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The Size of the Stag Determines the Level of Cooperation

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ABSTRACT

In the last 12,000 years, human societies have scaled up from small bands to large states of millions and even billions. Many modern societies and even groups of societies cooperate on large-scale projects with relatively low levels of conflict, but the scale and intensity of cooperation varies dramatically between societies. Here we attempt to formalize dynamics that may be driving this rapid increase in cooperation and the differences we see between societies. Our model extends an N-person stag hunt to include population growth dynamics, "stags" with different sized payoffs, and competition for these stags. An increasing number of cooperators is required to access larger stags. The payoff from these stags in turn increases carrying capacity, which increases competition for the stag. As population size increases, new cooperative thresholds are attainable, and as population size shrinks, previously attainable thresholds fall out of reach. Among other predictions, we show that when a new threshold is accessible to a population, the level of cooperation will increase to reach this threshold. However, when the next threshold is out of reach, cooperation decreases as individuals refrain from costly cooperation, preferring a smaller stag. This model offers a framework for understanding the rapid increase in the scale of human cooperation and decline of violence, differences between societies, and challenges to future cooperation.

INTRODUCTION

The evolution of cooperation and its role in human cultural evolution is a widely investigated topic (reviewed in Henrich & Muthukrishna, 2021). We theoretically explore how resource availability impacts the cooperation among individuals and make predictions about when to expect increasing and decreasing levels of cooperation. This work is connected to previous theoretical and empirical approaches that explore topics related to cooperation and competition (Ito & Doebeli, 2019; Pacheco et al., 2009). Here we investigate a setting in which individuals cooperatively produce rewards, in a N-person stag hunt cooperation game. In the game, each individual can decide to either cooperate or defect. If they chose to cooperate, then they pay a personal cost but help the community to come

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closer to receiving a communal reward. If they defect, then they do not pay any cost, but they still receive the communal reward if the cooperators are successful A communal reward is achieved if the number of individuals choosing to cooperate (k) meets a selected threshold (M). This threshold is the number of individuals required to access a resource. As an example, consider the increasing manpower and infrastructure needed to land a whale, mine coal or refine oil. If this threshold is reached then all members of the community receive an equal share of the communal reward, even those who choose to defect. In the model we present, the returns for cooperating grow exponentially with more cooperators. The exponential growth relationship models characteristic of various energy revolution humanity has gone through, for example hunter-gatherers mainly using fire over the systematic use of solar energy among horticulturalists and modern humans nuclear energy (Smil, 2015, 2017). Each of these energy revolutions lead to an exponential increase in energy returned on energy invested (EROI). However, to access these new energy technologies more cooperators than before working together on these technologies. This means that the returns for cooperating exponentially increase as the number of cooperators increases. In terms of animals, this is equivalent of saying that if you have enough hunters you can cooperate and catch a stag, such as is described in the classic stag hunt. However, if even more people cooperated you could hunt for a larger animal, such as a bear, and the meat from that bear would be twice as much as from a stag, and so on growing exponentially with more cooperators, or hunters in this example.

The chosen approach differs from the classic N-person stag hunt model (Pacheco et al., 2009) in the following ways:

1. There are multiple thresholds for the number of required cooperators to yield a reward. These thresholds increase in size and as larger thresholds are reached a larger reward is provided.
2. The size of the communal return is not scaled based on the number of cooperators. In the classic model this feature helps to incorporate the benefit of having more cooperators than needed to surpass the threshold. In our model, this benefit is included in the ability to reach a larger threshold.
3. The model population growth dynamics whereas in the classic N-person stag hunt model, population size is held constant.

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MODEL

Parameters and Variables

The parameters of our model are F, a multiplication factor which controls the size of the communal return ("the size of the stag") as a multiple of c, the cost of cooperating. M, the threshold values for cooperators required to reach the communal rewards. k, the number of cooperators in the community. N_0, the initial population size of the community. r_{max}, the maximum growth rate of the population. α the strength of average utility in determining carrying capacity. β, the factor by which communal returns increase, or how many times larger the current return is the return for reaching the next threshold.

The variables of our model are x, the likelihood of an individual to cooperate, and N, the population size. x's value is dependent on the difference between the utility of cooperators and defectors, where x is at equilibrium when cooperators and defectors have equal utility. N's rate of change is determined by r_{max} and the carrying capacity of the population. The carrying capacity is in turn dependent on the average utility in the community, where a larger average utility means a higher carrying capacity (see Section: Change in population size).

Parameter	Role	Range
F	Multiplication factor controlling the size of the communal return	$(1, \infty)$
K	Number of cooperators in the population	Integer, $k < N$
M	Number of cooperators required to achieve a communal reward	Integer, $M > 0$
r_{max}	Maximum population growth rate	$r_{max} > 0$
N_0	Initial population size	Integer, $N_0 > 0$
α	Control for the strength of average utility in determining carrying capacity	$0 < \alpha < \frac{N_0 N}{c(1-F)}$
β	The factor by which communal rewards increase (how many times larger than the previous reward is the current one)	$\beta \geq 1$
c	Cost of cooperating, or value of a hare	$c > 0$

Table 1: Parameters of the model

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Variable	Role	Dependent Upon
x	Cooperation rate	Expected cooperator and defector utility
N	Population size	r_{max}, α, average utility

Table 2: Variables of the model

General Idea

We specify the utility of an individual in the following way, separating the utility of defectors (U_D) and cooperators (U_C).

$$U_D(k) = \underbrace{\frac{F \cdot c}{N} \beta^{\left\lfloor \frac{k}{M} \right\rfloor}}_{\substack{Communal \\ return}} \tag{1}$$

$$U_C(k) = \underbrace{\frac{F \cdot c}{N} \beta^{\left\lfloor \frac{k}{M} \right\rfloor}}_{\substack{Communal \\ return}} \underbrace{-c}_{\substack{Cooperation \\ cost}} = \underbrace{U_D(k)}_{\substack{Communal \\ return}} \underbrace{-c}_{\substack{Cooperation \\ cost}} \tag{2}$$

Equation 1 and equation 2 both share a common portion, $\frac{F \cdot c}{N} \beta^{\left\lfloor \frac{k}{M} \right\rfloor}$, which we call the communal return. Every member of the community receives this return. However, as shown in equation 2, cooperators also pay an additional cost c to their utility. By cooperating in the stag hunt, cooperators miss the opportunity for an individual reward and thus pay a cost of c, for example for the lost hare.

The communal return can be separated into two parts: $\frac{F \cdot c}{N}$ and $\beta^{\left\lfloor \frac{k}{M} \right\rfloor}$. The term $\frac{F \cdot c}{N}$ represents the size of the communal return for a given induvial in the population. Again, in terms of stags and hares, $F \cdot c$ states that the utility of a stag is F times larger than the utility of a hare (c as stated above). This stag ($F \cdot c$) is then equally split across the entire population of N people, and so the utility of the stag is divided by N, or $\frac{F \cdot c}{N}$. As F is a parameter of our model, we manipulate the utility ratio of cooperating vs. defecting. The term $\beta^{\left\lfloor \frac{k}{M} \right\rfloor}$, in turn, specifies the level of communal return, which was reached, or the size of the animal caught in the cooperative hunt. That is, the stag is F times larger than the hare but then subsequent animals are β times larger than the previous animal. Thus, M is the threshold for the number of cooperators required to succeed at a hunt. For every M cooperators the next level of communal return is reached, with the size of the return increasing for each reached threshold. β represents how much larger the communal return gets for each new threshold reached. So for every

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M cooperators, the size of the communal return grows exponentially by some rate β. This dynamic is achieved in the floor function $\left\lfloor\frac{k}{M}\right\rfloor$, which is the division $\frac{k}{M}$ rounded down to the next smaller integer.

Summarising the dynamics for the communal return we can describe:

- $\frac{F \cdot c}{N}$ as the size of a communal return for one person
- $\beta^{\left\lfloor\frac{k}{M}\right\rfloor}$ as the level of the communal return based on which threshold is reached.

Overall, $\frac{F \cdot c}{N}\beta^{\left\lfloor\frac{k}{M}\right\rfloor}$ describes the communal return for one individual based on the current number of cooperators k. Notice, when $\beta = 1$, then the communal return does not increase with subsequent surpassing of the threshold, and so we return to a classic N-person stag hunt utility structure.

Using this formulation, we seek to answer the following research questions:

1. How does the current level of cooperation influence the future level of cooperation?
2. How does the cooperation level change when population size changes, if carrying capacity is a function of accessed resources?

To study these questions, we specify how the variables in our model change in the next section.

Variable cooperation rate

The cooperation rate x, so the likelihood of any given individuals in the population to cooperate, is specified by the replicator function in equation 3:

$$\dot{x} = x(1 - x)(f_C(x, N) - f_D(x, N)) \tag{3}$$

Equation 3 includes two other equations which we must still specify, $f_C(x, N)$ and $f_D(x, N)$. $f_C(x, N)$ and $f_D(x, N)$ represent the expected utility of cooperators and defectors respectively. They are both dependent on x, the cooperation rate of the society, as the communal return will vary with x. In equation 3, we can observe that the cooperation rate changes in such a way that the average utility of cooperators and defectors is the same.

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$$f_C(x,N) = \sum_{k=0}^{N-1} \underbrace{\binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k}}_{\substack{Likelihood\ of\ having \\ k\ cooperators}} \cdot \underbrace{U_C(k+1)}_{\substack{Utility\ of\ cooperating \\ with\ k\ others}} \tag{4}$$

$$f_D(x,N) = \sum_{k=0}^{N-1} \underbrace{\binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k}}_{\substack{Likelihood\ of\ having \\ k\ cooperators}} \cdot \underbrace{U_D(k)}_{\substack{Utility\ of\ defecting \\ with\ k\ cooperators}} \tag{5}$$

To determine the expected utility of cooperators and defectors, we calculate how likely it is to have k cooperators in the community for every possible value of k, given a population size of N and a cooperation rate of x, and multiply it by the utility of cooperating or defecting given those k cooperators (see equation 4 and equation 5). The likelihood is captured in $\binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k}$. This is the product of the possible permutations of k cooperators in the population of $N-1$ individuals. How likely given the cooperation rate of x is it to have k cooperators, and how likely given a defection rate of $1-x$ is it to have $N-1-k$ defectors (everyone not cooperating is defecting). The reason for only considering $N-1$ members of the population, rather than all N, is that we are interested in the payoff from cooperating or defecting given the actions of the other $N-1$ members of the population. The portion relating to the utility of cooperating or defecting is $U_C(k+1)$ and $U_D(k)$. $U_C(k+1)$ is the payout for cooperating with $k+1$ cooperators, the other k cooperators in your population plus yourself. $U_D(k)$ is the payout for defecting given that k other members of the population are cooperating. Combining both terms, we can calculate the expected utility of cooperating or defecting for a given cooperation rate of x and population size of N (see equation 4 and equation 5).

Variable population size

The change in population size is operationalized by a carrying capacity growth function. The carrying capacity is the largest population size that can be maintained by available resources.

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$$\frac{dN}{dt} = \underbrace{r_{max} \cdot N}_{\substack{Growth \\ rate}} \cdot \underbrace{\frac{g(U(x,N)) - N}{g(U(x,N))}}_{\substack{Carrying\ capacity \\ limiting\ factor}} \tag{6}$$

$$g(U(x,N)) = \underbrace{N_0}_{\substack{Base \\ population\ size}} \underbrace{+\alpha \cdot U(x,N)}_{\substack{Influence\ of\ average\ utility \\ on\ carrying\ capacity}} \tag{7}$$

$$U(x,N) = x \cdot f_C(x,N) + (1-x) \cdot f_D(x,N) \tag{8}$$

Equation 6 describes how the population size changes with time. It is the product of three different terms which influences the growth rate of the population. r_{max} is a parameter of the model which dictates the maximum possible growth rate of the population. N is the current population size. The population parameter N N s is included in the growth rate function because growth in larger populations is faster as there are more individuals who are able to reproduce and increase the population. Vice versa if the population is shrinking there are more people who can die and shrink the population faster. The final term, $\frac{g(U(x,N))-N}{g(U(x,N))}$, establishes the carrying capacity of the population size. Here, $g(U(x,N))$ specifies the carrying capacity, or the largest possible size of the population. When $g(U(x,N)) > N$ the carrying capacity is larger than the current population and so the community could support a growth in the population. In this case the population will increase until it reaches the carrying capacity. When $g(U(x,N)) < N$, the population shrinks until the carrying capacity is reached, as there are more people in the community than can be sustained by the available resources. This carrying capacity is itself detailed in equation 7. The carrying capacity can accommodate the initial population size N_0 plus some factor of the average utility in the community, as described by $U(x,N)$ in equation 8. Thus, the carrying capacity is directly related to the utility in the population. When the larger, cooperative hunt, there is more food to be shared, and the community can in turn accommodate a larger population. The carrying capacity increases.

A negative carrying capacity, in turn, is both intuitively and mathematically not sound. To ensure that the carrying capacity remains positive, we place an upper bound on α at $\alpha < \frac{N_0 N}{c(1-F)}$. Notice that the smallest possible value of the average utility $U(x,N)$ is $\frac{-c}{N}(1-F)$. This is the case when $x = 1$, meaning everyone in the population is cooperating. If $M > N$, so a successful hunt is impossible, then everyone pays the cooperation cost of c and receives the minimum cooperative reward of $\frac{F \cdot c}{N}$. So

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$U(1,N) = \frac{-c}{N}(1-F)$. This is the smallest possible value for $U(x,N)$ because if there were any defectors they would yield a larger return than the cooperators and so the average utility would be larger. In this case if $\alpha \geq \frac{N_0 N}{c(1-F)}$, then $\alpha \cdot U(1,N) \geq \frac{N_0 N}{c(1-F)}\left(\frac{-c}{N}(1-F)\right) = -N_0$. And so $g\big(U(1,N)\big) = N_0 + \alpha \cdot U \leq N_0 - N_0 = 0$, or $g\big(U(1,N)\big) \leq 0$. To ensure that this does not occur and that our carrying capacity is always positive we require $\alpha < \frac{N_0 N}{c(1-F)}$.

RESULTS

Level of cooperation when population size is fixed

We attempt to answer research question 1 by examining how the current level of cooperation influences future levels of cooperation. We do so by fixing the population size at N individuals, in effect treating it like a parameter. We then study the behaviour of $\dot{x}$ as described in equation 3. We observe a decrease in cooperation when $\dot{x} < 0$, and an increase in cooperation when $\dot{x} > 0$. We are interested in knowing under what conditions cooperation will either increase, decrease, or stay stable. From our definition of x being the cooperation rate, its value must be between 0 and 1 .The first two terms in equation 3 have no effect on the sign of $\dot{x}$ and $f_C(x,N) - f_D(x,N)$ dictates the sign of $\dot{x}$. If $f_C(x,N) > f_D(x,N)$ then $f_C(x,N) - f_D(x,N) > 0$ and cooperation is increasing, if $f_C(x,N) < f_D(x,N)$ then $f_C(x,N) - f_D(x,N) < 0$ and cooperation is decreasing. If the average utility of cooperators is larger than the average utility of defectors, then cooperation will increase, and vice versa. To study the relationship between $f_C(x,N)$ and $f_D(x,N)$, we will be rewriting $f_C(x) - f_D(x)$ (see Rewriting $\boldsymbol{f_C(x,N) - f_D(x,N)}$):

$$f_C(x,N) - f_D(x,N) = c\left(-1 + \frac{F}{N}Q(x)\right) \tag{9}$$

$$Q(x) = \sum_{l=1}^{\lfloor N/M \rfloor} \binom{N-1}{l \cdot M - 1} \cdot x^{l \cdot M - 1} \cdot (1-x)^{N - l \cdot M} \cdot \beta^{l-1}(\beta - 1) \tag{10}$$

By this we know that $f_C(x) - f_D(x) = 0$, and in turn $\dot{x} = 0$, for x_0 such that $Q(x_0) = \frac{N}{F}$.

Next, we explore conditions that will fix the cooperation rate. We fix the values for N, M, and β and then study the relationship between the cooperation rate x and the multiplication factor F which fixes

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cooperation (Figure 1). We are studying the function $F = \frac{N}{Q(x)}$ where x is the variable. In other words, for a certain population which cooperates at a rate of x, how large would the stag need to be for both cooperators and defectors to be equally satisfied and not want to switch sides? This is an interesting question, as it will enable us to give us the value for the multiplication factor$F$, which fixes the cooperation rate. This it provides insights into the level of cooperation relative to the size of the cooperative reward.

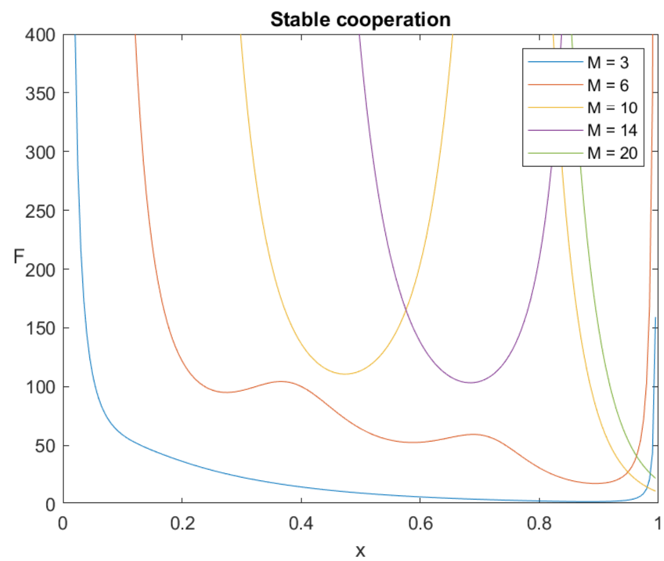

Figure 1. The F value needed to stabalize the cooperation rate (N=20, β=2).

Extreme levels of cooperation (x approaching 0 or 1) are only sustainable for very large cooperative rewards. This is explained in $\lim_{x \to 0,1} Q(x) = 0$, so $\lim_{x \to 0,1} F = \frac{F}{Q(x)} = \infty$. Intuitively, when cooperation is very low, near 0, then a successful hunt is extremely unlikely .The only reason to cooperate in that scenario is if the size of the stag is large enough to increase the expected utility albeit the low odds of a successful hunt. Similarly, when cooperation is very high, near 1, a successful hunt is nearly guaranteed. In this scenario, individuals are incentivized to defect and free-ride on the cooperative reward. However, if the stag is extremely large, then cooperating becomes viable as the added security of having an extra cooperator for such a large reward is more valuable than defecting to catch the hare. This way of thinking does not apply if having everyone cooperate is its own threshold, thus cooperating is still viable for a smaller stag (ex: the case of $M = 10$ with $N = 20$ in Figure 1).

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In general, the optimal value of the multiplicator factor F is at its lowest near cooperation rates equal to a multiple of $\frac{M}{N}$, or when the cooperation rate is such that it is likely to have just enough cooperators to reach a new threshold. For values of x slightly below or above a multiple of $\frac{M}{N}$, a larger stag is required to stabilize the cooperation rate. When the cooperation rate is below $\frac{M}{N}$, cooperators will want to defect as their cooperation is wasted seeing as they are unlikely to reach the next threshold while also comfortably remaining in the previous threshold even upon defecting. However, if the stag is slightly bigger then they are willing to remain cooperators because the payout from successfully reaching the next threshold is increasingly rewarding. Similarly, when the cooperation rate is above $\frac{M}{N}$, cooperators will want to defect as their cooperation is likely to be wasted effort. However, in this case the larger stag keeps cooperators as the security of remaining at a threshold is that much more important as the reward for doing so is larger. We can furthermore observe that the required F of the cooperative reward is generally decreasing. This is because with each new threshold reached, the communal return becomes exponentially larger and so the difference between cooperating and defecting is less relevant, as the cooperation cost c is fixed. Individuals are still satisfied even with a smaller multiplication factor F. As seen in Figure 1, if M is sufficiently small, in this case M=3, this pattern disappears. Instead the optimal value of F is always decreasing except at the very high extreme (for the same reasons explained above). When reaching the next threshold is within reach, cooperation should be always optimal.

We can derive two conclusions from these results. Firstly, people cooperate because it yields larger utility than defecting. When the likelihood of successfully cooperating is very low, individuals are only cooperative if the size of the reward offsets the low probability. If the returns from cooperating aren't sufficient for the risk involved, then individuals will defect instead. Secondly, when the current level of communal reward is larger, individuals are more willing to take a risk on cooperation.

Next, we consider how the current level of cooperation effects the future change in cooperation. We do so by once again fixing our population size N and studying the behaviour of $f_C(x) - f_D(x)$ with respect to x. We plot $f_C(x) - f_D(x)$ for multiple values of M, as M influences the general shape of this plot. We set $F = \frac{N}{Q(x_0)}$, where x_0 is the smallest number in (0, 1) such that $Q'(x_0) = 0$ and $Q''(x_0) < 0$ (Figure 2). As explained above, this value for F will ensure that $f_C(x_0) - f_D(x_0) = 0$ and by our selection of x_0, this stable level of cooperation will occur at the first spike in cooperation.

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We observe different patterns for $f_C(x) - f_D(x)$ depending on the relationship between M and N. Recall that cooperation is increasing when $f_C(x) - f_D(x) > 0$ and decreasing when $f_C(x) - f_D(x) < 0$. If $M \leq \frac{1}{2}N$, we observe multiple spikes and drops in the change of cooperation. This case shows how the level of cooperation may fluctuate between increases and decreases depending on the current level of cooperation. We note that the behaviour depicted in the graphs does not in fact depict how cooperation levels change across time, but rather depicts a snapshot of how the current level of cooperation will affect future levels of cooperation. Second, when the line in Figure 2 intersects the x-axis then $f_C(x) - f_D(x) = 0$ and that level of cooperation is fixed. As such this figure does not state that the level of cooperation will go through phases of rising and falling cooperation, but rather suggests that rises and falls in cooperation are finite. If $M > \frac{1}{2}N$ then there is only one possible threshold to be reached and so the behaviour mimics that of the original N-person stag hunt. In both cases, spikes in cooperation occur near values of x equal to a multiple of $\frac{M}{N}$. More specifically they occur at $x_0 \epsilon (0, 1)$ such that $Q'(x_0) = 0$ and $Q''(x_0) < 0$.

Intuitively, this behaviour of spikes and dips occurs because when a new cooperation threshold is within reach, individuals are most likely to switch from defecting to cooperating. As such, the change in cooperation rate is highest near this threshold. Similarly, the change in cooperation rate is lowest when directly between two thresholds when furthest from either one. This is because the increase in cooperation is driven by the reward of successfully reaching a threshold. If there is no threshold to be met realistically, there is no incentive to pay the cooperation cost c is low. Value is also placed on continuing to cooperate, when one threshold has just been met to add security to the cooperation and not accidentally fall below a threshold. We also see a general increase in future cooperation given an increase in current cooperation. This is because as the benefit of cooperation increases with new thresholds reached, then the comparative benefit of defecting is comparatively less valuable. Furthermore, the benefit of reaching an even further level, or of not falling below the current level, is itself larger and more valuable.

Similar to in Figure 1, there is a different behaviour when M is sufficiently small. Here we find that the change in cooperation is always increasing, until it becomes impossible to reach another threshold. The reason for this behaviour is that, when M is small enough, even after one threshold is successfully reached the next threshold is already within reach. Typically, after the rate of cooperators reaches a point such that one threshold is comfortably met, it becomes comparatively more worthwhile to defect

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as defectors can be confident in receiving the communal reward whereas there is no benefit for there being more cooperators. This behaviour continues until the next threshold is within reach, and then the payout for cooperating increases as there is now an added benefit for cooperating. However, in the case of a sufficiently small M, the next threshold is always close enough to be within reach and the utility for cooperators is increasing comparatively to the utility of defectors.

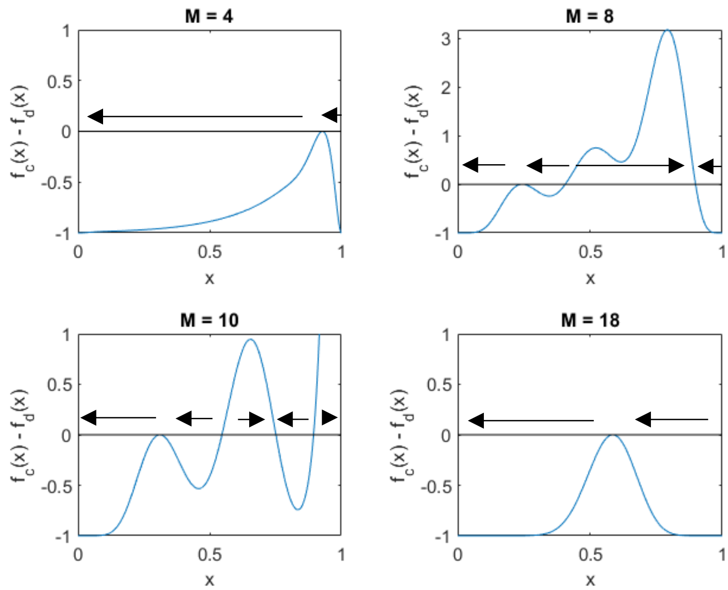

Figure 2. The effect of the current cooperation rate on future cooperation change (N=30, c=1, F=N/Q(x_0), β=2).

When the size of the population is fixed the change in cooperation generally increases, albeit with spikes and dips. In terms of the behaviour of F, this is seen in the fact that a stable population is available with a smaller F value near thresholds before rising again. However, over time the value of F appears to be generally falling. When it comes to the way current cooperation influences future cooperation, we find that cooperation increases most near a threshold. This is because when a new threshold is within reach then people will start cooperating more to reach this new threshold. Otherwise people are likely to defect because it is more personally beneficial.

Change in population size when cooperation rate is fixed

Before we can answer research question 2, we must examine how population size changes when the cooperation rate is fixed, as described in equation 6. Recall that the behaviour of $\frac{dN}{dt}$, and in turn how the population size changes, is dictated by $\frac{g(U(x,N))-N}{g(U(x,N))}$. As explained in the Variable population size

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section, we have selected α in such a way that $g(U(x,N)) > 0$ is always true. This means that the sign of $\frac{g(U(x,N))-N}{g(U(x,N))}$ is only controlled by $g(U(x,N)) - N$. If $N > 0$ and $r_{max} > 0$, then by equation 6, the sign of $\frac{dN}{dt}$ is only determined by $\frac{g(U(x,N))-N}{g(U(x,N))}$ and in turn the sign of $g(U(x,N)) - N$. Notice then that if $g(U(x,N)) > N$, then $\frac{dN}{dt} > 0$, if $g(U(x,N)) < N$, then $\frac{dN}{dt} < 0$, and if $g(U(x,N)) = N$, then $\frac{dN}{dt} = 0$. This relationship between the current population size and the community's carrying capacity determines how the population size changes.

It is important to note that the current population size also influences the carrying capacity. As described in equation 7, carrying capacity is a function of $U(x,N)$, the average utility of an individual in the community. This means that as the population size increases, each individuals' share of the communal reward decreases, and so average utility and the carrying capacity decreases. However, as the population size increases with the cooperation rate fixed, a share of new individuals in the population will be cooperators. Thus, more possible cooperators are available to reach the next threshold of the communal reward. This would yield a large increase to the average utility and the carrying capacity. As such we predict to see the carrying capacity go through a steady decline, followed by a large increase when a new threshold is met. This prediction is mirrored in the results shown in Figure 3. In Figure 4 shows how the carrying capacity translates into population change. Here when population growth is above the x-axis then the population is growing and when population growth is below the x-axis then it is shrinking. We see that in some conditions the population size is actually larger than the carrying capacity. Even though the carrying capacity is steadily increasing, it does at a slower rate than the population size. As a result, for some values of population size, the carrying capacity has stable points which it cannot surpass. We do see, however, that the carrying capacity continues to grow faster than the population size. This is because as the communal rewards grow exponentially, having a larger population with more cooperators is more beneficial then the cost in a sharing amongst more people.

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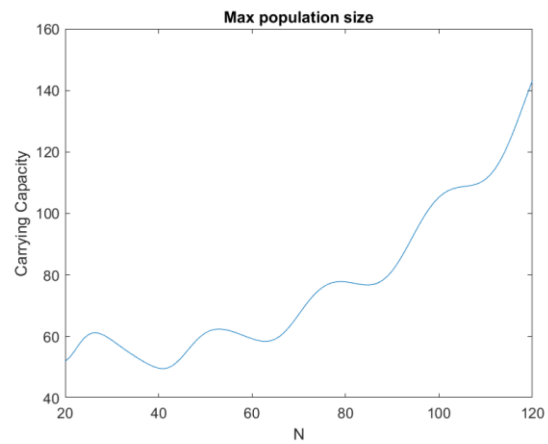

Figure 3. The effect of population size on carrying capacity (F=600, α=1, c=1, M=18, x=0.75, N_0=20, β=2).

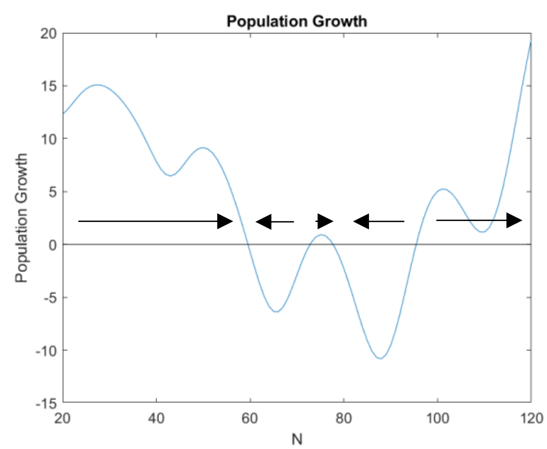

Figure 4. The effect of population size on population growth (F=600, α=1, c=1, M=18, x=0.75, N_0=20, β=2).

We also look at how different cooperation rates impact the carrying capacity of the population (Figure 5). To do so, we vary the cooperation rate and find the largest population size N, such that $g(U(x, N)) > N$. In this way we are considering how an increase in cooperation rate and an increase in population will influence the carrying capacity, with the later effect being described above. We find that as the level of cooperation increases, so does the carrying capacity. This increase is at first gradual but becomes steeper with time, because with a higher level of cooperation, more rewarding thresholds are reached. This provides larger increases to average utility and to the carrying capacity. There is the possibility that at some critical cooperation rate x_0, the carrying capacity increases towards infinity. This occurs when the increase in carrying capacity from reaching a new threshold introduces enough new cooperators into the community that the following threshold is itself reached.

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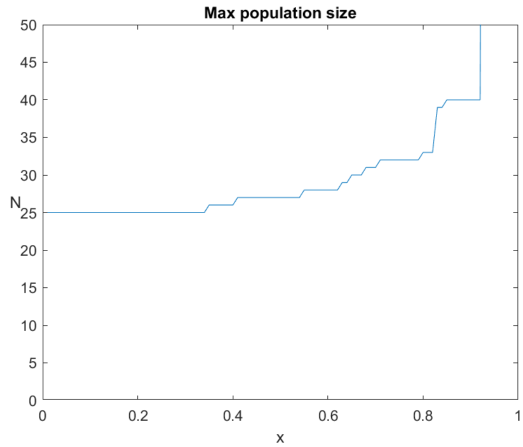

Figure 5. The effect of cooperation rate on the carrying capacity (F=100, α=1, c=1, M=8, N_0=20, β=2).

Given a rising level of cooperation or a rising population size, the carrying capacity of the community rises allowing for the population size to further increases. This behaviour is driven by the fact that as the number of cooperators becomes large enough that a new threshold for a communal reward is possible, then the average utility of each member of the community receives a large increase. This, in turn, also produces a large increase in the carrying capacity. However, in between thresholds, there is a slight drop in average utility and carrying capacity, as there are more individuals with whom to share resources.

Variable levels of cooperation rate and population size

Finally, we consider how the change in population size influences the change in cooperation rate and vice-versa. There are a couple ways we can examine this relationship. Firstly, we have already examined how different cooperation rates influences the carrying capacity in Figure 5. We can also examine the stable points of both cooperation rate and population size, drawing from equation 3 and equation 6. Here we are left with the following equations for these stable points:

(1) $f_C(x, N) - f_D(x, N) = 0$

(2) $g(U(x, N)) - N = 0$

And with a substitution for $g(U(x, N))$

(2) $N_0 + \alpha \cdot U(x, N) - N = 0$

(2) $N_0 + \alpha \cdot (x \cdot f_C(x, N) + (1 - x) \cdot f_D(x, N)) - N = 0$

To solve this, we first multiply (1) by $\alpha \cdot (1 - x)$ and then add it to (2)

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(3) $N_0 + \alpha \cdot x \cdot f_C(x,N) + \alpha \cdot (1-x) \cdot f_D(x,N) - N + \alpha \cdot (1-x) \cdot f_C(x,N) - \alpha \cdot (1-x) \cdot f_D(x,N) = 0$

(3) $N_0 + \alpha \cdot x \cdot f_C(x,N) - N + \alpha \cdot (1-x) \cdot f_C(x,N) = 0$

(3) $N_0 + \alpha \cdot f_C(x,N) - N = 0$

This means the relationship between x and N to fix both the cooperation rate and the population size is:

$$f_C(x,N) = \frac{N - N_0}{\alpha} \tag{11}$$

Note that this equation cannot be easily solved as N determines the upper limit of the sum in $f_C(x,N)$ and N must be an integer. We can however plot the relationship between N and x, however N cannot be the dependent variable for these same issues. We then have N as the independent variable and we plot the value of x which solves equation 11 (Figure 6). Recall also that equation 11 represents the combination of N and x which fixes both change in cooperation rate and change in population size. We see that as population increases, there is at first a spike in cooperation before gradually decreasing. This tells us that to ensure that both the population size and the level of cooperation are stable, the larger the population size, the less cooperation there will be. This is likely since as population size increases there are more people competing over resources and so defecting becomes a better solution to ensure that you receive the highest payout possible. As such, in larger populations, where there is more resource scarcity, cooperation is punished as cooperating requires one to pay a personal cost which is relatively more costly now that there are more individuals with whom to share the communal reward. This can also be interpreted by saying that as the population size increases, it is possible to transfer the burden of producing cooperators from the cooperation rate onto the size of the population without lowering the total number of cooperators. In this way its still the same threshold being reached as before, but now more people are defecting.

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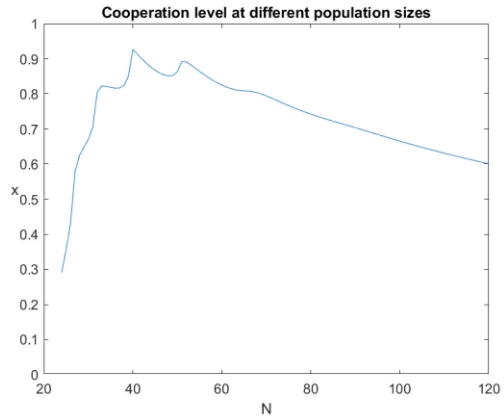

Figure 6. The solution to Equation 11 with N as the independent variable (F=100, α=1, c=1, M=10, N_0=20, β=2).

We can also perform the same analysis as in Figure 6, seeing how increasing our population size changes the level of cooperation, but by basing the level of cooperation on our equation for $\dot{x}$, rather than equation 11. Here we study how a change in population size impacts where cooperation is stable (Figure 7). This analysis does not take into account whether the population sizes are actually possible given our carrying capacity, which explains why the trend is different than in Figure 6, where the carrying capacity was a factor. Here we are simply interested in the way an increase in population size changes the cooperation rate. In Figure 7 we see that there are multiple stable levels of cooperation, which follow two trends. The first trend is a similar gradual decrease as is also depicted in Figure 6. The other trend is one of decrease followed by a return to total cooperation. This second trend always remains near a cooperation rate of 1. This suggests two equilibria for playing the game with an increasing population. One possibility is that as population size increases, cooperation drops as each individual's share of the communal reward is now smaller. Thus, individuals will prioritize themselves over their community. The other possibility is that as population increases, cooperation decreases, for the same reasons as the first trend, until reaching a point where a higher threshold can be reached and so cooperation increases back to complete cooperation, or $x = 1$.

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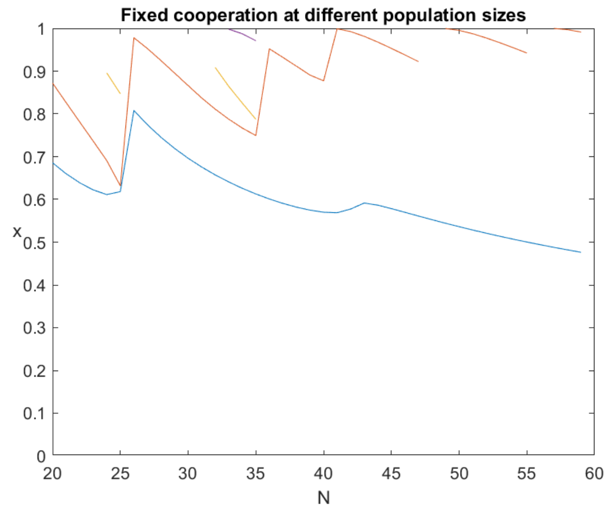

Figure 7. The multiple stable cooperation rates at increasing population sizes (F=75, α=1, c=1, M=8, N_0=20, β=2).

Another point to consider is how changing population size effects the relationship between current cooperation levels and change in cooperation (see Figure 2). Instead of keeping population size fixed, as in Figure 2, we set the population size to be at the current carrying capacity for the selected level of cooperation (Figure 8). For this exercise it is important to select values for our parameters such that the carrying capacity does not approach infinity, as seen in Figure 5, otherwise it would be impossible to set the population size to the carrying capacity. Here we see the same general behaviour as in Figure 2, but with more dramatic spikes and dips. The reason for the peaks being higher and the dips lower is explained by the fact that the population is at the carrying capacity and thus in a resource scarce environment. In the case where the next threshold is within reach, then increasing cooperation will create a large surplus of resources that will be able to maintain the large population. In the case where the next threshold is not within reach, we find a decrease in cooperation as the cost of cooperating is comparatively more expensive in a resource scarce environment. Furthermore, decreasing cooperation could also decrease the carrying capacity ensuring that there are less people with which to share resources. In all we see a pattern where, as cooperation increases, there is a general trend for cooperation to keep increasing in the future. However, this increase in cooperation is also surrounded by temporary decreases.

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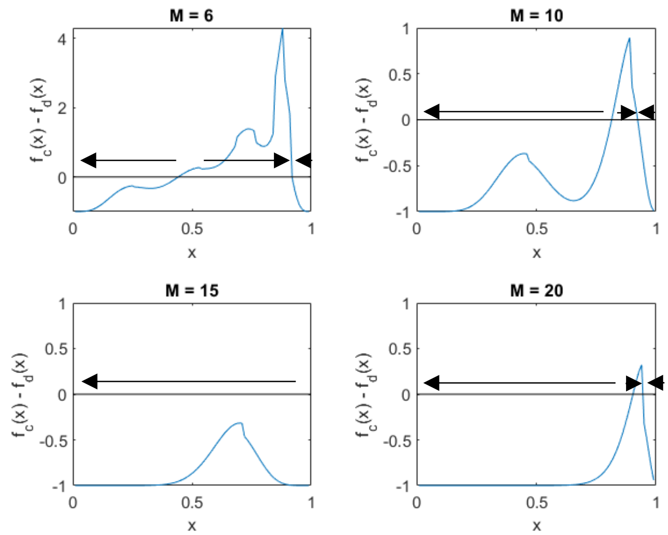

Figure 8. The effect of the current cooperation rate on future cooperation change with a population size equal to the carrying capacity (F=75, c=1, α=0.2, N_0=20, β=2).

DISCUSSION

Our model attempts to explain the mechanisms which drive cooperation. We build upon previous attempts to do so, namely the N-person stag hunt (Pacheco et al., 2009). We build our model upon the insight that that as more people cooperate, they are able to reach for larger rewards. Put differently, there is not only the option to defect and catch the hare or cooperate and catch the stag, but with a increasing group of cooperators even bears and whales can be caught. As such we see stages in cooperation. When there are enough cooperators to reach the next threshold, which would in turn greatly benefit the population, then cooperation increases. When the next threshold is out of reach then we see a decline in cooperation as it becomes beneficial to place oneself above others. When we consider the effects of a changing population size this behaviour becomes more dramatic. Larger populations place larger cooperative thresholds within reach, but there are also more people to share available resources with.

Something we notice in our model is that the trends tend to become weaker as higher cooperative thresholds are reached. In Figure 1 the multiplication factor F for a stable cooperation rate generally decreases when cooperation is higher and in Figure 3 the cost of a higher population to the average utility is reduced with time. This is tied to our model prediction that cooperation will be high when resources are plentiful and low when resources are scarce. As cooperation increases, the returns from cooperation are exponentially larger. This means that the utility of the population is much higher in

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these cases, and so even in situations where defection should be highest, between thresholds, resource scarcity is not so severe. Cooperation generally remains high. This same dynamic occurs when population increases. A larger population means larger cooperative returns, and so even though there are more people to share with, the population is generally better off.

There are two things we have not yet fully considered yet. Firstly, population size does not increase all at once. In our analysis we do not consider how populations gradually grow in size, but rather we imagine scenarios where the population size immediately reaches the carrying capacity. In a future simulation of the model, we could capture a more dynamic population size which grows in a more realistic manner. Secondly, resources in our model are always evenly split. This leads to population increases being generally favourable in our model, as the benefits of having a larger population and thus reaching a larger threshold outweigh the cost of sharing resources amongst more people. A more realistic assumption is that resources are not evenly shared. In that scenario, the effects of population increases may not be so straightforward, and there may be more severe movements in terms of cooperation rates in between thresholds.

There are 4 interactions to be studied in the model. How current cooperation effects future cooperation, how current cooperation effects future population, how current population effects future cooperation, and how current population effects future cooperation. The general trends are as follows:

- As the current cooperation increases then future cooperation will generally increase with some spikes and dips
- As the current cooperation increases then future population increases (through carrying capacity) with the possibility of a carrying capacity of infinity
- As the current population increases then future cooperation can either decrease or generally stay at near full cooperation
- As the current population increases then future population will generally increase with some spikes and dips

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APPENDIX

Rewriting $f_C(x,N) - f_D(x,N)$

$$f_C(x,N) - f_D(x,N) = \sum_{k=0}^{N-1} \binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k} \cdot U_C(k+1) - \sum_{k=0}^{N-1} \binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k} \cdot U_D(k)$$

$$f_C(x,N) - f_D(x,N) = \sum_{k=0}^{N-1} \binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k} \cdot \left(U_C(k+1) - U_D(k)\right)$$

$$f_C(x,N) - f_D(x,N) = \sum_{k=0}^{N-1} \binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k} \cdot \left(\frac{Fc}{N}\beta^{\left\lfloor\frac{k+1}{M}\right\rfloor} - c - \frac{Fc}{N}\beta^{\left\lfloor\frac{k}{M}\right\rfloor}\right)$$

$$f_C(x,N) - f_D(x,N) = \sum_{k=0}^{N-1} \binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k} \cdot \left(\frac{Fc}{N}\beta^{\left\lfloor\frac{k+1}{M}\right\rfloor} - \frac{Fc}{N}\beta^{\left\lfloor\frac{k}{M}\right\rfloor}\right) - c\sum_{k=0}^{N-1} \binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k}$$

With $\sum_{k=0}^{N-1}\binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k} = \left(x + (1-x)\right)^{N-1} = 1$, we get:

$$f_C(x,N) - f_D(x,N) = -c + \sum_{k=0}^{N-1} \binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k} \cdot \left(\frac{Fc}{N}\beta^{\left\lfloor\frac{k+1}{M}\right\rfloor} - \frac{Fc}{N}\beta^{\left\lfloor\frac{k}{M}\right\rfloor}\right)$$

$$f_C(x,N) - f_D(x,N) = c\left(-1 + \frac{F}{N}\sum_{k=0}^{N-1} \binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k} \cdot \left(\beta^{\left\lfloor\frac{k+1}{M}\right\rfloor} - \beta^{\left\lfloor\frac{k}{M}\right\rfloor}\right)\right)$$

So, we can say:

$$Q(x) = \sum_{k=0}^{N-1} \binom{N-1}{k} \cdot x^k \cdot (1-x)^{N-1-k} \cdot \left(\beta^{\left\lfloor\frac{k+1}{M}\right\rfloor} - \beta^{\left\lfloor\frac{k}{M}\right\rfloor}\right)$$

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Notice that because k must be an integer $\beta^{\left\lfloor\frac{k+1}{M}\right\rfloor} = \beta^{\left\lfloor\frac{k}{M}\right\rfloor}$ unless $k = lM - 1$ where $l \geq 1$ and l is an integer. In cases where $\beta^{\left\lfloor\frac{k+1}{M}\right\rfloor} = \beta^{\left\lfloor\frac{k}{M}\right\rfloor}$, then $\beta^{\left\lfloor\frac{k+1}{M}\right\rfloor} - \beta^{\left\lfloor\frac{k}{M}\right\rfloor} = 0$ and so has no influence on the result of the sum. Thus, we only need to examine the terms of the sum where $k = lM - 1$. By this observation we can rewrite $Q(x)$ as the following:

$$Q(x) = \sum_{l=1}^{\lfloor N/M \rfloor} \binom{N-1}{lM-1} \cdot x^{lM-1} \cdot (1-x)^{N-lM} \cdot \left(\beta^{\left\lfloor\frac{lM}{M}\right\rfloor} - \beta^{\left\lfloor\frac{lM-1}{M}\right\rfloor}\right)$$

$$Q(x) = \sum_{l=1}^{\lfloor N/M \rfloor} \binom{N-1}{lM-1} \cdot x^{lM-1} \cdot (1-x)^{N-lM} \cdot \beta^{l-1}(\beta - 1)$$