SPHERIOUSLY?

The challenges of estimating spherical pore size non-invasively in the human brain from diffusion MRI

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Abstract

The Soma and Neurite Density Imaging (SANDI) three-compartment model was recently proposed to disentangle cylindrical and spherical geometries, attributed to neurite and soma compartments, respectively, in brain tissue. The approach could also enable estimation of microstructure parameters such as the apparent size (radius) of the soma. There are some recent advances in diffusion-weighted MRI signal encoding and analysis (including the use of multiple so-called 'b-tensor' encodings and analysing the signal in the frequency-domain) that have not yet been applied in the context of SANDI. In this work, using: (i) ultra-strong gradients; (ii) a combination of linear, planar, and spherical b-tensor encodings; and (iii) analysing the signal in the frequency domain, four main challenges to robust estimation of soma size were identified:

- First, the Rician noise floor in magnitude-reconstructed data biases estimates of soma properties in a non-uniform fashion. It may cause overestimation or underestimation of the soma size and density. This can be partly ameliorated by accounting for the noise floor in the estimation routine.
- Second, even when using the strongest diffusion-encoding gradient strengths available for human MRI, there is an *empirical* lower bound on the spherical signal fraction and pore-size that can be detected and estimated robustly. For the experimental setup used here, the lower bound on the signal fraction was approximately 10%. We employed two different ways of establishing the lower bound for spherical radius estimates in white matter. The first, examining power-law relationships between the DW-signal and diffusion weighting in empirical data, yielded a lower bound of $7\mu m$, while the second, pure Monte Carlo simulations, yielded a lower limit of $3\mu m$.
- Third, if there is sensitivity to the transverse intra-cellular diffusivity in cylindrical structures, e.g., axons and cellular projections, then trying to disentangle two diffusion-time-dependencies using one experimental parameter (i.e., change in frequency-content of the encoding waveform) makes spherical pore-size estimates particularly challenging.
- Fourth, there is little differentiation in signal attenuation for low radii spherical pores, and it was found that mapping to R^4 helped in this regime and overall improved the estimation of spherical pore attributes.

We conclude that due to the aforementioned challenges spherical pore size estimates may be biased when the corresponding signal fraction is low, which must be considered when using them as biomarkers in clinical/research studies.

1 Introduction

Diffusion magnetic resonance imaging (dMRI) is a non-invasive technique widely used to study brain microstructure *in vivo*. Most dMRI methods are based on the conventional Stejskal-Tanner experiment [1] that applies a pair of pulsed field gradients along a single direction for each signal preparation, which we refer to here as 'linear' encoding. Using linear encoding, disentangling different microstructural properties such as their size, shape, and orientation is far from trivial [2, 3]. Such features may be entangled in the encoding process resulting in low specificity in their estimation. This is particularly problematic in dMRI where the image voxel is on the scale of a millimeter, and can therefore contain multiple microenvironments.

Biophysical modeling is often used to tackle the inverse problem of inferring relevant tissue features (such as cell size, shape, and orientation) from the measured dMRI signal [4, 5, 6, 7, 8]. Most contemporary dMRI models for neural tissue share some common assumptions and features. First, they separate the tissue into intra- and extra- neurite compartments. Second, the exchange between the compartments is considered to be negligible, such that each compartment has a fixed and time-invariant signal fraction f_i, where $\sum_i f_i = 1$. Third, most models treat the intra-neurite compartment as a 'stick' - that is a compartment in which the diffusivity perpendicular to the long axis of the compartment is assumed to be effectively zero. This assumption is based on the lack of sensitivity to the neurite diameter [9]. Different models have been used for the orientational dispersion of the compartments, represented by an orientation distribution function (ODF). Some models consider only purely parallel orientations (i.e., a delta function on the sphere ODF) [6, 8] while others use spherical harmonics [10] or a function such as the Watson distribution [7] to characterise orientation dispersion. A Gaussian anisotropic representation is most often used for the extra-neurite compartment. Its orientation is determined by the mean of the fiber ODF and is characterized by axial and radial diffusivities. In white matter, the extra-neurite compartment largely corresponds to the extra-axonal space (and possibly extra-astrocytic-process space). However, in gray matter, the extra-neurite compartment is comprised of the extracellular space (where the Gaussian representation is likely valid) and an additional contribution from the water restricted inside isotropic cellular structures, such as the cell bodies (soma) [11].

Recent studies have shown that the two-compartment model is not a good representation of the signal in gray matter [12, 13, 14, 15, 16, 17, 18]. This can be due to non-negligible water exchange processes occurring between intra- and extra-cellular compartments and between different intra-cellular compartments [19, 20], or the assumption that the water inside the soma behaves the same as water in extracellular space. [16, 21]. Addressing this model insufficiency, Palombo et al. [11] first demonstrated with non-trivial numerical simulations that, under specific experimental conditions, the contribution of soma to the total dMRI signal can be disentangled from that of neurite

and extra-cellular space, and then introduced a three-compartment model called Soma And Neurite Density Imaging (SANDI), which decomposes the measured dMRI signal into three main sources: neurites; cell bodies (namely soma); and extra-cellular space where, if there is any sensitivity to the size of that compartment, the diffusion MRI signal in each of these compartments will have a time-dependence. This enabled both soma size and soma density to be estimated non-invasively using dMRI for the first time.

However, the inverse problem of using complex multi-compartment models to infer microstructural information from the diffusion-weighted signal can be highly ill-posed and give rise to degeneracies in the model-parameter estimation, i.e., in the forward sense, completely different sets of model parameters predict the same dMRI signals [22, 23, 24, 2, 25, 26]. SANDI, like any other multi-compartment model [22], may suffer from the same degeneracy problems.

In SDE, the MR signal is sensitized to diffusion using a pair of gradient pulses that encode the position of the spins along the axis defined by the diffusion gradients. Double Diffusion Encoding (DDE) contains two pairs of pulsed-field gradients that are separated from each other with a mixing time τ [27, 28]. This approach has been utilized by several groups for extracting microstructural information [29, 30, 31, 32, 33, 34]. A framework called q-space trajectory imaging (QTI) was recently introduced by [35] to probe tissue using different gradient waveforms. The traditional, pulsed field gradient sequences attempt to probe a point in q-space but in q-space trajectory encoding, time-varying gradients are used to probe a trajectory in q-space and the b-matrix is defined as an axisymmetric second order tensor [36]. In this framework, SDE is just a special realization of linear tensor encoding (LTE) where the b-tensor has only one non-zero eigenvalue as all gradients are along the same axis. DDE is a special case of planar tensor encoding (PTE) as all gradients lie on a plane and the b-tensor has two non-zero eigenvalues. In spherical tensor encoding (STE) the gradients may point in all directions giving rise to a rank-3 b-matrix. Recently, b-tensor encoding has been used to resolve degeneracy problem [37, 34, 38, 39, 25].

While these studies have shown improvement in the accuracy of parameter estimates, a consideration of time-dependence of the diffusion-weighted signal can be used as another feature to add information. Notably, Gyori et al. [39] treated the signal coming from the spherical compartment as a simple mono-exponential with a fixed, time-invariant small diffusivity. This implicitly assumes that the spherical compartment would show the same signal behavior for linear and spherical tensor encoding for a given b-value. Lundell et al. [40] demonstrated that this is only true in restricted geometries when the LTE and STE waveforms have the same frequency power spectra.

In this study, we used b-tensor encoding with variable power spectra (including LTE and STE waveforms that were not spectrally matched to each other) to investigate whether and how it improves fitting of the SANDI model. Here, we exploit all three forms of b-tensor, i.e., LTE, STE, and PTE. We replace the stick component in the SANDI model with a cylinder to account for the

ultra-strong gradients employed in our *in vivo* experiments. We adopted van Gelderen's model of the spherical compartment [41], which explicitly includes both both Δ and δ. The challenge in using the free gradient waveforms, however, is that Δ and δ are poorly defined, and so the time-dependency of the obtained signal is not well-defined in the time domain. Therefore, to find a closed-form for the diffusion-weighted signal decay in the spherical compartment, we adopted this model to the frequency domain [9, 42, 40]. To the best of our knowledge, this is the first work that uses frequency-domain analysis to estimate the size of the cell-body using diffusion MRI.
The main findings of this paper are as follows:

- **Noise Sensitivity:** Even when complementing LTE with STE- and PTE-data, fitting the spherical pore properties remains challenging (when the signal fraction is small, i.e. $< 10\%$), with simulations showing biases in parameter estimates. Here we demonstrate that it is predominantly the Rician distribution of the noise (and associated noise floor) that impacts the estimation of spherical compartment properties. Such biases disappear when simulating purely Gaussian noise). However, if the Rician noise floor is accounted for in the model-fitting (albeit naively) – much of the noise-floor induced bias is ameliorated.

- **Lower Bound on Sphere Signal Fraction:** Here, by using the F-statistic to compare nested models (i.e., those that do or do not include a sphere fraction) in simulated data where the spherical pore properties are varied systematically, it was possible to identify a lower bound on the detectable MRI signal fraction limit. This was around 10% for data with SNR = 50.

- **Lower Bound on Spherical Pore Size in White Matter:** The empirical lower limit on spherical pore size in brain tissue was estimated by comparing exponents in power-law relationships between the dMRI signal and b-value fitted to simulated data, with exponents observed empirically *in vivo*. We employed two different ways of establishing the lower bound for spherical radius estimates in white matter. The first, examining power-law relationships between the DW-signal and diffusion weighting in empirical data, yielded a lower bound of $7\mu m$, while the second, pure Monte Carlo simulations, yielded a lower limit of $3\mu m$.

- **Fitting Landscape:** There is little differentiation in signal attenuation for low radii spherical pores (e.g. $R_s < 4\mu m$, Fig. 3 (a)), and it was found that mapping to R^4 helped in this regime

and overall improved the estimation of spherical pore attributes.

- **Sticks+Ball+Sphere *vs* Cylinders+Ball+Sphere:** In addition to the challenge of estimating sphere size, the fitting becomes even more challenging if we have cylinders instead of sticks. As shown by Veraart et al. [43], at 300 mT/m, and with appropriate diffusion times, we have sensitivity to the internal perpendicular diffusivity in cylindrical pores (demonstrated by a break from a power-law relationship between signal intensity and b-value). A challenge then arises when trying to disentangle two time-dependencies by varying the same experimental parameter (i.e, changing the frequency content of the gradient-encoding waveform). With currently-available pipelines, this prevents reliable estimates of spherical pore properties in white matter when there is sensitivity to intra-axonal radial diffusivity, and indeed may plague grey matter modelling if there is sensitivity to water in the astrocytic processes. We should note that most of the axons are smaller than 3 microns in radius, while the majority of soma are above 3 microns [44, 45, 46, 47, 48, 49]. Therefore, the ambiguity here is more relevant to WM voxels given a low soma density there.

2 Theory

Multi-compartment models express the diffusion-weighted signal as the sum of several compartments.

$$S = \sum_k f_k S_k \tag{1}$$

where f_k is the signal fraction ($\sum_k f_k = 1$) and S_k is the signal from the kth compartment.
For a general **B**-matrix, the diffusion-weighted MR signal is modeled as:

$$S(\mathbf{B})/S_0 = f_{\text{cyl}} \int_{\mathbb{S}^2} W(\kappa, \mathbf{n}) e^{-\mathbf{B}:\mathbf{D}_{\text{cyl}}(\mathbf{n},t)} d\mathbf{n} + f_{\text{sph}} S_{\text{sph}}(D_{\text{sph}}(t), \mathbf{B}) + f_{\text{ball}} e^{-bD_{\text{ball}}} \tag{2}$$

where f_{cyl}, f_{ball}, f_{sph}, $\mathbf{D}_{\text{cyl}}(\mathbf{n}, t) = (D^{||}_{\text{in}} - D^{\perp}_{\text{in}}(t))\mathbf{n}\mathbf{n}^T + D^{\perp}_{\text{in}}(t)I$, D_{ball} and D_{sph} are the cylinder, ball and sphere signal fractions and diffusivities, respectively [50]. $W(\mathbf{n})$ is the Watson orientation distribution function (ODF) and κ is the dispersion parameter. The diffusion weighting matrix $\mathbf{B}$ is given by $\mathbf{B} = \int_0^{TE} \mathbf{q}(t)\mathbf{q}^\intercal(t)dt$ where $\mathbf{q}(t) = \gamma \int_0^t \mathbf{g}(t')dt'$ [51, 35], and γ is the gyromagnetic ratio. Axial and radial elements in the diagonal axisymmetric b-tensor are $b_{||}$ and $b_\perp$ respectively, b-value, b is the trace of B-matrix and $b_\Delta = (b_{||} - b_\perp)/b$. For linear (LTE), planar (PTE) and spherical (STE) tensor encoding, $b_\Delta = 1, -1/2$, and 0 respectively [51].

To remove the effect of fiber orientation dispersion [30, 52], the acquired signal is averaged over all diffusion directions for each shell. This so-called 'powder-averaged' signal [53, 54] has less complexity than the orientation-dependent signal, and yields a signal whose orientationally-invariant aspects of diffusion are preserved but with an orientation distribution that mimics complete dispersion of anisotropic structures. Compartmental diffusion is represented with axisymmetric diffusion tensors which are described by isotropic diffusivity, $D_I = 1/3D_{||} + 2/3D_{\perp}$, and anisotropy, $D_{\Delta} = (D_{||} - D_{\perp})/(D_{||} + 2D_{\perp})$ where $D_{||}$ and $D_{\perp}$ are the axial and radial diffusivities, respectively. D_{Δ} changes between $-1/2$ for a planar tensor to 1 for a stick. The signal attenuation from the kth compartment is given by [26, 51]:

$$A_k(b, b_{\Delta}, D_{I;k}, D_{\Delta;k}) = exp(-bD_{I;k}[1 - b_{\Delta}D_{\Delta;k}]).g(3bD_{I;k}b_{\Delta}D_{\Delta;k}) \tag{3}$$

where

$$g(\alpha) = \int_0^1 exp(-\alpha x^2)dx = \sqrt{\frac{\pi}{4\alpha}}\,\mathrm{erf}(\sqrt{\alpha}) \tag{4}$$

and $\mathrm{erf}(.)$ is the error function [53]. Diffusion inside the sphere and ball is isotropic ($D_{\Delta;sph} = 0$, $D_{\Delta;ball} = 0$) while for the cylinder and stick it is anisotropic ($D_{\Delta;cyl} > 0$, $D_{\Delta;stick} = 1$). Therefore, the full signal equation is given by:

$$S/S_0 = f_{\mathrm{cyl/stick}}A_{\mathrm{cyl/stick}} + f_{\mathrm{sph}}A_{\mathrm{sph}} + f_{\mathrm{ball}}A_{\mathrm{ball}} \tag{5}$$

2.1 Two and three-compartment models

2.1.1 Cylinder+Ball+Sphere (Extended SANDI Model)

In the original SANDI framework, the dMRI signal in brain tissue is assumed to arise from three main non-exchanging compartments: (i) intra-neurite (modeled as diffusion in sticks); (ii) intra-soma (modeled as diffusion constrained to a sphere); and (iii) extra-cellular (modeled as isotropic Gaussian diffusion). Here we extend this model to consider the perpendicular diffusivity in the intra-neurite compartment, $D^{\perp}_{\mathrm{cyl}}(t)$, thereby modeling it with cylinders instead of sticks. We additionally explore the feasibility of modeling the intra-axonal perpendicular diffusivity and the additive spherical, $D_{\mathrm{sph}}(t)$, sensitivity simultaneously.

$$S/S_0 = f_{\mathrm{cyl}}A_{\mathrm{cyl}} + f_{\mathrm{sph}}A_{\mathrm{sph}} + f_{\mathrm{ball}}A_{\mathrm{ball}} \tag{6}$$

For complex gradient waveforms, the diffusion time is ill-defined. We therefore consider the diffusion spectrum $D^{\perp}_{\mathrm{cyl}}(\omega)$, $D_{\mathrm{sph}}(\omega)$ [42, 40] in our analyses of compartment size.

The restricted DW-signal inside the sphere and cylinder is $S = \exp(-\rho)$ where ρ is [42, 40]:

$$\rho = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{f}^T(\omega)\mathbf{D}(\omega)\mathbf{f}(-\omega)d\omega \tag{7}$$

where $\mathbf{f}(\omega) = \int_0^\tau \mathbf{f}(t)e^{-i\omega t}dt$, $\mathbf{f}(t) = \gamma \int_0^t \mathbf{g}(t')dt'$, $\mathbf{g}(t)$ is the gradient waveform. $\mathbf{D}(\omega)$ can be expressed with a rotation matrix $\mathbf{R}$ as $\mathbf{D}(\omega) = \mathbf{R}\Lambda(\omega)\mathbf{R}^{-1}$ where $\Lambda(\omega)$ is the diagonal matrix containing diffusion spectra $\lambda_j(\omega)$ along the restriction principal axes. The analytical expression for $\lambda_j(\omega)$ in the case of restricted diffusion in planar, cylindrical and spherical geometries, is the weighted sum of negative Lorentzians:

$$\lambda_j(\omega) = \sum_i B_i \frac{a_i D_0 \omega^2}{a_i^2 D_0^2 + \omega^2} \tag{8}$$

where for a cylinder, $\lambda_1(\omega) = \lambda_2(\omega) = \lambda(\omega)$ and $\lambda_3(\omega) = 0$ and

$$a_i = \left(\frac{\mu_i}{R_c}\right)^2 \quad and \quad B_i = 2\frac{(R_c/\mu_i)^2}{\mu_i^2 - 1} \tag{9}$$

where μ_i are the roots of $J_1'(\mu_i) = 0$ and $J_1'(.)$ is the Bessel function of the first kind and order [42, 9, 40] and R_c is the cylinder radius.
For a sphere, $\lambda_1(\omega) = \lambda_2(\omega) = \lambda_3(\omega) = \lambda(\omega)$ and

$$a_i = (\frac{\mu_i}{R_s})^2 \quad and \quad B_i = 2\frac{(R_s/\mu_i)^2}{\mu_i^2 - 2} \tag{10}$$

where μ_i are the roots of the derivatives of the first order spherical Bessel function $j_1'(\mu_i) = 0$ and R_s is the sphere radius. D_0 is fixed at $3\mu m^2/ms$ for the sphere, as proposed in [11] and $D_0 = D_{in}^{||}$ for cylindrical geometry.

2.1.2 Stick+Ball+Sphere ($R_{\text{cyl}} = 0$) (Original SANDI Model)

We define a three-compartment model, Stick+Ball+Sphere to investigate the sensitivity of the diffusion signal to the sphere radius and signal fraction. This model is the same as the original SANDI

model with the difference that here we use b-tensor encoding and frequency-domain analysis.

$$S/S_0 = f_{\text{stick}} A_{\text{stick}} + f_{\text{sph}} A_{\text{sph}} + f_{\text{ball}} A_{\text{ball}} \tag{11}$$

2.1.3 Stick+Ball [55] ($f_{\text{sph}} = 0$ and $R_{\text{cyl}} = 0$)

In this section, we compare the Stick+Ball+Sphere model with a Stick+Ball model and provide the range of sphere signal fractions and radii that make these two models significantly different. This latter model is the simplest model and does not have any time dependency.

$$S/S_0 = f_{\text{stick}} A_{\text{stick}} + f_{\text{ball}} A_{\text{ball}} \tag{12}$$

3 Method

Using both numerical simulations and *in vivo* experiments in healthy volunteers, we explore estimation of the soma size and density R_{sph} and f_{sph} using the combination of efficient gradient waveforms for LTE, PTE, and STE. Note: as we analyze the powder-averaged/orientationally-averaged signal, we do not need to estimate orientational dispersion. We study the challenges of the fitting landscape, the effect of noise, the lower limit on detectable signal fraction, the empirical lower limit on detectable pore size, and the challenge of disentangling two time-dependent properties (cylinder and sphere radius) of the model.

3.1 Fitting landscape

If we replace a_i and B_i from Eq. 9 into Eq. 8, we have:

$$\lambda_j(\omega) = \sum_i \frac{2}{\mu_i^2 - 1} \frac{D_0 \omega^2}{(\mu_i/R_c)^4 D_0^2 + \omega^2} \tag{13}$$

where $\lambda_j(\omega)$ is a function of R_c^4. There is little differentiation in signal attention for low radii ($R_s < 4\mu m$) so the mapping $r_c = R_c^4$ and $r_s = R_s^4$ helps with the estimation of low radii (Fig. 3(a)). It is demonstrated in Fig. 3(b-c) that the R^4 mapping improves the estimation of sphere size. When there is low sensitivity to some parameters, the numerical optimization algorithm terminates prematurely and therefore the estimates are not accurate.

3.2 Noise sensitivity

To explore the sensitivity of parameter estimation to noise perturbations, we simulated three different scenarios: (i) addition of Gaussian noise to the magnitude of the signal; (ii) addition of Gaussian noise to the real and imaginary channels which results in Rician-distributed magnitude signal; and (iii) addressing the noise-floor problem in case (ii) with a (simple) correction. In general when there is Gaussian noise in the signal, averaging improves the signal to noise ratio (SNR) and because of the orientational-averaging used in this work, we expect some improvement in the SNR in the first scenario and therefore better estimates of the model parameters. In the second scenario, the signal is corrupted by Rician-distributed noise and therefore the orientational-averaging that improved the SNR in case (i), does not remove the non-zero positive 'noise-floor' bias in Rician-distributed noise and therefore we expect some bias in the parameter estimates. In the third scenario, we use a simple correction for the Rician bias in case (ii). To estimate the standard deviation of the noise, we include the noise floor in the model so that the predicted signal is $S_n = \sqrt{S^2 + \sigma^2}$ where S is our original model prediction and S_n is the prediction after accounting for the noise floor [56, 57, 58]. We expect some improvement in the parameter estimates in case (iii) compared to case (ii) but the results of case (i) are expected to be better that all three cases.

3.3 Lower Bound on Resolvable Spherical Pore Signal Fraction

Here we considered that the framework had sensitivity to the sphere signal fraction if the inclusion of a sphere component to a stick+ball model was statistically supported by an F-test. To determine the lower bound on the spherical signal fraction and the pore size that can be detected using a diffusion-weighted signal, we systematically varied both parameters, while comparing the fit from two models: (i) the stick+sphere+ball model (three-compartments <u>including</u> a spherical component); and (ii) a stick+ball model (two-compartment <u>without</u> a spherical component). To test whether inclusion of the spherical compartment was needed to describe the signal (thereby showing sensitivity to this component) we considered the stick+sphere+ball model justified if the p-value from the F-test was less than 0.05 [59, 60]. Here, the F-statistic is calculated as $F = (SSR_1 – SSR_2)(N - M_2)/(SSR_2(M_2 – M_1))$ where SSR is the sum of squared residuals, M is the number of fitted parameters of the simplified (1) and full SANDI model (2), and N is the number of measurements. The p-value is estimated using $p = 1 - fcdf(F, M_2 - M_1, N - M_2)$ where $fcdf$ is the cumulative distribution function of F-distribution.

3.4 Stick+Ball+Sphere *vs* Cylinder+Ball+Sphere

If ultra-strong gradients and diffusion-time settings are such that we do, indeed, have sensitivity to the intra-neurite perpendicular diffusivity, then the intra-neurite compartment should be more correctly modeled using cylinders instead of sticks [43]. This introduces an additional challenge, as we now have two compartments (sphere and cylinder) with a diffusion-time dependence. To explore this, we conducted further simulations to investigate the impact of including a non-zero perpendicular intra-neurite diffusivity (or, rather, a cylinder with a finite radius of $R_{\text{cyl}} = 4\ \mu m$) on estimation of spherical pore-size.

3.5 Empirical Lower Bound on Spherical Pore Size

To identify the empirical lower bound on spherical pore size, we simulated signals for experiments with fixed diffusion time, $\Delta = 37.05ms$, and $\delta = 29.65ms$ (matching our *in vivo* experimental set-up), diffusivities $D_{in}^{||} = 2\ \mu m^2/ms$ and $D_{ec} = 2\ \mu m^2/ms$, but with variable sphere signal fractions $f_{\text{sph}} = (0.01 : 0.01 : 0.1, 0.15, 0.2 : 0.1 : 1)$, $f_{\text{ball}} = f_{\text{cyl}} = (1 - f_3)/2$, and sizes, $R_{\text{sph}} = 1 : 1 : 10\ \mu m$. For each set of signals, we fitted a power-law [15, 14, 2] to the direction-averaged signal from the LTE measurements for $b = 6, 7.5, 9, 10.5 ms/\mu m^2$ according to ($S/S_0 = \beta b^{-\alpha}$) and then compared the values of the exponent, α, with values observed empirically *in vivo* to establish a lower bound on the size of spherical pores. The rationale behind the choice of using the power-law to drive an empirical conclusion is that it is free of any model assumptions, and simply considers the rate of signal decay versus b-value. We know that the α value for a pure stick-like geometry is 0.5, and thus any deviation from this value is indicative of sensitivity to an additional compartment (a deviation from the stick-like geometry could be due to any shape that is not stick-like). The compartment that we choose to change is, in fact, a spherical compartment. By systematically increasing the size of the spherical compartment until such a deviation is detected, we can obtain an empirical lower bound on the spherical compartment. Any sensitivity to the intra-axonal perpendicular diffusivity would make the signal decay faster [43] but with the timing parameters used here, we do not expect any such sensitivity.

3.6 Simulations

The numerical simulations were performed using the model in Eq (2), with $f_{\text{sph}} = 0.01 : 0.01 : 0.1$, 0.15, $0.2 : 0.1 : 1$, $f_{ball} = f_{\text{stick}} = (1 - f_3)/2$, $D_{\text{in}}^{||} = 2\ \mu m^2/ms$, $D_{ball} = 0.6\ \mu m^2/ms$, $R_{\text{sph}} = 1 : 1 : 10\ \mu m$, and $R_{\text{cyl}} = 4\ \mu m$. The simulated protocol matched the *in vivo* protocol and comprised 10 $b = 0$ and 8 non-zero shells ($b = 1, 2, 3, 4.5, 6, 7.5, 9, 10.5\ ms/\mu m^2$)

in $(10, 31, 31, 31, 31, 61, 61, 61, 61)$ directions for LTE and 5 shells ($b = 1, 2, 3, 4.5, 6\ ms/\mu m^2$) in $(31, 31, 31, 31, 61)$ directions for PTE and 5 shells for STE ($b = 0.2, 1, 2, 3, 4.5\ ms/\mu m^2$) in $(6, 9, 9, 12, 15)$ directions and SNR = 50 with Rician noise. The 61 and 31 directions were optimized based on [61]. The noisy diffusion signal was modeled according to the following:

$$S_n = \sqrt{(S + N_r(0, \sigma))^2 + N_i(0, \sigma)^2} \tag{14}$$

where S_n and S are the noisy and noise-free signals, respectively, and N_r and N_i are the normal distributed noise in the real and imaginary images respectively with a standard deviation of σ [62, 56]. Here SNR level is defined as $1/\sigma$. For each b-tensor shape, and for each b-value, the diffusion signal was averaged over all directions in a shell.

3.7 *In vivo* Data

Two healthy participants who showed no evidence of a clinical neurologic condition were scanned with the approval of the Cardiff University School of Psychology Ethics Committee. Magnetization-prepared rapid gradient echo (MPRAGE) images were also acquired for anatomical reference. 192 sagittal slices with TE = 2.3 ms, TR = 1900 ms, TI = 900 ms, and a voxel size of 1 mm isotropic and 256×256 matrix size were acquired in 5 minutes.

Diffusion-weighted images were acquired with the protocol detailed in the simulation section 3.6 on a 3T Connectom MR imaging system with 300 mT/m gradients (Siemens Healthineers, Erlangen, Germany). Forty-two axial slices with $3mm$ isotropic voxel size and a 78×78 matrix size, TE = 88 ms, TR = 3000 ms, partial-Fourier = 6/8, were obtained for each individual. The total acquisition time was around one hour.

To take full advantage of q-space trajectory imaging, it is imperative to respect the constraints imposed by the hardware, while at the same time maximizing the diffusion encoding strength. Sjölund et al. [63] provided a tool for achieving this by solving a constrained optimization problem that accommodates constraints on maximum gradient amplitude, slew rate, coil heating, and positioning of radiofrequency pulses. The gradient waveform is optimized and Maxwell-compensated [64] based on a framework that maximizes the b-value for a given measurement b-tensor and echo time. Substantial gains in terms of reduced echo times and increased signal-to-noise ratio can be achieved, in particular as compared with naive planar and spherical tensor encoding.

3.8 Preprocessing

The data were corrected for positive signal bias, inherent to low-SNR magnitude MR data using the method proposed by Koay at al. [58]. Debiased images were subsequently corrected for Gibbs ringing [65]. We acquire some interleaved b0 images between the diffusion-weighted images (DWIs) to use for motion correction. In PTE and STE data, we registered the interleaved b0 images to the first b0 image and used the corresponding transformation to correct the motion in the DWIs. In LTE data the eddy current and subject motion were corrected by FSL EDDY [66] and finally the gradient nonlinearity was corrected by the method proposed by Rudrapatna et al. [67, 68]. We applied a 3D Gaussian filter with standard deviation of 0.5 to the preprocessed data to make the images smooth. We normalized the direction-averaged signal based on the b = 0 s/mm^2 signal in each voxel.

3.9 Regions of interest

We defined five regions of interest (ROIs) in the splenium and internal capsule as white matter regions and putamen, ventrolateral thalamus, and mediodorsal thalamus, as gray matter regions. To define anatomical regions of interest, publically-available atlases were co-registered to each participant's T_1-weighted MPRAGE image. The white matter ROIs were obtained from the JHU atlas [69], while the putamen ROI was obtained from the work of Tziorti et al. [70] and ventrolateral thalamus and mediodorsal thalamus from [71]. The MPRAGE image was co-registered to the diffusion-weighted data, and the resulting transform applied to the ROIs to translate them to the native diffusion-weighted space. The five ROIs are illustrated in Fig. 1.

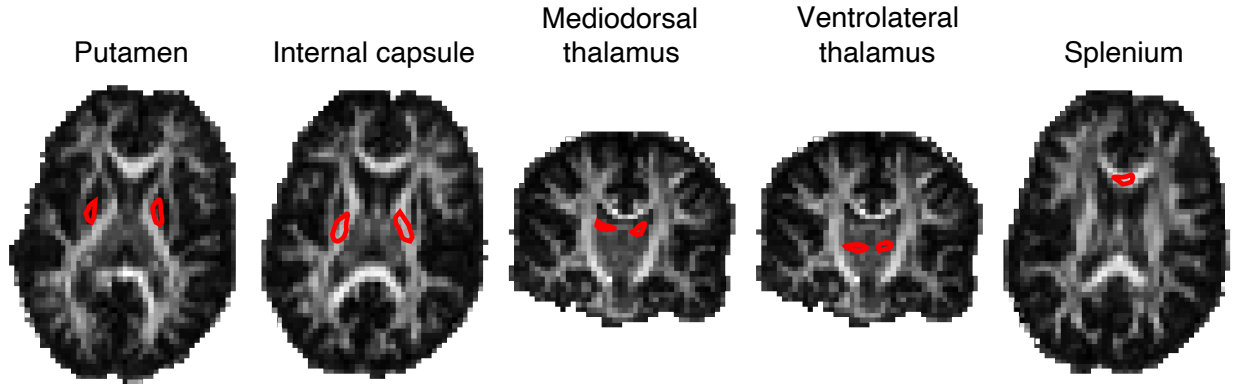

Figure 1: Location of the five ROIs used for the quantitative analysis of this study overlaid on the FA image of one subject. The posterior limb of the internal capsule, splenium, putamen, ventrolateral thalamus, and mediodorsal thalamus are illustrated as red ROIs on the FA map.

3.10 Goodness of fit for *in vivo* data

To check the stability of the model fit and that the global minima of the cost function had been found, we first fixed the signal fraction of the spherical compartment and estimated the remaining parameters (The same procedure was used by Lampinen et al. [26] for finding stick fraction that can be detected reliably). To assess the precision of parameter estimation, a metric of the goodness-of-fit (see below) was plotted for different values of sphere signal fraction, which was varied systematically between zero and one in 40 equal steps. If the model determined all parameters unequivocally, a clear optimum in the goodness-of-fit would be seen for some signal fractions. Conversely, a flat plot of the goodness-of-fit over a wide range of signal fractions would indicate degeneracy in the fitting, i.e. two or more sets of solutions yield a similarly good fit. For each ROI, all the voxels contained therein were concatenated to provide a sufficient number of data points for our estimation.

Goodness-of-fit was determined using normalized residual variance (NRV). The reduced chi-square [72] or NRV [26] was obtained by dividing the residual variance (σ_R^2) by the variance of noise (σ_{noise}^2):

$$NRV = \sigma_R^2/\sigma_{noise}^2 = [\sum n_i(S_i - S_i')^2/(n-k)]/(\sigma_{noise}^2/n_{voxel}) \tag{15}$$

where n_i is the number of directions for ith b and b_Δ. S_i and S_i' are the direction-averaged measured and predicted signals, n is the number of samples and k is the number of free parameters in the model. The standard deviation of the noise (σ_{noise}^2) was estimated for each ROI.

4 Results

4.1 Simulations

Fig. 2 (a) shows the gradient waveforms used for the linear, planar and spherical tensor encoding and their corresponding frequency spectra. Fig. 2 (b) shows the signal decay inside the spherical and cylindrical compartments using different encoding schemes. It is important to note that, for a given b-value, STE results in the most signal loss, followed by PTE and then LTE. LTE appears relatively insensitive for small radii spherical compartments.

4.1.1 Fitting landscape

Fig. 3 (a) shows the changes in apparent diffusivity of the spherical compartment (D_{sphere}) as a function of radius of the spherical compartment for the three b-tensor shapes. Clearly, there is a large difference in the sensitivity to pore size, with LTE being the least sensitive and PTE and STE

tracking each other closely in the plot of D_{sphere} *vs* R_{sph}. Notably, for all wave-forms, there is little differentiation in sphere signal attenuation for low radii, (e.g. $R_{sph} < 4\mu m$). In this regime, by mapping to R_s^4, it is easier for the optimizer to find the minima, but it does not change the noise propagation. For small radii, it is still the case that the noise dominates over measurable effects. The error bars in Fig. 3 (c) and (d) show the confidence interval. In the fitting, we use *lsqcurvefit* in MATLAB which returns the jacobian at the solution and then *nlparci* command is used to find the 95% confidence interval.

4.1.2 Noise sensitivity

Fig. 4 shows the results of fitting the sphere radius for different sphere signal fractions under different noise simulations. The figure also shows the p-value of the F-test between the two and three-compartment models in the presence of Gaussian, Rician, and corrected Rician noise. Here, we take $p < 0.05$ as an indication that the full model (three-compartment) is preferred over the simplified model (two-compartment). When the sphere radius or the signal fraction of the sphere is small ($R_{\text{sph}} < 2\mu m$ and $f_{\text{sph}} < 0.05$) the simplified model is preferred. Fig. 4 shows that the estimates are largely positively biased in the Rician-noise data, whereas their errors are symmetrically-distributed about the line of identity in Gaussian only. After Rician noise correction, it looks more like the Gaussian - in terms of symmetrical distribution around the line of identity.

4.1.3 Signal fraction resolution limit

The examination of the F-test measures indicates a lower bound on the signal fraction that can be detected or reasonably modeled from the diffusion-weighted signal. This is around 10% for SNR = 50 but this limit changes at different noise levels (Fig. 4). The changes of sphere radius estimates versus ground truth for a wider range of SNR values are shown in this link: `https://bit.ly/StickBallSph`. The lower limit scales inversely with SNR, i.e. as SNR increases, smaller sizes are detectable.

4.1.4 Stick+Ball+Sphere *vs* Cylinder+Ball+Sphere

In addition to the challenge of estimating sphere size (given the above 4 points), the fitting becomes more challenging if we have cylinders instead of sticks because in this model the time-dependence of the signal can now arise from two independent sources (the cylinder and the sphere). Fig. 5 shows the estimated sphere size when there is a non-zero diameter cylinder, (without any noise). For signal fractions smaller than 0.2 and sphere radius larger than 5 microns, the estimated sizes deviate from the ground truth. This behavior is observed where there is no noise in the simulated

signal. The deviation from the ground truth in the presence of noise, for different noise levels, is shown in the following link: `https://bit.ly/CylBallSph`.

4.1.5 Empirical lower limit on pore size

In addition to the lower bound imposed by the noise floor on the sphere signal fraction and size, there is an empirical lower limit on the pore size which can be derived from power-law measurements. Fig. 6 shows the effect of sphere signal fraction and size on the estimated exponent, α, in the power-law fit ($S/S_0 = \beta b^{-\alpha}$) experiments. As will be discussed below, in our *in vivo* data, we observe power-law exponents greater than 0.5 in the white matter. Fig. 6 shows that to observe such values of α the sphere radius needs to be larger than 7 microns. We will discuss the implications of this key result in the context of the *in vivo* data below.

A good 'sanity check' is that after reconstructing the signal from the estimated parameters the exponent α in the power-law fit ($S/S_0 = \beta b^{-\alpha}$) of the original and the modeled signal should not change considerably (see last row of Fig. 7), otherwise the parameters are representing the signal incorrectly. This approach has several limitations that should be acknowledged; The first limitation is that we used a pre-determined and fixed set of parameters in the simulation. If we had used a smaller diffusivity for the extra-cellular compartment, for the range of b-values used here ($6 < b < 10.5\ ms/\mu m^2$), there might have been some residual contribution from the extracellular compartment and therefore the exponent α and the behaviour of the signal would change. The second limitation is that we fixed the intra-neurite and extracellular signal fractions to be equal which may not reflect reality. Despite these limitations, we consider the result a useful empirical benchmark because the fixed diffusivities used in this experiment is close to the range of diffusivities estimated in previous works [73]. As such, the contribution from the extracellular compartment at high b-values will be negligible meaning that the only remaining signal contributions come from spheres and sticks.

4.2 *In vivo* results

Here we provide *in vivo* results from fitting the Stick+Ball+Sphere model. Fig. 7 shows the results of fitting the model to the signal by fixing the sphere signal fraction to different values. Results are shown for five different ROIs: Splenium; internal capsule; mediodorsal thalamus; ventrolateral thalamus; and putamen. Note that, for the fitting, the data points from all three voxels in the ROI are concatenated to provide enough data points for a stable fit, but in the figure, the average of these three voxels is shown. The flat valleys in NRV correspond to plausible sphere signal fractions in different ROIs of the brain, for example, 0-0.125 for Splenium and Internal Capsule and 0.2-0.3

for the Putamen (Fig. 7). When the valley is flat for a large range of sphere signal fractions, it means the data do not provide a unique solution and a range of parameters can represent the signal equally well. This may be related to a large range of acceptable parameters as shown by the second to the fourth row of Fig. 7. For example, acceptable $D_{in}^{||}$ values for the stick compartment ranged between 2.2 and 2.5 $\mu m^2/ms$ in the splenium and between 1.2 and 1.5 $\mu m^2/ms$ in the putamen. If there was no degeneracy in the estimation of parameters and the data could provide useful information about the underlying microstructure, then the plot would have a sharp valley at the local optimum. Among the five ROIs we selected in this experiment, we see a quite sharp minimum for the putamen and mediodorsal thalamus (with f_{sph} around 0.2-0.3), and the minima for white matter (splenium and internal capsule) clearly puts the sphere fraction in the sub 10% regime, which is were it is expected. The first column of Fig. 7 shows the parameter estimates for a synthetic signal generated with the following parameters; $f_{\text{sph}} = 0.5$, $f_{\text{ball}} = f_{\text{stick}} = 0.25$, $D_{\text{in}}^{||} = 2\ \mu m^2/ms$, $D_{\text{ball}} = 0.6\ \mu m^2/ms$, and $R_{\text{sph}} = 5\ \mu m$. There is a sharp minimum in NRV which shows there is only one set of parameters that fits the signal accurately.

Fig. 8 shows estimated parameter maps *in vivo*. When fitting on a voxel by voxel-level, the fitting is unstable and the resulting maps are not smooth. Besides, the large voxel size (3 mm) used here and the resulting problems with partial volume may affect the estimated parameters as well. Maps of the estimated parameters in Fig. 8 show a reasonable contrast that matches the results in [11]. The f_{stick} map has higher values in white matter tracts in the brain while f_{sph} values are higher in the gray matter. The f_{stick} values in cortical GM ranges from 0.1 to 0.2 which is in agreement with recent works on estimating neurite density in GM using b-tensor encoding [2, 26].

Fig. 9 shows the results of fitting a power-law ($S/S_0 = \beta b^{-\alpha}$) to the diffusion-weighted signal. CSF has the fastest decay and therefore no signal remains from CSF in the high b-value data and the alpha is close to zero. The decay in the GM is faster than the WM and therefore the estimated α values are correspondingly larger in GM than WM. Within WM, α is usually larger than 0.5. As noted by Veraart et al. [15], if there is only a stick compartment, then α = 0.5. The decay faster than the stick model may come from the exchange between compartments [6], sensitivity to the axon diameter [43] or the presence of a non-negligibly-sized third compartment [11] that makes the signal decay faster than 0.5. If the radius of the spherical compartment is less than approximately 7 microns then the exponent is smaller than 0.5. As will be seen in the map of α in Fig. 6, we do not observe values of α less than 0.5 in the white matter. This places a lower bound on the size of spherical pores in white matter of around 7 μm.

5 Discussion

The SANDI model [11] extends existing multi-compartment models that only consider two pools of water in brain tissue. [10, 7, 74, 75, 76, 3, 77]. Palombo et al. [16] suggested that the failure of the stick model in gray matter [14, 15] can be due to the abundance of cell bodies (namely soma). In previous works the contribution from soma was considered as part of the extracellular space [10, 78] because the exchange between the restricted water in soma and the hindered water in the extracellular space was assumed to be fast. However, recent studies [79] suggest that for a diffusion time much smaller than $500ms$ the signal from spins restricted in soma is non-negligible. The typical size of soma ranges from a few microns (for microglia and glia) to a few tens of microns (for big neurons) [80, 81].

In this study we found that based on the power-law fit to the signal in voxels containing white matter, the sphere radius is larger than $7\mu m$ (Fig. 6). While at first glance, one might not expect spherical compartments of this size in white matter *in vivo*, there are studies that report the size of astrocytes [44, 45, 46, 47] and oligodendrocytes [48, 49] and confirm that having effective soma radius $> 7\mu m$ is possible (data also publicly available on Neuromorpho.org). This suggests the presence of glial cells in the central nervous system whose soma can have such a size. Although in healthy conditions glial volume fraction in white matter has been estimated to be low ($5\% - 20\%$ [43, 82, 83],), the presence of large glial cells may become more relevant in pathological conditions, where glia may migrate and react undergoing hypertrophy of the whole cellular structure [84, 85].

In this work, we studied the minimal sphere signal fraction and radius that can be detected from diffusion MRI of water inside spheres. Our finding is in agreement with the results reported by Dhital et al. [86] (2% of the unweighted signal for moderate diffusion times using Prisma scanner) and Tax et al. [87] (isotropic signal fraction of 9.7% for the apparent diffusivity of $0.12\mu m^2/ms$). It should be noted that the protocol in this study is not specifically optimized for size estimation. For example intentionally varying the frequency spectra (e.g. to include high frequency components), might result in better sensitivity to smaller pore-sizes [88].

The estimation of stick signal fraction in [26] was challenging while in [25] Lampinen et al. provided reliable estimates of the two-compartment model parameters because the sequence was optimized in the latter. Comparing the estimation of sphere signal fraction in this work with the stick signal fraction in [26, 25] we conclude that to estimate the parameters accurately, the acquisition should be optimized toward the estimation of the model parameters.

Here we used a combination of linear, planar, and spherical tensor encoding to ameliorate the degeneracy problem that exists in the fitting of multi-compartment models. Our numerical sim-

ulations show that, when there is no noise, a small contribution of spheres with small sizes can be recovered. Our simulations (Fig. 3) show that for the timing parameters and the waveforms used here, adding planar or spherical tensor encoding to standard LTE encoding strategies helps to differentiate the signals between spherical pores with small radii. Nilsson et al. [9] reported that for the estimation of diameter in complete orientation dispersion (which we effect by powder-averaging the signal), from an SNR perspective it is advantageous to use oscillating gradient spin echo (OGSE) compared to standard SDE. The benefits of double diffusion encoding (DDE) for size estimation have been presented in other studies [31, 89, 90]. Our results suggest using multiple waveforms provides the best estimates.

In the case of cylinder diameter estimation, the resolution limit is determined by the amount of attenuation due to radial diffusion. This attenuation is estimated by the integral of the gradient squared and can be maximized by either a fat-pulse SDE or a rectangular oscillating pulse. However, when the long axis of the cylinder is not perpendicular to the direction of the applied gradient, the high b-values should be avoided because of the signal attenuation and decrease in SNR. To improve the SNR and the resolution limit for cylinder diameter estimation, waveforms should have more oscillations and hence lower b-values [9]. Here, we are targeting the sphere diameter, and therefore OGSE or SDE can be both useful.

In the estimation of spherical pore size from the diffusion-weighted signal, different confounding factors have to be considered. One of the challenges is that for small sphere radii (< 3 micron), the fitting landscape is flat and there is a negligible change in the signal for small sphere sizes (Fig. 2 (c)). Mapping R_s to R_s^4 makes these signal differences more detectable and therefore the estimation of the sphere size is more reliable.

Noise is another confounding factor that affects the estimation of parameters in both model-based and signal representation based techniques. Parameters obtained from multi-compartment models, (the stick+ball+sphere model in this paper) applied to noisy data are biased because of the effect of noise. Three different noise scenarios were simulated here: Gaussian noise, Rician noise, and corrected Rician noise. If the data were corrupted with purely Gaussian noise, then this could be removed to some extent through the act of computing the orientationally-averaged signal. However, as we invariably use the magnitude-reconstructed data, the noise has a Rician distribution, which presents a more challenging scenario because averaging does not remove the bias. This effect is more pronounced when there is a small contribution from the spherical compartment.

We wish to stress that the challenges we identified are mainly relevant to WM. The SANDI model was developed mainly for soma imaging in GM. Nothing in our results suggest that SANDI is unreliable in GM, and will indeed, benefit from the multi-waveform and frequency-domain approach presented here. The challenge in the gray matter is to determine whether the deviation

from the 'stick' model comes from the soma compartment, exchange between compartments [20] or both.

We note that the extracellular signal fraction obtained from the fit in the gray matter areas is around 0.35 which is higher than expected. This discrepancy can be explained by two factors: first, T_2 relaxation is not explicitly accounted for in our model and second, we consider the same proton density in both intra-axonal and extra-axonal spaces. The model presented here is only sensitive to relative signal amplitudes while differences in T_2 relaxation can impose different weights to the amplitudes of the pools [91, 92]. The specific assignment of nerve water T_2 components by simultaneously considering compartmental diffusion and transverse relaxation properties was already studied by Peled et al. [93] in myelinated nerves of the frog sciatic nerve tissue. More and more evidence is given for the T_2 relaxation time constants of intra- and extra-axonal water to be different from each other in case of slow exchange between the intra-axonal and extra-axonal pools [93, 94]. In case of fast exchange [91], the signal loss in all the pools would be weighted in the same manner. Although it is relatively easy to incorporate relaxation into the model and fit the experimental data with additional parameters (relaxation rates), such a model results in an unstable fit with the current protocol. To incorporate additional parameters (i.e., relaxation rates) in the model we need to obtain additional information in our experiment. For example, repeating the acquisition at different echo times as suggested by Lampinen et al. [25].

We highlight six limitations of the current study in the estimation of spherical compartment signal fraction and size. First, we assume that the intra-axonal water comes from straight axons, which is not the case in most of the white matter voxels [95, 59, 96, 97, 98]. Second, the extra-cellular component is modelled as a ball with isotropic diffusion. This assumption is not valid when there are coherently-oriented fibers (such as in the midline of the corpus callosum), where diffusion in the extra-axonal space can have a high anisotropy [99]. Third, at low frequencies, the time-dependency of diffusivity in the extracellular space can dominate over the time-dependency in the intra-axonal space [100, 9]. Fourth, it is assumed that the exchange between water environments is negligible. This assumption might be valid since previous studies have shown the exchange times in the white matter are of the order of seconds or longer [101] which is much larger than the time-scales that the effects of restricted diffusion can be observed. Fifth, in our tissue model, we have neglected the distribution of restricted dimensions (e.g. range of soma sizes, axon diameters). However, adding extra parameters to the model to account for this will make the fitting unstable. Six, the effect of T_2 relaxation is not considered in our model which may result in bias in the estimation of the other parameters of the model. In addition, the lack of a cerebrospinal fluid (CSF) component in the model is another limitation of this work.

Future directions. The fitting method used in this work is a nonlinear least-square fit that can be replaced with new deep learning approaches to improve the quality of fit [102]. From the acquisi-

tion perspective, the protocol that is used in this study is not optimized for the estimation of small sizes. Using a range of frequency spectra will help [88]. The protocol, used in this work, imposes a long acquisition time which can be minimized by optimizing the directions as well as the number of shells. In this paper, a simple arithmetic averaging is used for powder averaging which can be replaced with some other techniques such as weighted averaging [103, 104] to obtain a better orientation-invariant signal that improves the parameter estimates.

6 Conclusion

In this work, we have demonstrated key challenges and limitations in estimating spherical pore size non-invasively in the human brain from diffusion MRI. Our simulations show the effect of Rician bias on the estimation of pore size and identified the lower bound limit of the sphere signal fraction and size that can be detected from the diffusion-weighted signal from both an SNR and empirical perspective. We showed that for small signal fraction of soma, i.e. $< 10\%$, this is a problem. However, we know from detailed microscopy of brain cortex [105, 106], that in GM the soma signal fraction is above 20%. Therefore, reliable estimation of soma properties in GM is possible, while in WM it presents several challenges. The flat landscape of the fitting and mapping of the radius to enhance sensitivity to small sizes was also investigated. The results show that mapping R_s to R_s^4 improves the fitting of small radii. Using the ultra-strong gradients of the Connectom scanner, the diffusion signal in the white matter can be made sensitive to the axon diameter, and therefore the three-compartment model of stick+ball+sphere changes to cylinder+ball+sphere which has two time-dependencies, one for the diffusivity in the sphere and the other one for the diffusivity in the cylinder. Disentangling these two time-dependencies using only one sequence parameter (i.e., changing the frequency content of the encoding waveform) in the acquisition is challenging. Studying all these challenges prevents misinterpretation of the biased estimated parameters as the potential biomarkers in clinical studies.

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A Supplementary Material

Fig. S1 shows the estimated parameters of the stick+ball+sphere model on axial, coronal, and sagittal views of the *in vivo* brain image without applying the Gaussian kernel.

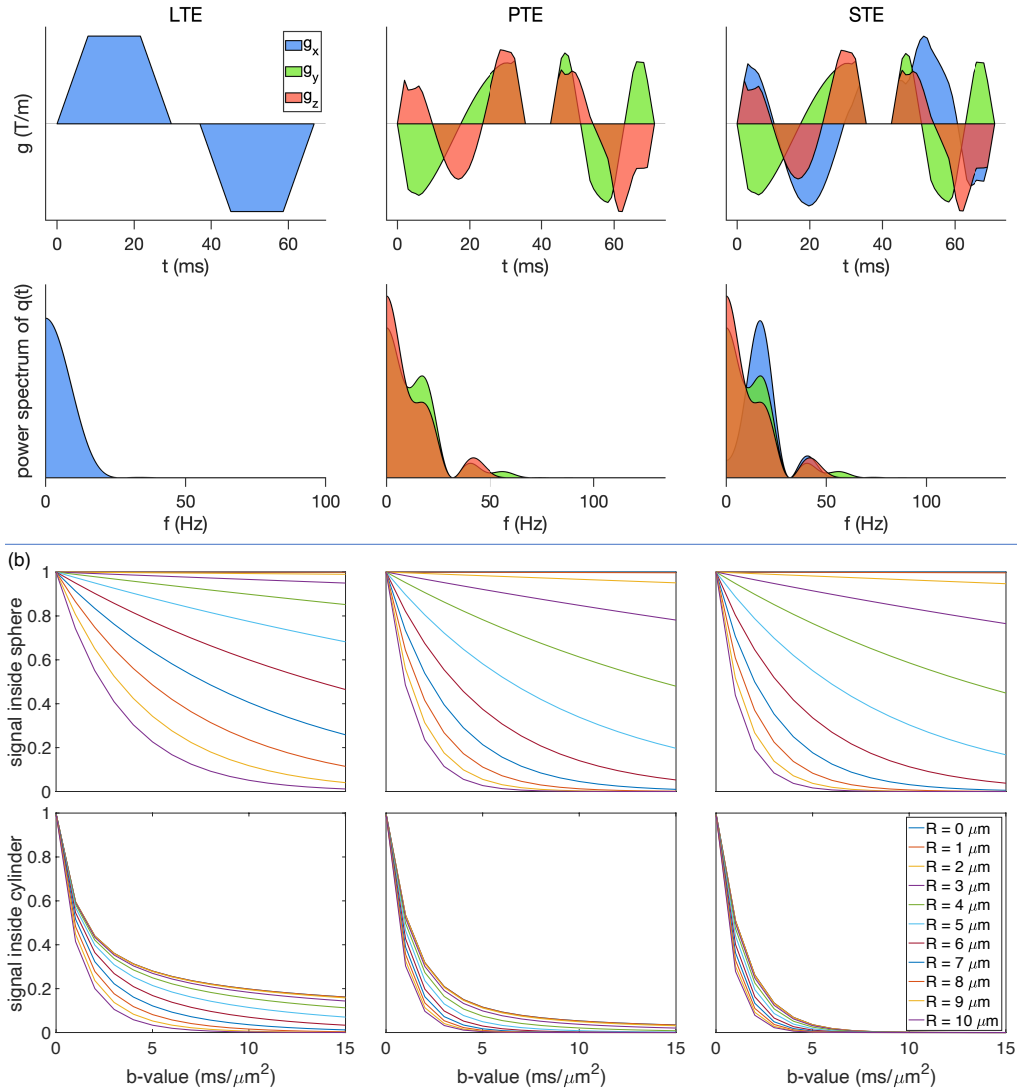

Figure 2: (a) The free gradient waveforms of the linear, planar, spherical tensor encoding and the corresponding frequency power spectra. (b) The signal decay inside the spherical and cylindrical compartments using different encoding schemes.

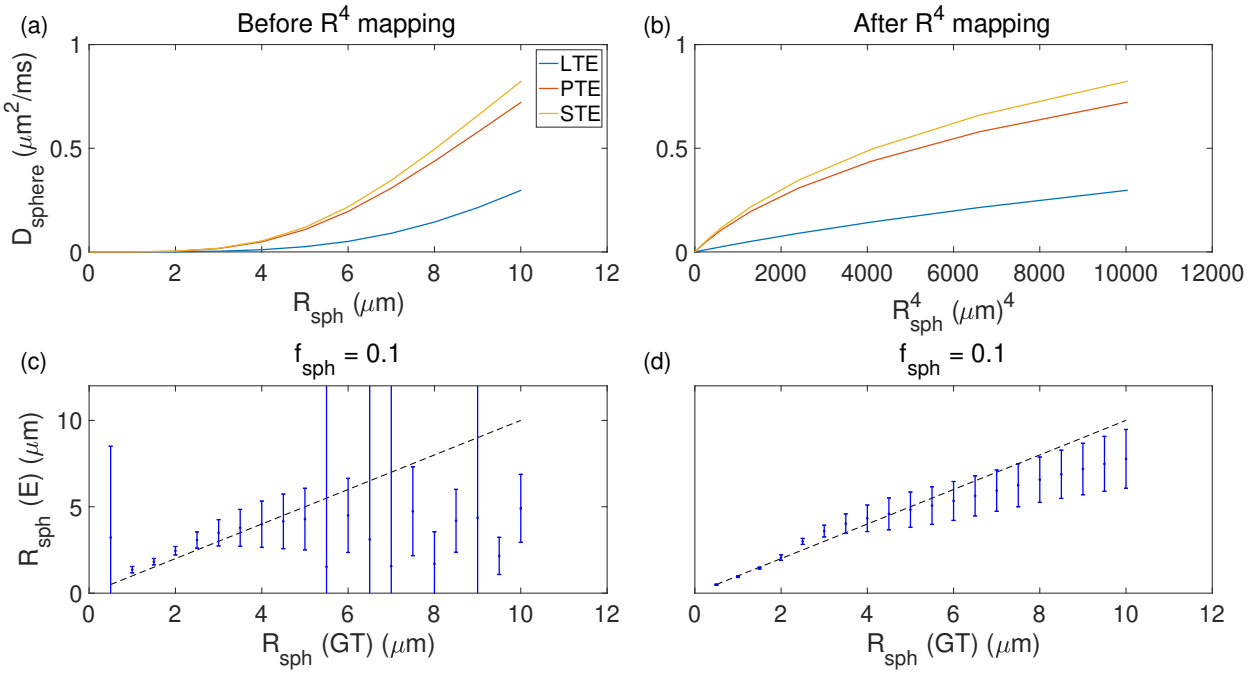

Figure 3: The effect of R^4 mapping on the apparent diffusivity of the sphere (D_{sphere}) and the estimated sphere size (The error bars show the confidence interval.). (a-b) The changes in the apparent diffusivity (D_{sphere}) *versus* the radius of the sphere (R_{sph}) for linear, planar and spherical tensor encoding (LTE, PTE, and STE). (a) D_{sphere} vs R_{sph} , (b) D_{sphere} vs R^4_{sph}, and (c-d) the results of sphere size estimation before and after R^4 mapping, respectively (dashed black line is the line of identity, GT = ground truth and E = estimated).

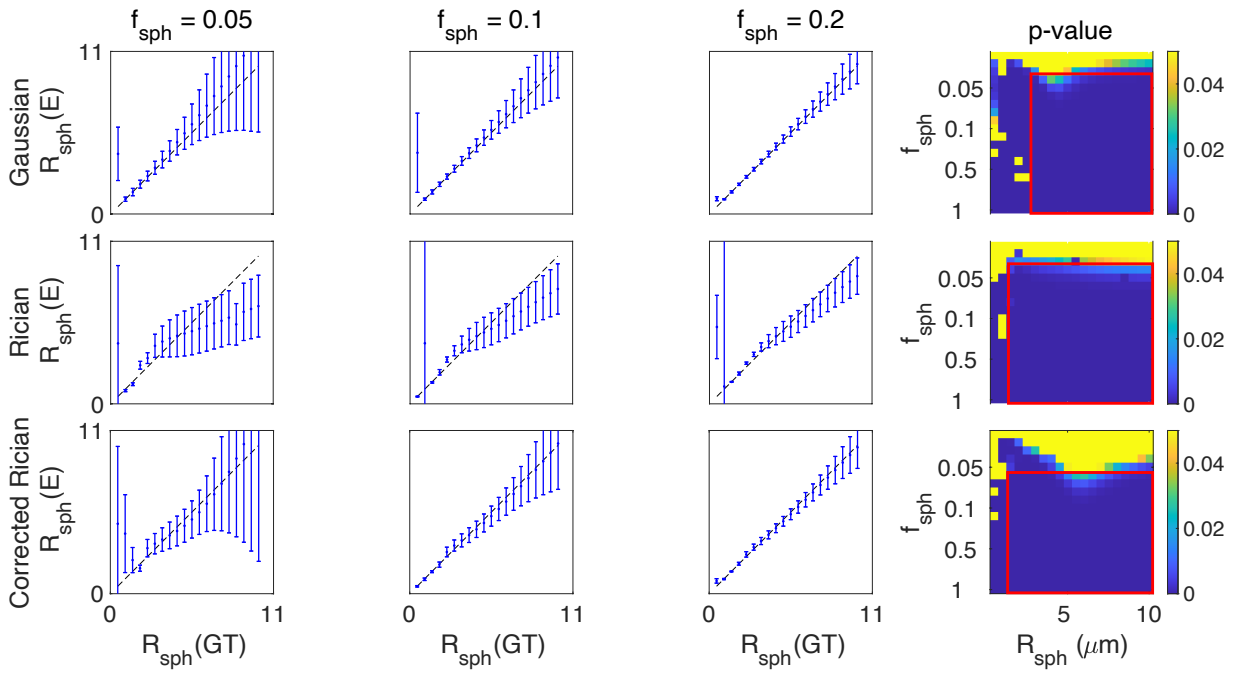

Figure 4: The results of fitting the sphere radius for different sphere signal fractions (GT = Ground Truth and E = Estimated). The figure also shows the p-value of the F-test in the presence of Gaussian, Rician, and corrected Rician noise respectively. The red rectangles in the right side plots show the areas that the three-compartment model is significantly different than the two-compartment model. Diagonal black line is the line of identity.

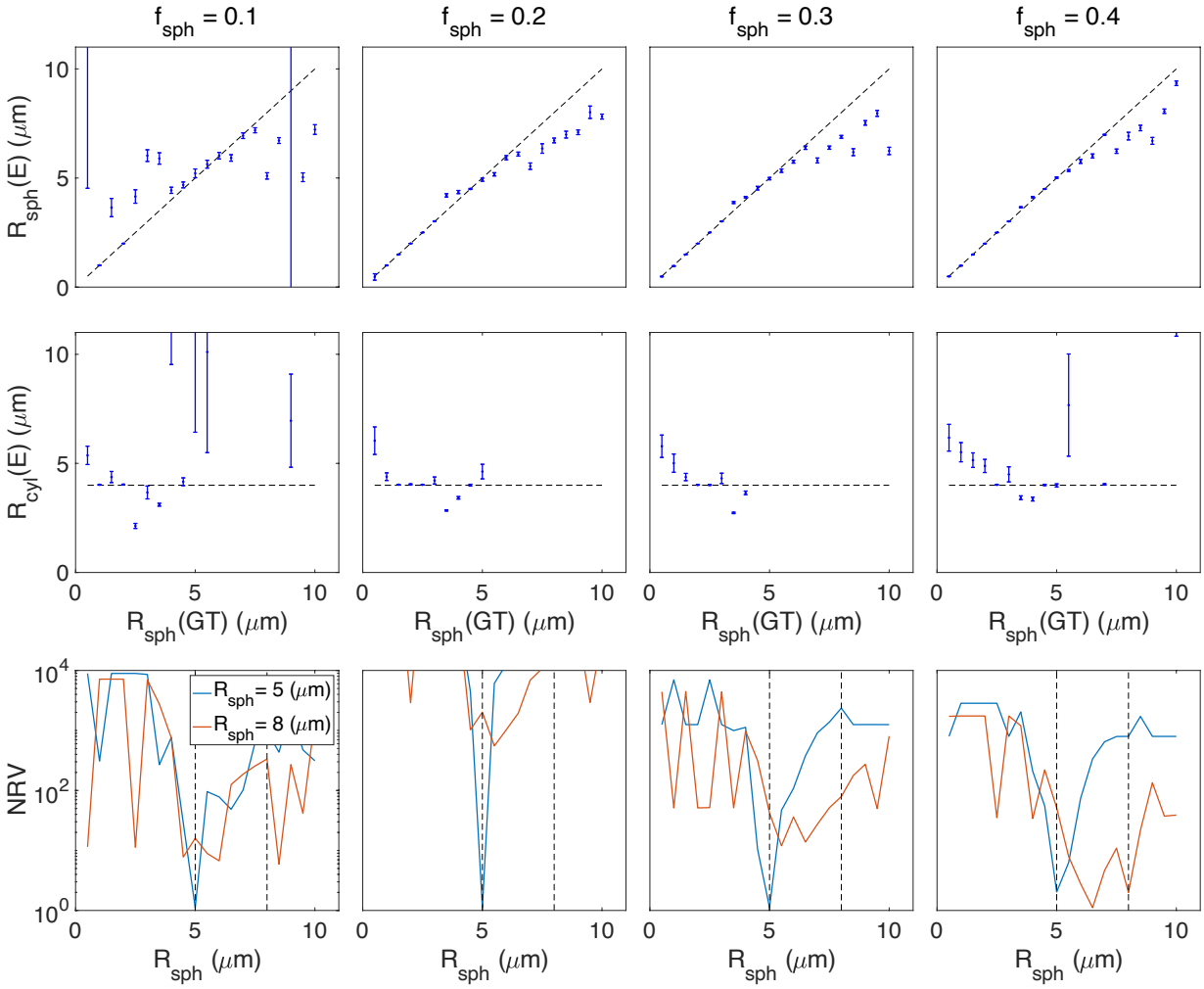

Figure 5: Estimated sphere and cylinder radii versus the ground truth sphere radius values for cylinder+ball+sphere model without noise ($SNR = \infty$). Third row shows the NRV value for two scenarios where the sphere radius is fixed to 5 and 8 μm, blue and red curves respectively. Note that there is a sharp minimum for $R = 5\mu m$ while the NRV curve has several local minima in the $R = 8\mu m$ case (GT = ground truth and E = estimated).

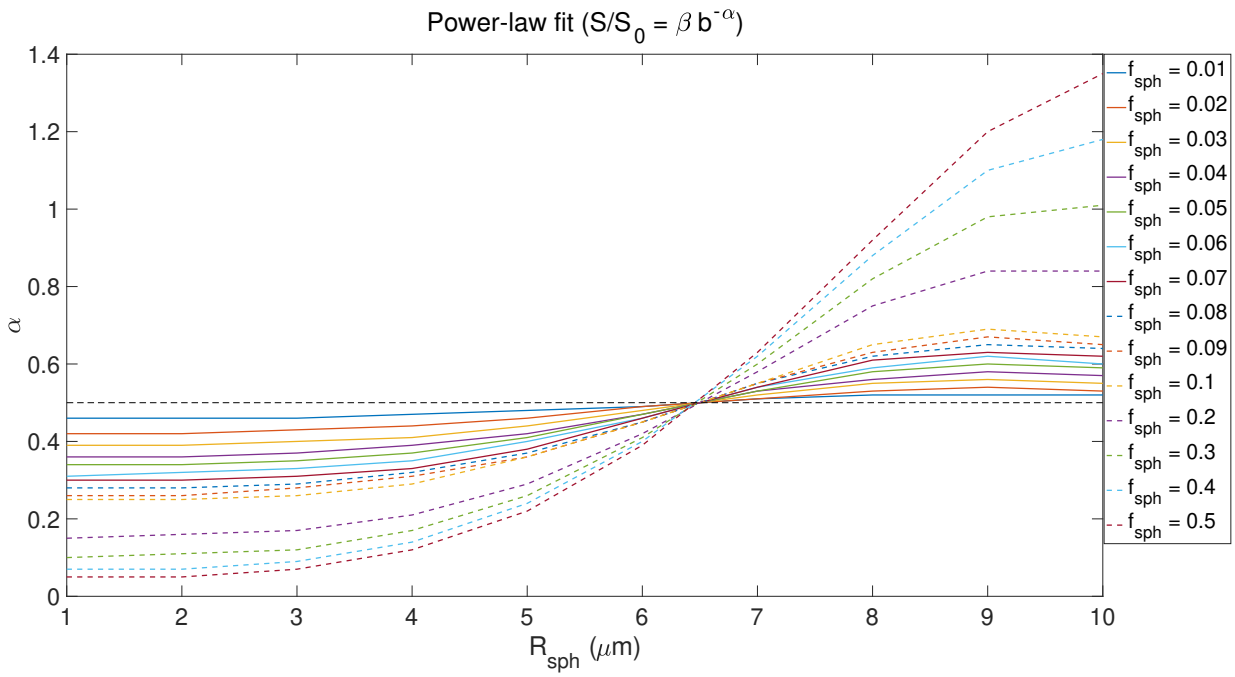

Figure 6: The effect of sphere size and signal fraction on exponent α (similar to Fig. 2 in [16]). ($f_{\text{sph}} = 0.01 : 0.01 : 0.1$, $f_{ball} = f_{\text{stick}} = (1 - f_{\text{sph}})/2$, $D_{\text{in}}^{||} = 2\ \mu m^2/ms$, $D_{\text{ball}} = 2\ \mu m^2/ms$, $R_{\text{sph}} = 1 : 1 : 10\ \mu m$, $\delta = 29.65\ ms$, and $\Delta = 37.05\ ms$).

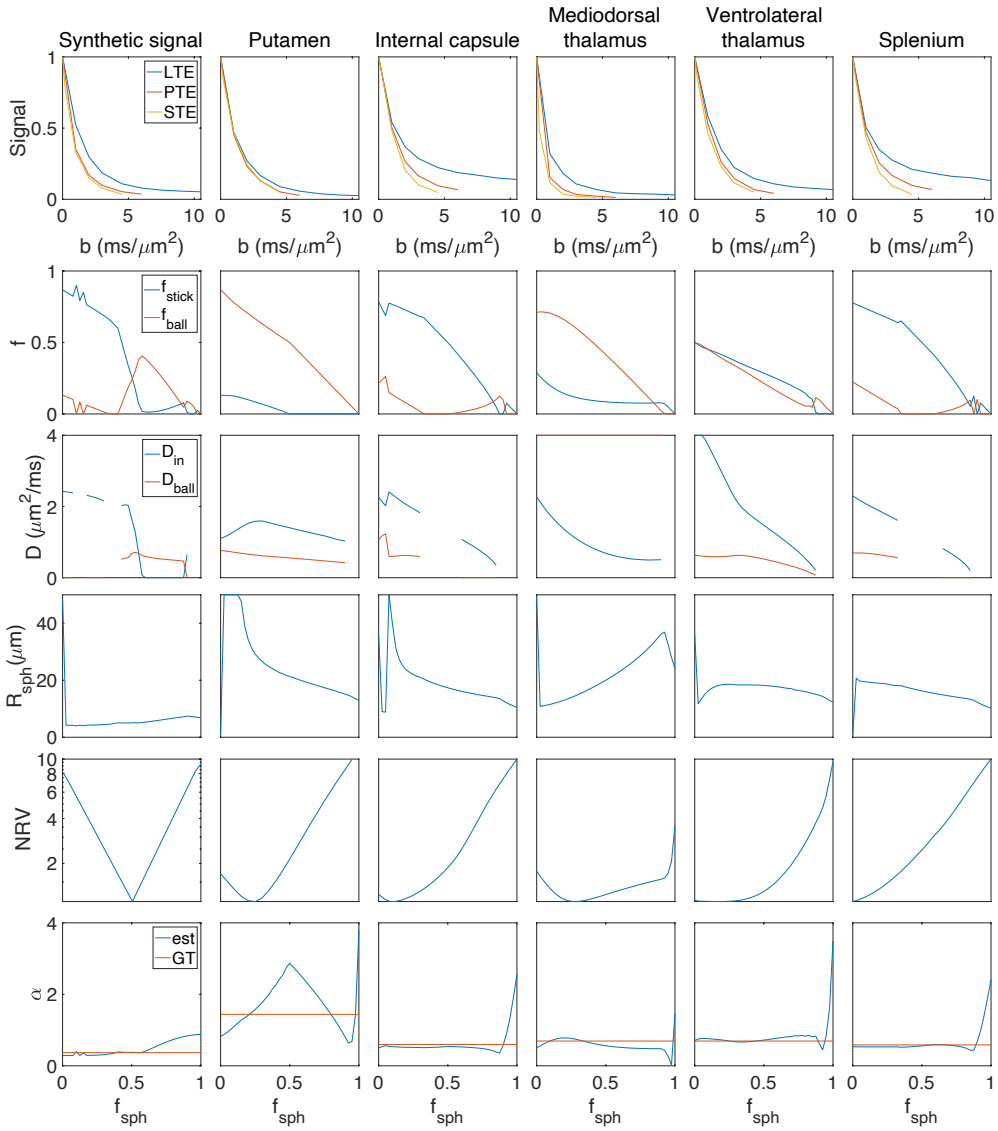

Figure 7: The results of fitting the stick+ball+sphere model to the diffusion-weighted signal by fixing the sphere signal fraction to different values. Five different ROIs of the brain are used here; putamen, internal capsule, mediodorsal thalamus, ventrolateral thalamus, and splenium. The mean value of the direction-averaged signal for each ROI is represented in the first row (in different columns). The second row shows the estimated signal fraction of stick (f_{stick}) and ball (f_{ball}) for different predefined sphere signal fractions (We used f as y-label here to show the signal fraction of both ball and stick in one plot.). The third row illustrates the parallel diffusivity of the stick ($D_{\text{in}}^{||}$) and the diffusivity of the ball (D_{ball}) for different ROIs. The estimated radius of the sphere is illustrated in the fourth row. And finally, the last two rows show how well this model can explain the signal for different predefined sphere signal fractions (f_{sph}) in terms of reduced chi-square and power-law. The first column in the figure shows the results of fitting a synthetic signal. Note that we do not estimate diffusivity of the compartment when the signal fraction is estimated as zero, this is the reason of discontinuity in the plots of estimated diffusivities.

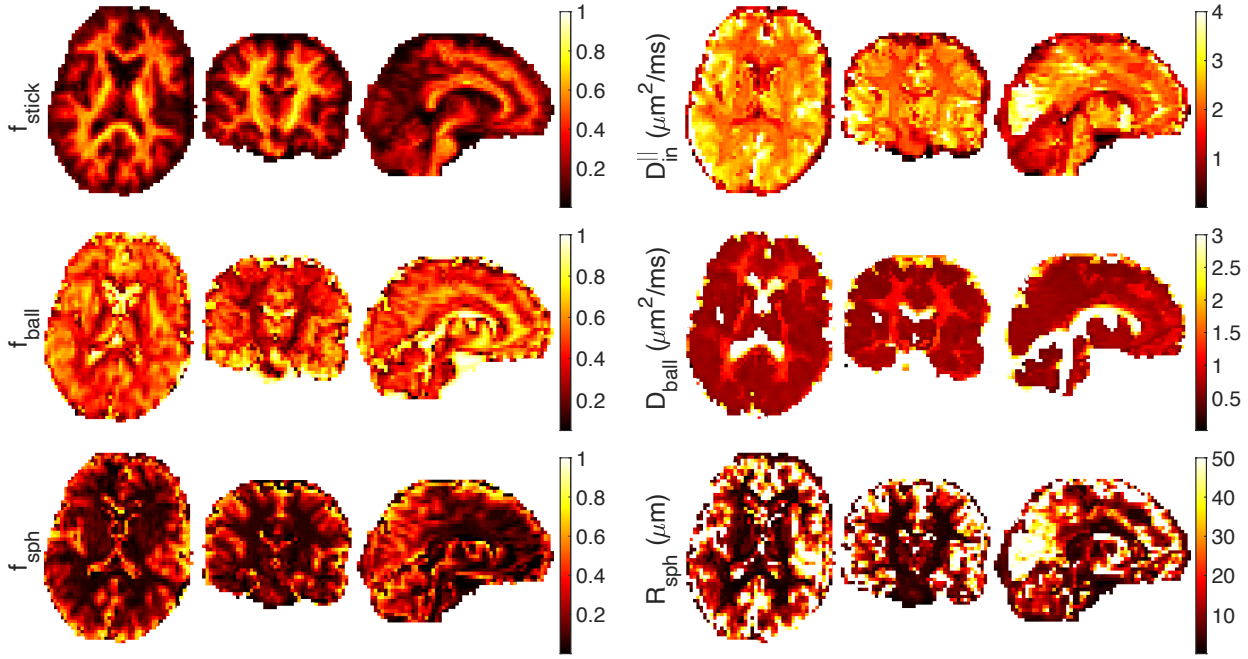

Figure 8: Estimated stick (f_{stick}), ball (f_{ball}), and sphere (f_{sphere}) signal fractions, intra-axonal parallel diffusivity ($D^{||}_{\text{in}}(\mu m^2/ms)$), extra-cellular diffusivity ($D_{\text{ec}}(\mu m^2/ms)$), and sphere radius ($R_{\text{sph}}(\mu m)$) on one slice of the smoothed brain image (A 3D Gaussian kernel with standard deviation of 0.5 is used for smoothing).

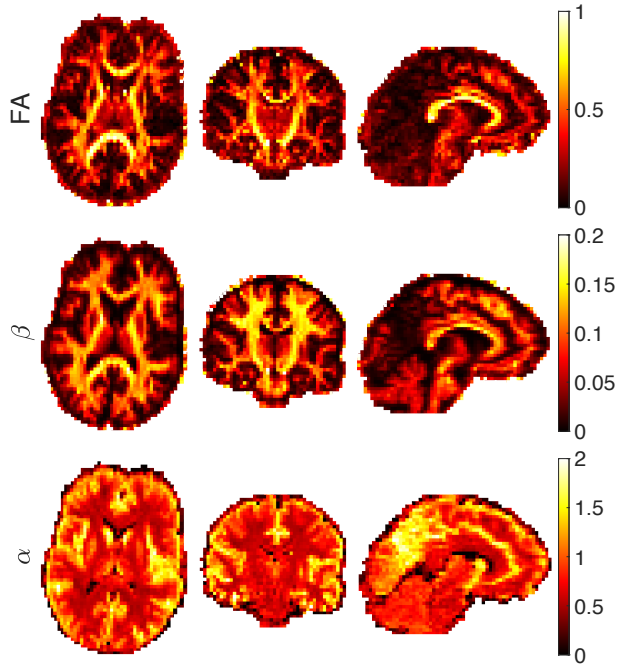

Figure 9: Estimated FA, parameter β and α of the power-law fit ($S/S_0 = \beta b^{-\alpha}$) from axial, coronal, and sagittal views of the brain image.

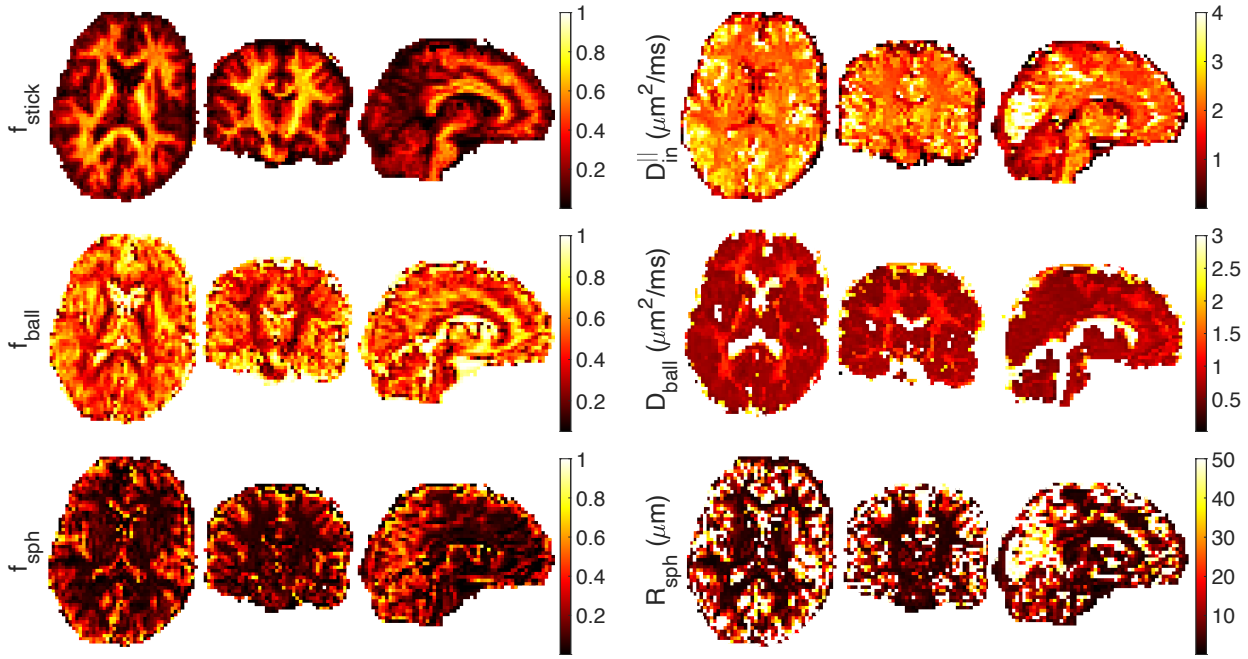

Figure S1: Estimated stick (f_{stick}), ball (f_{ball}), and sphere (f_{sphere}) signal fractions, intra-axonal parallel diffusivity ($D^{||}_{\text{in}}(\mu m^2/ms)$), extra-cellular diffusivity ($D_{\text{ec}}(\mu m^2/ms)$), and sphere radius ($R_{\text{sph}}(\mu m)$) on one slice of the brain image.