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2 **Simple muscle-lever systems are not so simple:**
3 **The need for dynamic analyses to predict lever mechanics that maximize speed**
4

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13 **Abstract**
14

15 Here we argue that quasi-static analyses are insufficient to predict the speed of an organism from
16 its skeletal mechanics alone (i.e. lever arm mechanics). Using a musculoskeletal numerical
17 model we specifically demonstrate that 1) a single lever morphology can produce a range of
18 output velocities, and 2) a single output velocity can be produced by a drastically different set of
19 lever morphologies. These two sets of simulations quantitatively demonstrate that it is incorrect
20 to assume a one-to-one relationship between lever arm morphology and organism maximum
21 velocity. We then use a statistical analysis to quantify what parameters are determining output
22 velocity, and find that muscle physiology, geometry, and limb mass are all extremely
23 important. Lastly we argue that the functional output of a simple lever is dependent on the
24 dynamic interaction of two opposing factors: those decreasing velocity at low mechanical
25 advantage (low torque and muscle work) and those decreasing velocity at high mechanical
26 advantage (muscle force-velocity effects). These dynamic effects are not accounted for in static
27 analyses and are inconsistent with a force-velocity tradeoff in lever systems. Therefore, we
28 advocate for a dynamic, integrative approach that takes these factors into account when
29 analyzing changes in skeletal levers.

30
31 **Introduction**
32

33 In this commentary we advocate for an integrative approach to analyzing changes in skeletal
34 lever mechanics. Biomechanists often infer the output speed of an organism's behaviour from the
35 geometry of its lever system using static or quasi-static analyses which ignore inertial properties
36 of the system (Anderson, 2010; Anderson and Westneat, 2009; Case et al., 2008; Copus and
37 Gibb, 2013; Olivier et al., 2021; Westneat, 1994). In these static conditions, the influence of a
38 simple lever geometry is straightforward. The large mechanical advantage of a crowbar (large
39 input lever and small output lever) for instance, amplifies the force and reduces the velocity of
40 the tip relative to the point of force application. As you decrease the mechanical advantage by
41 applying force closer to the fulcrum, the force amplification (output/input force) decreases while
42 the velocity amplification (output/input velocity) increases. Many have inferred from this that
43 there is a force-velocity tradeoff in lever systems, with greater mechanical advantage
44 (input/output lever arm) producing slower output velocities but greater output forces and smaller
45 mechanical advantage producing faster output velocities but lower output forces (Barel, 1983; M.
46 W. Westneat, 1994). We take issue with this last step of logic: that the ratio of output to input
47 velocity is sufficient to predict the speed of an organism's movement.

48
49 There are two assumptions buried in this inference from static intuitions. First, that one can
50 *increase* the output velocity of a behaviour while *decreasing* the force driving such behaviour.
51 While this is true for a 'quasi static' analysis (i.e., an analysis that does not take inertia into
52 account), considering mass results in a significantly more complicated relationship. If one
53 considers mass, and holds all else constant, *decreasing* the distance from the fulcrum at which
54 the force is applied (decreasing mechanical advantage) will *decrease* the torque
55 (Torque=Force*Distance) applied at the joint, *decreasing* output velocity, not increasing it. This
56 is because any increase in output velocity *must* be accompanied by an increase in output force.
57 Thus, for a system with mass, a constant force combined with decreasing mechanical advantage
58 can only decrease the output velocity of the system. Consequently, a mass inclusive analysis and
59 a quasi-static analysis can produce mutually exclusive predictions for the relationship between
60 moment arm and behaviour speed. The quasi-static analysis also makes a second assumption: for
61 the ratio of output to input velocity across a lever to predict the lever speed, there must be a one
62 to one relationship between changes in lever mechanics and changes in output speed (Alfaro et
63 al., 2004). This is not reliably true of lever systems actuated by muscles because this analysis

64 ignores dynamic interactions between muscle force, inertia, and behavioural kinematics
65 (Ackland et al., 2012; Clayton et al., 1998; Dunlop et al., 2004; Galantis et al., 2003; Hoy et al.,
66 1990; Marsh, 1999; McNeill et al., 1972; Nagano and Komura, 2003; Zajac, 1992). These two
67 assumptions are necessary to predict function from lever mechanical advantage alone and, as we
68 shall show, neither of them are valid. If only one message is internalized from this commentary,
69 let it be this: increasing the speed of a system necessitates increasing the force applied, so there
70 cannot be a monotonous trade-off between force and velocity in lever systems.

71
72 We are not the first to insist that the performance of biomechanical systems depend on the
73 dynamic interplay between multiple components (Dickinson et al., 2000; Nishikawa et al., 2007)
74 or that the relationship between form and function can be non-linear and complex (Koehl, 1996;
75 Wainwright, 2007). Many have argued against a reductionist approach to understanding
76 biomechanical systems and identified the key roles integrated dynamics play in determining
77 performance. For example, Josephson's "work loop technique" showed that the same muscle
78 could perform many different functions depending on the temporal pattern of applied strain and
79 activation. Likewise, numerous researchers have recognized that changing lever mechanics can
80 alter the strain rate applied to muscles (Holzman et al., 2008; Richards and Biewener, 2007) or
81 that muscle dynamics alter the function of a lever system (McHenry, 2011; Oufiero et al., 2012;
82 Roberts et al., 2018; Westneat, 2003) or both (Coombs, 1978; Galantis et al., 2003; Marsh, 1999;
83 Zajac, 1992). Yet, while some researchers recognize that a reductionist approach is not
84 sufficient, the dogma of a force-velocity tradeoff in lever systems persists (Arnold et al., 2011;
85 Bergmann and Hare-Drubka, 2015; Brusatte et al., 2012; Patek and Biewener, 2018; Vogel,
86 2013), particularly in the subfields of fish feeding (Alfaro et al., 2004; Cooper et al., 2017; De
87 Schepper et al., 2008; Evans et al., 2019; James Cooper et al., 2020; McGee et al., 2013; Olivier
88 et al., 2021; Oufiero et al., 2012; Roberts et al., 2018; Turingan et al., 1995; Westneat, 2003) and
89 bird beak biomechanics (Corbin et al., 2015; Herrel et al., 2009). Here we aim to clearly explain
90 why static intuitions that imply a monotonous force-velocity tradeoff are insufficient to predict
91 behaviour speed, and thus an integrative perspective that provides a more subtle understanding of
92 the complex dynamics is necessary.

93

94 The need to take an integrative approach will be demonstrated by using dynamic simulations of a
95 simple lever system (via OpenSIM, an open source musculoskeletal modeling program (Seth et
96 al., 2018)) driven by 100% activation of a single muscle. Across 22,572 simulations, we held
97 muscle volume and output lever length constant but varied muscle morphology (optimal fiber
98 length, pennation angle and starting normalized fiber length), input lever arm length (thus
99 varying mechanical advantage) and the inertia of the output lever (i.e. resistive forces, See
100 Supplementary Materials for model details). The additional mass was intended to account for the
101 influence of muscle mass, additional body segments or external forces like drag.

102

103 By analyzing the results of these simulations we will make 4 arguments for the integrative
104 approach. First, a single lever morphology can produce a wide range of maximum output
105 velocities if muscle properties and resistive forces vary. Second, different lever morphologies
106 can produce identical performance over a wide range of conditions. Third, mechanical
107 advantage is not the most significant determinant of performance in dynamic systems; resistive
108 forces, such as inertia, are more important for determining behaviour speed. Lastly, we look
109 across a range of moment arms to provide a mechanistic explanation of how the components of
110 a dynamic lever system interact.

111

112 **I: A single lever morphology can produce a wide range of output velocities**

113 To illustrate the range of function possible for a single lever morphology, we subset the results of
114 our simulations to those with a mechanical advantage of 1/8.28. Figure 1 shows the maximum
115 output velocities of 792 simulations with the same mechanical advantage but variable inertia and
116 muscle morphology. The resultant behaviour speeds were as low as 11.15 radians per second
117 and as high as 54.05 radians per second. This implies that the relationship between input and
118 output velocity (moment arm ratio) across a lever system is insufficient to determine the
119 maximum velocity of this system. There is thus not a one to one mapping from lever mechanics
120 to function, as has been suggested (Alfaro et al., 2004). Our simple lever can produce a wide
121 range of output velocities because this is a *lever system* composed of the lever, the driving
122 muscle and the resistive forces. As we will argue throughout this commentary, it is the
123 combination these elements that determine output velocity. This implies that a simple lever

124 system is not, in reality, “simple” and can produce a many to one mapping of morphology to
125 function just as more complex linkages do (Wainwright, 2007).
126

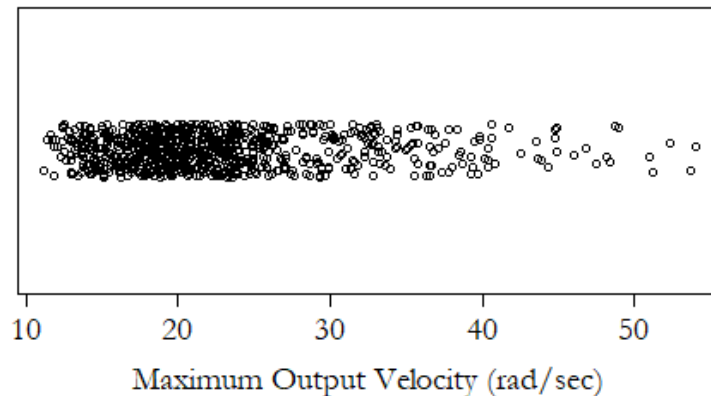


Figure 1. A single lever morphology can produce a wide range of maximum output velocities. Here we show the maximum output velocities of 792 unique lever systems with the same mechanical advantage but different muscle (i.e pennation angle, optimal fiber length and starting normalized fiber lengths) and resistive properties. Each dot represents the maximum velocity of a unique lever system. The resistive forces (inertia) acting on the system vary by $\pm 70\%$, the pennation angle ranges from zero to 40 degrees, and optimal fiber length and normalized muscle start length vary by $\pm 35\%$ and $\pm 20\%$ respectively. The data is jittered on the y-axis for clarity.

127

128 **II: Drastically different skeletal morphologies can generate the same output kinematics**

129 To contextualize the extent of variability displayed in our first analysis, we compared the
130 possible output kinematics of two drastically different skeletal morphologies with the same
131 variation in muscle and inertial properties. To do so, we subset the results of our simulations to
132 match the mechanical advantage of two example skeletons illustrated in Figure 2A; the forelimb
133 of the horse (mechanical advantage 1:13) and the forelimb of the armadillo (mechanical
134 advantage 1:4 (Smith and Savage, 1955)). While our first analysis aimed to show that a simple
135 lever can produce a one-to-many relationship between form and function, here we aim to
136 illustrate the reverse: that diverse lever morphologies can produce a many-to-one relationship
137 between form and function.

138

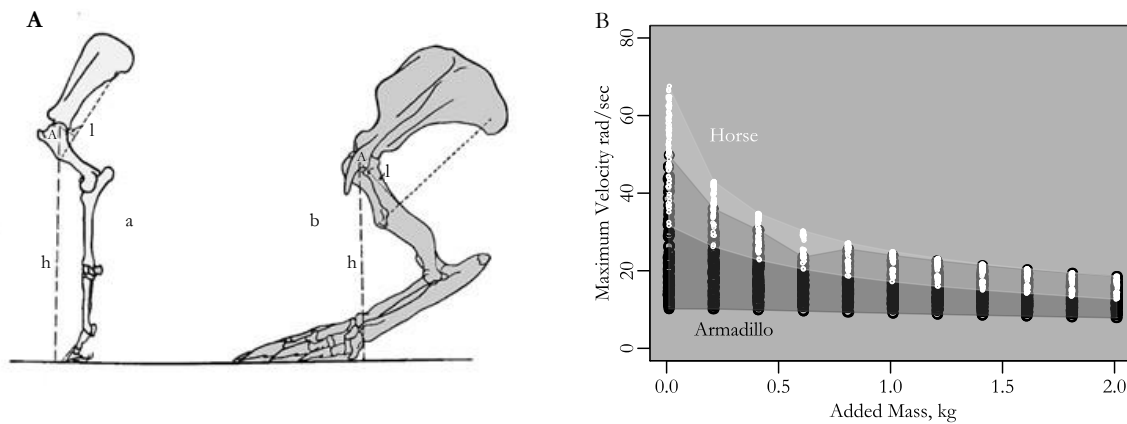


Figure 2. A) Left forelimbs of (a) *Equus* (mechanical advantage 1/13) and (b) *Dasypus* (mechanical advantage 1/4), to show the line of action of m. teres major. Text and figure adapted from Smith and Savage 1955.

B) The range of output velocities of lever systems with the mechanical advantage of *Equus* (light grey shaded regions and white points) vs *Dasypus* (dark grey shaded region with black points) is plotted as a function of the resistive forces acting on the systems. The functional overlap for the two different lever morphologies is broad and increases with increasing resistance.

139

140 In Figure 2B, we plot the possible output velocities of the two lever systems described above
141 against the resistance force encountered. Three main points should be taken from this figure. The
142 first reinforces the conclusions of our last analysis, namely that the lever mechanics of the horse
143 and armadillo can produce a wide range of output velocities, both across all conditions and even
144 when encountering the same resistive forces (described as 'Added Mass' in Figure 2B).

145 Secondly, for every driven mass, there is an overlapping region of output performance where the
146 two skeletal morphologies produce the same velocity. Changes in mechanical advantage alone
147 do not definitively determine the function of the system. Since variations in output velocity for
148 any driven mass are the result of changes in muscle morphology alone, the overlapping regions
149 are only possible because lever mechanics can be offset by changes in muscle morphology (Lee
150 and Piazza, 2009; Zajac, 1992). This is a viable biological path as skeletal and muscle
151 morphologies have been shown to evolve distinctly (Roberts et al., 2018).

152

153 The last point to be taken from Figure 2B is that as resistive forces (inertia) decrease, the region
154 of overlapping output between the two lever systems decreases. This means that changes in
155 mechanical advantage have a greater influence on the function of a system when the resistive
156 forces are small. Thus, the extent of muscle variation needed to compensate for changes in
157 mechanical advantage will be very high for small resistive forces and decrease for larger masses.
158 Taking this trend to its logical extreme, when resistive forces are ignored (as in static and quasi-
159 static analyses), the maximum velocity of a system would appear highly sensitive to changes in
160 mechanical advantage. Ignoring resistive forces, however, is not a reasonable simplification to
161 make given their impact on maximum velocity. Therefore, the one-to-one mapping between
162 changes in lever mechanics and function assumed in quasi-static analyses does not capture the
163 more complex dynamics of lever systems. Integrative analyses of variation in skeletal
164 morphology may be necessary to avoid significant errors when studying the morphological
165 variation that enables animals to move quickly.

166

167 **III: Mechanical advantage isn't the most important factor in determining output velocity**

168 Our first two analyses suggest that other factors, such as the resistive forces, may be more
169 important than mechanical advantage in determining the performance of a lever system. In this
170 third analysis, we quantify the relative contribution of muscle properties, mechanical advantage,
171 and resistive forces to the maximum velocity of our lever system. We first built linear statistical
172 models for each of the morphological elements in our lever system, (i.e. muscle force capacity,
173 muscle pennation angle, starting fiber length, mechanical advantage, and added mass (i.e.
174 inertia)) and evaluated the explanatory power of each predictor individually (R Core Team,
175 2017).

176

177 As we have repeatedly argued for the need to take an integrative approach to analyzing these
178 systems, we also built a multivariable linear regression model including all of the parameters
179 described above and their interaction effects. To determine which subset of the full list of
180 morphological parameters has the most explanatory power we performed a stepwise AIC
181 (Akaike Information Criterion) model comparison. We hoped first, to quantify the most
182 significant contributors to performance when the system is taken as a whole, then to compare the

183 explanatory power of this full model to the best individual predictor. See Supplemental
184 Materials for additional model details and results.

185
186 Table 1: Statistical models comparing predictive power of individual morphological variables (shaded grey region)
187 with a multivariate model including interaction effects.
188

Model	coeff	t-value	Adj. R²	P-value	AIC	delAIC
Null	21.83			<2e-16	169644	26651
Pennation Angle_rad	5.69	20.87	0.0189	<2.2e-16	169215	26221
Optimal Fiber Length	0.75	18.60	0.01505	<2.2e-16	169,303	26,309
Starting n. Fiber length	9.89	23.70	0.02425	<2.2e-16	169,091	26097
Moment arm_mm	-0.27	-35.8	0.05368	<2.2e-16	168,400	25406
Added Mass_kg	-10.96	-135.0	0.4469	<2.2e-16	156,278	13284
MomentArm * Added Mass* Pennation Angle * n.Fiber Length * Optimal Fiber Length			0.6933	<2.2e-16	142,994	0

189

190

191 When comparing the explanatory power of individual predictors, the adjusted R² values in our
192 first analysis reveal output velocities to be the most sensitive to resistive force (i.e. inertia).
193 Resistive forces explain 45% of the variation in the maximum velocity of our systems while
194 mechanical advantage, though the second most significant predictor, only explains 5%. The
195 relative importance of mass and moment arm can be seen visually in Figure 3A where we
196 compare the variation in maximum velocity across different moment arms for two different
197 resistive forces driven by the same muscle. The figure illustrates three points. First, in
198 conjunction with the statistical results, Figure 3A suggests that changing limb inertia can have
199 drastically larger effects on system kinematics than changing mechanical advantage. This implies
200 that the major error in quasi-static analyses is that they do not include the aspect of the system
201 (i.e. inertia) that most substantially alters kinematics.

202

203 Second, notice the inverted U-shape of the curves in
204 Figure 3A which would not be predicted from quasi-
205 static analyses. Figure 3A makes it particularly
206 clear that changes in output velocity do not change
207 *monotonically* with mechanical advantage.
208 Specifically, depending on the muscle and inertial
209 properties, the same change in mechanical
210 advantage could *increase* or *decrease* the maximum
211 output velocity. Importantly, for systems with mass,
212 there is often an optimal mechanical advantage that
213 will maximize output speed for a given set of
214 muscle and inertial conditions (Coombs, 1978).
215 Lastly, in agreement with the results of our previous
216 analysis, the magnitude of the functional change
217 resulting from an adjustment in lever mechanics
218 varies as a function of both mechanical advantage
219 and resistive forces.

220
221 As expected from the integrated nature of these
222 systems, the full model, incorporating all of the
223 morphological parameters, can explain 69% of the
224 variation in maximum velocity. To reiterate, the best
225 individual model (taking only mass into account)
226 only explained 44% of the variation. This again
227 highlights the integrated nature of these systems and
228 the need to study them as a whole.

229 230 **IV: Decreasing moment arm does not necessarily** 231 **increase output velocity**

232 We have tried to show the potential problems with using static or quasi-static analyses to predict
233 function from just skeletal morphology. But pointing out problems without providing

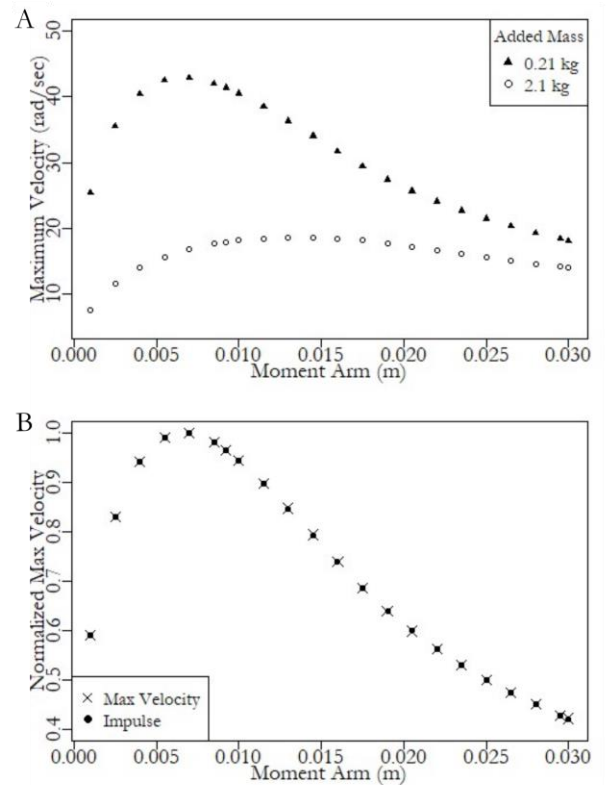


Figure 3. A) The maximum output velocities for lever systems driven by the same muscle but across a range of moment arms and encountering different resistive forces (black circles: lever inertia + 0.01 kg added mass, grey circles: lever inertia + 2 kg added mass). Note that output velocity does not increase linearly with decreasing mechanical advantage. Rather, there is an optimum mechanical advantage and that optimum value changes with resistive forces. Thus, the same change in mechanical advantage can either increase or decrease output velocity in different conditions. Further, the change in output velocity is more sensitive to changes in resistive forces than changes in moment arm in many conditions. B) Total output impulse and maximum system velocity exactly correspond when normalized by their maximum values. Any increase in output velocity must be accompanied by an increase in output force.

234 alternatives is not sufficient. A better goal, one we attempt, is to offer an alternative framework
235 to drive intuitions about the influence of changes in dynamic lever systems that can provide a
236 more subtle understanding of how components work together dynamically. In this section, we
237 present a more detailed results to illustrate the dynamic interactions that influence performance
238 metrics as lever mechanics vary. Specifically, in Table 2, we show the results of simulations
239 with the smallest, largest and optimal (i.e. producing highest output velocity, see figure 3A) for a
240 single driven mass and muscle morphology.

241
242 A static analysis would predict that decreasing moment arms would *increase* output velocities.
243 Instead, we found decreasing moment arm can sometimes *decrease* output velocity, as illustrated
244 in Figures 3A&B. For instance, when comparing the maximum velocities from Table 2,
245 decreasing the moment arm from 7 to 1.1 mm drops maximum velocity from 17 to 10 rad/s. To
246 make sense of this, it is important to consider how lever arms affect torque. As stated earlier, in
247 worlds where objects have mass, one cannot increase the output velocity of a limb without
248 increasing the force driving it as illustrated in Figure 3B. Any increase in velocity must be
249 driven by an increase in torque. Holding all else constant, a decrease in moment arm decreases
250 the torque available to drive the motion. Thus, in a dynamic system driven by an ideal actuator
251 (one that could produce constant force at any speed), a decrement in moment arm would
252 decrease both output velocity and output force. A second factor also limits output velocity at
253 small moment arms. In a system with a constrained range of motion, as we have here, a small
254 mechanical advantage also limits the work a muscle can produce. As moment arms decrease, the
255 available contractile distance over which a muscle can apply force will also decrease. Again, for
256 an ideal actuator, this decreases the energy (force*distance) that can be applied, and thus
257 decreases maximum output velocity. Output velocity, then, drops off as force is applied closer to
258 the point of rotation primarily because torques and muscle displacements are both limited by a
259 small input lever arm. Consequently, the intuition that low mechanical advantage necessarily
260 improves output velocity is often completely backwards.

261

262 Table 2: Morphological parameters and output from simulations varying moment arm but holding all else constant.

263

MA (mm)	Varying moment arm		
	Min	Opt	max
	1.0	7.0	29.5

Added Mass (kg)	2.01	2.01	2.01
Starting Normalized Fiber Length	1	1	1
Optimal Fiber Length (m)	0.07	0.07	0.07
Pennation Angle (deg)	0	0	0
Maxium Output Velocity (Rad/s)	10.00	17.60	9.52
Max Muscle Velocity (m/sec)	0.20	2.46	5.62
Muscle contraction distance/Optimal Fiber Length	0.05	0.37	0.56
In/Out Velocity Ratio	1.00e-03	7.00e-03	0.29e-03
Muscle Impulse (Ns)	1.17e+07	2.94e+06	3.78e+05
Output Impulse (Ns)	9.60e+04	1.69e+05	9.14e+04
Total muscle work (Nm)	3.06e+04	5.39e+04	1.05e+04
Total Output Power N/s	5.86e+04	1.81e+05	5.30e+04
Muscle Power/ Output Power	1	1	1
Time to Maximum Velocity (s)	0.53	0.26	0.31

264

265

266 The interesting question should really be, then, why does output velocity also drop at larger
267 moment arms despite increasing mechanical advantage? In our models, this drop off is primarily
268 due to muscle-force velocity effects, in agreement with predictions by others (Ilton et al., 2018;
269 Lee and Piazza, 2009; Nagano and Komura, 2003; Sutton et al., 2019; Zajac, 1992). In Table 2,
270 this can best be seen by noticing the increase in maximum muscle velocity as muscle impulse
271 decreases with increasing moment arm. Levers effectively alter the resistive forces and thus the
272 strain pattern acting on the muscle. Thus, we can explain the shape of the output velocity-
273 moment arm plot as the interaction of two opposing factors: those decreasing velocity at low
274 mechanical advantage (low torque and muscle work) and those decreasing velocity at high
275 mechanical advantage (force-velocity effects). These two effects will cause each lever system to
276 have an optimum mechanical advantage that minimizes these opposing factors, as noted by
277 others (Galantis et al., 2003; Josephson, 1985; Zajac, 1992).

278

279 It is important to note that the ratio of input to output velocity still changes linearly with
280 mechanical advantage (See Table 2). Yet, as we suggest in the introduction, this ratio does not
281 predict the maximum velocity of the whole motion. The output velocity is, instead, maximized
282 at an intermediate mechanical advantage value (See Figures 3A&B).

283

284 **What could replace quasi-static analyses?**

285

286 Our main goal with this commentary is to change the conceptual framework used to think about
287 the relationship between form and function in lever systems. Specifically, we argue for a need to
288 move away from the intuitions derived from reductionist static analyses which imply a force-
289 velocity tradeoff in lever mechanics. We propose shifting our thinking to an integrative
290 framework that acknowledges the dynamic interactions between the muscle, lever mechanics and
291 resistive forces (i.e. inertia) acting on the system. Within this new framework, there is a tradeoff
292 between constraints that limit output speed by limiting torque at low mechanical advantages and
293 constraints that limit output speed by limiting muscle force at high mechanical advantage
294 through muscle force-velocity properties.

295
296 In addition to changing the way we think about these systems, we also hope to provide an
297 example of how best to improve the accuracy of predicting the functional consequences of variations
298 in skeletal morphology. Given the highly integrated nature of these systems, we recommend
299 performing dynamic analyses that include direct measurements of muscle properties (pennation
300 angle, starting fiber length, ofl) and resistive forces to most accurately predict the influence of
301 changes in lever mechanics on kinematics. There are several open source musculoskeletal
302 modeling programs available to do this (Seth et al., 2018; Todorov et al., 2012) and published
303 examples of in house built models or studies using opensource software abound (De Schepper et
304 al., 2008; Farris et al., 2014; Hutchinson et al., 2015; Ilton et al., 2018; Richards and Eberhard,
305 2020; Roberts, 2003). See the supplemental materials for example code used in this manuscript.

306
307 If one does not have access to extant specimens from which to measure muscle and inertial
308 properties, we suggest building musculoskeletal-models that match the observable lever
309 mechanics and performing sensitivity analyses (Ackland et al., 2012; Anderson et al., 2007;
310 Hutchinson, 2004). This can be done by varying unknown muscle and inertial properties through
311 monte-carlo simulations (similar to our first two analyses) to get a measure of the uncertainty in
312 estimates of function. With this approach, one could test hypotheses about the functional
313 consequences of changes in skeletal morphology while accurately capturing the uncertainties.
314 Lastly, for back of the envelope estimates, our results suggest that shifting the focus from
315 changes in lever mechanics to changes in inertial properties and external resistive forces would
316 result in more accurate predictions of system maximum speed.

317

318 In summary, we suggest that it is not appropriate to assume a force-velocity tradeoff in muscle-
319 driven lever systems when looking across a whole motion because increasing output velocity
320 always requires increasing output force. We argue that inferences from changes in lever
321 mechanics alone to changes in function are error prone. This is for two reasons. First, quasi-static
322 analyses do not incorporate the most sensitive parameter, namely inertial effects. Second, the
323 influence of variation in individual parameters are highly interdependent. Thus, analyses that
324 predict changes in output from changes in a single input parameter will often predict inaccurate
325 functional consequences. Quasi-static analyses are, however, correct for systems in which the
326 inertial effects are very small because acceleration is negligible (for example, bite forces in fish).
327 As inertial effects get larger, however, the predictive power of quasi-static analyses weakens.

328

329 The analysis of lever systems which include the dynamic muscle and inertial effects yield an
330 improved quantitative framework to evaluate form and function. We hope this commentary will
331 both provide warning of the possible range of errors from the current framework and a roadmap
332 for how best to generate dynamic analyses to overcome these limitations.

333

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337 **Competing interests**

338 The authors have no competing interest to declare.

339

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344

345 **Data Availability**

346 The code used in the project and the generated data are available at DataDryad

347 <https://datadryad.org/stash/share/AqKLIaK7A1NgxKoLYGD7SX7Nab92qANnrhrInx-9RRc>.

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350 **References**

351 **Ackland, D. C., Lin, Y. C. and Pandy, M. G.** (2012). Sensitivity of model predictions of
352 muscle function to changes in moment arms and muscle-tendon properties: A Monte-Carlo
353 analysis. *J. Biomech.* **45**, 1463–1471.

354 **Alfaro, M. E., Bolnick, D. I. and Wainwright, P. C.** (2004). Evolutionary dynamics of
355 complex biomechanical systems: An example using the four-bar mechanism. *Evolution (N.*
356 *Y)*. **58**, 495–503.

357 **Anderson, P. S. L.** (2010). Using linkage models to explore skull kinematic diversity and
358 functional convergence in arthrodire placoderms. *J. Morphol.* **271**, 990–1005.

359 **Anderson, P. S. L. and Westneat, M. W.** (2009). A biomechanical model of feeding
360 kinematics for *Dunkleosteus terrelli* (Arthrodira, Placodermi) . *Paleobiology* **35**, 251–269.

361 **Anderson, A. E., Ellis, B. J. and Weiss, J. A.** (2007). Verification, validation and sensitivity
362 studies in computational biomechanics. *Comput. Methods Biomech. Biomed. Engin.* **10**,
363 171–184.

364 **Arnold, A. S., Richards, C. T., Ros, I. G. and Biewener, A. A.** (2011). There is always a trade-
365 off between speed and force in a lever system : comment on McHenry (2010). *Biol. Lett.* **7**,
366 878–879.

367 **Barel, C. D. N.** (1983). Towards a constructional morphology of cichlid fishes (teleostei,

- 368 perciformes). *Netherlands J. Zool.* **33**, 357–424.
- 369 **Bergmann, P. J. and Hare-Drubka, M.** (2015). Hindlimb muscle anatomical mechanical
370 advantage differs among joints and stride phases in basilisk lizards. *Zoology* **118**, 291–298.
- 371 **Brusatte, S. L., Sakamoto, M., Montanari, S. and Harcourt Smith, W. E. H.** (2012). The
372 evolution of cranial form and function in theropod dinosaurs: Insights from geometric
373 morphometrics. *J. Evol. Biol.* **25**, 365–377.
- 374 **Case, J. E., Westneat, M. W. and Marshall, C. D.** (2008). Feeding biomechanics of juvenile
375 red snapper (*Lutjanus campechanus*) from the northwestern Gulf of Mexico. *J. Exp. Biol.*
376 **211**, 3826–3835.
- 377 **Clayton, H. M., Lanovaz, J. L., Schamhardt, H. C., Willemen, M. A. and Colbornet, G. R.**
378 (1998). Net joint moments and powers in the equine forelimb during the stance phase of the
379 trot. **30**, 384–389.
- 380 **Coombs, W. P.** (1978). Theoretical Aspects of Cursorial Adaptations in Dinosaurs. *Q. Rev. Biol.*
381 **53**, 393–418.
- 382 **Cooper, W. J., Carter, C. B., Conith, A. J., Rice, A. N. and Westneat, M. W.** (2017). The
383 evolution of jaw protrusion mechanics is tightly coupled to benthic-pelagic divergence in
384 damselfishes (Pomacentridae). *J. Exp. Biol.* **220**, 652–666.
- 385 **Copus, J. M. and Gibb, A. C.** (2013). A forceful upper jaw facilitates picking-based prey
386 capture: Biomechanics of feeding in a butterflyfish, *Chaetodon trichrous*. *Zoology* **116**,
387 336–347.
- 388 **Corbin, C. E., Lowenberger, L. K. and Gray, B. L.** (2015). Linkage and trade-off in trophic
389 morphology and behavioural performance of birds. *Funct. Ecol.* **29**, 808–815.
- 390 **De Schepper, N., Van Wassenbergh, S. and Adriaens, D.** (2008). Morphology of the jaw

391 system in trichiurids : trade-offs between mouth closing and bit performance. *Zool. J. Linn.*
392 *Soc.* **152**, 171–736.

393 **Dickinson, M. H., Farley, C. T., Full, R. J., Koehl, M. A. R. R., Kram, R. and Lehman, S.**
394 (2000). How animals move: An integrative view. *Science* (80-.). **288**, 100–107.

395 **Dunlop, S. a, Tee, L. B. G., Stirling, R. V., Taylor, A. L., Runham, P. B., Barber, A. B.,**
396 **Kuchling, G., Rodger, J., Roberts, J. D., Harvey, A. R., et al.** (2004). Failure to restore
397 vision after optic nerve regeneration in reptiles: interspecies variation in response to
398 axotomy. *J. Comp. Neurol.* **478**, 292–305.

399 **Evans, K. M., Kim, L. Y., Schubert, B. A. and Albert, J. S.** (2019). Ecomorphology of
400 neotropical electric fishes: An integrative approach to testing the relationships between
401 form, function, and trophic ecology. *Integr. Org. Biol.* **1**,.

402 **Farris, D. J., Hicks, J. L., Delp, S. L. and Sawicki, G. S.** (2014). Musculoskeletal modelling
403 deconstructs the paradoxical effects of elastic ankle exoskeletons on plantar-flexor
404 mechanics and energetics during hopping. *J. Exp. Biol.* **217**, 4018–4028.

405 **Galantis, A., Woledge, R. C. and Vogel, S.** (2003). The theoretical limits to the power output of
406 a muscle-tendon complex with inertial and gravitational loads. *Proc. R. Soc. B Biol. Sci.*
407 **270**, 1493–1498.

408 **Herrel, A., Podos, J., Vanhooydonck, B. and Hendry, A. P.** (2009). Force-velocity trade-off in
409 Darwin’s finch jaw function: A biomechanical basis for ecological speciation? *Funct. Ecol.*
410 **23**, 119–125.

411 **Holzman, R., Day, S. W., Mehta, R. S. and Wainwright, P. C.** (2008). Integrating the
412 determinants of suction feeding performance in centrarchid fishes. *J. Exp. Biol.* **211**, 3296–
413 3305.

- 414 **Hoy, M., Zajac, E. and Gordon, E.** (1990). A musculoskeletal model of the human lower
415 extremity: The effect of muscle, tendon, and moment arm on the moment-angle relationship
416 of musculotendon actuators at the hip, knee, and ankle. *J. Biomech.* **23**, 157–169.
- 417 **Hutchinson, J. R.** (2004). Biomechanical modeling and sensitivity analysis of bipedal running
418 ability. II. Extinct taxa. *J. Morphol.* **262**, 441–461.
- 419 **Hutchinson, J. R., Rankin, J. W., Rubenson, J., Rosenbluth, K. H., Siston, R. A. and Delp,**
420 **S. L.** (2015). Musculoskeletal modelling of an ostrich (*Struthio camelus*) pelvic limb:
421 influence of limb orientation on muscular capacity during locomotion. *PeerJ* **3**, e1001.
- 422 **Iltou, M., Saad Bhamla, M., Ma, X., Cox, S. M., Fitchett, L. L., Kim, Y., Koh, J. sung,**
423 **Krishnamurthy, D., Kuo, C. Y., Temel, F. Z., et al.** (2018). The principles of cascading
424 power limits in small, fast biological and engineered systems. *Science* (80-.). **360**,.
- 425 **James Cooper, W., VanHall, R., Sweet, E., Milewski, H., DeLeon, Z., Verderber, A.,**
426 **DeLeon, A., Galindo, D. and Lazono, O.** (2020). Functional morphogenesis from embryos
427 to adults: Late development shapes trophic niche in coral reef damselfishes. *Evol. Dev.* **22**,
428 221–240.
- 429 **Josephson, B. Y. R. K.** (1985). Mechanical Power output from Striated Muscle during Cyclic
430 Contraction. *J. Exp. Biol.* **114**, 493–512.
- 431 **Koehl, M. A. R.** (1996). When does morphology matter? *Annu. Rev. Ecol. Syst.* **27**, 501–542.
- 432 **Lee, S. S. M. and Piazza, S. J.** (2009). Built for speed: Musculoskeletal structure and sprinting
433 ability. *J. Exp. Biol.* **212**, 3700–3707.
- 434 **Marsh, R. L.** (1999). How muscles deal with real-world loads: The influence of length trajectory
435 on muscle performance. *J. Exp. Biol.* **202**, 3377–3385.
- 436 **McGee, M. D., Schluter, D. and Wainwright, P. C.** (2013). Functional basis of ecological

- 437 divergence in sympatric stickleback. *BMC Evol. Biol.* **13**, 1–10.
- 438 **McHenry, M. J.** (2011). There is no trade-off between speed and force in a dynamic lever
439 system. *Biol. Lett.* **7**, 384–386.
- 440 **McNeill, P., Burrows, M. and Hoyle, G.** (1972). Fine structure of muscles controlling the strike
441 of the mantis shrimp, *Hemisquilla*. *J. Exp. Zool.* **179**, 395–415.
- 442 **Nagano, A. and Komura, T.** (2003). Longer moment arm results in smaller joint moment
443 development, power and work outputs in fast motions. *J. Biomech.* **36**, 1675–1681.
- 444 **Nishikawa, K., Biewener, A. A., Aerts, P., Ahn, A. N., Chiel, H. J., Daley, M. A., Daniel, T.**
445 **L., Full, R. J., Hale, M. E., Hedrick, T. L., et al.** (2007). Neuromechanics : an integrative
446 approach for understanding motor control. *ICB* **47**, 16–54.
- 447 **Olivier, D., Van Wassenbergh, S., Parmentier, E. and Frédérick, B.** (2021). Unprecedented
448 biting performance in herbivorous fish: How the complex biting system of pomacentridae
449 circumvents performance trade-offs. *Am. Nat.* **197**, E156–E172.
- 450 **Oufiero, C. E., Holzman, R. A., Young, F. A. and Wainwright, P. C.** (2012). New insights
451 from serranid fishes on the role of trade-offs in suction-feeding diversification. *J. Exp. Biol.*
452 **215**, 3845–3855.
- 453 **Patek, S. and Biewener, A. A.** (2018). *Animal Locomotion*. Second. Oxford.
- 454 **R Core Team** (2017). R: A language and environment for statistical computing. R Foundation
455 for Statistical Computing.
- 456 **Richards, C. T. and Biewener, A. A.** (2007). Modulation of in vivo muscle power output
457 during swimming in the African clawed frog (*Xenopus laevis*). *J. Exp. Biol.* **210**, 3147–59.
- 458 **Richards, C. T. and Eberhard, E. A.** (2020). In vitro virtual reality: An anatomically explicit
459 musculoskeletal simulation powered by in vitro muscle using closed-loop tissue-software

- 460 interaction. *J. Exp. Biol.* **223**,.
- 461 **Roberts, T. J.** (2003). Probing the limits to muscle-powered accelerations: lessons from jumping
462 bullfrogs. *J. Exp. Biol.* **206**, 2567–2580.
- 463 **Roberts, A. S., Farina, S. C., Goforth, R. R. and Gidmark, N. J.** (2018). Evolution of skeletal
464 and muscular morphology within the functionally integrated lower jaw adduction system of
465 sculpins and relatives (Cottoidei). *Zoology* **129**, 59–65.
- 466 **Seth, A., Hicks, J. L., Uchida, T. K., Habib, A., Dembia, L., Dunne, J. J., Ong, C. F.,
467 Demers, M. S., Rajagopal, A., Millard, M., et al.** (2018). OpenSim : Simulating
468 musculoskeletal dynamics and neuromuscular control to study human and animal
469 movement. *Comput. Biol.* **14**, e1006223.
- 470 **Smith, J. and Savage, R.** (1955). Some locomotory adaptations in mammals. *J. Linn. Soc.*
471 *London Zool.* **42**, 603–622.
- 472 **Sutton, G. P., Mendoza, E., Azizi, E., Longo, S. J., Olberding, J. P., Ilton, M. and Patek, S.
473 N.** (2019). Integrative and Comparative Biology Why do Large Animals Never Actuate
474 Their Jumps with Latch- Mediated Springs ? Because They can Jump Higher Without
475 Them. 1–10.
- 476 **Todorov, E., Erez, T. and Tassa, Y.** (2012). MuJoCo: A physics engine for model-based
477 control. *IEEE Int. Conf. Intell. Robot. Syst.* 5026–5033.
- 478 **Turingan, R. G., Wainwright, P. C., Hensley, D. A., Turingan, R. G., Wainwright, R. C.
479 and Hensley, D. A.** (1995). *Interpopulation variation in prey use and feeding biomechanics
480 in Caribbean triggerfishes*. Springer-Verlag.
- 481 **Vogel, S.** (2013). *Comparative Biomechanics*. Princeton, NJ: Princeton University Press.
- 482 **Wainwright, P. C.** (2007). Functional Versus Morphological Diversity in Macroevolution.

- 483 **Westneat, M. W.** (1994). Transmission of force and velocity in the feeding mechanisms of
484 labrid fishes (Teleostei, Perciformes). *Zoomorphology* **114**, 103–118.
- 485 **Westneat, M. W.** (2003). A biomechanical model for analysis of muscle force, power output
486 and lower jaw motion in fishes. *J. Theor. Biol.* **223**, 269–281.
- 487 **Zajac, F. E.** (1992). How musculotendon architecture and joint geometry affect the capacity of
488 muscles to move and exert force on objects: A review with application to arm and forearm
489 tendon transfer design. *J. Hand Surg. Am.* **17**, 799–804.
- 490
- 491