Membrane fission during bacterial spore development requires DNA-driven cellular inflation

Ane Landajuela[1,2†*], Martha Braun[2,3†*], Alejandro Martínez-Calvo[4], Christopher D. A. Rodrigues[5], Thierry Doan[6], David Z. Rudner[7], Ned S. Wingreen[8,9], Erdem Karatekin[1,2,3,10*]

Cellular and Molecular Physiology, Yale University, New Haven, CT, USA

Nanobiology Institute, Yale University, West Haven, CT, USA

Molecular Biophysics and Biochemistry, Yale University, New Haven, CT, USA

Grupo de Mecánica de Fluidos, Universidad Carlos III de Madrid, Spain

iThree Institute, University of Technology Sydney (UTS), Australia

Laboratoire d'Ingénierie des Systèmes Macromoléculaires, Aix-Marseille Université - CNRS UMR7255, Marseilles, France

Department of Microbiology, Harvard Medical School, Boston, Massachusetts,USA

Department of Molecular Biology, Princeton University, Princeton, NJ 08544, USA.

Lewis-Sigler Institute for Integrative Genomics, Princeton University, Princeton, NJ 08544, USA.

Université de Paris, SPPIN - Saints-Pères Paris Institute for the Neurosciences, Centre National de la Recherche Scientifique (CNRS), F-75006 Paris, France.

† These authors contributed equally

* To whom correspondence should be addressed

ane.landajuela@yale.edu

matha.braun@yale.edu

erdem.karatekin@yale.edu (lead contact)

SUMMARY

Bacteria require membrane fission for cell division and endospore formation. FisB
catalyzes membrane fission during sporulation, but the molecular basis is unclear as it
cannot remodel membranes by itself. Sporulation initiates with an asymmetric division
that generates a large mother cell and a smaller forespore that contains only 1/4 of its
complete genome. As the mother cell membranes engulf the forespore, a DNA
translocase pumps the rest of the chromosome into the small forespore compartment,
inflating it due to increased turgor. When the engulfing membranes undergo fission, the
forespore is released into the mother cell cytoplasm. Here we show that forespore
inflation and FisB accumulation are both required for efficient membrane fission. We
suggest that high membrane tension in the engulfment membrane caused by forespore
inflation drives FisB-catalyzed membrane fission. Collectively our data indicate that
DNA-translocation has a previously unappreciated second function in energizing FisB-
mediated membrane fission under energy-limited conditions.

HIGHLIGHTS

- Membrane fission during endospore formation requires rapid forespore inflation by
ATP-driven DNA translocation.

- Forespore inflation increases the tension of the engulfment and forespore membranes
to near lysis tensions.

- FisB catalyzes membrane fission, but only if the membrane tension is high.

- Membrane fission utilizes chemical energy transduced to mechanical energy during
DNA-packing into the forespore.

INTRODUCTION

When nutrients are scarce, spore-forming bacteria initiate a morphological process called sporulation, which produces dormant, stress-resistant spores that can remain dormant for many years until conditions are favorable for germination (Moir and Cooper, 2015; Setlow, 2003). The first morphological event during sporulation is the formation of an asymmetric septum that produces a small forespore and a larger mother cell (Figure 1A). The mother cell membranes then engulf the forespore in a process reminiscent of phagocytosis. When the edges of the engulfing membrane reach the cell pole, they undergo membrane fission to release the forespore (Figure 1A inset), now surrounded by two membranes, into the mother-cell cytoplasm. The mother cell then packages the forespore in a protective cortex and coat while the forespore prepares for dormancy. When the forespore is mature, the mother cell lyses, releasing the dormant spore into the environment. Before asymmetric division, the chromosomes are remodeled into an axial filament with the origins of replication at the cell poles and the termini at the mid-cell (Webb et al., 1997). Thus, immediately after asymmetric division, only ~1/4 of the forespore chromosome is trapped inside the forespore (Wu and Errington, 1994, 1998). The rest of the chromosome is pumped into the forespore by an ATPase, SpoIIIE (Bath et al., 2000; Wu and Errington, 1997), consuming one ATP molecule per every two base pairs translocated (Liu et al., 2015). As the chromosome is packed into the forespore, the forespore volume doubles from its initial size of $\sim 0.1\ \mu m^3$ by the time the engulfment membrane migrates completely around the forespore and continues to grow to $\sim 0.3\ \mu m^3$ about 3 h after septation (Lopez-Garrido et al., 2018). Forespore inflation is driven by osmotic forces which are mainly due to the counterions that are needed to neutralize the highly charged DNA (Lopez-Garrido et al., 2018). Upon complete DNA packing, the pressure difference between the forespore and mother cell reaches ~60 kPa (Lopez-Garrido et al., 2018). It has been suggested that DNA translocation can be seen as an energy transduction process, converting the chemical energy stored in ~1.5 million ATP molecules to mechanical energy, stretching the thin peptidoglycan layer between the forespore and engulfment membranes and smoothing their wrinkles (Lopez-Garrido et al., 2018). However, it is not clear if this stored mechanical energy is ever utilized by the nutrient-deprived, sporulating bacteria, and if so, how.

Here, we investigate the relationship between forespore inflation driven by DNA translocation and the membrane fission event that releases the forespore into the mother cell cytoplasm. We previously showed that this event is catalyzed by the protein FisB that accumulates at the neck of the engulfing membranes during sporulation in *Bacillus subtilis* (Doan et al., 2013; Landajuela et al., 2021). FisB, a ~250 residue protein that is conserved among endospore-forming bacteria, is produced after asymmetric division in the mother cell (Doan et al., 2013). It possesses a short N-terminal cytoplasmic domain, a single-pass transmembrane domain (TMD), and a large extracytoplasmic domain (ECD, Figure 1B) (Landajuela et al., 2021). In cells lacking FisB (*ΔfisB*), engulfment proceeds normally, but the membrane fission step is impaired (Doan et al., 2013; Landajuela et al., 2021). Membrane fission can be assessed during a sporulation time course by labeling cells with a lipophilic dye that crosses the cell membrane inefficiently (Figure 1C). In cells that have not yet undergone fission, the dye has access to the forespore, engulfing, and mother cell membranes, resulting in a

stronger signal where these three membranes are in close proximity. By contrast, in
cells that have undergone membrane fission, the dye only weakly labels the internal
membranes, allowing the quantification of cells that have undergone fission at a given
time point after initiation of sporulation (Landajuela et al., 2021) (Figure 1D). A
functional GFP-FisB fusion forms dim mobile clusters (DMC), each containing ~12 GFP-
FisB molecules. An intense spot at the engulfment pole (ISEP) containing ~40 copies of
GFP-FisB appears around the time of membrane fission (Figure 1E,F and (Landajuela
et al., 2021)). FisB localizes to the membrane neck that connects the engulfment
membrane to the rest of the mother cell membrane based only on lipid-binding, self-
aggregation, and the unique geometry encountered at the end of engulfment
(Landajuela et al., 2021). FisB self-oligomerizes and binds acidic lipids, bridging artificial
membranes, but does not appear to have other binding partners. During sporulation, the
only location where FisB can bridge membranes is at the membrane neck that
eventually undergoes membrane fission (Figure 1F), suggesting homo-oligomerization
and trans interactions that bridge membranes are sufficient to explain FisB
accumulation in the neck during late stages of engulfment (Landajuela et al., 2021).
However, how membrane fission occurs at the last step of engulfment is not known.

Here we show that forespore inflation and FisB accumulation at the membrane neck are
both required for efficient membrane fission. Interventions that slowed or prevented
forespore area expansion delayed or inhibited membrane fission, even when FisB had
accumulated at the membrane neck. Conversely, sporulating cells lacking FisB had
impaired membrane fission but their forespores expanded. We suggest that high
membrane tension in the engulfment membrane caused by forespore inflation drives
membrane fission catalyzed by FisB. Our results demonstrate that part of the
mechanical energy stored during the process of DNA packaging into the forespore is
used for membrane fission.

RESULTS

FisB does not remodel membranes, but forms a stable, extended network on GUV
membranes

The simplest mechanism of FisB-catalyzed membrane fission would involve direct
membrane remodeling by FisB, similar to eukaryotic membrane fission machineries
dynamin (Ferguson and De Camilli, 2012) or ESCRT-III (Schoneberg et al., 2017). To
test this idea, we studied interactions of the FisB extracellular domain (ECD) with model
GUV membranes. We fluorescently labeled recombinant purified FisB ECD with
iFluor555-maleimide at a cysteine introduced at residue 123 (iFluor-FisB ECD, Figure
2A), a substitution that does not affect FisB's function (Landajuela et al., 2021). GUV
membranes were labeled by including a fluorescent lipid in the lipid composition.
Membranes and the labeled protein were then visualized using spinning-disc confocal
(SDC) microscopy (Figure 2). We first incubated GUVs with various concentrations of
iFluor-FisB ECD in a closed chamber for 60 min, then imaged the membranes and FisB
ECD (Figure 2B). iFluor-FisB ECD did not bind GUVs lacking acidic lipids as reported
previously (Landajuela et al., 2021). By contrast, iFluor-FisB ECD bound GUVs
containing 30 mole % cardiolipin (CL). At 20 nM in solution FisB binding to GUVs was

barely detectable, but at ~75 nM, discrete FisB spots were visible. These spots were
diffused around the GUV membrane, reminiscent of the DMC in live cells. At 200 nM
and above, most GUVs were covered more uniformly and completely by iFluor-FisB
ECD. Consistent with previous work (Landajuela et al., 2021), and unlike many proteins
implicated in eukaryotic membrane fission (Campelo and Malhotra, 2012; Renard et al.,
2018; Simunovic et al., 2019), FisB did not cause any membrane deformations such as
tubulation or invaginations. Because such deformations are opposed by membrane
tension, we repeated these experiments using deflated GUVs with lower membrane
tension. Even at high coverage and when the GUVs were deflated, FisB did not cause
any remodeling of GUV membranes (Figure 2B). However, we noticed that deflated
GUVs covered with iFluor-FisB ECD did not display shape fluctuations typical of bare
deflated GUVs, suggesting FisB may form a fixed, extended network on the membranes
stabilizing the membrane shape.

We reasoned that if FisB ECD does indeed form an extended network on membranes,
its mobility should be low in such regions. To test this, we performed fluorescence
recovery after photobleaching (FRAP) experiments (Figure 2C). We incubated 1 µM
iFluor-FisB ECD with GUVs for 2 h. This allowed for a range of iFluor-FisB ECD
coverage of GUVs. Some GUVs were uniformly covered by iFluor-FisB ECD whereas
some others were covered only partially by iFluor-FisB ECD patches. We bleached the
iFluor-FisB ECD fluorescence in a rectangular region of interest (ROI) and monitored
the subsequent recovery on a given GUV. Recovery was faster for GUVs that had
partial, patchy iFluor-FisB ECD coverage, as new patches diffused into the bleached
region (Figure 2C upper row and Figure 2D). By contrast, there was virtually no
recovery for GUVs that were uniformly covered by iFluor-FisB ECD (Figure 2C lower
row and Figure 2D). To relate FisB ECD membrane coverage to mobility, we plotted the
mean iFluor-FisB ECD signal along the contour of the GUV, $F_{\mathrm{mean}}^{\mathrm{ECD}}$, against the fractional
fluorescence recovery 45 s after bleaching, as shown in Figure 2E. This analysis
showed that increasing FisB ECD coverage led to decreased FisB ECD mobility,
consistent with the idea that FisB ECD forms a denser, less dynamic network as
coverage increases.

To test how stable the FisB ECD network is, we dissolved the membranes after the
network had formed while observing iFluor-FisB ECD signals. Five to ten minutes after
addition of Triton X-100 the lipid signal disappeared, suggesting that the membranes
were dissolved. Remarkably, iFluor-FisB ECD signals were intact (Figure 2F), indicating
that once formed, the three-dimensional FisB ECD network was stable even after the
removal of the membrane.

Altogether, these data argue that FisB does not remodel membranes but is capable of
forming an extended, stable network in which individual FisB molecules are immobile.

Forespore inflation accompanies membrane fission

How can FisB catalyze membrane fission if it cannot remodel membranes by itself? We
reasoned that an additional cellular process should be involved, given that direct
interactions with other proteins could not be detected (Doan et al., 2013; Landajuela et
al., 2021). A process that could potentially influence membrane fission is the forespore

volume increase that occurs after asymmetric division due to translocation of DNA into
the forespore by the ATPase SpoIIIE (Lopez-Garrido et al., 2018) (Figure 3A). However,
whether and how this volume increase is related to membrane fission was not known.
We began by investigating whether there was a correlation between forespore inflation
and membrane fission. We used the lipophilic fluorescent dye 1-(4-
trimethylammoniumphenyl)-6-phenyl-1,3,5-hexatriene p-toluenesulfonate (TMA-DPH)
that allowed us to detect forespore contours of both cells that had undergone
membrane fission and those that had not (Landajuela et al., 2021). We used a semi-
automated active-contour fitting algorithm (Smith et al., 2010) and estimated the
membrane areas from the surface of revolution around the long axis of symmetry
(Methods). Three hours after initiation of sporulation via nutrient downshift, post-fission
forespores were visually larger than their pre-fission counterparts (Figure 3B). Pre- and
post-fission forespore areas decreased and increased as a function of time after the
nutrient downshift, respectively (Figure 3C), suggesting cells with rapidly growing
forespores underwent membrane fission, leaving behind smaller pre-fission forespores.
Similar trends were obtained using a soluble CFP forespore marker (Figure S1), ruling
out potential artifacts due to the use of a lipophilic membrane dye. The close link
between forespore inflation and membrane fission was more evident when we plotted
the percentage of cells having undergone membrane fission against the post-fission
forespore area, which yielded a nearly linear relationship (Figure 3D).

The forespore area difference between pre- and post-fission cells, ΔA_{FS}, increased
during sporulation to reach 1.3 ± 0.1 µm^2 (Figure 3E). Taking into account the 4
membrane leaflets that must grow together and assuming an area per lipid of ~ 0.7 nm^2,
$\sim 7.4 \times 10^6$ new lipids must be added to the engulfment and forespore membranes
during sporulation. Given that this growth occurs over ~2 h, lipids must be added at an
average rate of ~1,000 lipids/s to the forespore and engulfment membranes.
Interestingly, vegetative cell membrane areas grow at a comparable rate ($\dot{A}_{\mathrm{veg}} = 0.020$
μm^2/min, or about ~1000 lipids/s, taking into account two leaflets, 95% confidence
interval: 0.007-0.023 µm^2/min, n=8 cells), consistent with the finding that phospholipid
synthesis during sporulation occurs near the maximal rate associated with vegetative
growth (Pedrido et al., 2013). Thus, forespore inflation requires high rates of *de novo*
phospholipid synthesis despite energy-limiting conditions, and most of the newly
synthesized lipids are utilized for forespore area growth during late stages of engulfment
and after fission.

In summary, there is a clear correlation between membrane fission at the end of
engulfment and forespore inflation, suggesting that forespore growth may facilitate
membrane fission.

Hypothesis: increasing membrane tension during forespore inflation drives FisB-
dependent membrane fission

How can forespore inflation promote FisB-mediated membrane fission? We
hypothesized that increased membrane tension due to forespore inflation, coupled with
FisB oligomerization at the membrane neck, could work synergistically to drive
membrane fission. Because DNA pumping into the forespore leads to forespore inflation

through increased turgor and smooths membrane wrinkles (Lopez-Garrido et al., 2018),
there must be an associated increase in the tension of the forespore membrane.
Furthermore, because the forespore and engulfment membranes are separated through
a thin layer of peptidoglycan only ~20 nm wide (Khanna et al., 2019; Tocheva et al.,
2011), we expect the tension and the area of the engulfment membrane (which
eventually undergoes fission) to increase together with that of the forespore membrane.
The membrane tension of the forespore and engulfment membranes can be estimated
from the excess osmotic pressure in the forespore, ~60 kPa (Lopez-Garrido et al.,
2018). Assuming the two membranes that surround the forespore balance the excess
osmotic pressure, then according to Laplace's Law $\Delta P = 4\gamma/R_{fs}$, so that a forespore
radius of $R_{fs} = 400$ nm implies a membrane tension $\gamma \sim 6$ pN nm^{-1}. This is a very high
value, around values expected for membrane rupture (Evans et al., 2003; Needham and
Nunn, 1990; Ojkic et al., 2014). It is likely that the peptidoglycan cell wall surrounding
the forespore may actually be necessary during forespore inflation to avoid membrane
rupture (Lopez-Garrido et al., 2018).

The hypothesis that increasing membrane tension during forespore inflation drives FisB-
dependent membrane fission leads to the following predictions. First, interfering with
forespore inflation should hinder membrane fission. When forespore inflation is slowed
or blocked, FisB should be properly localized to the membrane neck (provided
engulfment is not compromised), because FisB's localization relies solely on its
interactions with itself and with membranes, and the geometry of the neck (Landajuela
et al., 2021). Second, in the absence of FisB, forespores should inflate but membrane
fission should be impaired.

Blocking lipid synthesis impedes forespore inflation and membrane fission

Lipid synthesis is required during vegetative growth (Yao et al., 2012; Zhang and Rock,
2008) and sporulation (Pedrido et al., 2013; Schujman et al., 1998) and fatty acid
availability determines average cell size (Vadia et al., 2017). When *B. subtilis* cells are
placed in a nutrient poor medium, lipid synthesis is initially downregulated, but with the
onset of sporulation, it returns close to the maximal rate (Pedrido et al., 2013). We
reasoned that blocking lipid synthesis at different time points after the initiation of
sporulation should block forespore area expansion at different stages. Furthermore, if
forespore inflation is needed for membrane fission, the percentage of forespores
blocked in inflation should correlate with those stalled at the pre-fission stage.

We blocked the synthesis of neutral lipids and phospholipids at different time points t
after nutrient downshift using cerulenin, a drug that inhibits *de novo* fatty acid
biosynthesis (Heath and Rock, 2004) (Figure S2A). We then probed the cells for
membrane fission at $t = 3$ h using TMA-DPH labeling. We found that the earlier lipid
synthesis was blocked, the smaller the fraction of cells that had undergone membrane
fission at $t = 3$ h (Figure S2B,C). Both pre-and post-fission forespore membrane areas
decreased with longer cerulenin application (Figure S2D). These results are consistent
with previous reports that de novo lipid synthesis is needed for engulfment and
forespore inflation (Lopez-Garrido et al., 2018; Pedrido et al., 2013). In addition to
perturbing engulfment (Figure S2E,F) FisB accumulation at the fission site was reduced

in the presence of cerulenin (Figure S2G), making it difficult to infer the role of forespore
inflation in membrane fission. In order to unmask the contribution of forespore inflation
on membrane fission, we focused on cells in which FisB had successfully accumulated
at the membrane fission site. We discovered that an increasing fraction of these cells
failed to undergo membrane fission with longer cerulenin application (Figure S2H).
Thus, although impaired engulfment and FisB accumulation at the fission site partially
account for the membrane fission defects observed, blocking lipid synthesis inhibited
membrane fission even for cells with proper FisB localization, suggesting forespore
inflation facilitates membrane fission.

Slowing DNA translocation slows membrane fission

In addition to lipid synthesis, forespore inflation requires ATP-dependent DNA
translocation into the forespore. Accordingly, in a complementary set of experiments we
took advantage of previously characterized mutations in the DNA translocase SpoIIIE to
investigate whether forespore inflation contributes to membrane fission.

We began by using an ATPase mutant (SpoIIIE36) that is translocation defective
(Pogliano et al., 1997). In the absence of DNA pumping, the forespores remained small
and engulfment was severely perturbed (Figure S3A), similar to the engulfment
phenotype of the ∆$spoIIIE$ mutant reported previously (Doan et al., 2013). Membrane
fission was severely impaired in these cells (Figure S3B) as was forespore area growth,
both for pre-fission and the small number of post-fission forespores (Figure S3C). The
fission defect can only be partly explained by deficient FisB localization, as 23±4 % of
the SpoIIIE36 cells had an ISEP at 3h after the nutrient downshift, compared to 70±6%
of WT cells (Figure S3G), yet only 5% of the SpoIIIE36 cells had undergone membrane
fission at this timepoint (Figure S3B). Importantly, among cells with an ISEP at the end
of engulfment, only ~16±4% of SpoIIIE36 cells had undergone membrane fission
compared to ~91±6% % of WT cells (Figure S3H). This difference was even more
pronounced when used a sensitized background in which FisB was expressed at ~8-
fold lower levels than wild-type (Doan et al., 2013; Landajuela et al., 2021). Under these
conditions, there were no post-fission SpoIIIE36 cells at hour 3 of sporulation among the
7% that had an ISEP. By contrast, 95% of the sporulating cells with SpoIIIE[WT] that had
an ISEP had undergone membrane fission in this FisB low-expression background
(Figure S3G,H). Altogether, these results suggest that membrane fission requires both
FisB accumulation at the neck to be cut (ISEP formation) and forespore inflation.

The severe engulfment defects in the SpoIIIE36 mutant have the potential to obscure
the contribution of forespore inflation to FisB localization and membrane fission.
Accordingly, we took advantage of a second mutant (SpoIIIE[D584A]) that translocates
DNA ~2.5-fold slower than wild-type (Burton et al., 2007) and does not have engulfment
defects (Figure 4). In sporulating cells expressing $spoIIIE$[D584A], forespore inflation was
delayed for post-fission cells and forespore area increased for pre-fission cells instead
of decreasing as for wild-type cells, suggesting slow forespore inflation slows
membrane fission (Figure 4A-C). This was more evident when the percentage of cells
that had undergone fission and post-fission forespore areas were plotted together as a

function of time (Figure 4D). The percentage of cells that underwent membrane fission increased with the increase in post-fission forespore area (Figure 4E). Importantly, engulfment (Figure 4F,G) and FisB localization (Figure 4H) were similar to wild-type. In fact, by hour 3 of sporulation, *spoIIIE*[D584A] cells accumulated ~1.4-fold more FisB at the cell pole than did wild-type cells (Figure 4I), yet only ~40% of the *spoIIIE*[D584A] cells had undergone membrane fission compared to ~75% of the wild type cells (Figure 4D), demonstrating that accumulating more FisB does not necessarily lead to more efficient membrane fission.

Together, these results strongly suggest that FisB accumulation at the membrane neck (ISEP formation) is *not* a sufficient condition for membrane fission; fission additionally requires forespore enlargement.

In the absence of FisB, forespores inflate but fission is impaired

Our data suggest that FisB accumulation at the fission site and forespore inflation work together to drive membrane fission. To investigate whether these two processes occur independently, we monitored forespore membrane area changes in cells expressing FisB at low levels or altogether lacking it.

We analyzed forespore area at different time points after a nutrient downshift, using TMA-DPH (Figure 5). As anticipated, most sporulating cells lacking FisB (*ΔfisB*) did not undergo membrane fission (Figure 1D) despite completing engulfment (Figure 5A) (Doan et al., 2013; Landajuela et al., 2021). Remarkably, the forespore areas of these pre-fission cells, $A_{\text{FS}}^{\text{pre}}$, increased as a function of time after the nutrient downshift, nearly doubling by $t = 3.5$ h, while those of wild-type cells decreased slightly during the same period (Figure 5B). The forespore areas of the small fraction of *ΔfisB* cells that underwent membrane fission (Figure 1), $A_{\text{FS}}^{\text{post}}$, were slightly larger than those of wild-type cells and followed a similar time course, increasing as a function of time (Figure 5C).

Sporulating cells expressing GFP-FisB at ~8-fold lower levels than wild-type undergo membrane fission more slowly (Figure S4). In these cells, post-fission forespore areas, A_{FS}^{post}, were slightly larger than those expressing GFP-FisB at native levels, and in both cases A_{FS}^{post} increased over time (Figure S4F). Similar to the *ΔfisB* mutant, the pre-fission forespore area increased as a function of time when GFP-FisB levels were reduced albeit the increase was more modest (Figure S4E).

Overall, these results indicate that forespore inflation occurs independently of FisB, but that forespore inflation is not sufficient to catalyze efficient membrane fission. When FisB is present, the fission process is greatly accelerated by forming an aggregate at the membrane neck to be severed, but only if the forespore is inflating (Figure 5D).

Modeling suggests increased engulfment membrane tension is the primary driver of FisB-catalyzed membrane fission

Next, we modeled the effect of increased membrane tension on membrane neck
geometry, including the roles of hydrodynamic drag due the DNA pumping and the
osmotic pressure difference between the cytoplasm and the lumen of the neck. We
developed a minimal model based on free energy minimization, considering the free
energy F of an axisymmetric and mirror symmetric membrane neck connecting two
membrane sheets, corresponding to the local geometry of the neck connecting the
forespore and mother cell membranes (see SI Appendix for details). The energy
functional F consists of a term accounting for membrane bending and tension, F_m,
another term, F_p, accounting for an osmotic pressure difference $\Delta p = p_\mathrm{cyto} - p_\mathrm{peri}$
between the cytoplasm and periplasm since the lumen of the neck is continuous with
the periplasmic space, and finally a term representing any pulling force f on the
forespore, e.g. due to DNA translocation, F_f.

We first explored the role of membrane tension, γ, on neck geometry. As shown
previously (Landajuela et al., 2021) using a minimal model for the neck as a cylinder of
radius r_cyl and length L_cyl, there are two regimes: (1) For a large neck (radius larger than
a critical radius r_crit), membrane tension drives the neck to become ever larger. (2) For
an initial $r_\mathrm{cyl} < r_\mathrm{crit}$, the neck will relax to a finite equilibrium radius set by the balance
between membrane tension and membrane curvature. Intuitively, these two regimes
follow because as the radius of the cylindrical neck grows, the area of the cylinder
grows $\sim L_\mathrm{cyl}\, r_\mathrm{cyl}$, but the associated flat membrane areas on the mother cell and
forespore *shrink* $\sim r_\mathrm{cyl}^2$. While the former term can dominate for small r_cyl leading to a
local minimum, the latter term eventually dominates for large r_cyl implying the neck will
open. The existence of a local minimum of neck radius is recapitulated by the full model
in SI Appendix Eq. 1. As seen in SI Appendix Fig. 2, in regime (2) increasing membrane
tension γ results in a narrowing of the equilibrium neck radius. While to our knowledge
there has been no direct measurement of the membrane tension in *B. subtilis*, we find
that reasonable values of $\gamma = 0.1 - 1$ pN nm^{-1} are sufficient to achieve a neck radius <
10 nm (Ojkic et al., 2014). It is likely that cell wall remodeling that drives engulfment
(Ojkic et al., 2016) brings the neck radius into regime (2), because a neck forms and a
small fraction of cells undergo membrane fission even in the absence of FisB (Figure 1
and Figure 5).

To examine the additional role of osmotic pressure difference between the cytoplasm
and the lumen of the neck, we also minimized F at finite Δp. While $\Delta p = 0.1$ atm has
little effect on the neck radius, $\Delta p = 1$ atm, which is realistic for *B. subtilis* (Lopez-
Garrido et al., 2018; Ojkic et al., 2014) results in a substantial further reduction of the
minimum neck radius (SI Appendix Fig. 2). This reduction makes sense, as higher
osmotic pressure in the mother cell will tend to compress the volume of the periplasmic
space contained within the neck.

By contrast, a pulling force on the forespore not only stretches the neck but also
increases its equilibrium radius (see SI Appendix Fig. 4). A pulling force of $f = 50 -$
100 pN, around the maximum force that can be applied by one or two SpoIIIE motors
translocating DNA into the forespore (Allemand and Maier, 2009; Liu et al., 2018),
increases the minimum neck radius less than 1 nm when $\Delta p = 1$ atm, $\gamma > 5$ pN nm^{-1}.

One can understand this effect simply by considering the total force in the neck at the
midpoint, which is the product of the circumference of the neck times the membrane
tension, $f = 2\pi r_{min}\gamma$. So at fixed membrane tension γ, an increase of the pulling force f
automatically implies an increasing r_{min}.

It is likely that the membrane tension in the rapidly expanding forespore and engulfment
membranes is higher than in the mother cell membrane. We estimated an upper limit for
this membrane tension gradient, by estimating the gradient that would be sufficient to
drive a flux of lipids in the neck matching the observed average increase in forespore
membrane area (see above and SI Appendix). Perhaps surprisingly, the resulting
difference is only $\sim 10^{-7}$ pN nm^{-1}, which is orders of magnitude less than the expected
overall membrane tension. Thus any flux of lipids between the mother cell and
forespore will have negligible effect on the equilibrium results presented above.
However, if this flux is impeded, e.g. by accumulation of FisB at the neck, the tension in
the engulfment membrane around the forespore would increase further, possibly
contributing to membrane fission (see below).

In summary, modeling suggests that the pulling force on the forespore exerted by DNA
translocation has a negligible effect on neck geometry. By contrast, a realistically high
membrane tension, possibly augmented by an osmotic pressure difference across the
membrane, is sufficient to drive the equilibrium neck radius down to $\leq$ 5-10 nm, with two
expected consequences. First, with a small neck radius, FisB can interact in *trans* and
accumulate in the neck, facilitating membrane fission (Landajuela et al., 2021). Second,
high membrane tension and a small neck radius likely both facilitate membrane fission.

Membrane fission due to high membrane tension

Our results so far suggest that increased membrane tension due to forespore inflation
drives FisB-catalyzed membrane fission. Increased membrane tension in the
engulfment membrane would provide a driving force for membrane flow from the mother
cell to the engulfment membrane through the neck connecing the two compartments.
One possibility is that by forming an oligomer at the membrane neck, FisB impedes lipid
movement through the neck, further increasing the tension in the engulfment
membrane, promoting membrane fission. Here we tested whether forespore inflation
leads to membrane movement between the mother cell and engulfment membrane
compartments.

We followed the kinetics of forespore area growth in individual cells and related the
kinetics to membrane fission. Using time-lapse microscopy, we imaged individual
sporulating cells labeled with the lipophilic dye FM4-64 (Lopez-Garrido et al., 2018) as
shown in Figure 6A,B (see Figure S5 for more examples). While TMA-DPH crosses the
cell membrane slowly, FM4-64 does not cross it, which allowed us to detect the time of
membrane fission in addition to monitoring membrane areas. The non-fluorescent FM4-
64 molecules in the aqueous phase exchange with the membrane-inserted fluorescent
ones (Cochilla et al., 1999). Membrane fission isolates the small inter-membrane space
from the bath, blocking exchange of bleached dyes with unbleached ones in the bath
and accelerating decay of the florescence signals arising from the forespore and

engulfment membranes. A plot of forespore contour fluorescence as a function of time
for individual cells shows a stable signal, which starts decreasing upon membrane
fission. We cross-correlated these individual fluorescence profiles with a model decay
function to determine the timing of membrane fission (Figure 6C). We defined t = 0 as
the frame just preceding fission (Figure 6B,C) and aligned all measurements with
respect to this time before averaging membrane areas as shown in Figure 6D,E. The
rate of forespore area increase slowed gradually after membrane fission (Figure 6D).
Remarkably, the mother cell area shrank before and grew after membrane fission
(Figure 6E).

Until FisB accumulation at the membrane neck and membrane fission, there are two
sources of lipids to support the growth of the forespore and engulfment membranes: de
novo lipid synthesis and lipid flux between the mother cell and engulfment membranes
(Figure 6F). After membrane fission, substantial expansion of the forespore and
engulfment membranes can only be supported by de novo lipid synthesis. Thus,
analysis of the rates of area changes can be informative about lipid flux through the
neck. We focused on the mother cell area changes for this analysis, because the lipid
synthesis rate in the forespore likely increases as a function of time as the lipid
synthase genes are gradually translocated into the forespore (Pedrido et al., 2013). The
mother cell area changed at rates of $-0.011\ \mu m^2/s$ (-262 lipids/s) and $+0.0032\ \mu m^2/s$
(+76 lipids/s) before and after fission, respectively. Assuming the rates of insertion into
the mother cell of newly synthesized lipids pre- and post-fission do not change
appreciably, we estimate ~340 lipids/s move through the neck toward the engulfment
membrane before FisB accumulation –a significant fraction of the maximal cellular lipid
synthesis capacity (see above).

Membranes lyse when stretched above only ~1 % in area (Evans et al., 2003). A deficit
of ~300 lipids/s would cause a 1% deficit of the $\sim 2.5\ \mu m^2$ engulfment membrane area
within a few minutes if the area expanded at fixed number of lipids. Thus, it is possible
that FisB oligomerization interferes with lipid flux through the neck, leading to a further
increase in the engulfment membrane tension, facilitating membrane fission.

DISCUSSION

Bacteria rely on membrane fission for every division cycle and during endospore
formation. FisB is the only molecule described so far with a dedicated role in membrane
fission in bacteria (Doan et al., 2013; Landajuela et al., 2021), but how FisB catalyzes
membrane fission has remained unclear. Here we report that FisB does not remodel
membranes directly but forms a stable network on them. Thus, another cellular process
must be involved to achieve membrane fission at the end of engulfment. Surprisingly,
we found that pumping of the chromosomal DNA into the forespore by the ATPase
SpoIIIE –a seemingly unrelated process– is a necessary condition for FisB-dependent
membrane fission. DNA pumping into the forespore leads to forespore inflation through
increased turgor, stretching the thin layer of peptidoglycan between the forespore and
engulfment membranes and smoothing membrane wrinkles (Lopez-Garrido et al.,

2018). FisB accumulation at the membrane fission site and forespore inflation are both
necessary conditions for efficient membrane fission.

We propose that increased membrane tension in the engulfment membrane due to
forespore inflation drives membrane fission (Figure 7). High membrane tension can
cause membrane fission through opening of a transient pore (Karatekin et al., 2003), as
was shown for fission catalysed by BAR domain proteins scaffolding a membrane tube
under extension (Simunovic et al., 2017). The presence of a protein scaffold on a
membrane tube has a dual role: slowing equilibration of membrane tension by impeding
membrane flow, and acting as a heterogeneous nucleation point for a membrane
defect, lowering the energy barrier for membrane poration (Simunovic et al., 2017).
Thus, FisB can similarly have a dual role in membrane fission. First, by oligomerizing at
the membrane neck, FisB impedes membrane flow from the mother cell membrane to
the engulfment membrane where tension is higher, leading to a further increase in
tension, making fission more likely. Second, the FisB oligomer-bare membrane interface
could provide a nucleation point for a membrane defect at high tension.

Membrane fission requires energy input. For all biological membrane fission reactions
described to date, membrane fission is energized and regulated locally. For the best
characterized cases, membrane fission proteins dynamin (Ferguson and De Camilli,
2012) or ESCRT III (Schoneberg et al., 2017) polymerize at the membrane fission site
and use the energy liberated by nucleoside triphosphate hydrolysis to reshape
membranes locally. By contrast, here we have found that membrane fission during
endospore formation in *B. subtilis* is energized indirectly through DNA translocation into
the forespore –a process that is required primarily for genetic inheritance and at first
sight bears little relation to the membrane fission that occurs at the opposite end of the
engulfment membrane. This situation is analogous to secondary transporters using
electrochemical gradients set by primary pumps such as the sodium-potassium ATPase
to energize secondary transport processes (Boron and Boulpaep, 2017).

Additionally, our results clarify the role of SpoIIIE in membrane fission during endospore
formation. It was originally proposed that in addition to DNA translocation, SpoIIIE
drives membrane fission because an ATPase deficient mutant resulted in membrane
fission defects (Liu et al., 2006; Sharp and Pogliano, 1999). However, SpoIIIE's role in
membrane fission remained unclear because the knock-out or ATPase dead mutants
(Figure S3 and (Doan et al., 2013)) cause severe engulfment defects. Since FisB
localization and membrane fission occur downstream of engulfment, any engulfment
defect would also produce FisB localization and membrane fission defects and would
not be particularly informative about membrane fission itself. Here we establish that
SpoIIIE does indeed play a critical role in membrane fission even in the absence of
engulfment defects, but this role is related to forespore inflation.

The membrane fission mechanism we have described here provides an elegant
example of how bacteria can use energy very efficiently under starvation conditions.
DNA translocation into the forespore is an energy-consuming process that cannot be
bypassed, but the byproduct of this process, namely increased turgor in the forespore,
can be harnessed to energize secondary processes. It will be interesting to see if

processes other than membrane fission are energized by mechanical energy stored in
the inflated forespore.

ACKNOWLEDGEMENTS

We thank all members of the Karatekin laboratory for fruitful discussions and Jeorg
Nikolaus (director of the Yale West Campus Imaging Core) for help with imaging. This
work was supported by National Institute of General Medical Sciences and National
Institute of Neurological Disorders and Stroke of the National Institutes of Health (NIH)
under award numbers R01GM114513 and R01NS113236 (to EK). The content is solely
the responsibility of the authors and does not necessarily represent the official views of
the National Institutes of Health. We gratefully acknowledge a Yale University
Predoctoral Fellowship to MB.

AUTHOR CONTRIBUTIONS

EK, MB, AL, and DZR conceived the study. MB carried out most of the experiments with
purified proteins and artificial membranes, AL carried out live cell imaging and analysis.
MB, EK wrote matlab scripts for image analysis, AL, MB analysed images. AMC+NSW
developed the mathematical model, DZR, NSW and EK supervised the work. CDAR
and TD provided reagents and intellectual input. EK wrote the manuscript, with input
from all authors.

CONFLICT OF INTEREST

544 The authors declare they have no conflicts of interest.

 FIGURE LEGENDS

Figure 1. FisB is required for efficient membrane fission during sporulation. **A.**
Schematic of sporulation stages. Upon starvation, sporulation is initiated by asymmetric
division into a mother cell (MC) and a forespore (FS). The MC then engulf the FS. At the
end of engulfment, a thin neck or tube connects the engulfment membrane to the rest of
the MC membrane (red box). Fission of the neck releases the FS, now surrounded by
two membranes, into the MC cytoplasm. Upon maturation, the FS turns into a spore and
the MC lyses to release it. Inset: membrane fission step marking the end of engulfment.
B. Domain structure of FisB (Doan et al., 2013; Landajuela et al., 2021). **C**. Membrane
fission assay. An aliquot of cells is labeled with the lipophilic dye 1-(4-
trimethylammoniumphenyl)-6-phenyl-1,3,5-hexatriene p-toluenesulfonate (TMA-DPH)
which fluoresces only once inserted into the membrane. The dye labels internal
membranes poorly. If a cell has not yet undergone membrane fission at the time of
TMA-DPH labeling, the dye has access to the space between the engulfment and FS
membranes, resulting in brighter labeling of the MC, FS, and engulfment membranes
when these are adjacent to one another (top row). After fission, the dye labels internal
membranes poorly (bottom row). **D**. Time course of membrane fission for wild-type
cells, *ΔfisB* cells, or *ΔfisB* cells complemented with mGFP-FisB expressed at native
levels (strain BAM003). Some membrane fission occurs even in the absence of FisB
(*ΔfisB*, gray markers). Mean±SEM from 3 independent experiments are shown (>300
cells were analyzed per point). **E.** Wide-field fluorescence microscopy images of cells
expressing mGFP-FisB at native levels (strain BAM003). Aliquots were taken from the
suspension at indicated times after nutrient downshift, labeled with TMA-DPH, and
images of mGFP-FisB and membranes were acquired sequentially. Examples of
sporulating cells with mGFP-FisB enriched at the septum (1.5 h), forming a dim mobile
cluster (DMC; 2.5 h) and with a discrete mGFP-FisB focus at the cell pole (intense spot
at engulfment pole, ISEP, 3 h) are highlighted with arrowheads. Scale bars represent
1μm. **F**. Schematic of FisB dynamics (Landajuela et al., 2021). At the end of
engulfment, ~40 FisB molecules accumulate at the neck into an immobile cluster to
catalyze membrane fission.

Figure 2. FisB does not remodel membranes. A. The soluble recombinant fragment
of FisB comprising its extracytoplasmic domain (ECD) used in the experiments with
GUVs. **B**. Interactions of FisB ECD with GUV membranes ("100% PC" and "30% CL"
membranes are composed of (all mole %) 99 PC, 1 NBD-PE, and 30 *E. coli* CL, 69
eggPC, 1 NBD-PE, respectively). At low concentrations, FisB ECD forms small mobile
clusters. With increasing concentration, labeling becomes more uniform. Even at high
coverage or with deflated GUVs (right), no membrane remodeling is evident (deflated
GUVs were composed of (all mole %) 25 *E. coli* PE, 5 *E. coli* CL, 50 *E. coli* PG, 19
eggPC and 1 DiD or NBD-PE (BS mix)). **C**. Mobility of FisB ECD decreases with
increasing coverage. GUVs composed of BS mix were incubated with 1 μM iFluor555-
FisB ECD for 2h. GUVs were covered with protein to varying degrees. Top row shows
fluorescence recovery of a GUV with low iFluor555-FisB ECD coverage. Bottom row
shows an example of a GUV fully covered with iFluor555-FisB ECD fluorescence that
does not recover after photobleaching. Photobleached regions are indicated with boxes.
D. Normalized fluorescence intensity from the boxed regions in C as a function of time.

E. Higher protein coverage ($\propto$ mean GUV membrane intensity) leads to lower protein
mobility. The fractional fluorescence recovery 45 s post-bleaching ($(F_{45} - F_0)/(1 - F_0)$,
where F_{45} is the fluorescence intensity at 45 s and F_0 is the intensity just after bleaching,
both relative to pre-bleach intensity) is plotted as a function of mean GUV membrane
intensity. Each dot represents a GUV. **F**. FisB ECD forms a fixed, stable network on the
GUV membrane that persists even after the membranes are dissolved with detergent. 1
µM iFluor555-FisB ECD was incubated with deflated GUVs for 2h, then Triton X-100
was added (1.7 mM final concentration, 7-8-fold above the critical micelle concentration
(Tiller et al., 1984)), and the sample imaged 5-10 min thereafter.

Figure 3. Forespore area increase during sporulation correlates with membrane
fission activity. A. Schematic of SpoIIIE-mediated pumping of the chromosome into
the forespore leading to forespore inflation. **B**. Images show wild-type strain (PY79) at
t=3 h after nutrient downshift to initiate sporulation. Membranes were visualized using
TMA-DPH. Forespores of cells that have undergone membrane fission (red contours on
the right panel) are larger than those that have not yet undergone membrane fission
(blue contours). Scale bars represent 1 µm. **C**. Quantification of forespore membrane
area for cells that have (red, "post") or have not (blue, "pre") undergone membrane
fission. **D**. The percentage of cells that have undergone membrane fission as a function
of post-fission forespore area. **E**. The difference between post- and pre-fission
forespore areas, $\varDelta A_{\mathrm{FS}}$, grows as a function of time. In C-E, mean$\pm$SEM of 3
independent experiments are shown, with 70 cells analyzed per data point.

Figure 4. Slower forespore inflation leads to slower membrane fission with no
adverse effect on engulfment or FisB localization. A. Fluorescence microscopy
images of TMA-DPH labeled wild-type SpoIIIE strain BAL039 (top row, times after
nutrient downshift are indicated). The lower row shows the same images, but with the
overlaid forespore contours for cells that have (red) or have not (blue) undergone
membrane fission. **B.** As in A, but for the slow DNA-pumping SpoIIIE strain BAL040
(SpoIIE[D584A]). **C**. The average forespore area for pre- (blue) and post-fission (red) cells
as a function of time into sporulation for the SpoIIE[D584A] (dashed lines, strain BAL040)
or wild-type (solid lines) cells (strain BAL039). (n=25-70 cells per data point). **D**. The
percentage of cells that have undergone membrane fission (left axis, black) followed a
very similar time course as the forespore area as a function of time (right axis, magenta)
for both WT (solid lines) and SpoIIE[D584A] (dashed lines) cells. Both membrane fission
and forespore inflation were delayed in SpoIIE[D584A] cells (>300 cells per data point). **E**.
The percentage of post-fission cells increased with increasing forespore area for both
WT (black) and SpoIIE[D584A] (magenta) cells. WT data is copied from Figure 3 for
comparison. **F**. Fluorescence microscopy images of WT or SpoIIE[D584A] cells expressing
mGFP-FisB at native levels in a *ΔfisB* background, probed 3h after the nutrient
downshift. Membranes were labeled with TMA-DPH. In cells expressing WT SpoIIIE
(top row) post-fission cells have larger forespores (red contours) and FisB is
accumulated at the membrane fission site (red arrowheads). At this stage, most cells
expressing SpoIIE[D584A] are still in the pre-fission stage, even if engulfment is visually
complete and FisB has accumulated at the membrane fission site. **G**. Percentage of WT
or SpoIIE[D584A] cells with engulfment membrane migration completed by t=3 h into
sporulation. Engulfment is not perturbed significantly by the slow DNA pumping by

SpoIIE^{D584A} (>300 cells per data point). **H**. FisB localization is not perturbed in
SpoIIE^{D584A} cells compared to WT cells. **I**. Distributions of mGFP-FisB fluorescence
intensities for the ISEP in willd-type ("WT") and the slow DNA pumping by SpoIIE^{D584A}
mutant strain ("slow"). Representative regions of interest (and regions taken for
background correction, "Bg") are shown on the example images on the left. The means
($\pm$ SD) are 70 ± 17 a.u. for WT and 99 ± 18 for SpoIIE^{D584A} ISEP intensities. Since for
WT this intensity corresponds to ~40 copies of FisB (Landajuela et al., 2021), we
conclude that nearly 60 copies are accumulated in SpoIIE^{D584A}. Scale bars represent 1
µm in A, B, and F. Means$\pm$SEM for 3 biological replicates are plotted in panels C-E,
G,H. See Figure S3 for results with an ATPase dead SpoIIIE mutant.

Figure 5. **In the absence of FisB, forespores inflate but fission is impaired. A**.
Representative images of wild-type (PY79) or *ΔfisB* cells (BDR1083) at the indicated
times after nutrient downshift. Membranes were visualized with TMA-DPH. Lower
panels show the FS contours for cells that have (red) or have not (blue) undergone
fission. Scale bars represent 1 µm. **B**. The average pre-fission forespore area, A_{FS}^{pre},
grows during sporulation for *ΔfisB* cells, while it decreases for the wild-type strain. **C**.
The average post-fission forespore area, A_{FS}^{post}, increases faster for *ΔfisB* cells. **D.**
Summary of results. FisB catalyzes membrane fission in rapidly growing forespores (top
row, "WT rapid inflation"). Slowly inflating forespores are less likely to undergo
membrane fission even in the presence of FisB (middle row, "WT slow inflation"). In the
absence of FisB, forespores inflate, but fission is rare. See Figure S4 for results with a
strain expressing FisB at reduced levels. In B, C, mean$\pm$SEM of 3 independent
measurements are shown (70 cells per data point).

Figure 6. Membrane flows between membrane compartments before and after
membrane fission. **A**. Schematic showing how the timing of membrane fission is
detected in time-lapse movies of individual cells. The dye (depicted as a star) becomes
highly fluorescent upon insertion into the membrane. The lipid-inserted and free dyes
exchange continuously, minimizing bleaching in pre-fission cells. Upon fission, the free
dye has no longer access to the inter-membrane space, leading to the onset of an
intensity decay. **B.** An example of a single cell, labeled with FM4-64, followed as a
function of time (see Figure S5 for more examples). Top row shows a montage of FM4-
64 fluorescence images as a function of time. The middle and bottom rows show the
detected forespore and mother cell contours respectively, using an active-contour fitting
algorithm. The frame preceding fission is indicated in the image (τ_{fiss}). Bar,1 µm. **C**. Plot
of the mean FS contour FM4-64 fluorescence (black) as a function of time for the
individual cell in B, displaying a relatively stable signal and a sudden decay upon
membrane fission. Cross-correlation of the fluorescence signal with a model decay
function (blue) was applied to determine the timing of membrane fission τ_{fiss}. **D**. Average
of forespore cell membrane surface area of wild-type sporangia as a function of time
(n=25). Time axes for individual cells were aligned such that $\tau_{fiss} = 0$ min. Error bars
represent SEM. The dashed blue and red lines are linear fits to the pre- and fission
periods, with slopes 10.6×10^{-3} and 7.9×10^{-3} µm^2/min. **E**. Similar to J, but for the
mother cell areas. The dashed blue and red lines are linear fits to the pre- and fission
periods, with slopes -10.5×10^{-3} and $+3.2 \times 10^{-3}$ µm^2/min. **F**. Schematic showing

sources and sinks of lipids. Before fission, newly synhtesized lipids are inserted into the
mother cell and engulfment membranes from the mother cell cytoplasm at rates M and
S_c, respectively. Lipids synthesized within the forespore are inserted at rate S_f, which
likely increases as lipid synthase genes initially located in the mother cell cytopolasm
are translocated into the forespore (Pedrido et al., 2013). Lipid flux through the neck (at
rate N) is only possible before fission.

Figure 7. Proposed model of how forespore inflation drives FisB-mediated
membrane fission. **A**. DNA translocation by the ATPase SpoIIIE inflates the forespore,
stretching the thin peptidoglycan layer between two and smoothing membrane wrinkles
(Lopez-Garrido et al., 2018). Increased membrane tension in the engulfment membrane
drives lipid flux through the neck between the engulfment and mother cell membranes,
partially supplying lipids needed for forespore inflation. **B**. FisB oligomerizes at the
membrane neck, impeding membrane flow and causing the engulfment membrane to
stretch further. **C**. Increased membrane tension drives membrane scission.

STAR★Methods

Key Resources Table

REAGENT or RESOURCE	SOURCE	IDENTIFIER
Chemicals, Peptides, and Recombinant Proteins		
TMA-DPH	Thermo Fisher Scientific	Cat#T204
FM4-64	Thermo Fisher Scientific	Cat#T13320
iFluor™ 555 maleimide	AAT Bioquest	Cat#1063
Cerulenin	Cayman Chemical	Cat# 10005647
Lipids	Avanti Polar Lipids	See Table S1 for a complete list of lipids used in this paper.
Experimental Models: Organisms/Strains		
Bacillus subtilis PY79		Tax. ID:1415167
See Table S2 for a complete list of strains used in this paper, including all the *Bacillus subtilis* PY79 variants	N/A	N/A
Recombinant DNA		
See Table S3 for a complete list of plasmids used in this paper. See Table S4 for a detailed explanation of their construction.	N/A	N/A
Oligonucleotides		
See Table S5 oligonucleotides	N/A	N/A
Software and Algorithms		
JFilament	(Smith et al., 2010)	http://athena.physics.lehigh.edu/jfilament/
SporeSize	This work	
Matlab	The MathWorks, Inc., Natick, MA	https://www.mathworks.com/

Contact for Reagent and Resource Sharing

699 Further information and requests for resources and reagents should be directed to and
700 will be fulfilled by the Lead Contact, Erdem Karatekin (erdem.karatekin@yale.edu)

Method details

Experimental Procedures

General *B. subtilis* methods

B. subtilis strains were derived from the prototrophic strain PY79 (Youngman et al.,
1983). Sporulation was induced in liquid medium at 37°C by resuspension according to
the method of Sterlini-Mandelstam (Sterlini and Mandelstam, 1969). *B. subtilis* strains
were constructed using plasmidic or genomic DNA and a 1-step competence method.
Site directed mutagenesis was performed using Agilent's Quick-change Lightning kit
following manufacturer's instructions and mutations were confirmed by sequencing.
Tables of strains (Table S2) and plasmids (Table S3) can be found in Supplemental
Information.

Fluorescence microscopy from batch cultures

500 µl samples at indicated times during sporulation were taken and concentrated ~50-
fold by centrifugation (3300xg for 30 s). Membranes were stained with 1-(4-
trimethylammoniumphenyl)-6-phenyl-1,3,5-hexatriene *p*-toluenesulfonate (TMA-DPH;
Molecular Probes) at a final concentration of 100 µM and the cells were then
immobilized on a 2% agarose pad made with sporulation buffer covered with a no.1.5
coverslip, after which the cells were immediately imaged live. In most cases
fluorescence microscopy was performed using a Nikon Ti microscope equipped with a
×100 Plan Apo 1.45 NA phase-contrast oil objective, an Orca-Flash4.0 V2 CMOS
camera (Hamamatsu Photonics, Shizuoka,Japan) and a Spectra X light engine
(Lumencor, Beaverton, OR, USA), all controlled by Nikon Elements software (Nikon
Corp. Tokyo, Japan). Excitation of TMA-DPH was achieved using the 395/25 nm band
of the SpectraX system and Chroma's ET 49000 single band filter set the DAPI filter set.
(λ_{ex}=395/25 nm; λ_{em}=431/28 nm). Excitation light intensity was set to 50% and exposure
times were 200 ms.

In experiments shown in **Figure *6*** and S3, cells were prepared following the same
procedure but visualized on a Leica DMi8 Wide-field Inverted Microscope equipped with
a HC PL APO 100× DIC objective (NA=1.40), and an Andor iXon Ultra 888 EMCCD
camera (Andor Technology Ltd., Belfast, Northern Ireland) and a Spectra X light engine
(Lumencor, OR,USA) controlled by the LAS X program (Leica Microsystems, Wetzlar,
Germany). Excitation of TMA-DPH was achieved using the 395/25 nm band of the
Spectra X system and the Leica DAPI filter set (λ_{ex}=395/25 nm; λ_{em}=460/50 nm).
Excitation light intensity was set to 50% and exposure times were 300 ms. .

Images were processed using the ImageJ software. Contours generated using

JFilament were coloured using Photoshop.

Time-lapse fluorescence microscopy

Cells were visualized on a Leica DMi8 Wide-field Inverted Microscope equipped with a
HC PL APO 100× DIC objective (NA=1.40) and an iXon Ultra 888 EMCCD Camera from
Andor Technology. Sporulation was induced at 37°C. To visualize the membranes,
0.5 µg/ml FM4-64 was added to the culture ~1 hours after sporulation induction and
incubation continued for another hour. Then a 10 µl sample was taken and transferred
to an agarose pad prepared as described (Lopez-Garrido et al., 2018). Pictures were
taken in an environmental chamber at 37°C every 5 min for at least 2 hours. Excitation
of FM4-64 was achieved using the 575/25 nm band of the SpectraX system and a
custom FM4-64 filter set (λ_{ex}=395/25 nm; λ_{em}>610nm). Excitation light intensity was set
to 5% to minimize phototoxicity and exposure times were 100 ms. For presentation
purposes, sporangia were aligned vertically (with forespore on top) by rotating them
using ImageJ. Contours generated with JFilament were colored using Photoshop.

Expression, purification, and labeling of recombinant proteins.

FisB ECD and mutants were purified as described in (Landajuela et al., 2021). Briefly,
His$_6$-FisB ECD was expressed in *E. coli* BL21 (DE3) (New England Biolabs,
Ipswich,MA,USA) and purified using HisPur™ Ni-NTA Resin (Thermo-Fisher Scientific,
Waltham, MA, USA). Protein expression was induced with 1 mM IPTG at OD$_{600}$ = 0.6
overnight at 16°C. Cells were harvested by centrifugation and the pellet was
resuspended in Lysis Buffer (20 mM HEPES, 500 mM NaCl, 0.5 mM TCEP, 20 mM
imidazole, 2% glycerol, 20 mM MgCl$_2$) and flash frozen in liquid nitrogen. Pellets were
thawed on ice and cells were lysed by 5 passes through a high-pressure homogenizer
(Avestin EmulsiFlex-C3, Ottawa, Canada). The lysate was spun down at 100,000 g and
the soluble fraction was incubated with HisPur™ Ni-NTA Resin for 2.5 h at 4°C while
rotating. The bound protein was washed sequentially with Lysis Buffer, Lysis Buffer
containing 50 mM and finally 100 mM Imidazole. The protein was eluted in Elution
Buffer (20 mM HEPES, 500 mM NaCl, 0.5 mM TCEP, 200 mM Imidazole, 2% glycerol,
20 mM MgCl2). The protein was concentrated using a Vivaspin® with a 10 kDa
molecular weight cutoff and the concentration determined by Bradford protein assay.
His6-FisB ECD G123C was labeled with iFluor555™ maleimide dye (AAT Bioquest,
Sunnyvale, CA, USA) by adding a 20x molar excess of dye to the protein and the
reaction was incubated overnight at 4°C. Free dye was removed using a HiPrep 20/10
(GE, Chicago, IL, USA) column.

Giant Unilamellar Vesicle (GUV) preparation

GUVs were formed by electroformation (Stockl et al., 2010). Briefly, chloroform-
dissolved lipids were mixed in a glass tube at desired ratios and spotted on two indium
tin oxide (ITO) coated glass slides. Organic solvent was removed by placing the lipid
films in a vacuum desiccator for at least 2 h. A short strip of copper conductive tape was

attached to each ITO slide which are then separated by a PTFE spacer and held
together with binder clips. The chamber was filled with 500 μl Swelling Buffer (SweBu, 1
mM HEPES, 0.25 M sucrose, 1 mM DTT) and sealed with Critoseal (McCormick
Scientific LLC, Saint Louis, MO, USA). GUVs were formed by applying a sinusoidal
voltage of 10 Hz and an amplitude of 1.8 V for at least 2 h at room temperature. GUV
membranes were labeled by including 1 mole % 1,2-dioleoyl-sn-glycero-3-
phosphoethanolamine-N-(7-nitro-2-1,3-benzoxadiazol-4-yl) (NBD-PE) or 1,1'-
dioctadecyl-3,3,3',3'-tetramethylindodicarbocyanine perchlorate (DiD) in the lipid
composition. In **Figure 2**, "100% PC" and "30% CL" membranes are composed of (all
mole %) 99% phosphatidylcholine (PC), 1% NBD-PE, and 30% *E. coli* cardiolipin (CL),
69% eggPC, 1% NBD-PE, respectively. Deflated GUVs were composed of (all mole %)
25% *E. coli* phosphatidylethanolamine (PE), 5% *E. coli* CL, 50% *E. coli*
phosphatidylglycerol (PG), 19% eggPC and 1% DiD or NBD-PE (BS mix). All lipids were
purchased from Avanti Polar Lipids (Alabaster, AL), except for DiD which was from
Thermo Fisher (Waltham, MA).

GUVs were imaged using a Nikon Eclipse TE2000-E microscope. Prior to adding the
GUVs, the imaging chamber was filled with a 5 mg/ml β-Casein (Sigma, Saint Louis, MI,
USA) solution to prevent attachment of the GUVs to the glass coverslip. The β-Casein
solution was removed and replaced with RB-EDTA.

Fluorescence recovery after photobleaching (FRAP)

FRAP measurements were conducted using a Leica SP8 inverted microscope in the
same open imaging chamber. As described above GUVs were diluted in RB-EDTA (25
mM HEPES at pH 7.4, 850140 mM KCl, 1 mM EDTA, 0.2 mM tris(2-carboxyethyl)
phosphine) and incubated with 1 μM FisB ECD for 2-3h. A rectangular area was chosen
to bleach as indicated in Figure 2C. Five images (laser power 0.7%) were recorded
before bleaching, 10 during bleaching (laser power 100%) and 60 after bleaching (laser
power 0.7%).

The mean GUV intensity was determined using ImageJ. A segmented line (10 pixels
wide) was drawn to manually follow the GUV membrane. The line was smoothed using
'fit spine' and the mean pixel intensity was calculated. The background was determined
as the mean pixel value of a 20 x 20-pixel box (close to but outside the GUVs) and
subtracted from the mean pixel intensity of the GUVs.

Image analysis

Determination of forespore and mother membrane surface area. Using the ImageJ
plugin Filament 2D (Smith et al., 2010), "snakes" were fitted to forespores and mother
cells (Figure S1A). Snakes were fitted to forespores using either the TMA-DPH
membrane stain (using ridges) or a soluble CFP marker (using gradients) expressed in
the forespores. Snakes were fitted to mother cells using the corresponding phase
contrast image using gradients. In all cases the snakes were fitted as contours. Fitting
parameters were as follows: $\alpha = 100$, $\beta = 100$, $\gamma = 800$, weight=0.5, stretch force=100,
deform iterations=50, point spacing=0.5, image smoothing=1.01.

To determine the surface area of forespores and mother cells, the snakes were then
further analyzed with MATLAB. First, an ellipse was fitted to the snakes to determine
the symmetry axis of the forespore or mother cell (Figure S1B). Half of the snake is then
rotated around the symmetry axis to create a surface of revolution from which the
membrane surface area for the forespore or mother cell was determined (Figure S1C).

For the analyses shown in **Figure 5**I, FisB foci were semi-automatically selected using
SpeckleTrackerJ (Smith et al., 2011). For each spot, the sum of pixel values in a 6
pixels $\times$ 6 pixels (0.5 µm$\times$0.5 µm) box around the center of the spot were calculated.
For each corresponding cell, the same operation was performed at a membrane area
where no clusters were present and subtracted from the individual FisB cluster intensity.

Determination of mean fluorescence intensity around the forespore. We used
ImageJ plugin Filament 2D to fit snakes around forespores (see above) to define
forespore contours. Mean contour intensity was calculated using Matlab, using snake
coordinates from Filament 2D**.**

836
837   **Modeling.** Please see SI Appendix for details.

 SUPPLEMENTARY FIGURE LEGENDS

Figure S1. Forespore area changes can also be detected using a soluble marker. Related to Figure 3. A. Membrane fission was assessed by fluorescent microscopy during a sporulation time course in wild type (BKM015) cells containing a fluorescent forespore reporter (P$_{spoIIQ}$-cfp). Membranes were visualized with the fluorescent dye TMA-DPH. Forespore contours in pre- and post-fission cells are indicated with blue or red contours, respectively. Time (in hours) after the initiation of sporulation is indicated as T2.5, T.3, etc.. Bar, 1 μm. **B.** Average post-fission forespore areas (A_{FS}) detected using either marker grow while average pre-fission FS areas shrink as a function of time into sporulation. Error bars represent SEM from three independent experiments (70 cells were analyzed per point).

Figure S2. Blocking lipid synthesis inhibits forespore inflation and membrane fission. Related to Figure 4. A. Cerulenin (Cer) inhibits *de novo* synthesis of neutral lipids and phospholipids by inhibiting *de novo* fatty acid biosynthesis. **B.** Representative images showing cells at *t* = 3 h after addition of 10 μg/ml cerulenin at indicated timepoints after nutrient downshift. Membranes were visualized with TMA-DPH. A cell that is blocked at the asymmetric division stage is highlighted (red arrowhead). Scale bars = 1 μm. **C.** Percentage of cells (wild-type strain, PY79) that have undergone membrane fission at *t* = 3 h after addition of cerulenin at the indicated timepoints. More than 300 cells were analyzed for each time point. **D**. Quantification of forespore membrane area for cells that have (red) or have not (blue) undergone membrane fission at *t* = 3 h after cerulenin addition at the indicated times after nutrient downshift (n=25-70 cells per data point). **E.** Fractional area occupied by the FS in pre- (blue) and post-fission (red) cells at t = 3 h. Cerulenin was added at the indicated times after nutrient downshift (n=25-70 cells per data point). **F**. Representative images showing cells (strain BAM003) at t=3 hr after the nutrient downshift, with Cer application for the indicated durations. Examples of cells with a discrete mGFP-FisB focus at the cell pole are highlighted with green arrowheads. Scale bars represent 1 μm. **G.** Engulfment is perturbed to a greater extent when lipid synthesis is inhibited earlier by cerulenin addition (>300 cells per data point). **H**. Percentage of cells with FisB accumulation (ISEP formation) at the membrane fission site, for cells that have visually completed engulfment. **I**. The fraction of post-fission cells among cells with correct FisB localization (ISEP formation) as a function of increasing cerulenin application time (n=90-200 cells were analyzed per point). Means±SEM of 3 independent experiments are shown for panels C-E and G-I.

Figure S3. Blocking forespore inflation inhibits membrane fission. Related to Figure 4. A. Engulfment and FisB localization in cells expressing wild-type (WT) SpoIIIE or ATP-ase deficient SpoIIIE36. Membranes were labeled with TMA-DPH and FisB was visualized using mGFP-FisB expressed at native levels on a *ΔfisB* background (Landajuela et al., 2021). Fluorescence microscopy images were acquired 3 h after nutrient downshift that initiated sporulation. Cells that have visually completed engulfment and had accumulated FisB a the membrane fission site are indicated by arrowheads. On the images on the right, detected FS contours are overlaid in blue and

red for pre- and post-fission cells. **B**. Percentage of cells that underwent membrane
fission as a function of time into sporulation, for WT and SpoIIIE36 cells. (For each data
point >300 cells were analyzed). **C**. FS areas as a function of time after nutrient
downshift, for pre- (blue) and post-fission (red) cells, expressing either WT (solid lines)
or ATPase deficient SpoIIIE (SpoIIIE36). (n=25-70 cells per data point) **D-F**. As in A-C,
but using cells expressing mGFP-FisB at ~8-fold lower levels (Landajuela et al., 2021).
G. FisB localization is affected when DNA translocation is completely blocked.
Percentage of cells displaying an intense spot at the engulfment pole (ISEP) for cells
expressing SpoIIIE^{WT} or SpoIIIE36 on a background with either native or reduced
mGFP-FisB levels. Scale bars = 1 μm. **H.** The fraction of post-fission cells among cells
with correct FisB localization (ISEP formation). (n=25-70 cells were analyzed per data
point). In panels B,C, E-H, mean$\pm$SEM of three independent biological replicates are
plotted for every point.

Figure S4. Pre-fission forespores inflate in a low expression FisB strain in which
membrane fission is delayed. Related to Figure 5. A. Summary of FisB copy number
quantification under native or low expression levels (Landajuela et al., 2021). Right.
Under native expression, on average, there are ~1,000 FisB molecules per cell at t=3 h.
Each DMC contains ~12 FisB molecules, while the ISEP contains ~40 FisB copies. In
the low-expression strain all the numbers are scaled down ~7-8-fold. **B.** Time course of
membrane fission for wild-type cells, *ΔfisB* cells, or *ΔfisB* cells complemented with
mGFP-FisB expressed at native (BAM003) or low levels (BAL003). (More than 300 cells
were analyzed per data point.) **C.** Fluorescence microscopy images of cells expressing
mGFP-FisB at the native level (BAM003) at 3 h into sporulation. Membranes were
visualized with TMA-DPH. Examples of a sporulating cell with a discrete mGFP-FisB
focus at the cell pole (intense spot at engulfment pole, ISEP) are highlighted with a red
arrowhead. In the panels on the right red contours indicate forespores that have
undergone fission whereas blue contours indicate forespores that have not yet
undergone membrane fission (70 cells per data point). Scale bars represent is 1 μm. **D.**
Similar to C but using a strain (BAL003) that expresses mGFP-FisB at lower levels in a
ΔfisB background. **E, F.** Under mGFP-FisB expression at low levels, both the average
pre- (E) and post-fission (F) forespore areas (A_{FS}) grow, in a manner qualitatively similar
to cells lacking FisB altogether (ΔFisB). By contrast, for cells expressing mGFP-FisB the
native level, the average pre- and post-fission FS areas decrease and increase as a
function of time into sporulation, respectively (70 cells were analyzed per data point).
Data for *ΔfisB* cells is copied from Figure 6 for comparison. In panels B, E, F,
mean$\pm$SEM of three independent replicates are plotted.

Figure S5. Examples of forespore area growth and membrane fission detected at
the single cell level. Related to Figure 6. Each top row of a triplet shows a unique cell
followed as a function of time in time-lapse fluorescence microscopy. Cell membranes
were labeled with FM4-64. Images are shown with an inverted look-up table. Frames
were acquired every 5 min. The middle and bottom rows of every triplet shows the
overlaid contour of the FS or the MC membrane, respectively. The intensity of the FS

membrane starts decreasing upon membrane fission, because bleached FM4-64
molecules are confined and can no longer exchange with the unlabeled dyes in the bath
after membrane fission. For each cell, the mean FS contour intensity and the FS area
are shown on the right. The time of membrane fission (τ_{fiss}) estimated from the contour
intensity time profile is indicated on the FS area plots.

REFERENCES

Allemand, J.F., and Maier, B. (2009). Bacterial translocation motors investigated by single
molecule techniques. FEMS Microbiol Rev *33*, 593-610.

Bath, J., Wu, L.J., Errington, J., and Wang, J.C. (2000). Role of Bacillus subtilis SpoIIIE
in DNA transport across the mother cell-prespore division septum. Science *290*, 995-997.

Boron, W., and Boulpaep, E. (2017). Medical Physiology, 3 edn (Philadelphia, PA:
Elsevier).

Burton, B.M., Marquis, K.A., Sullivan, N.L., Rapoport, T.A., and Rudner, D.Z. (2007). The
ATPase SpoIIIE transports DNA across fused septal membranes during sporulation in
Bacillus subtilis. Cell *131*, 1301-1312.

Campelo, F., and Malhotra, V. (2012). Membrane fission: the biogenesis of transport
carriers. Annu Rev Biochem *81*, 407-427.

Cochilla, A.J., Angleson, J.K., and Betz, W.J. (1999). Monitoring secretory membrane
with FM1-43 fluorescence. Annu Rev Neurosci *22*, 1-10.

Doan, T., Coleman, J., Marquis, K.A., Meeske, A.J., Burton, B.M., Karatekin, E., and
Rudner, D.Z. (2013). FisB mediates membrane fission during sporulation in Bacillus
subtilis. Genes Dev *27*, 322-334.

Evans, E., Heinrich, V., Ludwig, F., and Rawicz, W. (2003). Dynamic tension
spectroscopy and strength of biomembranes. Biophysical Journal *85*, 2342-2350.

Ferguson, S.M., and De Camilli, P. (2012). Dynamin, a membrane-remodelling GTPase.
Nat Rev Mol Cell Biol *13*, 75-88.

Heath, R.J., and Rock, C.O. (2004). Fatty acid biosynthesis as a target for novel
antibacterials. Curr Opin Investig Drugs *5*, 146-153.

Karatekin, E., Sandre, O., Guitouni, H., Borghi, N., Puech, P.H., and Brochard-Wyart, F.
(2003). Cascades of transient pores in giant vesicles: line tension and transport. Biophys
J *84*, 1734-1749.

Khanna, K., Lopez-Garrido, J., Zhao, Z., Watanabe, R., Yuan, Y., Sugie, J., Pogliano, K.,
and Villa, E. (2019). The molecular architecture of engulfment during Bacillus subtilis
sporulation. Elife *8*.

Landajuela, A., Braun, M., Rodrigues, C.D.A., Martinez-Calvo, A., Doan, T., Horenkamp,
F., Andronicos, A., Shteyn, V., Williams, N.D., Lin, C.X.*, et al.* (2021). FisB relies on homo-
oligomerization and lipid binding to catalyze membrane fission in bacteria. Plos Biol *19*.

Liu, N., Chistol, G., and Bustamante, C. (2015). Two-subunit DNA escort mechanism and
inactive subunit bypass in an ultra-fast ring ATPase. Elife *4*.

Liu, N., Chistol, G., Cui, Y., and Bustamante, C. (2018). Mechanochemical coupling and
bi-phasic force-velocity dependence in the ultra-fast ring ATPase SpoIIIE. Elife *7*.

Liu, N.J., Dutton, R.J., and Pogliano, K. (2006). Evidence that the SpoIIIE DNA
translocase participates in membrane fusion during cytokinesis and engulfment. Mol
Microbiol *59*, 1097-1113.

Lopez-Garrido, J., Ojkic, N., Khanna, K., Wagner, F.R., Villa, E., Endres, R.G., and
Pogliano, K. (2018). Chromosome Translocation Inflates Bacillus Forespores and
Impacts Cellular Morphology. Cell *172*, 758-770 e714.

Moir, A., and Cooper, G. (2015). Spore Germination. Microbiol Spectr *3*.

Needham, D., and Nunn, R.S. (1990). Elastic-Deformation and Failure of Lipid Bilayer-
Membranes Containing Cholesterol. Biophysical Journal *58*, 997-1009.

Ojkic, N., Lopez-Garrido, J., Pogliano, K., and Endres, R.G. (2014). Bistable Forespore
Engulfment in Bacillus subtilis by a Zipper Mechanism in Absence of the Cell Wall. Plos
Comput Biol *10*, e1003912.

Ojkic, N., López-Garrido, J., Pogliano, K., and Endres, R.G. (2016). Cell-wall remodeling
drives engulfment during Bacillus subtilis sporulation. Elife *5*.

Pedrido, M.E., de Ona, P., Ramirez, W., Lenini, C., Goni, A., and Grau, R. (2013). Spo0A
links de novo fatty acid synthesis to sporulation and biofilm development in Bacillus
subtilis. Mol Microbiol *87*, 348-367.

Pogliano, K., Hofmeister, A.E., and Losick, R. (1997). Disappearance of the sigma E
transcription factor from the forespore and the SpoIIE phosphatase from the mother cell
contributes to establishment of cell-specific gene expression during sporulation in Bacillus
subtilis. J Bacteriol *179*, 3331-3341.

Renard, H.F., Johannes, L., and Morsomme, P. (2018). Increasing Diversity of Biological
Membrane Fission Mechanisms. Trends Cell Biol *28*, 274-286.

Schoneberg, J., Lee, I.H., Iwasa, J.H., and Hurley, J.H. (2017). Reverse-topology
membrane scission by the ESCRT proteins. Nat Rev Mol Cell Biol *18*, 5-17.

Schujman, G.E., Grau, R., Gramajo, H.C., Ornella, L., and de Mendoza, D. (1998). De
novo fatty acid synthesis is required for establishment of cell type-specific gene
transcription during sporulation in Bacillus subtilis. Mol Microbiol *29*, 1215-1224.

Setlow, P. (2003). Spore germination. Current Opinion in Microbiology *6*, 550-556.

Sharp, M.D., and Pogliano, K. (1999). An in vivo membrane fusion assay implicates
SpoIIIE in the final stages of engulfment during Bacillus subtilis sporulation. Proc Natl
Acad Sci U S A *96*, 14553-14558.

Simunovic, M., Evergren, E., Callan-Jones, A., and Bassereau, P. (2019). Curving Cells
Inside and Out: Roles of BAR Domain Proteins in Membrane Shaping and Its Cellular
Implications. Annu Rev Cell Dev Biol *35*, 111-129.

Simunovic, M., Manneville, J.B., Renard, H.F., Evergren, E., Raghunathan, K., Bhatia,
D., Kenworthy, A.K., Voth, G.A., Prost, J., McMahon, H.T.*, et al.* (2017). Friction Mediates
Scission of Tubular Membranes Scaffolded by BAR Proteins. Cell *170*, 172-184 e111.

Smith, M.B., Karatekin, E., Gohlke, A., Mizuno, H., Watanabe, N., and Vavylonis, D.
(2011). Interactive, computer-assisted tracking of speckle trajectories in fluorescence
microscopy: application to actin polymerization and membrane fusion. Biophys J *101*,
1794-1804.

Smith, M.B., Li, H., Shen, T., Huang, X., Yusuf, E., and Vavylonis, D. (2010).
Segmentation and tracking of cytoskeletal filaments using open active contours.
Cytoskeleton (Hoboken) *67*, 693-705.

Sterlini, J.M., and Mandelstam, J. (1969). Commitment to Sporulation in Bacillus Subtilis
and Its Relationship to Development of Actinomycin Resistance. Biochem J *113*, 29-+.

Stockl, M., Nikolaus, J., and Herrmann, A. (2010). Visualization of lipid domain-specific
protein sorting in giant unilamellar vesicles. Methods Mol Biol *606*, 115-126.

Tiller, G.E., Mueller, T.J., Dockter, M.E., and Struve, W.G. (1984). Hydrogenation of triton
X-100 eliminates its fluorescence and ultraviolet light absorption while preserving its
detergent properties. Anal Biochem *141*, 262-266.

Tocheva, E.I., Matson, E.G., Morris, D.M., Moussavi, F., Leadbetter, J.R., and Jensen,
G.J. (2011). Peptidoglycan remodeling and conversion of an inner membrane into an
outer membrane during sporulation. Cell *146*, 799-812.

Vadia, S., Tse, J.L., Lucena, R., Yang, Z., Kellogg, D.R., Wang, J.D., and Levin, P.A.
(2017). Fatty Acid Availability Sets Cell Envelope Capacity and Dictates Microbial Cell
Size. Curr Biol *27*, 1757-1767 e1755.

Webb, C.D., Teleman, A., Gordon, S., Straight, A., Belmont, A., Lin, D.C., Grossman,
A.D., Wright, A., and Losick, R. (1997). Bipolar localization of the replication origin regions
of chromosomes in vegetative and sporulating cells of B. subtilis. Cell *88*, 667-674.

Wu, L.J., and Errington, J. (1994). Bacillus subtilis SpoIIIE protein required for DNA
segregation during asymmetric cell division. Science *264*, 572-575.

Wu, L.J., and Errington, J. (1997). Septal localization of the SpoIIIE chromosome
partitioning protein in Bacillus subtilis. EMBO J *16*, 2161-2169.

Wu, L.J., and Errington, J. (1998). Use of asymmetric cell division and spoIIIE mutants to
probe chromosome orientation and organization in Bacillus subtilis. Mol Microbiol *27*, 777-
786.

Yao, Z., Davis, R.M., Kishony, R., Kahne, D., and Ruiz, N. (2012). Regulation of cell size
in response to nutrient availability by fatty acid biosynthesis in *Escherichia coli*.
Proceedings of the National Academy of Sciences *109*, E2561.

Youngman, P.J., Perkins, J.B., and Losick, R. (1983). Genetic transposition and
insertional mutagenesis in Bacillus subtilis with Streptococcus faecalis transposon Tn917.
Proc Natl Acad Sci U S A *80*, 2305-2309.

Zhang, Y.M., and Rock, C.O. (2008). Membrane lipid homeostasis in bacteria. Nat Rev
Microbiol *6*, 222-233.

1044

FIGURE 1

FIGURE 2

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

A

	Membrane (TMA-DPH)	Membrane (TMA-DPH)	Spore marker	Merge
T2.5				
T3				
T3.5				

B

pre (TMA-DPH)
post (TMA-DPH)
pre (CFP)
post (CFP)

A_{FS} (μm^2)

time (h)

FIGURE S1

FIGURE S2

FIGURE S3

FIGURE S4

FIGURE S5

SI Appendix: Theoretical Modeling

In a previous work [1] we showed that a neck of radius $\leq 5 - 10$ nm in the region where the engulfment membrane meets the rest of the mother cell membrane (see sketch in Fig. 1a) is necessary for FisB to accumulate. This accumulation in turn favors membrane fission, as shown in the current main text. In this appendix, first, we develop a minimal model for the neck based on free-energy minimization. This minimal model assumes a highly simplified geometry of the membrane neck, as depicted in Fig. 1(b), that accounts for membrane tension, bending, and an osmotic pressure difference, $\Delta \tilde{p} = p_{\text{cyto}} - p_{\text{peri}}$, between the cytoplasm and periplasm since the lumen of the neck is continuous with the periplasmic space. Using this model, we explore if an increase of membrane tension in the neck, e.g. due to forespore inflation, along with the osmotic pressure difference are able to narrow the radius of the neck. Then, we consider a more realistic model by solving the complete Helfrich equation [2] including the possibility of a pulling force on the neck, e.g. due to DNA translocation through the SpoIIIE motor. For this more realistic model, the shape of the membrane neck is obtained as part of the solution.

MINIMAL MODEL FOR THE NECK: UNIFORM CYLINDER CONNECTING TWO PLANAR MEMBRANES

We first consider a highly simplified model based on the free energy $\mathscr{F}_0$ of an axisymmetric cylindrical membrane neck of radius R and length L that connects two planar membrane sheets, corresponding to the local geometry depicted in 1(b). We employ the classical Helfrich-Canham theory [2–7] for the energy of the membrane, $\mathscr{F}_0$, which reads

$$\tilde{\mathscr{F}}_0 = \int_{S_n} \mathrm{d}S_n \left[\frac{\kappa}{2} (2\tilde{H})^2 + \gamma \right] + \int_{S_s} \mathrm{d}S_s \gamma + \int_V \mathrm{d}V \Delta \tilde{p}, \quad (1)$$

where S_n and S_s are the surfaces of the membrane neck and sheets, and V is the volume inside the neck. The tildes denote dimensional variables which will subsequently be non-dimensionalized. Here $\tilde{H}$ is the mean curvature, κ is the bending modulus, γ is the membrane surface tension, and $\Delta \tilde{p} = p_{\text{cyto}} - p_{\text{peri}}$ is the osmotic pressure difference. The two surrounding membrane sheets are assumed to be planar since their curvature is much less than that of the neck, thus their only contribution to the energy comes from their area times the membrane tension. In this simplified model, we neglect the bending energy of junctions where the cylinder meets the membrane sheets. With these simplifications, the free energy $\mathscr{F}$ in Eq. (1) reads:

$$\frac{\tilde{\mathscr{F}}_0}{2\pi} = \left(\gamma \tilde{R} + \frac{\kappa}{2\tilde{R}} \right) L + \gamma (\tilde{R}_{\text{bc}}^2 - \tilde{R}^2) + \frac{\Delta \tilde{p}}{2} \tilde{R}^2 L, \quad (2)$$

where $\tilde{R}_{\text{bc}} > \tilde{R}$ is the outer boundary condition depicted in Fig. 1(b). Minimizing Eq. (2) with respect to the radius $\tilde{R}$

Figure 1. (a) Sketch of the engulfment membrane geometry during the last stage of forespore engulfment, showing the flow of lipids through the membrane neck, the osmotic pressure difference $\Delta \tilde{p} = p_{\text{cyto}} - p_{\text{peri}}$ between the cytoplasm and the periplasm, and the pulling force $\tilde{f}$ due to the SpoIIIE motors translocating DNA into the forespore. (b) Simplified model of the membrane neck, consisting of a straight cylinder of radius $\tilde{R}$ and length L, connecting two planar membranes of radius $\tilde{R}_{\text{bc}}$. (c) Schematic of the more realistic model of the membrane neck showing the parametrization of the surface, where s is the arc length along the contour and θ the angle with respect to the vertical.

yields

$$2\gamma \tilde{R}^2 - \kappa + 2\tilde{R}^3 \left(\Delta \tilde{p} - \frac{2\gamma}{L} \right) = 0. \quad (3)$$

To non-dimensionalize Eqs. (2)–(3) we introduce the natural length scale arising from the balance between bending and tension in Eq. (1) [8] as the characteristic length scale, $R_c = \sqrt{\kappa/(2\gamma)}$, i.e. $R = \tilde{R}/R_c$. The free energy is also made dimensionless as $\mathscr{F}_0 = \tilde{\mathscr{F}}_0/(\gamma R_c L)$. Hence, the dimensionless forms of Eqs. (2) and (3) read:

$$\frac{\mathscr{F}_0}{2\pi} = R + \frac{1}{R} + \frac{(R_{\text{bc}}^2 - R^2)}{L/R_c} + \frac{\Delta p}{2} R^2, \quad (4)$$

$$R^2 - 1 + R^3 \left(\Delta p - \frac{2}{L/R_c} \right) = 0, \quad (5)$$

where $\Delta p = \Delta \tilde{p} R_c / \gamma$.

Figure 2. (a) Dimensionless free energy $\mathscr{F}_0 = \tilde{\mathscr{F}}_0/(\gamma R_c L)$ from Eq. (4) as a function of neck radius R, highlighting the existence of a local free-energy minimum provide $L/R_c > 3^{3/2}$ (gray dashed curve). Here $R_{bc} = 6$. The inset shows the corresponding radius R_{min} of the local free-energy minimum as a function of L/R_c. (b) Same as in panel (a) but for different values of the dimensionless osmotic pressure difference Δp, for a fixed length of the neck, i.e. $L/R_c = 10$.

Zero osmotic pressure difference. The solution of Eqs. (4) and (5) when $\Delta p = 0$ reveals that for a long enough neck there is a metastable state of the neck at finite radius R_{min} [9, 10]. This local minimum arises from a balance between membrane bending energy, which always favors larger R, and membrane tension, which contributes a non-monotonic function of R to the membrane energy. The existence of this local minimum can be observed in Fig. 2, which shows the dimensionless free energy $\mathscr{F}_0$ as a function of R for different values of L/R_c, along with the location R_{min} of the local minimum. Intuitively, for R around R_{min}, increasing the radius R increases the total amount of membrane in the vicinity of the neck and so increasing R is opposed by surface tension. By contrast, for larger values of R, increasing R removes more membrane from the parallel sheets (removed area scaling $\sim R^2$) than is added to the neck (added area scaling $\sim LR$), so further increase of R is energetically favored (specifically when $R > \sqrt{3} + \sqrt{2}\sqrt{L/R_c - 3\sqrt{3}}/3^{3/4} + O(L/R_c)$). We find that, for lengths below a critical value $L/R_c < 3^{3/2}$, the energy functional $\mathscr{F}_0$ does not have a local minimum, so that expanding the radius of the neck always decreases the total energy of the system, implying the neck opens.

Finite osmotic pressure difference. Eq. (5) indicates that a positive osmotic pressure difference between the cytoplasm and the periplasm favors the formation of a locally stable, nar-

row membrane neck. Indeed, the critical neck length below which there is no locally stable narrow neck decreases as Δp increases, $L/R_c \leq 2 \cdot 3^{3/2}/(2 + 3^{3/2}\Delta p)$. Moreover, the local maxima shown in Fig. 2(a) for $\Delta p = 0$, disappears when $\Delta p > 2/(L/R_c)$, as shown in Fig. 2(b). This implies that the free energy $\mathscr{F}_0$ increases monotonically beyond the local minimum thereby the neck does not open up. Moreover, the minimum radius of the neck R_{min} always decreases as Δp increases.

EQUILIBRIUM SHAPES OF THE AXYSIMMETRIC MEMBRANE NECK

We now go beyond the minimal model described in the previous section by solving the complete Helfrich equation, where the shape of the neck is obtained as part of the solution. To this end, we describe the equilibrium shape of an axisymmetric and mirror symmetric membrane neck connecting two membrane sheets, corresponding to the local geometry of the neck connecting the forespore engulfing and mother cell membranes (see Figs. 1a and c). The energy functional $\tilde{\mathscr{F}}$ consists of a term accounting for membrane bending and tension, $\tilde{\mathscr{F}}_m$, another term accounting for the osmotic pressure difference between cytoplasm and periplasm, $\tilde{\mathscr{F}}_p$, and a term representing any pulling force f on the forespore,

Figure 3. (a) Dimensionless free energy from Eq. (6) as a function of the arc length s_{end} for different values of the radial boundary size R_{bc}. The insets show shapes of the neck for $R_{bc} = 4$ and two different values of s_{end}, corresponding to two local minima. The zoomed region depicts the sudden increase of free energy as the neck narrows. (b) Same as in panel (a) but displaying some of the multiple branches arising for sufficiently large values of s_{end}.

$\tilde{\mathscr{F}}_f$ [2–8, 11, 12]:

$$\tilde{\mathscr{F}} = \tilde{\mathscr{F}}_m + \tilde{\mathscr{F}}_p + \tilde{\mathscr{F}}_f = \int_A dA \left[\frac{\kappa}{2}(2\tilde{H})^2 + \sigma \right] + \int_V dV \Delta\tilde{p} - \tilde{f}L, \tag{6}$$

where $\Delta\tilde{p} = p_{cyto} - p_{peri}$, A is the area of the membrane neck, and V is the volume inside the neck.

For axisymmetric surfaces, it proves convenient to describe the surface in terms of the angle $\theta(s)$ between the contour and the ordinate axis, as parametrized by the arclength s (see sketch in Fig. 1c). To non-dimensionalize Eq. (6), we again introduce the natural length scale $R_c = \sqrt{\kappa/(2\gamma)}$ as the characteristic length. The definition of the radial $R(s)$ and axial $Z(s)$ coordinates, and the first variation of Eq. (6) yield the following nonlinear boundary-value problem [11, 13, 14]

$$R' = \cos\theta, \tag{7a}$$
$$Z' = -\sin\theta, \tag{7b}$$
$$\theta'' = -\frac{1}{2}(\theta')^2 \tan\theta - \frac{\cos\theta}{R}\theta' + \frac{\cos^2\theta + 1}{2R^2}\tan\theta +$$
$$\frac{1}{2}\tan\theta + \frac{\Delta p R}{4\cos\theta} - \frac{f}{R\cos\theta}, \tag{7c}$$

where derivatives are with respect to contour length s, $\Delta p = \Delta\tilde{p}R_c/\gamma$, and $f = \tilde{f}/(4\pi\gamma R_c)$. Concerning the boundary con-

ditions, we impose $R(s = s_{end}) = R_{bc}$, $Z(s = s_{end}) = 0$, $\theta(s = s_{end}) = 0$, and $\theta(s = 0) = \pi/2$. The above system of differential equations (7a)–(7c) is solved using the Matlab® built-in boundary-value problem solver bvp4c [15, 16].

Fig. 3 shows the dimensionless version of the free energy from Eq. (6), as a function of the arc length s_{end} for different values of the radial boundary size R_{bc} indicated in the legend. When $R_{bc} \gtrsim 3$, the free energy exhibits two local minima. One of them corresponds to a relatively small value of s_{end}, where the minimum neck radius is too large, i.e. the neck opens up. The other local minimum occurs at a larger value of s_{end} with a larger associated free energy, corresponding to a narrow neck. Increasing the value of s_{end} beyond the latter local minimum increases the total free energy drastically due to a sharp increase of the bending energy, which prevents membrane fission, as shown for $R_{bc} = 4$ (zoomed region in Fig. 3). When $R_{bc} \lesssim 3$, the free energy only displays one local minimum, which corresponds to that of a narrow neck.

Additionally, Fig. 4 shows the role of the pulling force f and the osmotic pressure difference Δp on the free energy and the shape of the membrane neck. Panel (a) depicts the dimensionless free energy $\mathscr{F}$ as a function of s_{end} with a zero and a non-zero value of f, i.e. $f = 1.5$ (see Table 1), and $\Delta p = 0$.

Figure 4. Dimensionless free energy from Eq. (6) as a function of the arc length s_{end} for $R_{\text{bc}} = 4$. Panel (a) shows two different values of the dimensionless pulling force f and $\Delta p = 0$, and panel (b) shows a case where $f = 1.75$ and $\Delta p = 1$. The insets show the shapes of the neck for different values of s_{end} indicated with dots.

Table I. Estimates of the physical and dimensionless parameters

Parameter	Value
κ	$20 k_{\text{B}} T$ [8]
γ	0.1–10 pN nm^{-1} [17–19]
L	40 nm
$\Delta \tilde{p}$	60 kPa [20]
$\tilde{f}$	50-100 pN [21, 22]
$R_{\text{c}} = \sqrt{\kappa/(2\gamma)}$	2–20 nm
L/R_{c}	9–90
$\Delta p = \Delta \tilde{p} R_{\text{c}}/\gamma$	0.012–12
$f = \tilde{f}/(4\pi\gamma R_{\text{c}})$	0.4–4

The insets, showing the shape of the axisymmetric membrane neck at different values of s_{end}, demonstrate that, for the same values of R_{bc} and s_{end}, the pulling force tends to increase both the length of the neck and the minimum neck radius. Panel (b) displays the free energy with a non-zero value of the pulling force and the osmotic pressure difference, i.e. $f = 1.75$ and $\Delta p = 1$. As expected, if we compare this result with that of panel (a) where $\Delta p = 0$, the osmotic pressure difference favors the narrowing of the neck.

We now estimate some of the physical and dimensionless

parameters used in the model to determine whether membrane tension and the osmotic pressure difference are able to reduce the neck radius, so that FisB proteins can interact and accumulate in the neck. This estimates are shown in Table I. In particular, we consider a pulling force of $\tilde{f} \sim 50 - 100$ pN, which corresponds to the SpoIIIE stall maximal force [21, 22], and an osmotic pressure difference of $\Delta \tilde{p} \sim 60$ kPa, as estimated in Ref. [20]. We assume that FisB can interact when the radius of the membrane neck is below ~ 10 nm [1], which is within therange of the characteristic neck radius shown in Table I, $R_{\text{c}} = \kappa\sqrt{(2\gamma)} \sim 2 - 20$ nm, using $\kappa \sim 20 k_{\text{B}} T$ [8], and $\gamma \sim 0.1 - 10$ pN nm^{-1} [17–19] (see last section of this Appendix for an upper estimate of the mem-brane tension). According to the model, when $\Delta p = f = 0$, the local minimum of $\mathscr{F}$ corresponding to the larger value of s_{end}, shown in the inset of Fig. 4(a), the dimensional mini-mum radius of the neck is $\tilde{R}_{\text{min}} \sim 0.1 - 1$ nm, which is signifi-cantly below the minimum radius required for FisB to interact. Moreover, the local free-energy minimum corresponding to a wider neck yields a minimum neck radius $\tilde{R}_{\text{min}} \sim 6.2 - 62$ nm, implying that, if membrane tension increases sufficiently, the neck radius can also be narrow enough for FisB to interact and accumulate inside the neck.

When $f = 1.5$ and $\Delta p = 0$, which corresponds to a range of $\gamma = 0.2 - 0.7$ pN nm^{-1} given that $\tilde{f} \sim 50 - 100$ pN, $\tilde{R}_{\min} \sim 7.8 - 15.6$ nm, corresponding to the membrane neck shown in the right bottom inset of Fig. 4(a). Hence, if we compare the range of the minimum neck radius with that of a null pulling force, it is evident that the radius of the neck increases due to the action of the pulling force. Nonetheless, as we point out above, if membrane tension is large enough, which corresponds to $\tilde{R}_{\min} \sim 7.8$ nm and $\gamma \sim 0.7$ pN nm^{-1} in the previous range of values, the minimum neck radius is small enough for FisB to interact. When $f = 1.75$ and $\Delta p = 1$, which corresponds to $\gamma \sim 0.53$ pN nm^{-1}, $\Delta \tilde{p} \sim 60$ kPa, and $\tilde{f} \sim 100$ pN, $\tilde{R}_{\min} \sim 8.8$ nm, i.e. the estimated value of the osmotic pressure difference only reduces slightly the minimum neck radius, as we also observe in our highly simplified model in Fig. 2(b), thereby favouring the accumulation of FisB in the neck.

FLUX OF LIPIDS

To estimate the membrane tension difference between the two ends of the neck that would drive a flux of lipids from the mother cell to the forespore of $\sim 10^3$ lipids s^{-1} (see *Forespore inflation accompanies membrane fission* in the main text), we balance the force due to the membrane tension difference $\Delta \gamma$ and the drag force on a cylinder (i.e. the neck in our highly simplified model), translating parallel to its axis in an outer bath of viscosity μ,

$$2\pi R \Delta \gamma = \frac{4\pi \mu U L}{\ln(L/R) - 1/2}. \tag{8}$$

Considering that 1 lipid occupies ~ 1 nm^2 and assuming a neck radius of $R \sim 5$ nm, the required lipid velocity along the neck is $U \sim 32$ nm s^{-1}. Taking $L \sim 40$ nm and $\mu \sim 10^{-3}$ Pa s, the estimated membrane tension difference is $\Delta \gamma \sim 3 \times 10^{-7}$ pN nm^{-1}. Hence, a flux of ~ 1000 lipids s^{-1} would require a relatively tiny excess membrane tension in the forespore engulfing membrane compared to the mother cell membrane. So a flux of lipids in the neck could readily provide ~ 1000 lipids s^{-1} to the forespore membranes.

COMPARING THE STALL FORCE OF SPOIIIE TO THE OSMOTIC PRESSURE DIFFERENCE

Here we analyze how the osmotic pressure difference between the mother cell and the forespore compares to the stall/disassembly force of SpoIIIE [21, 22]. According to Ref. [20], the excess pressure in the forespore is of the order of 60 kPa, which can be related to a salt concentration difference $\Delta \rho$ between the mother cell and the forespore via $\Delta P = N_A k_B T \Delta \rho$, yielding $\Delta \rho \sim 25$ mM. Taking the salt concentration in the mother cell to be $\rho_{mc} \sim 200$ mM, the salt concentration inside

the forespore is $\rho_{fs} \sim 225$ mM. We can now estimate the energy required to move one salt ion into the forespore as $E = k_B T \log(\rho_{fs}/\rho_{mc}) \sim 2.23 k_B T$. This estimate is of the same order as the energy available from SpoIIIE, which is its stall maximal force, ~ 50 pN [21, 22], multiplied by half the base spacing, i.e. ~ 0.17 nm, since each translocated base of DNA brings ~ 2 counterions, which yields $E_{SpoIIIE} \sim 2.1 k_B T$. This near equality of energies suggests that SpoIIIE work near its stall force, by creating the highest pressure difference it can in the forespore.

Additionally we can estimate the membrane tension required to balance this osmotic pressure difference. To this end we assume that the forespore is a sphere surrounded by two membranes, thus the equilibrium excess pressure is given by the Laplace law, $\Delta P = 4\gamma/R_{fs}$. Taking $R_{fs} \sim 400$ nm, $\gamma \sim 6$ pN nm^{-1}, which is slightly below the rupture tension, $\gamma_{rup} \sim 20$ pN nm^{-1} [17–19].

[1] A. Landajuela, M. Braun, C. D. A. Rodrigues, A. Martínez-Calvo, T. Doan, F. Horenkamp, A. Andronicos, V. Shteyn, N. D. Williams, C. Lin, N. S. Wingreen, D. Z. Rudner, and E. Karatekin, PLOS Biology 19, 1 (2021).
[2] W. Helfrich, Z. Naturforsch. C. 28, 693 (1973).
[3] P. B. Canham, J. Theor. Bio. 26, 61 (1970).
[4] O. Y. Zhong-Can and W. Helfrich, Phys. Rev. Lett. 59, 2486 (1987).
[5] O.-Y. Zhong-Can and W. Helfrich, Phys. Rev. A 39, 5280 (1989).
[6] U. Seifert and S. A. Langer, EPL 23, 71 (1993).
[7] U. Seifert, Adv. Phys. 46, 13 (1997).
[8] T. R. Powers, Rev. Mod. Phys. 82, 1607 (2010).
[9] A. Karlsson, R. Karlsson, M. Karlsson, A. S. Cans, A. Strömberg, F. Ryttsén, and O. Orwar, Nature 409, 150 (2001).
[10] P. Dommersnes, O. Orwar, F. Brochard-Wyart, and J. F. Joanny, EPL 70, 271 (2005).
[11] I. Derényi, F. Jülicher, and J. Prost, Phys. Rev. Lett. 88, 238101 (2002).
[12] T. R. Powers, G. Huber, and R. E. Goldstein, Phys. Rev. E. 65, 041901 (2002).
[13] F. Jülicher and U. Seifert, Phys. Rev. E 49, 4728 (1994).
[14] J. E. Hassinger, G. Oster, D. G. Drubin, and P. Rangamani, Proc. Nat. Acad. Sci. U.S.A 114, E1118 (2017).
[15] J. Kierzenka and L. F. Shampine, ACM Trans. Math. Softw. 27, 299 (2001).
[16] L. F. Shampine, J. Kierzenka, M. W. Reichelt, et al., Tutorial notes 2000, 1 (2000).
[17] D. Needham and R. S. Nunn, Biophys. J. 58, 997 (1990).
[18] E. Evans, V. Heinrich, F. Ludwig, and W. Rawicz, Biophys. J. 85, 2342 (2003).
[19] N. Ojkic, J. López-Garrido, K. Pogliano, and R. G. Endres, PLoS Comput. Biol. 10, e1003912 (2014).
[20] J. Lopez-Garrido, N. Ojkic, K. Khanna, F. R. Wagner, E. Villa, R. G. Endres, and K. Pogliano, Cell 172, 758 (2018).
[21] J.-F. Allemand and B. Maier, FEMS Microbiol. Rev. 33, 593 (2009).
[22] N. Liu, G. Chistol, Y. Cui, and C. Bustamante, eLife 7, e32354 (2018).