



22           **Abstract**

23           We propose a novel effective framework for analysis of the shared genetic  
24 background for a set of genetically correlated traits using SNP-level GWAS summary  
25 statistics. This framework called SHAHER is based on the construction of a linear  
26 combination of traits by maximizing the proportion of its genetic variance explained by the  
27 shared genetic factors. SHAHER requires only full GWAS summary statistics and matrices  
28 of genetic and phenotypic correlations between traits as inputs. Our framework allows both  
29 shared and unshared genetic factors to be to effectively analyzed. We tested our framework  
30 using simulation studies, compared it with previous developments, and assessed its  
31 performance using three real datasets: anthropometric traits, psychiatric conditions and lipid  
32 concentrations. SHAHER is versatile and applicable to summary statistics from GWASs  
33 with arbitrary sample sizes and sample overlaps, allows incorporation of different GWAS  
34 models (Cox, linear and logistic) and is computationally fast.

35

## 36            **Introduction**

37            There is a growing interest in studying the shared genetic background between  
38            genetically correlated traits<sup>1-5</sup> (see, for example, the number of PubMed search results by  
39            year for keywords related to "shared genetic background"). Studying this shared genetics  
40            between traits can help discover pleiotropic interactions, common genes and pathways, and  
41            identify genetic effects that are unique for each trait.

42            The problem of the decomposition of the variance of several traits into the  
43            shared/unshared genetic and environment components were first formulated by S. Wright in  
44            1921<sup>6</sup>. There are widely used classic twin designs to have this problem solved. They are  
45            based on structural equation modelling, in particular, multivariate pathway models assuming  
46            the existence of the genetic influences common for all traits and unique for each trait<sup>7</sup>.  
47            These designs are implemented only for the variance decomposition, but not for the  
48            identification of the genetic factors that determine these genetic impacts.

49            There are several terms for these common and unique genetic impacts. Hereafter we  
50            will call them the 'shared genetic impact' (SGI) and 'unshared genetic impacts' (UGI). The  
51            genetic factors that determine these impacts will be called 'shared genetic factors' (SGF) and  
52            'unshared genetic factors' (UGF), respectively. The heritability of each trait explained by  
53            SGF and UGF will be called 'shared heritability' and 'unshared heritability', respectively.

54            The application of different methods of multivariate analysis in genome-wide  
55            association studies (GWAS) allows the problem of SGF and UGF identification to be  
56            partially solved<sup>8-13</sup>. The multivariate methods involve complicated genetic or/and  
57            phenotypic correlation structures of traits in the analysis. In most cases, this increases the  
58            power of detection of the loci associated with several traits due to pleiotropic effects. If the  
59            detected locus has a pleiotropic effect on all studied traits, the locus could potentially be  
60            attributed to SGF, and if not, to UGF. However, a pleiotropic effect of the locus on all  
61            studied traits is necessary but insufficient for inclusion of this locus in SGF (at least effects  
62            should be also collinear between traits, see the model description below). Also, if a locus  
63            belonging in fact to SGF was not identified as having pleiotropic effects on all traits due to a  
64            limited statistical power of the analysis, then the locus can be erroneously assigned to UGF.  
65            Moreover, this approach of SGF identification assumes a manual classification of loci,  
66            which prevents the use of more sophisticated modern in-silico approaches for genetic  
67            analysis, for example, the ones that rely on GWAS summary statistics<sup>14</sup>. To our knowledge,

68 there is no specific method that could be good for both variance component decomposition  
69 and identification of SGF and UGF.

70 We had previously developed a method for obtaining genetically independent  
71 phenotypes (GIPs) <sup>2</sup>. This method is based on the calculation of the principal components  
72 using genetic rather than phenotypic correlations. We applied this method to genetically  
73 correlated pain phenotypes and aging related phenotypes and showed that the first GIP  
74 component, GIP1, that explains the largest proportion of the genetic variance probably could  
75 be interpreted as SGI <sup>2, 15</sup>. This makes GIP promising for identification of loci attributed to  
76 SGF. However, this method was not designed specifically for SGI analysis. In addition, no  
77 specific experiments have been performed to validate the approach or to estimate its  
78 statistical properties.

79 Here, we present a novel general framework for the estimation of shared and unshared  
80 heritability and identification of the shared and unshared genetic factors using the summary  
81 statistics of original traits. The essence of our approach is to find the optimum linear  
82 combination of traits which has the maximum proportion of its genetic variance explained  
83 by the SGF. We validated our framework using simulation studies under different scenarios,  
84 by comparing it with the developed GIP approach, and assessed its performance using three  
85 real datasets: anthropometric indices, psychiatric disorders and conditions, and lipid  
86 concentrations.

87

## 88 **Results**

### 89 **Abbreviations and terms**

90 SHAHER: a framework for the estimation of the shared and unshared heritability of studied  
91 traits and identification of the shared and unshared genetic factors using the summary  
92 statistics of original traits.

93 SGI: shared genetic impact.

94 UGI: unshared genetic impact.

95 SGF (shared genetic factors): genetic factors involved in the control of all studied traits and  
96 whose effects are collinear between all studied traits; SGI is due to SGF.

97 Shared heritability: the proportion of the trait variance explained by SGF.

98 SGIT (shared genetic impact trait): a trait defined as a linear combination of original traits  
99 maximizing its shared heritability.

100  $\alpha$ : the coefficients of an optimum linear combination of original traits for building the SGIT.

101 UGF (unshared genetic factors): the residual genetic factors of an original trait after  
102 exclusion of the SGF; UGI is due to UGF.

103 Unshared heritability: the proportion of the trait variance explained by UGFs.

104 UGIT (unshared genetic impact trait): an original trait after adjustment for the SGIT.

105  $\gamma$  the coefficients of a linear combination of original traits for building the UGIT.

106 MaxSH (MAXimization of Shared Heritability): a method for estimating the shared and  
107 unshared heritability of each trait and calculating the coefficients of the linear combination  
108 of the original traits:  $\alpha$ , to build the SGIT, and  $\gamma$ , to build the UGITs.

109 sumCOT (summary-level GWAS for linear Combination of Traits): a method to compute  
110 GWAS summary statistics for the linear combination of the original traits using their  
111 summary statistics.

112

### 113 **Shared heredity model**

114 We adopted a commonly used multivariate pathway model <sup>7</sup> in terms of SGF and  
115 UGF. We call it the 'shared heredity model'. For simplicity, we consider SGF and UGF as  
116 biallelic SNPs and consider a sample of  $N$  unrelated individuals measured for  $K$  traits and  
117 genotyped for  $M$  SNPs. For a standardized normal trait,  $y$  ( $N \times 1$ ), the traditional polygenic  
118 (null) model takes the form:  $y = G\beta + \varepsilon$ , where  $G$  is an ( $N \times M$ ) matrix of standardized  
119 genotypes;  $\beta$  ( $M \times 1$ ) and  $\varepsilon$  ( $N \times 1$ ) are genetic and non-genetic random effects, respectively;  
120  $\beta \sim N(\mathbf{0}, h^2 I_M)$  and  $\varepsilon \sim N(\mathbf{0}, (1-h^2) I_N)$ , where  $\mathbf{0}$  is a null mean vector,  $h^2$  is the trait  
121 heritability, and  $I$  is an identity matrix of the given dimension. For unrelated individuals, we  
122 expect  $y \sim N(\mathbf{0}, I_N)$ .

123 We propose to divide  $M$  SNPs into two non-overlapping SNP sets with sizes  $M_0$  and  
124  $M_1$  ( $M_0 + M_1 = M$ ). The set of  $M_0$  SNPs called 'SGF' includes only those SNPs whose effects  
125 on all traits are collinear. The set of  $M_1$  SNPs consists of the other SNPs, which do not have  
126 shared joint influence on all traits at once, this set being called 'UGF'. In accordance with  $M$ ,  
127  $G$  is divided into two matrices,  $G_0$  ( $N \times M_0$ ) and  $G_1$  ( $N \times M_1$ ). To decompose every trait into  
128 components explained by the SGF and UGF, we rewrote the traditional polygenic model in  
129 terms of  $G_0$  and  $G_1$

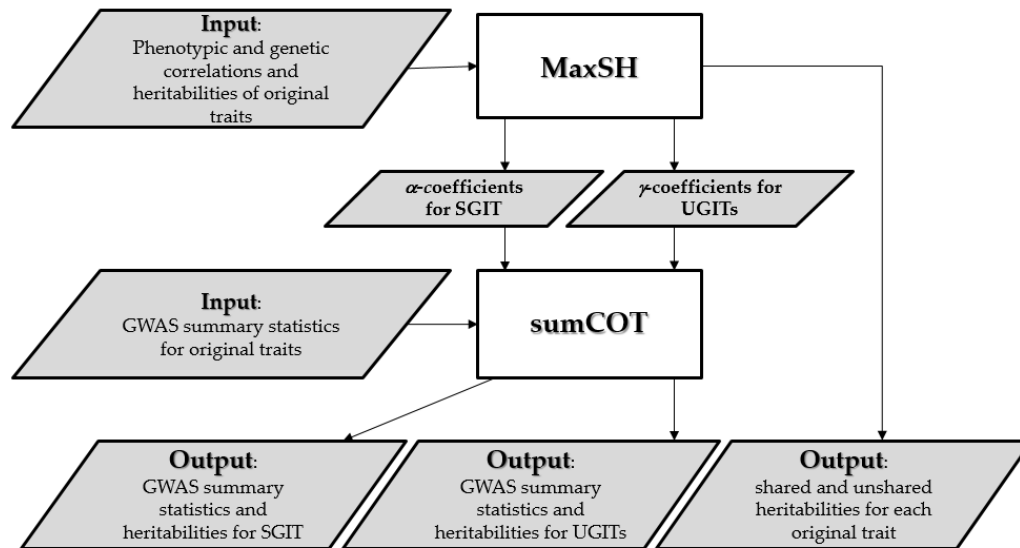
$$y_i = \underbrace{G_0 b_{0i}}_{\text{due to SGF}} + \underbrace{G_1 b_{1i}}_{\text{due to UGF}} + \varepsilon_i. \quad (1)$$

130 Here, the first and second terms are genetic components explained by SGF and UGF,  
 131 respectively, which are assumed independent. In the first term,  $b_{0i}$  is an  $(M_0 \times 1)$  vector of  
 132 non-zero SGF effects, which can be presented as  $\beta_0 w_i \sqrt{h_i^2}$ , where  $\beta_0$  is an  $(M_0 \times 1)$  non-zero  
 133 vector that is the same for all traits,  $\beta_0 \sim N(0, I_{M_0})$ , and  $w_i^2 h_i^2$  is the heritability of the  $i$ -th  
 134 trait explained by SGF. Here  $w_i$  is a non-zero trait-specific multiplier:  $w_i^2$  denotes the  
 135 proportion of  $h_i^2$  explained by SGF; the value of  $w_i$  can be positive and negative, indicating  
 136 the direction of the SGF effect on the  $i$ -th trait.  $G_0 \beta_0$  is the so-called shared genetic impact  
 137 or SGI. In the second term of Model (1),  $b_{1i}$  is an  $(M_1 \times 1)$  vector of UGF effects, which can  
 138 be presented as  $b_{1i} = \beta_{1i} \sqrt{(1 - w_i^2) h_i^2}$ ,  $\beta_{1i} \sim N(0, I_{M_1})$ . In contrast to  $\beta_0$ ,  $\beta_{1i}$  are different  
 139 for different traits, moreover they are not collinear. For illustrative purposes, we rewrote  
 140 Equation (1) as:

$$y_i = \underbrace{G_0 \beta_0}_{SGI} w_i \sqrt{h_i^2} + \underbrace{G_1 \beta_{1i}}_{due\ to\ UGF} \sqrt{1 - w_i^2} \sqrt{h_i^2} + \varepsilon_i.$$

141

## 142 Overview of the SHAHER framework



143

144 **Figure 1. Flowchart of the SHAHER framework.** Details are given in the text.

145

146 For analyses of the SGI and UGI on a set of correlated traits, we propose an effective  
 147 multi-stage framework named SHAHER (see Figure 1). The concept of the framework is

148 first to partition the genetic basis of each original trait into two components: one shared by  
 149 all the original traits and one shared not by all the original traits, and then to identify the  
 150 SNPs that contribute to these genetic components. To do this, we propose to construct new  
 151 traits: (1) an SGIT as a linear combination of original traits, which has the maximum  
 152 possible heritability explained by the SGF, and (2) UGITs as linear combinations of the  
 153 original traits, which are obtained by adjusting the original traits for the SGIT. This means  
 154 that the genetic basis of the UGITs is predominantly determined by the UGF.

155 Our framework requires matrices of phenotypic correlations ( $U_{phen}$ ) between the  
 156 original traits, the matrices of genetic correlations ( $U_{gen}$ ) between the original traits, the  
 157 heritabilities of the original traits and GWAS summary statistics of the original traits as  
 158 inputs. It is worth noting that  $U_{phen}$ ,  $U_{gen}$  and heritabilities could be estimated using GWAS  
 159 summary statistics of the original traits, for example, by the LD score regression method<sup>16</sup>.

160 SHAHER starts with a preliminary stage, which verifies the presence of SGI in a given  
 161 set of traits. This is achieved by checking the following requirements for  $U_{gen}$ : it must be  
 162 positive definite; the absolute values of its elements must be significantly more than a given  
 163 threshold, and the rank of the correlation matrix derived from  $U_{gen}$  by rounding its elements  
 164 to extremal correlation values, either -1 or 1, must be equal to one. If the requirements are  
 165 met, we turn to the basic stages of SHAHER.

166 *The MaxSH stage.* To determine the  $\alpha$  and  $\gamma$  coefficients for the linear combinations of  
 167 the original traits to build the SGIT and UGITs, we developed the MaxSH method, which is  
 168 based on the correlation component model given below. This model partitions the  
 169 phenotypic correlation matrix,  $U_{phen}$ , into environmental and genetic components,  $U_{env}$  and  
 170  $U_{gen}$ , respectively, the latter being further subdivided into two components caused by the  
 171 SGF and UGF:

$$\begin{aligned}
 U_{phen} &= \underbrace{\sqrt{H^2} U_{gen} \sqrt{H^2}}_{\text{genetic component}} + \underbrace{\sqrt{I - H^2} U_{env} \sqrt{I - H^2}}_{\text{environmental component}} \\
 U_{gen} &= \underbrace{W \mathbf{1} \mathbf{1}^T W}_{\text{due to SGF}} + \underbrace{\sqrt{I - W^2} U_{unsh} \sqrt{I - W^2}}_{\text{due to UGF}}
 \end{aligned}
 \tag{2}$$

172 Here  $W$  is a diagonal matrix, whose  $i$ -th diagonal element is  $w_i$ ;  $U_{unsh}$  is a matrix of genetic  
 173 correlations explained by UGF;  $H^2$  is a diagonal matrix, whose  $i$ -th diagonal element is  $h_i^2$ ,  
 174 and  $\mathbf{1}$  is a ( $k \times 1$ ) vector of units. Using this model, MaxSH solves several tasks.

175 First of all, using only the genetic correlation matrix,  $U_{gen}$ , we estimate the proportion  
 176 of heritability of every trait explained by SGF ( $W$ ). To do this, we minimize the difference  
 177 between  $U_{gen}$  and the auxiliary matrix  $V$ . This matrix is built using formula (2), with the  
 178 identity matrix used instead of  $U_{unsh}$ . The second task is to determine the  $\alpha$ -coefficients,  
 179 which is solved by maximizing the shared heritability of the SGIT. This task is analytically  
 180 solved as

$$a = \frac{U_{phen}^{-\frac{1}{2}} HW \mathbf{1}}{\sqrt{\mathbf{1}^T W H U_{phen}^{-1} H W \mathbf{1}}}.$$

181 It requires  $U_{phen}$ ,  $H^2$  and  $W$  as input data.

182 After determining the  $\alpha$ -coefficients and building the SGIT, we build a UGIT for every  
 183 trait using the residual regression equation  $UGIT_i = y_i - SGIT * c_i$ , where  $c_i$  is the impact of  
 184 the SGIT on the  $i$ -th original trait, defined as

$$185 \quad c_i = cov_{gen}(y_i, SGIT) / h_{SGIT}^2.$$

186 Here  $cov_{gen}$  denotes a genetic covariance. Note that we should use genetic rather than  
 187 phenotypic covariances, as our goal is to adjust only the genetic components of the original  
 188 traits.

189 Since the SGIT is the linear combination of the original traits, the UGITs are linear  
 190 combinations of the original traits, too. The coefficients of these linear combinations called  
 191 the  $\gamma$ -coefficients form the matrix of the  $\gamma$ -coefficients  $\Gamma = (I_K - \alpha \alpha^T)$ , where the  $i$ -th  
 192 column of  $\Gamma$  corresponds to linear combination coefficients for building the  $i$ -th UGIT.

193

194 *The sumCOT stage.* This stage is aimed at obtaining GWAS summary statistics for  
 195 the SGIT and UGITs using the previously determined  $\alpha$  and  $\gamma$  coefficients, GWAS summary  
 196 statistics (Z-scores, allele frequencies and sample sizes for each SNP) for the original traits  
 197 and the matrix of phenotypic correlations. The method can use Z scores obtained from any  
 198 regression model and allows for varying sample sizes and sample overlap between traits.  
 199 This sample overlap is incorporated into the estimation of the matrix of phenotypic  
 200 correlations. In short, the SNP effects for combined trait are calculated by summing effect  
 201 estimates from the individual trait GWASes, each multiplied by their corresponding linear  
 202 coefficient ( $\alpha$  or  $\gamma$ ), and standardized by the expected variance. The standard errors of the  
 203 SNP effect are calculated using variance-covariance arithmetic, taking into account the  
 204 phenotypic covariance between GWAS results to adjust for the sample overlap. Effective



205 sample sizes are then estimated based on the median Z statistic and allele frequencies by  
206 solving Equation (1) in <sup>17</sup>.

207 At the final stage, SHAHER checks for the correctness of the output. In particular, we  
208 anticipate that UGITs do not have a shared genetic basis. This is verified by applying  
209 MaxSH to the matrix of correlations between UGITs.

210 To summarize, our framework estimates shared and unshared heritabilities for each of  
211 the studied original traits and produces GWAS summary statistics for the SGIT and UGITs  
212 as outputs.

213 The full details and mathematical formulae of SHAHER are in *Supplementary*  
214 *Methods*.

## 215 **Simulation study**

216 To assess the MaxSH performance, we conducted simulation studies. We (1) assessed  
217 the accuracy of  $w$  estimates (using  $\Delta W$  metrics estimated as  $\left(\frac{w_0 - w_{est}}{w_0}\right)^2$ , where  $w_0$  and  $w_{est}$   
218 are modeled and estimated  $w$ , respectively) with respect to the loss function given in Fig. 3,  
219 (2) assessed the proportion of the shared heritability to the total heritability of the SGIT (the  
220  $Q$ -value) with respect to the loss function, and (3) compared the analytically predicted  
221 total/shared heritabilities of two traits: SGIT and the first component, GIP1, obtained by GIP  
222 method <sup>2</sup>. The  $Q$ -value can be interpreted as the specificity metrics of the SGIT: the closer  
223 the  $Q$ -value to 1, the lower the share of unshared heritability in the total heritability of the  
224 SGIT. The simulation scenarios were based on six varying parameters that describe the  
225 properties of the genetic and phenotypic correlation matrices. Under each scenario, we  
226 considered two situations, where all traits have the same  $w^2$  and different  $w^2$ 's. To distinguish  
227 between these situations, we will hereinafter write either ' $w^2$ ' or ' $w^2$ 's'. In total, we  
228 performed 10,000 iterations of simulations for each of 288 scenarios.

229 The results are presented in Supplementary Figures S1-18. For all scenarios, there are  
230 few general patterns: (1) the higher simulated  $w$  values, the higher the accuracy of the  $w$   
231 estimates, (2) the accuracy of the  $w$  estimates and the  $Q$ -value increase with an increasing in  
232 the number,  $K$ , of traits, (3) for all scenarios with  $w^2 > 0.8$ ,  $\Delta W$  was very low ( $< 0.025$ ) and the  
233  $Q$ -value was more than 90%.

234 For all scenarios with three traits, the accuracy of the  $w$  estimates was in general low:  
235  $\Delta W$  was not higher than 0.7 for scenarios with  $w^2 = 0.2$  and 0.3, although at  $w^2$  equal to or  
236 higher than 0.4  $\Delta W$  was less than 0.2. The  $Q$ -value was higher than 60% for almost all

237 scenarios with  $w^2 \geq 0.4$ . In almost all cases, the total and shared heritabilities of the SGIT  
238 were higher than the corresponding heritabilities of GIP1, except for the scenarios with  
239  $h^2=0.8$ .

240 For the scenarios with four and five traits, the accuracy of  $w$  estimates was higher:  
241  $\Delta W < 0.15$  for  $w^2 \geq 0.4$  and  $\Delta W < 0.05$  for  $w^2 \geq 0.5$ . For the scenarios with  $w^2 \geq 0.5$ , the  $Q$ -value  
242 was more than 70% for four traits and more than 80% for five traits. Again, the total and  
243 shared heritabilities of the SGIT were higher than the corresponding heritabilities of GIP1  
244 under all scenarios, except for the scenarios with  $h^2=0.8$ . In the scenarios with  $h^2=0.8$ , the  
245 total and shared heritabilities of the SGIT were higher than those of the GIP1 at  $w^2 \geq 0.5$ .

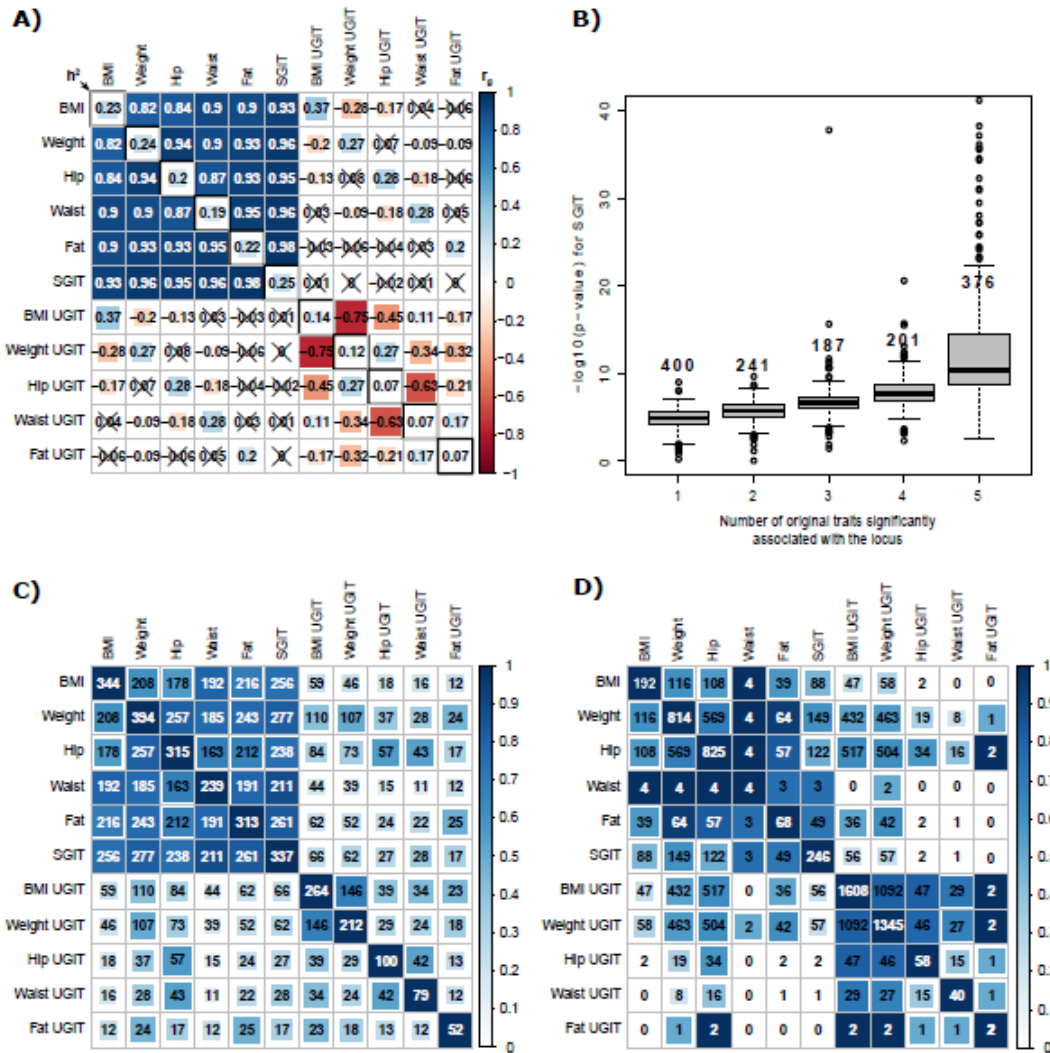
246 In summary, the performance of MaxSH was suitable at  $w^2 \geq 0.5$  and when the number  
247 of traits being higher than or equal to four. In the case of small  $w$  or three traits, the results of  
248 MaxSH should be interpreted with caution.

## 249 **Real data assessment**

250 We applied SHAHER to three datasets: anthropometric (five traits), psychometric  
251 (four traits) and lipid traits (three traits). We should note that the performance of SHAHER  
252 applied to three traits is limited (see simulation results), yet still passable, although the  
253 results should be interpreted with caution. We present SHAHER results for anthropometric  
254 traits in the main text as an example. The full results for the psychometric and lipid traits are  
255 presented in Supplementary Results.

256 At the first step, we confirmed that SGI exists for five traits. At the second step, we  
257 determined the  $\alpha$  and  $\gamma$  coefficients and their CI (see Supplementary Table 1a). At the third  
258 step, we applied sumCOT and obtained GWAS results for the SGIT and UGITs (see  
259 Supplementary Table 2a for heritability estimates and LD score regression intercepts).  
260 SHAHER results are presented in Figure 2.

261



262

263 **Figure 2. Results of the application of SHAHER to anthropometric traits.** A) The  
 264 heatmap of genetic correlations between the original, SGI and UGI traits. The number, color  
 265 strength and size of the squares in the matrix show the values of the correlation coefficients  
 266 between the traits. The diagonal elements represent heritabilities. Crossed out values indicate  
 267 insignificant correlations. B) Boxplots of  $-\log_{10}(\text{p-value})$  for the SGIT with respect to the  
 268 number of the original traits significantly associated with the locus. Two outliers for loci  
 269 with  $-\log_{10}(\text{p-value}) > 40$  are omitted. The number at the top of the boxplot corresponds to  
 270 the number of significant SNPs. C) The heatmap of the numbers of overlapping loci between  
 271 traits. The numbers in the cells represent the absolute numbers of overlapping loci. The color  
 272 strength and size of the squares in the cells show the relative scaled number of overlapping  
 273 loci (on the scale from 0 to 1). The diagonal elements represent the number of loci found for  
 274 every trait. D) The heatmap of the numbers of overlapping gene sets between traits. The  
 275 color strength and size of the squares in the cells show the relative scaled number of  
 276 overlapping gene sets (on the scale from 0 to 1). The diagonal elements represent the number  
 277 of gene sets found for every trait.

278

279

280 Figure 2A demonstrates genetic correlations between all pairs of the original  
281 anthropometric traits, SGIT and UGITs. All the original traits were positively correlated with  
282  $r > 0.82$ . We did not observe any significant genetic correlation between the SGIT and the  
283 UGITs. Moreover, we did not observe additional SGI among UGITs, which was up to  
284 expectation. The heritabilities of the UGITs varied from 0.07 to 0.14.

285 We revealed a dependence of the SGIT p-value from the number of the original traits  
286 significantly associated with the locus (Figure 2B). It clearly shows that the loci associated  
287 with all the original traits have lower SGIT p-values than the other loci.

288 Joint clumping of 11 traits (five original traits, five UGITs and SGIT) resulted in 820  
289 genome-wide significantly associated loci ( $p\text{-value} < 5 \times 10^{-8}$ , Supplementary Table 3a). If a  
290 locus was not significantly associated with any of the original traits, it was considered new.  
291 SGIT was significantly associated with 337 SNPs. We detected no new loci among SGIT  
292 loci. The clumping of UGITs revealed 422 loci, of which 199 were new. At the same time,  
293 the clumping of only original traits allowed 621 loci to be detected, of which 161 could not  
294 be detected by analyzing SGIT or UGITs. Thus, the joint analysis of SGIT and UGITs  
295 increased the number of associated loci by more than 32%. Figure 2C reflects the  
296 overlapping between significantly associated loci for 11 analyzed traits. There is a weak  
297 albeit non-zero overlap between loci for UGITs and SGIT, although the genetic correlation  
298 between them is zero. It could be due to the conservative settings of the clumping procedure,  
299 which tends to clump together closely located loci, and due to some level of unspecificity of  
300 the SHAHER.

301 Next, we checked how enriched gene sets overlap between the SGIT, UGITs and  
302 original traits (see Figure 2D). Significant results ( $FDR < 5\%$ ) of enriched gene sets and tissue  
303 enrichment analyses are presented in Supplementary Table 4. As expected, the heatmap of  
304 the overlapping gene sets looks similar to the heatmap of genetic correlations and the  
305 heatmap of the overlapping loci. Moreover, there were almost no overlap between SGIT and  
306 any UGIT. For the original traits, the number of enriched gene sets varied a lot: from 4 for  
307 the waist to 825 for the hip circumference. It should be noted that we observed a high  
308 number of enriched gene sets for the BMI UGIT, 1608, which was almost ten times the value  
309 for BMI (192).

310 Finally, we obtained GIP1 GWAS statistics and calculated the genetic correlations  
311 between the SGIT and GIP1. The genetic correlation was higher than 0.97.

312

## 313 **Discussion**

314 We developed a new fast and efficient framework, which allows us to decompose the  
315 heritability of each trait from a given set of traits into two components. One of them is  
316 explained by shared genetic factors common to all traits. Another one is explained by  
317 unshared genetic factors specific for each trait. The framework not only decomposes  
318 heritability, but also identifies SNPs associated with the shared and unshared genetic  
319 impacts. To our knowledge, this framework is unparalleled. It has an additional advantage: it  
320 uses GWAS summary statistics obtained for original traits and does not require raw  
321 genotype or phenotype data.

322 We compared the performances of MaxSH and GIP in identifying the shared genetic  
323 components. GIP calculates the linear combination coefficients via the eigenvalues of the  
324 genetic covariance matrix and can be considered a close approximation to MaxSH. In our  
325 simulations, GIP and MaxSH were similar in almost all scenarios, with MaxSH being  
326 somewhat superior in terms of the power (total heritability) and quality (shared heritability).  
327 If obtaining genetically independent phenotypes is not the aim, we suggest using SHAHER,  
328 because it is more robust and gives additional metrics like SGI contributions to the  
329 heritability of the original traits.

330 The framework is computationally effective. The stage using sumCOT is the most  
331 time consuming. However, it takes only several minutes for an average computer to conduct  
332 a GWAS of a linear combination of traits with 6M SNPs using a C++ implementation of the  
333 sumCOT. MaxSH, based on numerical optimization procedures, and the other parts of the  
334 framework take seconds.

335 The proposed sumCOT method can be applied as an independent tool to address  
336 additional tasks. One of them is making a summary-level adjustment of traits by other traits  
337 using the same scheme as was used for obtaining the UGIT GWAS statistics. This can be  
338 helpful, for example, for ridding the studied trait's genetic component of the genetic  
339 component that was caused by the confounding or unaccounted effects of assortative mating  
340 or family effects, which is quite a problem in GWAS at the biobank scale<sup>15, 18</sup>. Another task  
341 is a GWAS for the trait that appears as a linear combination of the original traits. The  
342 sumCOT method is robust to differences in sample sizes used for GWASs of original traits  
343 and is applicable to different GWAS models (Cox, linear or logistic).

344 The main interest in the application of the SHAHER framework lies in the possibility  
345 of obtaining novel biological insights into a trait's heritability composition. This can be

346 achieved by the application of a huge variety of *in-silico* follow-up techniques to the SGIT  
347 and UGITs. The SGIT is of interest by itself, but we also emphasize the importance of the  
348 comparison of shared and unshared impacts for each trait. In our real data application, the  
349 most remarkable case is BMI in the set of anthropometric traits (see Figure 3C). We found  
350 246 and 1608 significantly enriched gene sets for the SGIT and UGIT of BMI, respectively,  
351 with negligible overlapping between them of size 56. By analyzing only BMI, we would  
352 have detected only 192 enriched gene sets. By analyzing each of the impacts separately, we  
353 dramatically increased the number of observed unique gene sets (1798 in total for both SGI  
354 and UGI). It means that each sub-phenotype controlled by the SGF and UGF is less  
355 heterogeneous than the original trait. According to the significant gene sets, the UGIT of  
356 BMI (see Supplementary Tables 4) controls some structural changes in body compositions  
357 and bone formation, while the SGIT is involved in some general signaling pathways and  
358 pathways related to nervous system development and probably to general psycho-social  
359 aspects of BMI, obesity and other anthropometric traits <sup>19</sup>.

360 Although SHAHER is effective, it has several limitations. First, when trait-trait  
361 genetic correlations are weak, it is expected that the contributions of these traits to the shared  
362 heritability will be small, too. In this case, MaxSH may overestimate these contributions.  
363 Secondly, the framework is applicable only if the number of traits is no less than three. In the  
364 case of three traits, the performance is limited and the SHAHER results should be interpreted  
365 with caution. We have shown in simulations and real dataset examples that MaxSH works  
366 better at higher numbers of genetically correlated traits being analyzed. However, an  
367 increase in the number of weakly correlated traits leads to a decrease in the proportion of  
368 SNPs associated with all traits simultaneously and to a decrease in the efficiency of the  
369 framework. Thirdly, although the set of SNPs identified by the SGIT GWAS is enriched for  
370 the SGF, each SNP should be interpreted with caution for whether it is shared or not,  
371 because SHAHER has some level of unspecificity. Finally, if any confounding effects were  
372 included in the GWAS of the original traits, these effects are amplified in the SGIT <sup>15</sup>. The  
373 confounding effects can be controlled easily using special methods like LD score regression  
374 <sup>20</sup>, although this method fails to distinguish a polygenic component if the trait was measured  
375 in the sample with the assortative mating or family effects. Thus, we suggest a thorough  
376 check of the original GWAS for the presence of any effects of possible confounders before  
377 proceeding to SHAHER.

378           In conclusion, we propose a novel effective framework for analysis of the shared  
379 genetic background for a set of genetically correlated traits using GWAS summary statistics.  
380 The framework allows us to obtain novel biological insights into the trait's genetic impact  
381 composition. By analyzing shared and unshared genetic impacts separately, we increased the  
382 number of identified loci and observed unique gene sets, identified genetic mechanisms  
383 being common for all traits or specific for every trait. Of note, sumCOT can be used as a  
384 stand-alone method for obtaining GWAS results of the linear combination of the traits using  
385 their summary statistics.

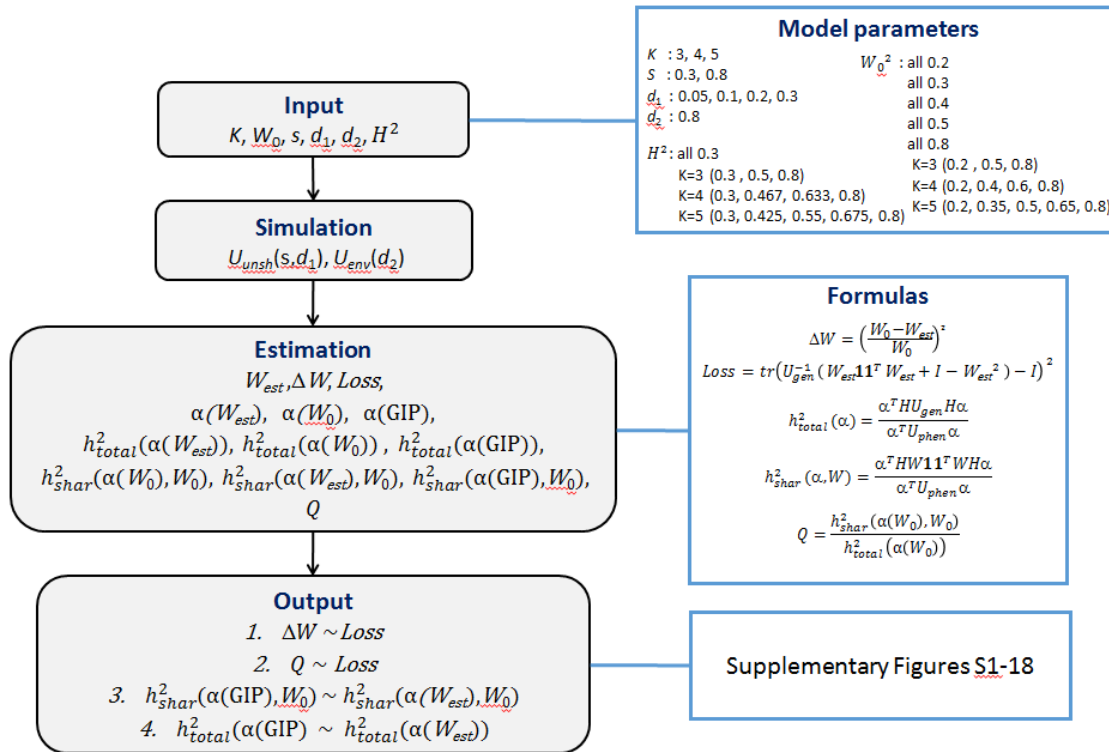
386

## 387 **Materials and Methods**

### 388 **Simulation study**

389           Under different scenarios, we designed simulations to assess the performance of  
390 MaxSH. We (1) assessed the accuracy of  $w$  estimates, (2) assessed the proportion of SGIT  
391 heritability explained by the SGF to the total heritability of the SGIT (the  $Q$ -value), and (3)  
392 compared the analytically predicted total and shared heritabilities of the SGIT and GIP1 with  
393 respect to the loss function. The design of our simulation experiment is shown in Figure 3.  
394 To generate the input for the MaxSH and GIP approaches, we used a six-parameter  
395 simulation model, in which  $K$  is the number of traits;  $W_0^2$  is a  $(K \times K)$  diagonal matrix, where  
396 the  $i$ -th diagonal element is  $w_i^2$  (the proportion of heritability explained by SGF);  $s$  is the  
397 proportion of zeros in the matrix  $U_{unsh}$ ;  $d_1$  is the amplitude of the uniform distribution for  
398 non-zero values of  $U_{unsh}$  and  $d_2$  is the amplitude of the uniform distribution for  $U_{env}$ ;  $H_2$  is the  
399 diagonal matrix with diagonal elements equal to the trait heritabilities. The parameters  
400 values used are given in Figure 3.

401



402

403 **Figure 3. A schematic depicting the overall workflow of a simulation study.** All details  
 404 are given in the text.

405

406 For each fixed number,  $K$ , of the original traits and fixed heritability,  $h_i^2$  ( $i=1, \dots, K$ ), of  
 407 each trait, we simulated  $U_{gen}$ . To do this, we separately modelled two its components caused  
 408 by SGF and UGF as  $W \mathbf{1} \mathbf{1}^T W$  and  $\sqrt{I - W^2} U_{unsh} \sqrt{I - W^2}$ , respectively (see the ‘Model’  
 409 box in Figure M1 of Supplementary Methods). Here  $\mathbf{1}$  is a  $(K \times 1)$  vector of units, and  $U_{unsh}$  is  
 410 a  $(K \times K)$  matrix randomly generated using the parameters  $s$  and  $d_1$  (see Supplementary  
 411 Methods). Then we randomly generated the trait-trait correlation matrix  $U_{env}$  explained by  
 412 the environmental factors, by giving the parameter  $d_2$  (see Supplementary Methods). Finally,  
 413 we modeled a matrix of phenotypic correlations by using Model (2) with regard to simulated  
 414 values  $W_0$ .

415 Using simulation data,  $U_{phen}$ ,  $U_{gen}$  and  $H^2$ , we estimated  $W_{est}$  and calculated its squared  
 416 relative difference with the simulated values of  $W_0$  ( $\Delta W$ ). We revealed a dependence of  $\Delta W$   
 417 on the loss function ( $Loss$ ). The  $Loss$  value characterizes the difference between  $U_{gen}$  and the  
 418 auxiliary matrix  $V$ .



419 Then we estimated  $\alpha$  in three ways: (1) using MaxSH and  $W_0$ , (2) using MaxSH and  
420  $W_{est}$ , and (3) using the GIP method <sup>2</sup>. On the basis of these estimates, we formed three traits  
421 being the linear combinations of the original traits. For these combined traits, we calculated  
422 the total heritability and the heritability explained by SGF.

423 The simulated experiments were repeated 10,000 times for each set of parameters.  
424 The model parameters and formulas for all calculated values are shown in Figure 3.

425

## 426 **Application to real data**

### 427 **Data sets**

428 We used three publicly available real data sets: anthropometric traits, psychiatric  
429 conditions and lipid concentrations, which contain five, four and three traits respectively.

430 The group of anthropometric traits consisted of UK Biobank GWAS summary  
431 statistics obtained from the Neale lab ([http://www.nealelab.is/blog/2017/7/19/rapid-gwas-of-](http://www.nealelab.is/blog/2017/7/19/rapid-gwas-of-thousands-of-phenotypes-for-337000-samples-in-the-uk-biobank)  
432 [thousands-of-phenotypes-for-337000-samples-in-the-uk-biobank](http://www.nealelab.is/blog/2017/7/19/rapid-gwas-of-thousands-of-phenotypes-for-337000-samples-in-the-uk-biobank)) for people of European  
433 ancestry: BMI (N = 336,107), weight (N = 336,227), hip (N = 336,601), waist circumference  
434 (N = 336,639) and whole body fat mass (N = 330,762).

435 The second dataset reflecting psychometric traits was constructed from GWAS  
436 results provided by the Psychiatric Genomics Consortium  
437 (<https://www.med.unc.edu/pgc/download-results/>) for bipolar disorder, BIP (N cases =  
438 20,352; N controls = 31,358) <sup>21</sup>, major depressive disorder, MDD (N cases = 43,204; N  
439 controls = 95,680; without UK Biobank and 23andMe data) <sup>22</sup> and schizophrenia, SCZ (N  
440 cases = 36,989; N controls = 113,075). Summary statistics for the fourth trait – subjective  
441 well-being (N = 110,935) – were derived from UK Biobank data from the Neale lab. All the  
442 psychometric trait GWASs were conducted using samples of white Europeans.

443 The last dataset corresponding to lipid traits was formed using GWAS data for  
444 European participants from the Global Lipid Genetics Consortium  
445 (<http://csg.sph.umich.edu/willer/public/lipids2013/>) for LDL cholesterol (N = 173,082),  
446 triglycerides (N = 177,860) and total cholesterol (N = 187,365).

447 Summary statistics for the three data sets were integrated and quality controlled by  
448 the GWAS-MAP platform developed by our group <sup>23</sup>. The GWAS-MAP database contains  
449 implemented software for quality control of GWAS results, estimation of phenotypic  
450 correlations and LD Score regression (LDSC) <sup>20</sup>.

451 We conducted the quality control of all data and unified them within the GWAS-  
452 MAP platform <sup>23</sup>. We filtered all summary statistics by minor allele frequencies  $\geq 0.01$ .  
453 Additionally, we filtered GWAS results for BIP by imputation qualities  $\geq 0.9$ . We did not  
454 apply this filter to the other traits due to the absence of imputation quality in summary  
455 statistics data. Finally, using GWAS-MAP, we performed a correction for genomic control  
456 for all traits (including the original traits, SGIT and UGITs) with an LDSC intercept greater  
457 than 1 <sup>20</sup>. Thus, we corrected all traits from the psychometric dataset apart from MDD, all  
458 original anthropometric traits and their SGIT and lipid SGIT as their LDSC intercept  
459 exceeded 1 (see Supplementary Tables 2a-c). Moreover, all SNPs with the p-value equal to 0  
460 were excluded from analysis.

#### 461 **Genetic analysis**

462 Pairwise phenotypic correlations between traits were computed using the GWAS-  
463 MAP platform described above. The used method is based on correlations between  
464 insignificant  $z$ -statistics for independent SNPs as previously described in <sup>9</sup>. SNP-based  
465 heritability and genetic correlation coefficients were estimated using the LD Score  
466 regression software <sup>16</sup> embedded in the GWAS-MAP platform. The significance threshold  
467 for genetic correlations was set at  $4.5 \times 10^{-4}$  ( $0.05/112$ , where 112 is the number of pairwise  
468 combinations between all original traits, their SGIT and UGITs in each dataset - between 11,  
469 9 and 7 traits for anthropometry, psychometric and lipid traits respectively).

470 SHAHER analysis included checking if there was an SGI or not, the application of  
471 MaxSH and conducting SGIT and UGIT GWASs. The threshold for confirming the  
472 existence of an SGI at the first stage was empirically set to 0.2.

473 For each dataset, we visualized the full genetic correlation matrices using the  
474 *corrplot()* function from the *corrplot* R package (v.0.84) <sup>24</sup>. We also placed the SNP-based  
475 heritability estimates on the diagonal and crossed out non-significant values.

476 Finally, we compared the GWAS results obtained for the SGIT by MaxSH and GIP  
477 (the principal component analysis on the matrix of genetic covariances)<sup>2</sup>.

#### 478 **Gene set and tissue/cell type enrichment analyses**

479 We performed a gene set enrichment analysis and a tissue/cell type enrichment  
480 analysis combined with a gene prioritization using the Data-driven Expression Prioritized  
481 Integration for Complex Traits (DEPICT) tool v.1.1, release 194 <sup>25</sup>. We selected genome-  
482 wide significant SNPs ( $p\text{-value} < 5 \times 10^{-8}$ ) from summary statistics before the genomic

483 control and applied the DEPICT software with default parameters  
484 (<https://data.broadinstitute.org/mpg/depict/>). The MHC region was excluded from analyses.

485 Next, for the gene set enrichment results, we calculated the number of significant  
486 enriched gene sets ( $FDR < 5\%$ ) and constructed an overlapping matrix, in which each cell  
487 represents the number of overlapping gene sets for each pair of traits. For each pair of traits,  
488 we scaled the number of overlapping gene sets by the minimum number of significant gene  
489 sets for this pair of traits. The resulting matrix was visualized using the corrplot R-package  
490 as described above.

#### 491 **The number of original traits associated with SGIT loci**

492 We performed a clumping procedure to search for loci associated with each of the  
493 original traits, SGIT and UGITs at a genome-wide significance level of  $5 \times 10^{-8}$ . The  
494 associated locus was defined as a genomic region spanning 500 kb in either direction of the  
495 lead SNP. Those loci that were significantly associated with SGIT, but not with the original  
496 traits, were assumed to be new loci.

497 We expected that the loci associated with all the original traits used to obtain SGIT  
498 are likely to be SGF. To test this expectation, for each dataset we selected all independent  
499 loci that were significantly associated with at least one of the original traits and calculated  
500 the number of the original traits significantly associated with these loci. For the original  
501 anthropometric and lipid traits, we empirically set the significance threshold at  $p\text{-value} =$   
502  $1 \times 10^{-5}$ . For the psychometric traits, it was set at  $1 \times 10^{-3}$ . We then analyzed the SGIT  $p\text{-values}$   
503 for the selected loci and constructed boxplots of  $-\log_{10}$  for them with regard to the number of  
504 the original traits significantly associated with these loci.

#### 505 **Data Availability**

506 The SHAHER framework is implemented as a set of R/C++ scripts and is freely  
507 available at [https://github.com/Sodbo/shared\\_heredity](https://github.com/Sodbo/shared_heredity).

#### 508 **Acknowledgements**

#### 509 **Funding**

510 The work of GRS was supported by the Russian Foundation for Basic Research  
511 (project 20-04-00464). The work of YAT and EEE was supported by the Russian

512 Foundation for Basic Research (project 19-015-00151). The work of SZS was supported by  
513 the Russian Ministry of Education and Science under the 5-100 Excellence Programme and  
514 by the Federal Agency of Scientific Organizations via the Institute of Cytology and Genetics  
515 (project 0259-2021-0009/AAAA-A17-117092070032-4). The work of PRHJT was  
516 supported by the Medical Research Council Human Genetics Unit (MC\_UU\_00007/10).

## 517 **Author contributions**

518 YAT, GRS and TIA conceived and oversaw the study. YAT, GRS, SZS and TIA  
519 contributed to the design of the study and interpretation of the results. GRS developed the  
520 MaxSH method, including the algorithm and program, and conducted simulation studies. PT  
521 developed the C++ version of sumCOMB and tested the developed framework. EST, EEE,  
522 SGF, SZS and YAT wrote the source code for the framework and performed real data  
523 analyses. All co-authors discussed the results and contributed to preparing the draft and final  
524 version of the manuscript.

## 525 **Conflict of interest**

526 P.R.H.J.T. is an employee of BioAge Labs. The remaining authors declare no conflict  
527 of interest.

## 528 **Supplementary Information legend**

529 Supplementary Tables 1a-c: linear combination coefficients and CIs of SGIT for real  
530 data sets

531 Supplementary Tables 2a-c: results of LD score regression for original traits from  
532 real data sets

533 Supplementary Tables 3a-c: clumping results for real data sets

534 Supplementary Tables 4-6: DEPICT results for real data sets

## 535 **References**

536 1. Jiang X, Finucane HK, Schumacher FR, Schmit SL, Tyrer JP, Han Y, et al. Shared  
537 heritability and functional enrichment across six solid cancers. *Nature communications*.  
538 2019;10(1):1-23.

- 539 2. Tsepilov YA, Freidin MB, Shadrina AS, Sharapov SZ, Elgaeva EE, van Zundert J, et  
540 al. Analysis of genetically independent phenotypes identifies shared genetic factors  
541 associated with chronic musculoskeletal pain conditions. *Communications biology*.  
542 2020;3(1):1-13.
- 543 3. Sampson JN, Wheeler WA, Yeager M, Panagiotou O, Wang Z, Berndt SI, et al.  
544 Analysis of heritability and shared heritability based on genome-wide association studies for  
545 13 cancer types. *JNCI: Journal of the National Cancer Institute*. 2015;107(12).
- 546 4. Brainstorm C, Anttila V, Bulik-Sullivan B, Finucane HK, Walters RK, Bras J, et al.  
547 Analysis of shared heritability in common disorders of the brain. *Science*. 2018 Jun  
548 22;360(6395). PubMed PMID: 29930110. Pubmed Central PMCID: PMC6097237.
- 549 5. Yang Y, Zhao H, Heath AC, Madden PA, Martin NG, Nyholt DR. Shared genetic  
550 factors underlie migraine and depression. *Twin Research and Human Genetics*.  
551 2016;19(4):341-50.
- 552 6. Wright S. Correlation and Causation. *JouMal of Agricultural Research*. 1921.
- 553 7. Rijdsdijk FV, Sham PC. Analytic approaches to twin data using structural equation  
554 models. *Briefings in bioinformatics*. 2002;3(2):119-33.
- 555 8. Galesloot TE, Van Steen K, Kiemeny LA, Janss LL, Vermeulen SH. A comparison  
556 of multivariate genome-wide association methods. *PloS one*. 2014;9(4):e95923.
- 557 9. Stephens M. A unified framework for association analysis with multiple related  
558 phenotypes. *PloS one*. 2013;8(7):e65245.
- 559 10. Yang X, Zhang S, Sha Q. Joint analysis of multiple phenotypes in association studies  
560 based on cross-validation prediction error. *Scientific reports*. 2019;9(1):1-10.
- 561 11. Turley P, Walters RK, Maghziyan O, Okbay A, Lee JJ, Fontana MA, et al. Multi-trait  
562 analysis of genome-wide association summary statistics using MTAG. *Nature genetics*.  
563 2018;50(2):229-37.
- 564 12. Fatumo S, Carstensen T, Nashiru O, Gurdasani D, Sandhu M, Kaleebu P.  
565 Complimentary methods for multivariate genome-wide association study identify new  
566 susceptibility genes for blood cell traits. *Frontiers in genetics*. 2019;10:334.
- 567 13. Ning Z, Tsepilov YA, Sharapov SZ, Grishenko AK, Feng X, Shirali M, et al. Beyond  
568 power: multivariate discovery, replication, and interpretation of pleiotropic loci using  
569 summary association statistics. *bioRxiv*. 2019:022269.
- 570 14. Pasaniuc B, Price AL. Dissecting the genetics of complex traits using summary  
571 association statistics. *Nature Reviews Genetics*. 2017;18(2):117-27.
- 572 15. Timmers PRHJ, Tiys ES, Sakaue S, Akiyama M, Kiiskinen TTJ, Zhou W, et al.  
573 Genetically independent phenotype analysis identifies LPA and VCAM1 as drug targets for  
574 human ageing. *bioRxiv*. 2021:2021.01.22.427837.
- 575 16. Bulik-Sullivan B, Finucane HK, Anttila V, Gusev A, Day FR, Loh PR, et al. An atlas  
576 of genetic correlations across human diseases and traits. *Nat Genet*. 2015 Nov;47(11):1236-  
577 41. PubMed PMID: 26414676. Pubmed Central PMCID: PMC4797329.
- 578 17. Winkler TW, Day FR, Croteau-Chonka DC, Wood AR, Locke AE, Mägi R, et al.  
579 Quality control and conduct of genome-wide association meta-analyses. *Nature protocols*.  
580 2014;9(5):1192-212.
- 581 18. Howe LJ, Lawson DJ, Davies NM, Pourcain BS, Lewis SJ, Smith GD, et al. Genetic  
582 evidence for assortative mating on alcohol consumption in the UK Biobank. *Nature*  
583 *communications*. 2019;10(1):1-10.
- 584 19. Marcellini F, Giuli C, Papa R, Tirabassi G, Faloia E, Boscaro M, et al. Obesity and  
585 body mass index (BMI) in relation to life-style and psycho-social aspects. *Archives of*  
586 *gerontology and geriatrics*. 2009;49:195-206.

- 587 20. Bulik-Sullivan B, Finucane HK, Anttila V, Gusev A, Day FR, Loh P-R, et al. An  
588 atlas of genetic correlations across human diseases and traits. *Nature genetics*.  
589 2015;47(11):1236.
- 590 21. Stahl EA, Breen G, Forstner AJ, McQuillin A, Ripke S, Trubetskoy V, et al.  
591 Genome-wide association study identifies 30 loci associated with bipolar disorder. *Nature*  
592 *genetics*. 2019;51(5):793-803.
- 593 22. Wray NR, Ripke S, Mattheisen M, Trzaskowski M, Byrne EM, Abdellaoui A, et al.  
594 Genome-wide association analyses identify 44 risk variants and refine the genetic  
595 architecture of major depression. *Nature genetics*. 2018;50(5):668-81.
- 596 23. Gorev D, Shashkova T, Pakhomov E, Torgasheva A, Klaric L, Severinov A, et al.,  
597 editors. GWAS-MAP: a platform for storage and analysis of the results of thousands of  
598 genome-wide association scans. *Bioinformatics of Genome Regulation and*  
599 *Structure\Systems Biology (BGRS\SB-2018)*; 2018.
- 600 24. Wei T, Simko V, Levy M, Xie Y, Jin Y, Zemla J. corrplot: visualization of a  
601 correlation matrix. R package v. 0.84. 2017.
- 602 25. Pers TH, Karjalainen JM, Chan Y, Westra H-J, Wood AR, Yang J, et al. Biological  
603 interpretation of genome-wide association studies using predicted gene functions. *Nature*  
604 *communications*. 2015;6(1):1-9.
- 605