

1 A modelling framework for the prediction of
2 the herd-level probability of infection from
3 longitudinal data

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30

Abstract

31 The collective control programmes (**CPs**) that exist for many in-
32 fectious diseases of farm animals rely on the application of diagnostic
33 testing at regular time intervals for the identification of infected an-
34 imals or herds. The diversity of these CPs complicates the trade of
35 animals between regions or countries because the definition of freedom
36 from infection differs from one CP to another. In this paper, we de-
37 scribe a statistical model for the prediction of herd-level probabilities
38 of infection from longitudinal data collected as part of CPs against
39 infectious diseases of cattle. The model was applied to data collected
40 as part of a CP against bovine viral diarrhoea virus (**BVDV**) infec-
41 tion in Loire-Atlantique, France. The model represents infection as a
42 herd latent status with a monthly dynamics. This latent status de-
43 termines test results through test sensitivity and test specificity. The
44 probability of becoming status positive between consecutive months is
45 modelled as a function of risk factors (when available) using logistic
46 regression. Modelling is performed in a Bayesian framework, using
47 either Stan or JAGS. Prior distributions need to be provided for the
48 sensitivities and specificities of the different tests used, for the proba-
49 bility of remaining status positive between months as well as for the
50 probability of becoming positive between months. When risk factors
51 are available, prior distributions need to be provided for the coeffi-
52 cients of the logistic regression, replacing the prior for the probability
53 of becoming positive. From these prior distributions and from the lon-
54 gitudinal data, the model returns posterior probability distributions
55 for being status positive for all herds on the current month. Data
56 from the previous months are used for parameter estimation. The im-
57 pact of using different prior distributions and model implementations
58 on parameter estimation was evaluated. The main advantage of this
59 model is its ability to predict a probability of being status positive in a
60 month from inputs that can vary in terms of nature of test, frequency
61 of testing and risk factor availability/presence. The main challenge
62 in applying the model to the BVDV CP data was in identifying prior
63 distributions, especially for test characteristics, that corresponded to
64 the latent status of interest, i.e. herds with at least one persistently in-
65 fected (**PI**) animal. The model is available on Github as an R package
66 (<https://github.com/AurMad/STOCfree>) and can be used to carry
67 out output-based evaluation of disease CPs.

68 1 Introduction

69 For many infectious diseases of farm animals, there are control programmes
70 (CPs) that rely on the application of diagnostic testing at regular time inter-
71 vals for the identification of infected animals or herds. In cattle, such diseases
72 notably include infection by the bovine viral diarrhoea virus (BVDV) or by
73 *Mycobacterium avium* subspecies *paratuberculosis* (MAP). These CPs are ex-
74 tremely diverse. Their objective can range from decreasing the prevalence of
75 infection to eradication. Participation in the CP can be voluntary or com-
76 pulsory. The classification of herds regarding infection status can be based
77 on a wide variety of testing strategies in terms of the nature of the tests used
78 (identification of antibodies vs. identification of the agent), the groups of
79 animals tested (e.g. breeding herd vs. young animals), number of animals
80 tested, frequency of testing (once to several times a year, every calf born...).

81 Even within a single CP, surveillance modalities may evolve over time. Such
82 differences in CPs were described by [van Roon *et al.* \(2020a\)](#) for programmes
83 targeting BVDV infections and by [Whittington *et al.* \(2019\)](#) for programmes
84 against MAP.

85 Differences in surveillance modalities can be problematic when purchas-
86 ing animals from areas with different CPs because the free status assigned
87 to animals or herds might not be equivalent between CPs. A standardised
88 method for both describing surveillance programmes and estimating confi-
89 dence of freedom from surveillance data would be useful when trading animals
90 across countries or regions. While inputs can vary between programmes, the
91 output needs to be comparable across programmes. This is called output-
92 based surveillance ([Cameron, 2012](#)). Probabilities measure both the chance
93 of an event and the uncertainty around its presence/occurrence. If well de-
94 signed, a methodology to estimate the probability of freedom from infection
95 would meet the requirements of both providing a confidence of freedom from
96 infection as well as of being comparable whatever the context.

97 Currently, a common quantitative method used to substantiate freedom
98 from infection to trading partners is the scenario tree method ([Martin *et al.*,
99 2007](#)). The method is applied to situations where there is a surveillance
100 programme in place, with no animals or herds confirmed positive on testing.
101 What is estimated with the scenario tree method is the probability that the
102 infection would be detected in the population if it were present at a chosen
103 *design prevalence*. The output from this approach is the probability that
104 infection prevalence is below the design prevalence given the negative test

105 results (Cameron, 2012). Therefore, this method is well suited for situations
106 where populations are free from infection and those who want to quantify
107 this probability of freedom from infection, e.g. for the benefit of trading
108 partners (Norström *et al.*, 2014).

109 In a context where disease is controlled but still present, it would only
110 be safe to trade with herds that have an estimated probability of freedom
111 from infection that is deemed sufficiently high or, equivalently, a probability
112 of infection that is deemed sufficiently low. Identifying these herds involves
113 estimating a probability of infection for each herd in the CP and then defining
114 a decision rule to categorise herds as uninfected or infected based on these
115 estimated probabilities.

116 In this paper, we propose a method to estimate herd level probabilities
117 of infection from heterogeneous longitudinal data generated by CPs. The
118 method predicts herd-month level probabilities of being latent status positive
119 from longitudinal data collected in CPs. The input data are test results, and
120 associated risk factors when available. Our main objective is to describe
121 this modelling framework by showing how surveillance data are related to
122 the *probabilities of infection* (strictly speaking, *probabilities of being latent*
123 *status positive*) and by providing details regarding the statistical assumptions
124 that are made. A secondary objective is to compare two implementations of
125 this modelling framework, one in JAGS (Plummer, 2003) and one in Stan
126 (Stan Development Team, 2021), for the estimation of these probabilities of
127 being latent status positive. The comparison is performed using surveillance
128 data collected as part of a CP against BVDV infection in Loire-Atlantique,
129 France. The challenges of defining prior distributions and the implications
130 of using different prior distributions are discussed. The functions to perform
131 the analyses described in this paper are gathered in an R package which is
132 available from GitHub (<https://github.com/AurMad/STOCfree>).

133 2 Materials and methods

134 2.1 Description of the model

135 2.1.1 Conceptual representation of surveillance programmes

136 Surveillance programmes against infectious diseases can be seen as imperfect
137 repeated measures of a true status regarding infection. In veterinary epidemi-
138 ology, the issue of imperfect testing has traditionally been addressed using

139 latent class models. With this family of methods, the true status regarding
140 infection is modelled as an unobserved quantity which is linked to test results
141 through test sensitivity and specificity. Most of the literature on the sub-
142 ject focuses on estimating both test characteristics and infection prevalence
143 (Collins & Huynh, 2014). For the estimations to work, the same tests should
144 be used in different populations (Hui & Walter, 1980), the test characteristics
145 should be the same among populations, and test results should be condition-
146 ally independent given the infection status (Toft *et al.*, 2005; Johnson *et al.*,
147 2009) ; although some of these assumptions can be relaxed in a Bayesian
148 framework. Latent class models can also be used to estimate associations
149 between infection, defined as the latent class, and risk factors when the test
150 used is imperfect (Fernandes *et al.*, 2019). In the study by Fernandes *et al.*
151 (2019), the latent class was defined using a single test, through the prior
152 distributions put on sensitivity and specificity. When using latent class mod-
153 els with longitudinal data, the dependence between successive test results
154 in the same herds must be accounted for. In the context of estimating test
155 characteristics and infection prevalence from 2 tests in a single population
156 from longitudinal data, Nusinovici *et al.* (2015) proposed a Bayesian latent
157 class model which incorporated 2 parameters for new infection and infection
158 elimination. The model we describe below combines these different aspects
159 of latent class modelling into a single model.

160 We propose using a class of models called Hidden Markov Models (HMM,
161 see Zucchini *et al.* (2017)). Using surveillance programmes for infectious dis-
162 eases as an example, the principles of HMMs can be described as follows:
163 the latent status (*class*) of interest is a herd status regarding infection. This
164 status is evaluated at regular time intervals: HMMs are discrete time mod-
165 els. The status at a given time only depends on the status at the previous
166 time (Markovian property). The status of interest is not directly observed,
167 however, there exists some quantity (such as test results) whose distribution
168 depends on the unobserved status. HMMs have been used for decades in
169 speech recognition (Rabiner, 1989) and other areas. They have also been
170 used for epidemiological surveillance (Le Strat & Carrat, 1999; Touloupou
171 *et al.*, 2020), although not with longitudinal data from multiple epidemiolog-
172 ical units such as herds. The model we developed is therefore a latent class
173 model that takes into account the time dynamics in the latent status. The
174 probability of new infection between consecutive time steps is modelled as a
175 function of risk factors.

176 Figure 1 shows how surveillance programmes are represented in the model

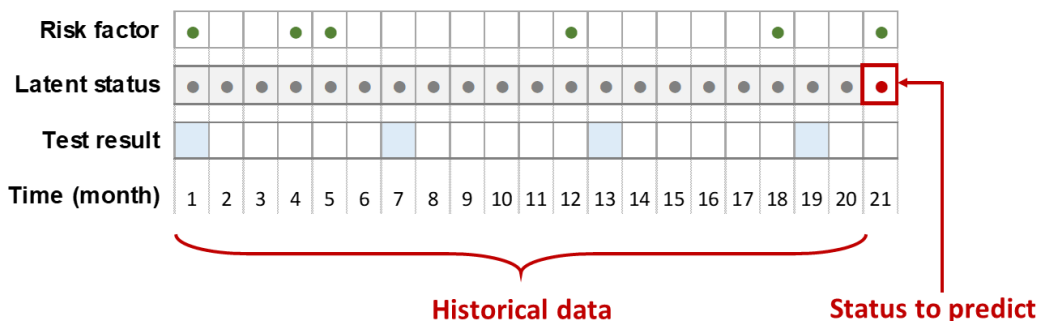


Figure 1: Conceptual representation of the implementation of a surveillance programme within a herd. The focus of the model is the latent status regarding infection, which is modelled at the herd-month level. This status partly depends on risk factors and determines test results. In this diagram, risk factors are represented as green dots when present and available test results as blue shaded squares. The model predicts a probability of infection for the most recent month in the surveillance programme using all the data collected for the estimation of model parameters.

177 as a succession of discrete time steps. The focus of this model is a latent
178 status evaluated at the herd-month level. This latent status is not directly
179 observed but inferred from its causes and consequences incorporated as data.
180 The consequences are the test results. Test results do not have to be available
181 at every time step for the model to work, although the estimation will be
182 more accurate with a large number of test results. The causes of infection are
183 risk factors of infection. The model estimates this latent status monthly, and
184 predicts it for the last month of data. These herd-month latent statuses will
185 be estimated/predicted from test results and risk factors recorded in each
186 herd.

187 2.1.2 Modelling framework, inputs and outputs

188 The model is designed to use longitudinal data collected as part of surveil-
189 lance programmes against infectious diseases. In such programmes, each herd
190 level status is re-evaluated when new data (most commonly test results, but
191 may also be data related to risk factors) are available. The model mimics

192 this situation by predicting the probability of a positive status for all herds
193 in the CP on the last month of available data. Data from all participating
194 herds up to the month of prediction are used as historical data for parameter
195 estimation (Figure 1).

196 The estimation and prediction are performed within a Bayesian frame-
197 work using Markov Chain Monte Carlo (MCMC). The model encodes the
198 relationships between all the variables of interest in a single model. Each
199 variable is modelled as drawn from a statistical distribution. The estimation
200 requires prior distributions for all the parameters in the model. These priors
201 are a way to incorporate either existing knowledge or hypotheses in the es-
202 timation. For example, we may know that the prevalence of herds infected
203 with BVDV in our CP is probably lower than 20%, certainly lower than 30%
204 and greater than 5%. There are different ways of specifying such constraints
205 using statistical distributions. We will briefly describe two that are used in
206 different places in our modelling framework. The first one consists in using a
207 Beta distribution. The Beta distribution is bounded between 0 and 1, with
208 2 parameters α and β determining its shape. With the constraints specified
209 above, we could use as a prior distribution $Beta(\alpha = 15, \beta = 100)$ ¹. The
210 second one consists in using a normal distribution on the logit scale. The
211 principle of the logit transformation is to map probabilities that are bounded
212 between 0 and 1 onto an interval that extends from $-\infty$ to $+\infty$. Quantities
213 defined on the logit scale, can be mapped back onto the probability scale us-
214 ing the inverse logit transformation². This is extremely convenient because
215 it allows the use of normal distributions on the logit scale, whose mean and
216 standard deviation have an intuitive meaning. With the constraints specified
217 above, we could use as a prior distribution a $Normal(\mu = -2, \sigma^2 = 0.09)$ ³.
218 If we do not know anything about this infection prevalence (which is rare), we
219 could use a $Beta(\alpha = 1, \beta = 1)$ prior, which is uniform between 0 and 1 ; or
220 a $Normal(\mu = 0, \sigma^2 = 10)$ on the logit scale. From the model specification,

¹The $Beta(\alpha = 15, \beta = 100)$ distribution has a mean of 0.13 and a standard deviation of 0.03. In R, it can be plotted using the following instructions `curve(dbeta(x, 15, 100))`

²The logit transformation is defined as $logit(p) = \ln(\frac{p}{1-p})$ and the inverse logit transformation is defined as $logit^{-1}(x) = \frac{e^x}{1+e^x}$. A value of 0 on the logit scale corresponds to a probability of 0.5.

³The $logit^{-1}Normal(\mu = -2, \sigma^2 = 0.09)$ distribution has a mean of 0.12 and a standard deviation of 0.03. In R, it can be plotted using the following instructions `curve(STOCfree::dnorm_logit(x, -2, .3))`.

221 the prior distributions and the observed data, the MCMC algorithm draws
222 samples from the posterior distributions of all the variables in the model.
223 These posterior distributions are the probability distributions for the model
224 parameters given the data and the prior distributions. MCMC methods are
225 stochastic and iterative. Each iteration is a set of samples from the joint pos-
226 terior distributions of all variables in the model. The algorithm is designed
227 to reach the target joint posterior distribution, but at any moment, there is
228 no guarantee that it has done. To overcome this difficulty, several indepen-
229 dent instances of the algorithm (i.e. several chains) are run in parallel. For a
230 variable, if all the MCMC draws from the different chains are drawn from the
231 same distribution, it can be concluded that the algorithm has reached the
232 posterior distribution. In this case, it is said that the model has converged.

233 The focus of our model is the monthly latent status of each herd. This
234 latent status depends on the data on occurrence of risk factors and it affects
235 test results. The data used by the model are the test results and risk factors.
236 At each iteration of the MCMC algorithm, given the data and priors, a herd
237 status (0 or 1) and the coefficients for the associations between risk factors,
238 latent status and test results are drawn from their posterior distribution.

239 In the next 3 sections, the parameters for which prior distributions are
240 required, i.e. test characteristics, status dynamics and risk factor parameters,
241 are described. The outputs of Bayesian models are posterior distributions for
242 all model parameters. Specifically, in our model, the quantities of interest
243 are the herd level probabilities of being latent status positive on the last
244 test month in the dataset as well as test sensitivity, test specificity, infection
245 dynamic parameters and parameters for the strengths of association between
246 risk factors and the probability of new infection. This is described in the
247 corresponding sections.

248 **2.1.3 Latent status dynamics**

249 Between test events, uninfected herds can become infected and infected herds
250 can clear the infection. The model represents the probability of having a
251 positive status at each time step as a function of the status at the previous
252 time step (Figure 2). For the first time step when herd status is assigned,
253 there is no previous status against which to evaluate change. From the second
254 time step when herd status is assigned, and onwards, herds that were status
255 negative on the previous time step have a certain probability of becoming
256 status positive and herds that were status positive have a certain probability

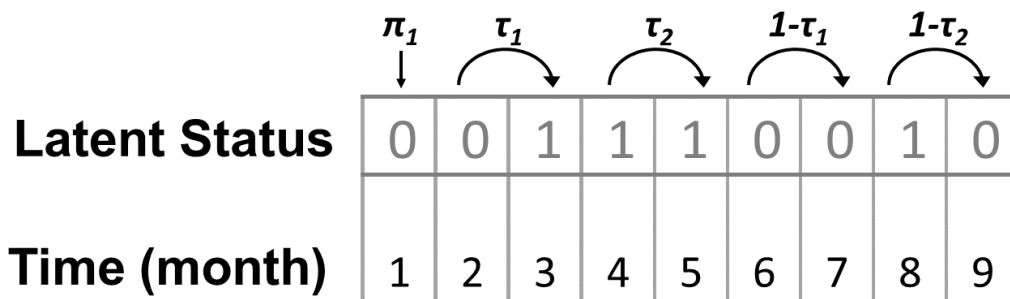


Figure 2: Modelling of infection dynamics. The diagram shows hypothetical latent statuses (0 for negative; 1 for positive) as a function of time in month, with examples of all possible transitions. $\pi_1 = p(S_1 = 1)$ is the probability of being status positive at the first point in time, $\tau_1 = p(S_t = 1|S_{t-1} = 0)$ is the probability of becoming status positive and $\tau_2 = p(S_t = 1|S_{t-1} = 1)$ is the probability of remaining status positive.

257 of remaining status positive.

258 These assumptions can be summarised with the following set of equa-
 259 tions⁴. The status on the first time step (S_1) is a Bernoulli event with a
 260 normal prior on the logit scale for its probability of occurrence:

$$S_1 \sim \text{Bernoulli}(\pi_1) \quad (1)$$

261

$$\text{logit}(\pi_1) \sim \text{Normal}(\mu_{\pi_1}, \sigma_{\pi_1}^2) \quad (2)$$

262 From the second time step when herd status is assigned, and onwards,
 263 a positive status is also a Bernoulli event (S_t) with a probability of occur-
 264 rence that depends on the status at the previous time step as well as on
 265 the probability of becoming status positive and the probability of remain-
 266 ing status positive. In this case, the probability of becoming status positive
 267 is $\tau_1 = p(S_t = 1|S_{t-1} = 0)$ and the probability of remaining positive is
 268 $\tau_2 = p(S_t = 1|S_{t-1} = 1)$.

$$S_t \sim \text{Bernoulli}(\pi_t) \quad (3)$$

⁴Statuses are estimated/predicted at the herd-month level. Herd is omitted from the notation to facilitate reading. S_t should be read as S_{ht} where h represents the herd.

269

$$\pi_t = \begin{cases} \tau_1 & \text{if } S_{t-1} = 0 \\ \tau_2 & \text{if } S_{t-1} = 1 \end{cases} \quad (4)$$

270

$$\text{logit}(\tau_1) \sim \text{Normal}(\mu_{\tau_1}, \sigma_{\tau_1}^2) \quad (5)$$

271

$$\text{logit}(\tau_2) \sim \text{Normal}(\mu_{\tau_2}, \sigma_{\tau_2}^2) \quad (6)$$

272 Therefore, the status dynamics can be completely described by π_1 , τ_1 and
273 τ_2 .

274 **2.1.4 Incorporation of information on risk factors for new infec-** 275 **tion**

276 The probability of new infection is not the same across herds. For example,
277 herds that introduce a lot of animals or are in areas where infection preva-
278 lence is high could be at increased risk of new infection (Qi *et al.*, 2019).
279 Furthermore, the association between a given risk factor and the probability
280 of new infection could be CP dependent. For example, the probability of
281 introducing infection through animal introductions will depend on the infec-
282 tion prevalence in the population from which animals are introduced. As a
283 consequence, estimates for these associations (as presented in the literature)
284 could provide an indication about their order of magnitude, but their preci-
285 sion may be limited. On the other hand, the CPs which are of interest in this
286 work usually generate large amounts of testing data which could be used to
287 estimate the strengths of association between risk factors and new infections
288 within a given CP. The variables that are associated with the probability of
289 new infection could increase the sensitivity and timeliness of detection.

290 When risk factors for new infection are available, the model incorporates
291 this information by modelling τ_1 as a function of these risk factors through
292 logistic regression, instead of the prior distribution for τ_1 .

$$\text{logit}(\tau_{1ht}) = X_{ht}\theta \quad (7)$$

293 where X_{ht} is a matrix of predictors for herd h at time t and θ is a vector
294 of coefficients. Normal priors are used for the coefficients of the logistic
295 regression.

$$\theta_i \sim \text{Normal}(\mu_i, \sigma_i) \quad (8)$$

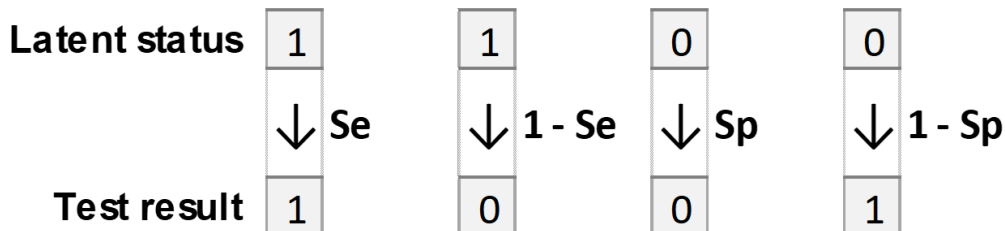


Figure 3: Relation of the model latent status to test result. Sensitivity is the probability of a positive test result in a status positive herd. Specificity is the probability of a negative test result in a status negative herd.

296 2.1.5 Test characteristics

297 The model allows the inclusion of several test types but for the sake of clarity,
 298 we show the model principles for only one test type. These principles can be
 299 extended to several tests by specifying prior distributions for all tests.

300 Tests are modelled as imperfect measures of the latent status (Figure 3).
 301 Test sensitivity is the probability of a positive test result given a positive
 302 latent status ($Se = p(T = 1|S = 1)$, refers to true positives) and test speci-
 303 ficity is the probability of a negative test result given a negative latent status
 304 ($Sp = p(T = 0|S = 0)$, refers to true negatives).

305 Test result at time t is modelled as a Bernoulli event with probability
 306 $p(T_t)$ of being positive.

$$T_t \sim \text{Bernoulli}(p(T_t)) \quad (9)$$

307 The relation between the probability of testing positive, the probability
 308 of a positive status, test sensitivity and test specificity is the following:

$$p(T_t) = \begin{cases} 1 - Sp & \text{if } S_t = 0 \\ Se & \text{if } S_t = 1 \end{cases} \quad (10)$$

309 Information or hypotheses regarding test characteristics are incorporated
 310 in the model as priors modelled by Beta distributions:

$$Se \sim \text{Beta}(Se_a, Se_b) \quad (11)$$

311

$$Sp \sim Beta(Sp_a, Sp_b) \quad (12)$$

312 2.1.6 Prediction of a probability of infection in JAGS

313 In JAGS, a specific step was needed in order to predict the final probability
 314 of being status positive given historical data and a test result on the month of
 315 prediction, when such a test result was available. In Stan, this step was not
 316 necessary because the forward algorithm directly predicted the probability
 317 of being status positive in the last month. In explaining how predictions are
 318 performed in JAGS, we use the following notation: \tilde{y} is the predicted value
 319 for y , $\hat{\beta}$ is the estimated value for β . The equation $\tilde{y} = \hat{\beta}.x$ means that the
 320 predicted value for y is equal to x (data) times the estimated value for β .

321 The model predicts herd-level probabilities of being latent status positive
 322 on the last month in the data mimicking regular re-evaluation as new data
 323 come in. If there is no test result available on this month, the predicted
 324 probability of being status positive (called $\tilde{\pi}_t^*$) is the predicted status on the
 325 previous month times $\tilde{\tau}_{1t}$ if the herd was predicted status negative or times
 326 $\hat{\tau}_2$ if the herd was predicted status positive (Table 1)⁵. This can be written
 327 as:

$$\tilde{\pi}_t^* = p(\tilde{S}_t | \hat{S}_{t-1}, \tilde{\tau}_{1t}, \hat{\tau}_2) = \begin{cases} \hat{\tau}_2 & \text{if } \hat{S}_{t-1} = 0 \\ \tilde{\tau}_{1t} & \text{if } \hat{S}_{t-1} = 1 \end{cases} \quad (13)$$

328 where:

$$\tilde{\tau}_{1t} = \text{logit}^{-1}(X_t \hat{\theta}) \quad (14)$$

329 If a test result was available, the prediction must combine information
 330 from the test as well as previous information. The way to estimate this
 331 predicted probability from $p(\tilde{S}_t^*)$ and test results can be derived from Table 1.
 332 The predicted probability of being status positive can be computed as:

$$p(\tilde{S}_t | T_t, \tilde{S}_t^*) = \begin{cases} \frac{(1-Se).p(\tilde{S}_t^*)}{(1-Se).p(\tilde{S}_t^*) + Sp.(1-p(\tilde{S}_t^*))} & \text{if } T_t = 0 \\ \frac{Se.p(\tilde{S}_t^*)}{Se.p(\tilde{S}_t^*) + (1-Sp)(1-p(\tilde{S}_t^*))} & \text{if } T_t = 1 \end{cases} \quad (15)$$

333 where $T_t = 1$ when the test at time t is positive, $T_t = 0$ when it is negative

⁵Here $\tilde{\tau}_{1t}$ is *predicted* from herd-month specific risk factors while $\hat{\tau}_2$ is the same for all herds and *estimated* from historical data.

Table 1: Probability of test result by herd status. Cells on the first row are test positive herds with true positives on the left-hand side and false positives on the right-hand side. Cells on the second row are test negative herds with false negatives on the left-hand side and true negatives on the right-hand side.

		Herd status _t	
		+	-
Test _t	+	$Se.\pi_t$	$(1 - Sp)(1 - \pi_t)$
	-	$(1 - Se).\pi_t$	$Sp.(1 - \pi_t)$

334 2.1.7 Model implementations

335 The pre-processing of the data and the analysis of the results of the Bayesian
 336 models were done in R (R Core Team, 2020). The HMM was implemented
 337 in both JAGS and Stan.

338 The model was initially implemented in JAGS, which performs Bayesian
 339 inference using Gibbs sampling (Plummer, 2003). The model equations were
 340 directly translated into JAGS code. The `runjags` R package (Denwood,
 341 2016) was used to interface R and JAGS.

342 The model was then implemented in Stan (Stan Development Team,
 343 2021). Stan is a newer and more efficient way of performing Bayesian infer-
 344 ence using Hamiltonian Monte Carlo. However, Stan does not allow latent
 345 discrete parameters to be modelled directly. Therefore, for the Stan imple-
 346 mentation of our model, the forward algorithm (Baum & Eagon, 1967) was
 347 adapted from Damiano *et al.* (2018). The `cmdstanr` R package (Gabry &
 348 Cešnovar, 2020) was used to interface R and Stan.

349 2.2 Application of the model to a control programme 350 for BVDV infection in cattle

351 2.2.1 Data

352 The model was evaluated on data collected for the surveillance of BVDV
 353 infection in dairy cattle in Loire-Atlantique, France. Under the programme,
 354 each herd was tested twice a year with a bulk tank milk (BTM) antibody

355 ELISA test. For each campaign of testing, tests were performed for all herds
356 over a few weeks. Data on the number of cattle introduced into each herd
357 with the associated date of introduction were also available. For the model
358 evaluation, test data of 1687 herds from the beginning of 2014 to the end
359 of 2016 were used. Risk factor data collected between 2010 and 2016 were
360 available to model (possibly lagged) associations between risk factors and the
361 latent status.

362 **2.2.2 Test results**

363 Test results were reported as optical density ratios (ODR). These ODR values
364 were discretised in order to convert them into either seropositive (antibodies
365 detected) or seronegative (no antibodies detected) outcomes. The choice of
366 the threshold to apply for the discretisation as well as the sensitivity and
367 specificity of this threshold for the detection of seropositivity were based on
368 the ODR distributions from test data collected outside of the study period.
369 The overall ODR distribution was modelled as a mixture of underlying ODR
370 distributions for seropositives and seronegatives. The details of the method
371 used are provided as supplementary material.

372 **2.2.3 Selection of risk factors**

373 A difficulty in the evaluation of putative risk factors was that Bayesian models
374 usually take time to run, especially with large datasets as used here. It was
375 therefore not possible to perform this selection with our Bayesian model.
376 To circumvent this problem, logistic models as implemented in the R glm
377 function ([R Core Team, 2019](#)) were used⁶. The outcome of these models was
378 seroconversion defined as a binary event, and covariates of interest were risk
379 factors for becoming status positive as defined through the τ_1 variable. All
380 herds with 2 consecutive test results whose first result was negative (ODR
381 below the chosen threshold) were capable of seroconverting. Of these herds,
382 the ones that had a positive result (ODR above the chosen threshold) on
383 the second test were considered as having seroconverted. The time of event
384 (seroconversion or not) was considered the mid-point between the 2 tests.

385 Two types of risk factors of new infection were evaluated: infection through
386 cattle introductions and infection through neighbourhood contacts ([Qi *et al.*](#),

⁶The functions used to perform this evaluation are included in the [STOCfree package](#).

387 2019). Cattle introduction variables were constructed from the number of an-
388 imals introduced into a herd on a given date. In addition to the raw number of
389 animals introduced, the natural logarithm of the number of animals (+1 be-
390 cause $\ln(0)$ is not defined) was also evaluated. This was to allow a decreasing
391 effect of each animal as the number of animals introduced increased. Regard-
392 ing the neighbourhood risk, the test result data were used. For each testing
393 campaign, the municipality-level prevalence of test positives (excluding the
394 herd of interest) was calculated, and is subsequently termed 'local preva-
395 lence'. It was anticipated that when local seroprevalence would increase, the
396 probability of new infection in the herd of interest would increase as well.

397 For all candidate variables, a potential problem was delayed detection,
398 which relates to the fact that a risk factor recorded at one point in time may
399 be detected through testing much later, even if the test is sensitive. For ex-
400 ample, if a trojan cow (a non-PI female carrying a PI calf) is introduced into
401 a herd, the lactating herd will only seroconvert when the PI calf is born and
402 has had contact with the lactating herd. Therefore, for each candidate vari-
403 able, the data were aggregated between the beginning of an interval (labelled
404 lag1, in months from the outcome measurement) and the end of this inter-
405 val (labelled lag2, in months from the outcome measurement). Models with
406 all possible combinations of time aggregation between lag1 and lag2 were
407 run, with lag1 set to 0 and lag2 set to 24 months. The best variables and
408 time aggregation interval were selected based on low AIC value, biological
409 plausibility and suitability for the Bayesian model.

410 2.2.4 Bayesian models

411 Four different Bayesian models were considered. For all models, historical
412 data were used for parameter estimation and the probability of infection on
413 the last month in the dataset was predicted.

414 **Model 1 - Perfect test, no risk factors:** in order to evaluate the monthly
415 dynamics of seropositivity and seronegativity, the Bayesian model was run
416 without any risk factors and assuming that both test sensitivity and test
417 specificity were close to 1. The prior distributions for sensitivity and speci-
418 ficity were $Se \sim Beta(10000, 1)$ (percentiles: 5 = 1, 50 = 1, 95 = 1) and
419 $Sp \sim Beta(10000, 1)$. Regarding infection dynamics, prior distributions were
420 specified for the prevalence of status positives (also test positives in this sce-
421 nario) on the first testing time $logit(p(S_1^+)) \sim \mathcal{N}(0, 10)$ (on the probability

422 scale - percentiles: $5 = 0$, $50 = 0.5$, $95 = 1$), the probability of becoming status
423 positive $\text{logit}(\tau_1) \sim \mathcal{N}(-3, 1)$ (percentiles: $5 = 0.01$, $50 = 0.047$, $95 = 0.205$),
424 and the probability of remaining status positive $\text{logit}(\tau_2) \sim \mathcal{N}(2.2, 0.05)$ (per-
425 centiles: $5 = 0.893$, $50 = 0.9$, $95 = 0.907$). The same prior distribution for τ_2
426 was used in all models. The motivation for this choice was the fact that tests
427 were performed every 6 months in all herds. The consequences of choosing
428 this prior was that infected herds had a small probability of changing status
429 between consecutive months (median probability = 0.1), but after 6 months,
430 the probability of still being positive was $0.9^6 = 0.53$, at which time the
431 status was updated with a new test result.

432 **Model 2 - Imperfect test, no risk factors:** the objective of this model
433 was to incorporate the uncertainty associated with test results in both pa-
434 rameter estimation and in the prediction of the probabilities of infection. The
435 priors for test sensitivity and specificity were selected based on the ODR dis-
436 tributions for seronegatives and seropositives identified by the mixture model.
437 The following prior distributions were used: $Se \sim \text{Beta}(10, 1)$ (percentiles:
438 $5 = 0.741$, $50 = 0.933$, $95 = 0.995$) and $Sp \sim \text{Beta}(10, 1)$. For the status
439 dynamics parameters, the same prior distributions as in Model 1 were used.

440 **Model 3 - Perfect test, risk factors:** in order to quantify the association
441 between risk factors and the probability of becoming status positive if the
442 test were close to perfect, the Bayesian model was run with the risk factors
443 identified as associated with seroconversion on the previous step, and using
444 the same priors for sensitivity, specificity and τ_2 as in Model 1. The priors for
445 risk factors were specified as normal distributions on the logit scale. The prior
446 for the intercept was $\theta_1 \sim \mathcal{N}(-3, 1)$ (on the probability scale - percentiles: 5
447 $= 0.01$, $50 = 0.047$, $95 = 0.205$). This represented the prior probability of a
448 new infection in a herd purchasing no animal and with a local seroprevalence
449 of 0. The priors for the other model coefficients were centred on 0 with a
450 standard deviation of 2. On the logit scale, values of -4 (2 standard deviations
451 in this case) correspond to probabilities close to 0 ($\text{logit}(-4) = (0.018)$) and
452 values of 4 to probabilities that are close to 1 ($\text{logit}(4) = (0.982)$).

453 **Model 4 - Imperfect test, risk factors:** in order to quantify the asso-
454 ciation between risk factors and the probability of becoming status positive
455 while incorporating test imperfection, the Bayesian model was run with the

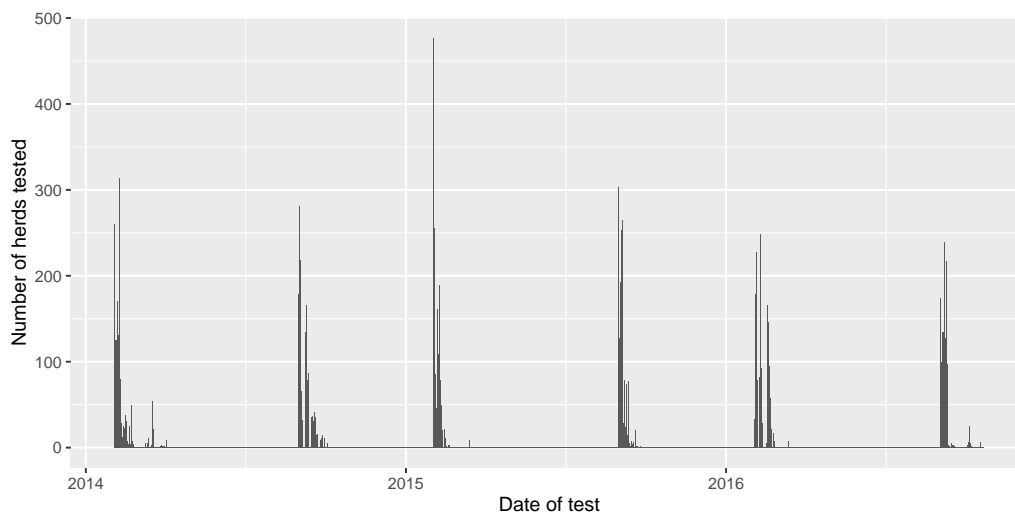


Figure 4: Distribution of the test dates between 2014 and 2017 in 1687 herds from Loire-Atlantique, France.

456 risk factors identified as associated with seroconversion using the same priors
457 as in Model 1 for tests characteristics and the same priors as in Model 3 for
458 infection dynamics and risk factors.

459 Each model was run in both Stan and JAGS. For each model, 4 chains
460 were run in parallel. For the Stan implementation, the first 1 000 iterations
461 were discarded (warmup). The model was run for 500 more iterations with
462 every iteration stored for analysis. This yielded 2 000 draws from the poste-
463 rior distribution of each parameter. For the JAGS implementation, the first
464 15 000 MCMC iterations were discarded (burn-in). The model was run for
465 10 000 more iterations of which 1 in 20 was stored for analysis. This yielded
466 2 000 draws from the posterior distribution of each parameter. For all mod-
467 els, convergence was assessed visually using traceplots. Each distribution was
468 summarised with its median and 95% credibility interval.

469 3 Results

470 3.1 Test results

471 Between the beginning of 2014 and the end of 2016, there were 9725 available
472 test results, reported as ODRs, from 1687 herds. Most herds were tested

473 in February and September (See Figure 4). The cut-off of 35 used in the
474 CP seemed to discriminate well between the distributions associated with
475 seronegative and seropositive herds respectively, and was therefore retained
476 in the remainder of the analysis. Using this threshold, there were 44.1%
477 of seropositive tests between 2014 and 2016. The associated estimated test
478 sensitivity and specificity were 0.978 and 0.949 respectively. However, in
479 the Bayesian models 2 and 4, because there was considerable uncertainty
480 regarding the assumptions made, sensitivity and specificity were modelled
481 using $Beta(10, 1)$ prior distributions (percentiles: 5 = 0.741, 50 = 0.933, 95
482 = 0.995).

483 3.2 Selection of risk factors

484 Risk factors related to animal introductions and seroprevalence were evalu-
485 ated with logistic models. The model outcome was a seroconversion event.
486 A first step of the analysis was, for each variable, to identify the time in-
487 terval that was the most predictive of an observed seroconversion. Figure 5
488 presents the AIC values associated with each possible interval for the vari-
489 ables $\ln(\text{Number of animals introduced} + 1)$ and local seroprevalence.

490 For the animal introduction variables, for the same time interval, the
491 AICs of the models of the untransformed number of animals were higher
492 than the ones for the log transformed values (not shown). It can also be
493 noted that considering longer intervals (further away from the diagonal) was
494 usually better than considering short intervals (close to the diagonal). It
495 may be that some herds never buy any animal while, on average, herds that
496 buy once have already done it in the past. In this case, it is possible that
497 the infection was introduced several times, while it is not possible to know
498 which animal introduction was associated with herd seroconversion. This
499 could explain the apparent cumulative effect of the number of introductions.
500 The cells that are close to the diagonal are associated with short intervals.
501 Considering one month intervals, the probability of infection was highest for
502 introductions made 8 months from the month of seroconversion.

503 Local seroprevalence was evaluated from data collected in 2 different test-
504 ing campaigns per year, as shown in Figure 4. For this reason, in the investi-
505 gation of lagged relationships between local seroprevalence and the probabil-
506 ity of seroconversion, the maximum local seroprevalence was computed, and
507 not the sum as for the number of animals introduced. The strength of as-
508 sociation between local seroprevalence and herd seroconversion was greatest

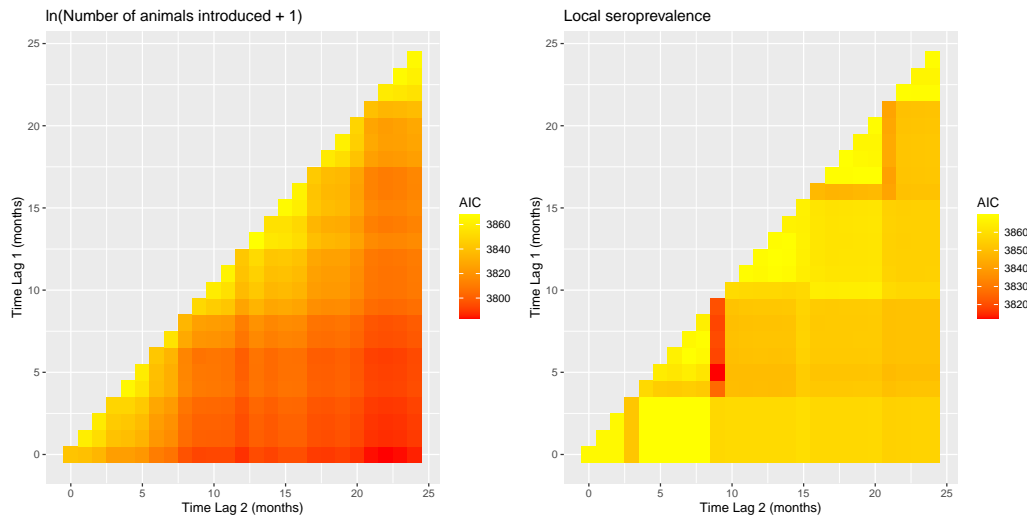


Figure 5: AIC values associated with logistic models of the association between 2 variables and the probability of seroconversion between 2 tests. Each coloured row represents the end of the interval that is closest to the current month and each column the end of the interval that is the furthest in the past. For example, the line at the bottom represents intervals that end on the current month: the first column is for the interval that started and ended on the current month (length of one month) and the last column is for the interval between 24 months ago and the current month. The variable evaluated on the left-hand side panel is the sum of the $\ln(\text{number of animals introduced} + 1)$ between lag1 and lag2. The variable evaluated on the right-hand side panel is the max of the local seroprevalence between lag1 and lag2.

509 for local seroprevalence 9 months prior to herd seroconversion.

510 A final multivariable logistic model with an animal introduction variable
511 and a local seroprevalence variable was constructed. In the choice of the
512 time intervals to include in this model, the following elements were consid-
513 ered. First, the Bayesian model runs with a monthly time step. Aggregating
514 data over several months would result in including the same variable sever-
515 al times. Secondly, historical data may sometimes be limited. Having
516 the smallest possible value for the end of the interval could be preferable.
517 For this reason, the variables considered for the final model were the nat-
518 ural logarithm of the number of animals introduced 8 months prior to the
519 month of seroconversion as well as the local seroprevalence 9 months prior

Table 2: Results of the final logistic model of the probability of seroconversion between consecutive tests. The risk factors retained in the model were the logarithm of the number of animals introduced in the herd 8 months before seroconversion and the local seroprevalence 9 months before seroconversion.

	lag1	lag2	Estimate	p-value
Intercept	-	-	-1.96	< 0.001
ln(Number animals introduced +1)	8	8	0.38	< 0.001
local seroprevalence	9	9	4.59	< 0.001

520 to the month of seroconversion. The results of this model are presented in
521 Table 2. All variables were highly significant. The model intercept was the
522 probability of seroconversion in a herd introducing no animals and with local
523 seroprevalence of 0 in each of the time intervals considered. The probabil-
524 ity of seroconversion between 2 tests corresponding to this scenario was of
525 0.124. Buying 1, 10 or 100 animals increased this estimated probability to
526 0.171, 0.866 and 1 respectively. Buying no animals and observing a sero-
527 prevalence of 0.2 (proportion of seropositives in the dataset) was associated
528 with a probability of seroconversion of 0.261.

529 3.3 Bayesian models

530 Running the different models for the 1687 herds with 3 years of data on
531 the first author’s laptop (CPU: Intel Core i5-8350U, RAM: 16 Go, Win-
532 dows 10) took significantly more time in JAGS (3 to 4.5 hours) than in Stan
533 (around 1 hour). In models 3 and 4, the candidate covariates were the nat-
534 ural logarithm of the number of animals introduced 8 months before status
535 evaluation/prediction as well as the local seroprevalence 9 months before.
536 The 95% credibility interval for the estimated coefficient associated with lo-
537 cal seroprevalence included 0. This variable was therefore removed from the
538 models and only cattle introductions were considered.

539 3.3.1 Model parameters

540 For Models 1 and 3, in which the test was assumed to be perfect, the 4 chains
541 of each model converged and mixed well regardless of the programme used
542 for Bayesian inference. For Models 2 and 4, in which wider distributions

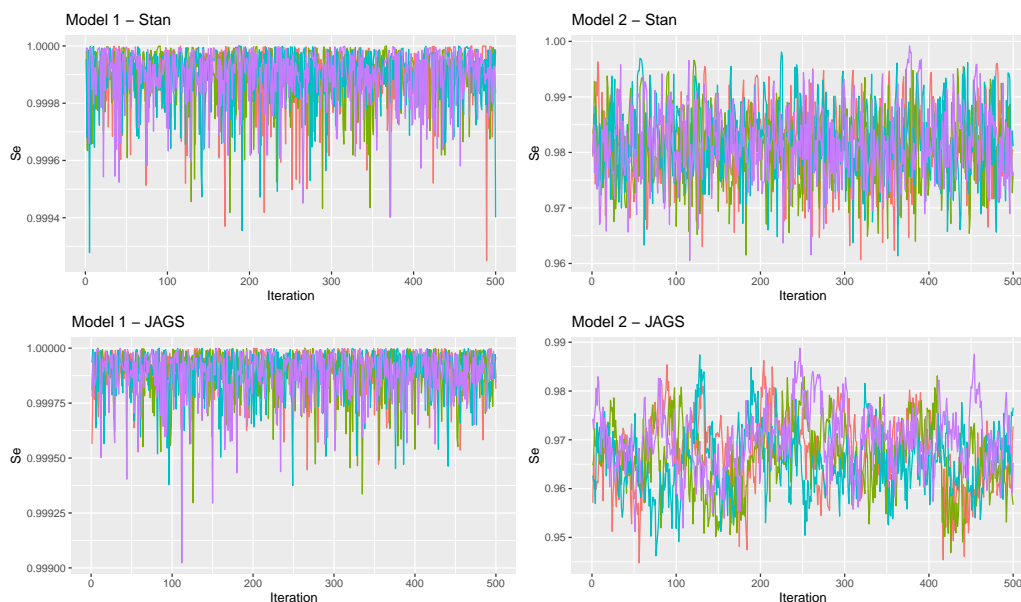


Figure 6: Traceplots for test sensitivity in Models 1 and 2 estimated in Stan and JAGS. Each color represents one of 4 chains run for each model.

543 were assumed for test characteristics, the chains converged and mixed well
544 for the Stan version, but mixing was poor for the JAGS version. As an
545 illustration, Figure 6 represents the traceplots for test sensitivity in Models
546 1 and 2 with both the Stan and JAGS version of the models. In the JAGS
547 version of Model 2, autocorrelation is visible in the traceplot for sensitivity,
548 despite the fact that only one iteration in 20 (thinning of 20) was kept for
549 analysis. Figure 7 and Table 3 show the distributions of model parameters
550 for the 4 models. Although the JAGS model tends not to converge as well,
551 the parameter estimates are similar between the Stan and JAGS versions of
552 the models.

553 In Model 1, the prior distribution put on sensitivity and specificity was
554 very close to 1. With this model, the latent status corresponded to the test
555 result. In effect, it modelled the monthly probability of transition between
556 BTM test negative and BTM test positive. In this case, the median (per-
557 centile 2.5 - percentile 97.5) probability of becoming status positive between
558 consecutive months was 0.032 (0.030 - 0.034). This represents a probability
559 of becoming status positive over a 12 month period of 0.323 (0.310 - 0.340).
560 For status positive herds, the monthly probability of remaining positive was

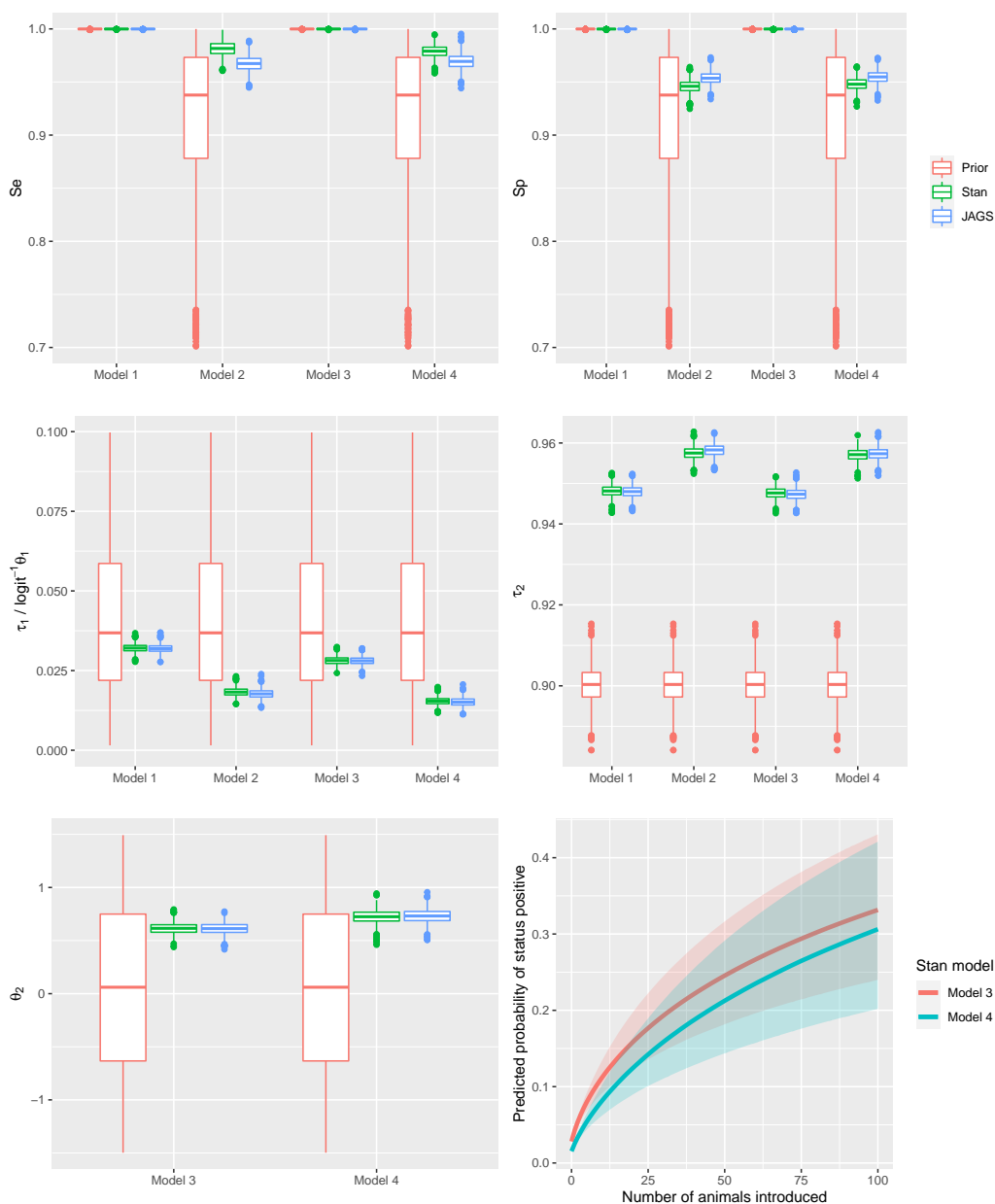


Figure 7: Parameters prior and posterior distributions for the 4 Bayesian models. Model 1: Perfect test, no risk factor; Model 2: Imperfect test, no risk factor; Model 3: Perfect test, risk factor; Model 4: Imperfect test, risk factor. The only risk factor included is the logarithm of the number of animals introduced + 1. In Models 1 and 2, the probability of becoming status positive is modelled with τ_1 . In Models 3 and 4, the probability of becoming positive is modelled using logistic regression. From these models, $\logit^{-1}\tau_1$ is the probability of becoming positive when no animal is introduced (i.e. model intercept). τ_2 models the increase in the probability of becoming positive with the number of animals introduced. The last row of the Figure represents the posterior distribution for τ_2 as well as the corresponding increase in the probability of becoming positive with the number of animals introduced.

Table 3: Median (2.5%, 97.5%) of the parameter posterior distributions used in the 4 Bayesian models evaluated. Model 1: Perfect routine test; Model 2: Perfect routine test and risk factors; Model 3: Imperfect routine test and risk factors; Model 4: Imperfect routine test, confirmatory testing and risk factors.

Model	Inference	Se	Sp	$\tau_1 / \text{logit}^{-1}\theta_1$	θ_2	τ_2
Model 1	Stan	1 (1-1)	1 (1-1)	0.032 (0.03-0.034)	-	0.948 (0.945-0.951)
	JAGS	1 (1-1)	1 (1-1)	0.032 (0.03-0.034)	-	0.948 (0.945-0.951)
Model 2	Stan	0.982 (0.968-0.994)	0.946 (0.934-0.957)	0.018 (0.016-0.021)	-	0.958 (0.954-0.96)
	JAGS	0.967 (0.953-0.982)	0.954 (0.943-0.965)	0.018 (0.015-0.02)	-	0.958 (0.955-0.961)
Model 3	Stan	1 (1-1)	1 (1-1)	0.028 (0.026-0.031)	0.615 (0.508-0.716)	0.948 (0.945-0.95)
	JAGS	1 (1-1)	1 (1-1)	0.028 (0.026-0.031)	0.613 (0.508-0.721)	0.947 (0.944-0.95)
Model 4	Stan	0.979 (0.966-0.989)	0.948 (0.937-0.959)	0.015 (0.013-0.018)	0.725 (0.596-0.842)	0.957 (0.954-0.96)
	JAGS	0.969 (0.956-0.982)	0.955 (0.943-0.965)	0.015 (0.013-0.018)	0.731 (0.606-0.856)	0.957 (0.954-0.96)

561 of 0.948 (0.945 - 0.951) which represents a probability of still being status
 562 positive 12 months later of 0.526 (0.507 - 0.547).

563 In models 2 and 4, a $Beta(10, 1)$ distribution was used as a prior for test
 564 sensitivity and specificity. Despite this distribution spanning a relatively
 565 large interval (percentiles: 5 = 0.741, 50 = 0.933, 95 = 0.995), all models
 566 converged to high values for both sensitivity and specificity. As noted above,
 567 convergence was not as good for the JAGS versions of the models, although
 568 the JAGS and Stan estimates are close. Interestingly, for model parameters
 569 related to status dynamics and risk factors, the Stan and JAGS estimates
 570 were almost identical for all models. Adding test imperfection to the models
 571 resulted in a decrease in the probability of becoming positive (from 0.032 to
 572 0.018 between models 1 and 2; from 0.028 to 0.015 between models 3 and 4)
 573 as well as in an increase in the probability of remaining positive (from 0.948
 574 to 0.958 between models 1 and 2; from 0.948 to 0.957 between models 3 and
 575 4). The most likely reason is that, in some herds, some negative tests arising
 576 in a sequence of positive tests were considered as false negatives resulting in
 577 longer sequences of positive status and, as a consequence, fewer transitions
 578 from negative to positive status.

579 In models 3 and 4, a risk factor of becoming status positive was incor-
 580 porated into the estimation. The model intercept (θ_1) was much lower than
 581 the estimate from the logistic model estimated in the variable selection step.
 582 This was due to the different time steps considered (1 month vs. half a year).
 583 On the other hand, the estimate for the association between the natural log-
 584 arithm of the number of animals introduced and the probability of becoming

585 positive was higher. This association is plotted in the bottom right-hand
586 side panel of Figure 7. The probability of becoming latent status positive
587 between 2 months goes from 0.015 when introducing no animal ($\text{logit}^{-1}\theta_1$
588 in Table 3) to greater than 0.3 for 100 animals introduced. This suggests
589 that including the number of animals introduced into the prediction of herd
590 statuses could increase the sensitivity of detection.

591 3.3.2 Predicted probabilities of infection

592 Figure 8 shows the distributions of herd-level probabilities of infection pre-
593 dicted by the 4 Bayesian models, using Stan and JAGS. These probability
594 distributions are bimodal for all models. The left-hand side corresponds to
595 herds that were predicted status negative on the month before the month of
596 prediction. These are associated to becoming status positive, i.e. τ_1 . The
597 right-hand side of the distributions corresponds to herds that were predicted
598 status positive on the month before the month of prediction. These are asso-
599 ciated to remaining status positive, i.e. τ_2 . Figure 9 shows the distributions
600 of the predicted probability of being status positive for 4 herds. It can be
601 seen that herds that were consistently negative (positive) to the test had
602 extremely low (high) probabilities of being status positive. Accounting for
603 the number of animals introduced increased the probability of infection in
604 the herds that were test negative. An important difference between JAGS
605 and Stan was that in JAGS latent statuses are explicitly represented as a
606 binary variable. As a consequence, herds can *jump* between status positive
607 and status negative on the month before the month to predict, leading to
608 bimodal distributions for the predicted probability of being status positive.
609 This does not happen with Stan where the latent status is represented by a
610 continuous variable. Therefore, the predicted distributions can be different
611 between the 2 models. This can be seen for the herd at the bottom left of
612 Figure 9.

613 4 Discussion

614 This article describes a statistical framework for the prediction of an infection
615 related status from longitudinal data generated by CPs against infectious
616 diseases of farm animals. The statistical model developed estimates a herd
617 level probability of being *latent status* positive on a specific month, based

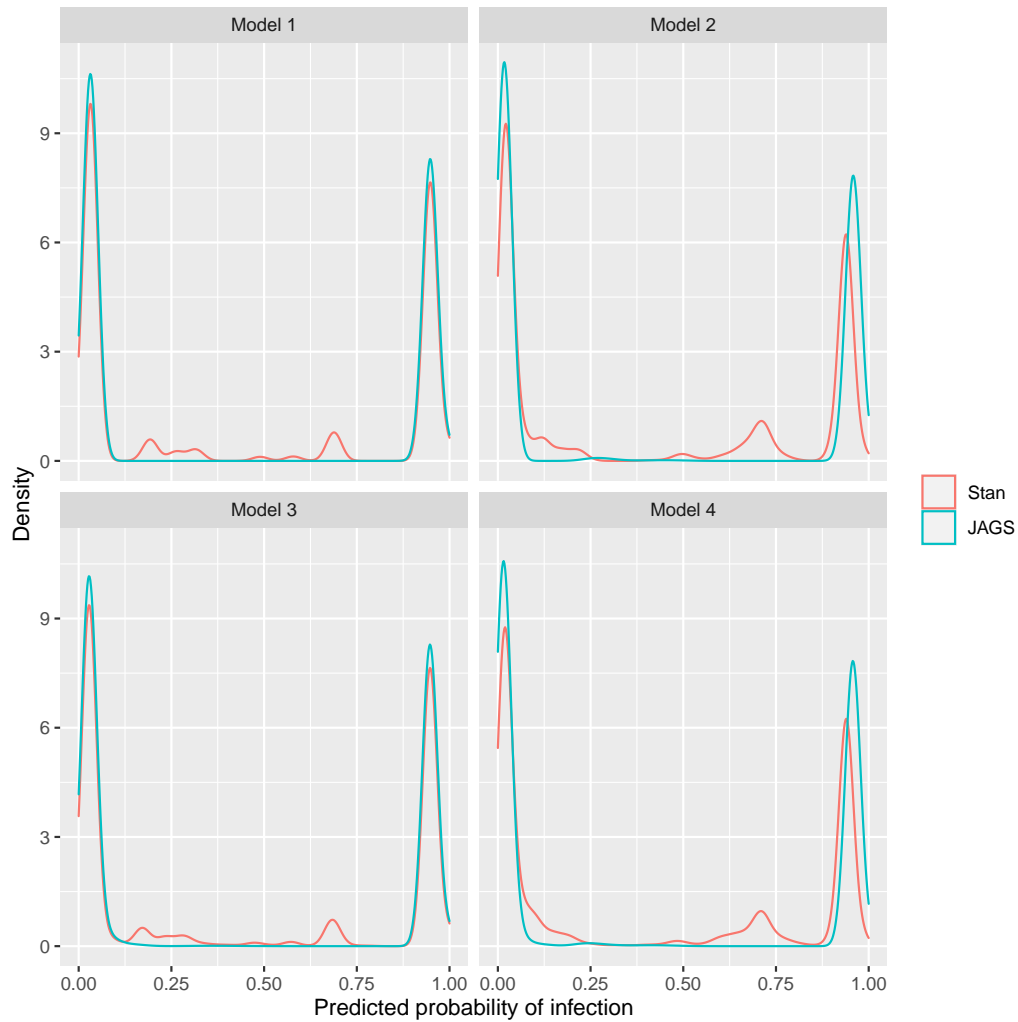


Figure 8: Distributions of predicted probabilities of being status positive for all herds with the 4 Bayesian models evaluated with Stan and JAGS. Model 1: Perfect test, no risk factor; Model 2: Imperfect test, no risk factor; Model 3: Perfect test, risk factor; Model 4: Imperfect test, risk factor.

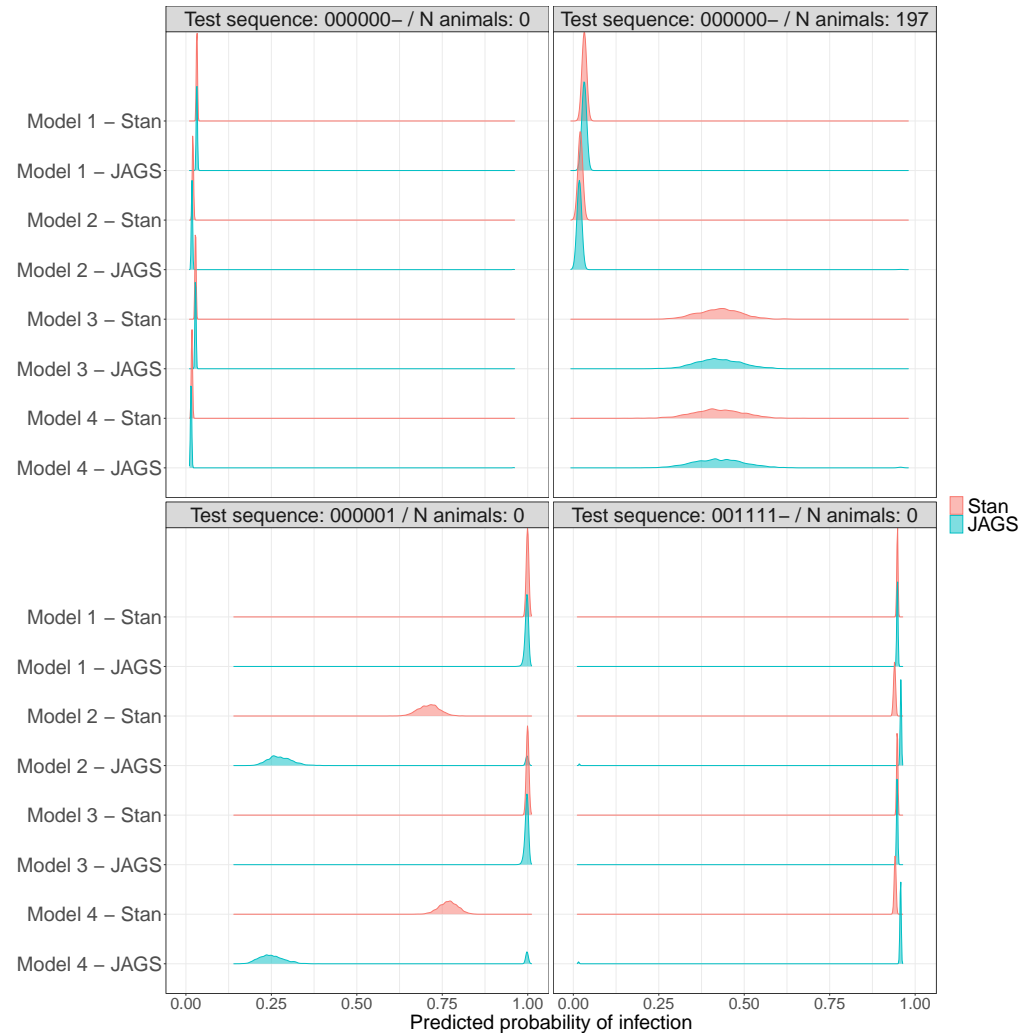


Figure 9: Distribution of predicted probabilities of being status positive on the month of prediction for 4 herds with the 4 models compared. Model 1: Perfect test, no risk factor; Model 2: Imperfect test, no risk factor; Model 3: Perfect test, risk factor; Model 4: Imperfect test, risk factor. The title of each panel corresponds to the sequence of test results (- indicates that a test result was available on the month before prediction), and the number of animals introduced 8 months before the month of prediction (risk factor).

618 on input data that can vary in terms of the types of test used, frequency
619 of testing and risk factor data. This is achieved by modelling the latent
620 status with the same discrete time step, regardless of the frequency with
621 which input data are available, and by modelling changes in the latent status
622 between consecutive time steps. This model therefore fulfils one of our main
623 objectives which was to be able to integrate heterogeneous information into
624 the estimation. However, in order to be able to compare the output of this
625 model run on data from different CPs, the definition of the latent status
626 should be the same.

627 The model was implemented in both Stan and JAGS. The first version
628 of the model was in JAGS, in which it was straightforward to translate the
629 model equations into computer code. However, with this JAGS model, con-
630 vergence was slow and the chains did not mix well when the prior distri-
631 butions put on sensitivity and specificity were slightly wide. This led us
632 to develop a Stan version of the model. Stan is a newer programme which
633 uses Hamiltonian Monte Carlo for performing Bayesian inference (Carpenter
634 *et al.*, 2017). It was more challenging to write the model in Stan, which does
635 not support latent discrete parameters. This was achieved by adapting a
636 Stan implementation of the forward algorithm developed by others (Dami-
637 ano *et al.*, 2018). The Stan implementation is by comparison much faster
638 and converges better, and should therefore be preferred.

639 When estimated in either JAGS or BUGS, discrete latent state models
640 such as HMMs are known to converge slowly; and the autocorrelation in the
641 draws from the posterior distributions is usually high. Yackulic *et al.* (2020)
642 showed that the marginalisation of the latent states considerably reduces the
643 time needed to estimate the parameters of such models while returning the
644 same estimates. We did not implement this approach in JAGS, although this
645 would have been possible using the *ones trick*, as explained in the article by
646 Yackulic *et al.* (2020). The forward algorithm is a type of marginalisation
647 that partly explains the better performance of the Stan version of the model.
648 However, Yackulic *et al.* (2020) also compared the speed of the marginalised
649 versions of their model in different programmes and observed that Stan was
650 orders of magnitude faster than JAGS.

651 In this model, the latent status is mostly defined by the prior distributions
652 put on the different model parameters. In setting the prior distributions
653 there are two issues: setting the distribution's central value (mean, median
654 ...) and setting the distribution width. Using a prior distribution that does
655 not include the true parameter value can lead to systematic error (bias) or

656 failure of convergence. Setting prior distributions that are too wide can lead
657 to a lack of convergence, when multiple combinations of parameter values
658 are compatible with the data. This was a problem in the initial modelling
659 when only the JAGS model was available. In this case, putting narrow prior
660 distributions on test sensitivity and test specificity allowed the model to
661 converge (results not shown). These narrow distributions imply very strong
662 hypotheses on test characteristics.

663 The definition of prior distributions for test characteristics that reflect the
664 latent status of interest is challenging (Duncan *et al.*, 2016). This was appar-
665 ent in our efforts to apply this approach to BVDV infection. For the trade
666 of animals from herds that are free from BVDV infection, the latent status
667 of interest was the *presence of at least one PI animal in the herd*. The test
668 data available to estimate the probability of this event were measures of bulk
669 tank milk antibody levels which were used to define seropositivity as a binary
670 event. Although milk antibody level is associated with the herd prevalence of
671 antibody positive cows (Beaudeau *et al.*, 2001), seropositive cows can remain
672 long after all the PIs have been removed from a herd. Furthermore, vaccina-
673 tion induces an antibody response which may result in vaccinated herds being
674 positive to serological testing regardless of PI animal presence (Raue *et al.*,
675 2011; Booth *et al.*, 2013). Therefore, the specificity of BTM seropositivity,
676 i.e. the probability for herds with no PI animals to be test negative, is less
677 than 1. More importantly, this specificity depends on the context; i.e. on the
678 CP. PI animals can be identified and removed more or less quickly depending
679 on the CP, the proportion of herds vaccinating and the reasons for starting
680 vaccination can differ between CPs. Test sensitivity can also be imperfect.
681 Continuing with the example of bulk tank milk testing, contacts between
682 PI animals present on the farm and the lactating herd may be infrequent,
683 which would decrease sensitivity. In this case, the sensitivity of the testing
684 procedure is the sensitivity of the test for the detection of seroconversion in
685 a group of animals multiplied by the probability that the tested group has
686 seroconverted if there is a PI animal in the herd. The probability of con-
687 tact between PI animals and the lactating herd depends on how herds are
688 organised, which could vary between CPs. This problem is alleviated when
689 newborn calves are tested because the group of animals tested is the group
690 in which the infectious animals are most likely to be present. Furthermore,
691 with BTM testing, the contribution of each seropositive cow to the BTM
692 decreases as herd size increases which can result in differences in BTM test
693 sensitivity associated with different herd sizes between CPs.

694 The effects of using different prior distributions for test characteristics on
695 latent status definition, parameter estimation and probability prediction were
696 evaluated. In models 1 and 3, the dichotomised BTM antibody test results
697 were modelled assuming perfect sensitivity and perfect specificity. With these
698 assumptions, the latent status was the dichotomised test results. In Models
699 2 and 4, the BTM antibody test was assumed to have lower sensitivity and
700 specificity, based on normal distributions associated with seronegativity and
701 seropositivity identified by a mixture model. The latent status in Models
702 2 and 4 can therefore be described as *seropositivity*. Because overall the
703 probability of changing status was small, assuming an imperfect sensitivity
704 led to isolated negative test results in sequences of mostly positive test results
705 being considered false negatives, as shown by the increase in the estimated
706 value for τ_2 between Models 1 and 2 and Models 3 and 4. This illustrates
707 that in addition to test characteristics, status dynamics will determine the
708 latent status within herds.

709 A way to obtain information on test characteristics as part of CPs could be
710 to incorporate data from confirmatory testing into the model. In CPs, herds
711 that test positive are usually re-tested in order to rule out a false positive
712 test, and to identify infected animals if needed. The testing procedure used
713 in confirmatory testing usually has a high sensitivity and a higher specificity
714 than routine testing in relation to the gold standard. When incorporated
715 into the model, this high quality information, in conjunction with wider
716 prior distributions on routine testing specificity, should allow the posterior
717 distribution of the specificity of routine testing to be revised towards the
718 gold standard. Indeed, if a confirmatory test comes back negative, then
719 the corresponding latent status will become negative with high probability.
720 Given the low probability of becoming status negative between consecutive
721 months, the latent status on the month of routine testing has an increased
722 probability of being negative, leading to a decrease in the specificity of routine
723 testing. Confirmatory testing data was not available for this study. We
724 attempted to evaluate the usefulness of confirmatory testing by simulating
725 confirmatory tests at random after an initial positive test result. The results
726 were not convincing, because simulating test results at random was often not
727 consistent with patterns of test results in individual herds.

728 Status dynamics contributed to the estimation of the latent status in
729 several ways. Negative test results interspersed with sequences of positive
730 test results will be classified as latent status positive (i.e. as false negatives)
731 more often as test sensitivity decreases and τ_2 increases. Positive test re-

732 sults interspersed with sequences of negative test results will be classified as
733 latent status negative (i.e. as false positives) with increased frequency as
734 test specificity and τ_1 each decrease. With a perfect test (sensitivity and
735 specificity equal to 1), the model can learn the values of τ_1 and τ_2 from the
736 data, and the prior distributions put on these parameters can be minimally
737 informative. With decreasing values for test sensitivity and specificity, the in-
738 formation provided through the prior distributions put on τ_1 and τ_2 becomes
739 increasingly important. The informative value of τ_1 and τ_2 will increase as
740 the probability of transition between latent status negative and latent status
741 positive decrease, i.e. when τ_1 is small and τ_2 is high.

742 When data on risk factors of new infection are available, the τ_1 param-
743 eter is modelled as a function of these risk factors using logistic regression.
744 In such a case, prior distributions are put on the parameters of the logistic
745 regression. In the application that we presented, we used a prior distribution
746 corresponding to a low probability of new infection in the reference category
747 (intercept: herds which introduced no animals) and we centred the prior dis-
748 tribution for the association with cattle introductions on a hypothesis of no
749 association (mean = 0 on the logit scale). This allowed the model to estimate
750 the association between the risk factor and the latent status from historical
751 data and to use the estimated association to predict probabilities of being
752 latent status positive on the month of prediction. The prior distributions put
753 on test characteristics had a moderate impact on the parameter estimates.
754 Between Model 3 and Model 4, considering an imperfect test resulted in a
755 slightly reduced impact of the number of cattle introduced on the probabili-
756 ty of becoming status positive (See curves at the bottom of Figure 7). The
757 most likely explanation for this is that Model 4 allowed the highest level of
758 discrepancy between dichotomised test result and latent status, while assum-
759 ing a low probability of changing status between months. This resulted in
760 negative test results in herds that were regularly positive to be classified as
761 latent status positive (false negatives, associated with lower test sensitivity,
762 see Table 3) thereby removing opportunities for new infections in herds that
763 were regularly positive while also buying animals. This would imply that
764 the estimated association from model 4 is more closely associated with new
765 infections than estimates from Model 3 because herds that are regularly test
766 positive have less weight in the estimation. It would also have been possible
767 to base the prior distributions for the model coefficients on published liter-
768 ature. Unfortunately, estimates of the strengths of association between risk
769 factors and the probability of new infection are not readily available from

770 the published literature or are hard to compare between studies (van Roon
771 *et al.*, 2020b). However, estimates from the literature could allow the prior
772 distributions to be bounded within reasonable ranges.

773 The identification of the most predictive time interval between risk factor
774 occurrence and seroconversion required the evaluation of the associations
775 between the probability of seroconversion on a given month and risk factor
776 occurrence over all possible intervals between this month and the 24 previous
777 months. Although there are several Bayesian methods for such variable selection
778 (O'Hara & Sillanpää, 2009), estimation using MCMC is time consuming
779 and was not feasible in our case. The variables included were therefore identified
780 with logistic models estimated by maximum likelihood for all possible
781 lags. The approach used is related to cross-correlation maps developed for
782 applications in ecology (Curriero *et al.*, 2005), and similar to work conducted
783 in veterinary epidemiology (Bronner *et al.*, 2015). This confirmed the importance
784 of animal introduction and neighbourhood contacts in new infections
785 (Qi *et al.*, 2019). However, in the Bayesian models, the 95% credibility for
786 the association between local seroprevalence and new infection included 0
787 and this variable was therefore not included. The reason for this was not
788 elucidated in this work. Other risk factors such as herd size, participation in
789 shows or markets, the practice of common grazing have shown a consistent
790 association with the probability of new infection by the BVDV (van Roon
791 *et al.*, 2020b). These variables were not included in our model because the
792 corresponding data were not available. One advantage of our approach is
793 the possibility to choose candidate risk factors to include in the prediction of
794 infection based on the data available in a given CP. The associations between
795 the selected putative risk factors and the probability of new infection can be
796 estimated from these data.

797 Given the reasonably good performance of tests for the detection of BVDV
798 infection, the main advantage of incorporating these risk factors was not to
799 complement the test results on a month a test was performed, but rather to
800 enhance the timeliness of detection. Risk factors that are associated with
801 new infection will increase the predicted probability of infection regardless
802 of the availability of a test result. Therefore, when testing is not frequent,
803 infected herds could be detected more quickly if risk factors of infection are
804 recorded frequently. If the available data on risk factors of new infection
805 captured all the possible routes of new infection, it would be possible to
806 perform tests more frequently in herds that have a higher probability of
807 infection as predicted by our model. In other words, our model could be

808 used for risk-based surveillance (Cameron, 2012).

809 In the CP from which the current data were used, herds are tested twice
810 a year. This could lead to a long delay between the birth of PI calves and
811 their detection through bulk tank milk testing. We addressed this problem
812 of *delayed detection* by proposing a method for the investigation of lagged
813 relationships between risk factor occurrence and new infections, and by in-
814 cluding lagged risk factor occurrences in the prediction of the probability of
815 infection. In our dataset, herds purchasing cattle were more likely to have
816 seroconverted 8 months after the introduction. In the Bayesian model, cattle
817 introduction was modelled as affecting the probability of becoming status
818 positive 8 months after the introduction. It can be argued that infection is
819 present but not detected during this period, as the expression *delayed detec-*
820 *tion* suggests, and that the probability of infection should increase as soon
821 as risk factor occurrence is recorded. Modelling this phenomenon would be
822 possible by decreasing the test sensitivity for a period corresponding to the
823 lag used in the current version of the model. This would imply that for this
824 duration, any negative BTM test result would not provide any information
825 about the true status regarding infection and that the herd would have an
826 increased predicted probability of infection. This could be incorporated in
827 future versions of the model.

828 There are several questions related to this modelling framework that
829 would require further work. The model outputs are distributions of herd
830 level probabilities of infection. Defining herds that are free from infection
831 from these distributions will require decision rules to be developed based on
832 distribution summaries (likely a percentile) and cut-off values. It would also
833 be possible to model the probability of remaining infected between consecu-
834 tive tests (τ_2) as a function of the control measures put in place in infected
835 herds. Another area that requires further investigations is the evaluation
836 of the modelling framework against a simulated gold standard to determine
837 whether it provides an added value compared to simpler methods. The avail-
838 ability of the model code as a Github repository allows interested users to
839 improve or suggest improvements to our modelling framework. The model
840 can be used to evaluate the output of disease CP thus aiding the use of
841 output-based surveillance.

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848 Conflict of interest disclosure

849 The authors of this article declare that they have no financial conflict of
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