

Connectome-Based Smoothing (CBS): Concepts, methods, and evaluation

Sina Mansour L.^{1,*}, Caio Seguin^{2,3}, Robert E. Smith^{4,5}, and Andrew Zalesky^{1,2}

¹Department of Biomedical Engineering, The University of Melbourne, Parkville, Victoria, Australia

²Melbourne Neuropsychiatry Centre, Department of Psychiatry, The University of Melbourne and Melbourne Health, Parkville, Victoria, Australia

³The University of Sydney, School of Biomedical Engineering, Sydney, Australia

⁴The Florey Institute of Neuroscience and Mental Health, Heidelberg, Victoria, Australia

⁵Florey Department of Neuroscience and Mental Health, The University of Melbourne, Parkville, Victoria, Australia

*To whom correspondence should be addressed. Email: sina.mansour.lakouraj@gmail.com

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Abstract

Structural connectomes are increasingly mapped at high spatial resolutions comprising many hundreds—if not thousands—of network nodes. However, high-resolution connectomes are particularly susceptible to image registration misalignment, tractography artifacts, and noise, all of which can lead to reductions in connectome accuracy and test-retest reliability. We investigate a network analogue of image smoothing to address these key challenges. Connectome-Based Smoothing (CBS) involves jointly applying a carefully chosen smoothing kernel to the two endpoints of each tractography streamline, yielding a spatially smoothed connectivity matrix. We develop computationally efficient methods to perform CBS using a matrix congruence transformation and evaluate a range of different smoothing kernel choices on CBS performance. We find that smoothing substantially improves the identifiability, sensitivity and test-retest reliability of high-resolution connectivity maps, though at a cost of increasing storage burden. For atlas-based connectomes (i.e. low-resolution connectivity maps), we show that CBS marginally improves the statistical power to detect associations between connectivity and cognitive performance, particularly for connectomes mapped using probabilistic tractography. CBS was also found to enable more reliable statistical inference compared to connectomes without any smoothing. We provide recommendations on optimal smoothing kernel parameters for connectomes mapped using both deterministic and probabilistic tractography. We conclude that spatial smoothing is particularly important for the reliability of high-resolution connectomes, but can also provide benefits at lower parcellation resolutions. We hope that our work enables computationally efficient integration of spatial smoothing into established structural connectome mapping pipelines.

Highlights:

- We establish a network equivalent of image smoothing for structural connectomes.
- Connectome-Based Smoothing (CBS) improves connectome test-retest reliability, identifiability and sensitivity.
- CBS also facilitates reliable inference and improves power to detect statistical associations.
- Both high-resolution and atlas-based connectomes can benefit from CBS.

Keywords: Structural connectivity | Connectome smoothing | High-resolution connectomics | Tractography

1. Introduction

Spatial smoothing is widely recognized as a crucial preprocessing step in many neuroimaging pipelines. It can increase the signal-to-noise ratio (SNR) by eliminating the high-frequency spatial components of noise [1–5] and is typically used in different neuroimaging modalities such as structural mag-

netic resonance imaging (MRI) [6–8], functional MRI [9–13], positron emission tomography (PET) [14–17], magnetoencephalography (MEG) [18, 19], electroencephalography (EEG) [20], and functional near-infrared spectroscopy (fNIRS) [21, 22]. As a result, options for spatial smoothing are provided in many neuroimaging toolboxes, such as AFNI [23], FreeSurfer [24], FSL [25], and SPM [26](Friston et

al., 1994).

Structural connectivity computed from diffusion MRI tractography can be used to construct structural connectomes [27–29], and there is considerable interest in performing statistical inference on this graph representation of the brain [30, 31]. However, unlike image-based statistical inference, such data are currently not explicitly smoothed. Most structural connectomes are studied at the resolution of large-scale brain atlases comprising tens to hundreds of regions. The process of assigning tractography streamlines to such large-scale regions manipulates the data in a manner somewhat akin to smoothing. Nonetheless, the potential impact of additional (explicit) smoothing has not yet been evaluated. Moreover, given that connectomes are spatially embedded graphs, conventional univariate smoothing methods are not directly applicable to connectomes, and so smoothing methods tailored to connectome data are required.

High-resolution connectomes are a subset of connectomes that investigate the connectivity structure of the brain at the resolution of cortical vertices/voxels [32]. Recent studies highlight the advantages of investigating structural connectomes at this higher spatial resolution than atlases with coarse parcellations [32–37]. For example, high-resolution structural connectivity maps robustly capture intricate local modular structures in brain networks and provide insightful connectome biomarkers of neural abnormalities [35, 36, 38]. We recently established a computationally efficient framework to map high-resolution structural connectomes, and found that these connectomes enabled accurate prediction of individual behaviors and neural fingerprinting [32]. As part of this recent work, we implemented a preliminary method for connectome-based smoothing, building on earlier structural connectome smoothing approaches [33].

In this study, we extend our earlier work by formalizing the principles of connectome-based smoothing (CBS), aiming to develop efficient computational methods to facilitate connectome smoothing and determine optimal smoothing parameters. We investigate the impact of smoothing on high-resolution and atlas-based connectomes, quantifying its benefits for reliability, identifiability and statistical power. We anticipate that CBS will become a common step in connectome mapping workflows.

2. Materials and methods

2.1. Connectome-based smoothing

Here, we develop an efficient and scalable method to enable network-based smoothing of spatially-embedded high-resolution connectivity matrices. Unlike conventional spatial smoothing algorithms that are defined in terms of a single smoothing kernel, CBS is inherently bivariate and involves a pair of spatially distant smoothing kernels operating at the two ends of each connection. The framework developed

here extends our recent work on high-resolution connectomes, where we first investigated the concept of connectome-based smoothing [32]. We also acknowledge the seminal work of Besson and colleagues, who found that connectome smoothing improved the reliability of high-resolution connectomes [33].

We use A to denote the symmetric connectivity matrix inferred from tractography, with size $v \times v$ where v is the total number of network nodes and element $A(i, j)$ stores the streamline count between nodes v_i and v_j . This matrix can be decomposed into two half-incidence matrices U and V , each of size $v \times n$, where n is the total number of streamlines. These matrices encode the connectivity endpoint information, such that the streamline endpoint pairs are mapped to the columns of U and V . For instance, if the k th streamline ends in nodes v_i and v_j , then the k th columns of U and V are vectors with a single non-zero element, with weight 1, respectively located at $U(i, k)$ and $V(j, k)$. This signifies that streamline k connects the endpoints v_i and v_j . Fig. 1A–C demonstrates the decomposition of streamlines encoded in a connectivity matrix and the half-incidence matrix representations. Mathematically, the symmetric connectivity matrix is decomposed as follows:

$$A = UV^T + VU^T \quad (1)$$

Since the columns of the half-incidence matrices each represent a single endpoint associated with a spatial coordinate, a conventional spatial smoothing kernel can be applied to those columns, resulting in a pair of smoothed half-incidence matrices U_s and V_s . As previously derived [32], a smoothed connectivity matrix can be constructed by combining the smoothed half-incidence matrices as follows:

$$A_s = U_s V_s^T + V_s U_s^T \quad (2)$$

Here, we propose a simplification of this formulation, which leads to improved computational tractability. Let F_s denote a spatial smoothing kernel of size $v \times v$, such that column i of F_s stores the weights for a smoothing kernel spatially centered at the i th node of the network. This smoothing kernel can be used to compute the smoothed half-incidences:

$$\begin{aligned} U_s &= F_s U \\ V_s &= F_s V \end{aligned} \quad (3)$$

Smoothing can thus be represented as a linear transformation of each half-incidence matrix. Under this simplification, CBS reduces to a matrix congruence between the smoothed and initial connectivity matrices, which can be efficiently computed without using half-incidence matrices. Specifically, we have that:

$$\begin{aligned} A_s &= U_s V_s^T + V_s U_s^T \\ &= F_s U V^T F_s^T + F_s V U^T F_s^T \\ &= F_s A F_s^T \end{aligned} \quad (4)$$

120 This equation shows that the smoothed connectivity
 121 A_s is a congruent transformation of the initial connectivity
 122 matrix A . Fig. 1D-F illustrates a simple example
 123 of this transformation. This simplification improves
 124 the computational feasibility since performing
 125 CBS is no longer dependent on the number of stream-
 126 lines n which is typically greater than the number of
 127 non-zero connectome edges. The precise derivation
 128 of smoothing kernel matrix F_s is presented later in
 129 Section 2.6. **Smoothing parameters.**

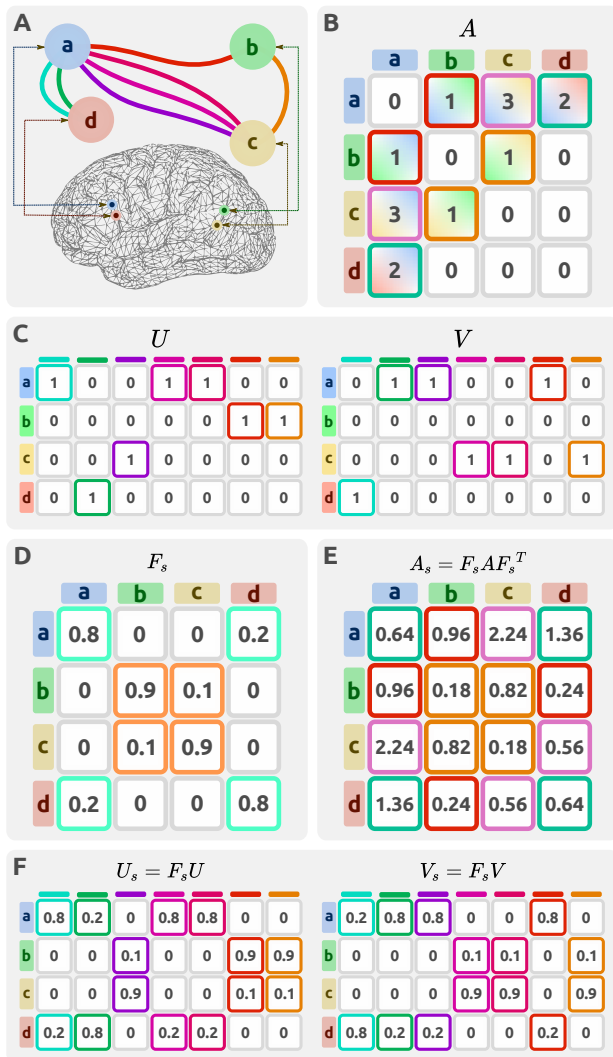


Fig. 1. Illustrative example demonstrating the decomposition of streamlines into connectivity and incidence matrices. (A) A hypothetical network in which 7 streamlines connect four brain regions/nodes. The nodes (a,d) and (b,c) are selected to be spatially proximal. (B) Matrix A encodes the network in a 4x4 connectivity matrix. (C) The network can be alternatively represented by two half-incidence matrices U and V . (D) A connectome smoothing kernel F_s can be defined based on the pairwise geodesic distances between nodes. (E,F) The network representations can be spatially smoothed using CBS to produce a smoothed connectivity matrix A_s or a pair of smoothed half incidence matrices U_s, V_s .

130 2.2. Study design

131 We investigated the impact of CBS on connectomes
 132 mapped at different node resolutions. As detailed
 133 below, high-resolution (60k nodes) and atlas-based
 134 (300 nodes) connectomes were mapped for individu-
 135 als using diffusion MRI and established whole-brain

136 tractography methods. Data from two diffusion MRI
 137 acquisitions for each individual were used, enabling
 138 evaluation of test-retest reliability and identifiability
 139 across different smoothing parameters. Fig. 2
 140 provides a brief overview of the study design. We next
 141 describe the diffusion MRI acquisition, whole-brain
 142 tractography and connectome mapping procedures,
 143 smoothing parameters, and the evaluation methodol-
 144 ogy used in this study.

145 2.3. Imaging data acquisition and pre- 146 processing

147 Imaging data were sourced from the Human Connec-
 148 tome Project (HCP) [39, 40]. We obtained the diffu-
 149 sion and structural MRI images from the 42 healthy
 150 young adults comprising the HCP test-retest cohort.
 151 For these individuals, two separate imaging sessions
 152 were conducted across two different days, with the
 153 intervening period between the test and retest scans
 154 ranging from 18 to 343 days. These duplicate individ-
 155 ual scans enabled the assessment of both intra- and
 156 inter-individual variations in the mapped connectiv-
 157 ity information. Diffusion MRI data were acquired
 158 using a 2D spin-echo single-shot multiband EPI se-
 159 quence with a multi-band factor of 3 and monopolar
 160 diffusion sensitization. The diffusion data consisted
 161 of three shells (b-values: 1000, 2000, 3000 s/mm^2)
 162 and 270 diffusion directions equally distributed within
 163 the shells, and 18 b=0 volumes, with an isotropic
 164 spatial resolution of 1.25mm [41]. We analyzed
 165 preprocessed diffusion data, where preprocessing
 166 was completed by the HCP team, using an estab-
 167 lished minimal preprocessing pipeline (v3.19.0).
 168 This included b=0 intensity normalization across
 169 scanning sessions, EPI and eddy-current-induced
 170 distortion corrections, motion correction, gradient
 171 nonlinearity correction, registration to native
 172 structural space, and masking the final data with
 a brain mask [42].

173 2.4. Connectome resolution

174 We mapped both high-resolution and atlas-based
 175 connectomes to evaluate the impact of CBS on differ-
 176 ent parcellation granularities. All high-resolution
 177 connectomes were mapped on the fsLR-32k standard
 178 surface mesh, comprising 32,492 vertices on each
 179 hemisphere [43]. This space is recommended for
 180 high-resolution cross-subject studies of diffusion
 181 MRI as it provides an accurate representation of
 182 the cortical surface with fewer vertices than the
 183 native mesh [42]. The combined left and right
 184 cortical surfaces consisted of 59,412 vertices after
 185 exclusion of the medial wall. The high-resolution
 186 maps were downsampled to a lower spatial resolu-
 187 tion defined by the HCP-MMP1.0 atlas comprising
 188 360 cortical regions [44]. The downsampling
 189 procedure is detailed in the Section 2.7. **CBS
 190 for atlas-based connectivity.** In brief, the high-
 191 resolution connectivity matrix was aggregated
 192 across all vertices belonging to each atlas region
 such that the connectivity weight between two

193 atlas nodes was equal to the sum of the connectivity
 194 weights over all high-resolution vertices connecting
 195 those atlas nodes. The subcortex was not included in
 196 either high-resolution or atlas-based connectomes.

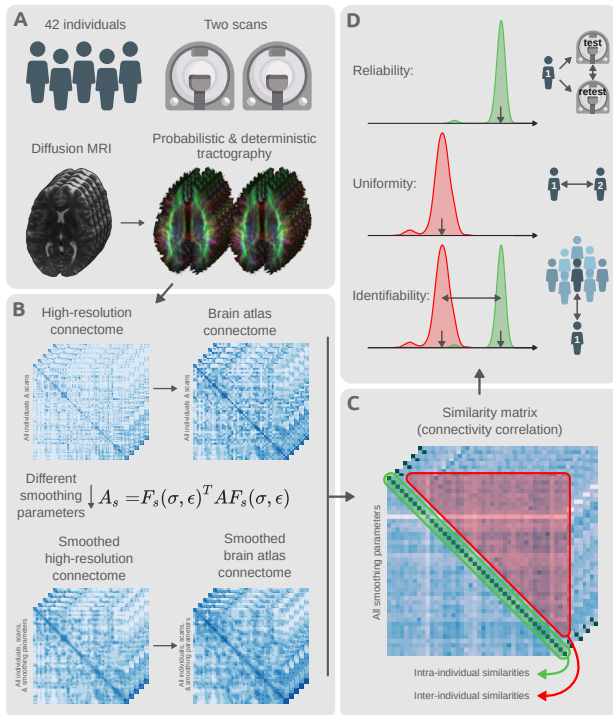


Fig. 2. Schema of study design and methodology. (A) Test-retest diffusion MRI scans of 42 individuals were sourced from the Human Connectome Project. This provided a duplicate scan of every individual. Probabilistic and deterministic tractography were utilized to estimate whole-brain white matter fiber trajectories for all individuals and scans. (B) Tractography results were used to map unsmoothed structural connectomes using the high-resolution fsLR-32k surface mesh. Different smoothing parameters were used to transform the unsmoothed high-resolution connectomes into various CBS smoothed alternatives. All variants of smoothed and unsmoothed connectomes were also downsampled to connectivity maps on the HCP-MMP1.0 brain atlas comprising 360 cortical regions [44]. (C) All mapped connectomes were used to evaluate the level of similarity between connectivity maps of different scans (test and retest) for each combination of parcellation resolution and set of smoothing parameters. Both intra- and inter-individual similarities were computed for all pairs of scans. (D) The computed similarities were used to evaluate the level of connectome reliability, uniformity, and identifiability: reliability quantifies the average similarity of connectomes belonging to scans of the same individual; uniformity quantifies the average conformity of connectomes belonging to different individuals; identifiability measures the extent to which scans of the same individuals are differentiable from the rest of the group.

197 2.5. Tractography & connectome construction procedure

198
 199 The impact of CBS was evaluated on both probabilistic
 200 and deterministic tractography algorithms. MR-
 201 trix3 software was used to perform tractography [45].
 202 An unsupervised method was used to estimate the
 203 white-matter (WM), grey-matter (GM), and cerebrospinal
 204 fluid (CSF) response functions [46] for spherical
 205 deconvolution [47]. The fiber orientation distribution
 206 (FOD) in each voxel was estimated using a Multi-
 207 Shell, Multi-Tissue Constrained (MSMT) spherical
 208 deconvolution, which improves tractography at tissue
 209 interfaces [48]. The fsLR-32k surface mesh was
 210 used to generate a binary voxel mask at the interface

211 between WM and cortical GM, from within which
 212 tractography streamlines were uniformly seeded at
 213 random coordinates from within this ribbon. Probabilistic
 214 tractography was performed by 2nd-order integration
 215 over fiber orientation distributions (iFOD2) [49].
 216 Deterministic tractography was performed using a
 217 deterministic algorithm that utilized the estimated
 218 FOD with a Newton optimization approach to locate
 219 the orientation of the nearest FOD amplitude peak
 220 from the streamline tangent orientation (“SD_Stream”) [50].
 221 Five million streamlines were generated for each
 222 tractography method for each scan.

223
 224 A streamlines propagation mask was generated using
 225 the intersection of voxels with non-zero white matter
 226 partial volume as estimated by FSL FAST [51] and
 227 voxels with non-zero sub-cortical grey matter volume
 228 as estimated by FSL FIRST [52]. The sub-cortical
 229 GM was included in the propagation mask to preserve
 230 long streamlines relaying through the sub-cortex,
 231 only terminating streamlines at the boundaries of
 232 cortical GM or CSF. The streamline endpoints were
 233 then mapped to the closest vertex of the individual’s
 234 WM surface mesh (fsLR-32k). Streamlines ending
 235 far from the cortical vertices ($>2\text{mm}$) were
 236 discarded. The remaining streamlines were used to
 237 generate a $59,412 \times 59,412$ high-resolution connectivity
 238 matrix for each of the two sessions for each individual.
 239 These data form the input for evaluation of CBS as
 240 described in the following sub-sections.

241 2.6. Smoothing parameters

242 The matrix of spatial smoothing kernels, F_s , determines
 243 the spatial distribution of smoothing weights. We use
 244 a Gaussian function to define kernel weights, $G(\delta)$,
 245 as a function of distance from the kernel center, δ ,
 246 as given by:

$$G(\delta) = \frac{1}{(\sqrt{2\pi}\sigma)^k} e^{-\frac{\delta^2}{2\sigma^2}} \quad (5)$$

247 Where k is the dimension of the spatial kernel. The
 248 parameter σ is the standard deviation of the Gaussian
 249 distribution which determines the strength of
 250 smoothing. In this study, smoothing was applied to
 251 the cortical surface mesh ($k = 2$) and was quantified
 252 by the geodesic distance over the surface mesh. Despite
 253 each subject possessing the same set of vertices, the
 254 smoothing kernel was computed separately for each
 255 scan, based on the precise inter-vertex geodesic
 256 distances on the white-matter surface mesh of each
 257 individual scan.

258 To compare the impact of different kernel standard
 259 deviations, smoothing kernels were computed with 1,
 260 2, 3, 4, 6, 8, and 10mm FWHM (full width at half
 261 maximum) ($\text{FWHM} = \sigma\sqrt{8\ln 2}$).

262 A second parameter that can impact smoothing is
 263 *truncation* of the kernel. As the Gaussian distribution
 264 decays exponentially with distance, the kernel is
 265 effectively zero for sufficiently large distances, and
 266 so contributions can be ignored with minimal loss of

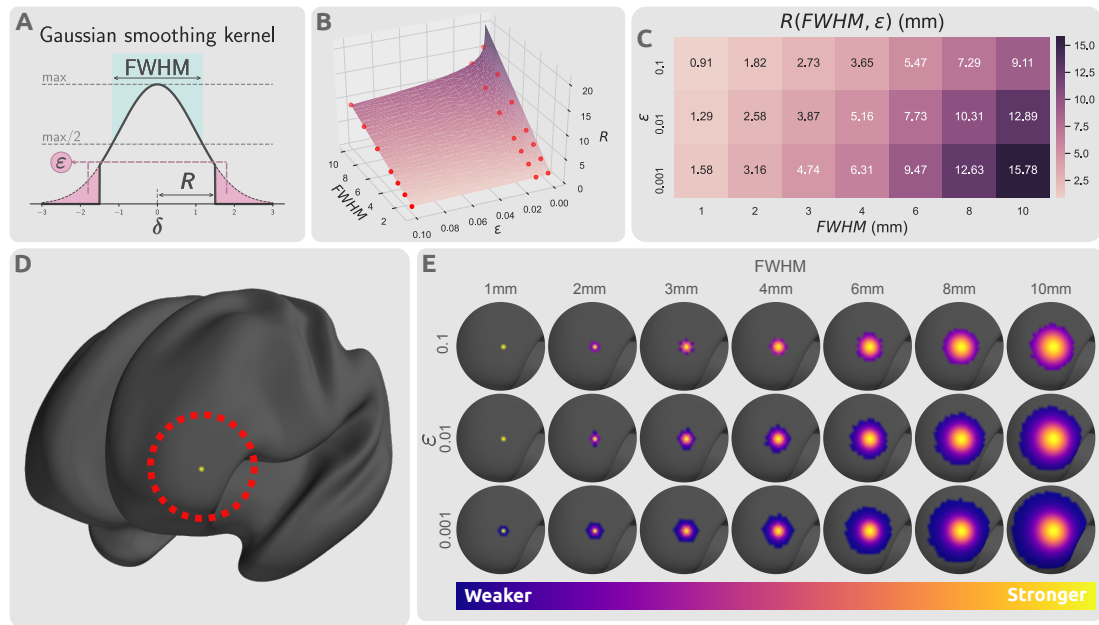


Fig. 3. Impact of kernel parameters on truncated kernels. (A) Distribution of a truncated Gaussian kernel with smoothing parameters FWHM, ϵ , and $R(\text{FWHM}, \epsilon)$. FWHM determines the standard deviation of the Gaussian kernel, ϵ dictates the proportion of the kernel that is truncated, and R determines the threshold radius beyond which the kernel is set to zero. The thick black line represents the smoothing kernel as a function of the smoothing parameters. (B) Smoothing parameter space of FWHM, ϵ , and R . The parameter space plane shows the value of smoothing kernel radius R as a function of kernel standard deviation, FWHM, and truncation threshold, ϵ . The radius is linearly related with FWHM, but log-linearly with the inverse of ϵ . The red points indicate the selected smoothing parameters from the parameter space that were used in this study. (C) The values of kernel truncation radius at the respective smoothing parameters selected for FWHM and ϵ . (D) A sample cortical vertex in the left frontal lobe of an inflated cortical mesh. (E) The respective column of the smoothing kernel F_s for the vertex shown in panel (D) with different smoothing parameter choices projected on the cortical surface.

267 precision. Truncation results in a sparse smoothing
 268 kernel, enabling computationally efficient smoothing
 269 of high-resolution connectomes. Here we studied the
 270 effect of the truncation threshold, ϵ , which is defined
 271 as the fraction of the kernel integral discarded as a re-
 272 sult of kernel truncation (Fig. 3A): for each value of
 273 FWHM, we generated three kernels for assessment,
 274 corresponding to $\epsilon = \{0.1, 0.01, 0.001\}$. This trun-
 275 cation can alternatively be expressed as a kernel ra-
 276 dius R (which has benefits both conceptually and pro-
 277 grammatically):

$$R(\text{FWHM}, \epsilon) = \text{FWHM} \sqrt{-\log_2 \epsilon} \quad (6)$$

278 Proof of this relationship is provided in the Sup-
 279 plementary Information Section S.1. **Thresholding ra-
 280 dius.**

281 Fig. 3A shows the influence of FWHM and ϵ on
 282 the truncated kernel. Fig. 3B,C show the relation-
 283 ship between standard deviation, truncation thresh-
 284 old and radius. Truncated kernels were generated
 285 with nonzero kernel weights only at locations with
 286 distance less than $R(\text{FWHM}, \epsilon)$ from the kernel cen-
 287 ter. Consequently, kernels were re-normalized such
 288 that for every vertex the column sum of F_s was 1.0
 289 despite truncation. Fig. 3D,E demonstrate the spatial
 290 distribution of a single row of this smoothing kernel
 291 over a sample cortical surface mesh.

292 2.7. CBS for atlas-based connectivity

293 As described in Section 2.4. **Connectome resolu-
 294 tion**, smoothed versions of the parcellation-based

295 atlas-resolution connectome can be computed by
 296 first applying smoothing to the high-resolution con-
 297 nectome, then aggregating the connectivity values
 298 within the vertices corresponding to each atlas par-
 299 cel. This approach however necessitates the high stor-
 300 age and computational complexity demands of high-
 301 resolution connectome data. We therefore derived a
 302 more computationally efficient procedure to perform
 303 CBS on atlas-based connectomes.

304 A brain parcellation atlas can be denoted by a bi-
 305 nary $p \times v$ matrix P , where p is the number of brain
 306 regions in the atlas, such that the i th row of P is a
 307 binary mask of vertices belonging to the i th atlas re-
 308 gion and each vertex belongs to at most one region
 309 (a “hard parcellation”). An atlas-based connectivity
 310 map A_p can be represented by the matrix multiplica-
 311 tion $A_p = PAP^T$: this operation reduces the $v \times v$
 312 high-resolution connectivity A to a $p \times p$ atlas
 313 connectivity map A_p . To smooth A_p , the high-resolution
 314 connectivity matrix A can be smoothed to A_s and
 315 then downsampled to create the smoothed atlas con-
 316 nectivity map A_{sp} . An equivalent approach is to first
 317 spatially smooth every row of the brain atlas P , and
 318 then normalize every column to produce a smoothed
 319 “soft parcellation” $P_s = PF_s$, where each region is
 320 now defined as a weighted probability map across ver-
 321 tices and vertices can have non-zero membership to
 322 multiple regions. This enables direct computation of
 323 smoothed parcellation-based connectome matrix A_{sp}
 324 without necessitating computation of the smoothed
 325 high-resolution connectome matrix A_s (see Supple-
 326 mentary Information Section S.2. **CBS for atlas-based
 327 connectivity** for detail):

$$\begin{aligned}
 A_{sp} &= PA_s P^T \\
 &= PF_s A F_s^T P^T \\
 &= (PF_s) A (PF_s)^T \\
 &= P_s A P_s^T
 \end{aligned} \tag{7}$$

2.8. Connectome similarity

To evaluate the potential advantages of smoothing, a measure of similarity based on Pearson's correlation was used to quantify the conformity of two connectivity maps [32, 53]. To compute the similarity between two networks A_1 and A_2 , first, Pearson's correlation was computed for all respective rows of the connectivity matrices, yielding v correlation coefficients, each indicating the connectivity similarity of a single node; these correlations were then averaged over all nodes to produce a single value indicating the similarity of two connectomes. This measure was used to quantify both intra- and inter-individual connectome matrix similarities.

2.9. Evaluation metrics

Direct connectome comparisons were performed within each combination of: tractography algorithm (deterministic and probabilistic); parcellation resolution; and network smoothing parameters. Within each of these configurations, smoothed structural connectomes were generated independently for the two scanning sessions for each of 42 participants. For each scan in session 1, its similarity to every session 2 scan (1 intra-individual and 41 inter-individual) was computed; aggregated across all individuals, this process yielded 42 values comparing connectomes of the same individual (intra-individual similarities), and 42×41 measuring the similarity between connectomes of different individuals (inter-individual similarities). The intra-individual similarities were averaged to form a measure of connectome reliability μ_{intra} , indicating the extent of consistency of mapped connectomes for an individual; similarly, the inter-individual similarities were averaged to yield a measure of population uniformity of the connectivity maps μ_{inter} . Ideally, connectomes should be reliable (i.e. high μ_{intra}) and preserve inter-individual differences (i.e. low μ_{inter}). Hence, high reliability and low population uniformity is desirable.

To evaluate the extent to which an individual's connectome is unique, we adopted an established identifiability framework [54]. *Identifiability* quantifies the extent to which an individual can be differentiated from a larger group based on a set of individual attributes. Here, identifiability was measured by the effect size of the difference in the means of intra-individual and inter-individual similarities [32]:

$$\text{identifiability} = \frac{|\mu_{intra} - \mu_{inter}|}{s} \tag{8}$$

Where μ_{intra} and μ_{inter} are the mean of the two intra- and inter-individual similarity distributions and s is the pooled standard deviation of the two distributions.

2.10. Evaluating statistical power with atlas-resolution smoothing

Generally, smoothing can result in a loss of effective spatial resolution, blurring, and shifting or merging of adjacent signal peaks [55–58], but is necessary to strike a compromise between sensitivity and specificity [59]. Hence, we investigated the impact of CBS on mass univariate significance testing of associations between cognitive performance and atlas-based structural connectivity. Given that structural connectivity and cognition are known to be associated [32], we tested whether the use of CBS would improve power to detect such associations. For each pair of regions in the parcellation atlas, Pearson's correlation coefficient was used to test for an association between connectivity strength and a previously established measure of overall cognitive performance [60]. This yielded a correlation coefficient for each pair of regions. Age and sex were regressed out from the cognitive measure as confounds. This was repeated across 100 bootstrap tests each including 90% of the sample ($N=35$) to increase the robustness of the comparisons against individual effects.

To generate a distribution of correlation coefficients under the null hypothesis of an absence of association between connectivity and cognitive performance, we randomized cognitive scores between individuals and recomputed all correlation coefficients; this was repeated for 1000 randomizations (10 randomizations within each bootstrap sample), yielding 1000 correlation coefficients representing the null distribution for

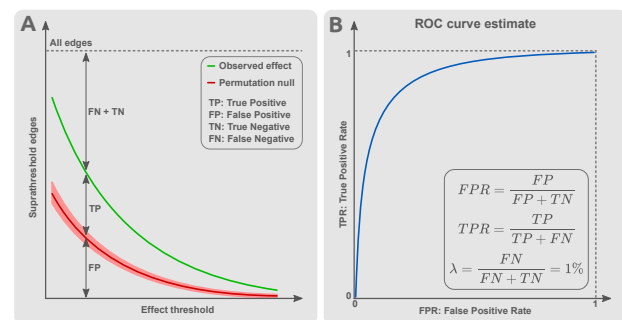


Fig. 4. Estimation of receiver operating characteristic (ROC) curves for mass univariate testing of associations between cognitive performance and structural connectivity. For each pair of regions in the parcellation atlas, Pearson's correlation coefficient was used to test for an association between connectivity strength and a previously established measure of overall cognitive performance. (A) For a given effect size threshold (horizontal axis), the number of suprathreshold connections (vertical axis) yielded the combined number of true positives (TP) and false positives (FP), indicated by the green line. The red line indicates the total number of FP , determined by randomizing cognitive scores between individuals and recomputing all correlation coefficients (1000 randomizations; mean & 95% confidence interval shown). (B) Assuming a constant value for the false omission rate ($\lambda = \frac{FN}{FN+TN} = 0.01$), an ROC curve can be estimated for different effect size (correlation coefficient) thresholds. TPR: true positive rate. FPR: false positive rate.

each connection. For a range of correlation coefficient thresholds from 0.1 to 0.5 (which indicate small to large effects according to Cohen’s conventions [61]), we counted the number of suprathreshold connections in both the empirical and randomized data (averaged across the 1,000 randomizations).

From these data, we generated an ROC (Receiver Operator Characteristic) curve as follows. The number of suprathreshold connections in the empirical data was assumed to give the combined number of true positives (TP) and false positives (FP), while the average total number of suprathreshold connections in the randomized data estimated the total number of FP (Fig. 4A). The combined number of false negatives (FN) and true negatives (TN) was determined by subtracting $TP+FP$ from the total number of connections. Finally, since the true underlying effect was unknown, we assumed a false omission rate of 1%, i.e., $\lambda = \frac{FN}{FN+TN} = 0.01$. This assumption enabled estimation of sensitivity ($\frac{TP}{TP+FN}$) and specificity ($\frac{TN}{TN+FP}$) that were used to generate the ROC curve. We ensured that our estimates were robust to the choice of λ (see Supplementary Information Section S.3. Replication of ROC curve estimates for detail). This process was repeated independently for various smoothing kernels, and for data generated using both deterministic and probabilistic tractography algorithms, to investigate the impact of CBS on the statistical power to detect associations between cognitive performance and connectivity.

Additionally, we tested the replicability of the suprathreshold effects in a test-retest comparison to evaluate the replicability of the observations before and after smoothing. At each utilized threshold value, for every edge that was suprathreshold in the data from either session 1 or session 2, we calculated the difference in correlation coefficient between the two sessions. This provided a distribution of effect differences observed across a range of effect thresh-

olds. Thus, a lower average effect difference indicated higher consistency of the connectivity-behavior observations and higher replicability of the findings.

3. Results

We investigated the utility of CBS for high-resolution and atlas-based connectomes, focusing on connectome reliability and identifiability as well as computational and storage requirements. We recommend optimal smoothing kernels for connectomes mapped with deterministic and probabilistic tractography, and we demonstrate that smoothing improves the statistical power to detect associations between connectivity and cognitive performance.

3.1. High-resolution connectome storage size

High-resolution connectomes require considerable storage and computational resources, and CBS can increase this burden, due to reductions in matrix sparsity. Fig. 5 summarizes the sizes of stored connectomes for various kernels. Kernels with larger FWHM and/or more lenient truncation thresholds incur greater storage demands for high-resolution connectomes. We found that the kernel radius $R(\text{FWHM}, \epsilon)$, which is dependent on both parameters, was a reasonable predictor of connectome size. We also observed that connectomes mapped using probabilistic tractography were approximately an order of magnitude larger than their deterministic counterparts both prior to smoothing (10MB for probabilistic and 1MB for deterministic) and after performing CBS with identical smoothing parameters.

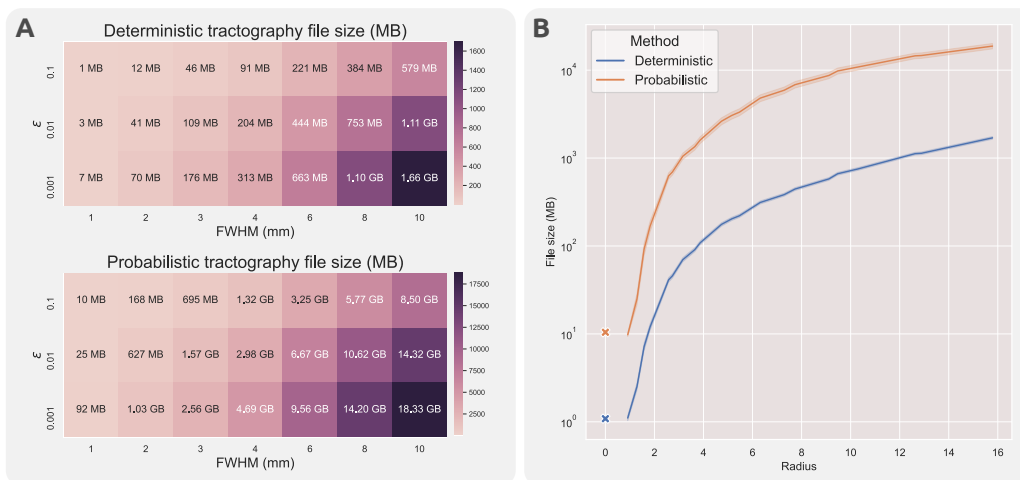


Fig. 5. Impact of CBS on connectome storage requirements. (A) Tables show the mean storage size of individual connectomes mapped using deterministic (upper) and probabilistic (lower) tractography following smoothing, as a function of truncation threshold and full-width at half maximum (FWHM) of smoothing kernel (B) The relationship between the kernel radius and file size of individual connectomes. Results for connectomes with no smoothing are marked with an x. File sizes are plotted using a logarithmic scale. Shaded bands indicate one standard deviation from the mean.

3.2. Identifiability and reliability

Fig. 6 summarizes the impact of CBS on the identifiability and reliability of high-resolution structural connectivity maps. We observed that both larger FWHM values and smaller truncation thresholds (i.e., larger R in both cases) consistently improved connectome reliability (mean intra-subject similarity). While high-resolution connectomes without smoothing had a relatively low reliability ($\mu_{intra} < 0.2$), CBS with kernels as little as 3-4mm FWHM resulted in a substantial increase in reliability ($\mu_{intra} > 0.5$), with reliability exceeding 90% ($\mu_{intra} > 0.9$) achieved in some scenarios.

CBS also impacted connectome identifiability. We observed that while CBS with a 2-4mm FWHM kernel improved the identifiability of connectomes, CBS with larger FWHM was detrimental for individual identifiability, such that CBS with a 10mm FWHM resulted in more than 50% reduction in identifiability. For both identifiability and reliability measures, CBS was more sensitive to a change in kernel FWHM in contrast to the truncation threshold ε . Increasing the truncation threshold from $\varepsilon = 0.01$ to $\varepsilon = 0.001$ had negligible impact on either measure.

Tractography algorithm choice also impacted reliability and identifiability. High-resolution connectomes mapped using deterministic tractography had relatively lower reliability (10-20% lower), but higher identifiability (20-30%), compared to their probabilistic counterparts with identical CBS parameters.

Fig. 7 shows the impacts of CBS with different ker-

nel parameters on an atlas-parcellation-based structural connectome. In agreement with the high-resolution analyses, we observed both that increases in FWHM and decreases in kernel truncation thresholds led to improved connectome reliability, and that this improvement in reliability comes at the expense of reduced identifiability. Without CBS, the atlas-based connectomes were already relatively reliable (deterministic: 92%, probabilistic: 98%). Use of the largest smoothing kernel increased these to 97% and 99%, respectively, albeit at the cost of a small reduction in identifiability (from 7.8 to 7.2 for deterministic and from 6.8 to 6.1 for probabilistic). Changing kernel extent from $\varepsilon = 0.01$ to $\varepsilon = 0.001$ again had no considerable impact on reliability or identifiability. The magnitude of influence of CBS on the atlas-resolution connectomes was comparatively smaller than the effects observed at the higher resolution.

All in all, we observed that the advantages of CBS for high-resolution were maximized with 3-6mm FWHM kernels; larger smoothing kernels (>6mm FWHM) could deteriorate high-resolution identifiability for the sake of reliability. In contrast, identifiability of the atlas-resolution maps were less sensitive to larger smoothing kernels, and thus kernels of 6-10mm FWHM can be used to improve reliability with proportionally smaller losses in identifiability. To achieve similar reliability and identifiability, connectomes generated using deterministic tractography were found to require CBS with larger smoothing kernels compared to their probabilistic counterparts.

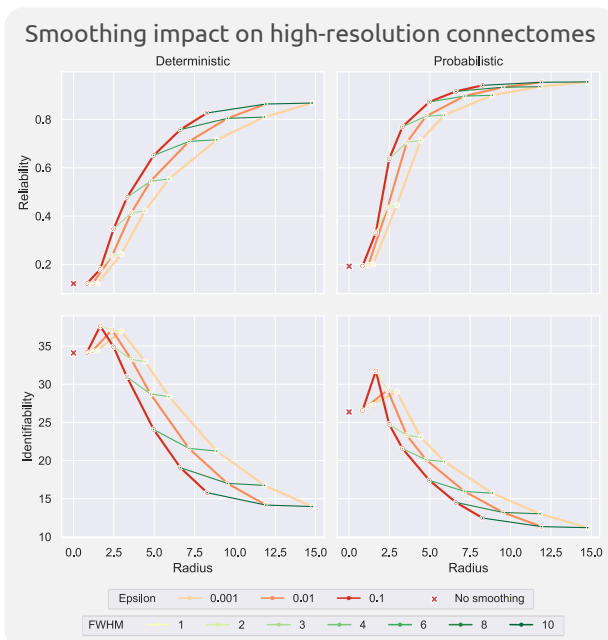


Fig. 6. Impact of CBS on high-resolution connectomes for a range of different kernel parameters. Reliability (first row) and identifiability (second row) are reported for deterministic (left column) and probabilistic (right column) structural connectomes mapped at the resolution of cortical vertices. Results for connectomes with no smoothing are marked with an x in each plot. Kernel truncation thresholds, ε , are colored using warm colors such that each line connects points with equal ε ; similarly, FWHM is colored using shades of green.

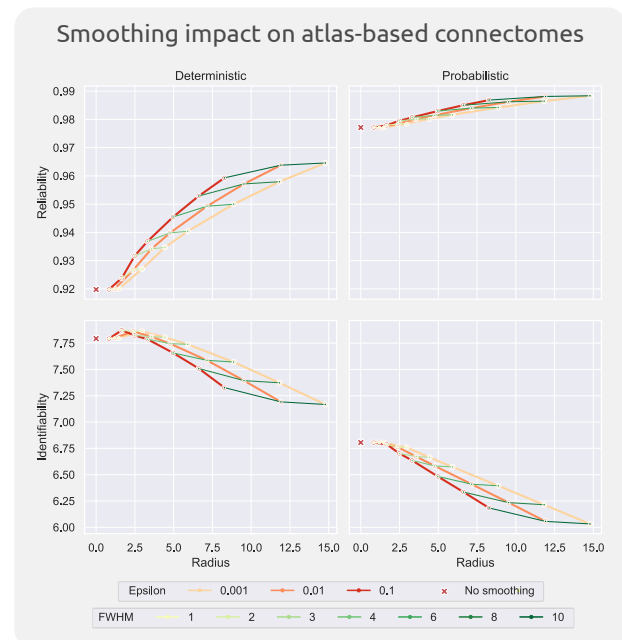


Fig. 7. Impact of CBS on atlas-based connectomes, for a range of different kernel parameters. The connectome reliability (first row) and identifiability (second row) are reported for deterministic (left column) and probabilistic (right column) structural connectomes mapped at the resolution of atlas parcels. The unsmoothed atlas-based connectivity results are marked with x in each plot. Kernel truncation thresholds, ε , are colored using warm colors such that each line connects points with equal ε ; similarly FWHM is colored using shades of green.

543 Finally, it should be noted that the optimal CBS kernel
 544 parameters essentially depend on the application
 545 for which the connectome will be used.

546 3.2.1. Case Study: Impact of CBS on statistical 547 power

548 Finally, we investigated whether CBS can improve
 549 statistical power to detect associations between struc-
 550 tural connectivity and cognitive performance. For
 551 this study we used $\text{FWHM} = 8\text{mm}$ and $\varepsilon = 0.01$,
 552 based on the results reported above. For this case
 553 study, we considered the mapped atlas-based connec-
 554 tomes and computed Pearson's correlation coefficient
 555 between streamline counts and cognitive performance
 556 for each pair of regions. ROC curves were then com-
 557 puted for each case, as described in the Methods, to
 558 determine whether CBS improved statistical power to
 559 identify associations between connectivity and cog-
 560 nitive performance.

561 First, we tested whether the magnitude of effect
 562 in the set of suprathreshold connections (i.e., connec-
 563 tions with a correlation coefficient exceeding a fixed
 564 threshold) were replicable between the test and retest
 565 datasets. We found that CBS improved replicability
 566 in suprathreshold connections, particularly more so
 567 for connectomes mapped with probabilistic tractog-
 568 raphy (Fig. 8A); this suggests that CBS can improve
 569 the reproducibility of mass univariate testing on con-
 570 nectomes.

571 Next, we enumerated the number of suprathresh-
 572 old connections as a function of the effect thresh-
 573 old (Fig. 8B). While the proportion of suprathresh-
 574 old connections increases following smoothing for the

empirical data, indicating a potential gain in sensi- 575
 tivity, a similar increase in the randomized (null 576
 distribution) data suggests that this may come at 577
 the expense of poorer specificity. For connectomes 578
 mapped with deterministic tractography, the num- 579
 bers of suprathreshold connections for the empiri- 580
 cal and randomized data are separated by a compar- 581
 able gap, irrespective of whether CBS was per- 582
 formed. For probabilistic tractography, the num- 583
 ber of suprathreshold connections for the randomized 584
 data was comparable with and without smoothing, 585
 whereas CBS resulted in a substantially greater pro- 586
 portion of suprathreshold connections for the empiri- 587
 cal data. This suggests that CBS can improve the 588
 statistical power of mass univariate testing per- 589
 formed on connectomes mapped with probabilistic 590
 tractography, without a substantial loss in specificity. 591

To further investigate these effects, we consid- 592
 ered precision ($\frac{TP}{TP+FP}$) as a function of effect size 593
 threshold (Fig. 8C); and from this, generated ROC 594
 curves (Fig. 8D). Performing CBS on connectomes 595
 mapped from probabilistic tractography improves the 596
 precision and sensitivity of the inference. This im- 597
 provement is also partially observed for connectomes 598
 mapped from deterministic tractography only for 599
 smaller effect thresholds ($r < 0.3$). Taken together, 600
 these results suggest that CBS is particularly bene- 601
 ficial to improving the statistical power of inference 602
 performed on connectomes mapped with probabilistic 603
 tractography; in contrast, for connectomes mapped 604
 with deterministic tractography, the benefit of CBS 605
 is marginal and possibly detrimental for larger effect 606
 size thresholds ($r > 0.3$). More importantly, CBS 607
 improved replicability with minimal impact on the 608

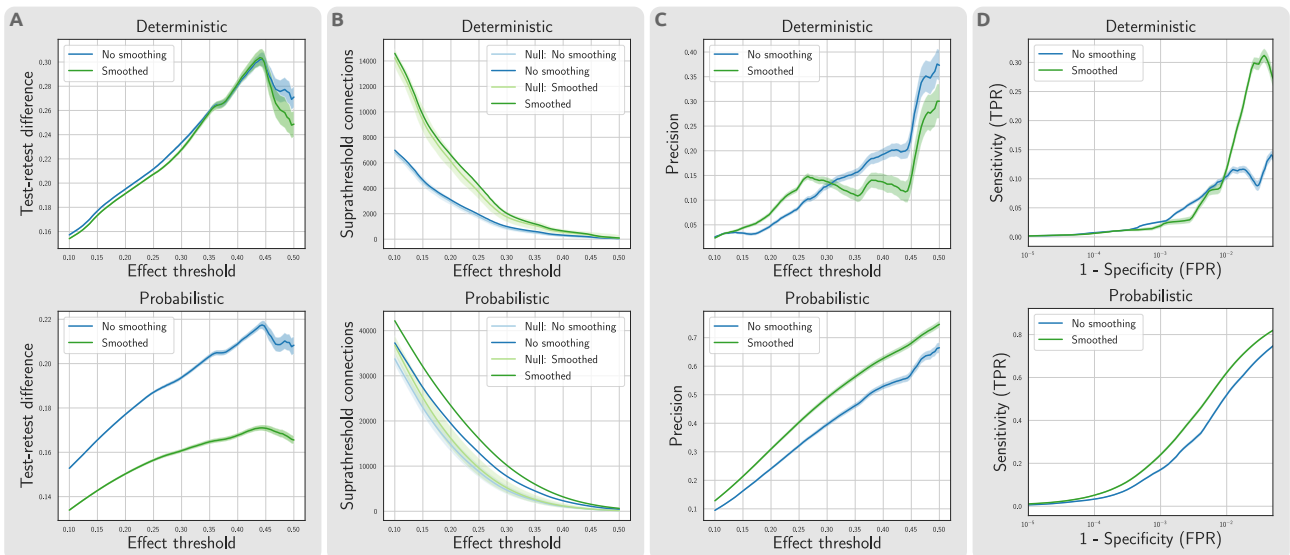


Fig. 8. Impact of CBS on statistical power of mass univariate testing on atlas-based connectomes, based on an exemplar dataset examining correlations between structural connectivity and cognitive performance. (A) Replicability of suprathreshold connections between test and retest datasets: a lower difference of the observed effect magnitude between test and retest is favorable in terms of replicability. (B) The number of suprathreshold connections as a function of the effect size threshold was compared with a null distribution from permutation. To assess the predictive utility of the connectomes, precision, sensitivity, and specificity was estimated from a comparison with the null. (C) Precision was calculated from the ratio of supra-threshold edges found in empirical data compared to the null model at different effect thresholds. (D) ROC curves were estimated to demonstrate the respective changes in sensitivity ($TPR = \frac{TP}{TP+FN}$) and specificity ($1 - FPR = \frac{TN}{TN+FP}$) of the edges selected at different effect thresholds. The analyses were repeated across bootstrap samples to provide a robust estimate of statistical power. Shaded lines indicate 95% confidence intervals. Abbreviations: TP: True Positive, FP: False Positive, TN: True Negative, FN: False Negative, TPR: True Positive Rate, FPR: False Positive Rate.

609 statistical power for connectomes mapped with both
610 tractography algorithms.

611 4. Discussion

612 In this study, we established a computationally
613 efficient formalism for connectome smoothing and
614 demonstrated that our connectome-based smoothing
615 (CBS) method can benefit the analysis of atlas-based
616 and high-resolution connectomes. Our results demon-
617 strate that CBS impacts different aspects of con-
618 nectivity mapping analyses, including individual re-
619 liability, inter-individual variability, and the inter-
620 scan replicability of brain-behavior statistical asso-
621 ciations, as well as computational storage demands.
622 The choice of smoothing kernel parameters involves
623 a trade-off between connectome sensitivity and speci-
624 ficity: larger kernels (higher FWHM and lower ϵ) im-
625 prove connectome sensitivity, but are detrimental to
626 connectome specificity. It is therefore important to
627 select a level of smoothing that strikes a balance be-
628 tween these competing factors. In the following sec-
629 tions, we provide some guidelines for selecting optimal
630 smoothing parameters and discuss the implications of
631 performing CBS for connectome reliability, identifi-
632 ability, storage requirements, and statistical power.

633 4.1. *Appropriate smoothing parameters*

634 Our results indicate that CBS differentially affects
635 the characteristics of structural connectivity matri-
636 ces mapped with different tractography methods and
637 parcellation resolutions. Although we cannot sug-
638 gest a one-size-fits-all smoothing kernel, our find-
639 ings can guide selection of appropriate CBS smooth-
640 ing kernels in future studies. Table 1 provides some
641 rules of thumb for selecting a level of spatial smooth-
642 ing which aims to achieve a balance between re-
643 liability and identifiability, while also considering
644 storage demands. In general, high-resolution con-
645 nectomes benefit from smaller FWHM compared to
646 atlas-based connectomes, and deterministic maps re-
647 quire larger FWHM than their probabilistic counter-
648 parts to achieve the same level of reliability. However,
649 the goals of the analysis at hand must be considered
650 when selecting the level of smoothing. For example,
651 if the goal is to identify an individual from a group
652 based on their connectome, deterministic tractogra-
653 phy and a smaller FWHM than recommended in Ta-
654 ble 1 may be desirable. On the other hand, if one
655 wishes to build a reliable consensus structural con-
656 nectome that is robustly consistent across individu-
657 als, a higher FWHM than recommended in Table 1
658 may be favored. A value of 0.01 is suggested univer-
659 sally for the kernel truncation threshold ϵ , as smaller
660 thresholds yield negligible impacts on identifiability
661 and reliability whilst incurring much greater storage
662 costs.

663 Without CBS, connectomes mapped from deter-
664 ministic tractography were found to yield higher

665 identifiability; conversely, connectomes mapped from
666 probabilistic tractography were more reliable. This is
667 in line with previous reports suggesting that proba-
668 bilistic tractography achieves higher sensitivity, lower
669 specificity, and lower interindividual variability, com-
670 pared to deterministic approaches [62–65]. Given
671 that many factors other than reliability and identi-
672 fiability would affect the choice of tractography al-
673 gorithm, we suggest that CBS could be leveraged to
674 achieve a balance between reliability and identifiabil-
675 ity of the selected tractography algorithm. Hence, we
676 could take advantage of a comparatively larger kernel
677 for deterministic tractography approaches to match
678 the reliability and identifiability of the probabilistic
679 counterpart.

Recommended smoothing parameters		FWHM	ϵ	R
High- resolution	Probabilistic	3mm	0.01	4mm
	Deterministic	6mm	0.01	8mm
Atlas	Probabilistic	8mm	0.01	10mm
	Deterministic	8mm	0.01	10mm

Table 1. Recommended smoothing parameters. This table provides rule of thumb recommendations for CBS smoothing kernels of different variants of structural connectomes. In general, connectomes at the resolution of a brain atlas can benefit from larger CBS kernels compared to high-resolution connectomes. High-resolution connectomes computed from probabilistic tractography are advised to be smoothed less than their deterministic counterparts. Reducing epsilon below 0.01 is unfavorable and computationally costly. The rounded values for kernel radius $R(\text{FWHM}, \epsilon)$ provide sensible approximations.

680 Our results highlight that CBS is a critical step
681 to improving the reliability of high-resolution connec-
682 tomes. High-resolution connectivity mapping is particu-
683 larly sensitive to noise, artefacts, and registration
684 misalignment, all of which can be alleviated—to a cer-
685 tain extent—with the new CBS formalism developed
686 here.

687 4.2. *Connectome reliability*

688 Structural connectivity maps are commonly used in
689 research to draw statistical inferences regarding as-
690 sociations between brain connectivity and different
691 aspects of human cognition, behavior, and mental
692 health [66–71]. The statistical power of such infer-
693 ences can depend on the reliability of the measure un-
694 der study: a connectivity measure that can be reliably
695 assessed for all individuals can potentially improve
696 the characterization of brain-behavior associations.
697 However, improvements in reliability achieved by in-
698 creasing the level of smoothing come at the expense
699 of poorer spatial specificity and increases in connec-
700 tome storage and computational requirements. CBS
701 enables researchers to balance this trade-off to match
702 the goals of the analysis at hand. Commonly used
703 atlas-based connectivity maps are comparatively reli-
704 able, even without any smoothing, since the reduced
705 spatial resolution of inter-subject correspondence im-
706 posed by a parcellation performs an operation compa-

707 rable to smoothing. Nevertheless, we found that CBS
708 could marginally improve the reliability of atlas-based
709 connectomes.

710 4.3. Individual identifiability

711 The concept of neural fingerprinting has emerged in
712 recent years which considers the challenge of identi-
713 fying an individual from within a large group of oth-
714 ers, based on their connectome or other neuroimaging
715 data [53]. While the efficacy of a measure at indi-
716 vidual identification does not necessitate existence of
717 behavioral and pathological biomarkers in individu-
718 als, it could still be conceived as an indicator of the
719 strength of such individual brain-behavior associa-
720 tions. By reducing the impact of noise and registra-
721 tion misalignments, CBS can enhance detection of
722 individual differences in connectivity maps, enabling
723 clearer differentiation of individuals and thus poten-
724 tially improve the accuracy of neural fingerprinting.
725 Our findings suggest that a minimal smoothing ker-
726 nel of 2mm FWHM improves both reliability and
727 identifiability of high-resolution connectivity matri-
728 ces. Implementing CBS with larger kernels (i.e. >
729 2mm FWHM) further enhances connectome reliabil-
730 ity substantially, but results in a gradual reduction in
731 identifiability due to loss of individual identifiers by
732 spatial blurring. Smoothing the high-resolution con-
733 nectivity maps beyond 6mm FWHM is unnecessary
734 because gains in reliability diminish, despite detri-
735 mental impacts on identifiability, spatial specificity,
736 and storage requirements.

737 4.4. Storage requirements

738 It is important to consider the storage demands
739 and associated computational burdens of handling
740 smoothed connectome data. If the connectome size is
741 larger than a gigabyte or so, handling the file (loading
742 into memory and conducting analyses) can become
743 unacceptably time-consuming. Even with the assis-
744 tance of high-performance computing infrastructure,
745 any benefits of using connectomes larger than a few
746 gigabytes might not outweigh the time and resources
747 required to process the larger files. This especially
748 limits the extent of smoothing for connectomes gener-
749 ated using probabilistic tractography, which can grow
750 to more than a few gigabytes when smoothed above
751 4-6mm FWHM. In contrast, connectomes mapped
752 with deterministic tractography can be smoothed fur-
753 ther whilst remaining highly computationally feasi-
754 ble. Nevertheless, if greater smoothing is essential
755 in a study, a high-performance computing platform
756 with access to adequate memory can be used to pro-
757 cess smoothed connectomes (potentially without use
758 of sparse matrix data structures), which may take
759 tens of gigabytes of memory per individual connec-
760 tome.

4.5. Implications on atlas resolution

761 Our results highlight the impact of CBS on structural
762 connectomes mapped both at the high resolution of
763 individual surface vertices, and the lower resolution
764 of a brain atlas. While the findings vary in terms of
765 magnitude of influence, a common pattern is visible
766 across resolutions: higher FWHM results in a more
767 reliable connectome, yet higher FWHM reduces the
768 identifiability of connectomes. We developed compu-
769 tationally efficient methods to perform CBS at both
770 resolutions. Atlas-based connectivity matrices have a
771 relatively small memory footprint (<1MB), and thus
772 they can be processed and stored efficiently, regard-
773 less of the level of smoothing. Similar to the high-
774 resolution connectomes, when using an atlas par-
775 cellation, probabilistic and deterministic tractogra-
776 phy approaches have complementary attributes when
777 comparing reliability and identifiability: connectomes
778 mapped from probabilistic tractography achieve bet-
779 ter reliability compared to their deterministic coun-
780 terparts, whereas deterministic connectomes can bet-
781 ter reveal individual differences. This is one possible
782 factor that can guide the choice between deterministic
783 and probabilistic tractography algorithms. However,
784 CBS can be used to increase the reliability of con-
785 nectomes mapped from deterministic tractography to
786 match the reliability of the probabilistic approach.
787

788 Finally, the atlas-based smoothing results suggest
789 that probabilistic maps are to a certain extent repre-
790 sentative of highly smoothed deterministic ones. In
791 other words, more smoothed deterministic maps were
792 analogous to less smoothed probabilistic maps, as the
793 probabilistic evaluation curves in Fig. 7 seem to be a
794 continuation of the deterministic curves. This obser-
795 vation is in agreement with prior expectations given
796 the mechanisms used to generate the data, as proba-
797 bilistic tractography-based connectivity has an intrin-
798 sic spatial smoothness due to the stochastic variabil-
799 ity in streamline propagation. The proposed method
800 to perform CBS on atlas-based connectomes does
801 not require construction of any intermediate high-
802 resolution connectomes and is a fast operation rela-
803 tive to the time required to perform whole-brain trac-
804 tography. Thus, while the benefits of spatial smooth-
805 ing for atlas-based connectomes were modest, we re-
806 commend including CBS in future connectome map-
807 ping workflows.

4.6. CBS and principals of spatial smoothing

810 Our proposed connectome-based spatial smoothing
811 approach is an extension of spatial signal smooth-
812 ing to networks, and hence, fundamental concepts
813 within the domain of spatial smoothing are applica-
814 ble to CBS. For instance, from a signal processing
815 perspective, the matched filter theorem states that
816 spatial smoothing by an appropriate Gaussian ker-
817 nel equalizes the voxel-wise standard deviation and,
818 in turn, yields an optimal sensitivity to detect effects

819 of unknown extent [72, 73]. Additionally, with re- 875
820 gards to single-subject inference, such smoothing fa- 876
821 cilitates the application of multiple comparison cor- 877
822 rection using random field theory [72–74] and finally,
823 smoothing mitigates residual anatomical variability
824 of individuals at the group-level. These concepts are
825 equally applicable to CBS, wherein, a matrix multi-
826 plication with a smoothing kernel achieves a similar
827 purpose to convolution of image data with a 3D spa-
828 tial smoothing kernel; as a result, CBS can be utilized
829 to (i) maximize connectivity SNR through appropri-
830 ate filter selection, (ii) improve single-subject infer-
831 ence, and (iii) improve the reliability of group level
832 connectivity analyses. This poses an interesting future
833 research direction to explore the benefits of CBS
834 for whole-brain high-resolution network inference in
835 which voxel-wise approaches [31, 74–76] are combined
836 with network-based approaches [30].

837 4.7. Concluding remarks

838 In this study, we developed a novel formalism for
839 connectome-based smoothing of structural connectiv-
840 ity matrices and demonstrated the wide-ranging ben-
841 efits of connectome smoothing. Our results indicate
842 that CBS with different kernel FWHMs and trun-
843 cation thresholds significantly impacts various char-
844 acteristics of structural connectivity matrices. In
845 high-resolution connectomes, smoothing up to 3-6mm
846 FWHM was deemed favorable, though the choice
847 of smoothing parameters imposes a trade-off be-
848 tween reliability and individual identifiability. We
849 provided recommendations for smoothing parameter
850 choices that achieve a compromise between reliability
851 and identifiability. Our connectome-based smooth-
852 ing method and associated recommendations can be
853 incorporated into future structural connectivity map-
854 ping pipelines, enabling more reliable and better pow-
855 ered connectome analyses. Moreover, high-resolution
856 structural connectivity overcomes the known uncer-
857 tainty and ambiguity in determination of brain par-
858 cellation, and so will be a powerful analysis frame-
859 work moving forward; our demonstrated and evalu-
860 ated smoothing framework is an essential tool in fa-
861 cilitating such, and we have made reasonable recom-
862 mendations for how others can use it.

863 Data and code availability

864 All imaging data used in this study was sourced
865 from the Human Connectome Project (HCP)
866 (www.humanconnectome.org). The bash scripts
867 used to perform tractography using MRtrix3 [45]
868 (www.mrtrix.org), as well as all Python code re-
869 quired to perform CBS and map smoothed connec-
870 tomes at either the resolution of vertices or an atlas,
871 are provided in our git repository. This code
872 repository can be accessed from github.com/sinamansour/connectome-based-smoothing. Addition-
873 ally, to facilitate future research, the codes for

smoothing connectomes at high-resolution and atlas-
resolution will be released as a standalone python
package (currently under development).

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Author contributions

902 **S.M.L.:** Conceptualization, Methodology, Formal
903 analysis, Data curation, Software, Writing - original
904 draft, Writing - review & editing **C.S.:** Conceptu-
905 alization, Writing - original draft, Writing - review
906 & editing **R.S.:** Conceptualization, Writing - original
907 draft, Writing - review & editing **A.Z.:** Supervision,
908 Conceptualization, Writing - original draft, Writing -
909 review & editing
910

Competing interests

911 The authors declare no competing interests.
912

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1229 Supplementary Information

1230 *S.1. Thresholding radius*

1231 This section provides the mathematical rationale behind the relationship between R , σ (or alternatively FWHM),
 1232 and ε presented in Equation 6. The truncation radius R was formulated as a function of σ and ε such that
 1233 the proportion of signal loss for a 2-dimensional Gaussian kernel with the strength of σ truncated at a radius
 1234 of R is equal to ε . Given that the Gaussian kernel was defined such that its total cumulative density is unity
 1235 ($\int_{-\infty}^{\infty} G(\delta) = 1$), the relationship between the smoothing parameters can be defined by the following integration
 1236 over the 2-dimensional surface area:

$$\varepsilon = 1 - \iint_{\delta < R} G(\delta) dA \quad (S1)$$

1237 This integration can be solved in polar coordinates by the following closed form equation:

$$\begin{aligned} 1 - \varepsilon &= \int_0^{2\pi} \int_0^R G(r, \theta) r dr d\theta \\ &= \int_0^{2\pi} \int_0^R \frac{1}{(\sqrt{2\pi}\sigma)^2} e^{-\frac{r^2}{2\sigma^2}} r dr d\theta \\ &= \int_0^R \frac{1}{\sigma^2} r e^{-\frac{r^2}{2\sigma^2}} dr \\ &= -e^{-\frac{r^2}{2\sigma^2}} \Big|_0^R \\ &= 1 - e^{-\frac{R^2}{2\sigma^2}} \end{aligned} \quad (S2)$$

1238 And this can be used to describe R as a function of σ and ε :

$$1 - \varepsilon = 1 - e^{-\frac{R^2}{2\sigma^2}} \rightarrow \boxed{R = \sigma \sqrt{-2 \ln \varepsilon}} \quad (S3)$$

1239 And given the relationship between FWHM and σ (FWHM = $\sigma \sqrt{8 \ln 2}$), this equation can be rewritten based
 1240 on FWHM:

$$\boxed{R(\text{FWHM}, \varepsilon) = \text{FWHM} \sqrt{-\log_2 \varepsilon}} \quad (S4)$$

1241 *S.2. CBS for atlas-based connectivity*

1242 In the main text, it was briefly mentioned that mapping the high-resolution connectivity is not necessary for
 1243 smoothing the connectivity matrices at an atlas resolution: alternatively, a smoothed version of an atlas-based
 1244 connectivity matrix can be derived from a soft parcellation, which is derived by applying spatial smoothing
 1245 to the parcels of the brain atlas (and normalizing each vertex to a unity sum of parcel memberships). In this
 1246 section, we provide the formal proof of this equivalence: first, downsampling a high-resolution connectivity
 1247 matrix to an atlas-based connectome matrix is formulated by linear algebraic formulations; these formulations
 1248 are then used to complete a formal proof of the equivalence.

1249 Following the prior nomenclature, A is a $v \times v$ matrix denoting the high-resolution connectivity matrix where
 1250 v is the number of vertices. According to Equation 4, the smoothed high-resolution connectivity matrix A_s
 1251 is calculated as follows:

$$A_s = F_s A F_s^T \quad (S5)$$

1252 Where F_s is a $v \times v$ column-normalized spatial smoothing kernel. A formal notion of a brain atlas can be
 1253 denoted by $p \times v$ matrix P , where p is the number of brain regions in the atlas. Elements $P(i, j)$ encode the
 1254 relationship between vertex/voxel v_i and region p_j .

$$P(i, j) = \begin{cases} 1, & \text{if } v_j \in p_i \\ 0, & \text{otherwise} \end{cases} \quad (S6)$$

An atlas-resolution connectome A_p is a $p \times p$ matrix, which is normally mapped from an atlas parcellation such that elements $A_p(i, j)$ encode the aggregate contribution from those streamlines for which one endpoint is assigned to region p_i and the other endpoint is assigned to region p_j (showing here the streamline count for simplicity):

$$A_p(i, j) = \sum_{v_k \in p_i \& v_l \in p_j} A(k, l) \quad (\text{S7})$$

This notion can be formalized by the following matrix representation which can be used to derive A_p from A and P :

$$A_p = PAP^T \quad (\text{S8})$$

Hence, the element $A_p(i, j)$ counts the overall connectivity between regions p_i and p_j by adding all high-resolution connectivity edges between them. Equations S5 and S8 yield the following definition for the smoothed atlas-based connectivity A_{sp} :

$$A_{sp} = PA_s P^T = PF_s A F_s^T P^T = (PF_s) A (PF_s)^T \quad (\text{S9})$$

The matrix PF_s can thus be treated as a $p \times v$ weighted soft parcellation map, i.e. a non-binary brain atlas. This soft parcellation can be used to generate smoothed connectomes based on an atlas parcellation (each streamline contributes to many connectome edges, based on all parcels with non-zero densities at both endpoints) (see Equation S8). A key benefit of this approach is that it obviates the need to create computationally cumbersome high-resolution connectomes as an intermediate step in construction of lower-resolution connectome matrices. A different approach to compute this soft parcellation, that additionally does not necessitate computation of high-resolution smoothing matrix F_s , is further described in the ensuing sections.

S.2.1. Column normalization

To describe the soft parcellation PF_s , a formal definition of normalizing every column should first be defined. Column normalization of an $l \times m$ matrix B can be defined by the matrix multiplication of B with a diagonal norm matrix constructed from column sums.

Definition S.1. $\langle |B| \rangle$ denotes an $m \times m$ diagonal column norm matrix constructed from B where $\langle |B| \rangle(i, i)$ is the sum of the elements of the i th column in B :

$$\langle |B| \rangle(i, j) = \begin{cases} \sum_{\forall k} B(k, j), & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases} \quad (\text{S10})$$

Hence,

$$\langle |B| \rangle = \begin{bmatrix} \sum_{\forall k} B(k, 1) & 0 & \cdots & 0 \\ 0 & \sum_{\forall k} B(k, 2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{\forall k} B(k, l) \end{bmatrix} \quad (\text{S11})$$

And a consequence of Definition S.1 is the statement in the next corollary.

Corollary S.1.1. Let $z_i \in \mathbb{R}^i$ denote the vector of ones, i.e. all i vector elements equal 1. The following is true for any diagonal norm matrix:

$$z_l B = z_m \langle |B| \rangle \quad (\text{S12})$$

Both sides of the equation above compute the column sums of B . Column normalization can be formally defined by the following theorem.

Theorem S.1. The row normalization is a matrix transformation of an $l \times m$ matrix B to an $l \times m$ normalized matrix $N(B)$, such that the sum of every column in $N(B)$ is equal to 1, i.e. $z_l N(B) = z_m$. $N(B)$ can be derived by the following matrix multiplication:

$$N(B) = B \langle |B| \rangle^{-1} \quad (\text{S13})$$

1286 *Proof.* Corollary S.1.1 can be used to prove $z_l N(B) = z_m$:

$$z_l N(B) = z_l B \langle |B| \rangle^{-1} = z_m \langle |B| \rangle \langle |B| \rangle^{-1} = z_m I_m = z_m$$

1287

□

1288 Where I_m is the $m \times m$ identity matrix. The following remarks are a consequence of the aforementioned
1289 definitions and theorems.

1290 **Remark.** The diagonal norm matrix of a brain atlas parcellation $\langle |P| \rangle$ is the $v \times v$ identity matrix I_v , as every
1291 vertex belongs to a single atlas region and thus the sum of any column of P equals 1:

$$\langle |P| \rangle = I_v \Rightarrow z_p P = z_v \quad (\text{S14})$$

1292 Thus, for any arbitrary $v \times v$ matrix X :

$$\langle |PX| \rangle = \langle |X| \rangle \quad (\text{S15})$$

1293 $\langle |PX| \rangle$, by definition, is a diagonal matrix:

$$z_v \langle |PX| \rangle = z_p PX \quad (\text{S16})$$

1294 and from Equation S14 we know that:

$$z_p PX = z_v X = z_v \langle |X| \rangle \quad (\text{S17})$$

1295 Therefore, $\langle |X| \rangle$ is the same diagonal matrix as $\langle |PX| \rangle$. In other words, the sum of the columns of PX is
1296 equal to the sum of the columns of X .

1297 **Remark.** The normalized high-resolution smoothing kernel F_s is defined from column normalization of the
1298 Gaussian kernel smoothing weights matrix F_G (from Equation S13):

$$F_s = N(F_G) = F_G \langle |F_G| \rangle^{-1} \quad (\text{S18})$$

1299 Where F_G is a symmetric $v \times v$ matrix yielded from the truncated Gaussian function calculated upon the
1300 surface mesh:

$$F_G(i, j) = \begin{cases} G(\delta_{ij}), & \text{if } \delta_{ij} < R(\text{FWHM}, \varepsilon) \\ 0, & \text{otherwise} \end{cases} \quad (\text{S19})$$

1301 S.2.2. Smoothed brain atlas

1302 Equation S9 showed that a smoothed soft parcellation $P_s = PF_s$ can be used to directly derive smoothed
1303 atlas connectivity maps from tractography. In this section, a formal proof will be provided for the following
1304 statement:

1305 **Theorem S.2.** The smoothed soft parcellation $P_s = PF_s$ can be computed in the absence of F_s , by separately
1306 smoothing every row of P , followed by normalizing every column of the smoothed parcellation:

$$P_s = N(PF_G) \quad (\text{S20})$$

1307 *Proof.* Using the previously derived equations, we prove that $P_s = N(PF_G)$:

$$\begin{aligned} P_s &= PF_s && \text{: from Equation S9} \\ &= PF_G \langle |F_G| \rangle^{-1} && \text{: from Equation S18} \\ &= PF_G \langle |PF_G| \rangle^{-1} && \text{: from Equation S15} \\ &= N(PF_G) && \text{: from Equation S13} \end{aligned}$$

1308

□

The proof above confirms that structural connectivity based on a parcellation atlas, incorporating CBS, can be constructed directly from a tractogram and soft parcellation, without necessitating computation of either the high-resolution smoothing matrix or the high-resolution connectome. To smooth an atlas-resolution connectome, the brain atlas P should first be transformed to a normalized smoothed soft parcellation $P_s = N(PF_G)$. PF_G is equivalent to independently smoothing the binary representation of each parcel, while the normalization of such ensures that the sum of parcel memberships of every vertex is 1. Hence, the soft-parcellation P_s can be computed by spatial smoothing and then be directly combined with the tractogram to produce a connectome: each streamline endpoint may have non-zero attribution to multiple parcels, and the contribution of the streamline to the connectome is therefore distributed across the set of edges associated with those two sets of parcels. This constitutes an approach to apply CBS on atlas-resolution connectomes that does not require any high-resolution connectomic computations.

S.3. Replication of ROC curve estimates

The computation of ROC curves reported in the manuscript relied on the assumption of a fixed false omission rate ($\lambda = \frac{FN}{FN+TN}$). To ensure that the findings were not biased by the selected value for λ , the same analyses was repeated for a range of plausible values of $\lambda \in \{10\%, 1\%, 0.1\%\}$. Fig. S1 presents the results of this evaluation. The findings indicate that CBS increases the sensitivity of the statistical analyses and the inference power, particularly for connectomes mapped from probabilistic tractography, regardless of the selection made for the false omission rate λ .

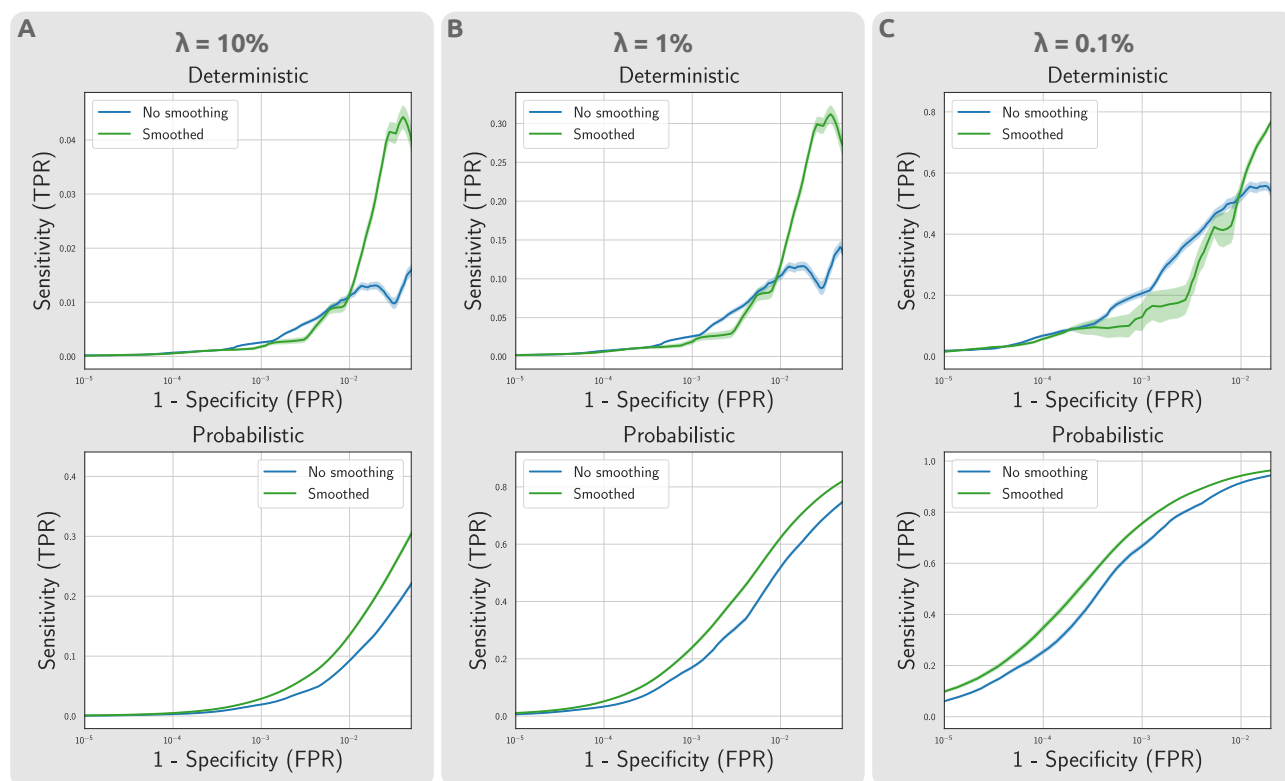


Fig. S1. Impact of CBS on statistical power of mass univariate testing on atlas-based connectomes, for different false omission rate assumptions. The estimated ROC curves demonstrate the respective changes in sensitivity and specificity of the suprathreshold edges at different effect thresholds. The analyses was repeated across a range of false omission rates to ensure the robustness of findings with regards to parameter selection.