

# Limits and Convergence properties of the Sequentially Markovian Coalescent

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1 **Abstract**

2 Many methods based on the Sequentially Markovian Coalescent (SMC)  
3 have been and are being developed. These methods make use of genome  
4 sequence data to uncover population demographic history. More recently,  
5 new methods have extended the original theoretical framework, allowing  
6 the simultaneous estimation of the demographic history and other biolog-  
7 ical variables. These methods can be applied to many different species,  
8 under different model assumptions, in hopes of unlocking the popula-  
9 tion/species evolutionary history. Although convergence proofs in par-  
10 ticular cases have been given using simulated data, a clear outline of the  
11 performance limits of these methods is lacking. We here explore the limits  
12 of this methodology, as well as present a tool that can be used to help  
13 users quantify what information can be confidently retrieved from given  
14 datasets. In addition, we study the consequences for inference accuracy  
15 violating the hypotheses and the assumptions of SMC approaches, such  
16 as the presence of transposable elements, variable recombination and mu-  
17 tation rates along the sequence and SNP call errors. We also provide a  
18 new interpretation of the SMC through the use of the estimated transi-  
19 tion matrix and offer recommendations for the most efficient use of these  
20 methods under budget constraints, notably through the building of data  
21 sets that would be better adapted for the biological question at hand.

22 **Keywords**— Hidden Markov Model, Ancestral Recombination Graph, Popu-  
23 lation Genetics

## 24 1 Introduction

25 Recovering the demographic history of a population has become a central theme in  
26 evolutionary biology. The demographic history (the variation of effective population  
27 size over time) is linked to environmental and demographic changes that existing  
28 and/or extinct species have experienced (population expansion, colonization of new  
29 habitats, past bottlenecks) [14, 43, 4]. Current statistical tools to estimate the de-  
30 mographic history rely on genomic data [51] and these inferences are often linked to  
31 archaeological or climatic data, providing novel insights on the evolutionary history  
32 [70, 33, 44, 1, 12, 26, 25]. From these analyses, evidence for migration events have  
33 been uncovered [26, 5], as have genomic consequences of human activities on other  
34 species [9]. Linking demographic history to climate and environmental data greatly  
35 supports the field of conservation genetics [10, 17, 40]. Indeed, using such approaches  
36 can help ecologists in detecting effective population size decrease [68], and thus serve  
37 as a guide in maintaining or avoiding the erosion of genetic diversity in endangered  
38 populations, and potentially predicting the consequences of climate change on genetic  
39 diversity [27]. In addition, studying the demographic histories of different species  
40 in relation to one another can unveil latent biological or environmental evolutionary  
41 forces [16], unveiling links and changes within entire ecosystems. With the increased  
42 accuracy of current methods, the availability of very large and diverse data sets and  
43 the development of new theoretical frameworks, the demographic history has become  
44 an information that is essential in the field of evolution [48, 6]. However, obtaining  
45 unbiased estimations/interpretations of the demographic history remain challenging  
46 [3, 8].

47 The most sophisticated methods to infer demographic history make use of

48 whole genome polymorphism data. Among the state-of-the-art methods, are those  
49 based on the theory of the Sequentially Markovian Coalescent (SMC) developed by  
50 McVean and Cardin[35] after the work of Wiuf and Hein [69], corrected by Marjoram  
51 and Wall [31] and first applied to whole genome sequences by Li and Durbin [26], who  
52 introduced the now well-known, Pairwise Sequentially Markovian Coalescent (PSMC)  
53 method. PSMC allows demographic inference of populations with unprecedented ac-  
54 curacy, while requiring only one sequenced diploid individual. This method uses the  
55 distribution of SNPs along the genome between the two sequences to account for and  
56 infer recombination and demographic history of a given population, assuming neu-  
57 trality and panmixia. Although PSMC was a breakthrough in demographic inference,  
58 it has limited power in inferring more recent events. In order to address this issue,  
59 PSMC has been extended to account for multiple sequences (*i.e.* more than two) into  
60 the method known as the Multiple Sequentially Markovian Coalescent (MSMC) [50].  
61 By using more sequences, MSMC better infers recent events and also provides the pos-  
62 sibility of inferring population splits using the cross-coalescent rate. MSMC, unlike  
63 PSMC, is not based on SMC theory [35] but on SMC' theory [31], therefore MSMC  
64 applied to only two sequences has been defined as PSMC'. Methods developed after  
65 MSMC followed suit, with MSMC2 [30] extending PSMC by incorporating pairwise  
66 analysis, increasing efficiency and the number of sequences that can be inputted (up to  
67 a hundred), resulting in more accurate results. SMC++ [63] brings the SMC theory to  
68 another level by allowing the use of hundreds of unphased sequences (MSMC requires  
69 phased input data) and breaking the piece-wise constant population size hypothesis,  
70 while accounting for the sample frequency spectrum (SFS). Because SMC++ incorpo-  
71 rates the SFS in the estimation of demographic history, it increases accuracy in recent  
72 time [63]. SMC++ is currently the state of the art SMC-based method for big data

73 sets ( $>20$  sequences), but seems to be outperformed by PSMC when using smaller  
74 data sets [45]. In a similar vein, the Ascertained Sequentially Markovian Coalescent  
75 (ASMC) [42] extends the SMC theory to estimate coalescence times at the locus scale  
76 from ascertained SNP array data, something that was made possible by the theory  
77 presented by Hobolth and Jensen [18].

78 More recently, a second generation of SMC-based methods have been devel-  
79 oped. New features have been added to the initial SMC theory, extending its ap-  
80 plication beyond simply inferring past demography [1, 53, 66]. The development of  
81 C-PSMC [16] allows the interpretation of estimated demographic history in the light  
82 of coevolution between species, making the first link between demographic history es-  
83 timated by PSMC and evolutionary forces (although biological interpretation remains  
84 limited). iSMC [1] extends the PSMC theory to account and infer the variation of the  
85 recombination rate along sequences, unlocking recombination map estimations. An im-  
86 pressive advancement is the development of MSMC-IM, which to some extent solves  
87 the population structure problem, allowing the accurate and simultaneous inference of  
88 the demographic history and population admixture [66]. eSMC [53] incorporates com-  
89 mon biological traits (such as self-fertilization and dormancy) and demonstrated the  
90 strong effect life-history traits can have on demographic history estimations. Results  
91 which could not be explained under the initial SMC hypotheses can now be explained  
92 by the potential presence of measurable phenomena.

93 New methods have been developed since PSMC, that have been either strongly  
94 inspired by the SMC [54, 62] or that are completely dissociated from it [58, 2, 49, 20,  
95 29, 19, 57, 65]. Though there are alternative approaches, methods based on the SMC  
96 are still considered state of the art, and remain widely used [32, 3, 59], notably in

97 human evolution studies [59, 45]. However, each described method has its specificity,  
98 being based on different hypothesis in order to solve a particular problem or shortcom-  
99 ings of existing methodology. Although all these methods allow a new and different  
100 interpretation of genomic data, none of these methods guarantees unbiased inference,  
101 and their limitations have underlined how crucial and challenging demographic infer-  
102 ence is, highlighting the complementarity and usefulness of applying several inference  
103 methods on a given dataset.

104 SMC-based methods display very good fits when using simulated data, espe-  
105 cially when using simple single population models based on typical human data param-  
106 eters [63, 50, 53, 66]. However, the SMC makes a large number of hypotheses [26, 50]  
107 that are often violated in data obtained from natural populations. When inputting  
108 data from natural populations, extracting information or correctly interpreting the  
109 results can become troublesome [8, 64, 3] and several studies address the consequences  
110 of hypothesis violation [15, 8, 49, 34, 52]. They bring to light how strongly population  
111 structure or introgression influence demographic history estimation if not correctly ac-  
112 counted for [15, 8]. Furthermore, some SMC-based methods require phased data (such  
113 as MSMC [50] and MSMC-IM [66]), and phasing errors can lead to a strong overes-  
114 timation of population size in recent time [63]. The effect of sequencing coverage has  
115 also been tested in Nadachowska et al. [37], showing the importance of high coverage  
116 in order to obtain trustworthy results, and yet, SMC methods seem robust to genome  
117 quality [45]. Selection, if not accounted for, can result in a bottleneck signature [52],  
118 and there is currently no solution to this issue within the SMC theory, though it could  
119 be addressed using different theoretical frameworks that are being developed [55, 38].  
120 More problematic, is the ratio of effective recombination over effective mutation rates

121  $\frac{\rho}{\theta}$ , which, if it is greater than one, biases estimations [63, 1, 53]. It is also important to  
122 keep in mind that there can be deviations between  $\frac{\rho}{\theta}$  and the ratio of recombination  
123 rate over mutation rate measured experimentally ( $\frac{r}{\mu}$ ), as the former can be greatly  
124 influenced by life-history, such as in organisms displaying self-fertilization, partheno-  
125 genesis or dormancy, and this can lead to issues when interpreting results (*e.g.* [53]).  
126 It is thus necessary to keep in mind that the accuracy of SMC-based methods depends  
127 on which of the many underlying hypothesis are prone to being violated by the data  
128 sets being used.

129         In an attempt to complement previous works, we here study the limits and  
130 convergence properties of methods based on the Sequentially Markovian Coalescent.  
131 We first define the limits of SMC-based methods (*i.e.* how well they perform theo-  
132 retically), which we will call the best-case convergence. In order to do this, we use  
133 a similar approach to [13, 41, 19], and compare simulation results obtained with the  
134 simulated Ancestral Recombination Graph (ARG) as input to results obtained from  
135 sequences simulated under the same ARG, so as to study the convergence properties  
136 linked to data sets in the absence of hypothesis violation. We test several scenarios  
137 to check whether there are instances, where even without violating the underlying hy-  
138 potheses of the methodology, the demographic scenarios cannot be retrieved because  
139 of theoretical limits (and not issues linked with data). We also study the effect of the  
140 optimization function (or composite likelihood) and the time window of the analysis  
141 on the estimations of different variables. Lastly, we test the effect of commonly vi-  
142 olated hypotheses, such as the effect of the variation of recombination and mutation  
143 rates along the sequence and between scaffolds, errors in SNP calls and the presence  
144 of transposable elements and link abnormal results to specific hypothesis violations.

145 Through this work, our aim is to provide guidelines concerning the interpretation of  
146 results when applying this methodology on data sets that may violate the underlying  
147 hypotheses of the SMC framework.

## 148 2 Materials and Methods

149 In this study we use four different SMC-based methods: MSMC, MSMC2, SMC++  
150 and eSMC. All methods are Hidden Markov Models and use whole genome sequence  
151 polymorphism data. The hidden states of these methods are the coalescence times  
152 (or genealogies) of the sample. In order to have a finite number of hidden states,  
153 they are grouped into  $x$  bins ( $x$  being the number of hidden states). The reasons for  
154 our model choices are as follows: *i*) MSMC, unlike any other method, focuses on the  
155 first coalescence event of a sample of size  $n$ , and thus exhibits different convergence  
156 properties [50], *ii*) MSMC2 computes coalescence times of all pairwise analysis from  
157 a sample of size  $n$ , and can deal with a large range of data sets [58], *iii*) SMC++  
158 [63] is the most advanced and efficient SMC method and lastly, *iv*) eSMC [53] is a re-  
159 implementation of PSMC' (similar to MSMC2), which will contribute to highlighting  
160 the importance of algorithmic translations as it is very flexible in its use and outputs  
161 intermediate results necessary for this study.

### 162 2.1 SMC methods

#### 163 2.1.1 PSMC', MSMC2 and eSMC

164 PSMC' and methods that stem from it (such as MSMC2 [30] and eSMC [53]) focus on  
165 the coalescence events between only two individuals (or sequences in practice), and,



166 as a result, do not require phased data. The algorithm goes along the sequence and  
167 estimates the coalescence time at each position. In order to do this, it checks whether  
168 the two sequences are similar or different at each position. The presence or absence of  
169 a segregating site along the sequence (determined by the population mutation rate  $\theta$ )  
170 is used to infer the hidden state (*i.e.* coalescence time). However, the hidden state is  
171 only allowed to change in the event of a recombination, which leads to a break in the  
172 current genealogy. Thus, the population recombination rate  $\rho$  constrains the inferred  
173 changes of hidden states along the sequence (for a detailed description of the algorithm  
174 see [50, 66, 53]).

### 175 **2.1.2 MSMC**

176 MSMC is mathematically and conceptually very similar to the PSMC' method. Unlike  
177 other SMC methods, it simultaneously analyses multiple sequences and because of  
178 this, MSMC requires the data to be phased. In combination with a second HMM,  
179 to estimate the external branch length of the genealogy, it can follow the distribution  
180 of the first coalescence event in the sample along the sequences. However, due to  
181 computational load, MSMC cannot analyze more than 10 sequences simultaneously  
182 (for a detailed description see [50]).

### 183 **2.1.3 SMC++**

184 SMC++ is slightly more complex than MSMC or PSMC. Though it is conceptually  
185 very similar to PSMC', mathematically it is quite different. SMC++ has a differ-  
186 ent emission matrix compared to previous methods because it calculates the sample  
187 frequency spectrum of sample size  $n + 2$ , conditioned on the coalescence time of two  
188 "distinguished" haploids and  $n$  "undistinguished" haploids. In addition SMC++ offers

189 features such as a cubic spline to estimate demographic history (*i.e.* not a piece-wise  
190 constant population size). The SMC++ algorithm is fully described in [63].

#### 191 **2.1.4 Best-case convergence**

192 Using sequence simulators such as msprime [21] or scrm [60], one can simulate the  
193 Ancestral Recombination Graph (ARG) of a sample. Usually the ARG is given through  
194 a sequence of genealogies (*e.g.* a sequence of trees in Newick format). From this ARG,  
195 one can find what state of the HMM the sample is in at each position. Hence, one  
196 can build the series of states along the genomes, and build the transition matrix.  
197 The transition matrix, is a square matrix of dimension  $x$  (where  $x$  is the number  
198 of hidden states) counting all the possible pairwise transitions between the  $x$  states  
199 (including from a given state to itself). Using the transition matrix built directly  
200 from the exact ARG, one can estimate parameters using eSMC or MSMC as if they  
201 could correctly infer the hidden states. Hence estimations using the exact transition  
202 matrix represents the upper bound of performance for these methods. We choose  
203 to call this upper bound the best-case convergence (since it can never be reached in  
204 practice). For this study's purpose, a second version of the R package eSMC [53]  
205 was developed. This package enables the building of the transition matrix (for eSMC  
206 or MSMC), and can then use it to infer the demographic history. The package is  
207 mathematically identical to the previous version, but includes extra functions, features  
208 and new outputs necessary for this study. The package and its description can be found  
209 at <https://github.com/TPPSellinger/eSMC2>.

### 210 2.1.5 Baum-Welch algorithm

211 SMC-based methods can use different optimization functions to infer the demographic  
212 parameters ( *i.e.* likelihood or composite likelihood). The four studied methods use  
213 the Baum-Welch algorithm to maximize the likelihood. MSMC2 and SMC++ imple-  
214 ment the original Baum-Welch algorithm (which we call the complete Baum-Welch  
215 algorithm), whereas eSMC and MSMC compute the expected composite likelihood  
216  $Q(\theta|\theta^t)$  based only on the transition matrix (which we call the incomplete Baum-  
217 Welch algorithm). The use of the complete Baum-Welch algorithm or the incomplete  
218 one can be specified in the eSMC package. The composite likelihood for SMC++ and  
219 MSMC2 is given by equations 1 and the composite likelihood for eSMC and MSMC  
220 by equation 2:

$$Q(\Theta|\Theta^t) = \nu_{\Theta^t} \log(P(X_1|\Theta)) + \sum_{X,Z} E(X, Z|\Theta^t) \log(P(X|Z, \Theta)) + \sum_{X,Y} E(Y, X|\Theta^t) \log(P(Y|X, \Theta)) \quad (1)$$

221 and :

$$Q(\Theta|\Theta^t) = \sum_{X,Z} E(X, Z|\Theta^t) \log(P(X|Z, \Theta)), \quad (2)$$

222 with:

- 223 •  $\nu_{\Theta}$  : The equilibrium probability conditional to the set of parameters  $\Theta$ .
- 224 •  $P(X_1|\Theta)$  : The probability of the first hidden state conditional to the set of  
225 parameters  $\Theta$ .
- 226 •  $E(X, Z|\Theta^t)$  : The expected number of transitions of X from Z conditional to  
227 the observation and set of parameters  $\Theta^t$ .

- 228 •  $P(X|Z, \Theta)$  : The transition probability from state  $Z$  to state  $X$ , conditional to  
229 the set of parameters  $\Theta$ .
- 230 •  $E(Y, X|\Theta^t)$  The expected number of observations of type  $Y$  that occurred during  
231 state  $X$  conditional to observation and set of parameters  $\Theta^t$ .
- 232 •  $P(Y|X, \Theta)$  : The emission probability conditional to the set of parameters  $\Theta$ .

### 233 2.1.6 Time window

234 Each tested SMC-based method has its own specific time window for which estima-  
235 tions are made. Note that hidden states are defined as discretized intervals, as a  
236 consequences of which the boundaries, length and number of states of the time win-  
237 dow do implicitly affect inferences. For example, the original PSMC has a time window  
238 wider than PSMC', so that estimations cannot be compared one to one. To measure  
239 the effect of choosing different time window parameters, we analyze the same data  
240 with four different settings. The first time window is the one used for PSMC' defined  
241 in [50]. The second time window is that of MSMC2 [66] (similar to the one of the  
242 original PSMC [26]), which we call "big" since it goes further in the past and in more  
243 recent time than that of PSMC'. We then define a time window equivalent to the first  
244 one (i.e. PSMC') shifted by a factor five in the past (first time window, *i.e.* hidden  
245 states, multiplied by five). The last one is a time window equivalent to the first one  
246 (i.e. PSMC') shifted by a factor five in recent time (i.e. first time window divided by  
247 five).

### 248 2.1.7 Regularization penalty

249 To avoid inferring unrealistic demographic histories with variations of population size  
250 that are too strong and/or too rapid, SMC++ introduced a regularization penalty

251 (<https://github.com/popgenmethods/smcpp>). This parameter penalizes population  
252 size variation. In SMC++, the lower value of the penalty the more the estimated size  
253 history is a line (*i.e.* constant population size). Regularization penalty was also imple-  
254 mented in eSMC. Setting the regularization penalty parameter to 0 is equivalent to no  
255 penalization, and the higher the parameter value, the more population size variations  
256 are penalized (<https://github.com/TPPSellinger/eSMC2> for more details). We tested  
257 the effect of regularization on inferences with both methods using simulated sequence  
258 data. The sequence data was simulated under sawtooth demographic histories with  
259 different amplitudes of population size variation.

260 All the command lines to analyze the data generated can be found in S2 of the  
261 Appendix.

## 262 **2.2 Simulated sequence data**

263 Throughout this paper we simulate different demographic scenarios using either the  
264 coalescence simulation program `scrm` [60] or `msprime` [21]. We use `scrm` for the best-  
265 case convergence as it can output the genealogies in a Newick format (which we use as  
266 input). We use `scrm`, which outputs simulated sequences in the `ms` format, to simulate  
267 data for eSMC, MSMC, MSMC2. We use `msprime` to simulate data for SMC++ since  
268 `msprime` is more efficient than `scrm` for big sample sizes [21] and can directly output  
269 `.vcf` files (which is the input format of SMC++).

### 270 **2.2.1 Absence of hypothesis violation**

271 We simulate five different demographic scenarios: saw-tooth (successions of population  
272 size exponential expansion and decrease), bottleneck, exponential expansion, expo-  
273 nential decrease and constant population size. Each of the scenarios with varying

274 population size is tested under four amplitude parameters (*i.e.* by how many fold the  
275 population size varies: 2, 5, 10, 50). We infer the best-case convergence under four  
276 different sequence lengths ( $10^7$ ,  $10^8$ ,  $10^9$  and  $10^{10}$  bp) and choose the per site mutation  
277 and recombination rates recommended for humans in MSMC’s manual, respectively  
278  $1.25 \times 10^{-8}$  and  $1 \times 10^{-8}$  (<https://github.com/stschiff/msmc/blob/master/guide.md>).  
279 When analyzing simulated sequence data, we simulate sequences of 100 Mb: two se-  
280 quences for eSMC and MSMC2, four sequences for MSMC and twenty sequences for  
281 SMC++.

## 282 2.2.2 Calculation of the mean square error (MSE)

283 To measure the accuracy of inferences we calculate the Mean Square Error (MSE).  
284 We first divide the time window (in log10 scale) of each analysis into ten thousand  
285 points. We then calculate the MSE by comparing the actual population size and the  
286 one estimated by the method at each of the ten thousand points. We thus have the  
287 following formula:

$$MSE = \frac{\sum_{i=1}^{10^4} (y_i - y_i^*)^2}{10^4} \quad (3)$$

288 Where:

- 289 •  $y_i$  is the population size at the time point  $i$ .
- 290 •  $y_i^*$  is the estimated population size at the time point  $i$ .

291 All the command lines to simulate data can be found in S1 of the Appendix.

### 292 **2.2.3 Presence of hypothesis violation**

293 **SNP calling:** In practice, SNP calling from next generation sequencing can yield  
294 different numbers and frequencies of SNPs depending on the chosen parameters for  
295 the different steps of analysis (read trimming, quality check, read mapping, and SNP  
296 calling) as well as the quality of the reference genome, data coverage and depth of  
297 sequencing, species ploidy [46]. Therefore, based on raw sequence data, the stringency  
298 of filters can lead to excluding SNPs (false negatives) or including spurious ones (false  
299 positives). When dealing with complex genomes or ancient DNA [56, 7], SNPs can  
300 be simultaneously missed and added. We thus simulate four sequences of 100 Mb  
301 under a "saw-tooth" scenario and then a certain percentage (5,10 and 25 % ) of SNPs  
302 is randomly added to and/or deleted from the simulated sequences. We then ana-  
303 lyze the variation and bias in SNP calling on the accuracy of demographic parameter  
304 estimations. As an additional analysis we test the effect of ascertainment bias on infer-  
305 ences (a prominent issue in microarray SNP studies) by simulating 100 sequences with  
306 msprime where only SNPs above a certain (Minor Allele Frequency) MAF threshold  
307 (1%,5% and 10%) are kept, then run SMC methods on a subset of the obtained data.  
308

### 309 **Changes in mutation and recombination rates along the sequence:**

310 Because the recombination rate and the mutation rate can change along the sequence  
311 [1], and chromosomes are not always fully assembled in the reference genome (which  
312 consists of possibly many scaffolds), we simulate short sequences where the recombi-  
313 nation and/or mutation rate randomly change between the different scaffolds around  
314 an average value of  $1.25 \times 10^{-8}$  per generation per base pair (between  $2.5 \times 10^{-9}$  and  
315  $6.25 \times 10^{-8}$ ). We simulate 20 scaffolds of size 2 Mb, as this seems representative of

316 the best available assembly for non-model organisms [28, 61]. We then analyze the  
317 simulated sequences to study the effect of assuming scaffolds share the same muta-  
318 tion and recombination rates. In addition, we simulate sequences of 40 Mb (assuming  
319 genomes are fully assembled) where the recombination rate along the sequence ran-  
320 domly changes every 2 Mbp (up to five-fold) around an average value of  $1.25 \times 10^{-8}$   
321 (the mutation rate being fixed at  $1.25 \times 10^{-8}$  per generation per bp) to study the  
322 effect of the assumption of a constant recombination rate along the sequence.

323         **Transposable elements (TEs):** Genomes can contain transposable ele-  
324 ments whose dynamics violate the classic infinite site mutational model for SNPs,  
325 and thus potentially affect the estimation of different parameters. Although methods  
326 have been developed to detect [39] and simulate them [24], understanding how their  
327 presence/absence influences demographic inferences remains unclear. TEs are usually  
328 masked when detected in the reference genome and thus not taken into account in the  
329 mapped individuals due to the redundancy of read mapping for TEs. Due to their  
330 repetitive nature, it can be difficult to correctly detect and assemble them if using  
331 short reads, as well as to assess the presence/absence polymorphism of individuals of  
332 a population [11]. In addition, the quality and completeness of the reference genome  
333 (*e.g.* using the reference genome of a sister species as the reference genome) can  
334 strongly affect the accuracy of detecting, assembling and masking TEs [47]. To best  
335 capture and mimic the effect of TEs unaccounted for in the data, we altered four sim-  
336 ulated sequences of length 20 Mb in four different ways. The first way to simulate the  
337 effect of unmapped and unaccounted TEs is to assume they exhibit presence/absence  
338 polymorphism, hence creating gaps in the sequence. For each individual, we remove  
339 small pieces of sequence of different length (1kb, 10 kb or 100kb), so that up to a



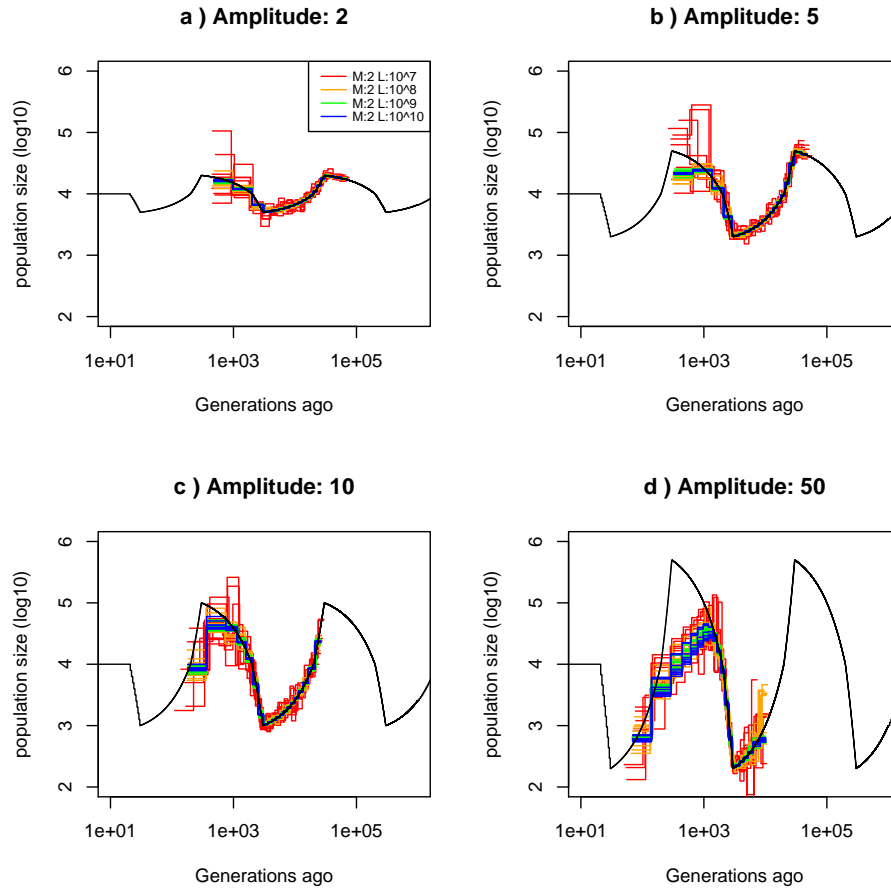
340 certain percentage (5,10,25,50%) of the original simulated sequence is removed, so  
341 as to shorten and fragment the whole sequence to be analyzed. The second way, is  
342 to consider unmasked TEs, done by randomly selecting small pieces of the original  
343 simulated sequence (1kb, 10 kb or 100kb), making up to a certain percentage of it  
344 (5,10,25,50%), and removing all the SNPs found in those regions (*i.e.* removing muta-  
345 tions from TEs). The removed SNPs are hence structured in many small regions along  
346 the genome. Thirdly, we test the consequences of simultaneously having both removed  
347 and unmasked TEs in the data set. Lastly, to measure the importance of detecting  
348 and masking TEs, we assume all TEs to be present and masked when building the  
349 multihetsep file (*i.e.* considering TEs as missing data).

## 350 **3 Results**

### 351 **3.1 Best-case convergence**

352 Results of the best-case convergence of eSMC under the saw-tooth demographic his-  
353 tory are displayed in Figure 1. Increasing the sequence length increases accuracy and  
354 reduces variability, leading to better convergence and reducing the mean square error  
355 (see Figures 1a-c and Supplementary Table 1). However, when the amplitude of popu-  
356 lation size variation is too great (here for 50 fold), the demographic history cannot be  
357 retrieved, even when using very large data sets (see Figure 1d). Similar results are ob-  
358 tained for the three other demographic scenarios (bottleneck, expansion and decrease,  
359 respectively displayed in Supplementary Figures 1, 2 and 3). The bottleneck scenario  
360 seems especially difficult to infer, requiring large amounts of data, and the stronger  
361 the bottleneck, the harder it is to detect it, even with sequence lengths equivalent to  
362  $10^{10}$ bp. In Supplementary Figure 4, we show that even when changing the number of

363 hidden states (*i.e.* number of inferred parameters), some scenarios with very strong  
364 variation of population size remain badly inferred.



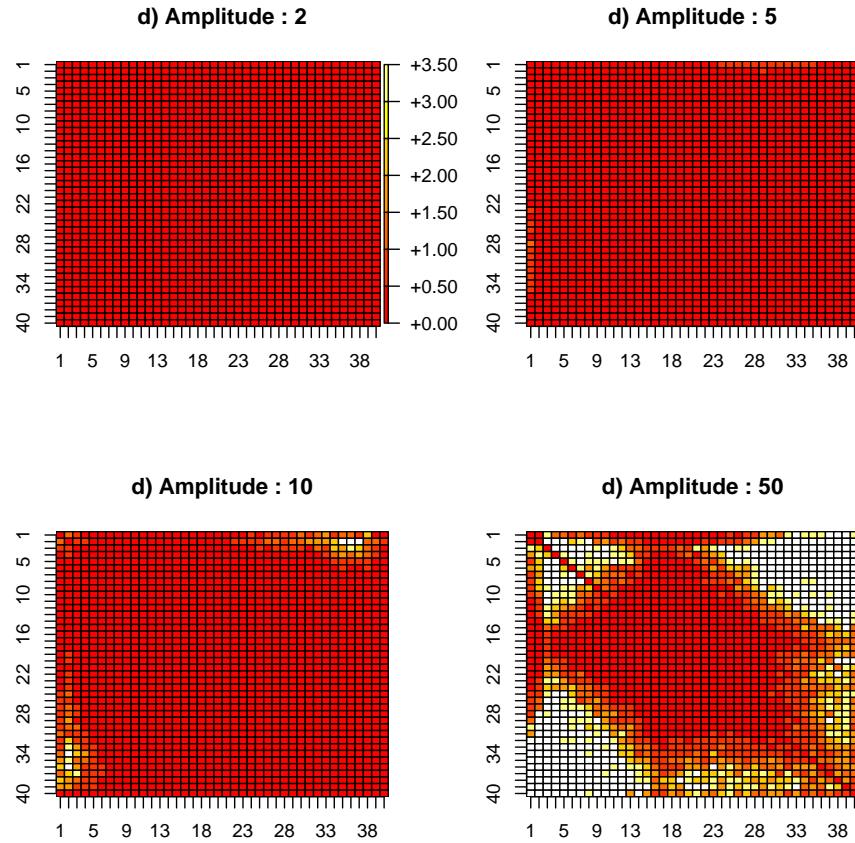
**Fig. 1. Best-case convergence of eSMC** Estimated demographic history using simulated genealogy over sequences of 10,100,1000,10000 Mb (respectively in red,orange, green and blue) under a saw-tooth scenario (original scenario in black) with 10 replicates for different amplitudes of size change: a) 2-fold, b) 5-fold, c) 10-fold, and d) 50-fold. The recombination rate is set to  $1 \times 10^{-8}$  per generation per bp and the mutation rate to  $1.25 \times 10^{-8}$  per generation per bp.

365

In Supplementary Figures 5, 6, 7 and 8, we show the best-case convergence of

366 MSMC with four genome sequences and generally find that these analyses present a  
367 higher variance than eSMC. However, MSMC shows better fits in recent times and is  
368 better able to retrieve population size variation than eSMC (see Supplementary Figure  
369 5d). Scenarios with strong variation of population size (*i.e.* with large amplitudes) still  
370 pose a problem (see Supplementary Figure 9), and no matter the number of estimated  
371 parameters, such scenarios cannot be correctly inferred using MSMC.

372         To better understand these results, we examine the coefficient of variation cal-  
373 culated from the replicates at each entry of the transition matrix. We can see that  
374 increasing the sequence length reduces the coefficient of variation (the ratio of the  
375 standard deviation to the mean, hence indicating convergence when equal to 0, see  
376 Supplementary Figure 10). Yet increasing the amplitude of population size variation  
377 decreases the number of some hidden state transitions leading to unobserved transi-  
378 tions. Unobserved transitions result from the reduced probability of coalescence events  
379 in specific time intervals (*i.e.* hidden states). In these cases matrices display higher co-  
380 efficients of variation and can be partially empty (Figure 2). This explains the increase  
381 of variability of the inferred scenarios, as well as the incapacity of SMC methods to  
382 correctly infer the demographic history with strong population size variation in specific  
383 time intervals independently of the amount of data available.



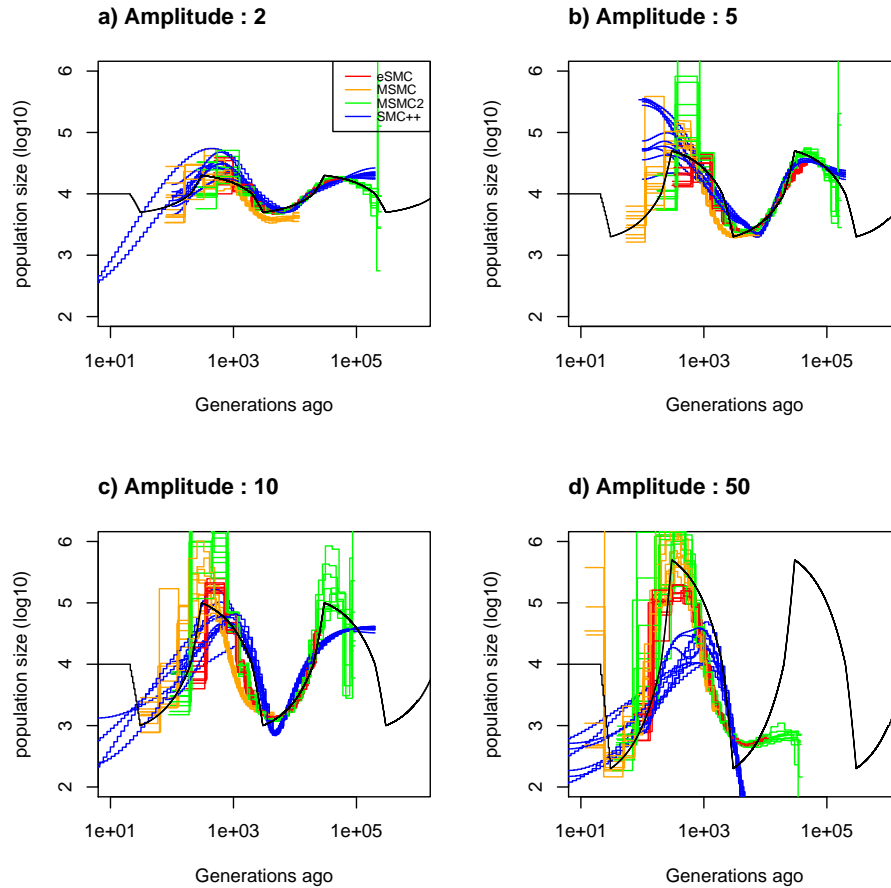
**Fig. 2. Estimated transition matrix in sharp saw-tooth scenario** Estimated coefficient of variation of the transition matrix using simulated genealogy over sequences of 10000 Mb under a saw-tooth scenario of amplitude 2, 5,10 and 50 (respectively in a, b, c and d) each with 10 replicates. Recombination and mutation rates are as in Figure 1. White squares indicate absence of observed transitions (*i.e. no data*).

## 384 **3.2 Simulated sequence results**

### 385 **3.2.1 Scenario effect**

386 In the previous section, we explored the theoretical performance limitations of eSMC  
387 and MSMC using trees in Newick format as input. In this section, we evaluate how  
388 these methods perform when inputting simulated sequence data using the same recom-  
389 bination and mutation rates. We first perform two benchmark analyses, the constant  
390 population size scenario (Supplementary Figure 11) and the sawtooth demographic  
391 scenario from [50] (Supplementary Figure 12). eSMC and MSMC2 retrieve the con-  
392 stant population size scenario although MSMC fails to retrieve it in the far past and  
393 SMC++ in recent time (Supplementary Figure 11). All methods can retrieve the saw-  
394 tooth demographic scenario although SMC++ displays high variance in recent times  
395 (Supplementary Figure 12). Secondly, we investigate the effect of amplitude of popu-  
396 lation size variation as in Figure 1. Results for the saw-tooth scenario are displayed in  
397 Figure 3, where the different models display a good fit, but are not as good as expected  
398 from the best-case convergence given the same amount of data (Figure 1 (orange line)  
399 and Supplementary Table 1 vs Figure 3 (red line) and Supplementary Table 2). As  
400 predicted by Figures 1 and 2, the case with the greatest amplitude of population size  
401 variation (Figure 1d) is the least well fitted (see Supplementary Table 2 for the MSE).  
402 All estimations display low variance and a relatively good fit in the bottleneck and  
403 expansion scenarios for small population size variation (see Supplementary Figures  
404 13a and 14a ). However, the strengths of expansions and bottlenecks are not fully  
405 retrieved in scenarios with population size variation equals to or is higher than tenfold  
406 the current population size (Supplementary Figures 13c-d, and 14c-d). To study the  
407 origin of differences between simulation results and theoretical results, we measure

408 the difference between the transition matrix estimated by eSMC and the one built  
409 from the actual genealogy. Results show that hidden states are harder to correctly  
410 infer in scenarios with strong population size variation, explaining the high variance  
411 (see Supplementary Figure 15). We demonstrate there that for the same amount of  
412 data, the simulation, and thus by extension the real data, shows additional stochastic  
413 behaviour than the best-case convergence (Figure 1).



**Fig. 3.** Estimated demography using simulated sequences as input. Estimated demographic history (black) under a saw-tooth scenario with 10 replicates using simulated sequences for different amplitudes of population size change: a) 2, b) 5, c) 10 and d) 50. Two sequences of 100 Mb for eSMC and MSMC2 (respectively in red and green), four sequences of 100 Mb for MSMC (orange) and 20 sequences of 100 Mb for SMC++ (blue) were simulated. Recombination and mutation rates are respectively set to  $1 \times 10^{-8}$  and  $1.25 \times 10^{-8}$ .



414           Increasing the time window in eSMC results in an increased variance of the  
 415 inferences (Supplementary Figure 16). In addition, shifting the window towards more  
 416 recent time leads to poor demographic estimations, but shifting it further in the past  
 417 does not seem to bias it (there are however consequences on estimations of the recom-  
 418 bination rates, see Table 1 for more details). Concerning the optimization function, we  
 419 find that the complete Baum-Welch algorithm gives similar results to the incomplete  
 420 one (Table 1).

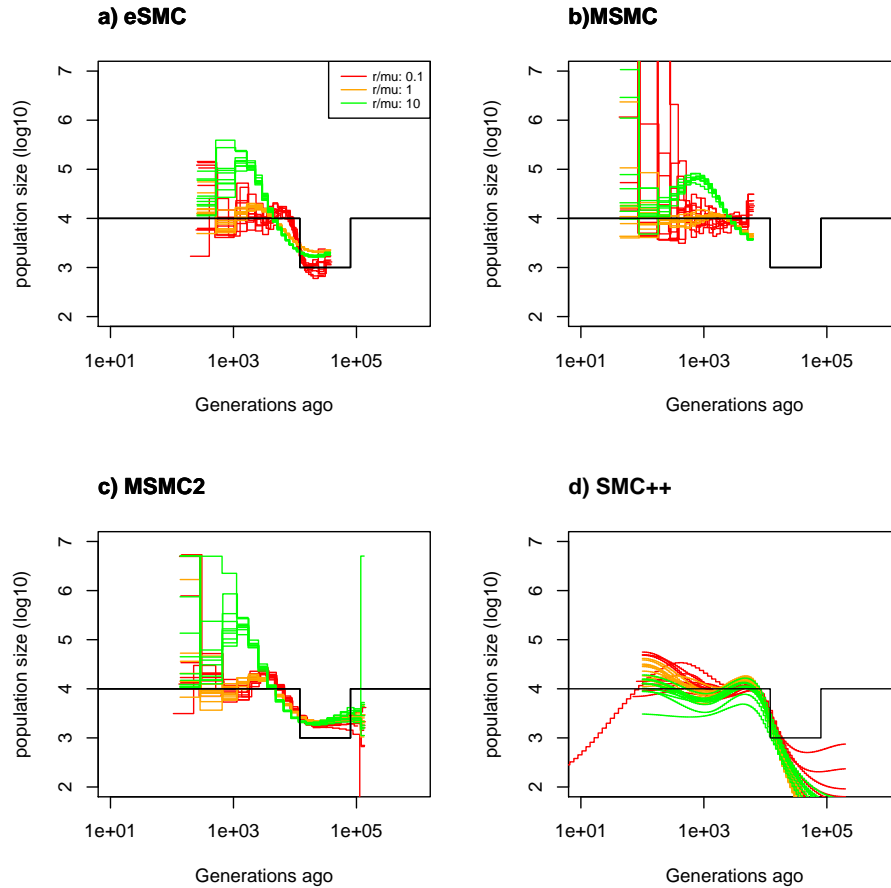
Optimization function	Scenario	real $\frac{\rho}{\theta}$	normal window $\frac{\rho}{\theta}^*$	Big Window $\frac{\rho}{\theta}^*$	Old window $\frac{\rho}{\theta}^*$	Recent window $\frac{\rho}{\theta}^*$
Incomplete Baum-Welch	Saw-tooth	0.8	0.79 (0.036)	0.72 (0.039)	0.72 (0.042)	0.94 (0.005)
Complete Baum-Welch	Saw-tooth	0.8	.79 (0.044)	0.72 (0.039)	0.72 (0.042)	1.56 (0.087)
Incomplete Baum-Welch	Constant	0.8	0.86 (0.019)	0.85 (0.020)	0.84 (0.019)	0.98 (0.002)
Complete Baum-Welch	Constant	0.8	0.86 (0.019)	0.85 (0.020)	0.84 (0.019)	1.06 (0.02)

Table 1: Average estimated values for the recombination over mutation ratio  $\frac{\rho}{\theta}$  by eSMC over ten repetitions for different sizes of the time window. The coefficient of variation is indicated in brackets. Four sequences of 50 Mb were simulated with a recombination rate set to  $1 \times 10^{-8}$  per generation per bp and a mutation rate to  $1.25 \times 10^{-8}$  per generation per bp.

421           Adding a regularization penalty to eSMC can drastically impact inferences  
 422 (Supplementary Figure 17) and reduces performance quality. When regularization is  
 423 added, eSMC fails to correctly capture the amplitude of population size variation and  
 424 with extreme regularization penalty, eSMC infers a constant population size. Yet,  
 425 adding regularization in SMC++ can increase performance and avoid spurious vari-  
 426 ation of population size (Supplementary Figure 18). However, strong regularization  
 427 can lead to the inference of constant population size and thus poor estimations.

### 428 **3.2.2 Effect of the ratio of the recombination over the mutation rate**

429 The ratio of the effective recombination over effective mutation rates ( $\frac{\rho}{\theta}$ ) can influence  
430 the ability of SMC-based methods to retrieve the coalescence time between two points  
431 along the genome [63]. Intuitively, if recombination occurs at a higher rate compared  
432 to mutation, then it renders it more difficult to detect any recombination events that  
433 may have taken place before the introduction of a new mutation, and thus bias the  
434 estimation of the coalescence time [53, 63]. Under the bottleneck scenario, we find  
435 that the lower  $\frac{\rho}{\theta}$ , the better the fit of the inferred demography by eSMC and SMC++  
436 in the past, but also the higher the variance of the inferences (see Figure 4 ). However  
437 each method displays the worse fit when  $\frac{\rho}{\theta} = 10$  (Supplementary Table 3). SMC++  
438 seems slightly less sensitive to  $\frac{\rho}{\theta}$  than other methods. When calculating the difference  
439 between the transition matrix estimated by eSMC and the one built from the actual  
440 genealogy (ARG), we find that, unsurprisingly, changes in hidden states are harder to  
441 detect when  $\frac{\rho}{\theta}$  increases, leading to an overestimation of hidden states on the diagonal  
442 (see Supplementary Figures 19, 20 and 21).



**Fig. 4. Effect of  $\frac{r}{\theta}$  on inference of demographic history.** Estimated demographic history under a bottleneck scenario with 10 replicates using simulated sequences. We simulate two sequences of 100 Mb for eSMC and MSMC2 (respectively in a and b), four sequences of 100 Mb for MSMC (c) and twenty sequences of 100 Mb for SMC++ (d). The mutation rate is set to  $1.25 \times 10^{-8}$  per generation per bp and the recombination rates are  $1.25 \times 10^{-9}$ ,  $1.25 \times 10^{-8}$  and  $1.25 \times 10^{-7}$  per generation per bp, giving  $\frac{r}{\theta} = 0.1, 1$  and  $2$  and the inferred demographies are in red, orange and green respectively. The demographic history is simulated under a bottleneck scenario of amplitude 10 and is represented in black.

443 It is, in some instances, possible to compensate for a  $\frac{\rho}{\theta}$  ratio that is not ideal  
 444 by increasing the number of iterations. Indeed, for eSMC, the demographic history  
 445 is better inferred (Supplementary Figure 22), although the correct recombination rate  
 446 cannot be retrieved (Table 2). MSMC is able to better infer the correct recombina-  
 447 tion rate than other methods despite  $\frac{\rho}{\theta} > 1$ , but poorly estimates the demographic  
 448 history. The past demographic inferences obtained using MSMC2 and SMC++ are  
 449 not improved when increasing the number of iterations (see Supplementary Figure 22  
 450 and Table 2).

method	real $\frac{\rho}{\theta}$	set 1 , $\frac{\rho^*}{\theta^*}$	set 2 , $\frac{\rho^*}{\theta^*}$	set 3 , $\frac{\rho^*}{\theta^*}$	set 4 , $\frac{\rho^*}{\theta^*}$	set 5 , $\frac{\rho^*}{\theta^*}$
eSMC	10	1.35 (0.026)	1.76 (0.047)	1.29 (0.027)	1.74 (0.048)	1.80 (0.041)
MSMC	10	2.70 (0.011)	6.58 (0.031)	2.68 (0.011)	6.57 (0.032)	6.62 (0.030)
MSMC2	10	1.27 (0.055)	1.65 (0.13)	1.26 (0.060)	1.75 (0.060)	1.60 (0.29)
SMC++	10	0.56 (0.38)	0.48 (0.38)	1.32 (0.15)	0.21 (0.62)	0.98 (0.24)

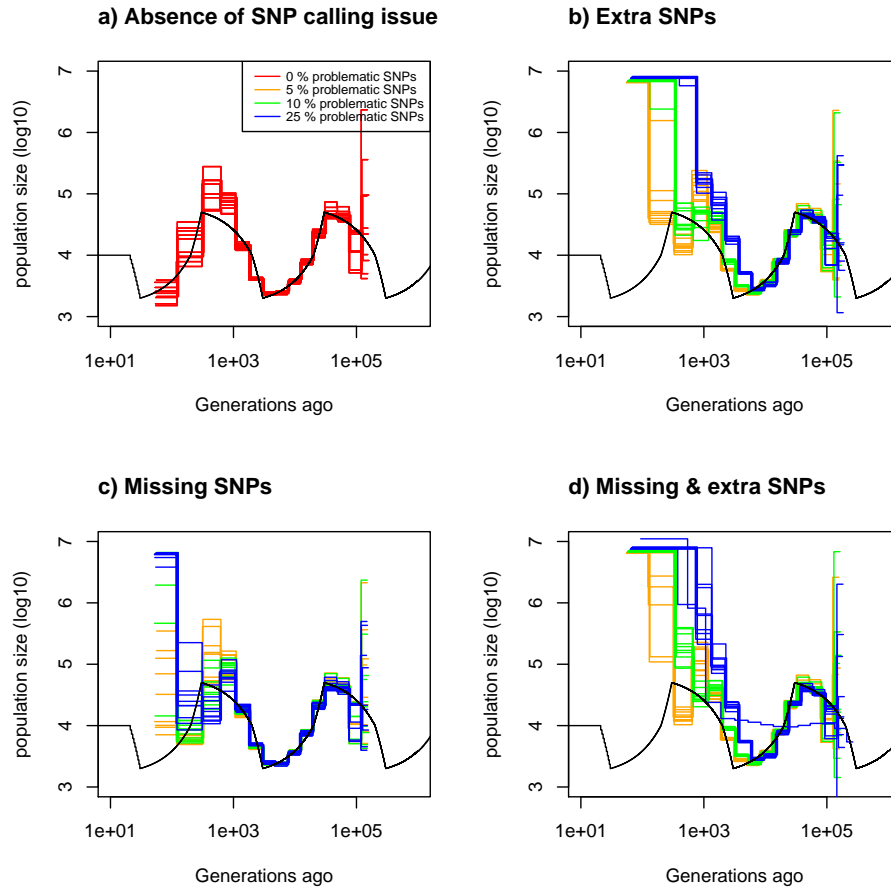
Table 2: Average estimated values for the recombination over mutation ratio  $\frac{\rho}{\theta}$  over ten repetitions. The coefficient of variation is indicated in brackets. For eSMC, MSMC and MSMC2 we have: set 1: 20 hidden states; set 2: 200 iterations; set3: 60 hidden states; set 4: 60 hidden states and 200 iterations and set 5: 20 hidden states and 200 iterations. For SMC++: set 1: 16 knots; set 2: 200 iterations; set 3: 4 knots in green; set 4: regularization penalty set to 3 and set 5: regularization-penalty set to 12.

### 451 3.3 Simulation results under hypothesis violation

#### 452 3.3.1 Imperfect SNP calling

453 We analyze simulated sequences that have been modified by removing and/or adding  
 454 SNPs using the different SMC methods. We find that, when using MSMC2, eSMC

455 and MSMC, having more than 10% of spurious SNPs (*e.g.* no quality filtering) can  
456 lead to a strong over-estimation of population size in recent time but that missing  
457 SNPs have no effects on inferences in the far past and only mild effects on inferences  
458 of recent time (see Figure 5 for MSMC2, Supplementary Figures 23 and 24 for eSMC  
459 and MSMC respectively). The mean square error is displayed in Supplementary Table  
460 4.

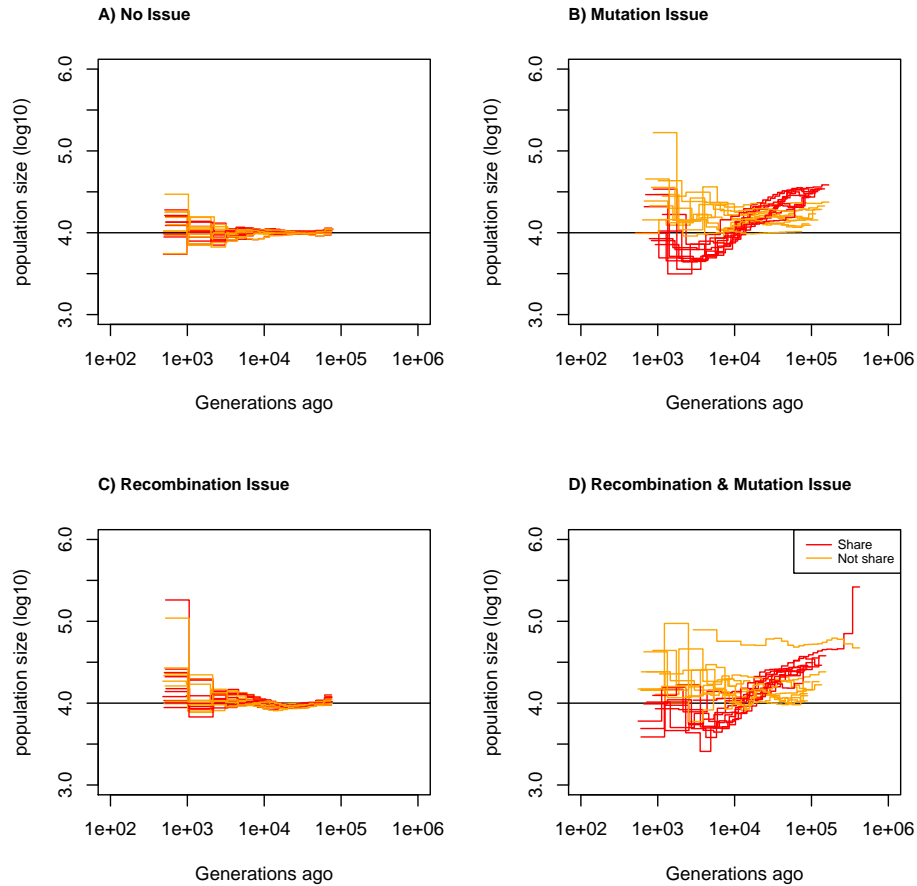


**Fig. 5. Consequences of SNP calling errors.** Estimated demographic history using MSMC2 under a saw-tooth scenario with 10 replicates using four simulated sequences of 100 Mb. Recombination and mutation rates are as in Figure 1 and the simulated demographic history is represented in black. a) Demographic history simulated with absence of SNP calling issue (red). b) Demographic history simulated with 5% (orange), 10% (green) and 25% (blue) missing SNPs. c) Demographic history simulated with 5% (orange), 10% (green) and 25% (blue) additional SNPs. d) Demographic history simulated with 5% (orange), 10% (green) and 25% (blue) of additional and missing SNPs.

461 As complementary analyses we analyze simulated sequences with a Minor Allele  
462 Frequency (MAF) threshold. Results are shown in Supplementary Figure 25. The  
463 more SNPs are removed, the poorer the estimations in recent time (Supplementary  
464 Figure 25), demonstrating the impact of severe ascertainment bias.

### 465 **3.3.2 Specific scaffold parameters**

466 We simulate sequence data where scaffolds have either been simulated with the same  
467 recombination and mutation rates or with different recombination and mutation rates.  
468 Data sets are then analyzed assuming scaffolds share or do not share the same re-  
469 combination and mutation rates. We can see in Figure 6 (and Supplementary Table  
470 5) that when scaffolds all share the same parameter values, estimated demography is  
471 accurate both when the analysis assumed shared or differing mutation and recomb-  
472 nation rates. However, when scaffolds are simulated with different parameter values,  
473 analyzing them under the assumption that they have the same mutation and recomb-  
474 nation rates leads to poor estimations. Assuming scaffolds do not share recombination  
475 and mutation rates does improve the results somewhat, but the estimations remain  
476 less accurate than when scaffolds all share with same parameter values. If only the  
477 recombination rate changes from one scaffold to another, the demographic history is  
478 only slightly biased, whereas, if the mutation rate changes from one scaffold to the  
479 other, demographic history is poorly estimated.



**Fig. 6. Estimating demographic history using scaffolds sharing or differing in mutation and recombination rates** Estimated demographic history using eSMC under a saw-tooth scenario with 10 replicates using twenty simulated scaffolds of two sequences of 2 Mb assuming scaffolds share (red) or do not share recombination and mutation rates (orange). The simulated demographic history is represented in black. a) Scaffolds share the same parameters, recombination and mutation rates are set at  $1.25 \times 10^{-8}$ , b) Each scaffold is randomly assigned a recombination rate between  $2.5 \times 10^{-9}$  and  $6.25 \times 10^{-8}$  and the mutation rate is  $1.25 \times 10^{-8}$ , c) Each scaffold is randomly assigned a mutation rate between  $2.5 \times 10^{-9}$  and  $6.25 \times 10^{-8}$  and the recombination rate is  $1.25 \times 10^{-8}$  and d) Each scaffold is assigned a random mutation and an independently random recombination rate, both being between  $2.5 \times 10^{-9}$  and  $6.25 \times 10^{-8}$ .

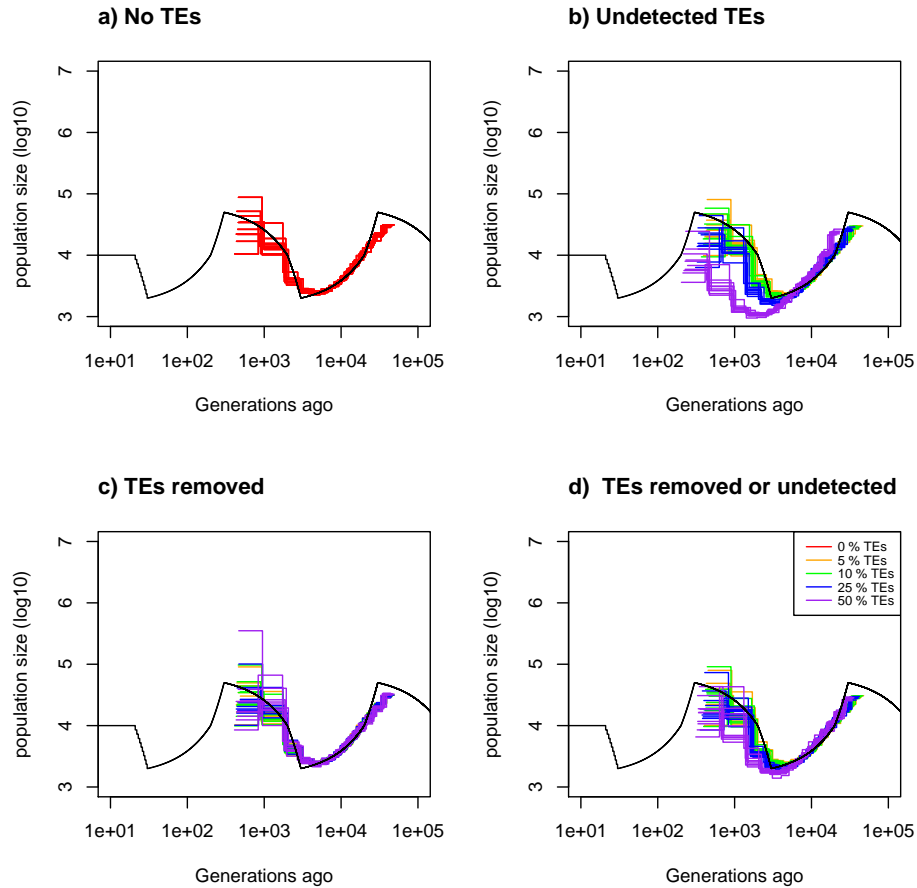


480 Even if chromosomes are fully assembled, assuming we here have one scaffold  
481 of 40 Mb (chromosome fully assembled), there may be variations of the recombination  
482 rate along the sequence, however this seems of little consequence when applying eSMC.  
483 As can be seen in Supplementary Figure 26, the demographic scenario is well inferred,  
484 despite an increase in variance and a smooth "wave" shaped demographic history  
485 when sequences simulated with varying recombination rates are compared to those  
486 with a fixed recombination rate throughout the genome. Overall we see that when  
487 recombination rate is heterogeneous along the genome by a factor 5, it is not untypical  
488 to falsely estimate a two-fold variation of  $N_e$  even though the true  $N_e$  is constant in  
489 time.

### 490 3.3.3 How transposable elements bias inference

491 Transposable elements (TEs) are present in most species, and are (if detected) taken  
492 into account as missing data by SMC methods [50]). Depending on how TEs affect  
493 the data set, we find that methods are more or less sensitive to TEs. If TEs are  
494 unmapped/removed from the data set, there does not appear to be any bias in the  
495 estimated demographic history when using eSMC (see Figure 7 and Supplementary  
496 Table 6), but there is an overestimation of  $\frac{\rho}{\theta}$  (see Table 3). We find that, the higher the  
497 proportion of sequences removed, the more  $\frac{\rho}{\theta}$  is over-estimated. For a fixed amount  
498 of missing/removed data, the smaller the sequences that are removed, the more  $\frac{\rho}{\theta}$  is  
499 over-estimated (Table 3). If TEs are present but unmasked in the data set (and thus  
500 not accounted for missing data by the model [50]), we find that this is equivalent to  
501 a faulty calling of SNPs, in which SNPs are missing, hence resulting in demographic  
502 history estimations by eSMC similar to those observed in Figure 5a. However, if  
503 the size of unmasked TEs increases, different results are obtained (see Supplementary

504 Figures 27 and 28). Indeed, in recent times there is a strong underestimation of  
505 population size and the model fails to capture the correct demographic history. The  
506 longer the TEs are, the stronger the effect on the estimated demographic history.  
507 Similar results are obtained with MSMC (Supplementary Figures 29, 30 and 31) and  
508 MSMC2 (Supplementary Figures 32, 33 and 34). However, when TEs are detected  
509 and correctly masked, there is no effect on demographic inferences (Supplementary  
510 Figures 35 and 36).



**Fig. 7. Consequences of masking or removing transposable elements (TEs) from data sets.** Estimated demographic history by eSMC under a saw-tooth scenario with 10 replicates using four simulated sequences of 20 Mb. The recombination and mutation rates are as in Figure 1 and the simulated demographic history is represented in black. Here the TEs are of length 1kbp. a) Demographic history simulated with no TEs. b) Demographic history simulated where TEs are removed. c) Demographic history simulated where TEs are masked. d) Demographic history simulated where half of the TEs are removed and SNPs on the other half are removed. Proportion of the genome made up by TEs is set to 0% (red), 5% (orange), 10% (green), 25% (blue) and 50% (purple).

TE length	method	real $\frac{\rho}{\theta}$	$\frac{\rho^*}{\theta}$ and 5% TEs	$\frac{\rho^*}{\theta}$ and 10% TEs	$\frac{\rho^*}{\theta}$ and 25% TEs	$\frac{\rho^*}{\theta}$ and 50% TEs
1 kb	eSMC	1	0.95 (0.021)	0.99 (0.022)	1.16 (0.10)	1.77 (0.36)
	MSMC	1	1.31 (0.098)	1.35 (0.11)	1.50 (0.088)	1.91 (0.11)
	MSMC2	1	0.87 (0.047)	0.88 (0.049)	1.0 (0.036)	1.35 (0.035)
10 kb	eSMC	1	0.96 (0.053)	0.98 (0.066)	1.10 (0.18)	1.36 (0.41)
	MSMC	1	1.38 (0.074)	1.41 (0.090)	1.54 (0.11)	1.68 (0.13)
	MSMC2	1	0.87 (0.064)	0.89 (0.067)	.99 (0.15)	1.13 (0.30)
100 kb	eSMC	1	0.95 (0.047)	0.95 (0.051)	0.98 (0.070)	1.0 (0.12)
	MSMC	1	1.36 (0.048)	1.36 (0.062)	1.40 (0.093)	1.49 (0.12)
	MSMC2	1	0.87 (0.056)	0.88 (0.050)	0.91 (0.079)	0.91 (0.073)

Table 3: Average estimated values for the recombination over mutation ratio  $\frac{\rho}{\theta}$  over ten repetitions. The coefficient of variation is indicated in brackets. TEs are of length 1kb, 10kb or 100 kb and are completely removed and the proportion of the genome made up by TEs is 5%,10% ,25% and 50%.

## 511 4 Discussion

512 Throughout this work we have outlined the limits of PSMC' and MSMC methodolo-  
513 gies, which had, until now, not been clearly defined. We find that, in most cases,  
514 if enough genealogies (*i.e.* data) are inputted then the demographic history is ac-  
515 curately estimated, tending to results obtained previously [13, 8], however, we find  
516 that the amount of data required for an accurate fit depends on the underlying demo-  
517 graphic scenario. The differences with previous works stems from estimations being  
518 made using the actual series of coalescence times [13, 8], whereas we use the series of  
519 hidden states built from the discretization of time summarized in a simple matrix. We  
520 also find that some scenarios are better retrieved when using either MSMC or meth-  
521 ods based on PSMC', indicating that there are complementary convergence properties

522 between these methodologies.

523           We develop a method to indicate if the amount of data is enough to retrieve  
524 a specific scenario, notably by calculating the coefficient of variation of the transition  
525 matrix using either real or simulated data, and therefore offer guidelines to build  
526 appropriate data sets (see also Supplementary Figure 8). Our approach can also be  
527 used to infer demographic history given an ARG (using trees in Newick format or  
528 sequences of coalescence events), independently of how the ARG has been estimated.  
529 Our results suggest that whole genome polymorphism data can be summarized in  
530 a transition matrix based on the SMC theory to estimate demographic history of  
531 panmitic populations. As new methods can infer genealogies better and faster [58, 22,  
532 36, 42], the estimated transition matrix could become a powerful summary statistic  
533 in the future. HMM can be a computational burden depending on the model and  
534 model parameters, and estimating genealogy through more efficient methods would  
535 still allow the use of SMC theory for parameter estimation or hypothesis testing (as  
536 in [67, 13, 19]). In addition, using the work of [66], one could (to some extent [23])  
537 extend our approach to account for population structure and migration.

538           We have also demonstrated that the power of PSMC', MSMC, and other SMC-  
539 based methods, rely on their ability to correctly infer the genealogies along the se-  
540 quence (*i.e.* the Ancestral Recombination Graph or ARG). The accuracy of ARG  
541 inference by SMC methods, however, depends on the ratio of the recombination over  
542 the mutation rate ( $\frac{\rho}{\theta}$ ). As this rate increases, estimations lose accuracy. Specifically,  
543 increasing  $\frac{\rho}{\theta}$  leads to an over-estimation of transitions on the diagonal, which explains  
544 the underestimation of the recombination rate and inaccurate demographic history es-  
545 timations, as shown in [63, 53]. As a way around this issue, in some cases it is possible

546 to obtain better results by increasing the number of iterations. MSMC's demographic  
547 inference is more sensitive to  $\frac{\rho}{\theta}$  but the quality of the estimation of the ratio itself is  
548 less affected. This once again shows the complementarity of PSMC' and MSMC. If  
549 the variable of interest is  $\frac{\rho}{\theta}$ , then MSMC should be used, but if the demographic his-  
550 tory is of greater importance, PSMC'-based methods should be used. The amplitude  
551 of population size variation also influences the estimation of hidden states along the  
552 sequences, with high amplitudes leading to a poor estimation of the transition ma-  
553 trix, distorting the inferred demography. We find that increasing the size of the time  
554 window increases the variance of the estimations, despite using the same number of  
555 parameters, as this results in a small under-estimation of  $\frac{\rho}{\theta}$ . In addition the complete  
556 and incomplete Baum-Welch algorithms lead to identical results, demonstrating that  
557 all the information required for the inference is in the estimated transition matrix.

558         Finally, we explored how imperfect data sets (due to errors in SNP calling,  
559 the presence of transposable elements and existing variation in recombination and  
560 mutation rates) could affect the inferences obtained using SMC-based methods. We  
561 show that a data set with more than 10% of spurious SNPs will lead to poor estimations  
562 of the demographic history, whereas randomly removed SNPs (*i.e.* missing SNPs) have  
563 a lesser effect on inferences. It is thus better to be stringent during SNP calling, as  
564 SNPs is worse than missing SNPs. Note, however, that this consideration is valid for  
565 demographic inference under a neutral model of evolution, while biases in SNP calling  
566 also affect the inference of selection (especially for conserved genes under purifying  
567 selection). However, if missing SNPs are structured along the sequence (as would be  
568 the case with unmasked TEs), there is a strong effect on inference. If TEs are correctly  
569 detected and masked, there is no effect on demographic inferences. It is therefore

570 recommended that checks should be run to detect regions with abnormal distributions  
571 of SNPs along the genome. Surprisingly, simulation results suggest that removing  
572 random pieces of sequences have no impact on the estimated demographic history.  
573 Taking this into account, when seeking to infer demographic history, it seems better  
574 to remove sections of sequences than to introduce sequences with SNP call errors or  
575 abnormal SNP distributions. However, removing sequences leads to an over-estimation  
576 of  $\frac{\rho}{\theta}$ , which seems to depend on the number and size of the removed sections. The  
577 removal of a few, albeit long sequences, will have almost no impact, whereas removing  
578 many short sections of the sequences will lead to a large overestimation of  $\frac{\rho}{\theta}$ . This  
579 consequence could provide an explanation for the frequent overestimation of  $\frac{\rho}{\theta}$  when  
580 compared to empirical measures of the ratio of recombination and mutation rates  $\frac{r}{\mu}$ .  
581 This implies, that in some cases, despite an inferred  $\frac{\rho}{\theta} > 1$ , the inferred demographic  
582 history can surprisingly be trusted. Note also that as discussed in [53], the discrepancy  
583 between  $\frac{\rho}{\theta}$  and  $\frac{r}{\mu}$  can be due to life history traits such as selfing or dormancy.

584         Simulation results suggest that any variation of the recombination rate along  
585 the sequence does not strongly bias demographic inference but slightly increases the  
586 variance of the results and leads to small waves in the demographic history (as a  
587 consequence of erroneously estimated hidden state transition events because of the  
588 non constant recombination rate along the sequence), as expected from previous works  
589 [26]. However, unlike Li and Durbin's results [26], if scaffolds do not share similar  
590 rates of mutation and recombination, but are analyzed together assuming that they  
591 do, estimations will be very poor. This could be due to the variation of mutation  
592 rate being within a scaffold in their study and the discrepancy between out and their  
593 results could suggest analyses based on longer scaffolds to be more robust. However,

594 this problem can be avoided if each scaffold is assumed to have its own parameter  
595 values, although this would increase computation time, it could provide useful insight  
596 in unveiling any variation in molecular forces along the genome, albeit in a coarser  
597 way than in [1]. As we found that non-accounted variation of the recombination rate  
598 along the sequence can lead to a spurious two-fold variation of population size, we  
599 here provide guidelines to test if small detected variations of population size are to be  
600 trusted. Since the consequences of a varying recombination rate might depend on the  
601 topology of the recombination map, one first needs estimate the recombination map  
602 (*e.g.* using iSMC [1]). If problematic regions are found they can be removed with  
603 almost no negative impact on the estimated demography (Figure 7). Otherwise, the  
604 recombination map can be used to simulate sequences *e.g.* using scrm [60]), which can  
605 be compared to results obtained for a constant recombination rate. Analyses can be  
606 run on both data sets to quantify the effect of the recombination map.

#### 607 **4.1 Guidelines when applying SMC-based methods**

608 Our aim through this work is to provide guidelines to optimize the use of SMC-based  
609 methods for inference. First, if the data set is not yet built, but there is some intuition  
610 concerning the demographic history and knowledge of some genomic properties of a  
611 species (*e.g.* recombination and mutation rates), we recommend simulating a data  
612 set corresponding to the potential scenarios. From these simulations, the transition  
613 matrix for PSMC' or MSMC-based methods can be built using the R package eSMC2.  
614 The results obtained can guide users when it comes to the amount and quality of data  
615 needed (sequence size and copy number) for a good inference. Beyond being used  
616 to guide the building of data sets, it is possible to assess trustworthiness of results  
617 obtained using SMC-based methods on existing data sets. If the estimated transition



618 matrix is empty in some places (*i.e.* no observed transition event between two specific  
619 hidden states; white squares in Figure 2), it could suggest a lack of data and/or strong  
620 variation of the population size somewhere in time. In order to test the accuracy of the  
621 inferred demography, the estimated demographic history can be retrieved and used to  
622 simulate a data set with more sequences and/or simulate a demographic history with  
623 a higher amplitude than the estimated one. The SMC method can then be run on  
624 the simulated data in order to check whether using more data results in a matching  
625 scenario or if a higher amplitude of population size can indeed be inferred, in which  
626 cases the initial results are most probably trustworthy.

627         As mentioned above, it is better to sequence fewer individuals, but to have  
628 data of better quality. It is also important to note, that more data is not necessarily  
629 always better, especially if there is a risk of spurious SNPs (see Figure 5). In some  
630 cases, methods are limited by their own theoretical framework, hence no given data set  
631 will ever allow a correct demographic inference. In such cases, other methods based on  
632 a different theoretical frameworks (*e.g.* SFS and ABC) might perform better [3, 51].

## 633 4.2 Concluding remarks

634 Here we present a simple method to help assess how accurate inferences obtained us-  
635 ing PSMC' and MSMC would be when applied to data sets with suspected flaws or  
636 limitations. We also provide new interpretations of results obtained when hypotheses  
637 are known to be violated, and thus offer an explanation as to why results sometimes  
638 deviate from expectations (*e.g.* when the estimated ratio of recombination over mu-  
639 tation is larger than the one measured experimentally). We propose guidelines for  
640 building/evaluating data sets when using SMC-based models, as well as a method

641 which can be used to estimate the demographic history and recombination rate given  
642 a genealogy (in the same spirit as Popsicle [13]). The estimated transition matrix is  
643 introduced as a summary statistic, which can be used to recover demographic history  
644 (more precisely the Inverse Instantaneous Coalescence Rate interpretation of popula-  
645 tion size variation, when assuming a panmictic population [8, 49]). This statistic could,  
646 in future, be used in scenarios with migration, without the computational load of Hid-  
647 den Markov models. When faced with complex demographic histories, or  $\frac{\rho}{\theta} > 1$ , we  
648 show that there are strategies that would allow those wishing to use SMC methodology  
649 to make the best use of their data.

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## 654 6 Competing Interest

655 The authors of this article declare that they have no financial conflict of interest with  
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