Title: A digitization theory of the Weber-Fechner law

Name of the author: Haengjin Choe

Affiliation of the author: Department of Physics, College of Natural Sciences, Chonnam National University, Gwangju, South Korea

Name of the corresponding author: Haengjin Choe

E-mail address of the corresponding author: eupjeong@gmail.com

Keywords: Weber-Fechner law; psychophysics; response information; perception entropy; percentropy

Abstract

Since the publication of the Shannon's article about information theory, there have been many attempts to apply information theory to the field of neuroscience. Meanwhile, the Weber-Fechner law describes the relationship between the external physical stimulus and the human response to that. It is not surprising that we assign the amount of information to the response in that law. But no one has succeeded in applying information theory directly to that law: the direct links between information theory and that response in the Weber-Fechner law have not yet been found. The proposed theory unveils a link between information theory and that response, and differs subtly from the field such as neural coding that involves complex calculations and models. Because our theory targets the Weber-Fechner law which is a macroscopic phenomenon, this theory does not involve complex calculations. The proposed theory is expected to mark a new era in the fields of sensory perception research. This theory must be studied in parallel with the field of microscopic scale such as neural coding.

Significance statement

Information theory of Shannon is originally a communication theory for communication engineers. Since the birth of information theory, this theory has affected diverse science fields, including neuroscience. A convergence of information theory and neuroscience is being proceeded. Information theory plays a big role in the research on stimulus and response. But past those researches involve complex calculations, because they target microworld such as neurons and cells. Our theory targets the Weber-Fechner law which is a macroscopic phenomenon. So our theory is simple. Our theory should be studied in parallel with the field of the microscopic scale. The author predicts that the new field of research will be created.

1 Introduction

1.1 Psychophysics of Weber and Fechner

In 1834, the German physiologist Ernst Heinrich Weber carried out an exciting experiment. Weber gradually increased the weight which the blindfolded man was holding. With doing so, Weber asked the subject when he first felt the increase of weight. Through this experiment, Weber found out that the least perceptible difference in weight was proportional to the starting value of the weight. For example, if a certain person barely feels the difference between 100 g and 104 g, then he barely feels the difference between 150 g and 156 g. In this case, $100:104 = 150:156$ holds. Weber's findings were elaborated in Weber (1851). Weber is considered one of the founders of experimental psychology.

Weber's empirical observations were expressed mathematically by Gustav Theodor Fechner who was a Weber's student (Fechner 1860). Fechner called his formulation the Weber's law. Fechner applied the result of the Weber experiment to the measurement of sensation (Ward 1876; Houstoun and Shearer 1930). The Weber-Fechner law established in the field of psychophysics attempts to mathematically describe the relationship between the magnitudes of a physical stimulus and the perceived intensities of the stimulus (Fechner 1860). In this fascinating law, physical stimuli are weight, sound frequency, etc. The Weber-Fechner law states that the relationship between stimulus and its perception is logarithmic. Therefore, if a

stimulus varies as a geometric progression, the corresponding perception varies as an arithmetic progression. Bear in mind that the stimuli in the Weber-Fechner law have physical units but that the responses in that law have no physical units. The best example of Weber-Fechner law is that many musicians adopt the equal temperament when they tune their musical instruments. Since the nineteenth century, musicians have tuned virtually all instruments in equal temperament (Berg and Stork 2005). Equal temperament in music theory is a tuning system in which the octave is divided into 12 semitones of equal size; note here that the size does not indicate frequency, but musical interval. In the equal temperament system, the musical instruments are tuned so that the frequencies of all notes form a geometric sequence. Such a tuning results in the fact that the musical intervals of every pair of two consecutive notes are the same (Hopkin 1996; Berg and Stork 2005). This example needs to be explained in more detail. Transposing or transposition in music means changing the key of a piece of music. When the musical-instrument accompaniment is too high or too low for us to sing to, we change the musical accompaniment into a lower or higher key. Even if we transpose the accompaniment into another key, as if by magic, we don’t feel awkward. There is an essential reason that the accompaniment does not sound awkward. This is because the musical instruments were tuned to the equal temperaments. Such a tuning gives an equal perceived step size because pitch is perceived as the logarithm of frequency. As another concrete example of Weber-Fechner law, the response of the human ear to change in sound intensity is logarithmic. Therefore, it is convenient to speak of the sound level defined as

$$(10\ \text{decibel})\log_{10}\frac{\text{intensity of a sound wave}}{\text{standard threshold of hearing intensity}}$$

above the standard threshold of hearing intensity (Kinsler et al. 2000). The standard threshold of hearing intensity is chosen as $10^{-12}\ \mathrm{W/m^2}$, which is near the lowest limit of the human range of hearing (Kinsler et al. 2000; Halliday et al. 2014). Note here that because the defining equation of the decibel scale has the same form as Weber-Fechner law, the perceived magnitude is proportional naturally to a number obtained after we drop decibel in the sound level. As a third concrete example of Weber-Fechner law, the response of the human eye to brightness of light is logarithmic. The stellar magnitudes m_1 and m_2 for two stars are related to the corresponding brightnesses b_1 and b_2 through the equation

$$m_2 - m_1 = \sqrt[5]{100}\log\frac{b_1}{b_2},$$

which results from the Pogson's proposal that a difference of five magnitudes should be exactly defined as a brightness ratio of 100 to 1 (Pogson 1856; Burke-Gaffney 1963; Mayer 1986; Young 1990; Hearnshaw 1992; Schulman and Cox 1997). Note here that the stellar magnitude corresponds to the response and hence has no physical unit.

1.2 Information theory of Shannon

Claude Elwood Shannon defined mathematically the concept of 'the amount of information' which is sometimes called 'the information content' (Shannon 1948).

Going one step further, he defined mathematically the concept of entropy (Shannon 1948). Due to information theory, we came to be able to quantify the amount of information transmitted over a communication channel.

1.3 Background of digitization theory

Is it possible to digitize the Weber-Fechner law? Digitizing the Weber-Fechner law in this article means the following: we define mathematically a new kind of the information content from the Weber-Fechner law, and going one step further we define mathematically a new kind of entropy. This digitization may be possible if we can discover links between the information content formula of Shannon and the Weber-Fechner formula. The author proposes a theory that we are able to digitize the Weber-Fechner law. The digitization procedure is as follows. We will add one more stimulus to the existing various stimuli applied to the Weber-Fechner law. This addition procedure is based on the analogy between the Weber-Fechner law and the information content. This additional stimulus and the corresponding perception are extremely extraordinary. We will digitize the Weber-Fechner law by extending and reinterpreting that law. Digitization of the Weber-Fechner law which is achieved by merging information theory with the Weber-Fechner law will lead us to two new concepts. Firstly, we define a new concept which the author calls ‘the amount of response information’. Ultimately, we end up defining another new concept which the author calls the ‘perception entropy’. The digitization theory of the Weber-Fechner

law treats external physical stimuli and responses, so what this theory says are the results from a macroscopic perspective.

2 Weber-Fechner law and the information content

2.1 Derivation of Weber-Fechner law

Weber-Fechner law is derived as follows (Nutting 1908). In the Weber's experiment, let S be weight at some instant and let $\mathrm{d}S$ be the differential increment in weight. Let $\mathrm{d}R$ be the differential change in perception. The equation to express the result of the Weber experiment is

$$\mathrm{d}R = k\frac{\mathrm{d}S}{S},$$

where k is a constant. (There is an important point in connection with k, which will be mentioned in Section 4.) Integrating both sides of this equation yields

$$R = k\ln S + C,$$

in which case C is an integration constant. Let us suppose that S_0 is the threshold of the stimulus below which a person does not perceive anything. Because $R = 0$ whenever $S = S_0$, C must be equal to $-k\ln S_0$. Therefore, we can finally obtain

$$R = k\ln\frac{S}{S_0} = K\log_2\frac{S}{S_0},$$

which is the mathematically expressed Weber-Fechner law. Let us note that R has no physical unit. But the physical stimulus S has, of course, a physical unit.

2.2 Definition of the information content

Shannon defined the amount of information of an event of the occurrence probability P as

$$I = -\log_2 P,$$

where the information content I is measured in bits (Shannon 1948). And he defined the entropy of a discrete random variable as the average amount of information of the random variable (Shannon 1948). In order to derive a new kind of the information content and hence a new kind of the entropy, let's redescribe the above equation as

$$I = -\log_2 \frac{P}{P_0},$$

where as a matter of course $P_0 = 1$.

3 Derivation and interpretation of digitization theory

3.1 Special derivation

In the equation

$$R = K \log_2 \frac{S}{S_0},$$

for all practical purposes $S \geq S_0$. In the equation

$$I = -\log_2 \frac{P}{P_0},$$

for all practical purposes $P \leq P_0$. We can interpret $P_0 = 1$ as the threshold of P. Most importantly, we can interpret I as a response to P; it would be nice for us to call P a bizarre kind of stimulus. However, this bizarre stimulus fundamentally differs from other ordinary physical stimuli in that it is simply a number which has no physical unit. Very important another fact is that the higher P is, the lower I is while the higher S is, the higher R is. But it is not essential that I is a strictly decreasing function with respect to P while R is a strictly increasing function with respect to S. The essentially important facts are that R and I with respect to S and P, respectively, are logarithmic and that both S and P have the threshold values. From these viewpoints, consequently, the author cannot choose but say that the occurrence probability is an anomalous kind of stimulus. It would be appropriate that we name the occurrence probability the ‘mathematical stimulus’ in that the occurrence probability has no physical unit. This mathematical stimulus is under the government of the Weber-Fechner law. By widening the range of applications of the Weber-Fechner law, we come to be capable of combining information theory with the Weber-Fechner law of psychophysics.

For simplicity, in current subsection we set K to 1 for all sensations. (We make complete descriptions in Subsection 3.2. In that subsection we make no

assumptions about K.) Then, we can say that when $S = S_0$, the amount of perception is 0 bits and that when $S = 2S_0$, the amount of perception is 1 bits. But there is one thing to be careful about. In reality, different people have different S_0. So, different people have different amount of response information for the identical S. In addition, as the body of a person ages, S_0 changes. So, even the response information content of a person changes with age for the identical S. Because of this fact, the response information content is quite different from the information content of Shannon. In defining equation of the information content of Shannon, P_0 is fixed at 1.

If the physical stimulus is the weight, the problem is simple. In order to define a new kind of entropy, let's consider the sound wave from now on. Because a pure tone which is a sound wave with a sinusoidal waveform is characterized by a frequency and a displacement amplitude, considering only either frequency or amplitude may be meaningless. Thus, we consider both simultaneously. For the sake of argument let's consider a pure tone of frequency f and amplitude A. Physics entropy and Shannon entropy are subject to the constraint that the total sum of probabilities is 1. On the other hand, there is not such a constraint in the defining equation of the new kind of entropy. The physical quantities A and f change much more freely. Therefore, it is impossible that we define the new entropy in the same form as the Shannon entropy. In order words, we must not define the new entropy as

$$\frac{A}{A_0}\log_2\frac{A}{A_0} + \frac{f}{f_0}\log_2\frac{f}{f_0},$$

which is of the form of Shannon entropy. Before we define the new entropy, we first define the total amount of response as

$$\log_2 \frac{A}{A_0} + \log_2 \frac{f}{f_0} = \log_2 \frac{Af}{A_0 f_0},$$

where $\log_2 \frac{Af}{A_0 f_0}$ is measured in bits. In other words, the total amount of response is defined so that it equals the sum of the amounts of each individual response. Consequently, we define the new entropy as the arithmetic mean of responses being considered; that is, new entropy is defined as

$$\left(\log_2 \frac{Af}{A_0 f_0}\right)/2.$$

The unit of this quantity is $\mathrm{bits/response}$. Please note that in the current situation there are two responses: one response due to the amplitude and the other due to the frequency. The author calls this new quantity the ‘perception entropy’ or the ‘percentropy’. The term ‘percentropy’ is a new compound word formed from perception and entropy. The percentropy does not correspond perfectly to the Shannon entropy. Nonetheless, it is useful to define the concept of percentropy because percentropy is linked directly with physics energy. We are easily able to understand such a fact. As an example, consider two pure tones. One tone has amplitude A_1 and frequency f_1. The other tone has amplitude A_2 and frequency f_2. The percentropy of the former tone is $\left(\log_2 \frac{A_1 f_1}{A_0 f_0}\right)/2$. The percentropy of the latter

tone is $\left(\log_2 \frac{A_2 f_2}{A_0 f_0}\right)/2$. The equality $A_1 f_1 = A_2 f_2$ is a necessary and sufficient condition for the fact that the percentropies of the two tones are the same. And $A_1 f_1 = A_2 f_2$ if and only if $(A_1 f_1)^2 = (A_2 f_2)^2$. We can therefore immediately conclude that the equality $A_1 f_1 = A_2 f_2$ is a necessary and sufficient condition for the fact that the energies of the two tones are the same. Let us now consider two compound tones. One compound tone consists of a pure tone which has amplitude A_{11} and frequency f_{11} and a pure tone which has amplitude A_{12} and frequency f_{12}. The other compound tone consists of a pure tone which has amplitude A_{21} and frequency f_{21} and a pure tone which has amplitude A_{22} and frequency f_{22}. The percentropy of the former compound tone is

$$\left(\log_2 \frac{A_{11}}{A_0} + \log_2 \frac{f_{11}}{f_0} + \log_2 \frac{A_{12}}{A_0} + \log_2 \frac{f_{12}}{f_0}\right)/4 = \left(\log_2 \frac{A_{11} f_{11} A_{12} f_{12}}{{A_0}^2 {f_0}^2}\right)/4.$$

The percentropy of the latter compound tone is

$$\left(\log_2 \frac{A_{21}}{A_0} + \log_2 \frac{f_{21}}{f_0} + \log_2 \frac{A_{22}}{A_0} + \log_2 \frac{f_{22}}{f_0}\right)/4 = \left(\log_2 \frac{A_{21} f_{21} A_{22} f_{22}}{{A_0}^2 {f_0}^2}\right)/4.$$

The equality $A_{11} f_{11} A_{12} f_{12} = A_{21} f_{21} A_{22} f_{22}$ is a necessary and sufficient condition for the fact that the percentropies of the two compound tones are the same. And $A_{11} f_{11} A_{12} f_{12} = A_{21} f_{21} A_{22} f_{22}$ if and only if $(A_{11} f_{11})^2 (A_{12} f_{12})^2 = (A_{21} f_{21})^2 (A_{22} f_{22})^2$. Therefore, in this compound-tones situation, the equality of the

two percentropies is a necessary and sufficient condition for the fact that the products of energies of the two pure tones composing each compound tone are the same.

3.2 General derivation

Now we do not assume that Ks are 1 for all sensations. Because Ks are simply constants, we just have to define the response information content for a single stimulus as

$$K \log_2 \frac{S}{S_0},$$

where $K \log_2 \frac{S}{S_0}$ is measured in bits. Let's consider a pure tone of frequency f and amplitude A. Let K_A and K_f be the corresponding constants. We first define the total amount of response as

$$K_A \log_2 \frac{A}{A_0} + K_f \log_2 \frac{f}{f_0} = \log_2 \left(\frac{A}{A_0}\right)^{K_A} \left(\frac{f}{f_0}\right)^{K_f},$$

where $\log_2 \left(\frac{A}{A_0}\right)^{K_A} \left(\frac{f}{f_0}\right)^{K_f}$ is measured in bits. In other words, the total amount of response is defined so that it equals the sum of the amounts of each individual response. And then we define the new entropy as the arithmetic mean of responses being considered; that is, new entropy is defined as

$$\left\{\log_2 \left(\frac{A}{A_0}\right)^{K_A} \left(\frac{f}{f_0}\right)^{K_f}\right\}/2.$$

The unit of this quantity is $\mathrm{bits/response}$. Because the amplitude and the frequency change independently of each other, we took the arithmetic mean. Notice that the percentropy is not linked directly with physics energy unless K_A equals K_f.

3.3 Utilization and interpretation

According to the proposed theory, it comes to be possible that we utilize percentropy (or the total amount of response information) for quantifying the sensitivities of the sensory organs of the humans; at this time the sensitivity is a function of the stimuli and their thresholds. This quantification allows us to compare the sensitivities of the sensory organs of people. It is best to explain an example. For simplicity, we set $K = 1$ for all sensations. Let us consider the case of the hearing. There are two people. For a person X, $A_0 = 33$ pm and $f_0 = 20$ Hz. For another person Y, $A_0 = 22$ pm and $f_0 = 60$ Hz. Let us suppose that two people hear a sound wave with an amplitude of 330 pm and a frequency of 1000 Hz. Then, the percentropy of the hearing organ of X is

$$\left(\log_2 \frac{Af}{A_0 f_0}\right)/2 = \left(\log_2 \frac{330 \times 1000}{33 \times 20}\right)/2 \approx 4.483 \ \mathrm{bits/response},$$

and the percentropy of the hearing organ of Y is

$$\left(\log_2 \frac{Af}{A_0 f_0}\right)/2 = \left(\log_2 \frac{330 \times 1000}{22 \times 60}\right)/2 \approx 3.983 \ \mathrm{bits/response}.$$

Thus the hearing organ of X is superior to that of Y. In fact, as we can very easily understand, we need only $A_0 f_0$ in comparing superiority of the hearing organs of people. In conclusion, we can say that the information-theoretical measure of the superiority of the hearing organs of people is $A_0 f_0$. Of course, we can easily devise $A_0 f_0$ as a measure of the superiority, without any knowledge appearing in this article. The key point here is that the measure $A_0 f_0$ is justified by the digitization theory of the Weber-Fechner law; i.e., $A_0 f_0$ is valid from the information-theoretical perspective. Please note that we have set $K = 1$ regardless of the type of sensation. If we had not set so, a more complex quantity other than $A_0 f_0$ would have been obtained as the measure of the superiority.

Even if we do not assume that $K = 1$ for all sensations, we can still compare the sensitivity of the sensory organs of people. But the measure of the superiority is no longer $A_0 f_0$. If we assume that the values of K_As for people are all identical and that the values of K_fs for people are all identical, then the measure of the superiority is ${A_0}^{K_A} {f_0}^{K_f}$.

So these are good. The question is then what $4.483\ \mathrm{bits/response}$ and $3.983\ \mathrm{bits/response}$ mean; equivalently, what 8.966 bits and 7.966 bits mean. It is clear that the response information content is proportional to the duration of the external stimulus. And, probably, the duration of the response to that stimulus is quantized. In order words, the duration of the response is always some integral multiple of elementary duration. And, probably, 8.966 bits and 7.966 bits is the

information content of elementary duration. According to this hypothesis, the information content during some time interval is equal to the information content during elementary duration multiplied by some integer. This hypothesis should be verified through other works.

4 Future challenges

The stimuli and responses that have been mentioned so far were about the external physical stimuli and responses to those; the stimuli and responses that have been mentioned so far have not included the stimuli and responses that neural coding handles. In order for the digitization theory of the Weber-Fechner law to be firmly established, we need to elucidate the relationships between microscopic and macroscopic theories on stimuli and responses.

Meanwhile, past information-theoretical researches on stimuli and responses have mainly focused on microanalysis (MacKay and McCulloch 1952; Rapoport and Horvath 1960; Werner and Mountcastle 1965; Stein 1967; Borst and Theunissen 1999; Dimitrov and Miller 2001; Koechlin and Summerfield 2007; Ikeda and Manton 2009). Those researches involve complex models and calculations. Because the Weber-Fechner law is a macroscopic phenomenon which is a manifestation of numerous microscopic outcomes, the digitization theory of the Weber-Fechner law involves far less complex calculations. In the future, macroanalyses as well as

microanalyses should be done together. Future researches will have to focus on elucidating the relationships between microscopic and macroscopic theories.

In particular, there is an important point in connection with K. K in psychophysics is originally an experimentally determined proportionality constant. But, K for each sense in our theory must be determined by comparing microscopic works and our theory. Additionally, author guesses that regardless of the type of sense, an expression for K includes the Boltzmann constant or the Boltzmann-like constant which serves to bridge the macroworld and the microworld. Besides the values of Ks, we must find the value of the elementary duration of response for each sense. Once Ks and the elementary durations are all determined, the field of research on stimuli and responses is expected to develop rapidly.

5 Conclusions

So as to achieve our goal, the links between the Weber-Fechner formula and the information content formula must be found. These links are the key idea behind the digitization theory of the Weber-Fechner law. A link presented in this article is an undiscovered link. Further insight into the defining equation of the information content is to recognize that we can redescribe the defining equation using the threshold of probability. Consequently, the definition of the information content for physiological response was chosen to be in analogy with the definition of the information content which was given by Shannon. Based on the analogy between the

defining equation of the information content and the formula describing the Weber-Fechner law, the author was able to theorize about the digitization of the Weber-Fechner law. On the one hand, the response information content has some features of conventional information theory. On the other hand, the response information content has the different features from conventional information theory. We also defined the perception entropy which means the average amount of the human response to external physical stimuli; the unit of the perception entropy is bits/response. The perception entropy is very information-theoretic, in the sense that its definition originates in the combination of information theory and the Weber-Fechner law.

The digitization theory of the Weber-Fechner law has the advantage of being powerful. The reason is that the Weber-Fechner law is a macroscopic phenomenon, and hence the calculations in this theory are straightforward. In order for our theory to be firmly established, we need to elucidate the relationships between microscopic and macroscopic theories on stimuli and responses. If the author's theory turns out to be of great usefulness, new field of research will be created. Then, that new field will find applications in many diverse fields such as physiology and medicine. It will be good for us to name this new field 'perceptional information theory' or 'perceptual information theory'.

References

1. Berg RE, Stork DG (2005) The physics of sound. Pearson Education, San Francisco
2. Borst A, Theunissen FE (1999) Information theory and neural coding. Nat Neurosci 2:947-957. https://doi.org/10.1038/14731
3. Burke-Gaffney MW (1963) Pogson's scale and Fechner's law. J Roy Astron Soc Can 57:3-8
4. Dimitrov AG, Miller JP (2001) Neural coding and decoding: communication channels and quantization. Network: Comput Neural Syst 12:441-472. https://doi.org/10.1080/net.12.4.441.472
5. Fechner GT (1860) Elemente der psychophysik. Breitkopf und Härtel, Leipzig
6. Halliday D, Resnick R, Walker J (2014) Waves—II. In: Principles of physics, 10th edn. John Wiley & Sons, Singapore, pp 423-453
7. Hearnshaw JB (1992) Origins of the stellar magnitude scale. Sky Telesc 84:494-499
8. Hopkin B (1996) Tuning systems & pitch layouts. In: Musical instrument design. See Sharp Press, Tucson, pp 21-28
9. Houstoun RA, Shearer JF (1930) Fechner's law. Nature 125:891-892. https://doi.org/10.1038/125891b0
10. Ikeda S, Manton JH (2009) Capacity of a single spiking neuron channel. Neural Comput 21:1714-1748. https://doi.org/10.1162/neco.2009.05-08-792

11. Kinsler LE, Frey AR, Coppens AB, Sanders JV (2000) The acoustic wave equation and simple solutions. In: Fundamentals of acoustics, 4th edn. John Wiley & Sons, New York, pp 113-148
12. Koechlin E, Summerfield C (2007) An information theoretical approach to prefrontal executive function. Trends Cogn Sci 11:229-235. https://doi.org/10.1016/j.tics.2007.04.005
13. MacKay DM, McCulloch WS (1952) The limiting information capacity of a neuronal link. Bull Math Biophys 14:127-135. https://doi.org/10.1007/BF02477711
14. Mayer A (1986) On the history of the stellar magnitude scale. J Am Assoc Var Star Obs 15:283-285
15. Nutting PG (1908) The derivation of Fechner's law. Science 27:510-511. https://doi.org/10.1126/science.27.691.510
16. Pogson NR (1856) Magnitudes of thirty-six of the minor planets for the first day of each month of the year 1857. Mon Not Roy Astron Soc 17:12-15. https://doi.org/10.1093/mnras/17.1.12
17. Rapoport A, Horvath WJ (1960) The theoretical channel capacity of a single neuron as determined by various coding systems. Inf Control 3:335-350. https://doi.org/10.1016/S0019-9958(60)90917-7
18. Schulman E, Cox CV (1997) Misconceptions about astronomical magnitudes. Am J Phys 65:1003-1007. https://doi.org/10.1119/1.18714

19. Shannon CE (1948) A mathematical theory of communication. Bell Syst Tech J 27:379-423. https://doi.org/10.1002/j.1538-7305.1948.tb01338.x
20. Stein RB (1967) The information capacity of nerve cells using a frequency code. Biophys J 7:797-826. https://doi.org/10.1016/S0006-3495(67)86623-2
21. Ward J (1876) An attempt to interpret Fechner's law. Mind 1:452-466. https://doi.org/10.1093/mind/os-1.4.452
22. Weber EH (1851) Der tastsinn und das gemeingefühl
23. Werner G, Mountcastle VB (1965) Neural activity in mechanoreceptive cutaneous afferents: Stimulus-response relations, weber functions, and information transmission. J Neurophysiol 28:359-397. https://doi.org/10.1152/jn.1965.28.2.359
24. Young AT (1990) How we perceive star brightnesses. Sky Telesc 79:311-313

Declarations

Funding: Not applicable

Conflicts of interest/Competing interests: Not applicable

Availability of data and material: Not applicable

Code availability: Not applicable