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Title: The energetic basis for smooth human arm movements

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Abstract

The central nervous system plans human reaching movements with stereotypically smooth

kinematic trajectories and fairly consistent durations. Smoothness seems to be explained by

accuracy as a primary movement objective, whereas duration seems to economize energy

expenditure. But the current understanding of energy expenditure does not explain

smoothness, so that two aspects of the same movement are governed by seemingly

incompatible objectives. Here we show that smoothness is actually economical, because

humans expend more metabolic energy for jerkier motions. The proposed mechanism is an

underappreciated cost proportional to the rate of muscle force production, for calcium

transport to activate muscle. We experimentally tested that energy cost in humans (N=10)

performing bimanual reaches cyclically. The empirical cost was then demonstrated to predict

smooth, discrete reaches, previously attributed to accuracy alone. A mechanistic,

physiologically measurable, energy cost may therefore explain both smoothness and duration

in terms of economy, and help resolve motor redundancy in reaching movements.

Introduction

Fig. 1. Goal-directed reaching tasks and optimization criteria. (A.) Typical experiments for point-to-point movements between targets. (B.) Hand speed trajectories vs. time. Kinematic objectives such as minimizing jerk or variance predict the observed smooth, bell-shaped profiles for hand speed. (C.) A number of effort-based objectives such as minimizing work or squared muscle force predict trajectories that are not smooth or not bell-shaped (Nelson 1983).

Upper extremity reaching movements are characterized by a stereotypical, bell-shaped speed

profile for the hand's motion to its target (Fig. 1A). The profile's smoothness seems to preserve

kinematic accuracy (Harris and Wolpert 1998), and have little to do with the effort needed to

produce the motion. But effort or energy expenditure appear to affect other aspects of

reaching (Huang et al. 2012; Shadmehr et al. 2019), and influence a vast array of other animal
behaviors and actions (Alexander 1996). It seems possible that effort or energy do influence the
bell-shaped profile, but have gone unrecognized because of incomplete quantification of such
costs. If so, then dynamic goals including effort could play a key role in movement planning.
39
The kinematic goal for accuracy may be expressed quantitatively as minimization of the final
endpoint position variance (Harris and Wolpert 1998). Non-smooth motions introduce
inaccuracy because motor noise increases with motor command amplitude, a phenomenon
termed signal-dependent noise (Matthews 1996; Sutton and Sykes 1967) . It predicts well the
speed profiles for not only the hand but also the eye. It explains why more curved or more
accurate motions need to be slower, and also subsumes an older theory for minimizing
kinematic jerk (Flash and Hogan 1985). The single objective of movement variance explains
multiple aspects of smooth movements, and makes better predictions than competing theories
(Diedrichsen et al. 2010; Haith et al. 2012; Todorov 2004).
49
There are nonetheless reasons to consider effort. Many optimal control tasks must include an
explicit objective for effort, without which movements would be expected to occur at maximal
effort ("bang-bang control," Harris and Wolpert 1998; Bryson and Ho 1975). In addition,
metabolic energy expenditure is substantial during novel reaching tasks, and decreases as
adaptation progresses (Huang et al. 2012). Such a cost also helps to determine movement
duration and vigor (Shadmehr et al. 2016), not addressed by the minimum-variance hypothesis.
Indeed, optimal control studies have long examined effort costs such as for muscle force
(Kolossiatis et al. 2016), mechanical work (Alexander 1997), squared force or activation (Nelson
1983; Ma et al. 1994), or "torque-change" (integral of squared joint torque derivatives; Uno et
al. 1989). But many such costs produce non-smooth velocity profiles (Fig. 1B), or lack
physiological justification, or both. Some studies have included explicit models of muscle energy
expenditure, but without testing such costs physiologically (Kistemaker et al. 2010). There is
good evidence that energy expenditure is relevant to reaching (Shadmehr et al. 2016), but no
physiologically tested cost function predicts the velocity profiles of reaching as well as the
minimum variance hypothesis.
65
The issue could be that metabolic energy expenditure for muscle is not quantitatively well-
understood. Energy is expended in proportion to force and time ("tension-time integral") in
isometric conditions (Crow and Kushmerick 1982), and in proportion to mechanical work in
steady work conditions (Barclay 2015; Margaria 1976), neither of which apply well to reaching.
There is, however, a less-appreciated cost for muscles that increases with brief bursts of
intermittent or cyclic action. It is due to active calcium transport to activate/deactivate muscle ,
observed in both isolated muscle preparations (Hogan et al. 1998) and whole organisms
(Bergstrom and Hultman 1988). It has also been hypothesized quantitatively (Doke and Kuo
2007), as a cost per contraction roughly proportional to the rate of change of muscle force.
Such a cost has indeed been observed in a variety of lower extremity tasks (Dean and Kuo 2011;
Doke et al. 2005; van der Zee and Kuo 2020). It has a mechanistic and physiological basis, is
supported by experimental evidence, and would appear to penalize jerky motions due to their
energetic cost. What is not known is whether this energetic cost can explain reaching.

79
We therefore tested whether there is an energetic basis for reaching movements. We did so by
measuring oxygen consumption during steady-state cyclic reaching motions. The expectation
was that the proposed force-rate cost would cost metabolic energy in excess of what could be
explained by mechanical work. We next applied the empirically derived cost for both force-rate
and work to an optimal control model of discrete, point-to-point reaching, and tested whether
it could predict the smooth, bell-shaped velocities normally attributed to minimum-variance. If
the proposed cost is observed as expected and predicts bell-shaped profiles, it could potentially
provide a re-interpretation of existing theories based on kinematics alone, and integrate energy
expenditure into a general framework for planning reaching movements.
89

Materials and Methods

There were three main components to
this study: (1) a simple cost model, (2) a
set of human subjects experiments with
cyclic reaching, and (3) an application of
the model to predict discrete reaching
trajectories. The model predicts that
metabolic cost should increase with the
hypothesized force-rate measure,
particularly for faster frequencies of
movement. Key to the experiment (Fig.
2) was to isolate the hypothesized force-
rate cost, by applying combinations of
movement amplitude and frequency that
control for the cost of mechanical work.
This primary test was accompanied by a
secondary, cross-validation test, with
different combinations of movement
amplitude and frequency. Finally, we
applied this same force-rate cost to the
prediction of discrete reaching

Fig. 2. Experiment for metabolic cost of cyclic reaching. (A) Cyclic reaching was performed bimanually and symmetrically in the horizontal plane, primarily about the shoulders. To isolate the hypothesized force-rate cost from the energetic cost of work, movements were varied to yield fixed mechanical power, by decreasing amplitudes with increasing movement frequency. (B) Movement data included shoulder angle, mechanical power, electromyography (EMG), and (not shown) metabolic energy expenditure via expired gas respirometry.

111 movement trajectories. This was to test whether the energetic cost, derived from continuous,
112 cyclic reaching movements, could also predict the smooth, discrete motions often found in the
113 literature.
114

Model predictions for force-rate hypothesis

We hypothesized that the energetic cost for reaching includes a cost for performing mechanical
work, and another for the rate of force production. These costs are implemented on a simple,
two-segment model of arm dynamics, actuated with joint torques. These torques perform work
on the arm, at an approximately proportional energetic cost (Margaria 1976) attributed to
actin-myosin cross-bridge action (Barclay 2015). The force-rate cost is hypothesized to result
from rapid activation and deactivation of muscle, increasing with the amount of force and

inversely with the time duration. It is attributable to active transport of myoplasmic calcium
(Bergstrom and Hultman 1988; Hogan et al. 1998), where more calcium is required for higher
forces and/or shorter time durations, hence force-rate (Doke and Kuo 2007).

For the simple motion employed here, the prediction of the total metabolic energy E consumed
per movement is the sum of costs for work and force-rate,

$$E = c_W W + E_{\text{FR}} \qquad 1$$

128 where W is the positive mechanical work per movement, c_W the metabolic cost per unit of
129 work, and E_{FR} is the hypothesized force-rate cost

$$E_{\text{FR}} = c_f \dot{F} \qquad 2$$

where $\dot{F}$ denotes the amplitude of force-rate (time-derivative of muscle force) per movement,
and c_f is the energetic cost for force-rate. This cost is to be distinguished from the earlier
torque-change hypothesis (Uno et al. 1989), which integrates a sum-squared force-rate over
time, and which had no hypothesized relationship to metabolic energetic cost. During cyclic
reaching, the peak force-rate $\dot{F}$ increases with both force amplitude and the frequency of cyclic
movement. Here, positive and negative work are performed in equal magnitudes, and so their
respective costs are lumped together into a single proportionality c_W. We assigned c_W a value
of 4.2, from empirical mechanical work efficiencies of 25% for positive work and -120% for
negative work (Margaria et al. 1963).

The work and force of the cyclic reaching movements about the shoulder are predicted by a
simple model of arm dynamics. In the horizontal plane of a manipulandum supporting the arm,

$$T(t) = I\,\ddot{\theta}(t) \qquad 3$$

with shoulder angle $\theta(t)$, shoulder torque $T(t)$ (treated as proportional to muscle force), and
rotational inertia I. Applying sinusoidal motion at amplitude A and movement frequency f (in
cycle/s),

$$\theta(t) = A \cos 2\pi f t.$$

The torque is therefore

$$T(t) = -4\pi^2 I\, A\, f^2 \cos 2\pi f t$$

and amplitude of mechanical power $\dot{W}$

$$\dot{W} \propto A^2 f^3 \qquad 4$$

154 We apply a particular movement condition, termed the *fixed power* constraint (Fig. 2A), where
155 the average positive mechanical power is kept fixed across movement frequencies, so that the
156 hypothesized force-rate cost will dominate energetic cost (Fig. 3A). This is achieved by
157 constraining amplitude to decrease with movement frequency (Fig. 3B),

$$A \propto f^{-\frac{3}{2}} \qquad 5$$

159 This fixed power condition also means that hand (endpoint) speed, proportional to $\dot{\theta}$, should
160 have amplitude varying with $f^{-1/2}$, and torque amplitude with $f^{1/2}$ (Fig. 3C, D).
161
162 Applying fixed power to the force-rate cost yields the energetic cost prediction. Torque-rate
163 amplitude $\dot{T}$ with Eqn. 5 and 2) yields
164

$$\dot{T} = b \cdot f^{\frac{3}{2}} \qquad \qquad 6$$

165
166 where b is a constant coefficient. The proportional cost per movement is therefore (Eqn. 2))

$$E_{\text{FR}} = c_f \cdot f^{\frac{3}{2}} \qquad \qquad 7$$

167
168 where c_f is a constant coefficient across conditions. Experimentally, it is most practical to
169 measure metabolic power $\dot{E}$ (Fig. 3a) in steady state. Multiplying E (cost per movement, Eqn. 2)
170 by f (movement cycles per time) yields the predicted proportionality for average metabolic
171 power,

$$\dot{E}_{\text{FR}} = c_f \cdot f^{5/2} \ . \qquad \qquad 8$$

172
173 The net metabolic rate $\dot{E}$ is expected to increase similarly, but with an additional offset for the
174 constant work cost $\dot{E}_W$ under the fixed-power constraint (Fig. 3A). Finally, the metabolic energy
175 per time associated with force-rate would be expected to increase directly with torque-rate per
176 time $f \cdot \dot{T}$,

$$\dot{E}_{\text{FR}} = c_t \cdot f \cdot \dot{T} \qquad \qquad 9$$

where movement frequency f represents cycles
per time, and coefficient c_t is equal to c_f divided
by b.

This force-rate coefficient is not specific to cyclic
movements alone. The general metabolic cost
model (Eqns. 1 and 2) is potentially applicable to
point-to-point and other motions, with different
amounts of work and force-rate. The force-rate
cost E_{FR} is independent of mechanical work, and
may be predicted using the cost coefficient
derived from cyclic experiments. The model may
therefore make testable predictions of energetic
cost even for movements that are acyclic and not
constrained to fixed power.

Experiments
We measured the metabolic power expended by
healthy adults (N = 10) performing cyclic
movements at a range of speeds but fixed power
(Eqn. 5). We tested whether metabolic power
would increase with the hypothesized force-rate
cost $\dot{E}_{\mathrm{FR}}$, in amount not explained by mechanical
work. We also characterized the mechanics of the

Fig. 3. Predicted cost and dynamics for cyclic reaching, as a function of movement frequency f. (A.) Force-rate cost is predicted to increase with $f^{5/2}$, whereas cost for mechanical work is predicted to remain constant for fixed power conditions. (B.) Fixed power is achieved by specifying movement amplitude A to decrease with frequency, according to $f^{-3/2}$. (C.) Torque amplitudes are expected to increase modestly, with $f^{1/2}$. (D) Peak hand speed is expected to decrease, with $f^{-1/2}$.

task in terms of kinematics, shoulder torque amplitude, and force-rate for shoulder muscles.
These were used to test whether the mechanics were consistent with the model of arm
dynamics, and whether force-rate increased as predicted (Eqns. (5-8). We first describe a
primary experiment with fixed power conditions, followed by an additional cross-validation
experiment. All subjects provided written informed consent, as approved by University of
Calgary Ethics board.

Subjects performed cyclic bimanual reaching movements in the horizontal plane, with the arms
supported by a robotic exoskeleton (KINARM, BKIN Technologies, Inc). The movements were
cyclic and bimanually symmetrical to induce steady energy expenditure sufficient to be
distinguished easily by expired gas respirometry. The exoskeleton was used to counteract
gravity in a low-friction environment (with no actuator loads), and to measure kinematics, from
which shoulder and elbow joint torques were estimated using inverse dynamics. Subjects were
asked to move each arm between a pair of targets, reachable mostly by medio-lateral shoulder
motion, with relatively little elbow motion (less than 1 deg average excursion across all
conditions). A single visual cursor (5 mm in diameter) was displayed for the right hand, along
with one pair of visual targets (circles 2.5 cm in diameter), all optically projected onto the
movement plane. To encourage equal bimanual motion, the cursor's position was not for one

hand alone, but rather computed as an average of right and left arm joint angles, making it
insufficient to move one arm alone.
222
223 Timing was set with a metronome beat for reaching each of the two targets, and amplitude by
224 adjusting the distance between the targets. Prior to data collection, subjects completed a 20-
225 minute familiarization session (up to 48 hours before the experiment) where they received task
226 instructions and briefly practiced each of the tasks.
227
228 The primary experiment was to test for the predicted energetic cost for reaching, in five
229 conditions of cyclic reaching at increasing frequency and decreasing amplitude. The frequencies
230 were 0.58, 0.78, 0.97, 1.17, 1.36 Hz, and amplitudes were 12.5, 8, 5.8, 4.4, 3.5°, respectively.
231 These cyclic movements were chosen to be of moderate hand speed, with peak speeds
232 between $0.4 - 0.6$ m/s.
233
234 We estimated metabolic rate using expired gas respirometry. Subjects performed each
235 condition for 6 minutes, analyzing only the final 3 minutes of data for steady-state aerobic
236 conditions, with standard equations used to convert O_2 and CO_2 rates into metabolic power
237 (Brockway 1987). We report net metabolic rate $\dot{E}$ for bimanual movement, defined as gross
238 rate minus the cost of quiet sitting (obtained in a separate trial, 98.6 ± 11.5 W, mean ± s.d.).
239
240 We also recorded arm segment positions and electromyographs simultaneously at 1000 Hz.
241 These included kinematics from the robot, and electromyographs (EMGs) from four muscles
242 (pectoralis lateral, posterior deltoid, biceps, triceps) in a subset of our subjects (5 subjects in
243 primary experiment, 5 in cross-validation). The EMGs were used to characterize muscle
244 activation and co-activation.
245
The metabolic cost hypothesis was tested using a linear mixed-effects model of net metabolic
power. This included the hypothesized force-rate cost (Eqn. 8) as a fixed effect, yielding
coefficient c_f for the force-rate term proportional to $f^{5/2}$. A constant offset was included for
each subject as a random effect. In addition, the force-rate cost $\dot{E}_{FR}$ was estimated by
subtracting the fixed mechanical work cost $\dot{E}_W$ from net metabolic power $\dot{E}$, and then
compared against torque rate amplitude per time (Eqn. 9). Sample size was appropriate to yield
a statistical power of 0.99 based on statistical characteristics of previous reaching studies of
metabolic cost (Wong et al. 2018). Both the main experiment and cross validation experiment
were performed a single time.
255
We tested expectations for movement amplitude and other quantities from kinematic data.
Hand velocity was filtered using a bi-directional lowpass Butterworth filter (1st order, 12 Hz
cutoff). Shoulder and elbow torques were computed using inverse dynamics, based on KINARM
dynamics (BKIN Technologies, Kingston), and subject-specific inertial parameters (Winter 1990).
The approximate rotational inertia of a single human arm and exoskeleton about the shoulder
was estimated as 0.9 kg·m². The positive portion of bimanual mechanical power was integrated
over total movement duration and divided by cycle time, yielding average positive mechanical

power. Linear mixed-effects models were used to characterize the power-law relations for mechanical power, movement amplitude, movement speed, torque amplitude, and torque rate amplitude (Fig. 3). The latter was estimated by integrating the torque rate amplitude per time (Eqn. 9) for each joint, and then summing the two. The force-rate hypothesis was also tested by comparing $\dot{E}_{FR}$ with torque rate per time (Fig 3A), assuming torque is proportional to muscle force.

Electromyographs were used to test for changes in muscle activation and co-activation. Data were mean-centred, low-pass filtered (bidirectional, second order, 30Hz cutoff), rectified, and low-pass filtered again (Roberts and Gabaldón 2008), from which the EMG amplitude was measured at peak and then normalized to each subject's maximum EMG across the five conditions. We expected EMG amplitude to increase with muscle activation, with simplified first-order dynamics between activation (EMG) and muscle force production (van der Zee and Kuo 2020). This treats the rate-limiting step of force production as a low-pass filter, so that greater activation or EMG amplitudes would be needed to produce a given force at higher waveform frequencies. The first-order dynamics mean that EMG would be expected to increase with torque rate $f^{3/2}$ rather than torque, as tested with a linear mixed-effects model. We also computed a co-contraction index for EMG, in which the smallest value of antagonist muscle pairs was computed over time, and then integrated for comparison across conditions (Gribble et al. 2003). All statistical tests were performed with threshold for significance of $P < 0.05$.

As a cross-validation test of the force-rate cost, we tested the generalizability of coefficient c_t against a second set of conditions with a separate set of subjects (also $N = 10$; two subjects participated in both sets). The conditions were slightly different: frequencies ranging 0.67 - 1.3 Hz and amplitudes 12.5 - 4.42°, which resulted in higher mechanical work and force-rate. We applied the model (Eqns. 1, 9) and c_t coefficient identified from the primary experiment to the cross-validation conditions. As a further test of the central hypothesis, we expected the model to roughly predict trends regarding mechanical and metabolic rates for the cross-validation conditions.

Estimation of elastic energy storage in shoulder muscles
We estimated the resonant frequency of cyclic reaching, to account for possible series elasticity in shoulder actuation. Series elasticity could potentially store and return energy and thus require less mechanical work from muscle fascicles. We estimated this contribution from resonant frequency, obtained by asking subjects to swing their arms back and forth rapidly at large amplitudes (at least 15°) for 20 s, and determining the frequency of peak power (PWelch in Matlab). We then used this to estimate torsional series elasticity, and the passive contribution to mechanical power.

Musculoskeletal model to simulate experimental conditions
We tested whether a Hill-type musculoskeletal model could explain the metabolic cost of cyclic reaching. The hypothesized force-rate is not explicitly included in current models of energy

expenditure, and would not be expected to explain the experimental metabolic cost. We
therefore tested an energetics model available in the OpenSim modeling system (Seth et al.
2018; Uchida et al. 2016; Umberger 2010), applied to a model of arm dynamics with six muscles
(Kistemaker et al. 2014). We used trajectory optimization to determine muscle states and
stimulations, with torques from inverse dynamics as a tracking reference. Optimization was
performed using TOMLAB/SNOPT (Tomlab Software AB, Sweden; (Gill et al. 2002)), to minimize
mean-squared torque error, squared stimulation level, and squared stimulation rate. The
optimized muscle states were then fed into the metabolic cost model (Umberger 2010).
314
315

Model of point-to-point reaching movements
The force-rate cost hypothesis was also used to predict point-to-point reaching movements and
their durations. Here we form an overall objective function J that includes the energetic cost
per movement E (Eqn. 1) as a physiological effort term, and a simple penalty proportional to
movement duration t_f. This may be regarded as an adaptation of Shamehr et al.'s (2016)
hypothesis that duration is a trade-off between effort and (the inverse of) a temporally
discounted reward. The overall objective is thus:
323

$$J(\theta(t), \tau(t)) = E(\theta(t), \tau(t)) + k \cdot t_f \qquad 10$$

324 where the energy expenditure is expressed a function of joint angle $\theta(t)$ and torque $\tau(t)$
325 trajectories, and duration is penalized with proportionality k. Minimization of this objective can
326 predict point-to-point reaching trajectories both of fixed duration (by constraining t_f) and of
327 free duration. We show that this objective predicts smooth, bell-shaped velocity profiles similar
328 to minimum-variance, as well as durations increasing with movement distance.
329

330 We used this objective in trajectory optimization of planar, two-joint reaching movements. For
331 fixed duration t_f, the objective J depends only on energetic cost E, with the mechanical work
332 and force-rate terms expressed as a time integral for both joints:
333

$$E(\theta(t), \tau(t)) = \sum_{i=1}^{2} \int_0^{t_f} \left(\left| \dot{\theta}_i \tau_i \right|_+ + \left| c_t \ddot{\tau}_i \right|_+ \right) dt \qquad 11$$

with joint torques τ_i ($i = 1, 2$ for elbow and shoulder, respectively). To compare with the
minimum variance model of Harris & Wolpert (1998), we used a similar straight reaching
movement of amplitude 30 cm and duration t_f of 650 ms. We also used the empirically-
estimated c_t from the cyclic reaching experiment (assuming the same coefficient for both
shoulder and elbow), along with a point-to-point constraint to have zero initial and final
acceleration of the hand, again using TOMLAB. The predicted hand velocity trajectory was
qualitatively compared with the empirical bell-shaped velocity from minimum variance (Harris
and Wolpert 1998).
342

343 We also examined movements of unconstrained duration, which have been shown to take
344 longer with greater movement distance (Reppert et al. 2018). We selected k so that the

350 ## Results

Table 1. Experimental results. Linear mixed effects models were used to test model predictions from data. Listed for each quantity: predicted power law, estimated coefficient, 95% confidence interval (CI), R^2, and P-value.

Quantity	Power law	Coefficient	95 % CI	R^2	P	Inter-cept
Metabolic Power $\dot{E}$ (W)	$f^{5/2}$	6.72	(4.58, 8.86)	0.50	9.70E-9	3.93
Movement amplitude A (°)	$f^{3/2}$	5.97	(5.66, 6.28)	0.97	1.02E-39	-0.47
Peak speed amplitude (m·s^{-1})	$f^{-1/2}$	0.43	(0.39, 0.47)	0.93	6.63E-29	0.01
Torque amplitude (N·m)	$f^{1/2}$	8.34	(5.77, 10.91)	0.52	4.10E-9	1.63
Positive mechanical power $\dot{W}$ (W)	f^0	1.20	(0.85, 1.55)			
Torque rate per time $f\dot{T}$ (N·m·s^{-2})	$f^{5/2}$	78.93	(72.37, 85.48)	0.94	2.19E-30	46.43
EMG amplitude: Pec	$f^{3/2}$	0.17	(0.12, 0.23)	0.65	1.1E-6	0.17
EMG amplitude: Delt	$f^{3/2}$	0.20	(0.11, 0.27)	0.56	1.5e-5	0.20

We found that the rate of metabolic energy expenditure increased substantially with movement
frequency, even as the rate of
mechanical work was nearly constant.
We first confirmed that cyclic reaching
was performed largely by sinusoidal
motions at the shoulder, across all
conditions (Fig. 4). These were
accompanied by approximately
sinusoidal torque and power, and fairly
consistent EMG profiles. Under such
conditions, subjects expended more
than triple (a factor of 3.56) the net
metabolic power for about twice the
frequency (a factor of 2.33), with 5.32 ±
2.73 W at the lowest frequency of 0.58
Hz, compared to 18.95 ± 6.02W at the
highest frequency of 1.36 Hz (Fig. 5A).
As predicted, metabolic rate increased

Fig. 4. Average bimanual reach trajectories and EMG. Mean (± s.d.; N=5 EMG subjects) angular displacement, shoulder torque, summed joint power, EMG, and torque rate-per-time for 5 conditions from Experiment 1. Right (blue) and left (red) arms are shown separately.

approximately with $f^{5/2}$ (Eqn.7; adjusted R^2 = 0.50; P = 1e-8; Fig. 4a; Table 1).

Other aspects of the cyclic reaching task were as prescribed and intended (Fig. 5B-E; Table 1).

Reach amplitudes decreased
according to the targets,
approximately with $f^{-3/2}$
(Fig. 5B). Shoulder torque
amplitude and endpoint
speed also changed with
respectively $f^{1/2}$ (Fig. 5C;
adjusted R^2 = 0.52; P = 4e-9)
$f^{-1/2}$ (Fig. 5D; R^2 = 0.93; P =
7e-29). Consistent with the
fixed-power condition,
average positive mechanical
power did not change
significantly with frequency
f (Fig. 5E; slope = 0.081 $\pm$
0.13 W·s^{-1}; mixed-effects
linear model with a fixed
effect proportional to f^1,
and individual subject

Fig. 5. Experimental results as a function of movement frequency f. (A.) Net metabolic power $\dot{E}$ vs. frequency f (means ± s.d., N = 10), with predicted power law $f^{5/2}$ (solid line). (B.) Movement amplitude and prescribed target $f^{-3/2}$. (C.) Torque amplitude and prediction $f^{1/2}$. (D.) Hand speed amplitude and prediction $f^{-3/2}$. (E.) Amplitude of torque rate per time and prediction $f^{5/2}$, and mechanical power amplitude $\dot{W}$ and constant power prediction.

offsets as random effects; P = 0.16). Amplitude of torque rate per time increased more sharply,
approximately with $f^{5/2}$ (Fig. 5E), with coefficient b of 78.93 ± 6.55 CI, 95% confidence
interval).

The net metabolic cost was also consistent with the hypothesized sum of separate terms for
positive mechanical work and force-
rate (Fig. 6). This is demonstrated
with metabolic power as a function
of movement frequency f, and as a
function of force-rate per time. With
positive mechanical work at a fixed
rate of about 1.2 W, the metabolic
cost of work was expected to be
constant at approximately 5 W. The
difference between net metabolic
rate and the constant work cost
yielded the remaining force-rate
metabolic power, increasing
approximately with $f^{5/2}$ (Fig. 6A).
This same force-rate cost could also
be expressed as a linear function of
the empirical torque rate per time,
with an estimated coefficient of

Fig. 6. Estimated metabolic cost contributions from work and force-rate. (A.) Net metabolic rate $\dot{E}$ vs. movement frequency f for cyclic reaching, with contributions from force-rate cost ($c_f f^{5/2}$) and mechanical work ($c_W \dot{W}$). Coefficient c_f was derived from experiment (Fig. 4), whereas c_W was specified as 4.2 to model a proportional cost for positive and negative mechanical work. (B.) Force-rate cost (metabolic power $\dot{E}_{FR}$) is linearly related to amplitude of torque rate per time $f\dot{T}$, by coefficient c_t determined from part A. and Fig. 4E.

414 c_t=8.5e-2 (Fig. 6B; see Eqn. 9); joint torque is treated as proportional to muscle force, assuming
415 constant shoulder moment arm.) In terms of proportions, mechanical power accounted for
416 about 94% of the net metabolic cost at 0.58 Hz, and 26% at 1.36 Hz. Correspondingly, force-rate
417 accounted for about 6% and 74% of net metabolic rate at the two respective frequencies.
418
Muscle EMG amplitudes increased with movement frequency (Fig. 7). Deltoid and pectoralis
both increased approximately
with $f^{3/2}$ (pectoralis: R^2= 0.65;
P = 1.1e-6; deltoid: R^2= 0.56; P
=1.5e-5), as did the co-
contraction index (R^2= 0.58; P =
0.0009). This was consistent
with
expectations of muscle
activation increasing faster than
torque for increasing
movement frequencies.
431

Fig. 7 EMG amplitude vs. movement frequency f during cyclic reaching. Inset figure depicts an example EMG rectified (black), filtered (blue), and amplitude (red). Pectoralis and deltoid EMGs (means ± s.d.; N = 10), with best-fit predictions curves (both $f^{3/2}$), $R^2 = 0.65$ and $R^2 = 0.56$, respectively.

Cross-validation of metabolic cost during cyclic reaching
Separate cross-validation trials agreed well with force-rate coefficients. The second group of
subjects moved with
slightly increasing
mechanical power, and
slightly higher metabolic
cost (Fig. 8). But applying
the cost coefficient c_t
derived from the primary
experiment, the model
(Eqns. 1 & 8) was
nevertheless able to
predict cross-validation
costs reasonably well (R^2 =
0.42; P = 2.7e-6).
447

Fig. 8. Cross-validation (CV) of force rate cost. (A.) Amplitude of cyclic reaching condition (compared with primary experiment) vs. movement frequency f and (B.) positive mechanical power $\dot{W}$ vs. f. (C.) Net metabolic rate $\dot{E}$ vs. movement frequency f for cross-validation (means ± s.d.; N = 10). Cross-validation conditions were such that average positive mechanical power W increased slightly with f, unlike the primary experiment. Predicted metabolic rate for CV was determined using c_t and c_W from primary experiment (solid line).

Passive elastic energy
storage during cyclic
reaching
The estimated natural frequency of cyclic arm motions was 2.83 ± 0.56 Hz. This suggests a
rotational stiffness about the shoulder joint of about 250 N·m·rad^{-1}, if series elasticity were
assumed for shoulder muscles. With passive elastic energy storage, the average positive
mechanical power of muscle fascicles would decrease slightly, from about 0.5 W per arm to
0.33 W. Thus, series elasticity would cause active mechanical power to decrease with
movement frequency, as energy expenditure increased.
457

Hill-type model does not predict experimentally observed energy cost
The Hill-type model's predicted net energy cost
increased approximately linearly with
movement frequency, from 33 W to 47 W. The
model dramatically over-predicted the net
metabolic cost for all movements (by up to a
factor of 6.2), and metabolic cost rose across
frequency by less than half as found
experimentally (a factor of 1.42 vs. empirical
3.56). Current musculoskeletal models do not
accurately predict the cost of cyclic reaching.
469

470 ## Force-rate-dependent cost predicts point-to-
471 point reaching motions and durations
472 We applied the energy cost from cyclic
473 reaching to predict discrete, point-to-point
474 reaching (Fig. 9) of fixed and free durations.

Fig. 9. Hand speed trajectories vs. time for point-to-point movements predicted by various objective functions, compared to empirical, bell-shaped profiles. Minimization objectives include metabolic energy expenditure ("min energy" according to model proposed here), error variance (Harris & Wolpert), torque-rate squared (Uno et al 1989, purple), and activation squared for a Hill-type muscle model. Minimum energy expenditure is the sum of work and force-rate costs (Eqn. 1), with coefficient c_t identified from the primary experiment (Fig. 4). All optima use the same initial and final targets and a fixed movement duration.

The prediction from trajectory optimization (Eqns. 10 and 11) was for a standard movement of
fixed duration and distance (0.65 s and 30 cm, respectively; Harris and Wolpert 1998), using the
energy cost coefficients c_W and c_t derived from the primary experiment. This yielded bell-
shaped velocities (Fig. 9) similar to the minimum variance model and to empirical data (Harris
and Wolpert 1998). Also compared were minimum torque-rate (Uno et al. 1989), and minimum
activation squared using a Hill-type muscle model (Kistemaker et al. 2014). Each objective
approximately reproduces the empirical bell-shaped profile, with metabolic cost (Eqn. 1),
torque-rate-squared, and activation-squared all having correlation coefficients above 0.8 (0.99,
0.98, and 0.82, respectively). The metabolic energy cost including empirically-tested work and
force-rate terms therefore predicts trajectories similar to other, non-energetic costs proposed
previously, and to human data.
486
Similar predictions were made for different distances, leaving duration unconstrained (Fig. 10).
The predicted, optimal durations increased
with movement distance, roughly similar to
human preferred durations (Reppert et al.
2018). The associated trajectories also
retained the bell-shaped velocities across all
distances. The proposed metabolic energy
cost, plus a penalty for long durations,
therefore predicts both trajectories and
durations roughly similar to human data.
497

Discussion
We tested whether the metabolic cost of
reaching movements is predicted by the
hypothesized energetic cost including force-
rate. Our experimental data showed a cost
increasing with movement frequency as
predicted with force-rate, more so than did
the mechanical work performed. The same
cost model was also cross-validated with a
separate set of reaching movements, and
predicts smooth reaching movements,

Fig 10. Hand speeds and movement time are predicted simultaneously by optimizing energetic cost and a (linear) cost of time. Hand speed as a function of time are plotted for 5 different reach distances. Inset: In recent empirical observations (adapted from Reppert et al. 2018), movement duration varies approximately linearly with reach distance.

509 similar to the minimum variance model. We interpret these findings as suggesting the force-
510 rate hypothesis as an energetic basis for reaching movements.
511
The force-rate hypothesis explains the observed metabolic energy cost increases better than
more conventionally recognized costs. For example, the cost of mechanical work alone cannot
explain the higher cost at higher movement frequencies, because the rate of work remained
fixed (Fig. 5). A possible explanation is that the energetic cost per unit of work (c_W in Eqn. 1)
could increase with faster movements, due to the muscle force-velocity relationship (Barclay
2015). But the conditions here actually yielded slower hand speeds with higher frequencies (Fig.

5D), and thus cannot explain the higher cost. Nor were our results explained by a current
musculoskeletal model (Umberger 2010), which drastically overestimated the overall cost and
underestimated the increases with movement frequency. The proposed force-rate hypothesis
thus explains these data better than previous quantitative models or relationships.
522
The force-rate hypothesis was also consistent with three other observations: (1)
electromyography, (2) cross-validation, and (3) point-to-point reaching. First, muscle EMGs
increased more rapidly (approximately with $f^{3/2}$; Fig. 7) with movement frequency than did
joint torques (approximately with $f^{1/2}$; Fig. 5C). The proposed mechanism is that brief bursts of
activation require greater active calcium transport (and thus greater energy cost), because
muscle force production has slower dynamics than muscle activation (van der Zee and Kuo
2020). Second, we cross-validated the primary experiment, by applying its cost coefficients (c_t
and c_W, Fig. 6) to predict an independent set of conditions. We found good agreement between
cross-validation data and the force-rate prediction (Fig. 8). The overall energy cost ($\dot{E}$ from Eqn.
1) depends on a particular combination of work, force, and movement frequency, yet only has
one degree of freedom (c_t). Third, the force-rate hypothesis also explains discrete, point-to-
point reaching. The characteristic bell-shaped velocity profile is predicted by optimal control,
using the cost coefficients derived from cyclic movements (Fig. 9). Moreover, movement
duration is predicted to increase with distance, approximately similar to human reaches (Fig
10). These observations serve as tests of the force-rate hypothesis, independently predicted by
a single model.
539
The force-rate cost is surely not the sole explanation for reaching. The optimal control approach
has been used to propose a variety of abstract mathematical objective functions that can
predict movement. But there may be multiple objectives that predict similar behavior. As such,
careful experimentation (Harris and Wolpert 1998; Kawato 1999) was required to disambiguate
minimum-variance from competing hypotheses such as minimum-jerk and -torque-change
(Kawato 1999). Similarly, the present study does not disambiguate force-rate from minimum-
variance, since both predict similar point-to-point movements. In fact, minimum-variance also
has some dependency on effort, albeit implicitly, due to the mechanism of signal-dependent
noise (Harris and Wolpert 1998). It also explains well the trade-off between movement speed
and endpoint accuracy, where energy expenditure is unlikely to be important.   However, the
ambiguity also means that both variance and force-rate could potentially contribute to
movement. It is quite possible that minimum variance dominates for fast and accurate
movements, and energy cost for the trajectory and duration of slower ones, with both
contributing to a unified objective for reaching.
554
555
Effort objectives have long been considered potential counterparts to the kinematic
performance objective. For example, the integrated squared muscle force or activation or
torque-change all emphasize effort and arm dynamics as explicit features for reaching (Uno et
al. 1989). Effort is also important for selection of feedback control gains (Kuo 1995; Todorov
and Jordan 2002), adaptation of coordination (Emken et al. 2007), identification of control
objectives from data (Vu et al. 2016) and determination of movement duration (Shadmehr et

al. 2016). The problem is that these manifestations of effort are abstract constructs with limited
physiological basis, justified mainly by their ability to reproduce bell-shaped velocities through
inverse optimization. However, multiple objectives can reproduce such velocities non-uniquely
Fig. 9), making additional and independent tests important for disambiguating them.
Accordingly, metabolic energy expenditure is a physiological, independently testable measure
of effort.The change in metabolic cost during adaptation (Huang et al. 2012) and the effect of
metabolic state on reaching patterns (Taylor and Faisal 2018) strongly suggest a role for energy
in reaching. The present study offers a means to incorporate a truly physiological effort cost
into optimal control predictions for smooth and economical movements.
571
572
There is a measurable and non-trivial energetic cost for cyclic reaching. Even though the arms
were supported by a planar manipulandum, at a movement frequency of 1.5 Hz, we observed a
net metabolic rate of about 19 W. For comparison, the difference in cost between continuous
standing and sitting is about 24 W (Mansoubi et al. 2015), making the reaching task nearly as
costly as standing up. For each half-cycle reaching action, analogous to a point-to-point
movement, the metabolic cost was about 3.5 J per arm. The cost may not be particularly high,
but the nervous system may nonetheless prefer economical ways to accomplish a reaching
task.
581
There are several limitations to this study. One is that energetic cost was experimentally
measured for the whole body, and not distinguished at the level of the muscle. Force-rate was
also estimated from joint torque and not from actual muscle forces. We therefore cannot
eliminate other physiological processes as possible contributions to the observed energy cost.
In addition, the hypothesized cost ($\dot{E}_{FR}$) is thus far a highly simplified, conceptual model for a
muscle activation cost. More precise mechanistic predictions of this cost would be facilitated
with specific models for muscle activation, myoplasmic calcium transport, and force delivery
are needed (e.g., Baylor and Hollingworth 1998; Ma and Zahalak 1991). We also tested the
force-rate cost in continuous reaches of fixed work, predominantly by the shoulder, and with
the arms supported against gravity. Further studies are needed to test more ecological
movements such as discrete reaching in arbitrary directions, and while interacting with objects.
593
The force-rate hypothesis suggests a substantial role for effort or energy expenditure in upper
extremity reaching movements. Some form of effort cost is often employed to examine
selection of feedback gains or muscle forces, and even generally expected for optimal control
problems where maximal-effort actions are to be avoided (Bryson and Ho 1975). And in the
experimental realm, energy expenditure is regarded as a major factor in animal life and
behavior (Alexander 1996), even to the small scale of a single neural action potential (Sterling
and Laughlin 2017). Under the minimum-variance hypothesis alone, reaching seems unusually
dominated by kinematics. But our results suggest that metabolic energy expenditure may have
been over-shadowed by the minimum-variance hypothesis, because it makes similar
predictions for point-to-point movements. There is need to both quantify and test the force-
rate hypothesis more specifically, perhaps in combination with minimum-variance.

Nonetheless, there is a meaningful energetic cost to reaching that can also explain the
smoothness of reaching motions.
607

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