The Horopter: Old and New

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ABSTRACT

The horopter's history may partly be responsible for today's ambiguous psychophysical definitions and obscured physiological significance. However, despite frequently expressed contempt when the horopter is mentioned, it is a useful clinical tool integrating physiological optics and binocular spatial perception. This article reviews basic concepts underlying binocular perception and stereopsis. Then, the horopter's old historical ideas that still influence today's research are discussed, showing their surprising contradictions with the conceptualized binocular vision. Two recent geometric theories of the horopter with progressively higher eye model fidelity resolve the contradictions. The first model corrects the 200-year-old Vieth-Müller circle as a geometric horopter by placing the nodal point in its anatomical location in a single refractive surface schematic eye. It provides the geometric horopter's analytical form for all the eyes' fixations for the first time. In the second geometric theory, the horopters closely resembling empirical horopters are constructed as conic sections in the binocular system with the asymmetric eye (AE) model. The AE comprises two parameters: the fovea's anatomical displacement angle from the eyeball's posterior pole and the crystalline lens's tilt angle. The foveal displacement and the cornea asphericity contribute to the eye's optical aberrations that the lens' tilt is partially compensating. This geometric theory corrects Ogle's classical but anatomically unsupported model of empirical horopters. Lastly, the geometric model of the empirical horopter is extended to the families of iso-disparity conic sections, which are used to study the global geometric aspects of phenomenal space in the Riemannian geometry framework.

1. Introduction

A group of scientists with extensive teaching records of both normal and abnormal binocular vision that involved both the underlying theory and its multitude of clinical applications described in [Ciuffreda et al. 2016] their experience as follows:

> *A question that has been posed to us on more than one occasion, frequently with a look ranging from inquisitiveness to disdain, is, "Why the horopter?" Certainly, this is a fair question. Unfortunately, the horopter seems to be misunderstood by many, hence the question.*

Of course, many older and newer publications contributed to this open contempt for the horopter, as the next two quotes illustrate. It was written in [Stevens 1905],

> *No question in physiologic optics has been more attractive to investigators in that department of research than that of the horopter, yet, although the ablest thinkers have devoted themselves to it with great earnestness, the subject has remained an unsolved enigma.*

Then, 70 years later, in [Richards 1975],

> *The significance of the horopter [...] has probably been exaggerated. Not only is its physiological significance obscure, but even its psychophysical definition has become ambiguous.*

This short article aims to help understand how it could come to such different attitudes toward the horopter. First, the basic concepts underlying binocular perception and stereopsis are introduced. Then, the 200-year-old history of the horopter that is still influencing today's research is presented. These two discussions display the inconsistencies between the physiologically understood binocular vision and still using the old horopter's ideas. Then, the newest published research papers that eliminate inconsistencies surrounding the horopter's conception are reviewed.

2. Binocular vision and stereopsis

Our two laterally separated eyes' retinal photoreceptors receive slightly different 2D projections of photons reflected off objects located in physical space. The brain's immense number of neurons process these proximal stimuli to enhance our spatial vision with depth, shape, and sensation of being immersed in the 3D environment. How is this feat achieved? To this end, two unique small retinal areas, each in a different eye, share a common subjective visual direction when separately stimulated. These retinal areas are called corresponding retinal elements determined by the nonius paradigm. We consider each pair of the binocular correspondence as having zero disparity. For a given binocular fixation, the horopter is the locus of points in space such that each point on the horopter projects to a pair of corresponding retinal elements. Thus, it is the distal stimuli curve of zero-disparity.

Further, a small object located away from the horopter has nonzero disparity and, in general, is perceived as two objects. However, the brain fuses its disparate images into a single percept for an object close to the horopter. Then the extracted disparity is used by the brain to create our sense of depth relative to the horopter. Even the objects located farther from the horopter, which leads to double vision, the disparity conveys the sense of depth [Ogle 1953].

The brain uses the disparity between two spatial points to sense the relative depth. Hence, our perception of objects' shapes and their relative locations in 3D space, stereopsis, and visual space geometry. Wheatstone's 1838 invention of the stereogram demonstrated that the brain uses horizontal retinal disparity to support stereopsis. Thus, stereopsis is supported mainly by the horizontal retinal correspondence and, hence, the longitudinal horopter.

The outcome of the above outlined physiological processes is the beholder's three-dimensional visual experience combined with other perceptual modalities and transcended by higher cognitive processing. However, Julesz's random-dot stereogram confirmed Wheatstone's observation and asserted that the spatial relations in visual space could be separated from psychological processes

[Julesz 1960]. This realization laid the ground for studies in the physiology basis of stereopsis, cf. [Hubel and Wiesel 1962, Barlow et al. 1967, Poggio and Talbot 1981].

Finally, human disparity contains fine and coarse disparity [Norcia et al. 1985]. Fine disparity allows us to determine the depth of objects in the central visual area where disparate images are fused into a single percept (Panum's fusional area). Coarse disparity provides stereopsis from disparities well beyond the fusible range. It is still useful for depth perception for moderately large disparities, even for double images but is only clearly signed with a vague impression of depth magnitude for very large disparity values [Blakemore 1970, Wilcox and Allison 2009]. Importantly, coarse disparity creates our sense of being immersed in the ambient environment [Barry 2013].

3. The Horopter's two-century-old history

This section is not a historical study of the horopter. It traces the original `geometric' definitions of the planar horopters, laboratory measurements of the empirical horopters, and their more recent analytical descriptions.

For a long time, two ideas prevailed about the horopter: geometric and empirical. The first circular horopter in which perceived equal angles along its points was deduced from Euclid's theorem in Aguilonius' Six Books of Optics, published in 1613. However, the credit for inventing circular horopters goes to Vieth (1818) and Müller (1826) for connecting the spatial horopter in the horizontal plane with the physiological retinal correspondence, the hallmark of the single vision [Shipley and Rawlings 1970a]. It asserted that each point of the horopter projects a pair of corresponding elements, each element of the pair in one of the two retinas if the corresponding elements are assumed to coincide when the retinas are precisely superimposed. Thus, the original horopter that invoked physiological correspondence was strictly geometric. This circular horopter is known as the Vieth-Müller circle (VMC).

The second idea was based on the actual experimental determination of the horizontal horopter made first by Hillebrand about 130 years ago. He measured the apparent frontal line and obtained the horopter’s curve positioned between the VMC and the objective fronto-parallel line, known as Hering-Hillebrand horopter deviation. One of possible explanations he gave, was that the corresponding elements are compressed in the temporal retinae relative to those in the nasal retinae. Later, the Hillebrand hypotheses was confirmed, and the precise correspondence’s distribution measured by the nonius paradigm introduced in the previous section. Over time many different notions of the horopter were introduced based on different measurement criteria [Tyler 2004]. Whether geometric or any empirical method, each gives a different horopter curve.

Despite different measurement criteria, the nonius method remains the most precise determination of the corresponding elements [Shipley and Rawlings 1970a]. However, given that the empirically measured distribution of the corresponding elements differs between people and cannot be readily achieved [Shipley and Rawlings 1970b, Noorden and Campos 2002], the geometric idea prevailed up to our time, establishing two types of horopters: the VMC as a theoretical and the measured horopter as an empirical.

In the geometrical description of the VMC as the horopter, one assumes the binocular system with a single refractive surface schematic eye model. Then, the center of the projection from the VMC to the retina should be identified with the nodal point located about 0.6 cm anterior to the eye's rotation center [Gulick and Lawson 1976]. However, the center of projection has not been precisely specified throughout history: Vieth placed it at the pupil and Müller at the lens center. However, the projection center is most often placed at the eye's rotation center as originally proposed by Volkmann (1836).

The analytic form of the VMC has been known for an arbitrary fixation only when the nodal point is placed in the eye's rotation center. In this case, when the eyes' are fixating on points on the VMC, it remains fixed. Otherwise, the VMC has only been known for symmetric fixations for all other nodal point locations, cf. [Turski2016]. Thus, the analytic form of the VMC is only known when it is physiologically inaccurate by the nodal point identified with the eye's rotation center. However, when Helmholtz in [Helmholtz 1867/1925] gave his final form of the shape of the 3D theoretical horopter based on geometric considerations, his horopter agreed with the VMC in the horizontal visual plane. Unfortunately, it established the anatomically incorrect horopter as the standard reference.

The problem with the VMC ambiguity has sometimes been `solved' peculiarly. For example, geometric conditions listed in Section 6.3 of [Stidwill and Fletcher 2011] that VMC must satisfy to be the horopter passing through the nodal points, included: `*And the eyes rotate about their nodal points and not their centers of rotation [sic].*' A similar statement was made in [Kalloniatis and Luu 2007], but neither paper refers to the source.

4. The Horopter's newest developments

The correct formulation of the geometric horopter for the symmetric eye model with a single nodal point, that is, a single refractive surface schematic eye model, was only given in 2016 [Turski 2016]. Its analytical form was fully described there, and its potential significance for fine aspects of binocular perception was discussed. I hypothesized there that the size and shape changes perceived in objects during eye movements might provide perceptual benefits, such as breaking camouflage but may also provide the aesthetic benefit of stereopsis [Ponce and Born, 2008]. Regardless of this result, the relative disparity is usually assumed independent of the eyes' position.

Next, in 2020, the geometric model of the empirical horopters as conic sections constructed in [Turski 2020] in the binocular system with asymmetric eyes (AEs) advanced the classical model of empirical horopters introduced for symmetric fixations as conic sections in an ad hoc way by Ogle in [Ogle 1932] and later extended to all fixations by Amigo in [Amigo 1965]. In contrast to Ogle's theory, horopters in [Turski 2020] are anatomically supported and correctly pass through the nodal points. Their geometry is integrated with the AEs' positions in the horizontal plane of bifoveal fixations. Their transformations are visualized in a computer simulation.

The AE model is introduced and comprehensively discussed in [Turski 2018] and slightly modified in Turski 2020]. The AE is comprised of the fovea's anatomical displacement from the posterior pole by angle $\alpha = 5.2°$ and the crystalline lens' tilt by angle β. Angle α is relatively stable in the human population, and angle β varies between $-0.4°$and $4.6°$. In the human eye, the fovea's

displacement from the posterior pole and the cornea's asphericity contribute to optical aberrations that the lens' tilt then partially compensates for [Artal 2014].

Further, this model of the empirical horopters is extended to the families of iso-disparity conics in a preprint [Turski 2021]. The lack of a family of iso-disparity curves that would more richly describe the structure of binocular visual space was mentioned in [Arditi 1986].

For almost all bifoveal fixations on points of the horizontal binocular field, the iso-disparity curves simulated in [Turski 2021] consist of either a family of hyperbolas or ellipses, depending on both the AE's two parameters: the fovea displacement from the posterior pole and the crystalline lens tilt relative to the cornea, and the location of the point of fixation. For every pair of such parameters of the AE, a unique symmetric fixation furnishes the family of iso-disparity frontal straight lines. This fixation is said to be at the abathic distance and is called the resting eyes posture.

For the eyes in the resting posture, the phenomenal metric as the Riemann metric is proposed based on numerical analysis of simulated iso-disparity lines and the topography of the retinal photoreceptors. In the Riemannian geometry framework, the geodesic differential equations are transformed into celebrated Ermakov-Pinney differential equations with general solutions despite being nonlinear [Ermakov 1880, Pinney 1950]. Interestingly, the geodesics in the gaze direction are incomplete and give finite distances to the horizon, which should be expected. Further, the Riemann curvature tensor vanishes such that the phenomenal field is flat. The sign of the curvature can be inferred from the global behavior of the iso-disparity ellipses and hyperbolas only when for the metric parameters for which the constant-depth frontal lines at the abathic distance fixations are geodesics.

5. Discussion

The VMC identified with the geometric horopter approximates only the longitudinal horopter curve at the fixation point, especially when compared to the measured empirical horopters. It also has simple geometric properties by the nodal point placed at the eye's rotation center. However, the nodal point's identification with the eye's rotation center degenerates perceptual functions by rendering relative disparity fixation independent.

When the nodal point is in anatomical location, the relative disparity depends on the eyes' posture. Although it should support the aesthetic functions of the eyes through the eyes' movements, the analytic form of the geometric horopter was only known before 2016 for symmetric fixations.

Similar concerns apply to the empirical horopter analytic modeling. Before 2020 there was one model of the empirical horopters proposed by Ogle in 1932 in which the conic sections are postulated ad-hoc with free parameters [Ogle 1932]. Further, they lacked the eye's anatomical underpinning, incorrectly passed through the eyes' rotation centers, and were not integrated with binocular eye movements.

Although the eccentric objects on the VMC may even be seen in crossed double images because they are away from the empirical horopter [Tschermak-Seysenegg 1952], the VMC has been used as the benchmark reference in the theoretical modeling of the horopter, I give below two examples.

Based on the VMC with the nodal point identified with the eye's rotation center, the theoretical point horopter is extended in [Schreiber et al. 2006] to a full surface. This extended horopter quantifies binocular alignment and visualizes its dependence on eye position. Thus, all results in this paper are conditioned by the assumption that the eye's nodal point is taken at the rotation center. However, the physiologically correct horopter undergoes rotation and deformation when the eyes change bifoveal fixation point, cf. [Turski 2020], and, therefore, its surface extrapolation will rotate and deform too. Thus, the dependence on the eye position is more complex than described by the VMC and, consequently, the analysis in [Schreiber et al. 2006] is specious.

It is suggested by [Sprague et al. 2015] that the empirical horopter deviation from the VMC is a result of the visual system allocating resources according to natural disparity statistics for binocular correspondence matches. However, given the studies in [Šikl and Šimeček 2014, Turski 2020], it is a highly unlikely suggestion. In [Šikl and Šimeček 2014], after the Mars-500 experiment, it was concluded that the long-term confinement does not affect visual space perception, while in [Turski 2020], it was asserted that the shape of empirical horopters is caused primarily by the misaligned eye's optical elements human healthy eyes [Chang et al. 2007, Schaeffel 2008].

Importantly, the geometrically constructed horopters in [Turski 2020] in the binocular system with the asymmetric eye model closely resemble empirical horopters. Their shape depends on the human eyes' natural asymmetry of optical elements. Thus, it erases the dichotomy between geometric horopter and empirical (nonius) horopter. Further, the geometric horopter theory in [Turski 2020] allows the straightforward extension to iso-disparity families of conic sections. The iso-disparity conics are ellipses, hyperbolas, or straight frontal lines depending on the AE's parameters and the point of bifoveal fixation [Turski 2021]. They provide the global geometric aspects of binocular visual space.

Of course, the VMCs compromise these global aspects of spatial perception because of their large difference from the empirical horopter hyperbolas for moderately far fixations; the coarse disparity in the near periphery will differ markedly. This difference will impact our sense of being immersed in the ambient environment.

Presumably, the time has come to reconsider the horopter's ideas taught by old masters that, unfortunately, contributed to ambiguous psychophysical definitions and obscured physiological significance.

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