A New Proposal of an Index for Regional Cerebral Perfusion Pressure – A Theoretical Approach with Fluid Dynamics

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Abstract

Cerebral blood flow (CBF) / cerebral blood volume (CBV) ratio derived by [^{15}O] H_2O/ CO_2 and CO positron emission tomography (PET) examination has been used empirically as an index for cerebral perfusion pressure (CPP). However, it lacks theoretical background and could not be confirmed to be proportionate to CPP, as measurement of local CPP is not practical.

We have developed a new index for CPP from Poiseuille equation based on a simple model. Our model implies that CBF/CBV^2 is proportionate to CPP. To estimate CPP, CBF/CBV^2 would be a preferable index to CBF/CBV theoretically.

keywords: positron emission tomography (PET), cerebral blood flow (CBF), cerebral blood volume (CBV), cerebral perfusion pressure (CPP), fluid mechanics

Introduction

Cerebral perfusion pressure (CPP) is driving force of cerebral blood flow (CBF), therefore, is very important factor to evaluate haemodynamic states. However, noninvasive measurement of local CPP has not been developed.

As the CBF/cerebral blood volume (CBV) ratio derived by [^{15}O] H_2O/ CO_2 and CO positron emission tomography (PET) examination reflected the artery patency, CBF/CBV was proposed as an index of haemodynamic reserve [1]. Then, as CBF/CBV showed relationship to oxygen extraction fraction [2] and mean arterial pressure in baboon [3], CBF/CBV has been treated as an index of CPP [4, 5]. When CPP decreases, CBV increases and CBF decreases, therefore, CBF/CBV may certainly serve as an index of CPP.

However, the index was empirically derived, and has no theoretical background. Certainly CBF/CBV may be treated as ordinal scale (*i.e.* the smaller CBF/CBV implies smaller CPP), however, there is no evidence of ratio scale (*i.e.* CBF/CBV is proportional to CPP). In this study, CPP was theoretically derived by CBF and CBV using fluid dynamics.

Theory

Assume that one cerebral small region, which contains one capillary. The capillary supplies blood flow to the entire region (Figure 1).

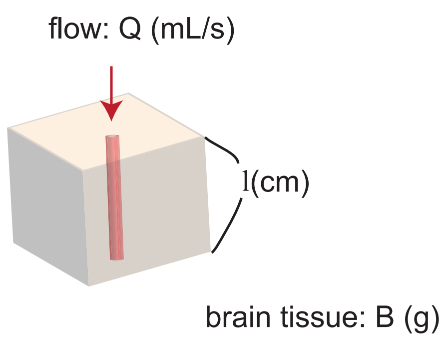

Figure 1: Model of a region of brain. The region contains one capillary. The capillary supplies blood flow to the entire

region.

Poiseuille equation, which can be derived by Navier-Stokes equations [6] describes incompressible fluid in lamina flow through a long pipe:

$$\Delta p = \frac{8\mu l Q}{\pi r^4} \tag{1}$$

where Δp denotes pressure difference between the two ends, *i.e.* local CPP (Pa), μ is dynamic viscosity (Pa·min), l is length of capillary, Q is volmetric flow rate (mL/min), and r is radius of the capillary.

CBF and CBV can be calculated as follows:

$$\mathrm{CBF} = Q/B \qquad (\mathrm{mL/g/min}) \tag{2}$$

$$\mathrm{CBV} = \pi r^2 l/B \qquad (\mathrm{mL/g}) \tag{3}$$

where B denotes local brain tissue weight including one capillary (g).

Therefore,

$$\mathrm{CPP} = \frac{8\pi\mu l^3}{B}\frac{\mathrm{CBF}}{\mathrm{CBV}^2} \tag{4}$$

As μ is constant and the volume of the brain region that one capillary perfuses is roughly determined, thus, CPP is demonstrated to be proportional to CBF/CBV^2.

Method

A simulation was executed to demonstrate how CBF/CBV and CBF/CBV^2 behave using a standard spreadsheet software, Excel (Microsoft Corporation, Redmond, WA, USA). It was run under the condition that drop rate of CBF after auto-regulation limit (Power's stage II [7]) was twice as much as CBV elevation before the limit (stage I).

Results

CBF/CBV^2 showed better linearity than CBF/CBV (Figure 2).

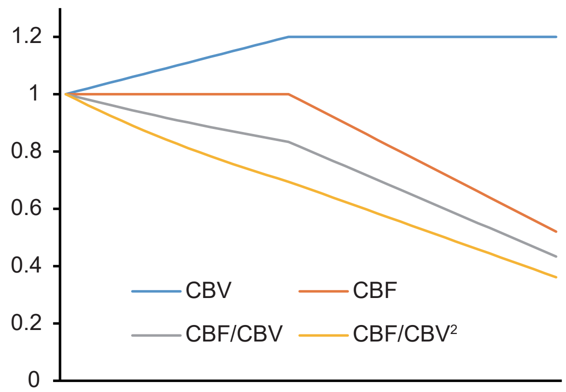

Figure 2: A simulation of CBF/CBV and CBF/CBV^2. CBF/CBV^2 showed better linearity than CBF/CBV.

Discussion

We have demonstrated theoretically that CBF/CBV^2 is an appropriate indicator of CPP. CBF/CBV is the reciprocal number of mean transit time [8], therefore, is proportionate to the mean velocity of blood, and certainly reflects CPP. However, our theoretical approach implies that CBF/CBV^2 would be better than CBF/CBV.

The linearity showed in Figure 2 was responsible to our choice that CBF dropping rate was twice as much as CBV elevation rate. However, it would be a reasonable assumption considering fluid dynamics equations (1, 2, 3, 4). Moreover, a previous study indicated a tendency that lower mean arterial pressure leads to lower CBF/CBV [3].

Poiseuille equation is applicable under the condition of laminar flow in a long tube. Our conclusion may be susceptible to turbulent flow, although we hope that the effect would be limited as Reynolds number $\left(\frac{2r\rho v}{\mu}\right)$ (v: velosity of fluid, ρ: the density of the fluid) of small capillary is small.

Competing interests

The authors declare that they have no competing interests.

Author's contributions

MK contributed to the conceptualization, creation of theory, and initial draft manuscript preparation. TM advised the project. All the authors discussed the project and have read and approved the final manuscripts.

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