Simulations of Lévy walk

Venkat Abhignan† and Sinduja Rajadurai⋆

†Department of Physics, National Institute of Technology, Tiruchirappali - 620015, India.
⋆ Tiruchirappali, India.

Abstract

We simulate stable distributions to study the ideal movement pattern for the spread of a virus using autonomous carrier.

Keywords— Stable distributions, Lévy walk, Simulations.

Introduction

Biological systems cannot be described only in the aspects of probabilistic distributions. However in the interest of studying interactions and common behaviours we consider systems to imitate similar laws at different scales [6]. Stable distributions have scale invariant behaviour, where linear combination of independent and identical distributions (IID's) with finite mean and variance leads to a normal distribution (ND) [5]. Similarly when we consider rescaled and reordered sum of IID's with non-finite variance it may converge to a Lévy distribution (LD). Though ND's are pervasive in most systems, LD's are found in biological systems mostly associated with optimal foraging behaviour [6, 8]. LD comprises of Lévy walk (LW) where multiple short steps are taken with long steps in-between, whereas the ND comprises of Brownian walk (BW) where multiple similar steps are taken. Observing the spread of present virus [2] we considered studying simulations of a simple model using stable distributions comprising of BW and LW to compare them. It can be observed that spread takes a Lévy like walk which seems to happen in different distance scales. Initially spread across different continents taking long steps followed by multiple short steps within the continent, then again long steps across different countries within the continent and followed by multiple short steps within the country, then different states within the country and so on, the distance scales keep changing but the behaviour nearly is the same. Previously such models have been extensively studied using many parameters to govern the spread [3]. Also other compartmental models typically forecast the rate of spread and behavioural count of the population [7, 4] (Such as the compartments of Susceptibles, Infectives and Recovered in the SIR model). We qualitatively concern our study only on which movement pattern is ideal for the spread of a virus on a macroscopic scale, without the consideration of various factors influencing on microscopic scales. We implement simple parameters to observe simulations based on the logic of steering behaviour developed by Craig Reynolds [9] and Daniel Shiffman [10].

Simulation model

We take a two dimensional canvas characterised by blue dots indicative of population densities as shown in Fig. 1. The population is highly dense in some regions and sparsely dense in other. A single autonomous carrier (blue triangle) moves around controlled by ND or LD spanning across the canvas infecting and spreading the virus indicated by turning the blue to red dots. As the autonomous carrier moves along in a direction it has a perceptive radius of few pixels where it infects only the blue dots within a definite boundary surrounding it. The perceptive radius imitates the realistic situation where a certain carrier can only infect a specific region around.

Simulations and discussion

We run multiple simulations to obtain time taken T_{50} by the autonomous carrier to infect fifty percent of the population density for different stable distributions. For the LD shown in Fig. 2(a) we obtain mean time T_{50} as 291 ± 21 seconds. Similarly for ND's Normal1(N1) and Normal3(N3) as shown in Figs. 2(a) and 2(b), we obtain mean T_{50} as 714 ± 173 and 412 ± 57 seconds respectively. We take the variance of N1 such that it is comparable with the perceptive radius of the autonomous carrier and the variance of N3 is comparable with the size of the canvas. We observe that the spread in LW is more patchy but spans the entire region (Fig. 3(a)). Whereas in BW with low variance such as N1, the spread is more thorough and confined to particular regions (Fig. 3(b)). It takes longer T_{50} compared to LW and is more non deterministic. We also measure T_{50} for other ND's with variance ranging between variance(N1) and variance(N3). Variance of ND Normal2(N2) is the geometric mean of variance(N1) and variance(N3) as shown in Figs. 2(a), 2(b). For ND with variance ranging between variance(N1) and variance(N2), T_{50} was obtained as 420 ± 64s. T_{50} for N2 was obtained as 400 ± 53s. Similarly for ND with variance ranging between variance(N2) and variance(N3), T_{50} was obtained as 372 ± 37s.

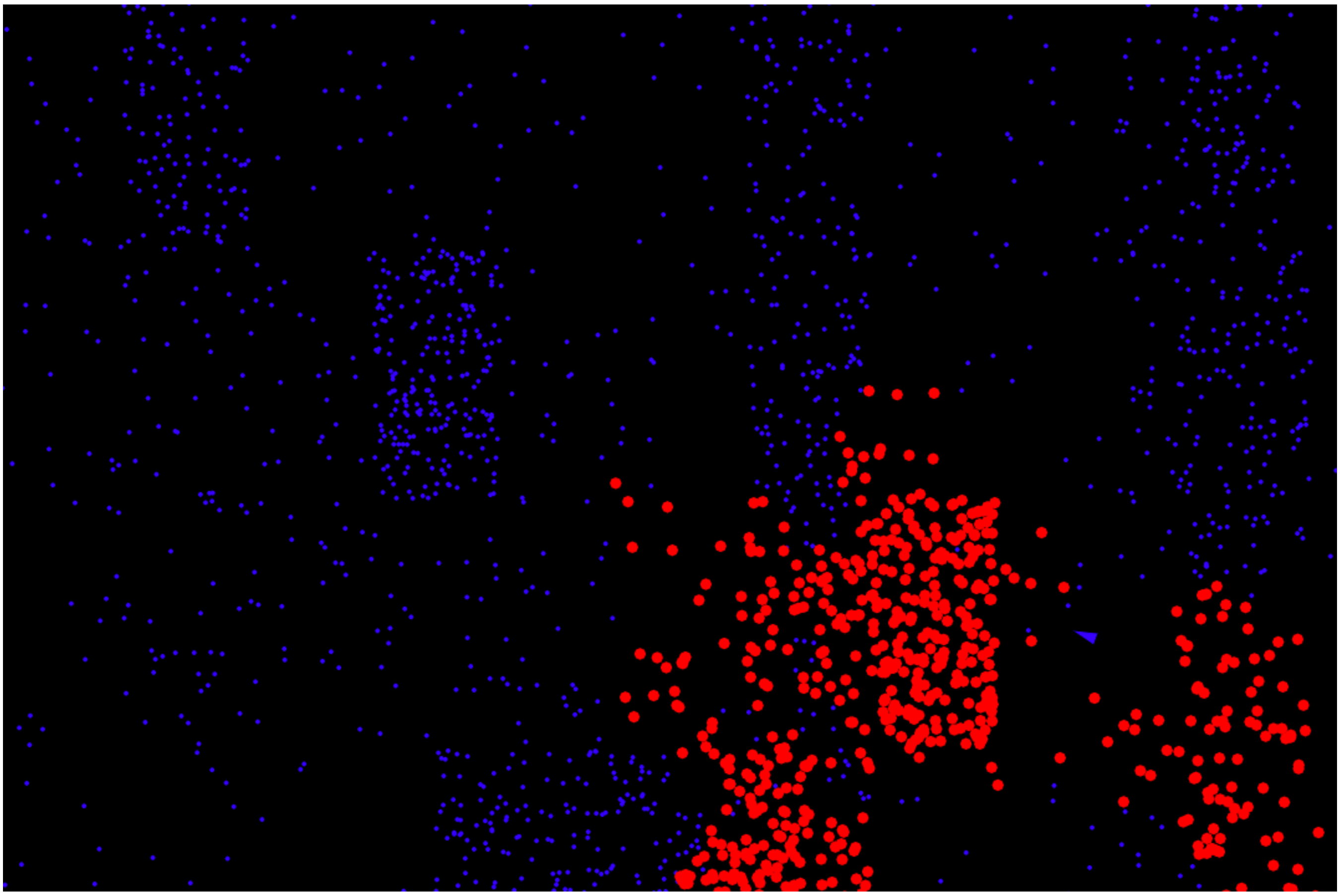
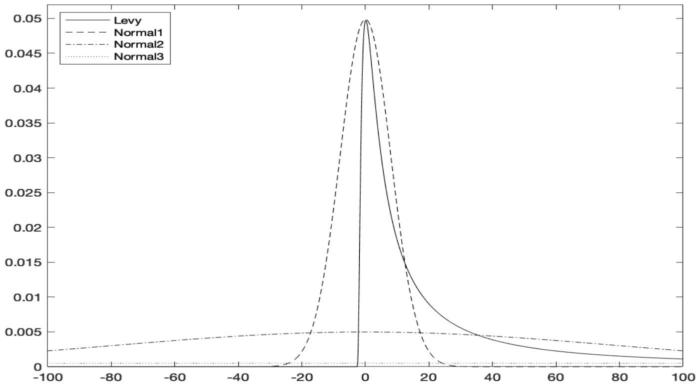
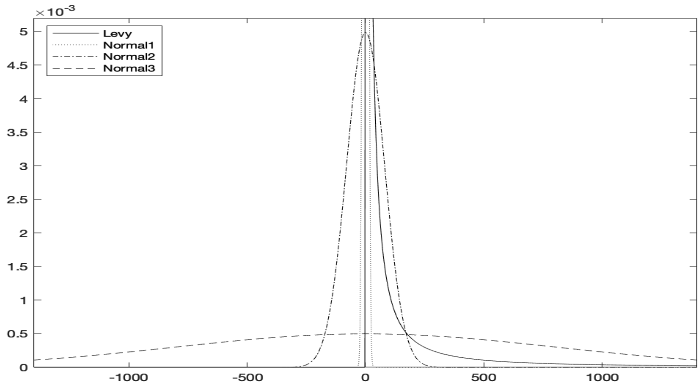


Figure 1: Simulation



(a) Comparing Lévy and Normal1 distribution



(b) Comparing Lévy and Normal3 distribution

Figure 2: Comparing probability distributions

We analyse that for our model, Lévy walk is the most ideal movement pattern for the spread. This may be related to the principle of least effort in the context of Zipf distribution [12], which have similar long-range correlations [11, 1]. It can perhaps be seen as, by following Lévy walk the autonomous carrier takes the least effort to achieve maximum spread compared to Brownian walk. Further simulations can be tried using more realistic parameters [3, 4] such as restricting the movement in infected zones and implementing multiple carriers.

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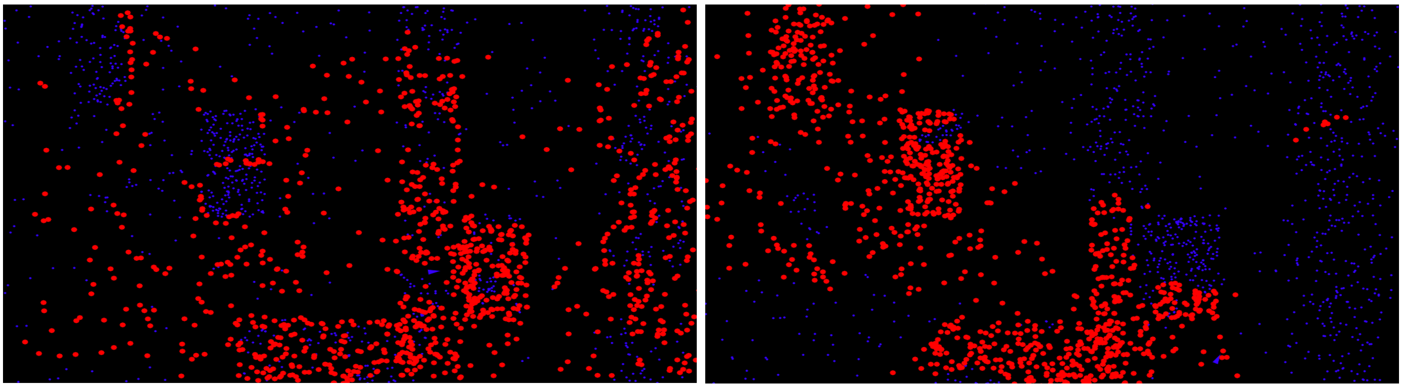

(a) Lévy walk (b) Brownian walk

Figure 3: Simulations of random walk

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