

1 **Multifractality in postural sway supports quiet eye training in** 2 **aiming tasks: A study of golf putting**

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23 24 **Supplementary materials**

25 **Supplementary Table S1.** Subset of Poisson regression model output including main and interaction
26 effects of time and perturbation of eye height.

27 **Abstract**

28 The ‘quiet eye’ (QE) approach to visually-guided aiming behavior invests fully in perceptual
29 information's potential to organize coordinated action. Sports psychologists refer to QE as the stillness
30 of the eyes during aiming tasks and increasingly into self- and externally-paced tasks. Amidst the
31 ‘noisy’ fluctuations of the athlete’s body, quiet eyes might leave fewer saccadic interruptions to the
32 coupling between postural sway and optic flow. Multifractality in postural sway is a robust predictor of
33 both visual and haptic perceptual responses. Postural sway generates optic flow centered on an
34 individual’s eye height. So, we manipulated the eye height part way through a golf-putting task for
35 participants trained in QE or trained technically as per conventional golf putting. We predicted that
36 perturbing the eye height by attaching wooden blocks below the feet would perturb the putting more so
37 in QE-trained participants than in those trained technically. We also predicted that responses to this
38 perturbation would depend on multifractality in postural sway. Specifically, we predicted that less
39 multifractality would predict more adaptive responses to the perturbation and higher putting accuracy.
40 Results supported both predictions. QE training and lower multifractality led to more frequent
41 successful putts, and the perturbation of eye height led to less frequent successful putts, particularly for
42 QE-trained participants. Models of radial error (i.e., the distance between the ball’s final position and
43 the hole) indicated that lower estimates of multifractality due to nonlinearity coincided with a more
44 adaptive response to the perturbation. These results challenge the past suggestions that reduced
45 multifractality might be a signature of diseased posture. Instead, they suggest that reduced
46 multifractality may act in a context-sensitive manner to restrain motoric degrees of freedom to achieve
47 the task goal.

48 **Keywords:** biotensegrity, degree of freedom, fractal, posture, tensegrity, visual adaptation

49 1. Introduction

50 Aiming our behaviors into the visible world requires an ongoing awareness of how our body fits in the
51 world around it. The difficulty of sorting out this bodily fit finds its classic encoding in the ‘degrees of
52 freedom’ problem: an explicit choreography of goal-directed movement that stipulates all individual
53 motoric degrees of freedom at fine ‘grains’ proves daunting and probably too rigid to be adapted to
54 change in task or context [1]. However, another perspective suggests that the abundance of degrees of
55 freedom is not a problem but a ‘blessing’ [2]. According to this view, there is no infinitude of motoric
56 variables to stipulate at very fine grains, and on the contrary, task or intention organizes the movement
57 system at the relatively coarse grain of the whole body and context [3]. This positive view of motor
58 abundance appears in recent studies of aiming behaviors like the golf swing, whereby a clearly defined
59 target in the context draws visual attention away from specific postures of the body, and indeed,
60 experts with their eyes on the target show no single optimized mode of execution [4]. Hence, instead of
61 requiring explicit choreography, the movement system’s organization may readily emerge from
62 perceptual linkages to the surrounding context [5–7].

63 1.1. Quiet eye versus technical training

64 Supporting task performance by emphasizing perceptual linkages versus explicit choreography is a
65 clear theme of research on ‘quiet eye’ (QE). Sports psychologists refer to QE as the stillness of the eyes
66 during aiming tasks, specifically the final gaze fixation 100 ms in duration within 3 degrees of visual
67 angle immediately before the movement is initiated [8]. In the case of a golf swing, ‘technical training’
68 that communicates an explicit choreography of the arm and leg placement leads to less accuracy than a
69 ‘quiet eye’ (QE) strategy. Experts show more frequent and longer gaze fixations on specific aspects of
70 the putting situation (e.g., golf ball, putting surface, target hole). Novices tend to fixate more
71 introvertedly (e.g., on the clubhead during the backswing phase) and focus on the ball only after it has
72 been hit with the club [9–13]. Visual guidance by focusing on the target leads to a more effective golf
73 swing than technical training of the postures and limb movements composing a golf swing. Indeed, in
74 contrast with technical training, QE training of golf putting leads to improved putting accuracy as well
75 as to a reduction in heart rate and muscle activity at the moment of impact [11]. Thus, it is believed that
76 instructing athletes by directing their perceptual linkages outward to the visual field supports higher
77 task performance as well as leads to more expert-like movement and physiology.

78 QE research opens a fascinating vantage point to situate ongoing research on linkages
79 anchoring organisms to their visual contexts and the synergies transforming visual information into
80 action (e.g., [14–16]). Research on QE has mostly examined the visual-cognitive bases for the
81 relatively more extraverted gaze into the task context [8,17,18], and the relationship of QE to the body,
82 the brain, and the nervous system has received relatively less attention [19–21]. To fill this gap, a
83 ‘postural-kinematic hypothesis’ has begun to complement the canonical ‘visual hypothesis’ that QE
84 was a primarily visual-cognitive mechanism [22]. So far, this postural-kinematic hypothesis
85 emphasizes only slower movement times, that is, longer duration offering the foundation for longer
86 fixations. This postural-kinematic hypothesis is a constructive first step. In this study, we aim to embed
87 more of the postural dynamics into this hypothesis, beyond simply movement duration. Additionally,
88 instead of reporting on gaze measurements, we will use QE as a training manipulation and focus on
89 investigating how postural sway supports the effect of the training.

90 1.2. Optic flow and postural sway can support the quiet eye strategy

91 A crucial consideration for postural-kinematic contributions to visually guided tasks is optic flow [5–
92 7]. Amidst the ‘noisy’ fluctuations of the athlete’s body, quieting the eyes might leave fewer saccadic
93 interruptions to the coupling between optic flow and postural sway. Optic flow consists of the
94 differential expansion, contraction, or rotation of visible objects and surfaces as an individual moves

95 about. It is specific to both the individual's movement and the contextual layout of objects and
96 surfaces. Indeed, virtual visual displays that impose artificial optic flow also engender anticipatory
97 behaviors (e.g., steering, braking) as if the objects and surfaces entailed by the optic flow are out there
98 —not just for humans but for other animal species and even robots [23]. Although most prominent in
99 locomotion-related contexts, postural sway is no less capable of generating informative optic flow in
100 non-locomotion contexts [24]. Because research on QE has mostly focused on the fixation of the eye
101 gaze, QE's relationship to optic flow remains a critical gap [25].

102 The present work aims to replicate the beneficial effects of QE on putting performance, and
103 more importantly, to understand how eye height and optic flow might support QE. Specifically,
104 assuming that QE might depend on the optic flow, we aim to test whether perturbing the optic flow
105 parameter of eye height perturbs the effect of QE training in golf putting more so than technical
106 training. Indeed, experimentally varying eye height is known to complicate optic flow in ways that
107 briefly perturbs visual perception (e.g., [26]). We used a single session in the style of Moore et al.'s
108 [11] golf-putting training paradigm to test for rapid attunement to changes in information from optic
109 flow and postural sway. For instance, manipulating eye height by adding wooden blocks to
110 participants' shoes alters their affordance judgments significantly, leading to an initial underestimation
111 of the optimal climbing height for a stair and sitting height for a chair. However, participants adjust to
112 the new eye height and make more accurate decisions later on [27,28]. Hence, we expect that adding
113 wooden blocks—(henceforth, 'clogs', to distinguish from 'blocks' of experimental trials)—to the feet
114 might upset putting accuracy in the QE-trained participants initially, but that this accuracy will rebound
115 as participants adjust to new eye height [8].

116 **1.3. Multiscaled organization of postural sway entails a multifractal structure**

117 Besides our manipulation of eye height, our expectations about the optic-flow foundation for QE also
118 include a hypothesis about how postural sway should moderate the effects of perturbed eye height. If
119 optic flow supports QE, we expect that QE-trained response to eye height perturbation will depend on
120 postural sway. Indeed, in the affordance research on reattunement to perturbation of eye height,
121 postural sway supported the adaptation of the relationship of eye height to judgments of optimal chair
122 height for sitting [28]. However, it was not simply that more sway facilitated this adaptation because,
123 for instance, an 'awkward stance' requiring more compensatory postural adjustments did not support
124 visual adaptation as much as comfortable standing did. An additional challenge in identifying what
125 portion of sway could support adaptation to altered eye height is that sway depends on events at
126 multiple timescales. At longer scales, QE training itself might influence sway by training an explicit
127 use of visual information. At shorter timescales, adding clogs might destabilize posture initially, and
128 then at the shortest timescales, sway might be implicated in adapting to this perturbation of eye height.
129 Hence, judiciously accounting the sway's contribution to visual adaptation due to perturbation of eye
130 height requires an appreciation of sway's multi-scaled organization.

131 The multi-scaled organization of sway may itself reveal a key to predicting—if not explaining
132 —how the movement system adapts its visually-guided actions. Interactions across multiple timescales
133 noted above are currently understood to entail a 'multifractal' structure in sway. Multifractality refers
134 to multiple (i.e., 'multi-') relationships between fluctuations and timescales, with these relationships
135 following power laws with potentially fractional (i.e., '-fractal') exponents. Interactions across
136 timescales engender a nonlinear patterning of fluctuations that multifractal analysis is well-poised to
137 diagnose and quantify [29]. Specifically, multifractal analysis estimates a spectrum of fractal exponents
138 whose width W_{MF} quantifies multifractality. When we compare measured series' W_{MF} to spectrum
139 widths of a set of best-fitting linear models ('linear surrogates') of those series, multifractal evidence of
140 nonlinearity manifests as a significant difference between W_{MF} for the original series and the average
141 W_{MF} for linear surrogates. The t -statistic, t_{MF} , comparing the original series' W_{MF} to the surrogates' W_{MF}

142 serves as an estimate of how much multifractality in the original series is due to nonlinear interactions
143 across timescales.

144 **1.4. Multifractal nonlinearity predicts perceptual responses to perturbations of task constraints**

145 Multifractality, W_{MF} , in postural sway is a robust predictor of perceptual responses in both visual and
146 haptic media [30–32]. Multifractal nonlinearity, t_{MF} , in postural sway predicts perceptual responses as
147 well, not just in the case of nonvisual judgments of length and heaviness of manually-wielded objects
148 [33] but also in visuomotor tasks [34,35] and in response to perturbations, such as those due to
149 prismatic goggles in a visual aiming task [36]. Now, we aim to test whether t_{MF} can predict the
150 responses to perturbations of eye height in QE-trained golf putting.

151 We expected that postural sway most robust to the perturbation of eye height would show
152 relatively narrow multifractal spectra that would still differ significantly from those of the surrogates.
153 So far, postural sway shows relatively narrower multifractal spectra (i.e., smaller W_{MF}) for younger
154 participants [37], those less likely to experience motion sickness [38], and those focusing on more
155 proximal surfaces in optic flow [39]. Hence, greater W_{MF} in postural sway might counteract any effects
156 of QE. All these findings are qualified by a failure of W_{MF} to correlate with canonical measures of sway
157 like standard deviation. However, postural stability is only ambiguously related to the standard
158 deviation of sway, requiring neither too much nor too little variability in sway [40,41]. Meanwhile,
159 both empirical and theoretical work suggests that t_{MF} is more closely related to postural stabilizing.
160 Evidence shows that postural stabilization following a perturbation to the base of support shows a rapid
161 reduction of t_{MF} [42]. The theoretical simulation also predicts that random perturbations to individual
162 scales of a nested system lead to multifractality due to nonlinearity and narrower multifractal spectra
163 (i.e., smaller W_{MF} and smaller-yet-significant t_{MF} ; [43]). Hence, incorporating the visual information
164 due to the perturbation of eye height adaptively into the movement system must involve exhibiting
165 smaller-yet-significant t_{MF} .

166 **1.5. Golf-putting task and hypotheses**

167 The participants were instructed to take golf putts towards a circular, hole-sized target painted on a
168 green putting surface in the laboratory—over four blocks of ten trials each. One group received QE
169 training, and the other group received technical training specifying how to move their limbs to make
170 the golf putt. Wooden clogs were attached to the feet of all participants to perturb their eye height.
171 Putting performance was treated both dichotomously (i.e., ‘making’ or ‘missing’ the putt) and
172 continuously (i.e., in terms of radial error). We tested hypotheses specific to these two treatments.

173 **1.5.1. Hypothesis-1: Effects on dichotomous treatment of putting performance**

174 Putting performance was dichotomized as a ‘make’ when the golf ball came to rest within the target or
175 2 feet directly beyond it, and as a ‘miss’ in all other cases. We predicted that QE training would lead to
176 more ‘makes’ (Hypothesis-1a) and that the eye height perturbation due to clogs would lead to more
177 misses following QE training than technical training (Hypothesis-1b). We also predicted that greater
178 sway W_{MF} would lead to fewer ‘makes’ (Hypothesis-1c).

179 **1.5.2. Hypothesis-2: Effects on continuous treatment of putting performance**

180 Putting performance was treated continuously in terms of unsigned radial distance from the target (i.e.,
181 unsigned radial error). We hypothesized that the eye height perturbation due to clogs would lead to
182 lower putting accuracy characterized by higher unsigned radial error following QE training than
183 technical training. In contrast to logistic modeling of a dichotomous dependent variable, which is
184 limited by the number of valid predictors (e.g., [44]), a continuous dependent variable allows testing
185 more complex hypotheses. We predicted that the above effects of clogs on QE would depend on the
186 multifractal nonlinearity, t_{MF} , of sway beyond any effects of multifractality (i.e., W_{MF}) alone

187 (Hypothesis-2a). We also predicted that lower t_{MF} of sway would be associated with more flexible
188 responses to the eye height perturbation, leading to a gradual increase but a subsequent decay of radial
189 error across trials within the blocks with the clogs on (Hypothesis-2b).

190

191 **2. Methods**

192 **2.1. Participants**

193 Twenty undergraduate students (11 women, *mean*±*s.d.* age = 19.5±1.2 years, all right-handed [45])
194 with normal or corrected vision participated in this study. All participants provided written informed
195 consent approved by the Institutional Review Board (IRB) at Grinnell College (Grinnell, IA).

196 **2.2. Procedure**

197 In an hour-long session, the participants completed a single putt on each of 40 trials. Prior to the
198 training trials, half of the participants received QE instructions (Table 1, left column) while the other
199 half received technical putting instruction (Table 1, right column), replicating Moore et al.'s [11]
200 methodology. The technical instructions mirrored the QE instructions but did not describe the eye-
201 related behaviors, hence mimicking standard golf instructions. After verbally confirming that they
202 understood the instructions, the participants watched a video demonstrating a QE-specific putting
203 stroke (<https://www.youtube.com/watch?v=lfWuL1klbsY&feature=youtu.be>) or a standard putting
204 stroke (<https://www.youtube.com/watch?v=HOYZn4MYmDI>). The researcher directed the quiet-eye
205 trained participants to the key features of the golfer's gaze control and the technically-trained
206 participants to the putting mechanics.

207 The participant performed the golf putts on a 6×15 feet standard indoor All Turf Mats™
208 artificial turf mat using a 1.68-inch diameter PING Sigma 2 Anser Stealth™ putter and regular-sized
209 white golf balls. The participants completed straight putts ten feet away towards a 2.5-inch diameter
210 circular orange dot, designed to fit the golf ball. This format allowed for more stringent measurement
211 and gave the participants a more focused target.

212 The participants completed 20 training trials (treated as two blocks of 10 trials per block in
213 regression modeling). Following a short 5-minute break, the participants completed two blocks of ten
214 test trials per block (i.e., 20 test trials total) in the absence of instructions. The participants were
215 randomly assigned to complete either their first or second block of ten trials while wearing 5-cm-raised
216 wooden blocks (i.e., clogs) attached to their shoes (they completed the other block of ten test trials
217 under the same conditions as the 20 training trials). Before the testing began, the participants were
218 allowed to walk around and adjust to the clogs to reduce fall risk.

219 **2.3. Measures**

220 The task performance was defined in terms of the radial error (the distance from the target at which the
221 ball stopped, in feet) and the number of putts successfully 'holed.' Because the target was not a real
222 hole, the putt was counted as holed in either of two events: the ball came to rest within the orange dot
223 on the floor or within 2 feet directly beyond the hole. In either of these cases, the radial error was
224 recorded as zero [46,47]. Postural sway was measured using a smartphone with the Physics Toolbox
225 Suite application (<http://vieryasoftware.net>) strapped to the participants' lower back. The app recorded
226 the acceleration of the torso along x -, y -, and z -dimensions at 100 Hz. An experimenter manually
227 started and stopped recording for each putt. Recording began after the participant set up over the ball
228 and stopped once the ball stopped moving.

229 **2.4. Multifractal analysis**

230 **2.4.1 Multifractal spectrum width of accelerometer displacement series**

231 For each putt, we computed an accelerometer displacement series for multifractal analysis as follows.
232 The accelerometer data encodes an N -length series of accelerations of the torso along x -, y -, and z -
233 dimensions centered on an origin on the smartphone. A displacement series of length $N-1$ was
234 computed by: 1) taking the first differences for the variable (e.g., if we use the letters i and j to indicate
235 individual values in sequence, $x_{diff}(j)=x(i)-x(i-1)$ for each i th value of x , for $2<i<N$,
236 $1<j=i-1<N-1$, 2) squaring and summing the differences of each dimension's displacements (i.e.,
237 $x_{diff}^2+y_{diff}^2+z_{diff}^2$), and 3) taking the square root of this sum (i.e., $\sqrt{(x_{diff}^2+y_{diff}^2+z_{diff}^2)}$).

238 Multifractal spectra $f(\alpha)$ were estimated for each accelerometer displacement series, using
239 Chhabra and Jensen's [48] method. The first step was to partition the series into subsets (or 'bins') of
240 various sizes, from four points to a fourth of series' length (figure 1, left). The second step was to
241 examine how the proportion of displacements within each bin changed with bin size or timescale
242 (figure 1, right). The third step was to estimate two distinct exponents: an exponent α describing
243 how the average proportion grows with bin size and an exponent f describing how Shannon's [49]
244 entropy of bin proportion changes with bin size. The later steps were all iterations of this third step,
245 calculating a mass μ by applying an exponent q to bin proportions and using $\mu(q)$ to weight
246 proportions selectively according to proportion size. q greater than 1 weights larger-proportion bins,
247 and q lesser than 1 weights smaller-proportion bins (figure 2, left). q -based mass $\mu(q)$ -
248 weighting generalizes single α and f estimates from earlier steps into continual $\alpha(q)$ and
249 $f(q)$ —with $\alpha(q)$ describing how mass-weighted average bin proportion changes with bin size
250 and $f(q)$ describing how Shannon's entropy of bin masses $\mu(q)$ with bin size (figure 2, top
251 right).

252 Multifractal analysis quantifies heterogeneity as variety in $\alpha(q)$ and $f(q)$. The ordered
253 pairs ($\alpha(q)$, $f(q)$) constitutes the multifractal spectrum (figure 2, bottom right). We included
254 $\alpha(q)$ and $f(q)$ only when mass-weighted proportions were defined and when both of their
255 corresponding relationships to bin size correlated at $r > 0.995$ on logarithmic plots, for bin sizes
256 4, 8, 12, ..., $N/4$ and $-300 < q < 300$, where N is the number of samples in the series.

257 2.4.2. Surrogate comparison

258 Multifractal spectrum width is sensitive to nonlinear interactions across timescales as well as to linear
259 temporal structure. Hence, evidence of nonlinear interactions across timescales requires comparing the
260 original series' multifractal spectrum width W_{MF} to those computed for a sample of linear surrogate
261 series (figure 3). These linear surrogate series retain the same value as the original series but in a
262 different order that preserves the original series' linear autocorrelation but destroys original sequence
263 (i.e., iterative amplitude-adjusted Fourier-transform or IAAFT [29,50]; figures 4a to d). Comparing
264 W_{MF} of the original and surrogate series (figures 4e, f) produces a t -statistic t_{MF} , a measure of nonlinear
265 interactions across timescales that is standardized across series with differing linear temporal structure.
266 t_{MF} was computed for each original series.

267 2.5. Mixed-effects modeling

268 The data were submitted to two kinds of mixed-effect modeling, namely, logistic and Poisson, using
269 the 'glmer' function in R package *lme4* [51]. The subsequent subsections provide details of the
270 predictors and the models.

271 2.5.1. Effects of time

272 The effects of time were modeled using trial number within a block ($\text{trial}_{\text{block}}$), block number (block),
273 and interactions of trial number with block number ($\text{trial}_{\text{block}} \times \text{block}$), and trial number across blocks
274 ($\text{trial}_{\text{exp}}$). The model used orthogonal polynomials of each of both predictors up to 3rd order (i.e., cubic).

275 The primary reason for the polynomial terms was to reveal how and when the perturbation of wearing
276 clogs might manifest and consequently subside. Cubic polynomials allow the simplest way to
277 accomplish this goal by allowing a peak and a flat, quiescent portion as part of the same continuous
278 function. Additionally, unlike the quadratic function, the cubic peak is not required to be symmetric
279 and occurring at the midpoint. Hence, the cubic function controls for any nonlinearities in the normal
280 course of trials without such perturbations. Whereas block is a linear effect across blocks, including
281 $\text{trial}_{\text{exp}}$ in the model allowed controlling for nonlinear change across blocks.

282 There are two important features to note about the treatment of time in this model. First, the
283 Poisson model encoded both classifications of trial numbers (i.e., $\text{trial}_{\text{exp}}$ and $\text{trial}_{\text{block}}$) in terms of an
284 orthogonal 3rd-order polynomial, using the function ‘poly’ in R , to include the linear, quadratic and
285 cubic growth of trial number with all three terms phase-shifted so as to be uncorrelated with one
286 another. Second, the lack of model convergence limited the appearance of block, such that block was
287 neither treated nonlinearly nor in interaction with perturbation. Attempting to include the orthogonal
288 3rd-order polynomials for block number led to a failure of model convergence, so block only appeared
289 as a linear term. The model also failed to converge when it included interactions of block and
290 perturbation because these two terms were strongly collinear.

291 2.5.2. Effects of manipulations

292 Manipulations included quiet-eye instruction, QE (QE = 1 for receiving QE instruction, and QE = 0 for
293 not receiving QE instruction) and perturbation of eye height, perturbation (perturbation = 1 for all ten
294 trials during the block when participants wore the clogs, and perturbation = 0 for all other trials).

295 2.5.3. Sway: Linear predictors

296 The accelerometer data encodes accelerations of the torso along x -, y -, and z -dimensions. Any single
297 posture corresponds to a specific point in this 3D space. Hence, sway corresponds to the deviation of
298 this 3D acceleration around the mean posture. The present analyses encoded this variability as MSD_{acc}
299 or RMS_{acc} , depending on which predictor supported a convergent model, along with mean and standard
300 deviation of 3D displacement ($Mean_{\text{disp}}$ and SD_{disp} , respectively).

301 2.5.4. Models

302 A mixed-effects logistic regression modeled the odds of a successful putt (i.e., a ‘make,’ vs. a ‘miss’),
303 which included all putts ending with the golf ball coming to rest within the orange dot on the floor or
304 within 2 feet directly beyond the hole. Predictors included QE \times perturbation (as well as lower-order
305 component main effects QE and perturbation; e.g., [52]) to test Hypotheses-1a and -1b, and W_{MF} to test
306 Hypothesis-1c, as well as block and RMS_{acc} to control for the effects of practice and linear structure of
307 postural sway. Additional predictors did not significantly improve model fit and hence were omitted. A
308 mixed-effects linear regression modeled the radial error (in inches) of all misses, with makes as defined
309 above treated as having a zero radial error. Predictors for this second model included QE \times perturbation
310 \times $\text{trial}_{\text{block}}$ \times t_{MF} to test Hypothesis-2 by specifically addressing trials within each block, QE \times $\text{trial}_{\text{block}}$
311 and QE \times $\text{trial}_{\text{exp}}$ to control for differences in learning due to QE across a linear progression of block; a
312 nonlinear function of trials, QE \times perturbation \times W_{MF} \times t_{MF} , to control for the effects of multifractal
313 spectral width and its relationship with t_{MF} ; and MSD_{acc} \times $Mean_{\text{disp}}$ \times SD_{disp} to control for the effects of
314 linear description of sway.

315

316 3. Results

317 **3.1. Testing hypothesis-1: Odds of a make increased with QE, decreased with the perturbation of**
318 **eye height, with a more negative effect of the perturbation for the QE group**

319 A mixed-effect logistic regression model of successful putting (i.e., the ‘makes’) returned the
320 coefficients reported in Table 2. Predictors for t_{MF} and for interactions of QE with all terms other than
321 perturbation failed to improve model fit significantly and so were omitted from the final model. This
322 model failed to return an effect of QE ($b = 0.20$, $s.e.m. = 0.30$, $p = 0.51$), failing to support Hypothesis-
323 1a. The perturbation failed to show a significant effect of the perturbation alone ($b = -0.22$, $s.e.m. =$
324 0.43 , $p = 0.60$). However, there was a moderately significant negative effect of QE \times perturbation ($b =$
325 -1.16 , $s.e.m. = 0.65$, $p = 0.07$), suggesting that the participants instructed to use the QE strategy had
326 marginally worse odds of a successful putt in the face of the perturbation, that is, 0.30 of the odds of
327 non-QE participants in the face of the perturbation. This result gave only marginal support to
328 Hypothesis-1b.

329 There were also effects of experience and trial-by-trial sway. Block had a positive effect ($b =$
330 0.24 , $s.e.m. = 0.13$, $p = 0.06$), suggesting that odds of a successful putt was 1.27 times higher with each
331 successive block of trials. RMS_{acc} showed a positive effect ($b = 4.05$, $s.e.m. = 1.34$, $p < 0.0001$), and
332 total multifractality W_{MF} showed a negative effect ($b = -7.25$, $s.e.m. = 2.94$, $p < 0.0001$), suggesting that
333 successful putts were more likely with more variable and less multifractal accelerations of sway. This
334 latter result supported Hypothesis-1c.

335 3.2. Poisson modeling of radial error

336 A Poisson model of trial-by-trial radial error (i.e., the Euclidean distance of the ball’s final resting
337 position from the hole’s center [in feet]) returned the coefficients in Supplementary Table S1.
338 Supplementary Table S1 describes the coefficients for the effects of time, perturbation of eye height,
339 and of interactions between the two. Table 3 describes the coefficients for trial-by-trial accelerations in
340 postural sway, including linear descriptors ($MSD_{acc} \times Mean_{disp} \times SD_{disp}$) and nonlinear descriptors
341 ($MSD_{acc} \times Mean_{disp} \times SD_{disp}$, and QE \times perturbation \times trial_{block} \times t_{MF}). The full model generated
342 predictions that correlated with the observed trial-by-trial radial error, $r = 0.54$. Figure 5 plots the
343 observed trial-by-trial radial error and trial-by-trial model predictions on the same axis for four
344 participants each from the technical and QE-trained groups, respectively. We detail the effects of time
345 and perturbation in the following sections. For brevity, we will only refer to significant coefficients in
346 text.

347 3.2.1. Non multifractal effects on radial error

348 This section details the non-multifractal effects described in Supplementary Table S1 and effects
349 preliminary to the test of Hypothesis-2 in Table 3. Radial error grew linearly and quadratically across
350 trials but showed negative cubic variation with trial—both for trials across the experiment and within a
351 block (figure 5a). The QE strategy prompted steady decay of error across the experiment, canceling out
352 the linear and cubic pattern of error over trials within a block (figure 5b). The perturbation of eye
353 height elicited no main effect but increased the growth of error over trials within a block, with the QE
354 instruction accentuating this nonlinear growth of error later in the block. The higher-order interaction
355 $MSD_{acc} \times Mean_{disp} \times SD_{disp}$ and its main component effects and lower-order interactions showed mostly
356 negative effects (only two of the two-way interactions showed positive coefficients), consonant with
357 the positive effect for RMS_{acc} predicting greater accuracy in the logistic model (Section 3.1.).

358 3.2.2. Total multifractality of sway, W_{MF} , increased radial error, but multifractality due to 359 nonlinearity, t_{MF} , attenuated radial error at the beginning and the end of blocks

360 Before considering what portion of multifractality is attributable to linear or nonlinear structure, the
361 total width W_{MF} of original series’ multifractal spectra is associated with greater radial error ($b =$
362 1.80×10^0 , $s.e.m. = 1.29 \times 10^{-1}$, $p < 0.0001$). This result is incidentally supportive of Hypothesis-1c,
363 resembling the negative effect on W_{MF} on the likelihood of a successful putt (Section 3.2.1.). We did
364 not find any main effect of multifractality due to nonlinearity, t_{MF} , nor any interaction of $W_{MF} \times t_{MF}$.

365 However, the negative effect for $t_{MF} \times \text{trial}_{\text{block}}$ (Quadratic; $b = -8.93 \times 10^{-2}$, $s.e.m. = 1.15 \times 10^{-2}$, $p <$
366 0.0001) indicated an association between multifractality due to nonlinearity with a significant reduction
367 of the positive quadratic profile for radial error over trials within a block (i.e., $\text{trial}_{\text{block}}$ (Quadratic);
368 Section 3.2.1.).

369 3.2.3. Testing Hypothesis-2a: Perturbation of eye height increased radial error only in 370 combination with QE training, with greater multifractality due to nonlinearity t_{MF} , or with both

371 Much like the effect of trials within a block (Section 3.2.3.), the multifractality of sway
372 promoted the tendency of QE to increase perturbation-related radial error. An important point to recall
373 at this point is that, unlike in the logistic regression (Section 3.1.), neither did we find a main effect of
374 perturbation nor an interaction effect of $QE \times \text{perturbation}$. Perturbation showed a significant effect
375 through its interactions with other factors, the earliest being its interaction with $\text{trial}_{\text{block}}$ (Section 3.2.3.);
376 the interaction of $\text{perturbation} \times W_{MF}$ ($b = -1.50 \times 10^0$, $s.e.m. = 3.12 \times 10^{-1}$, $p < 0.0001$) canceled out the
377 positive effect of W_{MF} on error (Section 3.2.2), suggesting that the perturbation reduced the error-
378 eliciting effect of W_{MF} . The interaction $QE \times W_{MF}$ reduced error by roughly half ($b = -1.04 \times 10^0$, $s.e.m.$
379 $= 2.03 \times 10^{-1}$, $p < 0.0001$), suggesting that QE promoted greater accuracy but only in the absence of
380 perturbation. However, the interaction with multifractality indicates that QE increased the error
381 attributable to the perturbation. For instance, the $QE \times \text{perturbation} \times W_{MF}$ ($b = 2.12 \times 10^0$, $s.e.m. =$
382 4.91×10^{-1} , $p < 0.0001$) reverses the apparently beneficial error-reducing interaction effect of
383 $\text{perturbation} \times W_{MF}$. Hence, QE reinstated the error due to total multifractality (Section 3.2.2), and it did
384 so only in the face of the perturbation.

385 Above and beyond the effects of W_{MF} in moderating the interaction effect of $QE \times \text{perturbation}$,
386 t_{MF} promoted the error-increasing effect of perturbation. Specifically, besides the moderating effect of
387 QE on the effect of $\text{perturbation} \times W_{MF}$, perturbation elicited even greater radial error with increases in
388 t_{MF} ($\text{perturbation} \times W_{MF} \times t_{MF}$: $b = 1.54 \times 10^{-1}$, $s.e.m. = 3.15 \times 10^{-2}$, $p < 0.0001$). QE did reverse this effect
389 ($QE \times \text{perturbation} \times W_{MF} \times t_{MF}$: $b = -2.21 \times 10^{-1}$, $s.e.m. = 4.97 \times 10^{-2}$, $p < 0.0001$). However, QE also
390 increased error in combination with perturbation and t_{MF} even in the absence of any moderating effect
391 of W_{MF} ($QE \times \text{perturbation} \times t_{MF}$: $b = 8.62 \times 10^{-3}$, $s.e.m. = 4.18 \times 10^{-3}$, $p < 0.05$). This result supports
392 Hypothesis-2a.

393 3.2.4. Testing Hypothesis-2b: the QE group with greater multifractal nonlinearity in sway 394 showed a greater initial but progressively slower increase in radial error due to the perturbation 395 of eye height

396 The interactions of $QE \times \text{trial}_{\text{block}}$ with multifractal nonlinearity followed a similar but weaker pattern as
397 the interactions of $QE \times \text{trial}_{\text{block}}$ with perturbation. Much like the perturbation of eye height, the
398 interaction of QE with t_{MF} accentuated the radial error late in a block of ten trials ($QE \times \text{trial}_{\text{block}} \times t_{MF}$):
399 it weakened the linear term ($b = -3.73 \times 10^{-2}$, $s.e.m. = 1.46 \times 10^{-2}$, $p < 0.05$), contributing to a negative-
400 quadratic peak in the middle of the block ($b = -1.85 \times 10^{-1}$, $s.e.m. = 1.66 \times 10^{-2}$, $p < 0.0001$) and a
401 positive cubic increase at the end of the block ($b = 1.13 \times 10^{-1}$, $s.e.m. = 1.73 \times 10^{-2}$, $p < 0.0001$). Hence,
402 t_{MF} and perturbation showed independent effects on the progression of error across trials in block.

403 However, the similarity of these effects independently on error across trials within a block did
404 not extend to their interaction. Indeed, most relevant to testing Hypothesis-2b is that the interaction of
405 perturbation and t_{MF} served to speed up the growth of error, and QE slowed it down across the block.
406 Coefficients for $\text{perturbation} \times \text{trial}_{\text{block}} \times t_{MF}$ indicated that in the face of perturbation, multifractal
407 nonlinearity removed the observed peak in error due to earlier negative quadratic effects ($b = 3.83 \times 10^{-1}$,
408 $s.e.m. = 5.28 \times 10^{-2}$, $p < 0.0001$) and increased error across the linear ($b = 4.07 \times 10^{-1}$, $s.e.m. = 5.20 \times 10^{-2}$,
409 $p < 0.0001$) and cubic terms ($b = 5.39 \times 10^{-1}$, $s.e.m. = 4.45 \times 10^{-2}$, $p < 0.0001$). On the other hand,
410 coefficients for $\text{perturbation} \times \text{trial}_{\text{block}} \times t_{MF}$ indicated that, in the face of perturbation for the QE

411 participants, the linear increase was shallower ($b = -2.71 \times 10^{-1}$, $s.e.m. = 6.11 \times 10^{-2}$, $p < 0.0001$), the
412 positive quadratic growth further removed the observed peak in error ($b = 4.24 \times 10^{-1}$, $s.e.m. = 6.14 \times 10^{-$
413 2 , $p < 0.0001$), and the cubic term was negative ($b = -7.90 \times 10^{-1}$, $s.e.m. = 5.79 \times 10^{-2}$, $p < 0.0001$).

414 The outcome of these differences distinguishes how multifractal nonlinearity predicted changes
415 in adaptive responses to the perturbation of eye height. Greater t_{MF} exhibited less adaptive response to
416 the perturbation in the QE case, and less-but-still-nonzero t_{MF} predicted a more gradual appearance of
417 error and then a subsequent decay of this error. Greater t_{MF} led the QE group to exhibit initially greater
418 error and a smaller change in error across trials, and lower levels of t_{MF} led the QE group to show a
419 delayed growth in error and subsequent decay in error (figure 5b), suggesting adaptation. It is evident
420 in figure 5b that although the error in the high- t_{MF} cases with the perturbation was smaller than the
421 delayed increase in the low- t_{MF} cases with the perturbation, the high- t_{MF} cases showed more sustained
422 error and no adaptation to the perturbation relative to high- t_{MF} cases without the perturbation. This
423 result supports Hypothesis-2b.

424

425 4. Discussion

426 We tested two hypotheses about the role of QE training in golf putting. Our first hypothesis was that,
427 for the dichotomous outcome of putting performance ('make' versus 'miss'), QE training would
428 improve the odds of a make, clogs (i.e., wooden blocks added to the feet) would reduce the odds of a
429 make in general and even more for QE trained participants, and multifractal spectrum width of sway,
430 W_{MF} , would reduce the odds of a make as well. Our second hypothesis was that, for the continuous
431 outcome of putting performance (radial error), the response of QE-trained participants to a perturbation
432 of eye height would depend on the multifractality attributable to nonlinear interactions across scales,
433 t_{MF} . We predicted that lower estimates of t_{MF} would coincide with a more adaptive response, that is,
434 with slower growth of radial error and then with subsequent decay of radial error over trials. Results
435 supported the first hypothesis only partially: we found no main effect of QE training, only a marginally
436 significant disadvantage due to QE with the clogs, and a significant main effect associating greater
437 multifractality W_{MF} with greater error. On the other hand, results strongly supported the second
438 hypothesis that accuracy with QE depends on the multifractal nonlinearity in postural sway. In fact, the
439 model testing second hypothesis in the Poisson model replicated the null effects for QE and its
440 interaction with clogs from the logistic regression. This second model not only replicated the
441 multifractal effect from the logistic model, but it also elaborated on a strong relationship of QE-trained
442 performance with multifractal aspects of sway.

443 These results offer two major insights for research into perception-action in visual aiming tasks.
444 First, it shows that QE training has clear roots in postural sway, vindicating and elaborating earlier
445 proposals that quiet eye is not simply about stabilizing the eyes [53,54]. Indeed, it has long been known
446 that simply stabilizing the retinal position compromises the persistence of a visual image [55–57]. And
447 contrary to a colloquial understanding of 'quiet eye,' more recent work confirms that those small
448 movements within gaze fixations called 'microsaccades' might prevent fading [58] and support visual
449 attentional processes [59]. We might draw a comparison between the 'quiet' suggested by 'quiet eye'
450 and that indicated by 'quiet standing.' Both 'quietudes' provide the needed instruction to a participant
451 or athlete to 'please move as little as possible,' but below the polite clarity for verbal instruction, both
452 'quietudes' depend upon a rich texture of fluctuations. Fluctuations throughout the body support
453 exploration during quiet standing [60], and quiet eye is no less an exception than quiet standing has
454 been [5–7]. The value of QE is not necessarily a net reduction of fluctuations but rather a way to train
455 athletes to orient their movements towards the optic flow generated by postural sway. Whereas the
456 postural-kinematic hypothesis had previously only referred to longer movement duration [22], we now
457 report that movements are not merely slower but evince a sort of nonlinearity constricting the degrees

458 of freedom across scales of the movement-system hierarchy. In one of the subsections below, we
459 discuss how this orienting towards optic flow might coordinate with mechanisms invoked by the more
460 elaborate visual hypothesis (e.g., [61,62]).

461 The second insight is that the multifractality in sway is an essential aspect of postural
462 adaptations contributing to visually-guided action. It was already known that damping out postural
463 perturbations could occasion a ratcheting down of multifractality in postural sway [42]. The present
464 results indicate that this reduction of multifractality is associated with more accurate responses in
465 visually-guided aiming. Note that this reduced multifractality remains different from corresponding
466 surrogates. Hence, nonlinear interactions across scales can constrict movement variability, making
467 reduced multifractality more adaptive [63–65]. This distinction is a departure from any shorthand
468 presumption that ‘more multifractal is better’ supported by early studies of heart rate variability (HRV)
469 [66]. Indeed, this departure reflects a growing clarity about the value of multifractal fluctuations for
470 predicting and explaining outcomes in perception-action.

471 **4.1. Clarifying the value of multifractal fluctuations for perceiving-acting systems**

472 The shorthand ‘more multifractal is better’ presumptions from early studies on HRV [66] have always
473 underestimated the more elaborate systematicity in the physiology of the human movement system
474 with upright posture. Early work proposed that a ‘loss of [fractal or multifractal] complexity’ would
475 occasion or coincide with aging or with pathology [67]. Indeed, early evidence showed that less healthy
476 outcomes co-occurred with more-than-fractal levels of temporal correlation [68]. However, the task has
477 always been known to influence multifractality in upright posture: multifractality of quiet standing can
478 reduce with pathology [69,70], but multifractality in gait can increase with pathology [71], as well as
479 with walking speeds faster or slower than self-selected comfortable speed [72]. Over the lifespan, the
480 maturation of gait involves loss of multifractality, beginning with the stabilization of toddler gait into
481 young adult gait and continuing into older age [73]—this lifelong progression actually is at odds with
482 more recent evidence that multifractality in HRV remains stable with healthy aging [74].

483 Militating strongly against simple ‘‘more multifractal is better’ kind of presumptions are a
484 variety of distinctions that are at once obvious but need recognition if we want to identify for what,
485 after all, multifractality is good. These distinct points include, for instance, that hearts are not upright
486 bipedal bodies, that age is not itself a pathology, and that walking is not standing. In the promise of
487 fractal/multifractal analysis, the discourse about ‘complexity,’ ‘dynamical stability,’ and the role of
488 fractality/multifractality in ‘optimality’ [75–77] can make no claims on all definitions of optimality and
489 so has not been responsible for acknowledging that some multifractality is not optimal [78]. Indeed,
490 studies in behavioral sciences have always justified the capacity of fractal variety to offer new insights
491 [79,80]. Any notion of a privileged level or direction of fractal variety [81] might be best understood as
492 a reaction to critical perspectives suggesting that the discourse in behavioral sciences should have
493 much less or perhaps zero fractality or multifractality in [82]. Measurements may wax and wane in
494 their multifractal structure. The only case in which we think more multifractality is always better than
495 less is not in all of our measurements but rather in the scientific discourse on the coordination of
496 movement systems engaged in perception-action.

497 **4.2. The value of multifractal fluctuation is specific to task constraints**

498 What is coming into focus is that the value of nonlinearity-driven multifractality depends on task
499 context. More of it is adaptive when tasks invite exploration and anticipation. Less of it is adaptive
500 when the task needs restraint for precise consistency and constraint. As an example of the former,
501 accumulating information as a reader can require keeping an open mind to follow the discourse
502 wherever it may lead, and more multifractality t_{MF} in the pacing of word-by-word reading supports
503 more fluent reading if the story has a twist in its plot [34]. Similarly, more multifractality W_{MF} in circle-

504 tracing behaviors make for poor tracing but is associated with better performance on
505 neurophysiological tests of flexibility with rule switching [83]. The freewheeling benefits of more
506 multifractality can also compete alongside the precision-, repetition-promoting benefit of less
507 multifractality. In a Fitts task, greater multifractality t_{MF} in hand and head movements predicted less
508 stable contact with the targets, but it became a predictor of more stable contact when participants had
509 the diffuse warning that they might be asked to close their eyes and continue the task [35]. Hence, what
510 would have upset task performance became a resource for exploring the task space for the eventuality
511 of a major loss of sensory information. And now, we see that a less multifractal postural system can be
512 more stable (e.g., smaller t_{MF} in [42]; and smaller W_{MF} in [38,39]) and, as present findings indicate,
513 more accurate in how it orients behaviors to a visually-guided aiming task.

514 Some of the task-specificity remains unclear, e.g., manual wielding of an object is more
515 accurate with greater multifractality in postural sway, but hefting an object to perceive heaviness
516 compared to a reference object is more accurate with less multifractality [33]. This difference could
517 have to do with the qualities of length and heaviness, but the simpler interpretation could be that
518 intending to perceive directs attention away from the center of pressure and intending to perceive
519 heaviness directs attention inwards toward the center of pressure, lending length perceptions and
520 heaviness perception to more and less postural instability. This issue of directing attention is again a
521 reason we will need to consider how to link this postural-kinematic work on QE back with the visual-
522 hypothesis work on QE.

523 The fact that reduced multifractality can be beneficial offers a unique distinction between
524 possibly two related but distinct interpretations of multifractal results: one as a biomarker of pathology
525 and the other as adaptation. Indeed, the loss of multifractality can be a biomarker for diagnosing a
526 pathology [84]. The physiological wisdom informing this usage is that the heart has to be, in effect,
527 poised to absorb and rebound from perturbations, avoiding fragility due to insult at any single
528 characteristic scale (e.g., [67]). Implicit in this wisdom is that, within the healthy body, the heart's task
529 is deeply anticipatory and exploratory, much like those task settings in which whole organisms benefit
530 from more multifractality. However, hastily concluding that loss of multifractality is just symptomatic
531 of postural systems with sensory deficits is premature and even casts too gloomy a connotation over
532 reduced multifractality [70]. The present results with a perturbation of eye height indicate that reduced
533 multifractality could be an adaptive response preserving stability and promoting task performance in
534 the face of abrupt change in the organism's mechanical relationship with the base of support. Hence,
535 far from being the signature of diseased posture, reductions in multifractality that maintain a significant
536 t_{MF} suggest nonlinear interactions across scales acting context-sensitively to restrain motoric degrees of
537 freedom so as to achieve the task goal.

538 Different task constraints prompt different modes of movement variability, and if multifractal
539 depictions of the body seem to physicalize or mechanize the movement system, they do so only in the
540 way that nonlinear-dynamical physical mechanisms might better explain context-sensitive behavior.
541 Surely, good-faith pursuit into nonlinear-dynamical complexity should avoid any catch-all
542 simplification (e.g., 'more is better, less is worse') and keep a keen eye on what sort of multifractality
543 is good for what end. Nonlinear dynamics has also profited from consideration of its measures in
544 different reference frames before: experimentally breaking apart the coincidence of relative-phase in
545 spatial frames from relative-phase in muscular frames revealed novel, more generic structure in the
546 Haken-Kelso-Bunz [85] law for multi-limb coordination [86]. Finally, considering the task frames for
547 our interpretation of multifractal estimates may be equally crucial for generalizing predictions for a
548 multifractal foundation for perception-action. Low multifractality in the heart signals a bad prognosis
549 to the clinician, but low multifractality in posture supports good task performance in visually-guided
550 aiming, particularly for an organism beset by malevolent experimenters uprooting their optic flow.

551 4.3. The value of multifractality is also specific to different tissues of the body.

552 Just as different tissues of the same body can differently support the same task, we expect that the
553 multifractal structure that spans across these different tissues supports overarching integrity to the
554 organismal-level behavior we measure in the laboratory [87]. Here, a comparison of present results
555 with similar work using a relatively less-mechanical, visual perturbation [36], may tee up
556 considerations for future work that could elaborate an explanation of QE by integrating the postural-
557 kinematic hypothesis with the visual hypothesis. The present results resemble those of Carver et al.
558 [36] who found that, without any training (i.e., neither QE nor technical), more multifractality in torso
559 sway led to greater error in a low-difficulty Fitts task (compare W_{MF} in Tables 1 and 3) led to smaller
560 error in the case of a visual perturbation (compare perturbation $\times W_{MF}$ in Table 3), that is,
561 multifractality in head sway became a stabilizing, accuracy-promoting asset to task performance when
562 the target was far out of reach. Future research could also probe the visual hypothesis for QE for
563 multifractal foundations, and based on Carver et al.'s [36] results, we predict that QE-trained
564 performance would improve with greater multifractality in head sway or eye movements. Monofractal
565 analysis of eye movements has successfully predicted visual attention to text [88], and such
566 fluctuations in quiet eye likely support an extension of the multifractal support for QE. The present
567 results do not include gaze data, but we hope it is informative, especially in support of the postural-
568 kinematic hypothesis, that the QE instructions enlist the nonlinear interactions across scales of the
569 movement system without drawing the athlete's attention to the body.

570 Ultimately, this task- and anatomically-specific portrait of multifractal fluctuations rests on the
571 even more generic facts that multifractal fluctuations spread across the body and that estimates of this
572 spread predict the accuracy of the perceptual outcomes [89,90]. Hence, rather than inventorying body
573 parts and task constraints prompting different values of multifractality, the longer view of investigating
574 visually-guided aiming must examine the flow of multifractal fluctuations. Carver et al. [36] found that
575 multifractality spread from hip to head, as well as from head and hip toward the throwing hand. Indeed,
576 in all these studies, the body appears to direct multifractal fluctuations towards those body parts most
577 clearly engaged in the focal tasks. We might expect similar results in golf putting. QE training may
578 promote this spread by reducing intentional movements of the head and eyes that might interfere with
579 upstream flows of multifractality. The direction of attention towards a distant target in the visual field
580 may depend on stabilizing the postural grasp of the surface underfoot and shunting multifractal
581 fluctuations toward the head and eyes. Indeed, executive functions directing attention to specific
582 features of the visual field benefits from a rich substrate of multifractal fluctuations in the head [91],
583 hand [92,93], and brain [94,95] that vary systematically with access to and mastery of the rules
584 supporting task performance. When we consider that eye movements show multifractality [96] and that
585 variability in the monofractal structure of eye movements can predict visual attention [88], we see early
586 glimpses of a multifractal bridge from the postural-kinematic support to the visual support for QE.

587 The novelty of multifractal modeling to behavioral sciences can make this promise of a
588 multifractal infrastructure supporting visually guided behaviors sound too ethereal and misty to be
589 practical. But we can root this proposal in the known physiology spanning the very same bones,
590 muscles, bones, joints, brain, and eyes whose coordination generates the winning golf putt.
591 Specifically, the connective tissues composing the fascia operate to coordinate context-sensitive
592 behavior across so wide a variety of scales (e.g. [97]) that multifractality is one of the few current
593 compelling frameworks for modeling how it might support perception-action [98,99]. Examining how
594 task constraints moderate a flow of multifractal fluctuations through the body could inform future
595 explanations for QE training and other visual aiming tasks.

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849 **Table 1.** Training instructions given to the quiet-eye and technical groups during the training phase.

quiet-eye (QE) training instructions	technical training instructions
assume your stance, and ensure your gaze is located on the back of the ball	take your stance with your legs shoulder-width apart
after setting up over the ball, fix your gaze on the hole	set your position so that your head is directly above the ball looking down
make no more than three fixations towards the hole	keep your clubhead square to the ball
your final fixation should be a quiet eye on the back of the ball. The onset of the quiet eye should occur before the stroke begins and last for 2 to 3s	allow your arms and shoulders to remain loose
ensure you direct gaze towards the clubhead during the putting stroke	the putting action should be pendulum-like, making sure that you accelerate through the ball
the quiet eye should remain on the green for 200 to 300ms after the club contacts the ball	after contact, follow through but keep your head still and facing down

850 Source: Adapted from [100].

851 **Table 2.** Logistic regression modeling of successful shots.

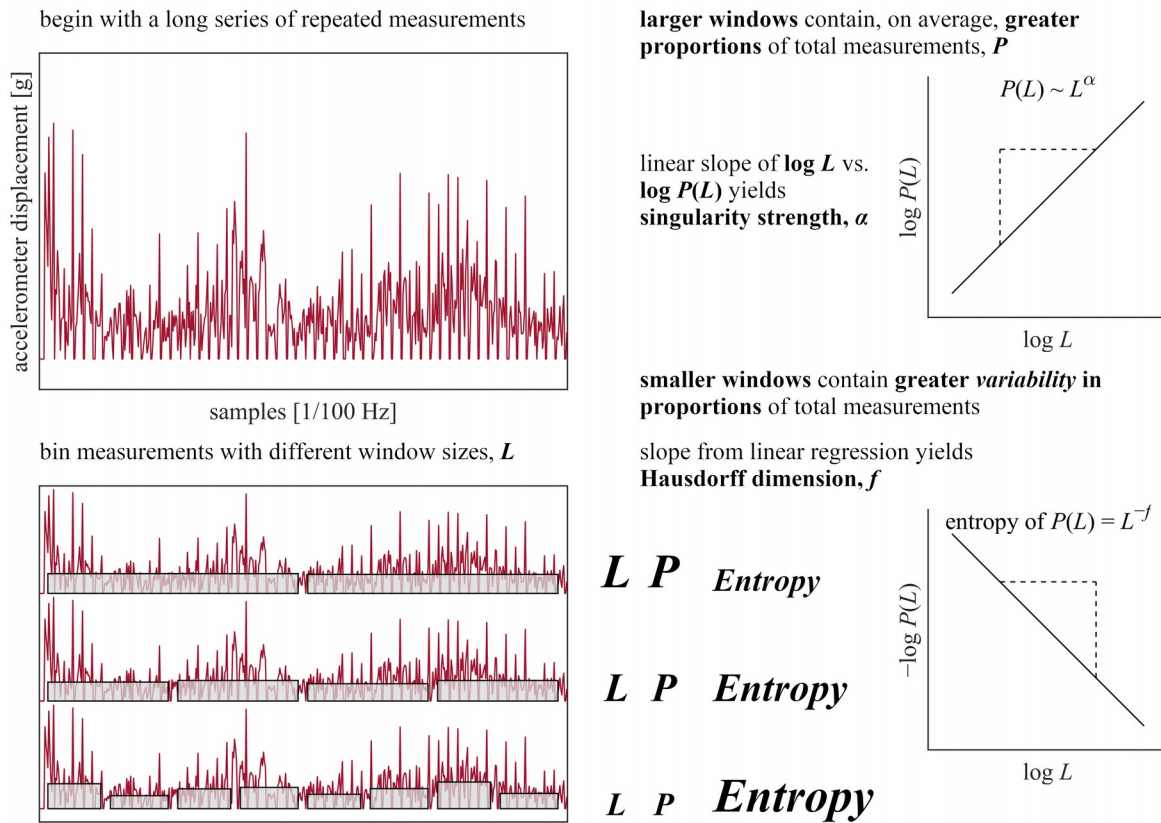
predictor	<i>b</i>	<i>s.e.m.</i>	<i>p</i>[*]
intercept	-2.85	0.48	< 0.0001
QE	0.20	0.30	0.51
perturbation	-0.22	0.43	0.60
QE × perturbation	-1.16	0.65	0.07
block	0.24	0.13	0.06
RMS_{Acc}	4.05	1.34	< 0.01
W_{MF}	-7.25	2.94	< 0.05

852 *boldfaced values indicate statistical significance at the alpha level of 0.05.

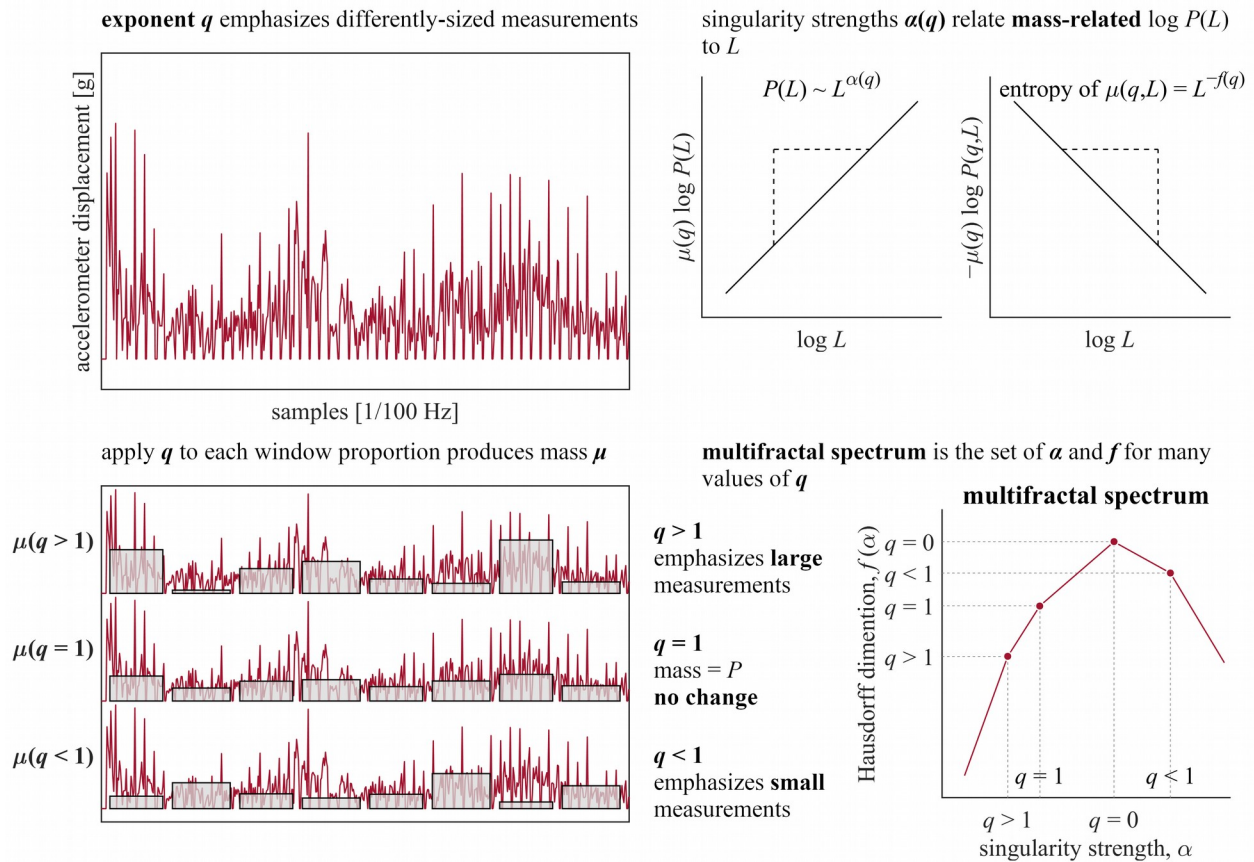
853 **Table 3.** Subset of Poisson regression model including main and interaction effects non-multifractal
 854 and multifractal measures of sway, perturbation of eye height, and time

predictor	<i>b</i>	<i>s.e.m.</i>	<i>p</i>[*]
<i>effects of non-multifractal measures of sway</i> (see Section 3.2.1.)			
<i>MSD</i> _{acc}	-8.80×10 ⁰	1.67×10 ⁻¹	< 0.0001
<i>Mean</i> _{disp}	-2.91×10 ²	1.33×10 ¹	< 0.0001
<i>SD</i> _{disp}	4.43×10 ⁰	1.64×10 ⁰	< 0.01
<i>MSD</i> _{acc} × <i>Mean</i> _{disp}	1.97×10 ⁴	1.00×10 ¹	< 0.0001
<i>MSD</i> _{acc} × <i>SD</i> _{disp}	-3.11×10 ³	1.03×10 ¹	< 0.0001
<i>Mean</i> _{disp} × <i>SD</i> _{disp}	2.56×10 ⁴	1.05×10 ¹	< 0.0001
<i>MSD</i> _{acc} × <i>Mean</i> _{disp} × <i>SD</i> _{disp}	-1.04×10 ⁵	1.08×10 ¹	< 0.0001
<i>effects of multifractal measures of sway and interactions with time</i> (see Section 3.2.2.)			
<i>W</i> _{MF}	1.80×10 ⁰	1.29×10 ⁻¹	< 0.0001
<i>t</i> _{MF}	2.21×10 ⁻⁴	1.05×10 ⁻³	0.83
<i>W</i> _{MF} × <i>t</i> _{MF}	-5.93×10 ⁻³	8.27×10 ⁻³	0.48
<i>t</i> _{MF} × trial _{block} (Linear)	1.36×10 ⁻²	9.82×10 ⁻³	0.17
<i>t</i> _{MF} × trial _{block} (Quadratic)	-8.93×10 ⁻²	1.15×10 ⁻²	< 0.0001
<i>t</i> _{MF} × trial _{block} (Cubic)	-1.08×10 ⁻²	1.14×10 ⁻²	0.34
<i>interactions of multifractal measures of sway with perturbation</i> (see Section 3.2.3.)			
QE × <i>W</i> _{MF}	-1.04×10 ⁻⁰	2.03×10 ⁻¹	< 0.0001
QE × <i>t</i> _{MF}	-1.85×10 ⁻³	1.30×10 ⁻³	0.16
QE × <i>W</i> _{MF} × <i>t</i> _{MF}	-4.59×10 ⁻³	1.09×10 ⁻²	0.67
perturbation × <i>W</i> _{MF}	-1.50×10 ⁰	3.12×10 ⁻¹	< 0.0001
perturbation × <i>t</i> _{MF}	4.57×10 ⁻³	2.85×10 ⁻³	0.11
perturbation × <i>W</i> _{MF} × <i>t</i> _{MF}	1.54×10 ⁻¹	3.15×10 ⁻²	< 0.0001
QE × perturbation × <i>W</i> _{MF}	2.12×10 ⁰	4.91×10 ⁻¹	< 0.0001
QE × perturbation × <i>t</i> _{MF}	8.62×10 ⁻³	4.18×10 ⁻³	< 0.05
QE × perturbation × <i>W</i> _{MF} × <i>t</i> _{MF}	-2.21×10 ⁻¹	4.97×10 ⁻²	< 0.0001
<i>interactions of multifractal measures with perturbation and time</i> (see Section 3.2.4.)			
QE × trial _{block} (Linear) × <i>t</i> _{MF}	-3.73×10 ⁻²	1.46×10 ⁻²	< 0.05
QE × trial _{block} (Quadratic) × <i>t</i> _{MF}	-1.85×10 ⁻¹	1.66×10 ⁻²	< 0.0001
QE × trial _{block} (Cubic) × <i>t</i> _{MF}	1.13×10 ⁻¹	1.73×10 ⁻²	< 0.0001
perturbation × trial _{block} (Linear) × <i>t</i> _{MF}	3.83×10 ⁻¹	5.28×10 ⁻²	< 0.0001
perturbation × trial _{block} (Quadratic) × <i>t</i> _{MF}	4.07×10 ⁻¹	5.20×10 ⁻²	< 0.0001
perturbation × trial _{block} (Cubic) × <i>t</i> _{MF}	5.39×10 ⁻¹	4.45×10 ⁻²	< 0.0001
QE × perturbation × trial _{block} (Linear) × <i>t</i> _{MF}	-2.71×10 ⁻¹	6.11×10 ⁻²	< 0.0001
QE × perturbation × trial _{block} (Quadratic) × <i>t</i> _{MF}	5.24×10 ⁻¹	6.14×10 ⁻²	< 0.0001
QE × perturbation × trial _{block} (Cubic) × <i>t</i> _{MF}	-7.90×10 ⁻¹	5.79×10 ⁻²	< 0.0001

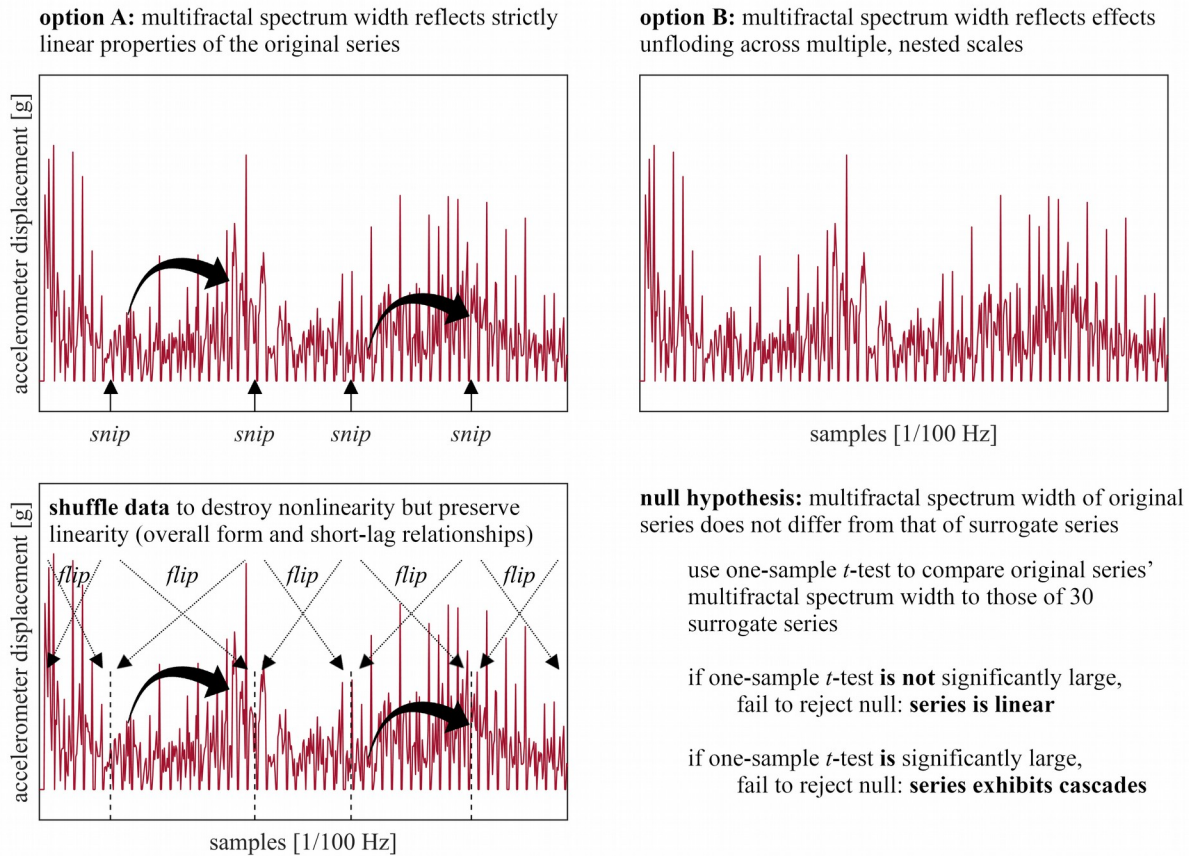
855 *boldfaced values indicate statistical significance at the alpha level of 0.05.



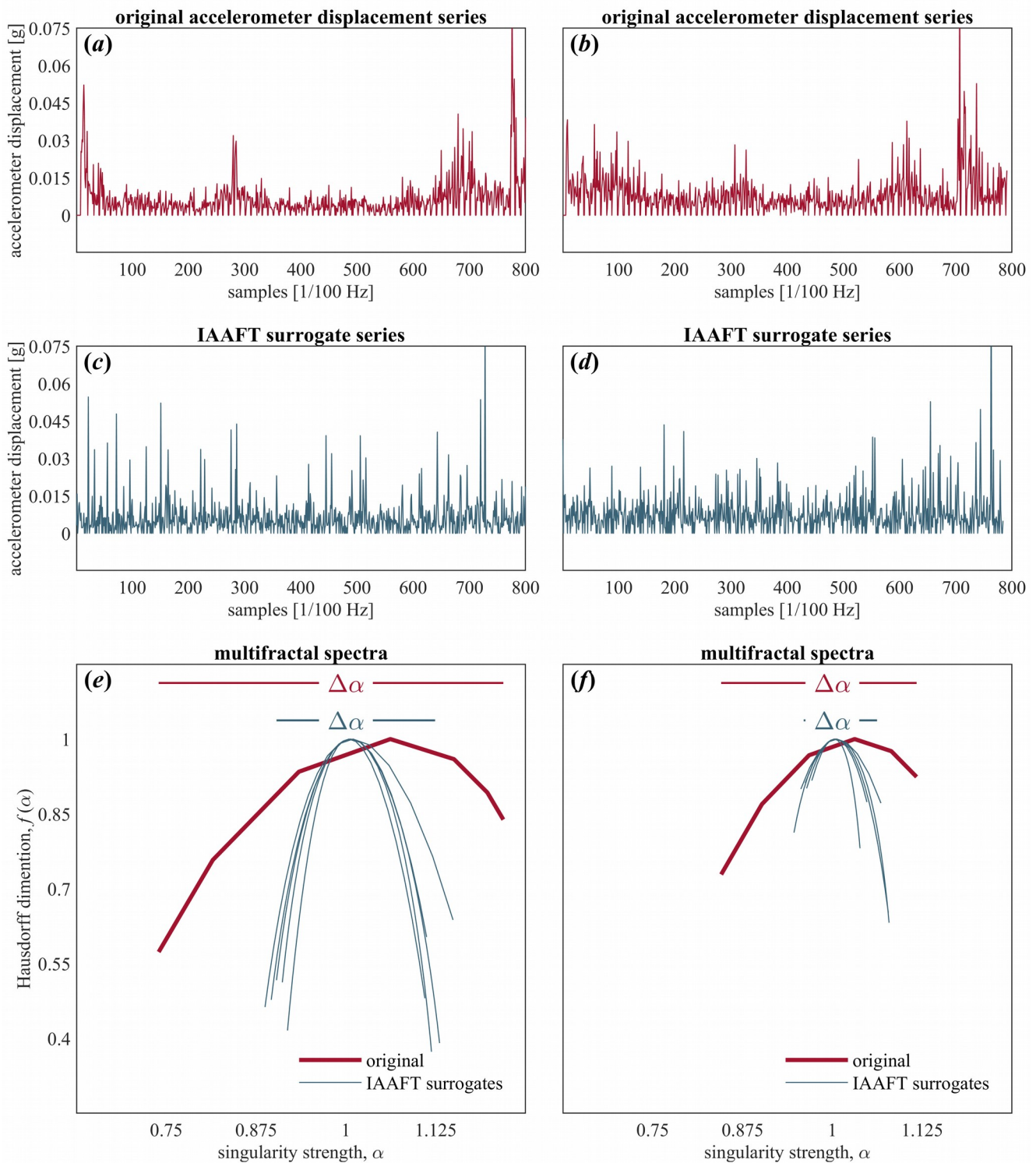
856 **Figure 1.** Schematic of Chhabra and Jensen's [48] method used to estimate multifractal spectrum
 857 $f(\alpha)$ (Part-1). See Section 2.4.1. for further details.



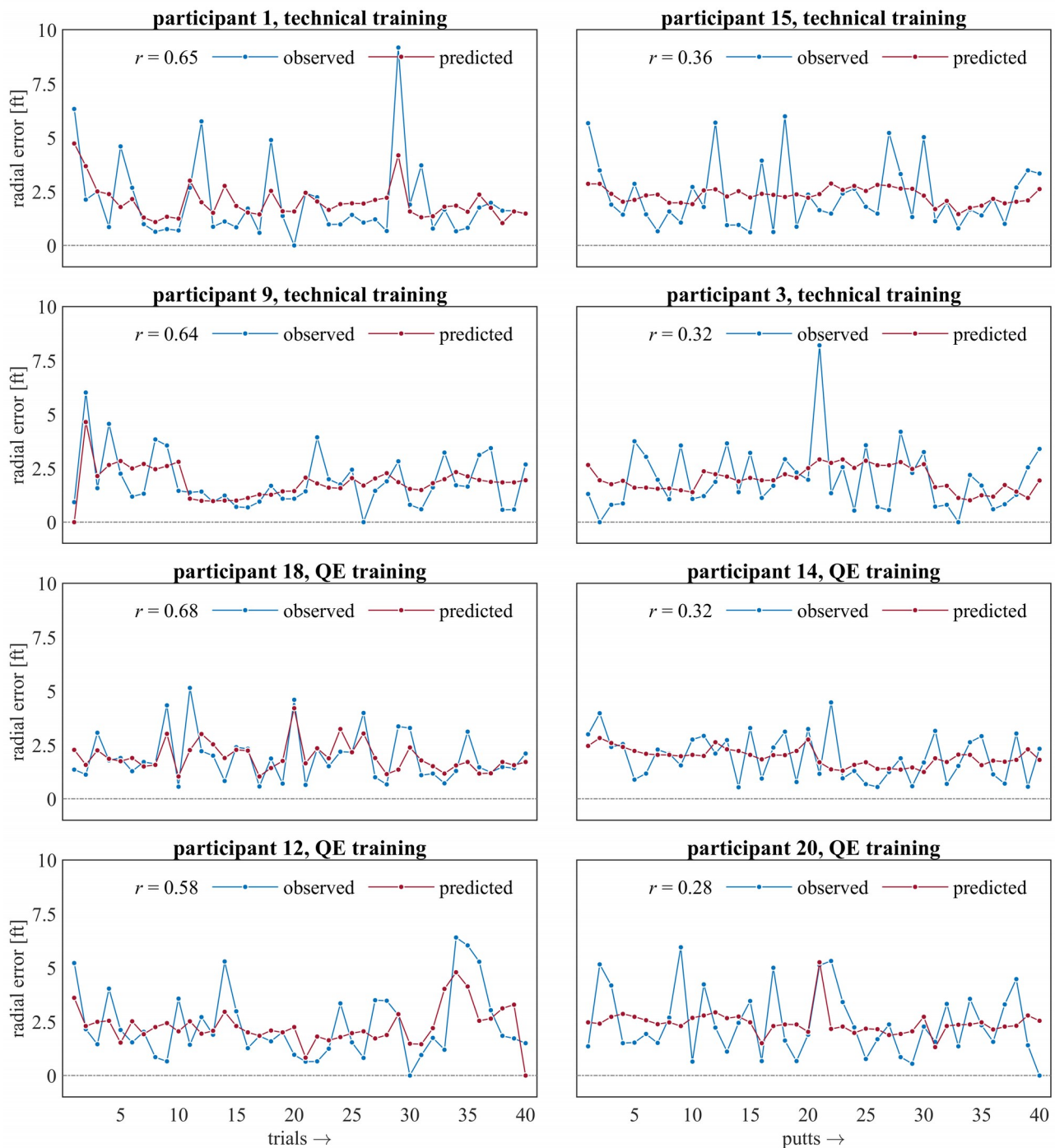
858 **Figure 2.** Schematic of Chhabra and Jensen's [48] method used to estimate multifractal spectrum
 859 $f(\alpha)$ (Part-2). See Section 2.4.1. for further details.



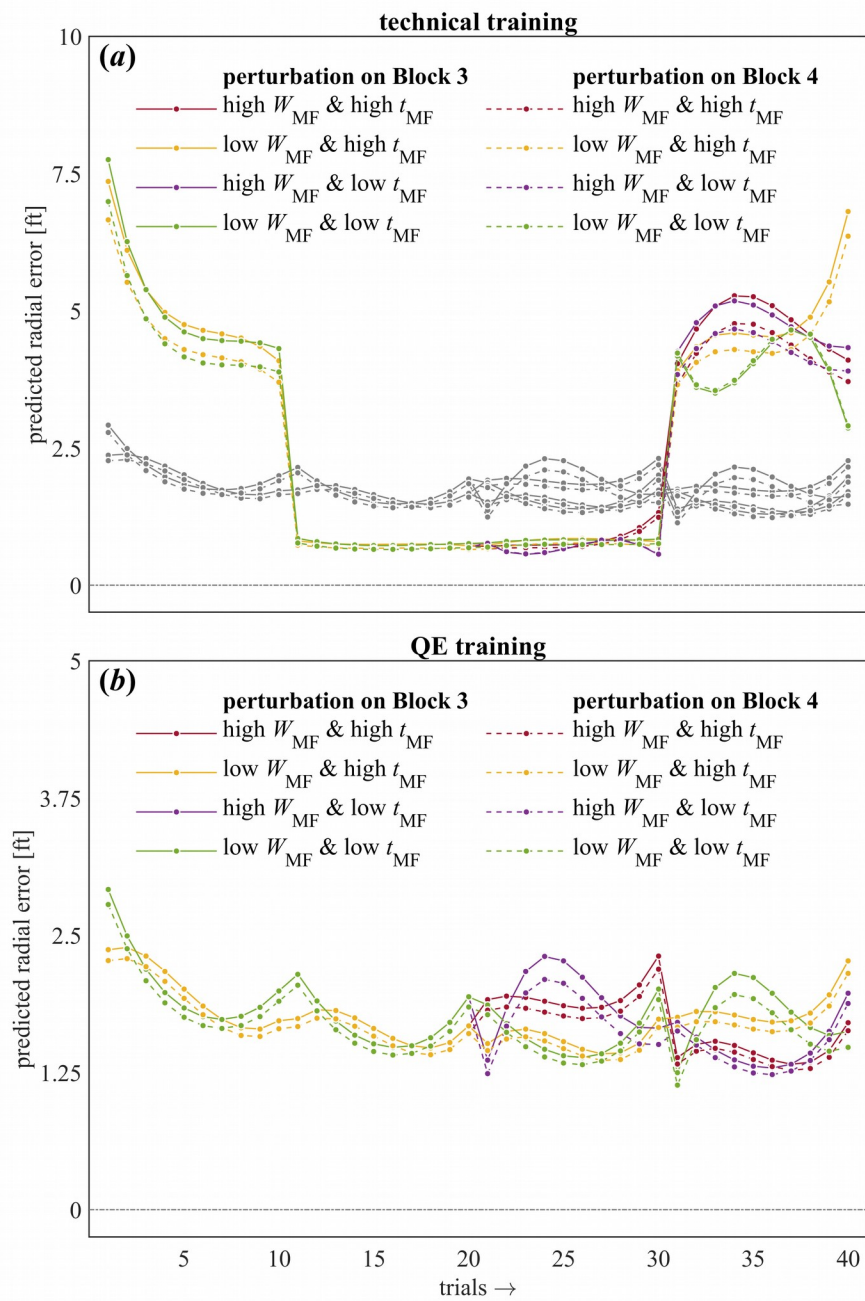
860 **Figure 3.** Surrogate analysis used to identify whether a series exhibits multifractality due to
861 nonlinearity (cascades). See Section 2.4.2. for further details.



862 **Figure 4.** Multifractal analysis. (a, b) Representative original accelerometer displacement series. (c, d)
863 Representative IAAFT surrogate series of the original accelerometer displacement series in a and b. (e,
864 f) Multifractal spectrum ($f(\alpha)$, α) of original series in a and b, and those of their five representative
865 IAAFT surrogates.



866 **Figure 5.** The observed trial-by-trial radial error and trial-by-trial model predictions on the same axis
867 for four participants each from the technical (top four panels) and QE-trained (bottom four panels)
868 groups, respectively. The representation includes two participants each with the highest (left panels)
869 and lowest (right panels) Pearson's correlation coefficient r indicating the strength of relationship
870 between the observed and predicted radial errors.



871 **Figure 6.** Trial-by-trial model predictions of radial error for the two participant groups. (a) Technical-
872 trained group. (b) QE-trained group.

873 **Supplementary Table S1.** Subset of Poisson regression model output including main and interaction
 874 effects of time and perturbation of eye height.

predictor	<i>b</i>	<i>s.e.m.</i>	<i>p</i>[*]
intercept	9.04×10 ⁰	3.29×10 ⁻¹	< 0.0001
trial _{exp} (Linear)	4.70×10 ¹	2.26×10 ⁰	< 0.0001
trial _{exp} (Quadratic)	3.08×10 ¹	1.20×10 ⁰	< 0.0001
trial _{exp} (Cubic)	-1.78×10 ⁰	1.54×10 ⁻¹	< 0.0001
trial _{block} (Linear)	2.17×10 ¹	9.34×10 ⁻¹	< 0.0001
trial _{block} (Quadratic)	9.60×10 ⁻¹	2.63×10 ⁻¹	< 0.001
trial _{block} (Cubic)	-1.62×10 ⁰	2.57×10 ⁻¹	< 0.0001
block	-1.41×10 ⁰	1.34×10 ⁻¹	< 0.0001
block × trial _{block} (Linear)	-1.36×10 ¹	5.83×10 ⁻¹	< 0.0001
block × trial _{block} (Quadratic)	-1.10×10 ⁰	1.10×10 ⁻¹	< 0.001
block × trial _{block} (Cubic)	6.40×10 ⁻¹	1.08×10 ⁻¹	< 0.0001
QE	-4.40×10 ⁰	4.81×10 ⁻¹	< 0.0001
QE × trial _{exp} (Linear)	-6.07×10 ¹	3.54×10 ⁰	< 0.0001
QE × trial _{exp} (Quadratic)	-3.21×10 ¹	1.86×10 ⁰	< 0.0001
QE × trial _{exp} (Cubic)	1.03×10 ⁰	2.18×10 ⁻¹	< 0.0001
QE × trial _{block} (Linear)	-2.41×10 ¹	1.41×10 ⁰	< 0.0001
QE × trial _{block} (Quadratic)	9.35×10 ⁻¹	3.79×10 ⁻¹	< 0.05
QE × trial _{block} (Cubic)	2.24×10 ⁰	3.54×10 ⁻¹	< 0.0001
QE × block	1.72×10 ⁰	1.96×10 ⁻¹	< 0.0001
QE × block × trial _{block} (Linear)	1.56×10 ¹	8.99×10 ⁻¹	< 0.0001
QE × block × trial _{block} (Quadratic)	1.31×10 ⁰	1.63×10 ⁻¹	< 0.0001
QE × block × trial _{block} (Cubic)	-5.57×10 ⁻¹	1.54×10 ⁻¹	< 0.01
Perturbation	3.90×10 ⁻²	3.01×10 ⁻¹	0.90
QE × perturbation	8.73×10 ⁻²	4.26×10 ⁻¹	0.84
perturbation × trial _{block} (Linear)	1.58×10 ⁰	3.02×10 ⁻¹	< 0.0001
perturbation × trial _{block} (Quadratic)	1.25×10 ⁰	3.08×10 ⁻¹	< 0.001
perturbation × trial _{block} (Cubic)	-2.42×10 ⁰	2.97×10 ⁻¹	< 0.0001
QE × perturbation × trial _{block} (Linear)	-1.16×10 ⁰	4.44×10 ⁻¹	< 0.01
QE × perturbation × trial _{block} (Quadratic)	-5.71×10 ⁰	4.43×10 ⁻¹	< 0.0001
QE × perturbation × trial _{block} (Cubic)	3.39×10 ⁰	4.31×10 ⁻¹	< 0.0001

875 *boldfaced values indicate statistical significance at the alpha level of 0.05.