating tasks (red lines indicate means, blue boxes indicate 95% c.i., error bars indicate range
308 of non-outlier data, red crosses indicate outliers, significance stars indicate: * $p<0.05$, ** $p<0.01$, *** $p<0.001$)

309

310 In addition to the gradual increase in average certainty from Rating1 to Rating3, we also checked
311 to see if there was a gradual decrease in average response time (RT). Because online testing is
312 often plagued by distractions that cause some trials to have exceptionally long response times,
313 we first removed all outlier trials. We defined an outlier as any trial in which RT was greater than

314 the within-participant median RT plus three times the within-participant median average
315 deviation. After cleaning the data in this way, we indeed found that average RT decreased from
316 one rating section to the next. The across-participant across-item mean RT for Rating1, Rating2,
317 and Rating3 was 4.87s, 3.87s, and 3.65s, respectively (see Figure 4). The decrease in RT between
318 Rating1 and Rating2 as well as between Rating1 and Rating3 was significant (both $p < 0.001$, two-
319 sided t-test), but the decrease between Rating2 and Rating3 was not ($p = 0.264$, two-sided t-test).



320

321 **Figure 4:** Across participants, the mean RT (across items) decreased with each additional rating. Left figure shows
322 the cross-participant mean of within-participant mean RT across trials, separate for each rating task. Right figure
323 shows the cross-participant mean of within-participant change in mean RT across trials, between rating tasks (red
324 lines indicate means, blue boxes indicate 95% c.i., error bars indicate range of non-outlier data, red crosses
325 indicate outliers, significance stars indicate: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$)

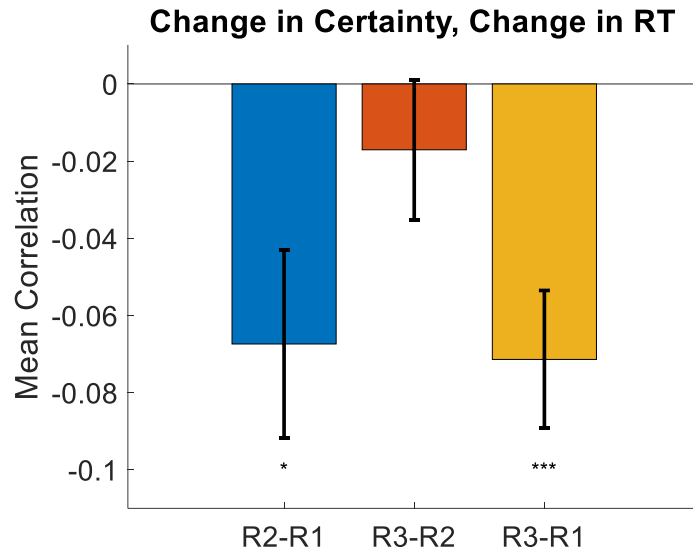
326

327 *Hypothesis 3: Certainty should negatively correlate with response time.*

328 For Rating1, the across-participant mean correlation between certainty and RT was negative, as
329 predicted (mean Spearman's rho = -0.121, $p < 0.001$, two-sided t-test). For Rating2, there was no
330 statistically significant correlation between certainty and RT (mean Spearman's rho = -0.009,

331 $p=0.752$, two-sided t-test). For Rating3, there was actually a positive correlation between
332 certainty and RT (mean Spearman's $\rho = 0.069$, $p=0.017$, two-sided t-test).

333 Recalling that overall RT decreased across rating sections, we thought that this might have hidden
334 the inherent relationship between certainty and RT. The idea is as follows. Initially (i.e., for
335 Rating1), some items are evaluated with high certainty, others with low certainty. The high
336 certainty evaluations are reported faster than the low certainty evaluations, thereby establishing
337 the negative correlation between certainty and RT. Eventually (i.e., for Rating2 and Rating3),
338 low-certainty evaluations become more certain (and thus more quickly evaluated). But, high-
339 certainty evaluations remain certain, and there is not much room for an increase in certainty for
340 these evaluations. Therefore, when averaging across the entire set of items, this would cause an
341 overall increase in certainty as well as an overall decrease in RT. This could deteriorate the initial
342 relationship between certainty and RT, as the set of items in effect shifts towards similarity (i.e.,
343 high certainty and low RT). To test this idea, we examined the evolution of certainty and RT on
344 an item-by-item basis. We first calculated, for each participant and each item, the change in both
345 the certainty and the RT from Rating1 to Rating2, and then calculated the within-participant
346 correlation between those variables across items. Across participants, the correlation between
347 Certainty Change and RT Change from Rating1 to Rating2 was indeed negative (mean Spearman's
348 $\rho = -0.067$, $p=0.010$, two-sided t-test). We then repeated this same analysis using the
349 differences from Rating2 to Rating3, and from Rating1 to Rating3. These correlations were both
350 negative as well, although the former was not significant (mean Spearman's $\rho = -0.017$,
351 $p=0.356$; mean Spearman's $\rho = -0.071$, $p<0.001$; two-sided t-tests). (see Figure 5).



352

353 **Figure 5:** Across participants, the change in value rating certainty negatively correlated with the change in value
354 rating RT. The idea here is that on an item-by-item basis, as certainty increases (across repeated ratings), it takes
355 the DM less time to decide upon a rating estimate. (error bars represent s.e.m., significance stars represent: *
356 $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$)

357

358 **Study 2**

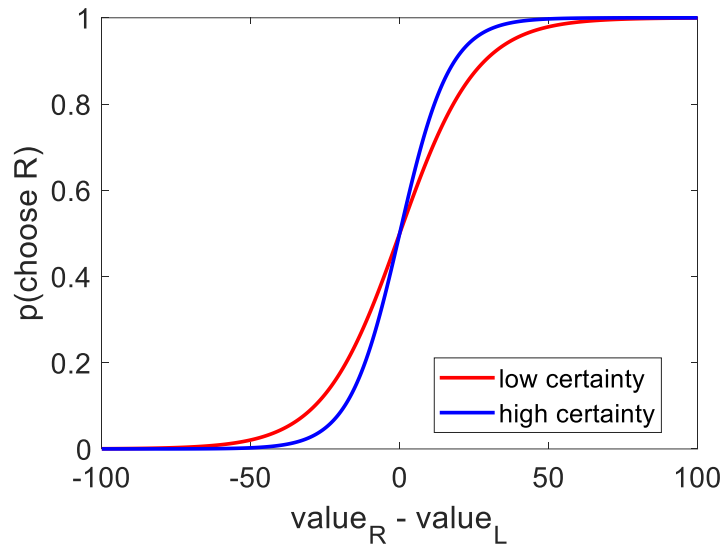
359 With Study 1, we demonstrated the reliability of our experimental measures of certainty
360 regarding subjective value estimates. With Study 2, we seek to further demonstrate the
361 importance of such measures by establishing their instrumental role in the decision making
362 process.

363 *Hypothesis 1: Choices will be more stochastic when value certainty is lower*

364 Value-based choice is primarily a function of the difference in the value estimates of the different
365 options in the choice set. The farther apart the value estimates are, the more likely it is that the
366 higher-rated item will be chosen; the closer together the value estimates are, the more likely it

367 is that the choice will appear to be random. Indeed, our data follow this pattern. For each
368 participant, we performed a logistic regression of choices against the difference in value ratings
369 of the paired options (choice = $\beta_0 + \beta_1 \cdot dV + \epsilon$). We found that this function fit the data
370 well above chance level, with a cross-participant average balanced accuracy of 77% ($p < 0.001$,
371 two-sided t-test). Across participants, there was no inherent bias for one side over the other
372 (mean $\beta_0 = -0.036$, $p = 0.350$, two-sided t-test) and there was a significant positive inverse
373 temperature parameter (mean $\beta_1 = 0.077$, $p < 0.001$, two-sided t-test).

374 What would be more interesting, however, would be to see how value estimate certainty
375 influences this choice model. We thus performed a similar logistic regression, for each
376 participant, except this time we also included an indicator variable that took the value of 1 if the
377 value certainty of a particular choice pair was greater than the median for that participant, and
378 0 otherwise (choice = $\beta_0 + \beta_1 \cdot dV + \beta_2 \cdot I \cdot dV + \epsilon$). Balanced accuracy remained at 77%
379 ($p < 0.001$, two-sided t-test). As with the previous model, there was no bias (mean $\beta_0 = -0.035$,
380 $p = 0.356$, two-sided t-test) and the inverse temperature for value difference was positive and
381 significant (mean $\beta_1 = 0.077$, $p < 0.001$, two-sided t-test). Notably, the regression coefficient
382 for the interaction of value difference and the high certainty indicator (i.e., the increase in choice
383 precision between low and high value certainty trials) was positive (mean $\beta_2 = 0.042$, $p = 0.054$,
384 one-sided t-test). (See Figure 6.)



385

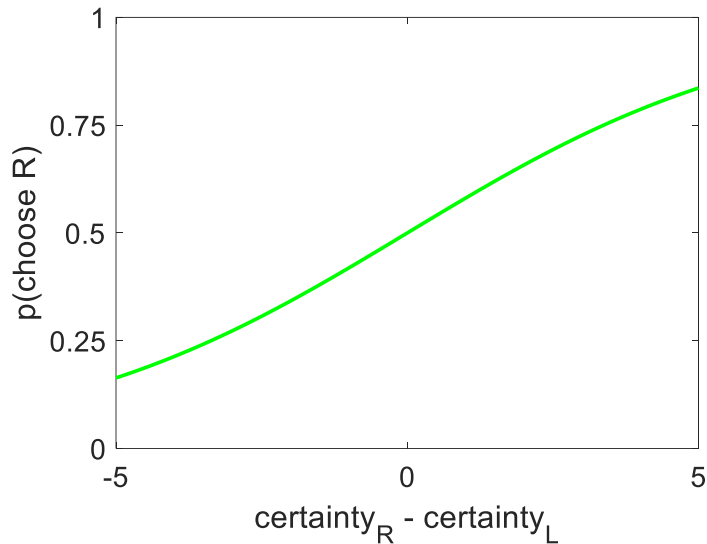
386 **Figure 6:** Across participants, the probability of choosing the option on the right increased as a function of the
387 value estimate difference (right option – left option). In particular: choices that were made between options with
388 low value certainty (red curve, within subject median split) were more stochastic than choices that were made
389 between options with high value certainty (blue curve) (left plot).

390

391 *Hypothesis 2: Options with higher value certainty will be chosen more often*

392 We posited that choices might be partially determined by how certain the individual value
393 estimates for each option were. We thus wondered how well choice could be predicted using
394 the difference in value certainty alone, without considering the difference in value estimates
395 themselves. For each participant, we ran a logistic regression of choices against the difference in
396 value estimate certainty (choice = $\beta_0 + \beta_1 \cdot dC + \epsilon$). Balanced accuracy was lower under
397 this model, as expected, but it was still well above chance level (cross-participant mean = 59%,
398 $p < 0.001$, two-sided t-test). Again, there was no bias (mean $\beta_0 = -0.044$, $p = 0.134$, two-sided t-
399 test). The inverse temperature for value certainty difference was positive and significant, as
400 expected (mean $\beta_1 = 0.326$, $p < 0.001$, two-sided t-test) (see Figure 7). This shows that choices

401 can indeed be predicted by the difference in the value certainty of the options under
402 consideration, without directly examining the difference in the value estimates themselves.



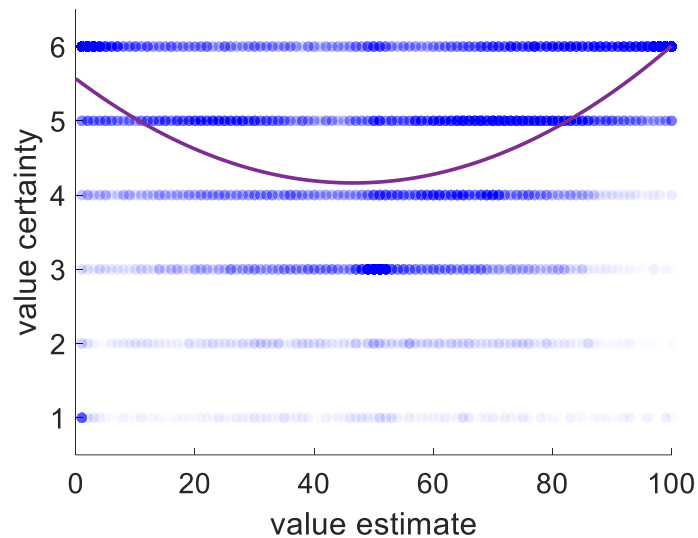
403

404 **Figure 7:** Across participants, the probability of choosing the option on the right increased as a function of the
405 value certainty difference (right option – left option).

406

407 Although we showed that choice could be predicted by value certainty even without considering
408 value estimate, we realized that there is generally a strong relationship between those two
409 variables. Supporting this notion, we found that value certainty correlated positively with value
410 estimate (mean Spearman's rho = 0.254, $p < 0.001$, two-sided t-test). Moreover, there was a clear
411 u-shaped relationship between value estimate and value certainty (see Figure 8). We note,
412 however, that the value certainty reports carried additional information beyond the value
413 estimate ratings themselves. The data clearly show that whereas very high or very low value
414 estimates almost always correspond to very high certainty, mid-range value estimates do not
415 necessarily correspond to relatively low certainty. It seems that sometimes participants

416 estimated an item's value to be mid-range because they were not certain about its true value,
417 but other times they were quite certain that its value was mid-range. This shows that value
418 certainty is partially constrained, but not fully determined, by value estimate itself.



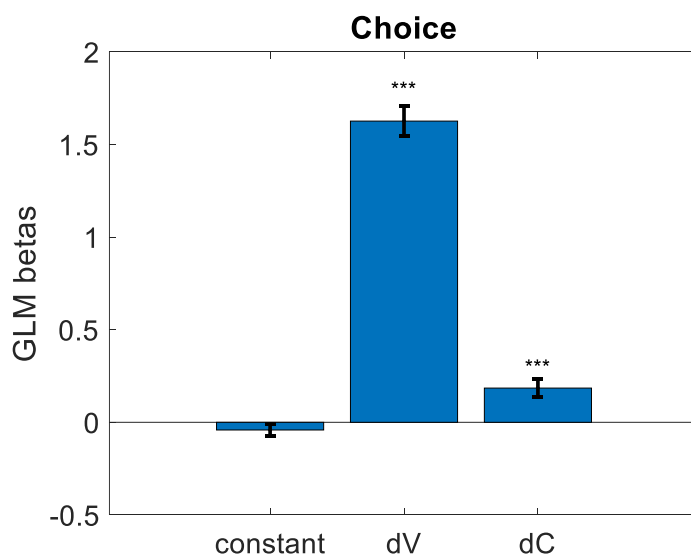
419

420 **Figure 8:** Value certainty is related to value estimate, with both a linear effect and a quadratic one. Blue dots
421 represent one item for one participant for one rating session. Purple curve represents the best linear + quadratic
422 fit across all trials and all participants.

423

424 Exploring further, we wondered if the predictive power of value certainty might be driven entirely
425 by its relationship with value estimate. That is, we wanted to check if the information contained
426 in the value certainty reports beyond what they convey about the value estimates themselves
427 would be useful in predicting choice. We predicted that, all else equal on a particular trial, the
428 option with the higher value estimate certainty would be the chosen option. To test this, we ran,
429 for each participant, a logistic regression of choices against the difference in value estimate
430 ratings as well as the difference in value certainty reports (choice = $\beta_0 + \beta_1 \cdot dV + \beta_2 \cdot dC$
431 + ϵ). Prior to running the regression, we first z-scored value estimate ratings and value certainty

432 reports separately for each participant. Balanced accuracy remained at 77% ($p < 0.001$, two-sided
433 t-test). As with the previous models, there was no bias (mean $\beta_0 = -0.042$, $p = 0.231$, two-sided
434 t-test), and the inverse temperature for value difference was positive and significant (mean β_1
435 = 1.626, $p < 0.001$, two-sided t-test). Notably, the inverse temperature for certainty difference
436 was also positive and significant (mean $\beta_2 = 0.185$, $p < 0.001$, two-sided t-test). (See Figure 9.)
437 The regression function we used orthogonalizes regressors sequentially when calculating beta
438 weights (i.e., here dC was orthogonalized to dV), so the impact of dC was truly separate from the
439 impact of dV. This suggests that not only did the participants consider the difference in value
440 estimates when choosing their preferred options, but they also considered the difference in value
441 certainty irrespective of the value estimates.



442

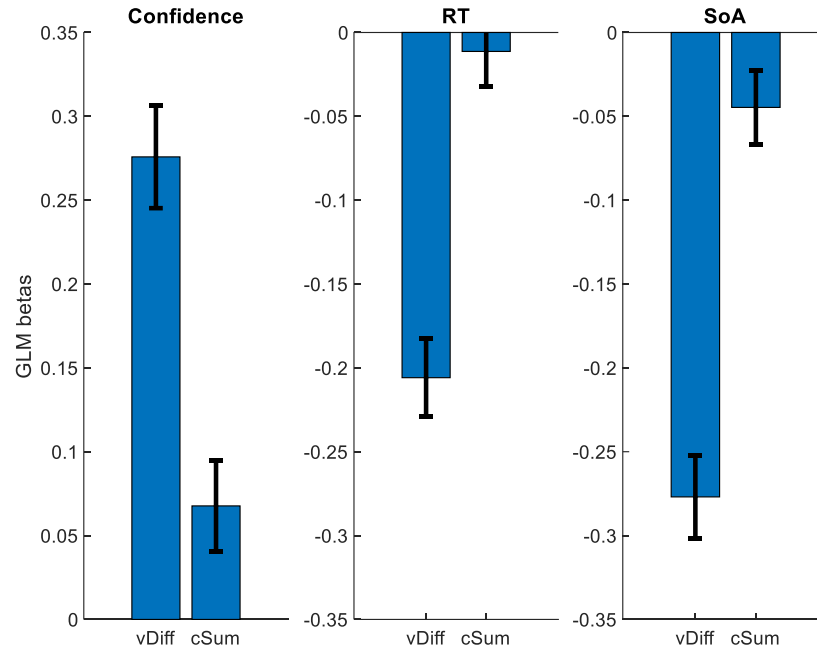
443 **Figure 9:** Cross-participant mean beta weights from GLM logistic regression of trial-by-trial value estimate
444 difference (dV) and value certainty difference (dC) onto choice. (error bars represent s.e.m.)

445

446

447 *Replication Results*

448 After testing our hypotheses, we next performed a series of analyses to try to replicate previously
449 reported results showing how value estimate certainty impacts a variety of dependent variables
450 during choice (Lee and Daunizeau, 2020a, 2020b). Specifically, we checked whether choice
451 confidence, response time, or choice-induced preference change changed as a function of value
452 estimate certainty. For our measure of choice-induced preference change, we used the
453 spreading of alternatives, defined as the post- minus pre-choice rating for the chosen option
454 minus the post- minus pre-choice rating for the rejected option. For each of the above dependent
455 variables, we ran a linear regression using absolute value estimate difference (vDiff) and summed
456 value estimate certainty (cSum) as regressors (response time outliers > median + 3*MAD
457 removed; all variables z-scored within participant). For choice confidence, we found that both
458 independent variables had positive beta weights, as predicted (mean for vDiff = 0.276, $p < 0.001$;
459 mean for cSum = 0.068, $p = 0.016$; two-sided t-tests). For response time, we found that both
460 independent variables had negative beta weights, as predicted, although only vDiff was
461 significant (mean beta for vDist = -0.206, $p < 0.001$; mean beta for cSum = -0.011, $p = 0.586$; two-
462 sided t-tests). For spreading of alternatives, we found that both independent variables had
463 negative beta weights, as predicted (mean beta for vDiff: -0.277, $p < 0.001$; mean beta for cSum:
464 -0.045, $p = 0.048$; two-sided t-tests). (See Figure 10.)



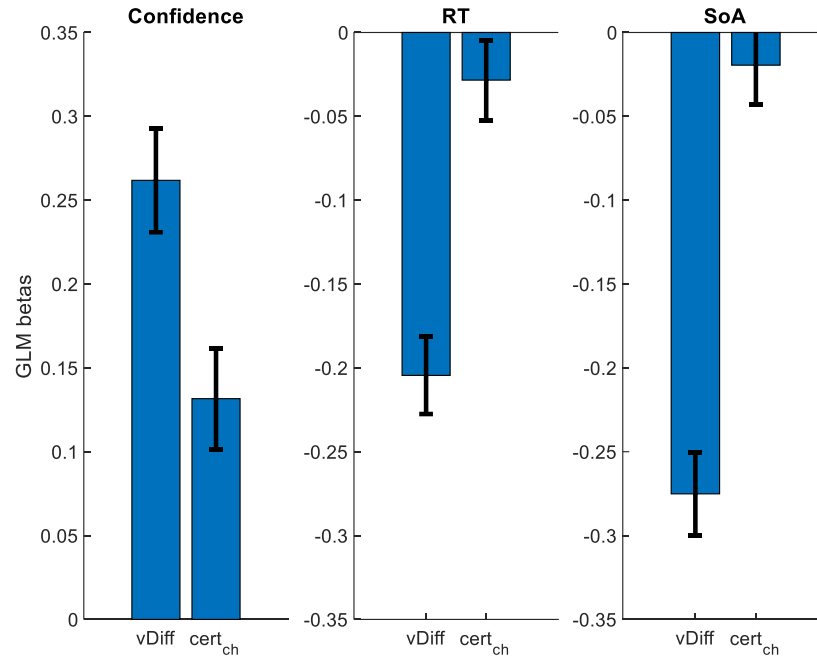
465

466 **Figure 10:** Cross-participant mean beta weights from GLM regressions of trial-by-trial absolute value estimate
467 difference (vDiff) and summed value estimate certainty (cSum) onto choice confidence (left figure), choice
468 response time (middle figure), and spreading of alternatives (right figure). (error bars represent s.e.m.)

469

470 The use of cSum to represent the relevant aspect of value estimate certainty during choice
471 deliberation was somewhat arbitrary. We therefore decided to examine other measures in the
472 place of cSum, specifically: certainty of the chosen option ($cert_{ch}$), certainty of the rejected option
473 ($cert_{rej}$), difference in certainty between the chosen and rejected options ($cert_{ch-rej}$). For each
474 participant, we repeated the same GLM regression as described above, replacing cSum with
475 $cert_{ch}$, $cert_{rej}$, and $cert_{ch-rej}$ in turn. We started with $cert_{ch}$. For choice confidence, we found that
476 both independent variables had positive beta weights, as predicted (mean for vDiff = 0.262,
477 $p < 0.001$; mean for $cert_{ch}$ = 0.132, $p < 0.001$; two-sided t-tests). For response time, we found that
478 both independent variables had negative beta weights, as predicted, although only vDist was

479 significant (mean beta for vDist = -0.205, $p < 0.001$; mean beta for cert_{ch} = -0.028, $p = 0.240$; two-
480 sided t-tests). For spreading of alternatives, we found that both independent variables had
481 negative beta weights, as predicted, although only vDiff was significant (mean beta for vDiff: -
482 0.275, $p < 0.001$; mean beta for cert_{ch}: -0.020, $p = 0.408$; two-sided t-tests). (See Figure 11.) The
483 regression analyses using cert_{rej} did not yield significant beta weights for the certainty term
484 (confidence: mean beta for vDiff: 0.275, $p < 0.001$; mean beta for cert_{rej}: -0.032, $p = 0.180$; RT:
485 mean beta for vDiff: -0.205, $p < 0.001$; mean beta for cert_{rej}: 0.010, $p = 0.659$; SoA: mean beta for
486 vDiff: -0.281, $p < 0.001$; mean beta for cert_{rej}: -0.039, $p = 0.108$; two-sided t-tests). The regression
487 analyses using cert_{ch-rej} yielded similar results as when using cert_{ch}, though with slightly lower
488 beta weights and slightly larger p-values (confidence: mean beta for vDiff: 0.263, $p < 0.001$; mean
489 beta for cert_{ch-rej}: 0.116, $p < 0.001$; RT: mean beta for vDiff: -0.204, $p < 0.001$; mean beta for cert_{ch-}
490 _{rej}: -0.021, $p = 0.402$; SoA: mean beta for vDiff: -0.282, $p < 0.001$; mean beta for cert_{ch-rej}: 0.021,
491 $p = 0.460$; two-sided t-tests).



492

493 **Figure 11:** Cross-participant mean beta weights from GLM regressions of trial-by-trial absolute value estimate
494 difference (vDiff) and value estimate certainty of the chosen option (cert_{ch}) onto choice confidence (left figure),
495 choice response time (middle figure), and spreading of alternatives (right figure). (error bars represent s.e.m.)

496

497

498 DISCUSSION

499 In this study, we have demonstrated the reliability of multiple measures of subjective value
500 estimate certainty, including self-reports, rating consistency, and response time. We have also
501 demonstrated the important role that value estimate certainty plays in choice itself, including its
502 positive impact on choice consistency and choice confidence, as well as its negative impact on
503 response time and choice-induced preference change. We might suggest that any contemporary
504 or future model of value-based decision making (and arguably, all types of decision making)
505 should consider including some measure of value estimate certainty for each of the options in
506 the choice set. At the present time, the only choice model that we are aware of that explicitly

507 includes a variable to represent value estimate certainty is the Metacognitive Control of Decisions
508 (MCD) presented by Lee and Daunizeau (2020b). This feature alone sets the MCD model apart
509 from the plethora of alternative models that abound in the literature. Yet it would not be
510 reasonable to claim that one class of model is inherently better than another simply because the
511 alternative failed to consider an important variable. Rather, we propose that the popular models
512 that already exist in the literature should be expanded to include value rating certainty. Only
513 then can a more complete and fair model comparison be made, and only then will we be able to
514 reach a better understanding of the cognitive mechanisms of decision making.

515 In particular, we call upon proponents of the so-called accumulation-to-bound models, such as
516 the Race Model (RM) and the Drift-Diffusion Model (DDM), to strongly consider revising their
517 models to include value estimate certainty. As it stands, most such models completely exclude
518 the possibility of item-specific certainty. These models typically (or exclusively) account for
519 stochasticity in the choice deliberation process at the system level, rather than at the option
520 level. This means that such models can explain or predict variations in observed behavior that
521 are dependent on choice context (e.g., clarity of perception, mental workload), but not on the
522 composition of the choice set itself. Given that stochasticity is one of the fundamental
523 components of evidence accumulation models (i.e., the diffusion parameter), it begs the question
524 as to why the nature of the stochasticity has not been more thoroughly explored.

525 Recent work has proven that an accumulation-to-bound process such as that represented by the
526 RM or DDM is an optimal policy, at least when optimality is defined as the maximization of reward
527 in a series of sequential decisions with a limited amount of time (Tajima et al, 2016, 2019). These

528 authors do indeed acknowledge the importance of certainty in their work, although it is not quite
529 of the same nature as that which we described in our study. In the work of Tajima et al (2016,
530 2019), pre-choice certainty about an option refers to the prior belief that a DM has about the
531 value distributions from which each option originates, rather than a belief about the value
532 estimates of the options themselves. However, we have shown that item-specific pre-choice
533 certainty is an important input to the choice deliberation process. Without a measure of item-
534 specific certainty, such a model cannot account for variations in choice behavior when the
535 different options originate from the same categorical set (e.g., snack foods). Tajima et al (2016)
536 suggest that evidence accumulation serves to increase the certainty about the option values, but
537 that the momentary evidence itself is uncertain. According to the authors, noise in the
538 momentary evidence itself could originate both externally (e.g., the stochastic nature of stimuli,
539 perceptual noise, ambiguity, incomplete knowledge) or internally (e.g., uncertain memory, value
540 inference that extends over time) (Tajima et al, 2016). Here, the authors seem to pave the way
541 for certainty measures that vary on an option-by-option basis, although they do not make this
542 explicit in their work.

543 Other recent work has suggested that the evidence accumulation process illustrated by a DDM is
544 influenced by attention (Sepulveda et al, 2020; Krajbich and Rangel, 2011; Krajbich et al, 2010).
545 Specifically, it has been proposed that during choice deliberation, evidence accumulates at a
546 higher rate for the option that is currently being gazed at, relative to the other option(s). This
547 evidence might support value estimation directly (Krajbich and Rangel, 2011; Krajbich et al, 2010)
548 or a more general goal-relevant information estimation (Sepulveda et al, 2020). However,
549 neither of these models include value estimate certainty. Indeed, these models explicitly assume

550 that both the prior uncertainty (i.e., variability in the environment from which the options
551 originate) and the evidence uncertainty (i.e., stochasticity in samples drawn from probability
552 distributions with fixed means) are identical across options. We have shown that such an
553 assumption is not reasonable, and thus likely impedes model performance. Furthermore, these
554 authors assume a causal link from gaze to information processing, without considering the
555 alternative. Specifically, it could be that information processing is what captures gaze, explained
556 as follows. The reason why a DM alternates his gaze between the options is to consider them in
557 turn—or in other words, to process information about each of them in turn. If relevant and useful
558 information is not encountered during a gaze fixation, the DM will likely divert his gaze to another
559 option. However, if information is encountered, the DM will likely linger on the currently-fixated
560 item in order to allow that information to accumulate. Under this explanation, there would
561 indeed be a causal link between gaze and information processing, but in the opposite direction
562 from that proposed by the above authors. Those authors model gaze shifts as random events
563 (Sepulveda et al, 2020; Krajbich and Rangel, 2011; Krajbich et al, 2010). Our proposal that gaze
564 shifts are driven away from scant information sources and towards rich information sources
565 seems more parsimonious. Further studies will be required to demonstrate the direction (or
566 even the existence) of the aforementioned causal link. We predict that gaze duration for each
567 choice option will positively correlate with (and actually enable) the observed increase in value
568 estimate certainty.

569

570 **ACKNOWLEDGEMENTS**

571 This manuscript has been released as a pre-print at

572 <https://www.biorxiv.org/content/10.1101/2020.06.16.155234v1>.

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624 **Supplementary Material**

625 *Data Quality Check*

626 **Study 1**

627 Before testing our hypotheses, we performed a number of simple data quality checks. First, we
628 assessed the test-retest reliability of value ratings. For each participant, we thus measured the
629 correlation between first rating (Rating1) and second rating (Rating2), across items. We found
630 that ratings were generally consistent (median Spearman's $\rho = 0.817$). Most participants
631 showed a correlation of greater than 60%. We then measured, for each participant, the
632 correlation between Rating1 and Rating3. We found that ratings were generally consistent
633 (median Spearman's $\rho = 0.818$).

634 Next, we performed a similar assessment of the test-retest reliability of certainty reports. Before
635 examining the certainty data, we first converted the qualitative reports to numbers ("not at all"
636 = 1, "slightly" = 2, "somewhat" = 3, "fairly" = 4, "very" = 5, "extremely" = 6). For each participant
637 we then measured the correlation between certainty for Rating1 (Certainty1) and certainty for
638 Rating2 (Certainty2), across items. We found that certainty reports were generally consistent
639 (median Spearman's $\rho = 0.352$), although much less so than value ratings. We then measured,
640 for each participant, the correlation between Certainty1 and Certainty3, across items. We found
641 that certainty reports were generally consistent (median Spearman's $\rho = 0.353$), although
642 much less so than value ratings.

643 Because rating certainty is a key variable for testing our hypotheses, we needed to be sure that
644 participants responded meaningfully to the rating certainty question. An analysis of how
645 certainty correlates with our other variables of interest would not be possible where there is
646 insufficient variability in the certainty data. For this reason, we calculated a score for how much
647 variance each participant had across certainty reports. The median certainty report variance was
648 0.935, 0.882, and 0.599 for Rating1, Rating2, and Rating3, respectively. We deemed that there
649 were no participants who were obvious outliers based on this score.

650 For each of the test-retest reliability measures described above, we searched for population
651 outliers. We defined an outlier for a specific measure as a participant whose score was more
652 than three median average deviations (MAD) away from the population median. This technique
653 yielded five outlier participants based on the Rating1-Rating2 test-retest reliability scores, and 10
654 outlier participants based on the Rating1-Rating3 test-retest reliability scores. There were no
655 outliers with respect to Certainty. All five of the outliers of the first type were also outliers of the
656 second type, which left us with a set of 10 total outlier participants for Study 1. We excluded
657 these participants from our reported analyses.

658 **Study 2**

659 Before testing our hypotheses, we performed a number of simple data quality checks. First, we
660 assessed the test-retest reliability of value ratings. For each participant, we thus measured the
661 pairwise linear correlation between first rating (Rating1) and second rating (Rating2), across
662 items. We found that ratings were generally consistent (median Spearman's $\rho = 0.803$,
663 $p < 0.001$, two-sided t-test).

664 Next, we performed a similar assessment of the test-retest reliability of certainty reports. Before
665 examining the certainty data, we first converted the qualitative reports to numbers (“not at all”
666 = 1, “slightly” = 2, “somewhat” = 3, “fairly” = 4, “very” = 5, “extremely” = 6). For each participant
667 we then measured the pairwise linear correlation between certainty for Rating1 (Certainty1) and
668 certainty for Rating2 (Certainty2), across items. We found that certainty reports were generally
669 consistent (median Spearman’s rho = 0.344, $p < 0.001$, two-sided t-test), although much less so
670 than value ratings.

671 Because rating certainty is a key variable for testing our hypotheses, we needed to be sure that
672 participants responded meaningfully to the rating certainty question. An analysis of how
673 certainty correlates with our other variables of interest would not be possible where there is
674 insufficient variability in the certainty data. For this reason, we calculated a score for how much
675 variance each participant had across certainty reports. The median certainty report variance was
676 0.971 and 0.818 for Rating1 and Rating2, respectively. We deemed that there were no
677 participants who were obvious outliers based on this score.

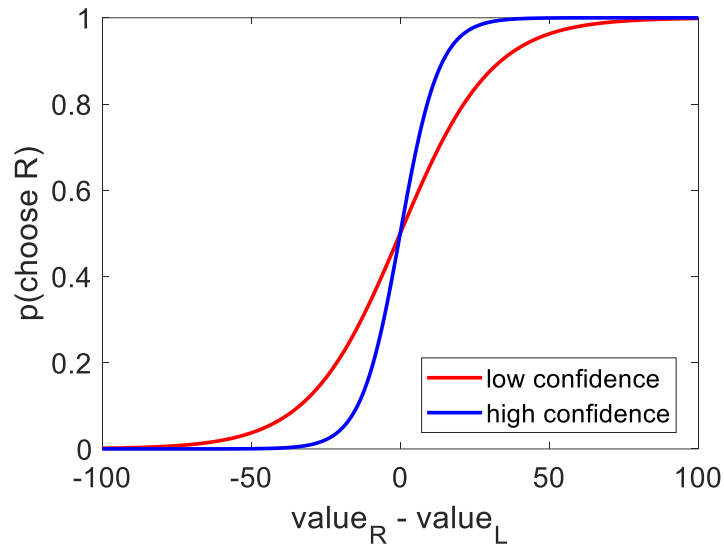
678 Finally, we checked whether choices were consistent with pre-choice ratings. For each
679 participant, we performed a logistic regression of choices against the difference in value ratings
680 of the paired options. We found that the balanced prediction accuracy was beyond chance level
681 (mean 77%), indicating participants were performing the choice task properly.

682 For each of the test-retest reliability measures described above, we searched for population
683 outliers. We defined an outlier for a specific measure as a participant whose score was more
684 than three median average deviations (MAD) away from the population median. This technique

685 yielded six outlier participants based on the Rating1-Rating2 test-retest reliability scores. This
686 left us with a set of six total outlier participants for Study 2. We excluded these participants from
687 our reported analyses.

688 We then explored a step further, postulating that choice confidence should modulate choice
689 consistency (often referred to as accuracy). The idea is that for high confidence choices, the DM
690 would more consistently distinguish the items, relative to low confidence choices. We thus
691 performed a similar logistic regression as we did in our main analysis (see Figure 6 in Results), for
692 each participant, except this time the indicator represented high choice confidence (within-
693 participant median split) instead of value certainty (choice = logistic[beta0 + beta1*dV +
694 beta2*Ind*dV]). Under this model, balanced accuracy was also 77% ($p < 0.001$, two-sided t-test).
695 Again, there was no bias (mean beta0 = -0.028, $p = 0.466$, two-sided t-test), and the inverse
696 temperature parameter remained positive and significant (mean beta1 = 0.065, $p < 0.001$, two-
697 sided t-test). Notably, the regression coefficient for the interaction of value difference and the
698 high confidence indicator (i.e., the increase in choice precision between low and high confidence
699 trials) was positive and significant (mean beta2 = 0.088, $p < 0.001$, two-sided t-test) (see Figure
700 S1). We thus confirmed a common observation that choice confidence and choice accuracy are
701 closely linked.

702



703

704 **Figure S1:** Across participants, the probability of choosing the option on the right increased as a function of the
705 value estimate difference (right option – left option). In particular: choices that were made with low confidence
706 (red curve, within subject median split) were more stochastic than choices that were made with high confidence
707 (blue curve).

708