# When is it Worth Working for Water? A Utility Maximization Theory 

Pamela Reinagel<br>Division of Biological Sciences, Neurobiology Section, University of California San Diego, La Jolla CA 92103<br>preinagel@ucsd.edu


#### Abstract

In nature, amount of work an animal must do to obtain a resource like water depends on conditions in the environment. Conditions change, so it would behoove animals to allocate effort flexibly such that they work enough, but not more than necessary. To study this, we maintained rats in an environment where all water was earned in a task. We varied the reward magnitude and measured voluntary effort and water consumption. The rats did more trials per day when the reward per trial was smaller, yet worked for more water per day when rewards were larger. We propose an analytic model based on utility maximization which can account for these behavioral observations. The model is fit with per-day total work and consumption, but provides insight into the timing of trials and implicates a candidate neural substrate.


Keywords: rodent; decision-making; reward; motivation; response vigor; neuroeconomics; consumer problem; demand curve; labor supply curve; wage-labor law; elasticity of demand; back-bending labor supply; classical economics; consumer problem; microeconomics; homeostasis; thirst; lamina terminalis (LT); subfornical organ (SFO); circumventricular organs.

## Introduction

When animals have two ways to get a resource like water, they tend to choose the way that gets them more water for less work. Neural mechanisms underlying this kind of comparative choice are wellstudied (1, 2). Similarly, when mice were offered only one way to get water at a time, they worked harder during the time blocks when the water reward was larger (3). This makes sense - save energy for when the work will pay off most - but it can't be the whole story. If motivation were driven entirely by expected reward, animals would be less motivated to work for water during a drought (because they would expect less reward per unit of effort) and might die of thirst. It helps matters that the perceived value of a reward is normalized according to recent experience, such that rewards that would have been considered small in a rich environment are perceived as large relative to a lean environment (4-6). But normalization would at best equalize motivation between rich and lean environments. If the difficulty of getting water changes slowly compared to the timescale of physiological necessity, animals must invest the most effort to gain it precisely when the reward for that effort is least.

To explore how animals adapt to this kind of challenge, we maintained rats in a live-in environment in which all their water was earned by performing a difficult sensory task. We varied the reward magnitude and measured rats' effort output and water consumption. As expected, rats did more trials per day when the reward per trial was smaller, thus maintaining healthy hydration levels regardless of reward size. More surprisingly, however, rats worked for more water per day (and far more than they needed) when it was easier to earn. This suggests that they can regulate their consumption dramatically (up to three-fold) to conserve effort when times are lean.

Classic animal behavior studies noted both these effects in experiments designed to test economic utility maximization theory (7-9). Here we revisit and extend that theoretical framework with the goal of relating utility maximization to behavioral dynamics and candidate neural mechanisms.

## Results

## Experimental approach

Rats performed a visual discrimination task to earn water rewards(10, 11). Briefly, an operant chamber was connected to each rat's cage as the rat's sole source of water. Rats could enter the chamber and initiate a trial at any time, upon which a visual motion stimulus was displayed on one wall of the chamber. The direction of motion indicated which of the two response ports would deliver a water reward; a response at the other port resulted in a brief time-out. The visual discrimination was difficult enough that rats made errors. We know rats consider this "work" because they will not do trials if there is no water reward, or if they are not thirsty. The reward volume was held constant for blocks of 1-2 weeks, and varied between time blocks. Below we use the term "reward size" to refer to the a priori expected reward of a trial (the volume of one reward multiplied by the probability of earning a reward). This value was stable for days and therefore known to the rat at the time of each decision to initiate a trial. We measured how many trials a rat performed and how much water reward a rat consumed per day. For further experimental details see Methods.

## Observations motivating the model

We measured the steady-state trial rate of $N=4$ rats which had access to the task 24 hours/day in their home cage and were tested for at least eight non-overlapping steady-state time blocks with reward sizes spanning at least a $50 \mu \mathrm{l}$ range, allowing us to ask how behavior correlated with reward size. Trial rate $L$ declined with reward size $w$ in each case (Figure 1A-D). This result confirmed our expectation that when reward size is held constant for days and no other water is available, rats will work harder for water when rewards are smaller. An obvious explanation of this could be that each rat performed the number of trials required to earn some fixed amount of water, regardless of reward size (Figure 1A-D, blue curves). We take this as the baseline hypothesis of the study.

We found, however, that total water intake was positively correlated with reward size (Figure 1E-H; Table 1). This is inconsistent with the fixed-intake baseline model (blue lines), and suggests that the rats took into consideration the cost of water (in effort) when deciding how much to consume. Note that for every reward size shown, the daily water intake was sufficient for the rat to sustain clinically normal hydration, weight, and apparent health and wellness long-term.


Figure 1. Rats worked harder for smaller rewards, but consumed more water when rewards were larger. Each panel shows results from one rat with 24 hour/day task access. Each symbol shows results averaged over a contiguous stretch of 4 to 7 days on a fixed reward condition, excluding the first day after a reward change. A-D: the expected reward $w$ ( $\mathrm{ml} /$ trial) vs. the number of trials performed per day $L$. Blue curves show the best fit to a fixed-intake model $L=k / w$. A. Rat with $N=12$ steady-state time blocks; the range of reward sizes spanned $0.07 \mathrm{ml} /$ trial; Fixed intake model $k=17.1 \mathrm{ml} /$ day. B. Rat with $\mathrm{N}=10$ blocks, range $0.05 \mathrm{ml} /$ trial, $k=18.4$. C. Rat with $\mathrm{N}=11$ blocks, range $0.08 \mathrm{ml} /$ trial, $k=17.3$. D. Rat with $\mathrm{N}=8$ blocks, range $0.08 \mathrm{ml} /$ trial, $k=$ 20.0. E-H: Water intake $H$ ( $\mathrm{ml} /$ day) vs the expected reward $w(\mathrm{ml} /$ trial), for the same data shown in AD. Blue lines indicate the best fit fixed-intake model. For statistics see Table 1 in Methods.

## Utility maximization model

We recognized these observations as analogous to phenomena first described in human economics. First, rats did less work per day when rewards were larger, just as in economics the labor supply declines as wage rates increase(8). In the case of humans, this so-called back-bending labor supply curve is attributed to workers preferring increased leisure time over increased income. Second, rats consumed more water per day when it was cheaper (in trials/ml), just as in economics the consumption of a commodity can be sensitive to price, an effect known as price elasticity of demand(7). In economics these patterns have classically been explained by utility maximization theory; therefore, we propose a utility maximization theory to explain our rats' behavior.

Utility is a measure of subjective value of anything, in arbitrary units of "utils". Utility can be either positive (for benefits) or negative (for costs). Utility maximization theory posits that individual choices are made to the individual's net utility, subject to external constraints such as wage rates or prices. The theory allows for utility to be subjective in the sense that preferences can be individually idiosyncratic, but assumes that an individual's preferences are stable and that individuals are able to determine what
behavioral choices will maximize their utility. To apply this theory to our task, we needed to develop a specific utility model for rats doing trials for water rewards.

## Utility of water

The equation we suggest for the utility of consuming H ml of water in a day has only a single free parameter, $\alpha$ :

$$
U_{H}(H)=(\alpha+1) \ln (H+1)-H \quad \text { where } H \geq 0 ; \alpha \geq 0 \quad \text { Equation } 1
$$

We chose this expression because it met the following criteria: $U_{H}(0)=0$ (consuming no water has no utility), it has a positive but decreasing slope (diminishing returns) up to a single maximum, beyond which it declines (consuming too much water is also bad). The maximum of $U_{H}(H)$ occurs at $H=\alpha$. Therefore $\alpha$ has a natural physical interpretation as the rat's most preferred quantity of water in the absence of any cost. The Marginal Utility of water, the derivative of Utility, is thus:

$$
M U_{H}(H)=U_{H}^{\prime}(H) d H=\frac{\alpha+1}{H+1}-1 d H
$$

Equation 1'

The amount of water consumed $H$ is determined by the number of trials done $L$ and the reward size or "wage rate" w: $H=w L$ (the budget line in economic terms). Thus, we can rewrite Equation 1 in terms of the number of trials done:

$$
U_{H}(L, w)=(\alpha+1) \ln (w L+1)-w L \text { where } \alpha \geq 0 ; L \geq 0 ; w \geq 0
$$

The maximum of $U_{H}(L)$ (Figure 2A, Equation 2) occurs at $w L=\alpha$. The Marginal Utility with respect to trial number $L$ is given by:

$$
M U_{H}(L)=U_{H}^{\prime}(L, w)=w\left(\frac{\alpha+1}{w L+1}-1\right) d L
$$

Equation 2'

Figure 2A shows curves of $U_{H}(L)$ for an example value of $\alpha$ and several different wage rates $w$. Figure 2 B shows the Marginal Utility curves $M U_{H}(L)$ vs $L$ on an expanded scale near the origin.

## Disutility of effort

The equation we suggest for the utility of the labor involved in performing $L$ trials in a day has only a single free parameter, $\beta$ :

$$
U_{L}(L)=-\beta L
$$

$$
\text { where } \beta \geq 0 ; L \geq 0
$$

Equation 3

We chose this expression because it was sufficient to meet the following criteria: $U_{L}(0)=0$ (doing no work has no cost), otherwise $U_{L}<0$ (working is a cost), and monotonically increasing (more work is more cost). The Marginal Utility with respect to trial number is thus very simply:

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$$
M U_{L}(L)=U_{L}^{\prime}(L)=-\beta d L \quad \text { Equation 3' }
$$

Figure 2C shows the relation of $U_{L}$ to $L$ for an example value of $\beta$; Figure 2D illustrates that we are making the approximation that the marginal utility of labor does not depend on $L$ in this model.

## Net utility

Putting these together, the net utility of performing $L$ trials to earn $H$ water is the sum of the utility of the water and the cost of the effort, as illustrated in Figure 2E.

$$
\begin{array}{lll}
U(L, w)=U_{H}(L, w) & + & U_{L}(L) \\
U(L, w)=(\alpha+1) \ln (w L+1)-w L & - & \beta L
\end{array}
$$

The Marginal Utility with respect to trial number - i.e., the increment in utility of performing $L+1$ trials compared to $L$ trials - is described by the equation (Figure 2F):

$$
\begin{array}{lrr}
M U(L, w)=M U_{H}(L, w) d L & \quad+M U_{L}(L) d L & \text { where } d L=1 \\
M U(L, w)=w\left(\frac{\alpha+1}{w L+1}-1\right) & -\beta &
\end{array}
$$

Note that the number of trials done $L$ can only be an integer. Therefore, we will evaluate these equations discretely with $d L=1$.

## Maximizing utility

In this setting, the utility maximization hypothesis, namely that rats choose the number of trials that will maximize total utility subject to the budget constraint, can be expressed as

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max
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with first order conditions expressed as

$$
M U_{L}(L, w)=M U_{H}(L, w) \quad \text { or equivalently } M U(L, w)=0
$$

and solution functions denoted by

$$
H^{*}(w), L^{*}(w)
$$

Second order conditions for a maximum are ensured by our curvature assumptions on $U_{H}(H)$ and $U_{L}(L)$. For the above choice of utility function, the null hypothesis that rats simply perform trials until they obtain a fixed target level of water corresponds to the parameter restriction $\beta=0$. The maximum utility solutions $L^{*}(w)$ and $H^{*}(w)$ are shown in Figure 2 G and H respectively for the example parameter choices. Note that as $w \rightarrow \infty$, the predicted earned water intake in the task approaches $\alpha$ (Figure 2 H , dashed line). The achieved utility $U\left(L^{*}, w\right)$ strictly increases with reward size (Figure 21 ). This reflects the fact that high-reward environments are always preferable to the rat.


Figure 2. An instantiation of the proposed utility model. Utility model evaluated for parameters $\alpha=25$, $\beta=0.01$. Wage rate (expected reward) is indicated by color, increasing from blue to red: $w=0.005,0.010$, $0.020,0.040,0.060,0.080,0.100$, or $0.200 \mathrm{ml} /$ trial. A. Utility of water earned by performing $L$ trials in one day, in arbitrary units of utils (c.f. Equation 2). B. Marginal utility of water with respect to trial number (the derivatives of curves in Panel A; c.f. Equation 2'), evaluated discretely at $d L=1$. Scale is expanded to show detail near origin. C. Utility of the work of performing $L$ trials in one day, which is negative and does not depend on $w$ (c.f. Equation 3). D. Marginal utility of labor with respect to trial number (the derivatives of curves in Panel C; c.f. Equation 3'). E. Net utility of performing $L$ trials in one day, equal to the utility of water (panel A) plus the utility of labor (panel C), c.f. Equation 4. F. Net marginal utility of performing $L$ trials, the derivatives of the curves in $E$, or the sum of the curves in B and D, c.f. Equation 4'. Note expanded scale. G. The predicted number of trials $L^{*}$ that will maximize utility, as a function of wage rate; compare to data in Figure 1A-D. H. The total water income $H^{*}$ earned by $L^{*}$ trials, as a function of wage rate; compare to data in Figure 1E-H. Dashed line indicates the parameter $\alpha$. I. The utility achieved by performing the optimal number of trials, as a function of wage rate.

## Fit of the model to the data

We fit the two free parameters of this model to the rat data as described in Methods. Figure 3 panels AD show the observed trials per day at the granularity of single-day observations (symbols), compared to the maximum utility solution of the model (red curves), for the same rats and experiments shown in Figure 1. The observed (symbols) and predicted (red curves) water consumption as a function of wage rate are shown in panels E-H. This serves to show that the proposed utility equations are compatible with the qualitative features of example rat data of the type we wish to explain. The fixed income model had the unrealistic implication that as reward size approaches 0 , trial rate would approach infinity (Figure 1 A-D, blue curves). The utility model accurately reflects that below some minimum wage rate, trial rates fall; and animals will not do any trials if there is no reward (at $w=0, L=0$ ).


Figure 3. Fit of utility-maximization model to rat data from the $\mathbf{2 4 h r} /$ day task. A-D: labor $L$ (trials/day) as a function of wage $w$ ( $\mathrm{ml} /$ trial) for the same experiments as Figure 1A-D shown at single-day resolution (symbols), compared with the utility maximization model fit to the data (red curve). E-H: income $H$ ( $\mathrm{ml} / \mathrm{day}$ ) as a function of wage $w$, for same data and utility model solution as A-D. I. Values of the parameter $\alpha$ fit to each rat. J. Values of the parameter $\beta$. K. The residual error of the fixed-income model (blue lines in Figure $1 \mathrm{E}-\mathrm{H}$ ) and of the utility model (red curves in panels $\mathrm{E}-\mathrm{H}$ ) with respect to water income $H$.

The fit parameter values are shown in Figure 31-J, and the residual error of the fit is compared with that of the fixed-income model for the same data points in Figure 3K. The parameter $\alpha$ predicts the rat's ad lib water satiety point. Although satiety was not measured in these rats, the values of $\alpha$ are plausible 24hour ad lib water consumption values based on measurements in other adult rats(12)

## Access schedule affects effort and consumption

Four rats were tested on both 24 hour/day and 2 hour/day schedules with a range of reward sizes on at least one schedule. In time-limited sessions, access to the task was limited to two hours/day and no water was given between sessions, such that the water earned in the task was still the rat's total water intake for the day. Rats can perform trials as rapidly as every four seconds, so in principle a rat could complete up to 1800 trials in a 2 -hour session. But the rats did fewer trials and therefore consumed less water per day on the 2-hour condition than the 24 -hour condition with a comparable wage rate (Figure 4A-H symbols), as we have noted previously(12).

Access schedule is known to have a strong effect on daily free water consumption of mice (13) and we have anecdotally observed this in rats as well (not shown). This implies that the parameter $\alpha$, which is equal to the free-water satiety point, should depend on the access schedule. To explore whether this effect alone could explain the effect of task scheduling on trials, we fit the utility equation to the data from both schedules jointly, with a distinct satiety parameter $\alpha$ for each access schedule ( $\alpha_{24}, \alpha_{2}$ ) and a shared parameter $\beta$. This approach was surprisingly successful at capturing the main structure in the data (Figure 4A-H curves; 4K).

The parameters of these fits (Figure $41, \mathrm{~J}$ ) are consistent with our unpublished observation that free water satiety point is both lower and less variable with 2 hour/day access than with 24 hour/day access. Two of the rats were much better explained by utility maximization than the fixed-income model, even if a different fixed-income target was used for each schedule (Figure $4 K$, rat B and D). The other two rats either had less elasticity of demand, and/or their elasticity was only expressed outside the range of reward sizes we tested. Although we fit three free parameters, empirical measurement of satiety points on each schedule could allow the model to be fit to both schedules with a single free parameter $\beta$.


Figure 4. Data and model fits for rats tested on two access schedules. Each symbol represents data from a single day during a steady state period, on either 24 hour/day (red) or 2 hour/day (purple) access schedules. Curves represent the model fit to both schedule conditions simultaneously. Panels A-D: observed effort $L$ (symbols) and utility maximizing effort $L^{*}$ (curves). Panels E-H: observed water consumption $H$ (symbols) and utility maximizing consumption $H^{*}$ (curves). Panels B-D and F-H are from the same rats as corresponding panels in Figures 1-2. Data in panels $\mathrm{A}, \mathrm{E}$ are from a different rat not shown above. I. The fit values of $\alpha$ for the $24 \mathrm{hr} /$ day (blue) or $2 \mathrm{hr} / \mathrm{day}$ (red) conditions. J. The fit values of $\beta$ (shared by both conditions). K. Residual error of the best-fit simple fixed income model (fixed target volume regardless of condition, 1 parameter) shown in light blue; the schedule-dependent fixed income model (different fixed target for each schedule, 2 parameters) in dark blue; and the utility maximization model (3 parameters) in yellow.

The utility equations we propose are based on the data presented; the data should not be taken as a test of the theory. We note that the model does not imply that all rats necessarily show elasticity of
demand (i.e. the parameter $\beta$ can be 0 for some rats). New rats including male rats should be tested to determine the distribution of elasticity of demand among rats in general.

## Dynamic interpretation of marginal utility

The utility maximization theory presented above is a static equilibrium model based on maximizing the total utility of doing $L$ trials and harvesting $H$ water in one day (given the wage rate, and a scheduledependent satiety parameter $\alpha$ ). We hypothesize, however, that the mechanism by which rats solve the utility maximization problem is temporally local: the rat continuously estimates the change in utility it would experience for doing one more trial, and initiates another trial if and only if the expected change in utility is positive. More strongly, we propose that the probability of doing a trial at any moment is a monotonic function of the net marginal utility with respect to trial number (Equation 4').

We explored temporal dynamics in 2-hour sessions because these are phase aligned by the start time of the sessions, and limited to a short fraction of a circadian cycle. We used the model parameters fit using the per-day trial counts for all schedules and wage rates (Figure 4) to generate marginal utility curves for the wage rate of interest in each case (Figure 5A-B). If $M U(L)$ represents the instantaneous drive to initiate a trial after the $L^{\text {th }}$ trial, the curve implies that the rats' trial rates should drop surprisingly steeply during a session.


Figure 5. Reinterpretation of the equilibrium model as a time-varying function. A. Marginal Utility as a function of trial number (Equation 4') for the $2 \mathrm{hr} /$ day schedule, based on the parameters ( $\alpha 2=20.0, \beta=0.014$ ) fit to the rat's daily trial counts on both schedules and all wage rates (Figure 4), and evaluated for $w=0.023$. B. Like panel A for a different rat and wage rate. Parameters $\alpha_{2}=19.9, \beta=0.034$, evaluated for $w=0.040$. C. Observed trial density over time in $\mathrm{N}=32$ 2-hour daily sessions with wage rate $w=0.023+/-0.002$, for the rat whose $M U(L)$ curve is shown in A. $\mathbf{D}$. Like C for the rat whose $M U(L)$ curve is shown in $\mathrm{B}, \mathrm{N}=14$ 2-hour daily sessions with $w=0.040+/-0.003$. E . The marginal utility at each time point (based on the average cumulative number of trials) is compared with the observed instantaneous rate of trial initiation, for the case analyzed in A and C. F. Like E, for the case analyzed in B and D. Examples were chosen as the two cases in which the same wage rate was tested for the most consecutive days in the 2-hour schedule. Different rats from any shown above.

In the examples shown, $M U(L)$ falls to half its initial value by trial 43 (Figure 5 A ) or by trial 25 (Figure $5 B$ ), in both cases corresponding to the rat having consumed only 1.0 ml of water. This prediction is qualitatively consistent with the observed timing of trials performed in the sessions with the simulated wage rate (Figure $5 \mathrm{C}-\mathrm{D}$ ). Recall that the timing of trials was not used in the fitting of the model parameters. The trial rate cannot be less than zero and has a physical upper limit of 0.25 trials $/ \mathrm{sec}$. Therefore, we expected a sigmoid rather than linear relationship between $M U(t)$ and trial rate, as observed in our data (Figure $5 \mathrm{E}, \mathrm{F}$ ).

## Quantitative predictions

A strength of the model is that it makes quantitative predictions, including some that go beyond experimental manipulations used in derivation of the model.

First, the parameter $\alpha$ predicts the rat's free water satiety point on the same access schedule. Therefore, $\alpha$ could be constrained by a free water satiety measurement, leaving only one free parameter $\beta$ to explain daily trial number and water consumption as a function of reward size on a schedule (e.g., Figure 6A, black curve).

Second, after fitting the parameter $\beta$ (e.g., Figure 6A, black curve), with no additional free parameters the model makes a quantitative prediction for the effect of a supplement or "endowment" of daily free water (Figure 6A, colored curves). The prediction is nontrivial because an endowment would shift the utility of water curve (Figure 2A) horizontally relative to the utility of labor curve (Figure 2C), resulting in a change in both trial rate and total income depending nonlinearly on the wage rate.

Third, the model predicts a forward-bending part of the labor supply curve ( $L$ initially increases with $w$ ), which we did not observe in our data. We attribute this to the fact that we could not test reward sizes that would have resulted in insufficient water intake. In the endowment experiment, however, the forward phase of the labor curve sometimes overlaps with conditions that provide adequate daily fluids.

Fourth, we further hypothesized that rats performed fewer trials on the 2-hour/day schedule because they were less thirsty, not because working faster was more aversive (Figure 4). If so, one could fit the parameter $\beta$ using data from one schedule (for example, the black curve in Figure 6A), and then predict the trial rate for any combination of schedule, wage rate, and endowment (i.e. all the other curves in all the panels of Figure 6) with no additional free parameters, using measured free water satiety points to constrain $\alpha$ for each schedule. The accuracy of this prediction could be compared to the alternative of fitting the parameter $\beta$ separately for each schedule. If the latter explains the data better (after penalizing for the additional free parameters), that outcome would favor the alternative hypothesis that both water satiety and labor aversion are schedule dependent.

In summary, the proposed theory has the potential to quantitatively explain the nonlinear interacting effects of three environmental variables (wage rate, schedule, and endowment) on rats' willingness to work for water, with as few as one free parameter. Any deviations from predictions will be informative
for revising the analytic form of them model, which would alter the shape of the utility curves and therefore update the predictions for neural dynamics.


Figure 6. Quantitative predictions of the model. A. Utility-maximizing trial rate $L$ as a function of reward size $w$. The single free parameter $\beta$ for a rat could be fit using observed daily trial counts from a range of reward sizes $w$ tested on a 24 hour/day schedule with no endowment, and experimentally measured 24 -hour free water consumption (here hypothetically $\alpha_{24}=30 \mathrm{ml} /$ day), producing the model curve shown in black. Without additional free parameters, the model predicts the trial rate for any reward size in the presence of any water endowment $H_{0}$ (ml/day, color key). B-C: Assuming measured free water consumption on two other schedules, here hypothetically 8 hours $/$ day $\alpha_{8}=18 \mathrm{ml} /$ day (B) and 2 hours/day $\alpha_{2}=12 \mathrm{ml} /$ day (C), the model further predicts the trial number for any novel combination of schedule, endowment, and reward size with no additional free parameters. D-F: The earned income $H_{\text {earned }}=w L$ corresponding to the trial numbers predicted in A-C. Note that the rat's total water intake, not shown, includes the endowment $\left(H_{\text {total }}=H_{0}+w L\right)$.

## A neural hypothesis

The recent explosion of progress in unravelling the neurobiology of thirst (14-28) provides an unprecedented opportunity to link behavioral motivation to known neural mechanisms within an economic theory framework. We have proposed that marginal utility $M U$ could be re-interpreted dynamically, and showed that this is consistent with the timing of behavior (Figure 5). On this hypothesis, the rat's task of solving the utility maximization problem reduces to simply detecting whether $M U>0$ at any given moment. This raises the question of where in the brain $M U$ is computed.

The time course of $M U(t)$ after the onset of task availability in rats (Figure 7A) is reminiscent of the time course of activity of SFO ${ }^{\text {GLUT }}$ neurons in the first minutes after thirsty mice access water(15, 18, 20, 29) (Figure 7D). The steep decline in the probability of rats initiating trials (Figure 7B) resembles the steep decline in the probability of the mice licking a water tube (Figure 7E). On average, SFO ${ }^{\text {GLUT }}$ activity falls off smoothly with time, but in any individual session SFO ${ }^{\text {GLUT }}$ activity oscillates around this average (not shown), such that the licks occur in bursts (Figure 7F). We observe similar bursts in the timing of trials in
individual rat sessions (Figure 7C). These analogies prompt us to advance the hypothesis that $M U$ is represented by $\mathrm{SFO}^{G L U T}$ in rats.


Figure 7. Hypothesis: SFO ${ }^{\text {GLUT }}$ activity is the neural representation of MU. A. Marginal Utility expressed as a function of time, based on the parameters fit to one rat's daily trial count as a function of wage rate and access schedule, c.f. Figure 5B,D,E. B. Observed trial rate as a function of time in 14 consecutive 2 -hour sessions by one rat tested on a 2 hour/day schedule at the wage rate modeled in A (c.f. Figure 5D). C. Times of trial initiation for the first hour of each 2hour session (rows), each point indicates the time of one trial. Panels D-F: data from mice at the onset of water access after water restriction, from Zimmerman et al (2019) and additional unpublished data of those authors. D. Activity of SFO ${ }^{G L U T}$ neurons at onset of drinking, measured by the change in GCaMP fluorescence (\%) relative to preceding baseline, averaged over $\mathrm{N}=15$ sessions in 15 different mice. E. Average licking rate in first 10 minutes after water access from the same experiments as D. G. Times of licks, where each row is an individual session and each lick is indicated by a point

A direct test of this hypothesis would entail recording from and causally manipulating these neurons during task performance. But the known properties of SFO ${ }^{\text {GLUT }}$ neurons allow some predictions that could be tested in much simpler behavioral experiments. First, non-hydrating fluids such as isotonic saline or oil are sufficient to trigger the rapid inhibitory feedback signals to SFO ${ }^{\text {GLUT }}$ that underlie the anticipatory or predictive drop in drinking behavior. In our task, occasional probe sessions using nonhydrating fluid rewards instead of water rewards should therefore exhibit the same rapid drop in $M U$ and in trial rate over time, despite the lack of any relief of the rat's dehydration. Conversely, hydrating fluids that bypass oral and gut sensory neurons fail to trigger the inhibitory feedback circuits to SFO ${ }^{\text {GLUT }}$. In our task, therefore, free water endowments provided as subcutaneous or IV fluids should not appreciably alter the trial rate dynamics or total trials in short sessions - in spite of appreciably relieving the rat's dehydration.

## Discussion

The affordances of the environment change over time. Therefore, animals must regulate their allocation of effort flexibly to ensure that each basic need is met without wasting time and energy. In our
experiment, rats did less work per day to earn water during protracted periods when water rewards were larger. Moreover the rats consumed substantially more water per day when it was easier to earn, consistent with elasticity of demand in economic theory (7).

These findings are in line with classic animal behavior studies showing that animals behave "rationally" in the sense required by utility maximization theory (30-41). Our study extends the literature in several respects. The "work" in our task was mental effort (difficult perceptual discriminations) rather than the mechanical effort (lever presses) studied previously, and the reward was water rather than food or food-and-water compound rewards. The classic experiments compared three wage conditions per experiment (baseline, uncompensated wage change, and compensated wage change), and assessed only a qualitative outcome (e.g. the direction of substitution and income effects). To our knowledge none of past studies explored the large number of different uncompensated wage changes sufficient to define the shape of the maximum utility curve as we do here. This was essential for assessing the shape of the utility equation curves. We introduce the idea of interpreting marginal utility as a dynamic variable, and our experiments recorded the time of every unit of labor and unit of water consumption, allowing us to relate MU to the temporal dynamics of behavior. Other recent studies have also fruitfully revisited and extended animal models of other aspects of consumer theory $(42,43)$.

An apparently contradictory recent study in rodents (3) showed that the vigor of response to a single available option is positively related to reward size. Our results can be reconciled with that finding. In that study, reward size was changed many times within a single 2-hour session. On this short timescale, fatigue from harvesting small rewards could directly reduce the animal's ability to harvest when rewards are larger; and animals can easily survive without any water for a few minutes while waiting for a better opportunity. The previous study was also constructed to equalize the state of thirst or satiety across compared reward conditions. Our experiments are complementary to these, in that we explored reward changes on much longer timescales, and in the presence of an intact feedback loop whereby behavioral choices had consequences for internal states that directly altered subsequent motivations.

Why would a rat consume more water than it needs? The capacity to store water in body tissue is limited; unused intake is soon eliminated as urine. Yet rats that were able to maintain good health on 8$10 \mathrm{ml} /$ day with small rewards were willing to do work to get three times that much water when rewards were large. This aligns with our previous finding that if water is rendered unpalatable, rats consume about $10 \mathrm{ml} /$ day and maintain health, but will consume $20-40 \mathrm{ml} /$ day when water is plain(12). We speculate that rats use this optional extra water in part to enable extra dry food consumption; unlike water, excess calories can be stored. This could be tested by measuring or restricting food consumption.

In summary, we have presented experimental data showing that when rats had to work to earn their water, they worked harder for smaller rewards, but worked for more total water when it was easier to get (Figure 1). We propose an analytic utility-maximization model (Figure 2) that is able to account for these observations (Figure 3) and that suggests an explanation for the effect of access schedule (Figure 4). We suggest a dynamic re-interpretation of marginal utility and relate this to the observed timing of behavior (Figure 5). The model makes testable quantitative predictions, with the potential to explain the dependence of behavior on three environmental variables (wage rate, schedule, and endowment)
with only one free parameter (Figure 6). We advance the hypothesis that SFO ${ }^{\text {GLUT }}$ neurons represent the model variable $M U$ (Figure 7). The model thus spans descriptive, quantitative, normative, algorithmic and mechanistic levels of explanation.

## Methods

## Experimental

All experiments were performed in strict accordance with all international, federal and local regulations and guidelines for animal welfare. Experiments were performed in AAALAC accredited facilities with the approval and under the supervision of the Institutional Animal Care and Use Committee at the University of California, San Diego (IACUC protocol \#S04135).

The eligible cohort for this study contained 16 female Long-Evans rats (Harlan Laboratories, Indianapolis,IN). Of these, ten had been previously tested for effects of ad lib citric acid water on trial rate (12), and six were naïve rats. Data were accrued up to a pre-scheduled end date, except that some older adults were removed from the study sooner due to age-related disease. The number of time blocks or distinct reward sizes after data curation differed between rats for many reasons: some but not all of rats had qualifying data that were collected in the course of the previous citric acid study; some reward sizes that were tested did not qualify as a steady-state run (e.g. runs interrupted by a day with too great a fluctuation in wage rate or task performance, or a deviation from standard access schedule, or if the rat received a water supplement); some rats had time off from the study to recover from weight loss (e.g., after a too-small reward size was tested); some older rats exited the study early for age-related health reasons; the naïve rats were added late in the study. The animals whose data are shown ( $\mathrm{N}=7$ across all analyses) were in each case all of the rats that had sufficient data for the analysis, by the indicated criteria. Details of how subjects in the cohort were screened for inclusion in each analysis, and the data from all animals whether or not included in any analysis, are provided in the data archive. Males remain to be tested in a future study.

The task the rats performed for work was a random dot motion visual discrimination task (10) conducted using a custom automated training and testing system $(11,44)$ whose control software is written in MATLAB (Mathworks, Natick MA). Briefly, an operant chamber was connected to the animal's home cage by a tube either chronically (for the 24 hour/day schedule) or for a daily timed session (for the 2 hour/day schedule). In the operant chamber there were three infrared beam-break lick sensors arrayed along the bottom edge of an LCD monitor visual display. The rat was required to lick the sensor at the horizontal center of the screen to initiate a trial, at which time the visual motion stimulus appeared and persisted until the rat licked a response sensor on the right or left side. A response lick on the side toward which the visual motion flowed was rewarded with a drop of water. Incorrect responses were punished by a 2 -second time-out. The rewarded side was selected randomly with equal probability independently each trial. Because the visual motion signal was embedded in noise, rats made errors and thus received rewards in approximately $75 \%$ of trials. Rats were individually caged during task access, but pair-housed between timed sessions for the 2hour/day condition. Dry rat chow was continuously available during and between sessions. The shaping sequence to train rats to perform the task has been described elsewhere ( 10,11 ).

Within the task software, reward volume was controlled by the duration a solenoid valve opened to allow flow from a gravity-fed water source (a $30-$ to $60-\mathrm{ml}$ calibrated syringe filled to standard level daily and positioned at a fixed height above the chamber). This nominal reward size (valve open time) was held constant for at least four and up to 14 consecutive days. Rats received daily health checks and were removed from the experiment immediately if they experienced $>10 \%$ weight loss or showed clinical signs of dehydration. Therefore, we only report results for reward sizes for which a rat was able to maintain body weight and clinically normal hydration for at least four consecutive days without water supplements.

Because the water volume of the reward was crucial to this study, we measured it in two ways. The left and right reward ports had separate water valves, which were chosen to be approximately matched for flow and then calibrated periodically to measure the (linear) relationship between open time in ms and volume delivered in $\mu$ l. The actual valve-open durations were recorded for each trial, and the expected reward (average $\mathrm{ml} /$ trial) was later computed on a per-day basis based on the number of rewards received on each side, the measured valve open times of those reward events, and the side-specific calibrated flow rates.

When rats lick a water tube to collect rewards, however, their tongues block the water flow part of the time, reducing the effective reward volume. To account for this, we also measured the total amount of water dispensed, by visually observing the change in water level of each station's individual watersupply syringe each day. Evaporative loss was prevented with loose-fitting covers. For most rats, the measured water consumption was about $75 \%$ of the volume predicted from calibrated flow, independent of the valve open time. This conversion factor was measured for each rat and used to correct calibrated measurements. We confirmed that the conclusions of the study were unaltered if the wage rate and intake were measured in 3-7 day blocks (as shown in Figure 1) instead of by day (Figure 3,4 ), or if calibrated reward volumes were used to estimate the wage rate instead of empirical measurements (not shown).

## Analysis

Analysis and modeling were performed in MATLAB version R2018b. The trial data were automatically recorded by task software, body weights and water consumption data were manually entered at the time of daily observations. These data were later aggregated by scripts which identified all consecutive stretches of dates with constant expected reward ( $\mathrm{ml} /$ trial) and access schedule ( $\sim 24$ hours/day or $\sim 2$ hrs/day), and no free water supplements. The first date after a change in either schedule or reward size was excluded from analysis to allow for possible transition effects. Any missing or inconsistent data points were resolved by inspection of written lab notebooks. The effective wage rate was estimated from the measured water consumption and observed trial number on a daily basis whenever direct measurements were available, or inferred from calibration otherwise, as detailed above.

To find the parameter combination for the utility model that minimized the mean squared error of prediction, we performed an exhaustive progressive grid search. The error surface was convex and the minimum squared error solution unique. The fit was deterministic and used all the available data points.

We started with a coarse grid uniformly sampling the plausible parameter value range ( $\alpha_{24}$ from 8-80 $\mathrm{ml} /$ day; $\alpha_{2}$ from $8-30 \mathrm{ml} /$ day; $\beta 0.01-0.15$ ). We evaluated the equations for every possible combination of parameter values, at each of the wage rates of the observed data points. We defined the residual error as the sum of the sum of squared differences between model and data for water intake H , divided by the degrees of freedom ( $\mathrm{N}-1$ ). We smoothed the error landscape by a narrow gaussian filter and then selected the parameter combination with the lowest residual error. The search grid was then recentered on the selected parameter combination, the span of the search grid was contracted and the resolution increased by a factor of 2 (if 3 parameters were being fit) or 3 (if only two parameters, i.e. for rats tested on only one schedule), excluding any illegal parameter values. This was repeated at 6 progressively finer resolutions. Although the minimum of the error landscape was unique, correlated perturbations of all parameters had a weak effect on the residual error.

To find the maximum utility solutions for a given parameter combination, Equation 4 was numerically evaluated for all integer values $L$ from 0 to $10^{5}$, at each experimentally tested wage rate $w$.

## Statistics

This was an exploratory study with a small number of animals. We do not generalize any conclusions to the population of rats as a whole, and we advisedly refrain from any statements of statistical significance. Rather we present the examples as existence proofs of the phenomena that motivated the model. The statistics we present quantify the evidence for within-individual trends.

In Figure 1 each symbol represents the average empirically measured reward size and average trial number or water consumption over 4-7 consecutive days in a steady state condition. These blocks were identified by first finding contiguous runs of at least 4 days in a row with the same reward and schedule, excluding the first day after a change in reward or schedule, and then breaking up the run into nonoverlapping blocks of 4 days; if <4 days remained at the end of the run they were included in the last block. The purpose of this was to average over daily fluctuations (compare symbols in Figure 1 to the same data shown at one-day granularity in Figure 4). The reward conditions were interleaved on a counterbalanced schedule. We show data from all $N=4$ rats that were tested in at least 8 such blocks with reward sizes that spanned at least a $50 \mu \mathrm{l}$ range of values. Three other rats had too few time blocks or too narrow a range of reward size to support a meaningful analysis of correlation, but their statistics are also shown for completeness (gray shading). Remaining rats in the study either had fewer than 3 independent time blocks or only one reward size ( $w$ range $<3 \mu$ l) and were not included in this analysis.

In panels A-D, our observation is that trial rate declined with increasing reward size, as opposed to being constant (as predicted by a fixed effort model) or increasing with reward size (as predicted if vigor increases with reward size). To quantify the evidence for this conclusion we used a one-tailed Spearman correlation vs. the null hypothesis that the correlation was $\rho \geq 0$. In panels $\mathrm{E}-\mathrm{H}$, our observation is that water consumption increased with increasing reward size, as opposed to being constant (as predicted by the fixed income model, blue) or declining (as predicted by either fixed effort or vigor increasing with reward). To quantify the evidence for this conclusion, we used a one-tailed Spearman correlation vs. the null hypothesis that the correlation was $\rho \leq 0$. The fixed income model is the average observed consumption $k=\bar{H}$, blue lines in E-H. The number of trials predicted by this model is simply $L=k / w$,
blue curves in A-D. We did not set a significance threshold because we do not intend to make a binary significance decision. Any $P$ values $<10^{-6}$ are reported as such because this exceeds the precision of the estimate of P. Spearman correlation was used because the relationships are monotonic but not necessarily linear.

|  | Time blocks | w range | k | w vs L |  |  | w vs H |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rat | N | $\mu \mathrm{l} /$ trial | ml/day | rho | $\mathrm{R}^{2}$ | p | rho | R ${ }^{1}$ | p |
| A | 12 | 68 | 17.1 | -0.94 | 0.88 | <1e-6 | 0.83 | 0.68 | 8.60e-04 |
| B | 10 | 54 | 18.4 | -0.96 | 0.93 | <1e-6 | 0.95 | 0.91 | <1e-6 |
| C | 11 | 84 | 17.3 | -0.98 | 0.96 | <1e-6 | 0.85 | 0.71 | 1.00e-03 |
| D | 8 | 81 | 20.0 | -0.98 | 0.95 | 2.00e-04 | 0.93 | 0.86 | 1.10e-03 |
| E | 9 | 16 | 15.2 | -0.68 | 0.47 | $2.50 \mathrm{e}-02$ | 0.37 | 0.13 | 1.70e-01 |
| F | 5 | 12 | 15.9 | -0.8 | 0.64 | $6.70 \mathrm{e}-02$ | 0.30 | 0.09 | 3.40e-01 |
| G | 3 | 37 | 19.7 | -1.00 | 1.00 | 1.70e-01 | 0.50 | 0.25 | 5.00e-01 |

Table 1. Statistics supporting Figure 1

## Data and Code Availability Statement

An interactive, executable replication code and data repository is available at CodeOcean (45). This repository includes documentation of intermediate analyses and contains all data not shown. Further data curation and provenance details along with the full dataset are permanently archived on Dryad (46). The MATLAB scripts that reproduce the data analysis, model implementation, parameter fitting, and figure generation are permanently archived at Zenodo (47).

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## Competing Interests

The author declares that there are no competing interests.

## Symbol Definitions

| Variable | Name | Units | Bounds | Description |
| :--- | :--- | :--- | :--- | :--- |
| w | Wage rate | $\mathrm{ml} /$ trial | $\geq 0$ | Expected reward (ml/reward x reward/trial) |
| L | Labor | trials/day | $\geq 0$ | Total trials performed by rat per 24-hour period |
| H 0 | Endowment | $\mathrm{ml} /$ day | $\geq 0$ | Free water given ( $\mathrm{H}_{0}=0$ in all experiments shown) |
| H | Income | $\mathrm{ml} /$ day | $\geq 0$ | Total water consumed by rat per 24-hour period, <br> $\mathrm{H}=\mathrm{H}_{0}+\mathrm{wL}$ (H=wL in all experiments shown) |
| $\alpha_{\mathrm{x}}$ | alpha, for <br> schedule x | $\mathrm{ml} /$ day <br> (hours/day) | $>0$ | Parameter for utility of water (schedule-specific); <br> equal to the free-water satiety point for schedule. |
| $\beta$ | beta | unitless | $\geq 0$ | Parameter for disutility of effort |

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