1 **Photoinduced Isomerization Sampling of Retinal in**
2 **Bacteriorhodopsin**

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Abstract
Photoisomerization of retinoids inside a confined protein pocket represents a critical
chemical event in many important biological processes from animal vision, non-visual
light effects, to bacterial light sensing and harvesting. Light driven proton pumping in
bacteriorhodopsin entails exquisite electronic and conformational reconfigurations
during its photocycle. However, it has been a major challenge to delineate transient
molecular events preceding and following the photoisomerization of the retinal from
noisy electron density maps when varying populations of intermediates coexist and
evolve as a function of time. Here I report several distinct early photoproducts
deconvoluted from the recently observed mixtures in time-resolved serial
crystallography. This deconvolution substantially improves the quality of the electron
density maps hence demonstrates that the all-*trans* retinal undergoes extensive
isomerization sampling before it proceeds to the productive 13-*cis* configuration. Upon
light absorption, the chromophore attempts to perform *trans*-to-*cis* isomerization at
every double bond coupled with the stalled *anti*-to-*syn* rotations at multiple single
bonds along its polyene chain. Such isomerization sampling pushes all seven
transmembrane helices to bend outward, resulting in a transient expansion of the
retinal binding pocket, and later, a contraction due to recoiling. These ultrafast

responses observed at the atomic resolution support that the productive photoreaction

in bacteriorhodopsin is initiated by light-induced charge separation in the prosthetic

chromophore yet governed by stereoselectivity of its protein pocket. The method of a

numerical resolution of concurrent events from mixed observations is also generally

applicable.

Introduction

Bacteriorhodopsin (bR) pumps protons outward from the cytoplasm (CP) against the

concentration gradient via photoisomerization of its retinal chromophore. The trimeric

bR on the native purple membrane shares the seven transmembrane helical fold and the

same prosthetic group (Fig. S1) with large families of microbial and animal rhodopsins

(Ernst et al., 2014; Kandori, 2015). An all-*trans* retinal in the resting state is covalently

linked to Lys216 of helix G through a Schiff base (SB), of which the double bond $C_{15}=N_\zeta$

is also in *trans*. Upon absorption of a visible photon, the all-*trans* retinal in bR

isomerizes efficiently and selectively to adopt the 13-*cis* configuration (Govindjee et al.,

1990). In contrast, an all-*trans* free retinal in organic solvents could isomerize about

various double bonds, but with poor quantum yields (Freedman and Becker, 1986;

Koyama et al., 1991).

A broad consensus is that the isomerization event takes place around 450-500 fs

during the transition from a blue-shifted species I to form a red-shifted intermediate J

(Herbst, 2002; Mathies et al., 1988). Various molecular events prior to the isomerization

have also been detected. Vibrational spectroscopy showed a variety of possible

motions, such as torsions about $C_{13}=C_{14}$ and $C_{15}=N_\zeta$, H-out-of-plane wagging at C_{14}, and

even protein responses (Diller et al., 1995; Kobayashi et al., 2001). Nevertheless, the

species I or a collection of species detected before 30 fs remain in a good *trans*

configuration about $C_{13}=C_{14}$ instead of a near 90° configuration (Zhong et al., 1996).

Recently, deep-UV stimulated Raman spectroscopy revealed strong signals of Trp and

Tyr motions in the protein throughout the I and J intermediates (Tahara et al., 2019).

Despite extensive studies, fundamental questions on the photoisomerization of retinal

remain unanswered at the atomic resolution. What is the quantum mechanical force

that causes the all-*trans* retinal to isomerize specifically to 13-*cis* after absorbing a

photon? Why not isomerize elsewhere in bR? How is the quantum yield of this specific

isomerization enhanced by the protein compared to those of free retinal in solution?

Does any isomerization sampling occur? This work addresses these questions by

solving a series of structures of the early intermediates based on the electron density

maps unscrambled from the published serial crystallography datasets using singular

value decomposition (SVD). These structures of "pure" photoproducts at atomic

resolution reveal widespread conformational changes in all seven helices prior to the

all-*trans* to 13-*cis* isomerization and after its completion, suggesting that isomerization

sampling takes place in bR, where rapid photoisomerizations and single bond rotations

are attempted everywhere along the polyene chain of the retinal before the only

successful one flips the SB at ~500 fs.

Several international consortiums carried out large operations of serial

crystallography at free electron lasers (XFELs). It is now possible to capture transient

structural species at room temperature in the bR photocycle as short-lived as fs

(Brändén and Neutze, 2021). Compared to cryo-trapping, authentic structural signals

from these XFEL data are expected to be greater in both amplitude and scope.

However, the signals reported so far do not appear to surpass those obtained by cryo-

trapping methods, suggesting much needed improvements in experimental protocols

and data analysis methods. Two major sources of data are used in this study (Table S1).

Nogly et al. captured retinal isomerization to 13-*cis* by the time of 10 ps and attributed

the specificity to the H-bond breaking between the SB and a water (Nogly et al., 2018).

Kovacs et al. contributed datasets at many short time delays (Kovacs et al., 2019). Those

sub-ps datasets demonstrate oscillatory signals at frequencies around 100 cm^{-1}. The

essence of this work is a numerical resolution of structural heterogeneity, a common

difficulty often encountered in cryo trapping and time-resolved serial crystallography.

To what extend a specific structural species can be enriched in crystals depends on the

reaction kinetics governed by many experimental parameters including but not limited

to the fluence, wavelength, and temperature of the light illumination. While it is

possible to reach higher fractional concentrations at specific time points for more stable

species such as K or M due to the ratio between the rates going into and exiting from

that species, transient species such as I and J are often poorly populated. If such

structural heterogeneity is not resolved, it is very difficult, if not impossible, to interpret

the electron density maps and to refine the intermediate structures (Ren et al., 2013).

An assumption in nearly all previous studies has been that each dataset, at a cryo

temperature or at a time delay, is derived from a mixture of a single photoinduced

species and the ground state. Therefore, the difference map reveals a pure intermediate
structure. This assumption is far from the reality thus often leads to misinterpretation
of the observed electron density maps. This work is yet another case study to
demonstrate the application of our analytical protocol based on SVD (Methods) that
makes no assumption on how many excited intermediates that contribute to the
captured signals at each time point (Ren, 2019; Ren et al., 2013; Yang et al., 2011). More
importantly, this work showcases that our resolution of structural heterogeneity enables
new mechanistic insights into the highly dynamic chemical or biochemical processes.
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Results and Discussion
A total of 24 datasets and 18 time points up to 10 ps are analyzed in this study (Table
S1). Difference Fourier maps at different time points and with respect to their
corresponding dark datasets are calculated according to the protocols previously
described (Methods). A collection of 126 difference maps at short delays ≤ 10 ps are
subjected to singular value decomposition (SVD; Methods) followed by a numerical
deconvolution using the previously established Ren rotation in a multi-dimensional
Euclidean space (Ren, 2016, 2019). Such resolution of electron density changes from
mixed photoexcited species in the time-resolved datasets results in four distinct
intermediate structures in the early photocycle, which are then refined against the
reconstituted structure factor amplitudes (Table S2; Methods).
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Low frequency oscillations observed upon photoexcitation
Ten out of 17 major components derived from the sub-ps delays of Kovacs et al. (Fig.
S2) describe five two-dimensional oscillatory behaviors at frequencies ranging from 60
to 400 cm^{-1} (Fig. S3). Compared to a bond stretching frequency commonly observed in
vibrational spectroscopy, these oscillations are at much lower frequencies. The lowest
frequency is 61±2 cm^{-1}, that is, a period of 550±20 fs (Fig. S3a), which matches exactly
the oscillation detected in transient absorption changes in visual rhodopsin (Wang et al.,
1994). Although these ten components follow the oscillatory time dependencies, they
do not show any association with the chromophore or the secondary structure of the
protein (Fig. S4). Similar oscillatory components were also present in the XFEL datasets
of MbCO (Ren, 2019). Therefore, the same conclusion stands that these low frequency
vibrations induced by short laser pulses often detected by ultrafast spectroscopy are the
intrinsic property of a solvated protein molecule, here specifically bacteriorhodopsin

(bR) (Johnson et al., 2014; Liebel et al., 2014). Interestingly, the isomerization sampling

and productive photoisomerization observed in this study occur within the first

oscillatory period at the lowest frequency. While such coincidence begs the question

whether the protein oscillation is required for isomerization (see below), direct evidence

is lacking in these XFEL data to support any functional relevance of these oscillatory

signals.

Intermediates I′, I, and expansion of retinal binding pocket

In contrast to the oscillating signals, three components U_{10}, U_{14}, and U_{17} reveal strong

light-induced structural signals in terms of both extensiveness and quality (Figs. 1ab

and S5). These signals originate exclusively from a few time points of Nogly et al., too

few to fit the time dependency with exponentials. Instead, a spline fitting through these

time points gives rise to the estimated coefficients c_{10}, c_{14}, and c_{17} in the linear

combination of $c_{10}U_{10} + c_{14}U_{14} + c_{17}U_{17}$ for reconstructing the electron density maps of the

states I, J, and their respective precursors I′, J′ (Fig. 2a). A reconstituted difference map

of I′ − bR (Fig. 1c) is located on the spline trajectory from the origin, that is, bR at the

time point of 0-, to the first time point of 49-406 fs (PDB entry 6g7i). This state is

denoted I′ as a precursor leading to the I state judged by the time point at ~30 fs.

However, this is not to say that a single species I′ exists around 30 fs. Quite the

opposite, the population of the time-independent conformational species I′ rises and

falls and peaks around 30 fs, while many other species during isomerization sampling

coexist with I′ at the same time (see below). The reconstituted difference map is used to

calculate a set of structure factor amplitudes that would produce this difference map of

I′ − bR (Methods). And the structure of I′ is refined against this reconstituted dataset

(beige; Figs. 1cd and S6). The same protocol is used to refine the structure of I state

(purple; Fig. S7) with a reconstituted difference map I − bR (Figs. 1a, 2ab, 3a, and S5).

This SVD-dependent refinement strategy extends the commonly used method based on

an extrapolated map to another level. This newly developed method is able to refine a

structure against any linear combination of signal components while eliminating noise

and systematic error components, and components identified as other intermediate

species mixed in the data. Therefore, this method enables the refinement of an

unscrambled, hence pure, structural species (Methods).

The all-*trans* retinal chromophore in the ground state of bR is largely flat except the

last atom C_{15} (Fig. 2c 2nd panel). In contrast, the side chain of Lys216 is highly twisted

forming two near-90° single bonds (Fig. 2c 4th panel), which results in a corner at C_ε that

deviates dramatically from the plane of the all-*trans* retinal (Fig. 2c 2nd panel). The

refined geometry of the retinal in I' retains a near perfect all-*trans* configuration,

including the Schiff base (SB) double bond $C_{15}=N_\zeta$, while various single bonds along the

polyene chain deviate from the standard *anti* conformation significantly (Fig. 2c 4th

panel). The torsional deviations from *anti* are in a descending order from the β-ionone

ring to the SB. These torsional changes result in an S-shaped retinal shortened by ~4%

(Fig. 2c 3rd panel). The distal segment C_6-C_{12} moves inboard up to 0.9 Å and the

proximal segment C_{13}-C_ε, including the SB, moves outboard up to 1.6 Å (Fig. 2c 1st and

2nd panels; see Fig. S1 for orientations in bR). This creased retinal observed here at

around 30 fs (Fig. 1d) is attributed to the direct consequence of a compression under an

attraction force between the β-ionone ring and the SB (see below).

The refined structure of the I state (Fig. S7) shows that the retinal remains in near

perfect all-*trans*, including the SB, and as creased as its precursor I' (Fig. 3c). The

torsional deviations from *anti* single bonds become even more severe compared to the I'

state and remain in a descending order from the β-ionone ring to the SB (Fig. 2c 4th

panel). The major difference from its precursor is that the single bond N_ζ-C_ε now

adopts a perfect *syn* conformation (Figs. 2c 4th panel and 3c), and the anchor Lys216 has

largely returned to its resting conformation. Such a lack of substantial change between

the ground state and the intermediate I was previously noted by a comparison of a

chemically locked $C_{13}=C_{14}$ with the native retinal (Zhong et al., 1996).

Remarkably, the major component $\boldsymbol{U}_{10}$ reconstituted into the difference map of I – bR

contains widespread signal associated with all seven helices (Fig. 2b). The reconstituted

map clearly shows collective outward motions from the center (Fig. 3a) suggesting an

expansion of the retinal binding pocket at hundreds of fs, which is confirmed by the

refined structure of the I state (Fig. 3d top panel). For example, the distance between

the C_α atoms increases by 0.8 Å between Arg82 and Phe208 and by 0.7 Å between Tyr83

and Trp182. It is noteworthy that similar protein signals are present in the raw

difference map calculated from the time point of 457-646 fs from Nogly et al. (6g7j) prior

to an SVD analysis (Fig. S8).

Transient bleaching at near UV of 265-280 nm was observed before 200 fs and

attributed to structural changes in the retinal skeleton and the surrounding Trp residues

(Schenkl et al., 2005). Recent deep-UV stimulated Raman spectroscopy also

demonstrated that motions of Trp and Tyr residues start to emerge at 200 fs and remain

steady until the isomerization is over at 30 ps (Tahara et al., 2019). Here the refined

structure of the I state with displaced helices and an expanded retinal binding pocket

offers an explanation for the stimulated Raman gain change at hundreds of fs.

However, it is unclear why and how such extensive protein responses take place even

before the retinal isomerization. According to the broadly accepted concept of

proteinquake, initial motions are generated at the epicenter where the chromophore

absorbs a photon and then propagated throughout the protein matrix (Ansari et al.,

1985). It is plausible that these ultrafast protein responses are the direct consequence of

isomerization sampling in a confined protein pocket. It was observed in organic

solvents using high-pressure liquid chromatography (HPLC) that all-*trans* retinal could

isomerize at various double bonds along the polyene chain to adopt 9-, 11-, and 13-*cis*

configurations, but with rather poor quantum yields (Freedman and Becker, 1986;

Koyama et al., 1991). This intrinsic property of the all-*trans* retinal would behave the

same even when it is incorporated in the protein except that the protein matrix herds

the chromophores on the right track of the productive photocycle and keeps the

concentrations of the attempted byproducts low. These byproduct conformations of the

retinal during isomerization sampling are too numerous and too minor to be observed

experimentally. Nevertheless, they cause a common effect, an expansion of its binding

pocket, since the all-*trans* retinal in the resting state is tightly boxed by massive side

chains all around (Fig. 3e). Any attempt to isomerize would push against this box one

way or another. For instance, triple attempts to isomerize simultaneously at 11, 13, and

15 positions were suggested by a quantum mechanics/molecular mechanics simulation

(Altoè et al., 2010). When the retinal binding pocket is altered in mutants, the quantum

yield of each isomerization byproduct is expected to increase resulting in an impaired

productive pathway (see below).

Intermediates J', J and productive isomerization of retinal

The time point of 10 ps of Nogly et al. (6g7k) differs from the previous time point of

457-646 fs (6g7j) by negating the component of U_{10} (Fig. 2ab), which leads to a

restoration of the normal retinal binding pocket in J′ from an expanded one in the I state

followed by a contraction in J (Fig. 3d bottom panel). Two time-independent structures

of J′ (green; Fig. S9) and J (gray; Fig. S10) are refined based on the respective

reconstituted difference maps with the same protocol (Methods). Their populations

peak at the approximate time of ~700 fs and ~20 ps, respectively. The observed

contraction of the retinal binding pocket is likely due to an elastic recoiling of the seven

helical bundle following its transient expansion caused by the isomerization sampling.

The creased retinal persists in both the J′ and J structures (Fig. 2c 2nd panel and Fig.

3c). The difference map of J′ – bR clearly shows the 13-*cis* configuration (Fig. 3b).

Indeed, near perfect 13-*cis* is successfully refined in both structures (Fig. 2c 4th panel).

While the SB double bond $C_{15}=N_\zeta$ is momentarily distorted from the *trans* configuration

in J′ with a torsion angle of 133°, a perfect *trans* configuration at $C_{15}=N_\zeta$ is promptly

restored in J (Fig. 2c 4th panel). The refined structures of this series of early

intermediates show that the SB N_ζ is rotating clockwise in the entire process of the

isomerization of I′ → I → J′ → J, if the retinal is viewed from the proximal to distal

direction (Fig. 2c). It seems that the isomerization starts in an expanded retinal binding

pocket and finishes in a tighter one. Whether the pocket expansion and contraction are

required for the productive isomerization and what role the low frequency oscillations

play in isomerization will need more time points at short delays to further isolate the

molecular events temporally.

Coulomb attraction as driving force of isomerization sampling

The fundamental questions remain: What is the driving force that causes the all-*trans*

retinal to isomerize after a photon absorption, at several double bonds if not restrained

but exclusively at $C_{13}=C_{14}$ in bR? How does the protein environment enhance the

quantum yield of the isomerization to 13-*cis*? Here I hypothesize that a Coulomb

attraction between the β-ionone ring and the SB at the Frank-Condon point, 0+ time

point, provides the initial driving force upon a photon absorption. The electric field

spectral measurements (Mathies and Stryer, 1976) and the quantum mechanics

simulation (Nogly et al., 2018) suggested that a charge separation occurs along the

polyene chain at the excited state of bR. Such a dipole moment was also detected

through a transient bleaching signal at near UV region (Schenkl et al., 2005). It can be

shown that a plausible charge separation of ±0.1e between the β-ionone ring and the SB

would cause an attraction force > 1 pN. If calibrated with the measured range of dipole

moment of 10-16 D (Mathies and Stryer, 1976), the charge separation could reach the

level of ±0.16e to ±0.26e, giving rise to an attraction force of 3.5-9 pN between the β-

ionone ring and the SB. This attraction force is evidently sufficient to crease the flat all-

trans retinal into an S-shape and to compress it slightly within tens of fs as observed in

I′ and I states (Figs. 1d, 2c 2nd and 3rd panels, and 3c). In the meanwhile, this very

attraction force also triggers simultaneous attempts of double bond isomerizations and

single bond rotations along the polyene chain that cause the expansion of the retinal

binding pocket as observed at hundreds of fs. Following the only successful

isomerization at $C_{13}=C_{14}$, the chromophore segment from C_{15} to C_δ is attracted to the β-

ionone ring; and these two parts become significantly closer (Fig. 2c 3rd panel). None of

the single bond rotations can complete under the restraints of the protein. Especially,

the segment closer to the midpoint of the retinal is more confined due to the steric

hinderance of Thr90 and Tyr185 from the inboard and outboard sides, respectively (Fig.

3e). Therefore, the single bonds deviate from *anti* less and less towards the midpoint

(Fig. 2c 4th panel). The effect of charge separation seems eased gradually as the reaction

proceeds beyond the J state as indicated by the slow restoration of the *anti* conformation

(Fig. 2c 4th panel).

Apparently, the same charge separation and the attraction force upon photon

absorption also take place in a solution sample of free retinal. Compared to the retinal

embedded in protein, photoisomerization in solution is nonspecific, resulting in a range

of byproducts, since an isomerization at any position would bring the SB significantly

closer to the β-ionone ring. It is understandable that each of the byproducts could only

achieve a poor quantum yield (Freedman and Becker, 1986; Koyama et al., 1991) as

rotations at multiple single bonds driven by the same attraction force and achieving a

similar folding of the polyene chain would further sidetrack the double bond

isomerizations thus diminishing their quantum yields. However, these byproducts due

to single bond rotations are short-lived beyond detection by HPLC as they

spontaneously revert back in solution. The protein environment in bR plays a major

role in enhancing the quantum yield of the isomerization to 13-*cis* by shutting down all

other reaction pathways triggered by the charge separation. This is further elucidated

by the mutant functions below.

Isomerization byproducts permitted by mutant protein environments

The structure of a double mutant T90A/D115A (3cod) showed little difference from the

wildtype (Joh et al., 2008) while the single mutants T90V and T90A retain < 70% and <

20% of the proton pumping activity, respectively (Marti et al., 1991; Perálvarez et al.,

2001). These observations illustrate that some nonproductive pathways of the

isomerization sampling succeed more in the altered retinal binding pocket. In the

wildtype structure, Thr90 in helix C points towards the $C_{11}=C_{12}-C_{13}-C_{20}$ segment of the

retinal from the inboard with its C_γ atom 3.7 Å from the retinal plane. Given the van

der Waals radius r_C of 1.7 Å, only 0.3 Å is spared for the H atoms of the C_γ methyl group

thereby effectively shutting down the nonproductive pathways of the isomerization

sampling. Any motion of the retinal would have to push helix C toward inboard

causing an expansion of its binding pocket. Missing this close contact in T90A increases

the room to 1.9 Å for isomerization byproducts, which would greatly reduce the

quantum yield of the 13-*cis* productive isomerization thus retain < 20% of the activity.

In addition to 13-*cis*, the retinal in the light adapted T90V mutant showed 9- and 11-

cis configurations at the occupancies of 3% and 18%, respectively, while these

configurations were not detected in light adapted wildtype (Marti et al., 1991). Then

why would a Val residue at this position with an equivalent C_γ atom permit the

formation of some isomerization byproducts? In wildtype bR, the side chain of Thr90

engages two strong H-bonds Trp86O-Thr90O_γ-D115O_δ so that its C_γ methyl group is

aligned toward the retinal. Without these H-bonds in T90V, the isopropyl group of

Val90 is free to adopt other rotameric positions so that neither of the C_γ methyl groups

has to point directly to the retinal, which increases the available room for the formation

of some isomerization byproducts. Compared to the light adapted state, these

isomerization byproducts could reach even higher percentages during active

photocycles thus reduce the proton pumping activity below 70%.

From the outboard, the side chain of Tyr185 in helix F is nearly parallel to the retinal

plane with a distance of 3.5 Å. This close contact of a flat area from C_8 to C_{14} of the

retinal prevents any significant motion of the retinal toward the outboard. Even slight

motions would push helix F away as observed here in the expansion of the retinal

binding pocket. The mutant Y185F largely retains the flat contact so that its proton

pumping activity does not reduce much (Hackett et al., 1987; Mogi et al., 1987).

However, it is predictable that various single mutants at this position with smaller and
smaller side chains would promote more and more isomerization byproducts and
eventually shut down proton pumping.
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Two massive side chains of Trp86 and 182 from the EC and CP sides respectively do
not seem to play a significant role in suppressing byproduct formation as shown by the
mutant W182F that retains the most of the wildtype activity (Hackett et al., 1987), since
the motions involved in isomerization sampling are oriented more laterally. The
transient expansion and contraction of the retinal binding pocket (Fig. 3d) indicate that
the tight box surrounds the mid-segment of the retinal (Fig. 3e) is not completely rigid.
Rather, its plasticity must carry sufficient strength to prevent isomerization byproducts.
Presumably, this strength originates from the mechanical property of the helical bundle.
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In summary, this work reveals the transient structural responses to many
unsuccessful attempts of double bond isomerization and single bond rotation by a
numerical resolution from the concurrent pathways, which are otherwise difficult to
observe. These findings underscore an important implication, that is, a nonspecific
Coulomb attraction provides the same driving force for the isomerization sampling
with and without a protein matrix. A productive isomerization at a specific double
bond is guided by the incorporation of the chromophore in a specific protein
environment. The productive pathway is selected from numerous possibilities via
stereochemical hinderance. Nevertheless, this nonspecific Coulomb attraction force
may not be directly applicable to the photoisomerization of retinal from 11-*cis* to all-
trans in the activation of visual rhodopsins. The key difference is bR as an energy
convertor versus a visual rhodopsin as a quantum detector (Lewis, 1978).
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Competing interests

ZR is the founder of Renz Research, Inc. that currently holds the copyright of the computer software dynamiX™.

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Figures and Legends

Figure 1. Shortened retinal in S-shape since earliest intermediate I'. (a) Cross sections

of component map U_{14}. The middle cross section is an integration ±0.2 Å around the

surface through the retinal. The top cross section is an integration 1.2-1.8 Å outboard

from the retinal surface and the bottom one is an integration 0.8-1.2 Å inboard. See Fig.

S1 for definitions of inboard, outboard, and other orientations in bR molecule. Green

and red indicate electron density gain and loss, respectively. Nearly the entire retinal is

in negative densities. The proximal segment and three waters are in intense negative

densities. On the other hand, strong positive densities flank the proximal and distal

segments from the outboard and inboard, respectively. Such signal distribution results

in the S-shaped retinal by the refinement shown in (d). (b) Cross sections of component

map U_{17}. The middle cross section is an integration ± 0.2 Å around the surface through

the retinal. The top panel is an integration 0.5-0.9 Å outboard and the bottom is an

integration 0.8-1.2 Å inboard. Negative and positive densities flank the retinal from the

outboard and inboard, respectively. (c) Difference map of I′ − bR reconstituted from U_{14}

and U_{17} (a and b). The map is contoured at $\pm 3\sigma$ in green and red mesh, respectively.

The opposite displacements of the distal and proximal segments of the retinal are

obvious. Extensive signals indicate changes in the water network and Asp85 and 212.

(d) Refined retinal conformation in beige overlaid on the resting state in white. This

view is orthographical to (c). The marked distances from C_3 to N_ζ show a shortened

retinal creased into an S-shape. C_{20} methyl group is tilted 33° toward outboard from its

resting state bR. Wat402 remains in H-bonds with both Asp85 and 212.

Figure 2. Early intermediates identified in SVD space. (a) Multi-dimensional spaces of
SVD. The SVD analysis of difference Fourier maps at short delays ≤ 10 ps results in
time-dependent coefficients $c_k(t)$, where $k = 1, 2, \ldots$, each corresponding to a time-
independent components U_k. Each raw difference map at a time delay t can be closely
represented by a linear combination of these components, $c_1(t)U_1 + c_2(t)U_2 + \ldots$, that is
called a reconstituted difference map. Each of these components U_k and the
reconstituted difference maps can be rendered in the same way as an observed
difference map. The coefficient set $c_k(t)$ is therefore a trace of the photocycle trajectory,
when these time-dependent functions are plotted in a multi-dimensional space or
plotted together against the common variable t. Coefficients corresponding to
components U_{10}, U_{14}, and U_{17} are plotted in three orthographical views. Three time
points from Nogly et al. in red contain U_{14} equally. These time points vary in U_{10} and
U_{17}. Datasets from Kovacs et al. in yellow do not carry any of these signals, therefore
cluster near the origin. The component map of U_{10} is displayed in (b) and U_{14} is
displayed in Figs. 1a and S5. U_{17} is displayed in Fig. 1b. Several apices of the spline
fitting are chosen as the potential pure states of I′, I, J′, and J marked by large dots. This
choice is only an approximate due to the insufficient number of time points observed.
(b) Component map U_{10}. The main chain and side chains of the protein are rendered
with ribbons and sticks, respectively. The retinal and Lys216 are in purple sticks.
Several key waters are in blue spheres. Parts of the structure are omitted to reveal more
of the interior. The map is contoured at $\pm 3\sigma$ in green and red, respectively. Three
orthographical views of U_{10} clearly show that the signals are distributed around the
middle segment of the molecule and taper off to both CP and EC surfaces. The signals
also concentrate at all seven helices. (c) Conformational parameters calculated from the
refined chromophore. The chemical structure of the chromophore on top is aligned to
the horizontal axis. Double bonds are shaded in gray. Atomic displacements of each
intermediate from the resting state show greater changes in the proximal segment (top
panel). A plane is least-squares fitted to C_4 through C_{14} of the resting state. The
distances of all atoms to this plane in the inboard and outboard directions show the
curvature of the chromophore. The creased retinal in early intermediates and the
inboard protruding corner at C_ε in the resting state are clearly shown (2nd panel).
Distances to atom C_4 are calculated for all refined chromophores. Changes in these
distances with respect to the resting state show the shortened chromophore in I′ and I.
Once isomerization to 13-*cis* occurs, the segment from C_{15} to C_δ around the SB becomes

significantly closer to the β-ionone ring due to the Coulomb attraction force, while the

distal segment of the retinal from C_{14} and beyond stretches (3rd panel). The torsion

angles of single and double bonds quantify *anti/syn* or *trans/cis* for the ground state and

all intermediates (4th panel). Only a single bond can be twisted with its torsion angle

near 90°. A twisted double bond would be energetically costly. Each double bond is

least-squares fitted with a plane. The interplanar angle between a double bond and the

corresponding one in the ground state measures the local tilting of the retinal (bottom

panel).

Figure 3. Intermediates I, J', J, and retinal binding pocket expansion and contraction.
(a) Reconstituted difference map I – bR from U_{10} and U_{14} (Figs. 1a, 2ab, and S5). The map is contoured at $\pm 2.5\sigma$ in green and red mesh, respectively. The difference map at three middle segments of helices B, C, and G show main chain displacements toward inboard or outboard as indicated by the arrows marking the negative and positive pairs of densities. These difference densities are the direct evidence of the expansion of the retinal binding pocket. The refined structure of I is in purple and the resting state is in white. (b) Reconstituted difference map J' – bR from U_{14} and U_{17} (Figs. 1ab, 2a, and S5). The map is contoured at $\pm 3.5\sigma$ in green and red mesh, respectively. These difference densities are the direct evidence of isomerization at hundreds of fs. The refined structure of J' in 13-*cis* is in green. (c) The refined 13-*cis* retinal conformation compared

with the resting state in white. I, J′, and J are in purple, green, and bluish gray,

respectively. The creased S-shape is easing gradually (Fig. 2c 2nd panel). (d) The refined

structures of I and J compared with the resting state viewed along the trimer three-fold

axis from the EC side. Atomic displacements in the main chain from bR to I and J are

color coded and marked by arrows with lengths 20× of the actual displacements. All

seven helices in I move away from the center except a small segment in helix C showing

an expansion of the retinal binding pocket (top panel). However, all seven helices in J

move closer to one another showing a contraction with respect to the resting state bR

(bottom panel). This contraction is much more significant if compared directly with the

expanded I state. (e) Two orthographical views of the retinal tightly boxed at its middle

segment. The closest contact is Thr90 and Tyr185 on the inboard and outboard sides of

the retinal plane, respectively. The minimum distance between them is 7.0 Å = $4r_C + 0.2$

Å, where $r_C = 1.7$ Å is the van der Waals radius of C. See also (Kandori, 2015).

Photoinduced Isomerization Sampling of Retinal in Bacteriorhodopsin

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Methods

From the outset, the key presumption is that every crystallographic dataset, at a given temperature and a given time delay after the triggering of a photochemical reaction, captures a mixture of unknown number of intermediate species at unknown fractions. Needless to say, all structures of the intermediates are also unknown except the structure at the ground state that has been determined and well refined by static crystallography. A simultaneous solution of all these unknowns requires multiple datasets that are collected at various temperatures or time delays so that a common set of intermediate structures are present in these datasets with variable ratios. If the number of available datasets is far greater than the number of unknowns, a linear system can be established to overdetermine the unknowns with the necessary stereochemical restraints (Ren et al., 2013). The analytical methods used in this work to achieve such overdetermination have been incrementally developed in the past years and recently applied to another joint analysis of the datasets of carbonmonoxy myoglobin (Ren, 2019). Time-resolved datasets collected with ultrashort pulses from an X-ray free electron laser were successfully analyzed by these methods to visualize electron density components that reveal transient heating, $3d$ electrons of the heme iron, and global vibrational motions. This analytical strategy is recapped below.

The methodological advance in this work is the refinement of each pure intermediate structure that has been deconvoluted from multiple mixtures. Structure factor amplitudes of a single conformation free of heterogeneity are overdetermined.

Given the deconvoluted structure factor amplitude set of a pure state, the standard

structural refinement software with the built-in stereochemical constraints is taken full

advantage of, e.g. PHENIX (Adams et al., 2010; Liebschner et al., 2019). In case that the

computed deconvolution has not achieved a single pure structural species, the

structural refinement is expected to make such indication.

Difference Fourier maps

A difference Fourier map is synthesized from a Fourier coefficient set of F_{light}-$F_{reference}$

with the best available phase set, often from the ground state structure. Before Fourier

synthesis, F_{light} and $F_{reference}$ must be properly scaled to the same level so that the

distribution of difference values is centered at zero and not skewed either way. A

weighting scheme proven effective assumes that a greater amplitude of a difference

Fourier coefficient F_{light}-$F_{reference}$ is more likely caused by noise than by signal (Ren et al.,

2001, 2013; Šrajer et al., 2001; Ursby and Bourgeois, 1997). Both the dark and light

datasets can serve as a reference in difference maps. If a light dataset at a certain delay

is chosen as a reference, the difference map shows the changes since that delay time but

not the changes prior to that delay. However, both the dark and light datasets must be

collected in the same experiment. A cross reference from a different experimental

setting usually causes large systematic errors in the difference map that would swamp

the desired signals. Each difference map is masked 3.5 Å around the entire molecule of

bacteriorhodopsin (bR). No lipid density is analyzed.

Meta-analysis of protein structures

Structural meta-analysis based on singular value decomposition (SVD) has been

conducted in two forms. In one of them, an interatomic distance matrix is calculated

from each protein structure in a related collection. SVD of a data matrix consists of

these distance matrices enables a large-scale joint structural comparison but requires no

structural alignment (Ren, 2013a, 2013b, 2016). In the second form, SVD is performed

on a data matrix of electron densities of related protein structures (Ren, 2019; Ren et al.,

2013; Schmidt et al., 2003, 2010). Both difference electron density maps that require a

reference dataset from an isomorphous crystal form and simulated annealing omit

maps that do not require the same unit cell and space group of the crystals are possible

choices in a structural meta-analysis (Ren, 2019; Ren et al., 2013). The interatomic

distances or the electron densities that SVD is performed on are called core data. Each

distance matrix or electron density map is associated with some metadata that describe

the experimental conditions under which the core data are obtained, such as

temperature, pH, light illumination, time delay, mutation, etc. These metadata do not

enter the SVD procedure. However, they play important role in the subsequent

interpretation of the SVD result. This computational method of structural analysis takes

advantage of a mathematical, yet practical, definition of conformational space with

limited dimensionality (Ren, 2013a). Each experimentally determined structure is a

snapshot of the protein structure. A large number of such snapshots taken under a

variety of experimental conditions, the metadata, would collectively provide a survey of

the accessible conformational space of the protein structure and reveal its rection

trajectory. Such joint analytical strategy would not be effective in early years when far

fewer protein structures were determined to atomic resolution. Recent rapid growth in

protein crystallography, such as in structural genomics (Chandonia and Brenner, 2006,

2012) and in serial crystallography (Glynn and Rodriguez, 2019; Schaffer et al., 2021),

has supplied the necessarily wide sampling of protein structures for a joint analytical

strategy to come of age. The vacancies or gaps in a conformational space between well-

populated conformational clusters often correspond to less stable transient states whose

conformations are difficult to capture, if not impossible. These conformations are often

key to mechanistic understanding and could be explored by a back calculation based on

molecular distance geometry (Ren, 2013a, 2016), the chief computational algorithm in

nucleic magnetic resonance spectroscopy (NMR), and by a structure refinement based

on reconstituted dataset, a major methodological advance in this work (see below).

These structures refined to atomic resolution against reconstituted datasets may reveal

short-lived intermediate conformation hard to be captured experimentally.

Unfortunately, a protein structure refined against a reconstituted dataset currently

cannot be recognized by the Protein Data Bank (PDB). Because crystallographic

refinement of a macromolecular structure is narrowly defined as a correspondence from

one dataset to one structure. A never-observed dataset reconstituted from a collection

of experimental datasets does not match the well-established crystallographic template

of PDB; let alone a refinement of crystal structure with the NMR algorithm.

A distance matrix contains M pairwise interatomic distances of a structure in the

form of Cartesian coordinates of all observed atoms. An everyday example of distance

matrix is an intercity mileage chart appended to the road atlas. Differences in the

molecular orientation, choice of origin, and crystal lattice among all experimentally

determined structures have no contribution to the distance matrices. Due to its

symmetry, only the lower triangle is necessary. A far more intimate examination of

protein structures in PDB is a direct analysis of their electron density maps instead of

the atomic coordinates. M such (difference) electron densities, often called voxels in

computer graphics, are selected by a mask of interest. In the case of difference maps,

only the best refined protein structure in the entire collection supplies a phase set for

Fourier synthesis of electron density maps. This best structure is often the ground state

structure determined by static crystallography. Other refined atomic coordinates from

the PDB entries are not considered in the meta-analysis. That is to say, a meta-analysis

of difference electron density maps starts from the X-ray diffraction data archived in

PDB rather than the atomic coordinates interpreted from the diffraction data, which

removes any potential model bias.

Singular value decomposition of (difference) electron density maps

An electron density map, particularly a difference map as emphasized here, consists of

density values on an array of grid points within a mask of interest. All M grid points in

a three-dimensional map can be serialized into a one-dimensional sequence of density

values according to a specific protocol. It is not important what the protocol is as long

as a consistent protocol is used to serialize all maps of the same grid setting and size,

and a reverse protocol is available to erect a three-dimensional map from a sequence of

M densities. Therefore, a set of N serialized maps, also known as vectors in linear

algebra, can fill the columns of a data matrix **A** with no specific order, so that the width

of **A** is N columns, and the length is M rows. Often, $M \gg N$, thus **A** is an elongated

matrix. If a consistent protocol of serialization is used, the corresponding voxel in all N

maps occupies a single row of matrix **A**. This strict correspondence in a row of matrix

A is important. Changes of the density values in a row from one structure to another

are due to either signals, systematic errors, or noises. Although the order of columns in

matrix **A** is unimportant, needless to say, the metadata associated with each column

must remain in good bookkeeping.

SVD of the data matrix **A** results in $\mathbf{A} = \mathbf{U}\mathbf{W}\mathbf{V}^{\mathrm{T}}$, also known as matrix factorization.

Matrix **U** has the same shape as **A**, that is, N columns and M rows. The N columns

contain decomposed basis components $\boldsymbol{U}_k$, known as left singular vectors of M items,

where $k = 1, 2, \ldots, N$. Therefore, each component U_k can be erected using the reverse

protocol to form a three-dimensional map. This decomposed elemental map can be

presented in the same way as the original maps, for example, rendered in molecular

graphics software such as Coot and PyMol. It is worth noting that these decomposed

elemental maps or map components U_k are independent of any metadata. That is to

say, these components remain constant when the metadata vary. Since each left

singular vector U_k has a unit length due to the orthonormal property of SVD (see

below), that is, $|U_k| = 1$, the root mean squares (rms) of the items in a left singular

vector is $1/\sqrt{M}$ that measures the quadratic mean of the items.

The second matrix W is a square matrix that contains all zeros except for N positive

values on its major diagonal, known as singular values w_k. The magnitude of w_k is

considered as a weight or significance of its corresponding component U_k. The third

matrix V is also a square matrix of $N \times N$. Each column of V or row of its transpose V^T,

known as a right singular vector V_k, contains the relative compositions of U_k in each of

the N original maps. Therefore, each right singular vector V_k can be considered as a

function of the metadata. Right singular vectors also have the same unit length, that is,

$|V_k| = 1$. Effectively, SVD separates the constant components independent of the

metadata from the compositions that depend on the metadata.

A singular triplet denotes 1) a decomposed component U_k, 2) its singular value w_k,

and 3) the composition function V_k. Singular triplets are often sorted in a descending

order of their singular values w_k. Only a small number of n significant singular triplets

identified by the greatest singular values w_1 through w_n can be used in a linear

combination to reconstitute a set of composite maps that closely resemble the original

ones in matrix A, where $n < N$. For example, the original map in the ith column of

matrix A under a certain experimental condition can be closely represented by the ith

composite map $w_1 v_{1i} U_1 + w_2 v_{2i} U_2 + \ldots + w_n v_{ni} U_n$, where $(v_{1i}, v_{2i}, \ldots)$ is from the ith row of

matrix V. The coefficient set for the linear combination is redefined here as $c_{ki} =$

$w_k v_{ki}/\sqrt{M}$. The rms of the density values in a map component, or the average magnitude

measured by the quadratic mean, acts as a constant scale factor that resets the modified

coefficients c_{ki} back to the original scale of the core data, such as Å for distance matrices

and $e^-/Å^3$ for electron density maps if these units are used in the original matrix A.

Practically, an electron density value usually carries an arbitrary unit without a

calibration, which makes this scale factor unnecessary. In the linear combination $c_{1i}U_1$ +

$c_{2i}U_2$ + … + $c_{ni}U_n$, each component U_k is independent of the metadata while how much of

each component is required for the approximation, that is, c_{ki}, depends on the metadata.

Excluding the components after U_n in this approximation is based on an assumption

that the singular values after w_n are very small relative to those from w_1 through w_n. As

a result, the structural information evenly distributed in all N original maps is

effectively concentrated into a far fewer number of n significant components, known as

information concentration or dimension reduction. On the other hand, the trailing

components in matrix $\mathbf{U}$ contain inconsistent fluctuations and random noises.

Excluding these components effectively rejects noises (Schmidt et al., 2003). The least-

squares property of SVD guarantees that the rejected trailing components sums up to

the least squares of the discrepancies between the original core data and the

approximation using the accepted components.

However, no clear boundary is guaranteed between signals, systematic errors, and

noises. Systematic errors could be more significant than the desired signals. Therefore,

excluding some components from 1 through n is also possible. If systematic errors are

correctly identified, the reconstituted map without these significant components would

no longer carry the systematic errors.

The orthonormal property of SVD

The solution set of SVD must guarantee that the columns in $\mathbf{U}$ and $\mathbf{V}$, the left and right

singular vectors U_k and V_k, are orthonormal, that is, $U_h \bullet U_k = V_h \bullet V_k = 0$ (ortho) and $U_k \bullet U_k$

$= V_k \bullet V_k = 1$ (normal), where $h \neq k$ but both are from 1 to N. The orthonormal property

also holds for the row vectors. As a result, each component U_k is independent of the

other components. In other words, a component cannot be represented by a linear

combination of any other components. However, two physical or chemical parameters

in the metadata, such as temperature and pH, may cause different changes to a

structure. These changes are not necessarily orthogonal. They could exhibit some

correlation. Therefore, the decomposed components U_k not necessarily represent any

physically or chemically meaningful changes (see below).

Due to the orthonormal property of SVD, an N-dimensional Euclidean space is

established, and the first n dimensions define its most significant subspace. Each

coefficient set $c_i = (c_{1i}, c_{2i}, \ldots, c_{ni})$ of the ith composite map is located in this n-dimensional

subspace. All coefficient sets for $i = 1, 2, \ldots, N$ in different linear combinations to

approximate the N original maps in a least-squares sense can be represented by N

points or vectors $c_1, c_2, \ldots, c_N$ in the Euclidean subspace. This n-dimensional subspace is

essentially the conformational space as surveyed by the jointly analyzed core data. The

conformational space is presented as scatter plots with each captured structure

represented as a dot located at a position determined by the coefficient set c_i of the ith

observed map. When the subspace has greater dimensionality than two, multiple two-

dimensional orthographical projections of the subspace are presented, such as Fig. 2a.

These scatter plots are highly informative to reveal the relationship between the

(difference) electron density maps and their metadata.

If two coefficient sets $c_i \approx c_j$, they are located close to each other in the

conformational space. Therefore, these two structures i and j share two similar

conformations. Two structures located far apart from each other in the conformational

space are dissimilar in their conformations, and distinct in the compositions of the map

components. A reaction trajectory emerges in this conformational space if the temporal

order of the core data is experimentally determined (Fig. 2a). Otherwise, an order could

be assigned to these structures based on an assumed smoothness of conformational

changes along a reaction trajectory (Ren, 2013a, 2013b, 2016). Causation and

consequence of structural motions could be revealed from the order of the structures in

a series, which may further lead to structural mechanism. In addition, an off-trajectory

location in the conformational space or a location between two clusters of observed

structures represents a structure in a unique conformation that has never been

experimentally captured. Such a hypothetical structure can be refined against a

reconstituted distance matrix using molecular distance geometry (Ren, 2013a, 2013b,

2016) or a reconstituted electron density map with the method proposed below.

Rotation in SVD space

Dimension reduction is indeed effective in meta-analysis of protein structures when

many datasets are evaluated at the same time. However, the default solution set of SVD

carries complicated physical and chemical meanings that are not immediately obvious.

The interpretation of a basis component U_k, that is, "what-does-it-mean", requires a

clear demonstration of the relationship between the core data and their metadata. The

outcome of SVD does not guarantee any physical meaning in a basis component.

Therefore, SVD alone provides no direct answer to "what-does-it-mean", thus its

usefulness is very limited to merely a mathematical construction. However, the

factorized set of matrices **U**, **W**, and **V** from SVD is not a unique solution. That is to say,

they are not the only solution to factorize matrix **A**. Therefore, it is very important to

find one or more alternative solution sets that are physically meaningful to elucidate a

structural interpretation. The concept of a rotation after SVD was introduced by Henry

& Hofrichter (Henry and Hofrichter, 1992). But they suggested a protocol that fails to

preserve the orthonormal and least-squares properties of SVD. The rotation protocol

suggested by Ren incorporates the metadata into the analysis and combines with SVD

of the core data. This rotation achieves a numerical deconvolution of multiple physical

and chemical factors after a pure mathematical decomposition, and therefore, provides

a route to answer the question of "what-does-it-mean" (Ren, 2019). This rotation shall

not be confused with a rotation in the three-dimensional real space, in which a

molecular structure resides.

A rotation in the n-dimensional Euclidean subspace is necessary to change the

perspective before a clear relationship emerges to elucidate scientific findings. It is

shown below that two linear combinations are identical before and after a rotation

applied to both the basis components and their coefficients in a two-dimensional

subspace of h and k. That is,

$$c_h U_h + c_k U_k = f_h R_h + f_k R_k, \tag{1}$$

853   where c_h and c_k are the coefficients of the basis components U_h and U_k before the

854   rotation; and f_h and f_k are the coefficients of the rotated basis components R_h and R_k,

855   respectively. The same Givens rotation of an angle θ is applied to both the components

856   and their coefficients:

$$\begin{cases} R_h = U_h\cos\theta - U_k\sin\theta; \\ R_k = U_h\sin\theta + U_k\cos\theta. \end{cases} \tag{2}$$

859 Obviously, the rotated components R_h and R_k remain mutually orthonormal and
860 orthonormal to other components. And
861

$$\begin{cases} f_h = s_h t_h = c_h \cos\theta - c_k \sin\theta; \\ f_k = s_k t_k = c_h \sin\theta + c_k \cos\theta. \end{cases} \quad (3)$$

Here $s_{h|k} = \sqrt{\sum f_{h|k}^2}$ are the singular values that replace w_h and w_k, respectively, after the

rotation. They may increase or decrease compared to the original singular values so
that the descending order of the singular values no longer holds. $T_{h|k} = (t_{h|k1}, t_{h|k2}, \ldots,$
$t_{h|kN}) = (f_{h|k1}, f_{h|k2}, \ldots, f_{h|kN})/s_{h|k}$ are the right singular vectors that replace V_h and V_k,
respectively. T_h and T_k remain mutually orthonormal after the rotation and
orthonormal to other right singular vectors that are not involved in the rotation.
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870 To prove Eq. 1, Eqs. 2 and 3 are combined and expanded. All cross terms of sine and
871 cosine are self-canceled:
872

$f_h R_h + f_k R_k = (c_h \cos\theta - c_k \sin\theta)(U_h \cos\theta - U_k \sin\theta) + (c_h \sin\theta + c_k \cos\theta)(U_h \sin\theta + U_k \cos\theta)$

$\qquad = c_h U_h \cos^2\theta + c_k U_k \sin^2\theta + c_h U_h \sin^2\theta + c_k U_k \cos^2\theta \pm c_h U_k \sin\theta\cos\theta \pm c_k U_h \sin\theta\cos\theta$

$\qquad = c_h U_h(\cos^2\theta + \sin^2\theta) + c_k U_k(\sin^2\theta + \cos^2\theta)$

$\qquad = c_h U_h + c_k U_k$

A rotation in two-dimensional subspace of h and k has no effect in other dimensions,
as the orthonormal property of SVD guarantees. Multiple steps of rotations can be
carried out in many two-dimensional subspaces consecutively to achieve a multi-
dimensional rotation. A new solution set derived from a rotation retains the
orthonormal property of SVD. The rotation in the Euclidean subspace established by
SVD does not change the comparison among the core data of protein structures. Rather
it converts one solution set $\mathbf{A} = \mathbf{UWV}^T$ to other alternative solutions $\mathbf{A} = \mathbf{RST}^T$ so that
an appropriate perspective can be found to elucidate the relationship between the core
data and metadata clearly and concisely.
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For example, if one physical parameter could be reoriented along a single dimension
k but not involving other dimensions by a rotation, it can be convincingly shown that
the left singular vector U_k of this dimension illustrates the structural impact by this

physical parameter. Before this rotation, the same physical parameter may appear to

cause structural variations along several dimensions, which leads to a difficult

interpretation. Would a proper rotation establish a one-on-one correspondence from all

physical or chemical parameters to all the dimensions? It depends on whether each

parameter induces an orthogonal structural change, that is, whether structural

responses to different parameters are independent or correlated among one another. If

structural changes are indeed orthogonal, it should be possible to find a proper rotation

to cleanly separate them in different dimensions. Otherwise, two different rotations are

necessary to isolate two correlated responses, but one at a time.

For another example, if the observed core datasets form two clusters in the

conformational space, a rotation would be desirable to separate these clusters along a

single dimension k but to align these clusters along other dimensions. Therefore, the

component U_k is clearly due to the structural transition from one cluster to the other.

Without a proper rotation, the difference between these clusters could be complicated

with multiple dimensions involved. A deterministic solution depends on whether a

clear correlation exists between the core data and metadata. A proper rotation may

require a user decision. A wrong choice of rotation may select a viewpoint that hinders

a concise conclusion. However, it would not alter the shape of the reaction trajectory,

nor create or eliminate an intrinsic structural feature. A wrong choice of rotation cannot

eliminate the fact that a large gap exists between two clusters of observed core datasets

except that these clusters are not obvious from that viewpoint. A different rotation may

reorient the perspective along another direction. But the structural conclusion would be

equivalent. See example of before and after a rotation in (Ren, 2016).

This rotation procedure finally connects the core crystallographic datasets to the

metadata of experimental conditions and accomplishes the deconvolution of physical or

chemical factors that are not always orthogonal to one another after a mathematical

decomposition. SVD analysis presented in this paper employs rotations extensively

except that no distinction is made in the symbols of components and coefficients before

and after a rotation except in this section. This method is widely applicable in large-

scale structural comparisons. Furthermore, Ren rotation after SVD is not limited to

crystallography and may impact other fields wherever SVD is used. For example, SVD

is frequently applied to spectroscopic data, images, and genetic sequence data.

Structural refinement against reconstituted dataset

The linear combination $\Delta\rho(t) = f_1(t)\mathbf{R}_1 + f_2(t)\mathbf{R}_2 + \ldots + f_n(t)\mathbf{R}_n$ after a rotation reconstitutes

one of the observed difference maps at a specific time point t. This time-dependent

difference map depicts an ever-evolving mixture of many excited species. A

reconstituted difference map $\Delta\rho(E)$ for a time-independent, pure, excited species E =

intermediate I′, I, J′, and J deconvoluted from many mixtures would take the same form

except that only one or very few coefficients remain nonzero if a proper rotation has

been found (Table S2). In order to take advantage of the mature refinement software for

macromolecular structures with extensive stereochemical restraints, a set of structure

factor amplitudes is needed. Therefore, it is necessary to reconstitute a set of structure

factor amplitudes that would produce the target difference map $\Delta\rho(E)$ based on a

known structure at the ground state. First, an electron density map of the structure at

the ground state is calculated. This calculated map is used as a base map. Second, this

base map of the ground state is combined with the positive and negative densities in the

target difference map $\Delta\rho(E)$ so that the electron densities at the ground state are skewed

toward the intermediate state. Third, structure factors are calculated from the

combined map. Finally, the phase set of the calculated structure factors is discarded,

and the amplitudes are used to refine a single conformation of the intermediate species

E that $\Delta\rho(E)$ represents.

This protocol following the SVD and Ren rotation of components achieves a

refinement of a pure structural species without the need of alternative conformations.

Several points are noteworthy. First, the minimization protocol in this refinement is

performed against a numerically reconstituted amplitude set that has never been

directly measured from a crystal. This reconstituted dataset could be considered as an

extrapolated dataset "on steroids" if compared to the traditional extrapolation of small

differences, such as, the Fourier coefficient set to calculate a 3Fo-2Fc map, a technique

often used to overcome a partial occupancy of an intermediate structure. An

extrapolation of small differences is not directly observed either but computed by an

exaggeration of the observed difference based on an assumption that the intermediate

state is partially occupied, such as the doubling of the observed difference in 3Fo-2Fc =

Fo + 2(Fo-Fc). In contrast to the conventional technique of extrapolation, the

deconvolution method applied here is an interpolation among many experimental

datasets rather than an extrapolation. Secondly, the deconvolution is a simultaneous

solution of multiple intermediate states mixed together instead of solving a single

excited state.

Second, a map calculated from the ground state structure is chosen as the base map

instead of an experimental map such as Fo or 2Fo-Fc map. If the second step of the

protocol is skipped, that is, no difference map is combined with the ground state map,

the refinement would result in an R factor of nearly zero, since the refinement is

essentially against the calculated structure factors (bR in Table S2). This is to say, the

residuals of the refinement are solely due to the difference component instead of the

base map. This is desirable since errors in the static structure of the ground state are

gauged during its own refinement. On the other hand, if an experimental map is

chosen as a base map, the refinement R factors would reflect errors in both the base map

and the difference map, which leads to a difficulty in an objective evaluation of this

refinement protocol.

Third, the combination of the base map and a difference map is intended to

represent a pure intermediate species. Therefore, alternative conformations in

structural refinement that model a mixture of species would defeat this purpose.

However, this combined map could be very noisy and may not represent a single

species without a proper rotation. This is particular the case, if the target difference

map $\Delta\rho$ is not derived from an SVD analysis and Ren rotation. The SVD analysis

identifies many density components that are inconsistent among all observed difference

maps and excludes them, which greatly reduces the noise content. Therefore, this

refinement protocol may not be very successful without an SVD analysis. Another

source of noise originates from the phase set of the structure factors. Prior to the

refinement of the intermediate structure, the phase set remains identical to that of the

ground state. This is far from the reality when an intermediate structure involves

widespread changes, such as those refined in this study. If the rotation after SVD is not

properly selected, the target difference map would remain as a mixture minus the

ground state. Therefore, the refinement of a single conformation would encounter

difficulty or significant residuals, as judged by the R factors, the residual map, and the

refined structure. A proper solution to this problem is a better SVD solution by Ren

rotation rather than alternative conformations. A successful refinement of near perfect

993 *trans* or *cis* double bonds is a good sign to indicate that the reconstituted amplitude set

994 after a rotation reflects a relatively homogeneous structure. If a double bond could not

995 be refined well to near perfect *trans* or *cis* configuration, the dataset of structure factor

996 amplitudes is likely from a mixture of heterogeneous configurations, which occurred

997 frequently in previous studies of bR and photoactive yellow protein (Jung et al., 2013;

998 Lanyi and Schobert, 2007; Nogly et al., 2018). It has been a great difficulty in

999 crystallographic refinement in general that a heterogeneous mixture of conformations

1000 cannot be unambiguously refined even with alternative conformations. This difficulty

1001 becomes more severe when a mixture involves more than two conformations or when

1002 some conformations are very minor.

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1004 Lastly, the refinement protocol proposed here could be carried out in the original

1005 unit cell and space group of the crystal at the ground state. However, this is not always

1006 applicable as the original goal of the meta-analysis is a joint examination of all available

1007 structures from a variety of crystal forms. It would be highly desirable to evaluate

1008 difference maps of the same or similar proteins from non-isomorphous crystals together

1009 by SVD. Alternatively, the refinement protocol could also be performed in the space

1010 group of P1 with a virtual unit cell large enough to hold the structure, which is the

1011 option in this study (Table S2). This is to say, the entire analysis of SVD-rotation-

1012 refinement presented here could be extracted and isolated from the original crystal

1013 lattices, which paves the way to future applications to structural data acquired by

1014 experimental techniques beyond crystallography, most attractively, to single particle

1015 reconstruction in cryo electron microscopy.

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1018 **Supplementary Tables**

Table S1. Datasets analyzed in this work

Publication	PDB	Label	Resolution	Main conclusions	New findings in this work
Nogly et al. Science 361, eaat0094, 2018	6g7h	dark6	1.5 Å	Retinal fully isomerizes by 10 ps. But the SB water dissociates earlier.	The short-delay datasets contribute to the structures of I′ → I → J′ → J. Photoisomerization in J′; retinal binding pocket expansion before 1 ps in I and contraction at 10 ps in J
	6g7i	49-406fs	1.9 Å		
	6g7j	457-646fs	1.9 Å		
	6g7k	10ps	1.9 Å		
	6g7l	8.33ms	1.9 Å		
Kovacs et al. Nat. Commun. 10, 3177, 2019	6ga1	dark1	1.7 Å	The exceedingly high power density of the pump laser causes two-photon absorption. Vibrational motions were observed.	The sub-ps datasets exhibit extensive vibrations at various frequencies. The vibrational signals are widespread over the entire bR molecule and not associated with any structural elements. Therefore, it is concluded that these global vibrations are intrinsic properties of bR induced by short laser pulses. The vibrational signals are more prominent under higher power density of the laser pulses. However, these vibrations are irrelevant to the light-driven proton pumping function of bR.
	6ga2	dark2	1.8 Å		
	6rmk	dark3	1.8 Å		
	6ga7	240fs	1.8 Å		
	6ga8	330fs	1.8 Å		
	6ga9	390fs	1.8 Å		
	6gaa	430fs	1.8 Å		
	6gab	460fs	1.8 Å		
	6gac	490fs	1.8 Å		
	6gad	530fs	1.8 Å		
	6gae	560fs	1.8 Å		
	6gaf	590fs	1.8 Å		
	6gag	630fs	1.8 Å		
	6gah	680fs	1.8 Å		
	6gai	740fs	1.8 Å		
	6ga4	1ps	1.8 Å		
	6ga5	3ps	1.9 Å		
	6ga6	10ps	1.8 Å		
	6ga3	33ms	2.1 Å		

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Table S2. Refinement statistics

Intermediate	bR	I′	I	J′	J
Time period	0-	< 50 fs	40-700 fs	0.5-2 ps	1-30 ps
Coefficient c_{10}	0	0	3,300	0	-4,200
Coefficient c_{14}	0	2,000	2,700	2,700	2,000
c_{17}	0	3,000	0	-1,300	-300
Starting model	PDB 6g7h				
Resolution range	50-2.1 Å				
Space group	P1				
Unit cell	$a = b = 62.32$ Å; $c = 111.10$ Å; $\alpha = \beta = 90°$; and $\gamma = 120°$				
Unique reflections	80,354 in working set + 4,236 in test set = 84,590 total				
Completeness	95% in working set + 5% in test set = 100% reconstituted				
R (%)	1.8	29.4	31.0	29.1	30.0
R_{free} (%)	1.9	31.1	32.4	30.4	30.7
Refined content	230 protein residues + retinal + water molecules				
Number of atoms	1,798	1,795	1,798	1,796	1,795
Water molecules	8	5	8	6	5
RMSD bonds (Å)	0.005	0.009	0.009	0.009	0.009
RMSD angles (°)	0.793	1.206	1.105	1.085	1.068
Rama. favored (%)	98.7	96.5	95.6	96.1	96.5
Rama. outliers (%)	0.0	0.0	0.4	0.4	0.4
Clash score	4	9	5	4	6

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1023 **Supplementary Figures and Legends**

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Figure S1. Orientations in bacteriorhodopsin. (a) Bacteriorhodopsin (bR) trimer viewed

from the extracellular (EC) side along the three-fold axis. (b) An orthographical view to

(a) looking from the outside of the trimer. (c and d) Two orthographical views of the

retinal chromophore looking along the three-fold and normal to the three-fold axis. The

plane of retinal is largely parallel to the three-fold axis. Therefore, two sides of the

plane are called inboard and outboard with respect to the three-fold axis. The direction

toward the anchor Lys216 is called proximal. The β-ionone ring direction is therefore

distal. (e) Chemical structure of retinal incorporated to its anchor Lys216. The atom

numbers and various segment names are marked.

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Figure S2. SVD applied to difference Fourier maps. Difference Fourier maps at the

short delays $t \leq 10$ ps are decomposed into component maps. Each difference map at a

time delay t can be represented by a linear combination of these components, $c_1(t)\boldsymbol{U}_1 +$

$c_2(t)\boldsymbol{U}_2 + ...,$ where $\boldsymbol{U}_k$ are the time-independent components and $c_k(t)$ are their

corresponding time-dependent coefficients (Methods). (a and b) Two example plots

show circular correlations between c_1 and c_2, c_9 and c_{11}. These circular correlations

1043 indicate two-dimensional oscillations. Each colored trace represents difference maps in

1044 a time series calculated with a common reference. Those time series with a dark

1045 reference are plotted with thick lines. Other series are in thin lines. (c) Singular values

1046 before and after Ren rotation (Ren, 2016, 2019) (Methods). Singular values derived from

1047 SVD indicate the significance of the components. 17 of them stand out.

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Ren: Isomerization sampling

Figure S3. Oscillations of SVD components. The coefficients of ten components c_1, c_2; c_4,

c_7; c_{15}, c_{16}; c_9, c_{11}; and c_{12}, c_{13} are found oscillating at frequencies ranging from 60 to 400

cm^{-1}. Each pair of the coefficients oscillate at a common frequency. These frequencies

are 61±2, 150±3, 224±7, 334±8, and 396±3 cm^{-1}, respectively. These coefficients are

plotted against the time delay t (left) and against each other in a pair (right). Each

coefficient is fitted with a sine function around a straight baseline $c_k = a \sin\left(\frac{2\pi t}{T} + \varphi\right) +$

$b + ct$. Both the fitted function and the baseline are plotted. The amplitude a for the

last pair of coefficients c_{12} and c_{13} are replaced with a Gaussian function $a -$

$\exp\left(-\frac{(t-t_0)^2}{\tau^2}\right)$ to implement a node at $t_0 = 509\pm5$ fs (e).

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a. U_1 b. U_2 c. U_4 d. U_7 e. U_9

f. U_{11} g. U_{12} h. U_{13} i. U_{15} j. U_{16}

Figure S4. No structural signal in oscillating components. Ten oscillating components are contoured at $\pm 3\sigma$ in green and red, respectively. The main chain and side chains of the protein are rendered with ribbon and sticks, respectively. The retinal and Lys216 are in purple sticks. Several key waters are in blue spheres. Parts of the structure are omitted to reveal more of the interior. Despite that the time-dependent coefficients to these components contain strong oscillatory signals (Figs. S2 and S3), these components themselves display no obvious association with any structural features such as the retinal or secondary structures. They are in stark contrast to the signal distributions of the non-oscillating components (Figs. 1ab, 2b, and S5).

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Figure S5. Two orthographical views of component map U_{14}. The main chain and side
chains of the protein are rendered with ribbons and sticks, respectively. The retinal and
Lys216 are in purple sticks. Several key waters are in blue spheres. Parts of the
structure are omitted to reveal more of the interior. The map is contoured at $\pm 3\sigma$ in
green and red, respectively. The signals are largely associated with the chromophore
and its immediate vicinity.
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Figure S6. Two orthographical views of the 2Fo-Fc map of I′ contoured at 3.5σ. Here Fo is the reconstituted structure factor amplitudes rather than observed amplitudes (Table S2). Fc is the structure factor amplitudes calculated from the refine structure (Methods).

1087   The same protocol applies to the Fourier synthesis of 2Fo-Fc maps of other

1088   intermediates (Figs. S7, S9, and S10).

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1093 Figure S7. Two orthographical views of the 2Fo-Fc map of I contoured at 3σ. Here Fo is
1094 the reconstituted structure factor amplitudes rather than observed amplitudes (Table
1095 S2). Fc is the structure factor amplitudes calculated from the refine structure (Methods).
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a

6g7j – 6g7h

b

Helix B   Helix C   Helix G

Ren: Isomerization sampling

Figure S8. Raw difference Fourier map at 457-646 fs. This difference Fourier map is

calculated from the dataset 6g7j at the time point of 457-646 fs by subtracting the dark

dataset 6g7h. The map is contoured at $\pm 3\sigma$ in green and red, respectively. This map is

prior to SVD analysis. Compared with $\boldsymbol{U}_{10}$ (Fig. 2b) and the reconstituted map (Fig. 3a),

it is clear that this is the original source of the widespread signals except that the σ

value of this map is higher than those after SVD. (a) The raw difference map contoured

in the entire molecule shows the association of the signals with the structural elements

at an excellent signal-to-noise ratio. (b) Details of the raw difference map show

displacements of helices. The raw difference map is largely the same as the

reconstituted map (Fig. 3a).

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1112   Figure S9. Two orthographical views of the 2Fo-Fc map of J′ contoured at 4σ. Here Fo
1113   is the reconstituted structure factor amplitudes rather than observed amplitudes (Table
1114   S2). Fc is the structure factor amplitudes calculated from the refine structure (Methods).
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Figure S10. Two orthographical views of the 2Fo-Fc map of J contoured at 5σ. Here Fo is the reconstituted structure factor amplitudes rather than observed amplitudes (Table S2). Fc is the structure factor amplitudes calculated from the refine structure (Methods).