LISA: Learned Indexes for Sequence Analysis

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ABSTRACT

Background: Next-generation sequencing (NGS) technologies have enabled affordable sequencing of billions of short DNA fragments at high throughput, paving the way for population-scale genomics. Genomics data analytics at this scale requires overcoming performance bottlenecks, such as searching for short DNA sequences over long reference sequences.

Results: In this paper, we introduce LISA (Learned Indexes for Sequence Analysis), a novel learning-based approach to DNA sequence search. We focus on accelerating two of the most essential flavors of DNA sequence search—*exact search* and *super-maximal exact match (SMEM) search*. LISA builds on and extends FM-index, which is the state-of-the-art technique widely deployed in genomics tools. Experiments with human, animal, and plant genome datasets indicate that LISA achieves up to 2.2× and 10.8× speedups over the state-of-the-art FM-index based implementations for *exact search* and *super-maximal exact match (SMEM) search*, respectively.

Code availability: https://github.com/IntelLabs/Trans-Omics-Acceleration-Library/tree/master/LISA.

KEYWORDS

FM-index, pattern search, short read mapping, DNA sequencing, machine learning, learned index structures, multi-core architectures, data locality

1 BACKGROUND

The latest high-throughput DNA sequencers can read terabases of DNA sequence data per day. For example, the Illumina NovaSeq 6000 sequencer, introduced in January 2017, can read up to 6 terabases of DNA sequence data in a 44-hour run [1]. In particular, it can sequence nearly 20 billion paired-reads, each of length 150 base-pairs, at a cost as low as $600 per genome. The sequencing throughput is increasing and the cost is decreasing at an exponential rate. The recently unveiled MGI DNBSEQ-TX sequencer, which came just 3 years after Novaseq 6000, can sequence at a rate of up to 20 terabases/day, generating reads of length 150, potentially

enabling a $100 cost per genome [2]. Already today, a growing number of public and private sequencing centers with hundreds of NGS deployments are paving the way for population-level genomics. However, realizing this vision in practice heavily relies on building scalable systems for downstream genomics data analysis.

A significant portion of time during the downstream processing of DNA sequence data is spent in searching for DNA sequence queries in a database of reference DNA sequences. This is typically done by building an index of the database to accelerate the search. Recently, in the databases domain, machine learning based index structures (a.k.a., learned indexes) have been shown to accelerate database queries [3]. In this work, we explore the use of learned indexes to accelerate DNA sequence analysis, in particular, DNA sequence search, and present our results for a subset of problems that can potentially benefit from using learned indexes.

Reference-guided assembly plays a critical role in downstream analysis. It is performed by piecing together the short reads (with length of a few hundred bases) by mapping each individual read to a long reference genome (e.g., the human genome consisting of 3 billion bases). Thus, a fundamental step of downstream analysis is mapping of millions of short reads (DNA query sequences) to a long reference sequence. BWA-MEM and Bowtie2 are two of the most widely used tools for sequence mapping [4, 5]. The key operation that has been shown to constitute a significant performance bottleneck during this mapping process is the search for *exact* matches of substrings of reads over the given reference sequence [5–10]. In this work, we focus on two variants of that: 1) *exact search*: search of matches of fixed length substrings of a read in the reference sequence and 2) *super-maximal exact match (SMEM) search*: for every position in the read, search of the longest exact match covering the position. Thus, SMEM search produces exact matches of variable length by definition. We specifically chose these two variants as these are the key kernels in BWA-MEM and Bowtie2.

The state-of-the-art techniques to perform DNA sequence search are based on building an FM-index over the reference sequence [11]. The FM-index implicitly represents the lexicographically sorted order of all suffixes of the indexed sequences. The key idea behind an FM-index is that, in the lexicographically sorted order of all

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suffixes of the reference sequence, all matches of a short DNA sequence (a.k.a., a "query") will fall in a single region matching the prefixes of contiguously located suffixes. Over the years, many improvements have been made to make the FM-index more efficient, leading to several state-of-the-art implementations that are highly cache-and processor-optimized [5, 7–10, 12–19]. Hence, it becomes increasingly more challenging to further improve this critical step in the genomics pipeline to scale with increasing data growth.

In this paper, we propose a machine learning based approach to improving the sequence search performance: LISA (Learned Indexes for Sequence Analysis). The core idea behind LISA, which enables a new machine learning enhanced algorithm for DNA sequence search, is to speed up the process of finding the right region of suffixes in the FM-index by learning the distribution of suffixes in the reference. Recent work on learned index structures has introduced the idea that indexes are essentially models that map input keys to positions and, therefore, can be replaced by other types of models, such as machine learning models [3]. For example, a B-tree index maps a given key to the position of that key in a sorted array. Kraska et al. show that using knowledge of the distribution of keys, we can produce a learned model, that outperforms B-trees in query time and memory footprint [3]. Taking a similar perspective, the FM-index can be seen as a model that maps a given query sequence to the single region matching the prefixes of contiguously located suffixes. In a recent preprint, we introduced LISA with some preliminary results for *exact search* [20]. In this paper, we have extended LISA to *SMEM search* and developed fully architecture-optimized implementations for both problems using multi-threading, vectorization, and efficient cache utilization.

More specifically, we make the following contributions in this paper.

- We demonstrate how *exact search* and *SMEM search* problems can be solved using learned indexes. LISA is the first ever work to do so.
- Since the state-of-the-art algorithms have implementations that are well tuned to the underlying architecture, for a fair comparison, we have developed a fully architecture-optimized implementation of our approach as well. We focus our efforts on the CPU, as that is the most widely available architecture for DNA sequence search.
- We demonstrate the benefits of LISA on an Intel® Xeon® Platinum 8280 processor. LISA achieves up to 2.2× and 10.8× speedups over the state-of-the-art implementations for *exact search* and *super-maximal exact match (SMEM) search*, respectively.

2 RESULTS

We demonstrate the efficacy of LISA by comparing the throughput (million-reads/sec) with FM-index based *exact search* and *SMEM search*. For the baseline comparison, we use the Trans-Omics Acceleration Library (TAL), which provides the architecture optimized implementations for traditional FM-index based *exact search* and *SMEM search* [18, 21, 22]. The optimized SMEM kernel from TAL is also used in BWA-MEM2 [21], an architecture-optimized implementation of BWA-MEM [4]. In order to establish TAL as the appropriate baseline, we first show a comparison with Sapling, a

Table 1: System Configuration

	Intel® Xeon® Platinum 8280 Processor (CLX)
Sockets × Cores × Threads	1 × 28 × 2
AVX register width (bits)	512, 256, 128
Vector Processing Units (VPU)	2/Core
Base Clock Frequency (GHz)	2.7
L1D/L2 Cache (KB)	32/1024
L3 Cache (MB) / Socket	38.5
DRAM (GB) / Socket	96
Bandwidth (GB/s) / Socket	128
Compiler Version	ICC v. 19.1.3.304

learned index based approach for *exact search* that was recently published [19].

2.1 Experimental Setup

2.1.1 System Configuration. We evaluate our solution on a single socket of Intel® Xeon® Platinum 8280 processor as detailed in Table 1 and referred to as CLX from here on. To force all memory allocations to one socket, we use the *numactl* utility [23]. For multi-threaded runs, we use 2 threads per core to get the benefit of hyper-threads. Optimizing file I/O is beyond the scope of this paper. Therefore, we do not include file I/O time in any of our results.

Table 2: Reference Sequences

Reference Sequence	Length (Million bases)	Version
Human	3101	human_g1k_v37
Asian Rice	387	IR64 (IRRI)
Zebra Fish	1679	GRCz11

Table 3: Read datasets used for *SMEM search* experiments

Read Datasets	Organism	Length	No. of Reads	Source (NCBI-SRA)
H1	Human	151	5 million	ERR2990063
H2	Human	151	5 million	ERR3239330
H3	Human	151	5 million	SRR7733443
H4	Human	101	5 million	SRR622461
H5	Human	101	5 million	SRR622457
A1	Asian Rice	151	5 million	SRR10724346
A2	Asian Rice	151	5 million	SRR10838753
A3	Asian Rice	151	5 million	SRR8241153
Z1	Zebra Fish	151	5 million	ERR2624531
Z2	Zebra Fish	151	5 million	ERR3333446
Z3	Zebra Fish	151	5 million	SRR10958316

Table 4: Seed datasets used for *exact search* experiments

Seed Datasets	Length	No. of Seeds	Original Read Dataset
S1	21	50 million	H1
S2	21	50 million	H2
S3	21	50 million	H3
S4	21	50 million	H4
S5	21	50 million	H5
S6	21	50 million	A1
S7	21	50 million	A2
S8	21	50 million	A3
S9	21	50 million	Z1
S10	21	50 million	Z2
S11	21	50 million	Z3

2.1.2 Datasets. We use three reference sequences - Human, Asian Rice, and Zebra Fish, as detailed in Table 2. For each of these reference sequences, we use multiple real read datasets (H1-H5 for Human, A1-A3 for Asian Rice, and Z1-Z3 for Zebra Fish) downloaded from sequence read archive [24] (Table 3). All of these read datasets consist of 5 million reads each. The read datasets for Asian Rice and Zebra Fish have reads of length 151. For Human reference, we use two types of reads datasets: H1-H3 contain of 151 length reads and H4-H5 are of 101 length. The older sequencing technologies produce reads of length 101, so we use these 101 length datasets to show the compatibility of LISA with the older sequencing technologies. For *exact search*, we use 21 length seeds generated from the read datasets as that is the default seed length used in Bowtie2. Seeds are the small fixed-sized substrings generated from a read sequence. For generating seeds, we followed the same strategy as Bowtie2 and generated 50 million seeds for each of the read datasets [5].

2.2 Correctness

In all cases, we have verified that output of the LISA-based approach is identical to that of the traditional FM-index based approach.

2.3 Establishing the Baseline

Here, we compare the execution time of LISA and TAL with the recently published Sapling for *exact search* [19]. Sapling demonstrated a speedup of over 2× over Bowtie, Mummer, and an optimized implementation of binary search. Therefore, we omit any comparison with Bowtie, Mummer, and binary search. Moreover, Sapling is single threaded. Therefore, we compare the performance of the three implementations using only a single thread. We have used the same evaluation method as used in the Sapling paper. We use the scripts provided with the Sapling source code to generate 50 million seeds of length 21 for the three reference sequences [25]. The script ensures that there is at least one match of the generated seeds.

Figure 1 shows the comparison. Note that the time reported for Sapling here is more than 2× less than that reported in [19] potentially due to a difference in architecture. Moreover, Sapling does not return the full interval in the suffix array in which the query matches, but rather returns one position in the interval – one needs to perform a search on both sides of the position to get

Figure 1: Performance of the *exact search* for Sapling, TAL, and LISA using a single thread.

the interval. So, the time required for Sapling based method for realistic usage would be higher than reported here. We could not run Sapling for human genome as it ran out of memory even when run on a different machine with 256 GB DRAM. The time reported for TAL and LISA is the time spent in getting the full interval.

Figure 1 shows that TAL is 3.5× and 4.2× faster than Sapling and LISA is 6.4× and 7.4× faster than Sapling, respectively, for Asian Rice and Zebra Fish. Therefore, for the rest of the paper, we only compare LISA with TAL.

2.4 Performance Evaluation

*2.4.1 **Exact Search**.* Figures 2a and 2b show the throughput achieved by LISA and TAL for *exact search* on a single thread and single socket, respectively, across different reference sequences and the read datasets. The x-axis represents the reference sequence and the datasets, and the throughput is shown on the y-axis (the higher the better). Observe that LISA outperforms TAL across all datasets.

Exact search finds all end-to-end matches of 21-length seeds. Recall that the traditional FM-index based search matches one base at a time against the reference sequence and therefore takes 21 steps for end-to-end matching of 21-length seed. LISA processes a whole 21-length seed in one shot and finds its matches in a single step. As a result, LISA achieves $1.3 - 2.2\times$ higher throughput than TAL.

*2.4.2 **SMEM Search**.* Figures 2c and 2d show the throughput comparison for *SMEM search*. LISA achieves $3.9 - 10.8\times$ higher throughput than TAL. On a single threaded execution, LISA achieves, on an average, 7.1× speedup over TAL across all datasets. On a multi-threaded execution, LISA achieves, on an average, 5.1× speedup over TAL.

Although LISA outperforms TAL across all datasets, the performance gain varies across datasets. The nature of reads and the reference sequences affect the overall performance gain. For instance, a read dataset with longer matching SMEMs is a better candidate for LISA than the one with the shorter matches. In Figures 2c and 2d the average length of the matches in H1 is 45, whereas in H3, the average length is 26.

Figure 2: Performance evaluation for exact search and SMEM search problem on a single thread and single socket with 28 cores. Throughput gain for each dataset is shown on the top of LISA bars.

3 CONCLUSIONS AND FUTURE WORK

Creating an index of the database appears as a motif in many areas in computational biology including genomics, transcriptomics, and proteomics. In this paper, we presented LISA - a machine learning based approach to index a database of DNA sequence to accelerate DNA sequence search. We demonstrated the benefits of our approach through two specific variants of DNA sequence search - *exact search* and *SMEM search*, and show up to 2.2× and 10.8× speedup, respectively. As future work, we plan to extend the ideas presented in this paper to many other problems in computational biology in which an index is created to accelerate search through a database. In particular, hash tables are a prime candidate for acceleration through the learned approach.

4 METHODS

In this section, we first define the notations and formally define the two DNA sequence search problems: *exact search* and *SMEM search*. Subsequently, we briefly describe the prior work and established methods to solve these problems. We provide the background on learned indexes followed by description of how we use learned indexes to solve the two DNA sequence search problems. For both exact search and SMEM search problems, an index of the reference sequence is required. Therefore, for all the algorithms in this section, the applicable index over the reference sequence is provided as input even if not explicitly stated, and we skip listing them for the sake of brevity.

4.1 Notation

We use upper case letters like X, Y to denote a DNA sequence, modeled as a string over the alphabet $\Sigma = \{A,C,G,T\}$, representing the four bases, and lower case letters like x, y to denote bases. Let $|X|$ denote the length of X, $X[i]$ denote the base at position i, and $X[i, j]$ ($i \leq j$) denote the substring $X[i]X[i + 1]X[i + 2] \cdots X[j]$. We use $y.X$ and $X.y$ to represent prepending and appending, respectively, of base y to DNA sequence X and $X.Y$ to denote the DNA sequence formed by concatenating sequences X and Y. We use y^n to denote a sequence composed of n contiguous copies of base y; e.g. T^3 is TTT.

4.2 The Exact Search Problem

Given a reference sequence R and a query sequence Q, the goal of exact sequence search is to find exact end-to-end matches of Q in R. More formally, an occurrence of Q is a position p in R, such that

$$0 \leq p, p + |Q| \leq |R|, \forall j \in [0, |Q|), R[p + j] = Q[j] \qquad (1)$$

Exact search finds all such occurrences of Q in R. Typically, $|R| \approx 10^9$ bases; e.g., the length of the human genome is nearly 3×10^9. On the other hand, $|Q|$ typically ranges from a few tens to a few hundred bases; e.g., the default query length in Bowtie2 for *exact search* is 21.

4.3 FM-Index

The most widely used method to perform DNA sequence search is by creating an FM-index of the reference and searching for the query using the FM-index.

Figure 3: FM-index, $F = \langle S, B, D, O \rangle$ and BW-Matrix for sample reference sequence $R \leftarrow$ ATACGAC\$. The lexicographical ordering is \$$< A < C < G < T$ [18].

Fig. 3 depicts the construction of the FM-index for an example reference sequence R. First, we append R with the character $\$ \notin \Sigma$ which is lexicographically smaller than all characters in Σ. Subsequently, we obtain all the rotations of R (Rotations(R)). The lexicographically sorted order of the rotations forms the BW-Matrix (Burrows Wheeler Matrix). The Burrows Wheeler Transform, a.k.a. BWT (B), is the last column of the BW-Matrix. The original positions in R of the first bases of these rotations constitute the suffix array (S).

All the exact matches of a query can be found as prefixes of the rotations in the BW-Matrix. Since the BW-Matrix is lexicographically sorted, these matches are located in contiguous rows of the BW-Matrix. Therefore, for a query, all the matches can be represented as a range of rows of the BW-Matrix. This range is called the *SA (Suffix Array) interval* of the query. For example, in Fig. 3, the *SA interval* of query *"AC"* is [1, 3). The values of the suffix array in the SA interval are 5 and 2. Indeed, the sequence *"AC"* is found at positions 5 and 2 in the reference sequence.

The FM-index is used to expedite search for the SA interval [11]. FM-index of a reference sequence R, F be defined as the tuple $\langle S, B, D, O \rangle$, consisting of the suffix array S and the BWT B, as well as D and O data structures. $D(x)$ is the count of bases in $R[0, |R|-1]$ that are lexicographically smaller than $x \in \Sigma$. $O(x, i)$ is the count of occurrences of base x in $B[0, i]$. Note that the BW-Matrix is not stored. For further information on the FM-index, see [11].

Algorithm 1 Exact Search Algorithm using FM-index

Input: FM-index F ($\langle S, B, D, O \rangle$) of reference R; Query Q
Output: SA interval of Q in R, $[l, r]$

1: **function** BACKWARD_SEARCH(F, Q)
2: $\quad i \leftarrow |Q| - 1$
3: $\quad l, r \leftarrow 1, |R|$
4: $\quad$ **while** $i \geq 0$ **do**
5: $\quad\quad i, l, r \leftarrow i - 1, f(Q[i], l), f(Q[i], r)$
6: $\quad\quad$ **if** $l \geq r$ **then return** ϕ
7: $\quad$ **return** $[l, r]$

4.4 Exact Search using FM-Index

Exact search based on FM-index is performed using backward search algorithm (Alg. 1) [10]. The query is processed from the end to the beginning. For each value of i, the algorithm finds the SA interval $[l, r)$ of $Q[i, |Q| - 1]$ in R by performing backward extension (line 5) on *SA interval* of $Q[i + 1, |Q| - 1]$ by $Q[i]$. The backward extension is performed using the function $f : (\Sigma, \mathbb{Z}) \rightarrow \mathbb{Z}$ supported by the FM-index that takes a base y and an integer location l that is the lower bound of the em SA interval of a DNA sequence X, and finds in $O(1)$ time the lower-bound of the em SA interval of a DNA sequence $y.X$, such that

$$f(y, l) = D(y) + O(y, l - 1) \tag{2}$$

The width of the *SA interval* either shrinks or remains the same at each step. If the algorithm terminates before reaching the beginning of the query, Q does not occur in R (line 8). Otherwise, the range corresponds to the positions of exact occurrences of Q in R (line 9). Clearly, backward search algorithm requires at most $|Q|$ backward extensions, each of which consumes constant time. Hence, the time complexity of the backward search algorithm is $O(|Q|)$.

4.5 Learned Indexes

Recent work on learned index structures has introduced the idea that indexes are essentially models that map inputs to positions and, therefore, can be replaced by other types of models, such as machine learning models [3]. For example, a B-tree index maps a given key to the position of that key in a sorted array. Kraska et al. show that using knowledge of the distribution of keys can produce a learned model, e.g., the recursive model index (RMI), that outperforms B-trees in query time and memory footprint. Following [3], other learned index structures have been proposed [26–29].

Taking a similar perspective, the FM-index can be seen as a model that maps a given query sequence to the SA interval for that query sequence. Based on this insight, in LISA, we use knowledge of the distribution of subsequences within the reference sequence to create a learned index structure that enables faster exact and SMEM search.

4.6 Exact-LISA: Exact Search using LISA

In this section, we describe how LISA combines a new index data structure with learned indexes to improve the performance of exact search. We define our LISA index used for exact search as the tuple, $L = \langle F, IPBWT, RMI \rangle$, where F is the FM-index and we will define *IPBWT* and *RMI* below.

Algorithm 2 Exact-LISA: Exact Search Algorithm using IPBWT

Input: $f^K : (\Sigma^K, \mathbb{Z}) \to \mathbb{Z}$, a function that finds the lower-bound location of $(\Sigma^K, \mathbb{Z})$ in the IPBWT of reference R; Query Q
Output: SA interval of Q in R, $[l, r]$.

1: **function** LISA_BACKWARD_SEARCH(f^K, Q)
2:     $l, r \leftarrow 0, |R|$
3:     split Q into $\left\lceil \frac{|Q|}{K} \right\rceil$ chunks, each of length K, with the last chunk possibly shorter than K
4:     **for** Y in reversed order of chunks **do**
5:         **if** $|Y| < K$ **then**   ▷ when last chunk has length $< K$
6:             $Y_l \leftarrow Y.\$.A^{K-|Y|-1}$
7:             $Y_r \leftarrow Y.T^{K-|Y|}$
8:             $l, r \leftarrow f^K((Y_l, l)), f^K((Y_r, r))$
9:         **else**
10:            $l, r \leftarrow f^K((Y, l)), f^K((Y, r))$
11:         **if** $l \geq r$ **then return** ϕ
12:     **return** $[l, r)$

Backward-search algorithm performs exact search of a query Q using the FM-index by iterating through the query sequence in backwards order, one base at a time, thereby consuming $O(|Q|)$ steps. The key idea of LISA is to iterate backwards through the query sequence in chunks of K bases at a time, so that exact search takes $O(\frac{|Q|}{K})$ steps. Similar to f, we need to support a function, $f^K : (\Sigma^K, \mathbb{Z}) \to \mathbb{Z}$, that takes in a length-K DNA sequence Y and the lower bound, l, of the *SA interval* of a DNA sequence X and returns the lower bound, l', of the *SA interval* of a DNA sequence $Y.X$. FM-Index can be redesigned to process K bases at a time by replacing Σ with Σ^K as the alphabet. However, it would require memory proportional to 4^K, and hence is infeasible.

4.6.1 **IPBWT**. In LISA, to enable the search for K bases at a time, we designed a new data structure named **Index-Paired BWT (IPBWT)**. For each row of the BW-Matrix, there is an entry in IPBWT consisting of a $\langle \Sigma^K, \mathbb{Z} \rangle$ pair. The first part is the first K bases of the corresponding BW-Matrix row. The second part is the BW-Matrix location of the DNA sequence with the first K and the last $n-K$ bases swapped. For two IPBWT entries $\langle X, l \rangle$ and $\langle X', l' \rangle$, we define

$$\langle X, l \rangle = \langle X', l' \rangle \Leftrightarrow X = Y \text{ and } l = l' \tag{3}$$

$$\langle X, l \rangle < \langle X', l' \rangle \Leftrightarrow X < Y \text{ or } X = Y \text{ and } l < l' \tag{4}$$

Note that, since the BW-Matrix is in a lexicographically sorted order, the IPBWT is also in a sorted order. Our desired function $f^K : (\Sigma^K, \mathbb{Z}) \to \mathbb{Z}$ is now equivalent to finding the lower bound location of an input $\langle Y, l \rangle$ in the IPBWT; i.e.; the first entry in IPBWT that does not compare less than $\langle Y, l \rangle$. We are free to choose any implementation for how to find that lower-bound location; for example, since the IPBWT is sorted, we could do a binary search over the entries of the IPBWT. Fig. 4a shows how to create an IPBWT with $K = 3$.

Given f^K, we can define Alg. 2 to perform *exact search* using IPBWT. For example, using the reference sequence and IPBWT from Fig. 4a, let the query sequence be ATTA. We split this into two chunks: ATT and A (line 3). We first use the RMI to find the

lower bound locations of $(A\$A, 0)$ and $(ATT, |R|)$, which are 1 and 5, respectively. Subsequently, we use the RMI to find the lower bound locations of $(ATT, 1)$ and $(ATT, 5)$, which are 3 and 5. Our algorithm gives the interval $[3, 5)$. We can confirm that ATTA can be found in position 3 and 4 of the BW-Matrix.

4.6.2 **Faster Chunk Processing using RMI**. Using the IPBWT, we are able to process the query sequence in chunks of K bases at a time. However, when processing each chunk, we must evaluate the function f^K. Using a binary search over the IPBWT takes $O(\log |R|)$ time. Therefore, the overall runtime of *exact search* using IPBWT with binary search for f^K is $O(\frac{|Q|}{K} \log |R|)$. For large reference sequences, this might be slower than *backward-search* using the FM-index. Therefore, we use a learned approach to accelerate f^K. In particular, f^K is a model that maps an input key – $\langle Y, l \rangle$ – to its position in the sorted IPBWT. Therefore, it can be replaced by a machine learning model. Based on this insight, in LISA, we use knowledge of the distribution of subsequences within the reference sequence to create a learned index structure to model f^K. We experimented with three of the prominent learned index structures – Recursive Model index (RMI) [3, 30], Piecewise Geometric Model Index (PGM) [27, 31], and Radix Spline index (RS) [32]. We found that a 2-layer RMI worked the best for our application, which is in line with the recently published benchmarking study comparing various learned indexes [33, 34].

Therefore, we model f^K using an RMI, which is a hierarchy of models that are quick to evaluate [3]; the RMI conceptually resembles a hierarchical mixture of experts [35]. A 2-layer RMI – that only has root and leaf layers – has a single model at the root layer. For any input key, the model at root layer is used to predict the correct model to use at the leaf layer. The predicted model at the leaf layer is used to predict position of the key in a sorted array and a range of positions in which the key is guaranteed to occur. If the key is not found in the predicted position, a last mile search is conducted in the provided range to find the key. Fig. 4b illustrates, with an example, how we use a 2-layer RMI to implement f^K in three steps:

(1) Since the RMI only accepts numbers as keys, we first convert the input $\langle Y, l \rangle$ into a number. Since there are only 4 bases, each base can be represented using 2 bits. Therefore, we convert the DNA sequence Y into an integer, $I(Y)$, with $2K$ bits by concatenating the bits of the individual bases together and creating the key as $I(Y) + \frac{l}{\text{maximum value of } l}$, which is a double precision (64 bit) floating point number.
(2) We give the encoded key to the RMI and traverse down the layers of the RMI to a leaf model. The leaf model predicts the position in the IPBWT where it expects to find the input key and a range in which we are guaranteed to find the input key.
(3) If the predicted position does not contain the input pair, we use binary search over the range to find the actual position of the key.

Note that this learning-based approach to modeling f^K guarantees correctness; LISA will produce exactly the same results as using backward search with FM-index. Instead of a floating point key, we can also create an integer representation as key by just

(a) Conversion of a BW-Matrix to IPBWT on reference CATTATTAGGA, where $K = 3$.

(b) Using a 2-layer RMI to evaluate f^K for input $(GCC, 39)$. **The RMI predicts position** 52, **and binary search finds the correct position** 54. **The solid lines show the traversal path down the RMI.**

Figure 4: Components of the LISA-based *exact search* algorithm

concatenating the bits of l to $I(Y)$, but that may result in keys of length greater than 64 bits and will need more than one step to compare – comparing 64 bits in each step. Another option is to use the most significant 64 bits of the integer representation as key, but that results in losing differentiating information when the most significant 64 bits are zero. Therefore, we represent the key as a double-precision (64 bits) floating point number and train the model over an array where each entry is a double-precision representation of the corresponding entry in the IPBWT. However, we perform last-mile search over the original IPBWT so as to ensure correctness. Note that we have a special case for handling the sentinel letter $ while maintaining this 2-bit encoding.

*4.6.3 **Parameter Selection**.* The number of leaf nodes is an important parameter for the RMI, as shown in Table 5. Larger number of leaf nodes results in better accuracy and smaller ranges for last mile search – thereby, smaller number of iterations of the binary search and better throughput – at the cost of larger memory consumption. Another important parameter is K, as shown in Table 6. For this experiment, we used a seed length that is divisible by the values of K to ensure that no length is at a disadvantage. Increasing K results in smaller number of steps of binary search, but would consume more memory. Moreover, 64 bits may not be sufficient to represent the corresponding keys for very large values of K, which might lead to information loss and worse model accuracy resulting in longer ranges to perform binary search in.

*4.6.4 **Performance Comparison with TAL using Synthetic Queries**.* In case of real datasets, few queries may not find a match in the reference. In those cases, we may not need to process the entire query. The FM-index based approach (Alg. 1) can stop as soon as an additional base results in no matches, while the LISA-based approach can stop only after a chunk is processed and returns no matches, thereby, processing more bases than the FM-index based approach. While LISA gets significant performance gain over FM-index based approach for real queries as shown in Sec. 2, it

Table 5: Effect of number of RMI leaf nodes. For this experiment, we use synthetic seeds of length 21 **that are sampled from the corresponding reference genome sequence. We report the throughput in Million Seeds Per Second (MSPS), average number of binary search steps per last-mile search and memory footprint of RMI with respect to the increase in the number of RMI leaf nodes.**

Reference genome	# RMI leaves	Throughput (MSPS)	# Binary search steps	Memory footprint (GB)
Human	2^{25}	6.8	7.38	0.8
	2^{26}	7.1	6.74	1.6
	2^{27}	7.5	6.28	3.2
	2^{28}	7.8	5.82	6.4
	2^{29}	8.0	5.53	12.8
Asian Rice	2^{21}	7.3	7.14	0.05
	2^{22}	7.8	6.45	0.10
	2^{23}	8.2	5.89	0.20
	2^{24}	8.6	5.37	0.40
	2^{25}	8.9	5.00	0.80
Zebra Fish	2^{24}	6.7	7.58	0.40
	2^{25}	7.1	7.08	0.80
	2^{26}	7.4	6.53	1.6
	2^{27}	7.7	6.15	3.2
	2^{28}	8.0	5.75	6.4

can achieve even higher speedup if all the queries have a match. Therefore, to showcase the maximum benefit of LISA, here we perform experiments using synthetic queries directly extracted from the corresponding reference sequence. We use queries of length 21, as that is the default length of seeds for Bowtie2. Fig. 5 shows that LISA achieves up to 2.3× and 2.8× speedup over TAL on a single thread and single CPU socket, respectively.

(a) Single thread	(b) Single socket (28 cores)

Figure 5: Performance evaluation for *exact search* problem on synthetic queries. For each experiment, we extracted 50 Million synthetic seeds of length 21 from the corresponding genomes. The speedup of LISA over TAL is shown at the top of each LISA bar. $K = 21$ and # of leaf nodes is 2^{27}, 2^{23} and 2^{26} for Human, Asian Rice and Zebra Fish, respectively.

Table 6: Effect of K. For this experiment, we compare three values of K (12, 15 and 20) by using synthetic seeds of length 120 (which is divisible by all three values) that are sampled from the corresponding reference genome sequence. We report the throughput in Million Seeds Per Second (MSPS), average number of binary search steps per seed and memory footprint of IPBWT with respect to the increase in the value of K.

Ref. Seq	K	Throughput (MSPS)	# Binary search steps	Memory footprint
Human	12	0.75	113	25 GB
	15	0.91	98	28 GB
	20	1.20	75	31 GB
Asian Rice	12	0.81	108	3.1 GB
	15	1.00	91	3.4 GB
	20	1.30	71	3.8 GB

4.7 The SMEM Search Problem

SMEM search finds all the SMEMs between a query and a reference sequence as defined in the following. A *maximal exact match* (MEM) is an exact match between substrings of two sequences that cannot be further extended in either direction. An SMEM is a MEM that is not contained in any other MEMs on the query sequence [36]. For reference sequence R and query sequence Q, we say $Q[i, j]$ is an SMEM exactly when $Q[i, j]$ has one or more matches in R but $\forall i', j'$ such that $i' < i, j' > j$, it is the case that $Q[i', j]$, $Q[i, j']$ and $Q[i', j']$ have no matches. Therefore, no two SMEMs can share the same upper-bound or lower-bound.

Typically, $|R| \approx 10^9$ bases; e.g., the length of the human genome is nearly 3×10^9. On the other hand, $|Q|$ is typically a few hundred bases (< 300 bases); e.g., a common query length in BWA-MEM for *SMEM search* is 151. To cover both the strands of the DNA, we look for matches of $|Q|$ in both R and the reverse complement of R.

4.8 Prior work related to SMEM Search

The algorithm for *SMEM Search* [4, 36] in BWA-MEM uses a bidirectional FM-index in which R is concatenated with the reverse complement of R and the FM-index of the concatenated sequence is built. The algorithm performs the following steps.

(1) It begins at the end of the query, backward extending by one letter at a time until there is at least one exact match. For backward extension, it uses FM-index employing an algorithm similar to Alg. 1. At the end of backward extension, it returns the corresponding match found – as an SMEM.
(2) Then it starts building the next SMEM from the position right next to the SMEM found. First, it forward extends using the benefit of bidirectional FM-index to reduce the problem of forward extension to that of backward extension. It performs forward extension until there is at least one match, while maintaining all the intermediate forward extensions. For each of the intermediate forward extensions, it performs backward extension and outputs the longest match as SMEM.
(3) It performs step 2 repeatedly until the entire query is processed.

Due to step 2, the algorithm has $O(|Q|^2)$ time complexity.

The search of Maximal Exact Matches (MEM), a closely related problem, has been well studied in literature using suffix arrays, sparse suffix arrays, FM-indexes, k-mer indexes, bloom filters, etc. [37–44]. In particular, [37] proposes the use of enhanced suffix arrays [45], a combination of FM-index and lcp-interval tree, to search for MEMs. For our LISA based SMEM algorithm – we (1) adapt the algorithm proposed in [37] to the SMEM search problem to develop an $O(|Q|)$ algorithm to search for SMEMs – to the best of our knowledge, no prior work has proposed an $O(|Q|)$ algorithm for SMEM search, (2) apply the learned approach and (3) develop an efficient hardware-aware implementation – to achieve significant performance gains.

4.9 SMEM-LISA: SMEM Search base on LISA

In this section, we first introduce a few properties of SMEM key to our algorithmic discussion. Subsequently, we briefly outline our new algorithm along with the high-level idea behind it. Finally, we give a detailed description of our algorithm. We define our LISA index used for SMEM search as the tuple, $L = \langle F, IPBWT, RMI, T, LCP \rangle$, where F, $IPBWT$, and RMI are as defined above and T and LCP will be defined in this section.

4.9.1 Monotonicity and Consistency Properties of SMEMs.
Compared to the FM-index based SMEM algorithm, where SMEMs are found by extending candidate substrings forward and backward, our new algorithm extends as well as 'shrinks' substrings to allow for efficient SMEM traversal. Before we describe the algorithm, we present a few key observations about the underlying structure of SMEMs:

DEFINITION. *For query sequence Q, we denote a substring of Q, $Q[i, j]$, and its SA interval, $[l, r)$, as a tuple $\langle i, j, l, r \rangle$.*

LEMMA (THE MONOTONICITY PROPERTY OF SMEM). *For any two different SMEMs $Q[i_1, j_1]$ and $Q[i_2, j_2]$, we have $i_1 < i_2 \iff j_1 < j_2$.*

PROOF. Without loss of generality, say $i_1 < i_2$. If $j_1 \geq j_2$, then $Q[i_2, j_2]$ is contained in $Q[i_1, j_1]$ and thus, can not be an SMEM. □

From this, we know that SMEMs appear in 'strictly increasing' order; i increases exactly if j increases. Therefore we can order them with either i or j (both are equivalent), and define the notion of adjacency.

DEFINITION. *We say two different SMEMs $Q[i_1, j_1]$, $Q[i_2, j_2]$ are* **adjacent** *if there doesn't exist another SMEM $Q[i_3, j_3]$ such that i_3 is between i_1 and i_2. If $i_1 < i_2$, we say $Q[i_1, j_1]$ is the* **left-adjacent** *SMEM of $Q[i_2, j_2]$, and $Q[i_2, j_2]$ is the* **right-adjacent** *SMEM of $Q[i_1, j_1]$.*

LEMMA (THE CONSISTENCY PROPERTY OF SMEM). *Consider any two adjacent SMEMs $Q[i_1, j_1]$ and $Q[i_2, j_2]$, where $i_1 < i_2$. For all $j \in [j_1 + 1, j_2]$, we have that $Q[i_2 - 1, j]$ has no match in R.*

PROOF. If $Q[i_2 - 1, j]$ has a match in R, then there exists some SMEM $Q[i^*, j^*]$ s.t. $i^* \leq i_2 - 1$ and $j^* \geq j$. By the monotonicity property, $i^* \leq i_2 - 1 \implies j^* < j_2$. Also, because $Q[i_1, j_1]$ is left-adjacent to $Q[i_2, j_2]$, we have $j^* < j_2 \implies j^* \leq j_1$. But $j^* \geq j \geq j_1 + 1$. □

The Consistency Property is the most crucial observation to our new algorithm. It says that if we begin with $Q[i = i_2, j = j_2]$ and start decrementing j, the lower-bound i_2 will remain the lowest possible bound to form a match, until we reach $j = j_1$. This suggests an efficient way to traverse adjacent SMEMs. Say we are at $Q[i = i_2, j = j_2]$ and want to move to its left-adjacent SMEM $Q[i_1, j_1]$. What we can do is keep decreasing j while testing if $Q[i - 1, j]$ still has no match in R. The first j yielding one or more matches for $Q[i - 1, j]$ in R must be j_1. Thus we have reached $Q[i = i_2, j = j_1]$. Then we can simply apply backward extension to reach $Q[i = i_1, j = j_1]$.

Algorithm 3 SMEM Algorithm using Extend and Shrink

Input: Query Q
Output: The sets of all SMEMs of Q in R, $\{Q[i, j]\}$, and the corresponding SA intervals $\{[l, r)\}$

1: **function** LISA_SMEM_SEARCH(Q)
2:     $SMEMS \leftarrow \emptyset$
3:     $i, j, l, r \leftarrow |Q|, |Q| - 1, 0, |R|$ ▷ note that $Q[|Q|, |Q| - 1]$ is an empty string
4:     $\langle i, j, l, r \rangle \leftarrow$ EXTEND-PHASE-K-AT-A-TIME($Q, \langle i, j, l, r \rangle$)
5:     $\langle i, j, l, r \rangle \leftarrow$ EXTEND-PHASE-1-AT-A-TIME($Q, \langle i, j, l, r \rangle$)
6:     $SMEMS \leftarrow SMEMS \cup \{\langle i, j, l, r \rangle\}$
7:     **while** $i \neq 0$ **do**
8:         $\langle i, j, l, r \rangle \leftarrow$ SHRINK-PHASE($Q, \langle i, j, l, r \rangle$)
9:         $\langle i, j, l, r \rangle \leftarrow$ EXTEND-PHASE-1-AT-A-TIME($Q, \langle i, j, l, r \rangle$)
10:        $SMEMS \leftarrow SMEMS \cup \{\langle i, j, l, r \rangle\}$
11:     **return** $SMEMS$

4.9.2 The New SMEM Algorithm.
Alg. 3 utilizes the Consistency Property to find SMEMs of a query. Say Q has n SMEMs, in increasing order of i: $Q[i_1, j_1], Q[i_2, j_2], \ldots, Q[i_n, j_n]$, where $i_1 = 0$ and $j_n = |Q| - 1$. We start from $i = |Q|$ and $j = j_n = |Q| - 1$ (line #3). We start with an extend phase (lines #4-5) and then alternate between a shrink phase (line #8) and an extend phase (line #9) in a while loop (lines #7-10). Each extend phase, given $Q[i_p, j_{p-1}]$, finds us $Q[i_{p-1}, j_{p-1}]$, and shrink phase, given $Q[i_p, j_p]$, finds us $Q[i_p, j_{p-1}]$. Therefore, the while loop traverses all SMEMs in reverse order.

4.9.3 Extend Phase.
The task of the extend phase is fairly straightforward. Since the upper-bound j of the SMEM is known, we can simply extend leftward until right before no match. The first extend phase may be long as we are starting from scratch and need to extend until we find one SMEM. All subsequent extensions are done after a shrink phase and start with a partial match already and thus, may not need long extensions. Based on this insight, in the first extend phase, we use the LISA index to extend in chunks of K bases at a time until we can not extend by K bases anymore (line #4). Subsequently, we extend by one base at a time until there are matches (line #5). For extensions after shrink phases, we only use the FM-index based one base at a time method (line #9). Alg. 4 and Alg. 5 detail how we extend one base at a time using FM-index and K bases at a time using LISA, respectively. These are similar to Alg. 1 and Alg. 2, respectively, but instead of returning ϕ when they can't match the entire query, they return the positions up to which they could find matches.

4.9.4 Shrink Phase.
Given an SMEM, $Q[i, j]$, according to the Consistency Property, we need to find the largest index $j' < j$ such that $Q[i, j']$ is left extendable (that is, $Q[i - 1, j']$ has a match in R) and the suffix interval of $Q[i, j']$. This requires us to support two functionalities:

- For any $Q[i, j]$, find if it is extendable to the left.
- Given $Q[i, j]$ and its SA interval $[l, r)$, find the SA interval of $Q[i, j']$.

LEMMA (THE EXTENDABILITY PROPERTY). *For a string $Q[i, j]$ with SA interval $[l, r)$, if we keep removing bases from the right, its left*

Algorithm 4 Algorithm for extending one base at a time

Input: Query Q; $\langle i, j, l, r \rangle$ where $Q[i, j]$ has a match in R with SA interval $[l, r)$ and there exists an SMEM, $Q[i', j]$; clearly, $i' \leq i$
Output: SMEM $Q[i', j]$ and its SA interval $[l', r')$

1: **function** Extend-Phase-1-at-a-time($Q, \langle i, j, l, r \rangle$)
2:     $i', l', r' \leftarrow i, l, r$
3:     **while** $i' \geq 0$ and $f(Q[i'], l') < f(Q[i'], r')$ **do**
4:         $i', l', r' \leftarrow i' - 1, f(Q[i'], l'), f(Q[i'], r')$
5:     **return** $\langle i', j, l', r' \rangle$

Algorithm 5 Algorithm for extending K bases at a time

Input: Query Q; $\langle i, j, l, r \rangle$ where $Q[i, j]$ has a match in R with SA interval $[l, r)$ and there exists an SMEM, $Q[i', j]$; clearly, $i' \leq i$
Output: SMEM $Q[i', j]$ and its SA interval $[l', r')$

1: **function** Extend-Phase-K-at-a-time($Q, \langle i, j, l, r \rangle$)
2:     $i', l', r' \leftarrow i, l, r$
3:     **while** $i' - K \geq 0$ and $f^K(Q[i' - K, i' - 1], l') < f^K(Q[i' - K, i' - 1], r')$ **do**
4:         $i', l', r' \leftarrow i' - K, f^K(Q[i' - K, i' - 1], l'), f^K(Q[i' - K, i' - 1], r')$
5:     **return** $\langle i', j, l', r' \rangle$

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extendability remains the same if the SA interval remains the same. In other words, for any j', such that $Q[i, j']$ has the same SA interval as $Q[i, j]$, $Q[i - 1, j']$ has a match in R if and only if $Q[i - 1, j]$ has a match in R.

Proof. Without loss of generality, if $j' < j$ and $Q[i, j]$ and $Q[i, j']$ have the same SA interval, that means $Q[i, j']$ is always followed by $Q[j' + 1, j]$ in R. So, if $Q[i - 1, j]$ has a match in R, $Q[i - 1, j']$ also has a match and if $Q[i - 1, j]$ does not have a match in R, $Q[i - 1, j']$ also does not have a match. □

Therefore, the extendability property only depends on the SA interval $[l, r)$ and the base $Q[i-1]$. The corollary of the extendability property is that for shrink phase, we need to remove bases from the right of $Q[i, j]$ until the SA interval changes and then check if the resultant string is left extendable. Our solution to this problem is based on Enhanced Suffix Arrays (ESA) [45] as proposed in Section 4 of [37].

Enhanced Suffix Arrays: Fig. 6 shows the ESA consisting of the BW-Matrix, LCP array, suffix tree and lcp-interval tree of our example reference sequence. The LCP (Longest Common Prefix) array, LCP, is an array of size $|R| + 1$ consisting of integers in the range $[0, |R|]$. $LCP[i]$ is defined as the length of the longest common prefix of BW-Matrix entries at i and $i - 1$; $LCP[0]$ and $LCP[|R|]$ are set to 0 as there are no BW-Matrix entries for positions -1 and $|R|$ [45]. A suffix tree [46] of a DNA sequence is a compressed trie containing all the suffixes of the sequence as keys and positions in the sequence as their values. Each node in the suffix tree represents a substring of the reference sequence, formed by concatenating the labels of the edges from the root node up to that node. Each leaf node represents a suffix of $|R|$. Therefore, number of leaf nodes is equal to $|R|$. An lcp-interval tree can be created by replacing each node label of a suffix tree with the SA interval of the corresponding substring it represents.

LCP		BW-Matrix	
00	-1	00	$CATTATTAGGA
01	0	01	A$CATTATTAGG
02	1	02	AGGA$CATTATT
03	1	03	ATTAGGA$CATT
04	4	04	ATTATTAGGA$C
05	0	05	CATTATTAGGA$
06	0	06	GA$CATTATTAG
07	1	07	GGA$CATTATTA
08	0	08	TAGGA$CATTAT
09	2	09	TATTAGGA$CAT
10	1	10	TTAGGA$CATTA
11	3	11	TTATTAGGA$CA
12	-1		

(a) The LCP array and BW-matrix of CATTATTAGGA$

(b) The suffix tree of CATTATTAGGA$

(c) The lcp-interval tree of CATTATTAGGA$

Figure 6: The Enhanced Suffix Array, Suffix Tree, and Interval Tree of $R \leftarrow$ CATTATTAGGA$.

represents. For all possible strings with non-zero matches, its SA

interval can only be one of the 19 possibilities corresponding to the 19 nodes in Fig. 6c. Moreover, these 19 intervals form a certain hierarchical tree, where an edge from x to y means that y is the first non-x SA interval we get if we keep removing bases on the right. Each non-leaf node is exactly a union of its children nodes, and the intervals of these children nodes are disjoint. Given $Q[i, j]$ and its SA interval $[l, r]$, the SA interval of $Q[i, j − 1]$ could either stay as $[l, r]$ or it could be the SA interval corresponding to the parent node of the node with label $[l, r]$.

Figure 7: The binary lcp-interval tree of $CATTATTAGGA$**. The dummy nodes are colored gray.**

We make the following enhancements to the design of the lcp-interval tree for its efficient representation and traversal.

- **Binary lcp-interval tree**. To store the lcp-interval tree, we need to store the parent node at each of the node requiring significant additional memory. Instead, we opt to insert a few "dummy" nodes to make it binary while maintaining the properties of the lcp-interval tree (Fig. 7). It is easy to see that there are now exactly $|R| − 1$ non-leaf nodes and $|R|$ leaves. To store all non-leaf nodes of the binary tree, we build a size-($|R| − 1$) array, called T, and each non-leaf node is stored at the position where the intervals of its two children meet. For example, the node $[1, 5)$ will be stored at position 2. This is equivalent to storing these nodes in the order of the in-order traversal. This also means that for each non-leaf node stored at position p, $LCP[p]$ is the length of the string represented by the node. For example, for the node corresponding to the interval $[10, 12)$, its children $[10, 11)$ and $[11, 12)$ meet at 11. So, $[10, 12)$ is stored at position $p = 11$. $LCP[11]$ is 3 and from Figures 6b and 6c, we can see that $[10, 12)$ represents

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Algorithm 6 Algorithm for one Shrink Operation

Input: An SA interval $[l, r]$
Output: Parent interval of $[l, r]$ in T, and the position at which the parent is stored

1: **function** SHRINK($[l, r]$)
2:     **if** $T[l].size[l] < T[r].size$ **then**
3:         **return** $\langle[r − T[l].size, r), l\rangle$
4:     **else**
5:         **return** $\langle[l, l + T[r].size), r\rangle$

---

the string TTA of length 3. Later, we will show that we don't need to store the leaves.

- **Storing interval size instead of interval**. In the array T, for each non-leaf node, we don't need to store the interval. Instead to save memory, we store only the size of the node's interval. The p-th such entry is denoted as $T[p].size$. The interval can be reconstructed from size and p. This size alone is sufficient to climb up the tree (Alg. 6). Suppose we are at some node $[l, r]$. We know that its parent must be stored at either index l or index r. Moreover, the index out of l and r not storing the parent stores some further ancestor, whose interval size must be larger than that of the parent. Using this, to find the parent of $[l, r]$, we simply compare $T[l].size$ and $T[r].size$. If the former is smaller, then the parent node is stored at index l, so the parent's interval is $[r − T[l].size, r)$; otherwise the parent node is stored at index r, so its interval is $[l, l + T[r].size)$. Therefore, Alg. 6 allows us to remove the minimum number of bases such that the SA interval changes and also provides the corresponding SA interval. In order to quickly check if the resultant shrunk string is left extendable, for each entry in T, we store a 4 bit entry called ext to store whether it is left extendable by A, C, G and T. For the dummy nodes, these 4 bits are all set to false. Now, given SMEM $Q[i, j]$ and its SA interval $[l, r]$, the objective of the shrink phase becomes finding the nearest ancestor of $[l, r]$ in the binary interval tree such that the ext bit for $Q[i − 1]$ is set to true.
- **Not storing leaves**. Finally, we note that the leaves are not stored. This is because the SMEM $Q[i, j]$ is not left extendable by definition. So its SA interval must always climb up at least one edge to reach the shrink phase's answer, and thus leaves are never the answer.

Implementing Shrink Phase Using Shrink Operation: Our algorithm for shrink phase is presented in Alg. 7. We start by one shrink operation (line #2) and while the ext bit corresponding to $Q[i − 1]$ is still false, we keep performing shrink operations (lines #3-4). Since $LCP[p]$ is the length of the string represented by the node at position p, we can get j' from i and $LCP[p]$ (line #5).

4.10   Time Complexity of the SMEM-LISA Algorithm

For this analysis, let us first assume that we do not use learned approach to extend K bases at a time, but instead use FM-index based 1 base at a time extension. Therefore, for extend phase, every decrement of i' (Alg. 4, line 4) takes $O(1)$ time. As for shrink phase,

---

Algorithm 7 Proposed Algorithm for the Shrink Phase

Input: Query Q; $\langle i, j, l, r \rangle$ where $Q[i, j]$ is an SMEM with SA interval $[l, r)$
Output: $\langle i, j', l', r' \rangle$, where j' is the upper-bound of the left-adjacent SMEM of $Q[i, j]$ and $[l', r')$ is the SA interval of $Q[i, j']$

1: **function** SHRINK-PHASE($Q, \langle i, j, l, r \rangle$)
2:     $\langle [l', r'), p \rangle \leftarrow$ SHRINK($T, [l, r)$)
3:     **while** $Q[i-1]$-th bit of $T[p].ext$ is false **do**
4:        $\langle [l', r'), p \rangle \leftarrow$ SHRINK($T, [l', r')$)
5:     $j' \leftarrow i + LCP[p] - 1$
6:     **return** $\langle i, j', l', r' \rangle$

---

each shrink operation (Alg. 7, line 4) takes $O(1)$ time. If there were no dummy nodes, the structure of suffix tree guarantees that j' decreases by at least 1 per shrink operation. After dummy nodes are inserted, because each true node can have at most $|\{A, C, G, T, \$\}| = 5$ children, parents and children are separated by at most 5 or $O(1)$ dummy nodes. So, it takes at most $O(1)$ extra calls for shrink to reach the true parent. Hence the time complexity of each decrement of j' is $O(1)$. There can be at most $|Q|$ decrements to i' and j', so the total time complexity is $O(|Q|)$.

When we use learned approach for the first phase of extension, the worst case complexity of an extension by K bases is $O(log|R|)$. This is because we need to perform binary search on the interval returned by the RMI, which could be the entire IPBWT in the worst case if we have a terrible model. However, as shown for exact search, extension by K using learned approach is faster than extension by K letters using FM-index, thus achieving lower than $O(K)$ complexity in practice. Therefore, our LISA based SMEM algorithm is sub linear in $|Q|$ in practice.

Figure 8: Performance comparison of TAL, our ESA based $O(|Q|)$ SMEM algorithm (Linear) and SMEM-LISA algorithm (LISA) for *SMEM search* problem using a single socket (28 cores) of CPU. We use Human, Asian Rice and Zebra Fish genomes as reference sequences and H1, A1 and Z1 as read datasets. The speedup of LISA/Linear over TAL is shown at the top of each LISA/Linear bar. $K = 21$ and # of leaf nodes is 2^{28}, 2^{24} and 2^{27} for Human, Asian Rice and Zebra Fish, respectively.

4.10.1 ***Quantifying the benefits of ESA and learned approach***. In these experiments, we compare TAL with two versions of our implementation: (1) Linear: the ESA based linear algorithm with the learned approach based enhancement turned off and 2) SMEM-LISA: ESA based linear algorithm enhanced with learned approach. For a fair comparison with the implementation in TAL that is well tuned for the underlying CPU, we developed hardware-efficient implementations for Linear and SMEM-LISA and use those for comparison. Fig. 8 shows that our linear algorithm achieves significantly high speedups of $4.2 - 4.7\times$ over TAL. The learned approach achieves additional performance gain, achieving a total speedup of $5.5 - 7.3\times$.

4.10.2 ***Performance Comparison with TAL using Synthetic Reads***. In these experiments, we compare our performance with TAL with respect to variation in read error rates by generating synthetic reads of various error rates ranging between $1 - 10\%$ using wgsim [47]. Fig. 9 shows that we achieve up to $8.7\times$ and $6.1\times$ speedups over TAL on a single thread and a single socket of CPU, respectively. This is due to a combination of using enhanced suffix arrays, learned index based approach and an efficient hardware aware implementation. While we achieve significant speedups even with high error rates, the speedup is higher when the error rate is lower. This is because LISA is used to extend K bases at a time only for the first extension. The smaller the error rate, the higher the chances of the first extension to be longer and LISA to be used for a larger portion of the read. Note that, for the latest short-read sequencers, a majority of bases have error rates of just 0.1%. This is reflected in higher speedup achieved by our approach on real datasets (Sec. 2).

4.11 Hardware-aware Design and Implementation

In this section, we describe our design for efficient utilization of the underlying CPU hardware so as to achieve maximum throughput for our LISA-based algorithms.

4.11.1 ***Motivation for using Software Prefetching***. A common pattern across all the algorithms discussed in this paper is that they all perform frequent memory accesses over large data structures (that cannot fit in cache) and very little compute, making them memory access bound. CPUs have hardware prefetchers that can prefetch data from memory in advance if there is a predictable pattern of memory addresses. However, for these algorithms, the addresses of the memory accesses are pseudo-random — i.e.; there is no clear pattern across subsequent accesses — and computation in one step decides the memory addresses for the next step, making it impossible for the hardware prefetchers to prefetch them. For example, in Alg. 1, in each iteration of the loop, two positions of O are accessed and the computation decides which two positions would be accessed in the next iteration. The positions of O accessed in subsequent iterations can be vastly different from each other. Similarly, for the computation of f^K shown in Fig. 4b, the computation of the level 1 of the RMI reveals which model of level 2 has to be used and the parameters of that model have to be read. In the last mile search, we use binary search, where each iteration reveals which half of the search space would be processed next.

(a) Single thread

(b) Single socket (28 cores)

Figure 9: Effect of varying error rate in the read datasets on throughput for *SMEM search* problem. For each experiment, 5 **Million synthetic reads of length** 151 **were generated using wgsim [47]. The speedup of LISA over TAL is shown at the top of each LISA bar.** $K = 21$ **and # of leaf nodes is** 2^{28} **and** 2^{24} **for Human and Asian Rice, respectively.**

Therefore, for every pseudo random memory access, we have to pay the cost of getting data from memory (memory latency) – making the algorithms memory latency bound. This necessitates the use of software prefetching — software instructions used to specify which memory addresses should be prefetched to cache in advance. These instructions are non-blocking; i.e.; they start the process of prefetching and return the control back so that other instructions can be run in the meantime, thus, allowing for overlap of memory access with computation. Each step decides the memory addresses for the next step and the little compute that is there in each step can not hide the memory latency for the prefetch of the next step. Therefore, we use the design described in [18] for backward search algorithm for *exact search* (Alg. 1) and apply it to the Exact-LISA (Alg. 2) and SMEM-LISA (Alg. 3) algorithms. Compared to backward search algorithm, LISA based algorithms are a lot more complex with many varied steps each requiring a different number of memory accesses making it particularly challenging.

4.11.2 ***Our Design for Software Prefetching***. Here, we describe a generic design that we have applied to each LISA-based algorithm. There is no data dependency across queries and the queries can be processed in parallel. On the other hand, processing of a query consists of several steps, which need to be executed in a sequential manner because of data dependency across steps. We start by formally defining a step. A step is a unit of work that receives a set of memory addresses as input, reads data from the memory addresses, performs compute on the data and generates a new set of memory addresses for the next step. The first and the last step are a little different. The first step (setup step) does not receive addresses to access data from memory and the last step produces the query output instead of memory addresses. Important to note that no step should access a memory address that is generated by the same step and needs software prefetching. All the work performed for a query can be modeled into a list of steps as defined here. Each thread gets M queries to execute at a time. On a single thread, if the queries are executed in sequential manner, each step of a query has to pay

the memory latency cost in the memory access phase. Instead, we process a batch of m queries simultaneously.

Let q_i denote the i-th query in a batch. We process the queries in a round robin fashion – processing one step at a time and prefetching for the next step. We first execute the setup step of q_0 and start software prefetching the memory addresses for the next step. While the data is getting prefetched, we process the setup step of q_1 and start its prefetching. We do this for all the queries in the batch and then return to q_0 for the next step. By this time, data for next step of q_0 is already in cache and we can perform the compute. We keep repeating this for all the steps for the queries. The queries may have different number of steps as in the case of binary search. When a query is done, we replace it with a new query to keep the batch size fixed. The batch size, m, should be big enough such that by the time we come back to a query, its data is already prefetched to cache. On the other hand, it should be small enough so that the data corresponding to all the queries in a batch can fit in cache. We theoretically calculate the range of batch sizes and then empirically find the best batch size near the theoretical range.

4.11.3 ***Vectorization***. The last mile search (Fig. 4b) is performed using a binary search. However, once we have less than 8 elements left to search, we compare with all of them simultaneouly using SIMD instructions.

4.11.4 ***Multi-threading***. We use a simple multi-threading design where we divide the set of queries into blocks of M queries and use dynamic scheduling to distribute the blocks of queries across threads.

4.11.5 ***Benefit of Hardware-aware Optimizations***. For these experiments, we use human genome as the reference sequence. For query datasets, we use S1 and H1 (described in Tables 4 and 3) for *exact search* and *SMEM search*, respectively.

Tables 7 and 8 show the benefits of hardware-aware optimizations for *exact search* and *SMEM search*, respectively, on a single thread. We report performance of TAL and three versions of LISA:

Table 7: Effect of hardware-aware optimizations for *exact search* on a single thread. Reference sequence: Human Genome. Query dataset: S1. We report throughput in Million Seeds Per Second (MSPS), instruction count and percentage of loads hitting in L1 cache (L1 hit rate) for TAL and three versions of LISA.

	TAL	LISA		
		Un-opt	+SW pref.	+vectorization
Throughput (MSPS)	4.49	2.29	7.69	8.82
Inst. count	9.2E+10	2.7E+10	4.1E+10	3E+10
L1 hit rate	99.6%	85%	99.6%	99.6%

Table 8: Effect of hardware-aware optimizations for *SMEM search* on a single thread. Reference sequence: Human Genome. Query dataset: S1. We report throughput in Million Reads Per Second (MRPS), instruction count and percentage of loads hitting in L1 cache (L1 hit rate) for TAL and three versions of LISA.

	TAL	LISA		
		Un-opt	+SW pref.	+vectorization
Throughput (MRPS)	0.04	0.14	0.54	0.55
Inst. count	3.9E+11	3E+10	4.4E+10	3.8E+10
L1 hit rate	91%	80.7%	97.7%	98.2%

Table 9: Memory bandwidth utilization (GB/s) on a single CPU socket (28 cores) for *exact search* and *SMEM search* for TAL and two versions of LISA.

	TAL	LISA Un-opt	LISA Optimized
Exact search	79.6	15	78.2
SMEM search	21.5	24	65.8

(1) without any hardware-aware optimizations (Un-opt), (2) with application of software prefetching to Un-opt (+SW pref.) and (3) with application of vectorization in addition to SW prefetching (+vectorization).

Compared to TAL, the number of instructions executed by our un-optimized implementations are 3.4× and 13× less, respectively, for *exact* and *SMEM search*. This shows the benefit of our algorithmic improvements to reduce the work required for sequence search. However, the un-optimized implementations suffer from significantly low L1 cache hit rate, resulting in poor throughput. Application of software prefetching to un-optimized LISA implementations dramatically improves the L1 hit rate, thus improving throughput, despite increasing the number of instructions. More specifically, software prefetching improves throughput by 3.36× and 3.86× for *exact search* and *SMEM search*, respectively. Through vectorization of the last few iterations of the binary search, we reduce the instructions to get a further 15% improvement in throughput for *exact search*. For *SMEM search*, the benefit of vectorization is only 2% because the last few iterations of binary search form only a small fraction of the total time. This shows that improvements in the algorithm and implementations that are well tuned to the hardware are both significant in LISA achieving high performance gains.

Table 9 reports the memory bandwidth achieved by TAL and un-optimized and fully optimized versions of LISA on a full CPU socket. It is clear that our optimizations improve the bandwidth utilization of LISA. While LISA consumes significantly high memory bandwidth, given that it needs significantly less time than TAL, it needs to read much less data from memory.

Comparing the performance of TAL with fully optimized LISA version, it is clear that the main benefit of LISA comes from reducing the work required to solve the problem, as evident in the reduced instruction counts and memory access requirements. In addition, the hardware-aware implementation ensures that the reduced amount of work translates to better performance.

5 RELATED WORK

5.1 DNA Sequence Search

Search of short DNA sequences in a reference sequence is usually done by using an index of the reference sequence. Several data structures are proposed in the literature for the index — look up table (LUT), FM-index based on Burrows Wheeler transform, suffix trees, prefix tries, and prefix directed acyclic word graph (DAWG). Out of these, LUT (MAQ [48], SOAP [49]) and FM-index (Bowtie [7], Bowtie2 [5], SOAP2 [8], BWA [10], BWA-MEM2 [21]) are the most popular, while suffix trees were used by a few earlier tools (e.g., Mummer [50]). In general, FM-index is the most popular in the community due to its low computational complexity and memory footprint [51].

5.2 Acceleration of FM-index based Search

A number of approaches have been proposed to accelerate FM-index based methods for modern multicore CPUs [5, 7, 8, 10, 12, 15, 17–19, 52], manycore CPUs [53, 54], GPUs [9, 13, 14, 55, 56], and FPGAs [16, 57]. A majority of them have targeted the simpler, exact search algorithm. We mention a few notable approaches here.

Compression of FM-index, first introduced in [11], is a widely used technique that reduces the memory requirement of FM-index. Compression of FM-index by, say a factor h, is achieved by storing the array O only for every h-th position and recomputing for the rest of the positions every time by using the closest position that is a multiple of h and the BWT. Compression also enables look-ups for more positions to hit in the same cache line, thereby, reducing the memory bandwidth requirement.

Trans-Omics Acceleration Library (TAL) [18, 21, 22] also uses a compressed FM-index and provides efficient hardware-aware implementations for CPUs. TAL demonstrates significant performance gains on *exact search*, *inexact search*, and *SMEM search* with application of software prefetching and vectorization on the FM-index based backward search algorithm, inexact search algorithm [10], and the *SMEM search* algorithm [36]. For *exact search* and *inexact search*, the software prefetching design used by TAL is similar to that presented in Sec. 4.11.

Chacon *et al.* [12, 13] developed a clever n-step FM-index strategy for *exact search* that allows processing n bases at a time. This is achieved by using Σ^n as alphabet instead of Σ. It reduces the number of memory accesses by a factor of n while increasing the number of executed instructions. Moreover, the memory requirement of this strategy is exponential in n, thus, only allowing $n \leq 4$ on most systems. They showed significant performance gains on CPU and GPGPU. However, their application of software prefetching on CPU for exact search yielded minimal benefit due to an inefficient design, resulting in their performance being memory latency bound.

For SMEM, [41, 45] use Enhanced Suffix Array to perform the shrink operation like us and uses FM-index for the extension. They lack the learned index based extension and efficient architecture-aware implementations of extend and shrink operations.

5.3 Learning-based Search

Sapling is a recently published learned index based solution for *exact search* that showed promising results [19]. Sapling is single threaded and does not provide architecture-aware optimizations. Given a query Q, Sapling outputs any one position in the BW-Matrix where the query matches instead of outputting the entire SA interval. Sapling learns a machine learning model for queries of fixed length k. Sapling paper proposes an artificial neural network (ANN) based model and a piece-wise linear model (PWL) and shows that PWL outperforms ANN. For the PWL model, Sapling divides the space of 4^k possible queries into equal sized intervals and divides the BW-Matrix into those intervals based on matching prefix. Within the interval, it uses a PWL model to predict the position of the query. It stores the maximum error for each interval to provide the bounds of last mile search (using binary search). The learned index used by Sapling is similar to the RadixSpline [32] with linear splines replaced with fixed length intervals. As mentioned in the Sapling paper, the fixed length intervals lead to very large bounds for binary search in some intervals. To alleviate that, Sapling also stores the 95th percentiles of errors to reduce the iterations of binary search. However, this still results in very large bounds for binary search. The best bound reported for Sapling is 653, which results in 10 iterations of binary search. RadixSpline resolves this by using linear splines to have a bound on the size of each interval. LISA uses RMI learned index structure that has empirically been shown to perform better than RadixSpline [32] and Piecewise Geometric Model (PGM) [27].

For queries with length less than k, they need to be appended by A's to make their length k. For queries with length greater than k, if v is the integer representation of $Q[0, k-1]$, it uses a floating point number between v and $v+1$ based on $Q[k, |Q|-1]$ to evaluate the PWL function on. However, in the last mile search the entire length of the query needs to be compared in each step making it inefficient for long queries. On the other hand, LISA divides a query into chunks of size K and uses IPBWT to be able to efficiently handle queries of all lengths.

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