An experimental prediction: granule cells can perform linearly non-separable computations

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Abstract

Multiple studies show how dendrites might extend some neurons' computational capacity. These studies leave a large fraction of the nervous system unexplored. Here we demonstrate how a modest dendritic tree can allow cerebellar granule cells to implement linearly non-separable computations. Granule cells' dendrites do not spike and these cells' membrane voltage is isopotential. Conjunction of Boolean algebra and biophysical modelling enable us to make an experimental prediction. Granule cells can perform linearly non-separable computations. The standard neuron model used in the artificial network, aka the integrate and fire, cannot perform such type of computations. Confirming the prediction we provide in the present work would change how we understand the nervous system.

Keywords

Dendrites, computation, linearly non-separable, neuroscience

Introduction

We show here how dendrites can extend the computational capacity of all neurons, even the tiniest. We already knew that dendrites might extend the computational capacity of some pyramidal neurons; their extended dendrites capable of dendritic spikes changed the way we saw them (see [2] for one of the first article presenting this idea). Later suggested that we should model these neurons as a two layers network [7]. This theoretical model was further consolidated by experiments showing that we can see a pyramidal neuron as a collection of non-linear subunits [8]. Certain non-linearities can even allow a dendrite to implement the exclusive or (XOR) which is impossible for a point neuron [10]. A similar kind of non-linearity, non-monotonic, was found in human pyramidal neurons [4]. But what about all other neurons with modest dendrites incapable of spiking? Pyramidal neurons only represent a fraction of all neurons. The dendrites of cerebellar stellate cells cannot emit spikes, but they do saturate [1] and they can be decomposed into multiple independent subunits turning them into two-stage units like the pyramidal neuron [9]. Previously we have shown that dendrites enable a neuron with passive dendrites to perform linearly non-separable computations, for instance, the feature binding problem [3]. We focus here on granule cells which have on average four small (15μm) dendrites and an isopotential voltage [5]. In the present study, we demonstrate that these neurons can implement a linearly separable computation. This opens the possibility to falsify the point neuron. This experiment could be realised with the existing techniques.

Methods

The exclamation point neuron

We used a leaky integrate and fire (LIF) with a passive dendrite to respectively model a 10μm soma and a dendrite 15μm long and with a 1μm diameter. We employed the standard LIF model and the Brian software to perform our simulation, all the code is freely available on the git repository attached with this paper.
Synaptic current I_s arises from synapses placed at soma and the dendritic tip. It is described by

$$I_s = g_s(E_s - V) \tag{1}$$

E_s is the synaptic reversal potential while g_s is the synaptic conductance which is bounded between 0 and 10nS. It jumps up instantaneously to its maximal value for each incoming spike and decays exponentially with time constant $\tau_s = 1\,\text{ms}$ otherwise:

$$\frac{dg_s}{dt} = -\frac{g_s}{\tau_s} \tag{2}$$

Notably, we use two independent g_s: one for the soma and one for the dendrite. This compartmentalisation of g_s extends the classic LIF model.

Boolean algebra reminder

First, let's present Boolean functions:

Definition 1. *A Boolean function of n variables is a function on $\{0,1\}^n$ into $\{0,1\}$, where n is a positive integer.*

Importantly, we commonly assume that neurons can only implement linearly separable computations:

Definition 2. *f is a linearly separable computation of n variables if and only if there exists at least one vector $w \in \mathbb{R}^n$ and a threshold $\Theta \in \mathbb{R}$ such that:*

$$f(X) = \begin{cases} 1 & \text{if } w \cdot X \geq \Theta \\ 0 & \text{otherwise} \end{cases}$$

where $X \in \{0,1\}^n$ is the vector notation for the Boolean input variables.

Results

The compact feature binding problem (cFBP)

In this section, we demonstrate a compact linearly non-separable computation used afterwards in our work. This computation is compact because it has four input/output lines. The standard neuron model cannot implement such computation making it linearly non-separable.

Inputs				Output
0	0	1	1	0
1	1	0	0	0
0	1	0	1	1
1	0	1	0	1

Table 1. The truth table of a non-linearly separable computation

The computation described in Tab. 1 is non-linearly separable. We call this computation the compact feature binding problem (cFBP).

Proof. The neuron stays silent for two disjoint couples (1,2) and (3,4) of active inputs. It means that $w_1 + w_2 \leq \Theta$ and $w_3 + w_4 \leq \Theta$ we can add these two inequalities to obtain $w_1 + w_2 + w_3 + w_4 \leq 2\Theta$. However, the neuron fire for two other couples made of the same active inputs (1,3) and (2,4). It means that $w_1 + w_3 > \Theta$ and $w_2 + w_4 > \Theta$ we can add these two inequalities to obtain $w_1 + w_2 + w_3 + w_4 > 2\Theta$. This yield a contradiction proving that no weights set exist to solving this set of inequalities. □

This computation is simple in two ways:

- It has a small number of lines - the same number as the famous XOR (exclusive or)
- Contrary to the XOR it can be implemented with excitatory inputs and a monotone transfer function.

Notably, a neuron with a single dendrite can easily implement this computation. We proved how a ball-and-stick biophysical model can implement this computation in a previous study and we are going to use a similar method in the next section to implement the cFBP[3].

We can interpret the 0s and 1s in the truth table in at least two ways: (1) either the pre or post-synaptic neurons activates (2) or they reach a given spike frequency. In the following section, we will use the latter interpretation.

Implementing the cFBP in a granule cell

We use local saturation in dendrites to implement the cFBP in a granule cell, the tiniest neurons in the brain. Their compactness makes them isopotential so we might wonder how local saturation can arise. Saturation has two origins. A reduction in driving force can cause saturation like in [1], but it can also be due to the intrinsic limitations in conductance. To better understand, imagine the neuron as a lake, the water level of the lake can depends on two things: (1) the level difference between the lake and rivers feeding into it, aka the driving force (2) the width of the river connected to the river. We are going to use this latter source of saturation to implement the cFBP in a granule cell.

We model a granule cell as an integrate and fire with a unique passive dendritic compartment (15μm in length and 1μm in diameter) as shown in Fig 1A. These cells receive high-frequency inputs from mossy fibres in the kHz range. Consequently, we consider a pre-synaptic input active when it fires a 1kHz regular spike-train and inactive if it didn't reach this frequency. We stimulate our model in four different ways to reproduce the truth table from the previous section. You can observe on Fig 1B that when we bound g_s, it becomes possible to overcome granule cells compactness and to implement the cFBP. When we locally bound g_s the total current flowing into the neuron grows larger as we double the number of open gates if synaptic inputs distribute over the neuron. Having two distinct g_s is key here. If we have a unique g_s the neuron remains a point and cannot identify inputs' origins: it always fires at the same frequency. Saturation of two distinct g_s therefore enables the implementation of the cFBP.

Conclusion/Discussion

We detailed in this brief report an experimental prediction: granule cells can implement linearly non-separable computations. These neurons have on average four dendrites, but we have shown here that a single suffices.

As we mentioned multiple times, locally bounded g_s are crucial. In all cases, g_s is going to saturate but we need a local saturation g_s to implement the cFBP.

The experience demonstrating this prediction seems straightforward. One would need to stimulate four distinct groups of mossy fibres following our different scenarios. An experimentalist could then record how a group of granule cell respond using for instance optogenetics reporting. We predict that a significant part of Granule cells might implement the cFBP. This prediction could reveal the true potential of single neurons. The next step consists of looking at the network level as it was already done with spiking dendrites [**?**]. A network of exclamation point neurons might have unsuspected behaviours.

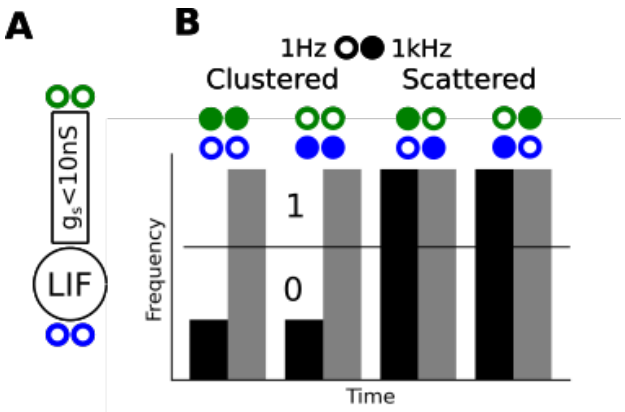

Figure 1. A granule cell implementing a linearly non-separable computation. (A) A leaky integrate and fire (LIF) with a dendritic compartment and two distinct g_s which cannot exceed 10nS, circles correspond to four input sites (B) Four stimulation scenarios, filled circles stand for a 1kHz input spike train. Below the response of our model versus the response of a standard LIF. We purposely removed the ticks label as the frequencies depend on the parameter of the biophysical model and input regularity (in our simulation 860Hz and 900Hz). These parameters can vary largely without affecting the outcome. [6]

Acknowledgements

I used "we" as science is a collective endeavour and this article would not have been possible without proximal interactions with A. Trah-Van-Min, T. Abrahamson, B. Gutkin and D. Digregorio. Discussions on a similar prediction had begun in 2013. I also want to acknowledge and A. Marini-Audouard for the proof-reading before submission.

Author contributions statement

R.C. wrote the initial draft, initiated the project, and made the initial simulations and figures.

Competing interests

Author declares no competing interest.

References

[1] Therese Abrahamsson, Laurence Cathala, Ko Matsui, Ryuichi Shigemoto, and David A. DiGregorio. Thin dendrites of cerebellar interneurons confer sublinear synaptic integration and a gradient of short-term plasticity. 73(6):1159–1172.

[2] Mel Bartlett. The clusteron: towards a simple abstraction to a complex neuron. 4.

[3] Romain Daniel Cazé, Mark Humphries, and Boris Gutkin. Passive dendrites enable single neurons to compute linearly non-separable functions. 9(2):e1002867.

[4] Albert Gidon, Timothy Adam Zolnik, Pawel Fidzinski, Felix Bolduan, Athanasia Papoutsi, Panayiota Poirazi, Martin Holtkamp, Imre Vida, and Matthew Evan Larkum. Dendritic action potentials and computation in human layer 2/3 cortical neurons. 367(6473):83–87.

[5] Elizabeth P. Lackey, Detlef H. Heck, and Roy V. Sillitoe. Recent advances in understanding the mechanisms of cerebellar granule cell development and function and their contribution to behavior. 7:1142.

[6] Raoul-Martin Memmesheimer and Marc Timme. Non-additive coupling enables propagation of synchronous spiking activity in purely random networks. 8(4):e1002384.

[7] Panayiota Poirazi, Terrence Brannon, and Bartlett W. Mel. Pyramidal neuron as two-layer neural network. 37(6):989–999.

[8] Alon Polsky, Bartlett W Mel, and Jackie Schiller. Computational subunits in thin dendrites of pyramidal cells. 7(6):621–627.

[9] Alexandra Tzilivaki, George Kastellakis, and Panayiota Poirazi. Challenging the point neuron dogma: FS basket cells as 2-stage nonlinear integrators. 10(1):3664.

[10] Anthony M Zador, Brenda J Claiborne, and Thomas H Brown. Nonlinear pattern separation in single hippocampal neurons with active dendritic membrane. page 8.