

***Full Title: Simulated hemiparesis increases optimal spatiotemporal gait asymmetry but not metabolic cost***

*Short Title: Spatiotemporal gait asymmetries with simulated hemiparesis*

Russell T. Johnson<sup>1</sup>, Nicholas A. Bianco<sup>2</sup>, James M. Finley<sup>1,3,4</sup>

Affiliations:

1. Division of Biokinesiology and Physical Therapy, University of Southern California, Los Angeles CA, USA
2. Department of Mechanical Engineering, Stanford University, Palo Alto CA, USA
3. Department of Biomedical Engineering, University of Southern California, Los Angeles CA, USA
4. Neuroscience Graduate Program, University of Southern California, Los Angeles CA, USA

Corresponding Author: Russell T. Johnson, [rtjohnso@usc.edu](mailto:rtjohnso@usc.edu)

**Key Words:** Post-stroke, Muscle Strength, Musculoskeletal modeling, Optimal control, Locomotion

## 1 **Abstract**

2 Several neuromuscular impairments, such as weakness (hemiparesis), occur after an individual has a stroke,  
3 and these impairments primarily affect one side of the body more than the other. Predictive musculoskeletal  
4 modeling presents an opportunity to investigate how a specific impairment affects gait performance post-  
5 stroke. Therefore, our aim was to use to predictive simulation to quantify the spatiotemporal asymmetries  
6 and changes to metabolic cost that emerge when muscle strength is unilaterally reduced. We also  
7 determined how forced spatiotemporal symmetry affects metabolic cost. We modified a 2-D  
8 musculoskeletal model by uniformly reducing the peak isometric muscle force in all left-limb muscles. We  
9 then solved optimal control simulations of walking across a range of speeds by minimizing the sum of the  
10 cubed muscle excitations across all muscles. Lastly, we ran additional optimizations to test if reducing  
11 spatiotemporal asymmetry would result in an increase in metabolic cost. Our results showed that the  
12 magnitude and direction of effort-optimal spatiotemporal asymmetries depends on both the gait speed and  
13 level of weakness. Also, the optimal metabolic cost of transport was 1.25 m/s for the symmetrical and 20%  
14 weakness models but slower (1.00 m/s) for the 40% and 60% weakness models, suggesting that hemiparesis  
15 can account for a portion of the slower gait speed seen in people post-stroke. Adding spatiotemporal  
16 asymmetry to the cost function resulted in small increases (~4%) in metabolic cost. Overall, our results  
17 indicate that spatiotemporal asymmetry may be optimal for people post-stroke, who have asymmetrical  
18 neuromuscular impairments. Additionally, the effect of speed and level of weakness on spatiotemporal  
19 asymmetry may explain the well-known heterogenous distribution of spatiotemporal asymmetries observed  
20 in the clinic. Future work could extend our results by testing the effects of other impairments on optimal  
21 gait strategies, and therefore build a more comprehensive understanding of the gait patterns in people post-  
22 stroke.

23

24

25

26

27 **Author Summary**

28

29 A stroke causes damage to the brain. This typically results in several changes to the nervous and muscular  
30 (neuromuscular) system that change how people post-stroke tend to walk and perform other tasks.

31 Individuals post-stroke tend to walk with an asymmetrical motion and expend more energy while walking

32 than other age-matched individuals. We still do not understand how each specific change to the

33 neuromuscular system is linked with changes in walking patterns, in part because it is difficult to test one

34 individual change at a time in people. Instead, we can use a mathematical model of the musculoskeletal

35 system that represents the individual changes to the muscular system that occur in people post-stroke. In

36 this study, we modeled how a common change in people post-stroke (muscle weakness) can impact walking

37 patterns. We found that the level of weakness and the walking speed affect the asymmetrical walking

38 patterns of our models, but do not change the total energy cost. Overall, our study is one step towards better

39 understanding how neuromuscular changes in people post-stroke affects walking patterns. This knowledge

40 could be applied to identify rehabilitation strategies that are most likely to improve walking in people post-

41 stroke.

42

## 43 1. Introduction

44

45 Many neuromuscular impairments occur after an individual has a stroke (1–3), such as muscle  
46 spasticity (4,5), reductions in muscle strength (6,7), and generation of abnormal patterns of muscle  
47 coordination (8–10). These changes often preferentially affect the side of the body that is contralateral to  
48 the side of the neural damage. The lateralized neuromuscular impairments result in marked gait deviations:  
49 people post-stroke walk slower, spend more time in double support, and typically display asymmetrical  
50 step lengths, step times, and joint kinematics (11–13). In addition to slower preferred gait speeds and  
51 spatiotemporal asymmetries, people post-stroke also walk with a greater metabolic cost of transport (COT)  
52 than age- and speed-matched control subjects (11,14). An ongoing objective for clinicians and researchers  
53 is to determine the causal associations relating neuromuscular impairments, gait deviations, and the  
54 energetic cost of walking (11,15–18). This knowledge could potentially be used to identify intervention  
55 targets that are most likely to improve rehabilitation outcomes.

56 Determining causal links between neuromuscular impairments and gait performance is challenging  
57 because it is currently impossible to independently modulate all known impairments in people post-stroke  
58 and then observe the effects of these changes on gait deviations or metabolic cost. Instead, some researchers  
59 have investigated the effects of select gait deviations on metabolic energy cost. When neurotypical  
60 individuals walk with increased spatiotemporal asymmetries, their metabolic COT increases compared to  
61 symmetrical walking patterns (19,20). As such, some common rehabilitation interventions have targeted  
62 these spatiotemporal asymmetries to try to improve outcome measures of gait performance in people post-  
63 stroke. However, recent studies using biofeedback have shown little or no improvement in metabolic COT  
64 when people post-stroke walk with more symmetric step lengths (21–25). Therefore, it is still unclear how  
65 impairment interacts with observable gait deviations to cause the increase in metabolic COT in this  
66 population.

67 Musculoskeletal modeling and predictive simulation provide the opportunity to generate  
68 predictions about how specific neuromuscular impairments affect gait performance as researchers can

69 modify muscle parameters in a systematic way while keeping other parameters unchanged (26,27). Previous  
70 researchers have used musculoskeletal modeling and predictive simulation to predict gait patterns for both  
71 healthy and clinical populations, such as people with cerebral palsy or people who walk with lower limb  
72 prosthetics (26,28–30). This approach has led to valuable insight into principles of motor control (28,31,32),  
73 the effect of impairments on gait mechanics and energetics (26,27,33,34), and the effect of gait patterns on  
74 joint loading (35,36). Overall, these studies, along with recent advancements in both computational  
75 efficiency and accessibility (37–39), have allowed predictive modeling and simulation studies to help reveal  
76 important principles of gait mechanics (40). Therefore, we can apply the methodology of musculoskeletal  
77 modeling and predictive simulation to understand the optimal gait patterns for people with neuromuscular  
78 impairments.

79         The aim of this study was to use predictive simulations to explore how a common impairment in  
80 people post-stroke, hemiparesis, should impact patterns of spatiotemporal asymmetry and metabolic cost  
81 for gait patterns that minimize muscular effort. Hemiparesis is defined as a decrease in muscular strength  
82 that primarily affects one side of the body, the effects of which are distinct from other associated post-  
83 stroke impairments like abnormal co-activation across muscles or muscle spasticity. To simulate  
84 hemiparesis in our musculoskeletal model, we systematically reduced the peak isometric muscle strength  
85 for all the muscles on the left limb of our model while keeping the right limb constant and evaluated patterns  
86 of spatiotemporal asymmetry and metabolic cost that emerged across a wide range of speeds while  
87 minimizing muscle excitations. We hypothesized that asymmetric walking patterns would emerge as being  
88 optimal as we increased the magnitude of unilateral weakness. We then asked how enforcing symmetric  
89 step lengths and step times in models with unilateral weakness impacted the metabolic cost of transport.  
90 Here, we expected that enforcing symmetry would lead to marked increases in the metabolic cost of  
91 transport when compared to models where symmetry was not enforced. Overall, this work will allow us to  
92 gain insight into how muscle strength impairments impact gait deviations and the metabolic energy cost of  
93 walking, independent of other neuromuscular changes that occur post-stroke.

94

95 **2. Methods**

96

97 *2.1 Musculoskeletal Model:*

98 A two-dimensional, sagittal-plane musculoskeletal model with 11 degrees-of-freedom (DOF) was  
99 used (Fig 1A) within the OpenSim Moco software to generate optimal control simulations of walking (37).  
100 The model had a pelvis that translated and rotated relative to the ground with 3 DOF and the torso was  
101 rigidly attached to the pelvis. Each hip, knee, ankle, and toe joints were each modeled as 1 DOF pin joints.  
102 The model was actuated with 24 Hill-type muscle-tendon units (12 per limb) based on the  
103 *DeGrootefregly2016Muscle* model with compliant series elastic elements (41). The foot-ground  
104 interactions were simulated using five contact spheres per foot, represented with smooth and continuous  
105 functions (42). We used this musculoskeletal model with symmetrical strength for the base conditions, and  
106 then created three other models where the peak isometric strength of each muscle on the left limb was  
107 reduced from the base model by 20%, 40%, and 60% for the three hemiparetic conditions (Fig 1B; Table  
108 1). This unilateral reduction in peak strength was intended to simulate the loss of force production capacity  
109 that commonly results from stroke. Although the weakness resulting from acute stroke results from a loss  
110 of descending drive from the brain, we would need to simulate large reductions in feasible excitations for  
111 this loss to impact the patterns of muscle activation observed in a sub-maximal task like walking. In  
112 addition, because the magnitude of a given loss of descending drive to a set of muscles is less practical to  
113 measure empirically than the resulting weakness, therefore this approach can be more easily extended to  
114 personalize predictive simulations.

115

116

117

118

119

**Table 1:** Peak isometric strength (in N) for each of the 12 muscles for each model. The right limb strength remained unchanged (Base values) while the left limb strength was modified for the 20%, 40%, and 60% weakness models.

	<b>Symmetrical</b>	<b>20%</b>	<b>40%</b>	<b>60%</b>
Biceps long head	4105	3284	2463	1642
Biceps short head	557	445.6	334.2	222.8
Gluteus maximus	4450	3560	2670	1780
Psoas	2448	1958.4	1468.8	979.2
Rectus femoris	2192	1753.6	1315.2	876.8
Vastus Intermedius	9593	7674.4	5755.8	3837.2
Gastrocnemius	4691	3752.8	2814.6	1876.4
Soleus	6194	4955.2	3716.4	2477.6
Tibialis anterior	1227	981.6	736.2	490.8
Extensor hallucis longus	889	711.2	533.4	355.6
Flexor digitorum longus	1331	1064.8	798.6	532.4
Flexor digitorum brevis	938	750.4	562.8	375.2

## 120 2.2 Optimal Control Problem for Nominal Conditions

121 We ran several sets of optimal control problems in Moco, which were solved by minimizing the  
122 sum of the integrated muscle excitations cubed divided by the horizontal displacement of the center of mass  
123 (Eq. 1), which is hypothesized to represent minimizing muscle fatigue (28,43). While it is unclear what  
124 objective function best explains features of human gait, several studies have suggested that humans select  
125 gait patterns that reduce the muscular effort, muscle fatigue, or metabolic energy associated with the task  
126 (44–49). These studies provide evidence that effort optimization may explain why we choose certain  
127 spatiotemporal features of our gait during steady-state walking and when exposed to novel task demands.

128 Direct collocation was used to solve for the set of states and controls needed to produce a full gait  
129 cycle, subject to the objective function and constraints. The set of states,  $x$ , were 11 generalized coordinates  
130  $q(t)$  corresponding to each joint and the three degrees of freedom for the pelvis, 11 generalized velocities  
131  $u(t)$ , 24 muscle activations  $a(t)$ , and 24 normalized tendon forces  $\tilde{F}^T(t)$ . The set of controls consisted of  
132 24 muscle excitations  $e(t)$  and 24 auxiliary variables representing the derivative of normalized tendon force  
133  $\dot{\tilde{F}}^T(t)$ , which were necessary for enforcing the muscle-tendon equilibrium equations in implicit form (Eq.  
134 2; 41). The states and controls were discretized on a grid of 201 evenly distributed nodes over a complete  
135 gait cycle. Optimal control problems were solved for each of the four models for speeds of 0.25, 0.50, 0.75,

136 1.00, and 1.25 m/s. Each problem was formulated to generate a full stride of walking by finding the set of  
 137 model states, controls, and final time  $t_f$ , subject to the objective function (Eq. 1),

$$J_1 = \frac{\sum_{i=1}^{24} \int_0^{t_f} e_i^3(t) dt}{q_{pelvis_x}(t_f) - q_{pelvis_x}(0)} \quad (1)$$

138 where  $e_i$  is the excitation of the  $i$ th muscle and  $q_{pelvis_x}$  is the horizontal position of the pelvis such that the  
 139 denominator represents the displacement of the model during the gait cycle.

140 Each optimal control problem was solved by minimizing  $J_1$  subject to constraints enforcing skeletal  
 141 kinematics, skeletal dynamics, muscle activation dynamics, and implicit tendon compliance dynamics (Eq.  
 142 2; 37,41),

$$\begin{aligned} u(t) &= \dot{q}(t) \\ \dot{u}(t) &= f(q(t), u(t), a(t), \widetilde{F}^T(t)) \\ \dot{a}(t) &= f(e(t), a(t)) \\ f(a(t), \widetilde{F}^T(t), \dot{\widetilde{F}}^T(t)) &= 0 \end{aligned} \quad (2)$$

143 Bounds were placed on the states, controls, and auxiliary variables (Eq. 3).

$$\begin{aligned} q_{LB}(t) &\leq q(t) \leq q_{UB}(t) \\ u_{LB}(t) &\leq u(t) \leq u_{UB}(t) \\ 0.001 &\leq e(t) \leq 1 \\ 0.001 &\leq a(t) \leq 1 \\ 0 &\leq \widetilde{F}^T(t) \leq 1.8 \\ -1000 &\leq \dot{\widetilde{F}}^T(t) \leq 1000 \end{aligned} \quad (3)$$

144 where  $q_{LB}$ ,  $q_{UB}$ ,  $u_{LB}$  and  $u_{UB}$  represent the lower and upper bounds on each kinematic value and speed,  
 145 respectively. Excitations ( $e(t)$ ) and activations ( $a(t)$ ) were bounded between 0.001 and 1. Normalized  
 146 tendon forces ( $\widetilde{F}^T(t)$ ) were bounded between 0 and 1.8 and their derivative ( $\dot{\widetilde{F}}^T(t)$ ) were bounded between



147 -1000 and 1000. Lastly, the optimization was also subject to periodicity constraints, where the final states  
 148 and controls must equal the initial states and controls, except for the horizontal position of the pelvis  
 149  $q_{pelvis_x}(t)$ , which must account for horizontal translation relative to the target velocity  $v$  and  $t_f$  (Eq. 4).

$$\begin{aligned}
 q(t_f) &= q(0) \\
 q_{pelvis_x}(t_f) &= q_{pelvis_x}(0) + (v * t_f) \\
 u(t_f) &= u(0) \\
 e(t_f) &= e(0) \\
 a(t_f) &= a(0) \\
 \widetilde{F}^T(t_f) &= \widetilde{F}^T(0)
 \end{aligned} \tag{4}$$

150 The final time of the gait cycle was allowed to vary within the optimization between 0.3 and 2 seconds,  
 151 which allowed the optimization to use the optimal stride length and stride time which satisfied the target  
 152 gait speed for each condition (Fig 1C).

153 We used three different initial guesses for each of the 20 conditions (four models and five speeds  
 154 each). The first initial guess was the result of a tracking optimization which used averaged gait data from  
 155 healthy control subjects walking at 1.4 m/s (31). The reference data were normalized to 101 time points to  
 156 align the data with the collocation points of the optimizations. The first initial guesses for each speed were  
 157 generated by minimizing a weighted combination of the total integrated muscle excitations cubed and the  
 158 difference between experimental and simulated kinematics and ground reaction forces (Eq. 5), while  
 159 matching the average gait speed (i.e., 0.25, 0.50, 0.75, 1.00, and 1.25).

$$J_{track} = J_1 + \int_0^{t_f} \left[ \left( w_1 * \sum_{i=1}^{11} \left( \frac{q_i(t) - \widehat{q}_i(t)}{\sigma_{qi}} \right)^2 + \left( \frac{u_i(t) - \widehat{u}_i(t)}{\sigma_{ui}} \right)^2 \right) + \left( w_2 * \sum_{j=1}^4 \left( \frac{GRF_j(t) - \widehat{GRF}_j(t)}{\sigma_j} \right)^2 \right) \right] dt \tag{5}$$

160 For Equation 5, the term  $J_1$  represents the muscular effort (Eq. 1). The second and third terms represent  
 161 deviations from the experimental kinematics and ground reaction forces (GRFs), respectively.  $q_i(t)$  and  
 162  $u_i(t)$  are the values and velocities of model coordinate  $i$  at time  $t$ ,  $\widehat{q}_i(t)$  and  $\widehat{u}_i(t)$  are the reference

163 kinematic data from Miller (2014) of coordinate  $i$  at time  $t$ , and  $\sigma_{qi}$  and  $\sigma_{ui}$  are the standard deviations of  
164 the reference data, averaged across the stride.  $GRF_j(t)$  represents the model ground reaction forces at time  
165  $t$  for the horizontal and vertical forces for the left and right side, and  $\widehat{GRF}_j(t)$  and  $\sigma_j$  represent the reference  
166 ground reaction force data means and standard deviations for each direction and limb. This initial guess  
167 was used for each of the four models for the corresponding walking speed (e.g., the tracking solution for  
168 0.75 m/s was used as an initial guess for each of the four models at 0.75 m/s speed). While the reference  
169 data from Miller, 2014 was at a different gait speed than the speeds used in this study, the tracking problem  
170 solutions produced realistic gait mechanics across speeds and were reasonable initial guesses for our  
171 predictive simulations.

172 The other two initial guesses were chosen heuristically from solutions of other conditions, either  
173 for different speeds or for different simulated hemiparesis models. For example, the 20% weak model at  
174 0.75 m/s had the following initial guesses: (1) tracking solution at a speed of 0.75, (2) the solution to the  
175 20% weak model at 1.00 m/s, and (3) the 40% weak solution at 0.75 m/s. If the solutions differed between  
176 initial guesses, we chose the optimal solution for each condition with the lowest objective function value,  
177 and this solution was then used to calculate the spatiotemporal asymmetry and metabolic cost (see Section  
178 2.4).

179

### 180 *2.3 Step Time and Step Length Symmetry Conditions*

181 Many of the optimal gait patterns observed for models of varying levels of weakness were  
182 spatiotemporally asymmetric. Since reducing these asymmetries is a common rehabilitation objective for  
183 people post-stroke (50), we conducted a second set of optimizations to determine how reducing the step  
184 length and step time asymmetry would impact metabolic cost. Therefore, we added two terms to the  
185 objective function ( $J_2$ ; Eq. 6) that allowed us to reduce step time symmetry or step length symmetry for the  
186 different models.

$$J_2 = J_1 + (w_1 * SLA_{goal}) + (w_2 * STA_{goal}) \quad (6)$$

187 The step length symmetry goal ( $SLA_{goal}$ ) was formulated to minimize the step length asymmetry  
188 (SLA; Eq. 7) during the stride. Theoretically, we would want to compute the step length for the non-paretic  
189 ( $SL_{NP}$ ) and paretic ( $SL_P$ ) sides across the gait cycle for each iteration. However, this approach does not  
190 work well for predictive simulations because gradient-based optimization approaches require smooth,  
191 differentiable objective function terms in order to achieve reliable convergence. Therefore, we instead  
192 approximated step length asymmetry by performing the following steps. We first set the total stride length  
193 to be equal to the stride length of the respective nominal condition (i.e., the stride length from the 0.75 m/s  
194 speed with 20% weakness), such that the stride length is matched within a speed/model condition. Then, to  
195 target a symmetrical step length, we computed the length for the right and left step that would both result  
196 in symmetrical step lengths during the gait cycle. To enforce these step lengths during the optimization, we  
197 set penalties that accumulate if distance between the feet exceed the targeted step length distance for each  
198 foot. Therefore, we are indirectly setting the step length for the paretic and non-paretic step using smoothing  
199 functions that work well for the optimization algorithm (see supporting information).

$$SLA = \frac{SL_{NP} - SL_P}{SL_{NP} + SL_P} \times 100 \quad (7)$$

200 The step time symmetry goal ( $STA_{goal}$ ) was designed to reduce the step time asymmetry during  
201 the gait cycle (Eq. 8). As with the  $SLA_{goal}$ , we needed to find an approximate form of step time asymmetry  
202 for this goal to work well within our optimization framework. Therefore, for this goal, we computed an  
203 approximation of step time asymmetry by detecting the number of time points each foot was in contact with  
204 the ground and computed the asymmetry index by normalizing by the total number of time points. Time  
205 points when both feet were in contact were assigned to the leading foot. Once we have the total number of  
206 nodes for non-paretic ( $ST_{NP}$ ) and paretic step time ( $ST_P$ ), respectively, we can then compute the asymmetry  
207 index based on the number of nodes and the stride time (see supporting information).

$$STA = \frac{ST_{NP} - ST_P}{ST_{NP} + ST_P} \times 100 \quad (8)$$

208           Since each of these symmetry goals are approximations of step length and step time asymmetries,  
209 we then computed the actual asymmetry indices to check if the optimal solution achieves the intended  
210 symmetrical pattern. If there were large discrepancies, we would then be able to adjust settings on the SLA  
211 or STA goal and re-run the optimization. We chose to run this symmetry sub-analysis on the 0.75 m/s speed  
212 because the nominal results had a consistent pattern of increasing step length asymmetry with greater  
213 hemiparesis. First, we solved a new set of optimal control problems to reduce the step time symmetry for  
214 each model at 0.75 m/s. For the first set of optimizations,  $w_1$  was set to 8 and  $w_2$  was set to 0, such that  
215 reducing the step length asymmetry was part of the objective, but step time asymmetry.

216           Finally, we solved another set of optimal control problems at 0.75 m/s with the added goal of  
217 reducing both step length and step time asymmetry. In this case, the weighting for  $J_2$  were set as  $w_1$  set to  
218 8 and  $w_2$  set to 5. These weightings were chosen heuristically such that we were able to reduce the  
219 asymmetry in each domain while getting good convergence. We noticed that if either  $w_1$  or  $w_2$  were too  
220 large, the optimization would not converge in a reasonable amount of time.

221

#### 222 *2.4 Optimization Settings*

223           All optimal control problems (28 total: four models x five speeds, plus four more for step time  
224 symmetry and four more for step length symmetry) were solved with 201 evenly spaced grid points, with  
225 Hermite-Sampson discretization (51), using CasADi (52) and IPOPT (53). Termination settings for the  
226 optimization were:  $1e-4$  constraint violation tolerance and  $1e-4$  convergence tolerance. Optimizations were  
227 solved on a four-core desktop computer with a 3.3 GHz Intel Core i5-6600 processor. In total, we ran 84  
228 optimizations for the nominal conditions (28 conditions with 3 initial guesses each), plus 18 additional  
229 optimizations for the symmetrical goal conditions. Each of the individual optimizations was solved on our  
230 desktop computer in times ranging from 2 to 14 hours, depending on the condition and initial guess.

231

232

233

234 *2.5 Outcome Measures and Evaluations*

235 The step length for the non-paretic (right) limb was calculated by taking the anterior-posterior (AP)  
236 distance between the ground-contact elements on the heels at the instant of non-paretic foot-strike where  
237 foot-strike was defined as the point when the vertical GRF was greater than 20 N (54). The step length for  
238 the parietic (left) limb was calculated similarly at the instant of parietic foot-strike. Step time for the non-  
239 parietic limb was calculated as the time from parietic foot-strike to non-paretic foot strike, and step time for  
240 the non-paretic limb was calculated as the time from non-paretic foot-strike to parietic foot-strike.

241 The metabolic cost of transport (COT) for each condition was also estimated based on the  
242 *Umberger2010MuscleMetabolicsProbe* accessed through OpenSim (55,56). The COT was calculated for  
243 each condition while holding muscle mass constant, to replicate weakness in the limb without muscle  
244 atrophy. In addition to predicting the change in COT between conditions, the positive and negative muscle  
245 fiber work performed for each muscle was also calculated (Eq 11), since muscle fiber work is a component  
246 of the metabolic cost model. First, the muscle fiber power was calculated for each muscle across time, then  
247 the positive and negative portions of the power curve were separately integrated to calculate positive and  
248 negative mechanical work, respectively for each limb. For the non-paretic side, positive mechanical power  
249 ( $W_{NP}^+$ ) was calculated by:

$$W_{NP}^+ = \sum_{m=1}^{12} \int_0^{t_f} P_{NPm}^+ dt \quad (11)$$

250  
251 where  $m$  is each of the 12 muscles of the limb,  $t_f$  equals the total time of the stride and  $P_{NPm}^+$  is the positive  
252 mechanical power for the  $m$ th muscle on the non-paretic side. Negative mechanical work was calculated  
253 based on the negative power of each non-paretic muscle. The positive and negative mechanical work for  
254 the parietic limb was calculated the same way as above, but with power data from the parietic limb. As with  
255 the metabolic COT calculations, we normalized both the positive and negative mechanical work by dividing  
256 by displacement to compute the positive and negative mechanical COT.

257           Lastly as different metabolic cost models can result in different predictions of metabolic cost (57),  
258 we evaluated whether our results would change with a different metabolic cost model. Therefore, we  
259 computed the metabolic COT using the Bhargava model (58) for each of the nominal conditions to compare  
260 with the results from the Umberger model.

261

## 262 **3. Results**

263

### 264 *3.1 Overview and Model Validation*

265           Overall, the kinematic and GRF results for the base model across speeds shared broad similarities  
266 in patterns with previous experimental data, which gave us confidence that the modeling and optimization  
267 methods we used produced sensible results. For example, peak knee flexion angle occurred earlier in the  
268 gait cycle as speed increased (Fig 2B), which aligns with experimental results (59). The peak ankle  
269 plantarflexion angle at push-off also occurred earlier in the gait cycle at faster speeds compared with slower  
270 speeds (Fig 2C), also matching with experimental data (59). Generally, the GRFs also matched  
271 experimental data, with greater peak vertical GRFs (Fig 2D) and greater posterior GRFs during early stance  
272 (Fig 2E) for faster speeds (60). Across all speeds, the step time and step length variables were each close  
273 to symmetrical for the base model, demonstrating that the optimal gait patterns for symmetrical models  
274 were symmetrical (Fig 3A and 3D).

275

### 276 *3.2 Effects of Simulated Hemiparesis on Spatiotemporal Asymmetry*

277           Generally, for models with muscle strength asymmetries, the optimal solution resulted in  
278 asymmetrical spatiotemporal patterns where the magnitude and direction of the asymmetries depended on  
279 both the level of muscle weakness and the gait speed (Fig 3A and 3B). For slower gait speeds (e.g., 0.25  
280 and 0.50 m/s), the optimization resulted in a positive step time asymmetry for both the 40% and 60%  
281 weakness models which corresponds to the model taking longer to transition from the paretic to non-paretic  
282 limb than vice versa. At faster speeds (e.g., 1.00 or 1.25 m/s) the optimal gait pattern had a negative step

283 time asymmetry for those same models. This suggests that the direction of step time asymmetry (positive  
284 or negative) is impacted by the gait speed that an individual walks and the degree of muscle weakness.  
285 Generally, the step times for the non-paretic side (Fig 3B) were about 0.40 seconds across most speeds and  
286 conditions, while the step times for the paretic side (Fig 3C) across the conditions were greater at faster  
287 speeds than slower speeds (i.e., ~0.40 seconds at slower speeds and ~0.50 seconds at faster speeds).

288 The direction and magnitude of step length asymmetry also varied depending on both the level of  
289 muscle weakness and the gait speed (Fig 3D). Marked step length asymmetries were generally observed  
290 once strength was reduced by 40%. For most speeds, the optimal solution for the model with 40% weakness  
291 was a positive step length asymmetry, with the largest asymmetry being observed in the weakest model at  
292 the slowest speed. These positive step length asymmetries correspond to a gait pattern where longer steps  
293 are taken with the paretic limb than the non-paretic limb. An exception to this trend occurred in the 1.25  
294 m/s condition which resulted in a negative step length asymmetry. The optimal solutions produced greater  
295 step lengths on both the non-paretic (Fig 3E) and paretic (Fig 3F) sides for faster speeds compared with  
296 slower speeds. The 40% weakness model at 1.25 m/s resulted in step times and step lengths that stood out  
297 from other similar conditions, with a much shorter right step time and shorter right and left step lengths  
298 than other similar conditions (i.e., 40% weakness at 1.00 m/s or 60% weakness at 1.25 m/s).

299

### 300 *3.3 Metabolic and Mechanical Cost of Transport for Nominal Conditions*

301 Despite the optimal solutions having marked spatiotemporal asymmetries, the computed metabolic  
302 COT was relatively consistent within speeds across different model weaknesses (Fig 4A). At speeds  
303 between 0.25 and 0.75 m/s, the metabolic COT was similar across all models, but there were subtle  
304 differences across the models in metabolic COT at 1.00 and 1.25 m/s. The metabolic COT was greater in  
305 the 40 and 60% weakness models than the symmetrical and 20% weakness models at 1.25 m/s, whereas at  
306 1.00 m/s the pattern was the opposite. Gait speed had a substantial effect on the computed metabolic COT  
307 across the speeds we tested, with greater COT at slower gait speeds than faster gait speeds. At faster gait  
308 speeds (e.g., 1.00 and 1.25 m/s), the metabolic COT was between 3-4 J/kg/m, while at slower speeds (e.g.,

309 0.25 and 0.50) the metabolic COT was close to 10 J/kg/m, which matches closely with the trend of  
310 metabolic COT across speeds in neurotypical individuals (11,47). The consistency of the COT for a given  
311 speed resulted from complementary changes in limb-specific metabolic COT. Metabolic COT on the non-  
312 paretic limb increased with greater weakness levels (Fig 4B) while COT was reduced proportionally on the  
313 paretic limb (Fig 4C).

314 One important factor to consider when determining metabolic energy consumed by muscles is the  
315 amount of mechanical work done by each muscle during walking (61,62). Therefore, we evaluated how  
316 positive and negative mechanical work changed across speeds and conditions. Overall, the optimal solutions  
317 resulted in gait patterns that had greater positive mechanical COT at slower gait speeds (Fig 4D).  
318 Additionally, for models with greater muscle weakness there was a decrease in the magnitude of the positive  
319 mechanical COT. While the non-paretic limb had a similar level of positive mechanical COT across the  
320 different models within a speed (Fig 4F), the reduction in total positive mechanical work was driven by a  
321 considerable decrease in the magnitude of positive mechanical COT by the paretic limb with increasing  
322 weakness (Fig 4F). These trends were also seen for negative mechanical COT, with greater negative  
323 mechanical COT at slower speeds and a decrease in negative mechanical COT with greater muscle  
324 weakness.

325 The second important factor in determining metabolic energy consumed by muscles is the level of  
326 muscle activation throughout the gait cycle. Total muscle activation increases at both faster gait speeds and  
327 greater levels of simulated hemiparesis (Fig 4G). At the individual limb level, the muscle activations for  
328 the non-paretic and paretic limbs increase with speed and weakness as well (Fig 4H and 4I). Overall, the  
329 metabolic energy cost does not significantly vary across level of simulated hemiparesis, which is likely a  
330 result of a decrease in mechanical muscle fiber work which offsets the increase in muscle activation with  
331 greater muscle weakness.

332 We observed a typical U-shaped pattern of metabolic COT with respect to gait speed within a model  
333 (Fig 5). Using these results, we can identify the “effort-optimal” solutions that resulted in the minimum  
334 metabolic COT within a model across the speeds. For both the base and 20% weakness models, the



335 minimum metabolic COT occurred at a gait speed of 1.25 m/s, which is similar to what is observed  
336 experimentally in neurotypical individuals (47,63,64). For the 40% and 60% weakness models, the  
337 minimum metabolic COT instead occurred at a slower speed of 1.00 m/s.

338 To evaluate the sensitivity of our results and conclusions to the metabolic model chosen, we  
339 computed the metabolic COT using the Bhargava metabolic model (58). The Bhargava model resulted in  
340 slightly greater metabolic COT (average offset of  $0.37 \pm 0.17$  J/kg/m) than the Umberger model, but the  
341 trends across the conditions were similar which suggests that our conclusions would not change with either  
342 the Umberger or Bhargava energetics models.

343

#### 344 *3.4 Effects of Minimizing Step Length and Step Time Asymmetry in a Model of Hemiparesis*

345 Finally, we evaluated whether the predicted metabolic COT would change when enforcing step  
346 length or step time symmetry for models with simulated hemiparesis. To perform this analysis, we added a  
347 term to the objective function which produced gait patterns with approximately symmetrical step lengths  
348 across the different hemiparetic models at a speed of 0.75 m/s. The optimal solutions for the 20%, 40%,  
349 and 60% models had step length asymmetry indices less than 1.5%, which was close to the symmetrical  
350 target of 0% asymmetry (Fig 6B). However, this resulted in greater step time asymmetry for these gait  
351 patterns (Fig 6A). Minimizing step length symmetry resulted in a slightly greater metabolic COT than the  
352 nominal conditions by 1-4% (Fig 6C). We then added a third term to the objective function, which in  
353 combination with the others, resulted in gait patterns that had approximately symmetrical step lengths  
354 (maximum asymmetry = 2.6%; Fig 6B) and step times (maximum asymmetry = -1%; Fig 6A). Despite  
355 adopting a gait that was nearly symmetric overall, the metabolic COT for these conditions was less than  
356 1% greater compared to when step length asymmetry was minimized alone, and only deviated by 5% from  
357 the nominal conditions.

358

359

360

## 361 4. Discussion

362

### 363 4.1 Overview of Key Results

364 The purpose of this study was to quantify the spatiotemporal asymmetries and changes to metabolic  
365 cost that emerge from effort-optimal predictions of gait with models of simulated hemiparesis across a wide  
366 range of speeds. Predicting the optimal gait pattern for models with simulated hemiparesis allows us to gain  
367 insight into the isolated effect of unilateral muscle weakness on gait asymmetry and metabolic COT. We  
368 found that the magnitude and direction of spatiotemporal asymmetry depended on the level of muscle  
369 weakness as well as the gait speed, with greater asymmetries corresponding to a greater level of muscle  
370 weakness. We then compared the metabolic COT across all optimal gait patterns to assess whether  
371 simulated hemiparesis and spatiotemporal asymmetries correspond with an increase in metabolic cost. We  
372 found that the predicted metabolic COT varied little with increasing levels of weakness, but scaled with  
373 speed as expected based on experimental data (e.g., 47). Finally, we evaluated whether enforcing step length  
374 or step time symmetry for the hemiparetic models would increase the metabolic COT. The metabolic COT  
375 only changed by a maximum difference of 5% between the nominal and symmetrical conditions. Overall,  
376 our results suggest that the spatiotemporal asymmetries seen in people post-stroke might be derived in part  
377 from optimal adaptations to underlying impairments such as hemiparesis. However, the presence of gait  
378 asymmetries does not necessarily contribute to the increase in metabolic cost that is observed relative  
379 neurotypical individuals.

380 Overall, a greater level of hemiparesis resulted in a more asymmetrical gait pattern in both step  
381 time and step length across all speeds, however the direction of the asymmetry depended on the gait speed.  
382 The 20% weakness model had step time asymmetries ranging from -5 to 2% and step length asymmetries  
383 ranging from -10 to 6%, while the 60% weakness models had step time asymmetries ranging from -20 to  
384 10% and step length asymmetries ranging from -3 to 70%. This suggests that the effort-optimal gait patterns  
385 for individuals with mild unilateral strength deficits may be closer to symmetrical, compared to individuals

386 with much greater muscle strength impairments who may have effort-optimal gait patterns with greater  
387 spatiotemporal asymmetries.

388 We also found that metabolic COT is relatively consistent across levels of simulated hemiparesis.  
389 Metabolic cost is impacted by the level of muscle activation and the magnitude of mechanical work done  
390 throughout the movement. While muscle activation increased with greater levels of simulated hemiparesis,  
391 the positive and negative work performed by the muscle fibers decrease with greater levels of simulated  
392 hemiparesis. The decrease in work performed by the muscle fibers is driven by a decrease in work done by  
393 the paretic limb due to less muscle force generation. Overall, this effectively creates a trade-off between  
394 muscle activation and mechanical work, resulting in a similar level of metabolic cost across models within  
395 a gait speed.

396 There were substantial speed effects on both spatiotemporal asymmetry and metabolic cost. The  
397 magnitude and direction of step length asymmetry was also dependent upon the gait speed. For hemiparetic  
398 gait, a positive step length asymmetry indicates shorter steps with the paretic limb than the non-paretic  
399 limb. This pattern can be expected of a person post-stroke if they are unable to swing their paretic limb  
400 forward. Many of the effort-optimal gait patterns fall into this category of having positive step length  
401 asymmetry with a couple of exceptions at the 0.25 and 1.25 m/s conditions. For the 60% weak model, there  
402 were greater positive step length asymmetries at slower speeds than for faster speeds. The overall task  
403 demands for the slow speed may allow the model to take extremely short paretic side steps, while still  
404 matching the task speed. The heterogeneity and relative ranges of both positive and negative step length  
405 asymmetries largely mirrors what has been measured in previous experiments of people post-stroke  
406 (25,65,66).

407 Our results suggest that the direction of step time asymmetry depends on the gait speed rather than  
408 the level of hemiparesis. At 1.25 m/s speed, the effort-optimal gait patterns resulted in negative step time  
409 asymmetries across the three hemiparetic models, while for the 0.25 m/s speed, the effort optimal gait  
410 patterns result in positive step time asymmetries. Paretic step time here was defined as the time from non-  
411 paretic foot strike to paretic foot strike, so it contains the duration of the double support time with the

412 leading, non-paretic limb and the paretic side swing time. Therefore, a negative step time asymmetry  
413 (greater paretic step time than non-paretic step time) would suggest that the model is using its non-paretic  
414 limb for body weight support and propulsion for a greater period than the paretic limb, which is what one  
415 might expect since weakness occurs in the paretic limb. In contrast, a positive step time asymmetry may be  
416 beneficial at the slow speeds because the paretic swing limb may be less able to take advantage of passive  
417 dynamics relative to what happens at faster speeds, resulting in a greater activation during swing phase to  
418 swing the leg forward.

419

#### 420 *4.2 Clinical Implications for Post-Stroke Rehabilitation*

421 The results from our simulation study have generated a couple of testable hypotheses. First, that  
422 the direction of spatiotemporal asymmetries depends on the severity of hemiparesis and walking speed. We  
423 know that spatiotemporal asymmetries, such as step length asymmetry, in people post-stroke are  
424 heterogeneous, with some individuals having longer step lengths with their paretic limb than their non-  
425 paretic and others having shorter step lengths with their paretic limb than non-paretic limb (25,66), however  
426 what drives these responses remains unclear (65). Our results suggest that one of the driving factors of  
427 asymmetry could be the level of hemiparesis.

428 Furthermore, the hypothesized relationship between severity of hemiparesis and the effort-optimal  
429 gait patterns (i.e., direction and magnitude of spatiotemporal asymmetries) suggests that clinical  
430 rehabilitation programs could be individually tailored based on measures of between-limb differences in  
431 strength. Typical goals for gait rehabilitation programs for people post-stroke are to have the individuals  
432 walk more symmetrically (50,67). However, it's still an open question whether this goal could be  
433 considered the optimal way to walk for people post-stroke, with recent data suggesting that symmetrical  
434 step lengths do not improve measures of metabolic COT (23,25) and result in other kinematic and kinetic  
435 asymmetries (22). The results of our predictive simulations are consistent with these studies: when we  
436 enforced step length symmetry for the 0.75 m/s speed, the resulting gait pattern had significant step time  
437 asymmetry, with a small increase in metabolic COT. The experimental results in combination with our

438 simulated results are reasonable given the underlying anatomical system is asymmetric, therefore optimal  
439 gait patterns are also likely to be asymmetric. However, that the symmetry conditions resulted in only small  
440 increases in metabolic COT suggest that there could be multiple gait strategies for people post-stroke that  
441 result in similar consumption of metabolic energy, therefore, patients who prioritize a symmetrical  
442 appearance of gait may be able to perform their preferred gait pattern without a large penalty on their  
443 endurance.

444 Another important research question for gait in people post-stroke is what are the factors that  
445 contribute most to the slower gait speed observed in this population compared to age-matched controls  
446 (16,25,68,69)? Our data suggest that moderate levels of hemiparesis could contribute to the reduction in the  
447 energy optimal gait speed, as minimum metabolic COT occurred at 1.00 m/s for the 40% and 60% weakness  
448 models compared with a minimum at 1.25 m/s for the base and 20% weak models. However, this slight  
449 reduction in speed does not match the difference in preferred gait speed in people post-stroke relative to  
450 control (i.e., ~0.7 m/s; 25). So, while unilateral muscle weakness may partially contribute to the reduction  
451 in gait speed, other types of impairments such as spasticity and abnormal muscle co-activation patterns or  
452 different priorities during gait (e.g., balance and comfort) may have additional contributions.

453 Lastly, our data present another potential explanation (besides metabolic COT) for slower walking  
454 speed in people post-stroke: preferred gait speed for people post-stroke may also be impacted by an  
455 individual's desire to minimize observable spatiotemporal asymmetries. Our data suggest that moderate  
456 speeds of 0.50 and 0.75 m/s result in effort-optimal gait patterns with moderate levels of gait asymmetry  
457 (<10% asymmetry indices), while slower (0.25 m/s) or faster speeds (1.00 or 1.25 m/s) can result in levels  
458 of gait asymmetry greater than 10%. Therefore, we can hypothesize that people post-stroke may walk with  
459 slower gait speeds than would be optimal from a metabolic COT point of view because this allows them to  
460 maintain lower levels of gait asymmetry than would be necessary for faster/optimal COT speeds. Further  
461 explanation of the preferred gait patterns in people post-stroke could be explained by factors like balance  
462 or comfort, though how people post-stroke sense and perceive of these factors is an important question that  
463 needs to be addressed but is beyond the scope of this study.

#### 464 *4.3 Modeling Decisions and Limitations*

465           Our results build upon several modeling choices that we made throughout the study, and as such,  
466 there are a few limitations to our project that should be considered in future research. First, we constrained  
467 our model to move only in the sagittal plane because our primary kinematic variables of interest for this  
468 project were step length and step time asymmetry, which are also in the sagittal plane, and we wanted to  
469 simulate several conditions (across speeds and across hemiparetic models) in a timely manner.

470           Additionally, we chose an objective function that minimizes the integrated sum of muscle  
471 excitations cubed across all muscles, which has been proposed to be a representation of minimizing muscle  
472 fatigue, or maximizing muscle endurance (28). This objective is formulated from experimental data that  
473 suggests the muscle force-endurance relationship is approximately cubic (43,70). While we evaluated the  
474 predicted metabolic COT from the results, we did not use metabolic COT in the objective function.  
475 Therefore, there could be different gait patterns that would further reduce the metabolic cost across each of  
476 the conditions. However, while different objective functions will result in different gait patterns from an  
477 optimal control solution (28,36,71), we believe that using our chosen objective function is appropriate for  
478 this study as fatigue is likely to be especially relevant for people with neuromuscular impairments.

479           Finally, the modeling of hemiparesis in our project involved several decisions. First, we decided to  
480 model hemiparesis instead of other impairments such as abnormal muscle coordination patterns or other  
481 changes in muscle properties because hemiparesis is straightforward to model by reducing peak muscle  
482 forces in the model. Future work should add other impairments, either on their own or in combination with  
483 hemiparesis, to better understand the independent or combinatory effects of the array of impairments after  
484 a stroke. Further, we decided to simulate hemiparesis by reducing the peak isometric muscle forces instead  
485 of modifying the maximum excitation values, which would relate to the decrease in central drive in people  
486 post-stroke. Modeling decreased central drive in a predictive simulation paradigm is impractical from a  
487 computational standpoint, as it would only affect the results when the maximum allowable excitation in the  
488 simulated weakness condition is exceeded in the base conditions. In gait, most muscles operate in a  
489 submaximal state, oftentimes far below full excitation. Therefore, setting a threshold excitation of 60% of

490 baseline, for example, would likely result in no changes to the gait strategy since none of the muscles  
491 surpassed 60% excitation level during the baseline stride. Furthermore, modeling decreased central drive is  
492 challenging from a physiological standpoint because it is difficult to assess the magnitude of impairment in  
493 central drive *in vivo*, instead, it's much simpler to measure muscle strength in the limbs using a  
494 dynamometer (e.g., 10). Therefore, our choice of reducing peak isometric muscle force as a representation  
495 of a reduction in the ability to produce force allowed for a reasonable way to model hemiparesis and is  
496 applicable since measuring the magnitude of impairment can be easily done in clinical settings. Another  
497 important modeling decision was to keep muscle mass constant across all hemiparetic models to simulate  
498 muscle weakness without muscle atrophy. Muscle mass is used for computing metabolic COT, because the  
499 metabolic energy consumed by muscles depends on activation levels and the volume of muscle activated  
500 (56). If we instead modeled muscle weakness alongside muscle atrophy, it would result in a reduction in  
501 metabolic cost for the hemiparetic models compared with our results due to a reduction in activated muscle  
502 mass.

503

#### 504 *4.4 Conclusion*

505 In this study, we predicted the effort-optimal spatiotemporal patterns for gait with simulated  
506 hemiparetic musculoskeletal models. We found that the magnitude and direction of spatiotemporal  
507 asymmetry is affected by the level of hemiparesis and the gait speed, which may explain the well-known  
508 heterogenous distribution of spatiotemporal asymmetries observed in clinical data. However, the greater  
509 metabolic COT observed in people post-stroke compared to controls does not appear to be driven by  
510 hemiparesis, and instead may be driven by factors like muscle co-activation or abnormal muscle synergy  
511 patterns. Further, our data predict that hemiparesis is one aspect that could lead to slower self-selected gait  
512 speeds in people post stroke, but other neuromuscular impairments or preferences may drive gait speed  
513 even slower than what was predicted to be energy optimal in our simulations. Lastly, our data provide  
514 additional theoretical support for the idea that asymmetrical gait patterns may be optimal when aspects of  
515 the underlying control system is asymmetrical. Overall, our study is a step towards a better understanding

516 of how specific impairments in people post-stroke affect gait patterns and metabolic COT. Since it is  
517 difficult to study how distributed, unilateral muscle weakness alone affects gait with human participants,  
518 our predictive modeling approach can allow for these tests since we can build custom models that represent  
519 the type of impairment we are focused on testing. Future studies should extend the work presented here to  
520 explore the effects of additional impairments and build a more comprehensive understanding of how a range  
521 of impairments influence post-stroke gait.

522

### 523 *Acknowledgements*

524 The authors thank Natalia Sánchez for discussions on the content of the paper.

525

### 526 *Availability of data and materials*

527 All models and code are available at <https://simtk.org/projects/post-stroke-sym>

528

### 529 *Competing interests*

530 The authors declare that they have no competing interests

531

### 532 *Funding*

533 NIH NCMRR R01HD091184

534

535



536 **References**

537

538 1. Gracies JM. Pathophysiology of spastic paresis. II: Emergence of muscle overactivity. *Muscle and*  
539 *Nerve*. 2005;31(5):552–71.

540 2. Gracies JM. Pathophysiology of spastic paresis. I: Paresis and soft tissue changes. *Muscle and*  
541 *Nerve*. 2005;31(5):535–51.

542 3. Twitchell TE. The restoration of motor function following hemiplegia in man. *Brain : a journal of*  
543 *neurology*. 1951 Dec;74(4):443–80.

544 4. Kuo C-L, Hu G-C. Post-stroke Spasticity: A Review of Epidemiology, Pathophysiology, and  
545 Treatments. *International Journal of Gerontology*. 2018 Dec 1;12(4):280–4.

546 5. Li S. Spasticity, Motor Recovery, and Neural Plasticity after Stroke. *Front Neurol*. 2017 Apr  
547 3;8:120–120.

548 6. Neckel N, Pelliccio M, Nichols D, Hidler J. Quantification of functional weakness and abnormal  
549 synergy patterns in the lower limb of individuals with chronic stroke. *J Neuroeng Rehabil*. 2006 Jul  
550 20;3:17.

551 7. Sánchez N, Acosta AM, López-Rosado R, Dewald JPA. Neural Constraints Affect the Ability to  
552 Generate Hip Abduction Torques When Combined With Hip Extension or Ankle Plantarflexion in  
553 Chronic Hemiparetic Stroke. *Frontiers in Neurology*. 2018;9:564.

554 8. Cruz TH, Dhaher YY. Evidence of abnormal lower-limb torque coupling after stroke: an isometric  
555 study. *Stroke*. 2008 Jan;39(1):139–47.

- 556 9. Dewald JP, Pope PS, Given JD, Buchanan TS, Rymer WZ. Abnormal muscle coactivation patterns  
557 during isometric torque generation at the elbow and shoulder in hemiparetic subjects. *Brain*. 1995  
558 Apr;118 ( Pt 2):495–510.
- 559 10. Sánchez N, Acosta AM, Lopez-Rosado R, Stienen AHA, Dewald JPA. Lower Extremity Motor  
560 Impairments in Ambulatory Chronic Hemiparetic Stroke: Evidence for Lower Extremity Weakness  
561 and Abnormal Muscle and Joint Torque Coupling Patterns. *Neurorehabilitation and neural repair*.  
562 2017/08/08 ed. 2017 Sep;31(9):814–26.
- 563 11. Finley JM, Bastian AJ. Associations Between Foot Placement Asymmetries and Metabolic Cost of  
564 Transport in Hemiparetic Gait. *Neurorehabil Neural Repair*. 2017 Feb 1;31(2):168–77.
- 565 12. Olney SJ, Richards C. Hemiparetic gait following stroke . Part I : Characteristics. *Gait and Posture*.  
566 1996;4(2):136–48.
- 567 13. Perry J, Burnfield J. *Gait Analysis: Normal and Pathological Function*. Perry J, Burnfield JM,  
568 editors. *Journal of Sports Science & Medicine*. 2010 Jun;9(2):353.
- 569 14. Platts MM, Rafferty D, Paul L. Metabolic cost of over ground gait in younger stroke patients and  
570 healthy controls. *Med Sci Sports Exerc*. 2006 Jun;38(6):1041–6.
- 571 15. Cruz TH, Lewek MD, Dhaher YY. Biomechanical impairments and gait adaptations post-stroke:  
572 multi-factorial associations. *Journal of biomechanics*. 2009/05/20 ed. 2009 Aug;42(11):1673–7.
- 573 16. Hsu A-L, Tang P-F, Jan M-H. Analysis of impairments influencing gait velocity and asymmetry of  
574 hemiplegic patients after mild to moderate stroke. *Archives of Physical Medicine and*  
575 *Rehabilitation*. 2003;84(8):1185–93.

- 576 17. Nadeau S, Arsenault AB, Gravel D, Bourbonnais D. Analysis of the clinical factors determining  
577 natural and maximal gait speeds in adults with a stroke. *Am J Phys Med Rehabil*. 1999  
578 Apr;78(2):123–30.
- 579 18. Suzuki K, Imada G, Iwaya T, Handa T, Kurogo H. Determinants and predictors of the maximum  
580 walking speed during computer-assisted gait training in hemiparetic stroke patients. *Arch Phys Med*  
581 *Rehabil*. 1999 Feb;80(2):179–82.
- 582 19. Ellis RG, Howard KC, Kram R. The metabolic and mechanical costs of step time asymmetry in  
583 walking. *Proc Biol Sci*. 2013 Feb 13;280(1756):20122784–20122784.
- 584 20. Stenum J, Choi JT. Disentangling the energetic costs of step time asymmetry and step length  
585 asymmetry in human walking. *J Exp Biol*. 2021 Jun 15;224(12).
- 586 21. Nguyen TM, Jackson RW, Aucie Y, de Kam D, Collins SH, Torres-Oviedo G. Self-selected step  
587 length asymmetry is not explained by energy cost minimization in individuals with chronic stroke.  
588 *Journal of NeuroEngineering and Rehabilitation*. 2020 Aug 26;17(1):119.
- 589 22. Padmanabhan P, Rao KS, Gulhar S, Cherry-Allen KM, Leech KA, Roemmich RT. Persons post-  
590 stroke improve step length symmetry by walking asymmetrically. *Journal of NeuroEngineering and*  
591 *Rehabilitation*. 2020 Aug 3;17(1):105.
- 592 23. Roemmich RT, Leech KA, Gonzalez AJ, Bastian AJ. Trading Symmetry for Energy Cost During  
593 Walking in Healthy Adults and Persons Poststroke. *Neurorehabil Neural Repair*. 2019 Aug  
594 1;33(8):602–13.
- 595 24. Ryan HP, Husted C, Lewek MD. Improving Spatiotemporal Gait Asymmetry Has Limited  
596 Functional Benefit for Individuals Poststroke. *Journal of Neurologic Physical Therapy [Internet]*.  
597 2020;44(3). Available from:

598 [https://journals.lww.com/jnpt/Fulltext/2020/07000/Improving\\_Spatiotemporal\\_Gait\\_Asymmetry\\_H](https://journals.lww.com/jnpt/Fulltext/2020/07000/Improving_Spatiotemporal_Gait_Asymmetry_H)  
599 [as.5.aspx](https://journals.lww.com/jnpt/Fulltext/2020/07000/Improving_Spatiotemporal_Gait_Asymmetry_H)

600 25. Sánchez N, Finley JM. Individual Differences in Locomotor Function Predict the Capacity to  
601 Reduce Asymmetry and Modify the Energetic Cost of Walking Poststroke. *Neurorehabil Neural*  
602 *Repair*. 2018 Aug 1;32(8):701–13.

603 26. Falisse A, Serrancolí G, Dembia CL, Gillis J, Jonkers I, De Groot F. Rapid predictive simulations  
604 with complex musculoskeletal models suggest that diverse healthy and pathological human gaits  
605 can emerge from similar control strategies. *Journal of The Royal Society Interface*.  
606 2019;16(157):20190402.

607 27. Song S, Geyer H. Predictive neuromechanical simulations indicate why walking performance  
608 declines with ageing. *The Journal of physiology*. 2018/03/02 ed. 2018 Apr;596(7):1199–210.

609 28. Ackermann M, van den Bogert AJ. Optimality principles for model-based prediction of human gait.  
610 *Journal of Biomechanics*. 2010 Apr;43(6):1055–60.

611 29. Anderson FC, Pandy MG. Dynamic optimization of human walking. *Journal of Biomechanical*  
612 *Engineering*. 2001 Oct;123(5):381–90.

613 30. Lin Y-C, Walter JP, Pandy MG. Predictive Simulations of Neuromuscular Coordination and Joint-  
614 Contact Loading in Human Gait. *Annals of Biomedical Engineering*. 2018 Aug 1;46(8):1216–27.

615 31. Miller RH. A comparison of muscle energy models for simulating human walking in three  
616 dimensions. *Journal of Biomechanics*. 2014 Apr;47(6):1373–81.

617 32. Nguyen VQ, Johnson RT, Sup FC, Umberger BR. Bilevel Optimization for Cost Function  
618 Determination in Dynamic Simulation of Human Gait. *IEEE Transactions on Neural Systems and*  
619 *Rehabilitation Engineering*. 2019 Jul 1;27(7):1426–35.

- 620 33. Fregly BJ, Reinbolt JA, Rooney KL, Mitchell KH, Chmielewski TL. Design of patient-specific gait  
621 modifications for knee osteoarthritis rehabilitation. *IEEE Trans Biomed Eng.* 2007 Sep;54(9):1687–  
622 95.
- 623 34. Ong CF, Geijtenbeek T, Hicks JL, Delp SL. Predicting gait adaptations due to ankle plantarflexor  
624 muscle weakness and contracture using physics-based musculoskeletal simulations. *PLoS Comput*  
625 *Biol.* 2019 Oct;15(10):e1006993.
- 626 35. Koelewijn AD, van den Bogert AJ. Joint contact forces can be reduced by improving joint moment  
627 symmetry in below-knee amputee gait simulations. *Gait Posture.* 2016 Sep;49:219–25.
- 628 36. Miller RH, Brandon SCE, Deluzio KJ. Predicting sagittal plane biomechanics that minimize the  
629 axial knee joint contact force during walking. *J Biomech Eng.* 2013 Jan;135(1):011007.
- 630 37. Dembia CL, Bianco NA, Falisse A, Hicks JL, Delp SL. OpenSim Moco: Musculoskeletal optimal  
631 control. *PLOS Computational Biology.* 2021 Dec 28;16(12):e1008493.
- 632 38. Falisse A, Serrancolí G, Dembia CL, Gillis J, De Groote F. Algorithmic differentiation improves the  
633 computational efficiency of OpenSim-based trajectory optimization of human movement. *PLOS*  
634 *ONE.* 2019 Oct 17;14(10):e0217730.
- 635 39. Nitschke M, Dorschky E, Heinrich D, Schlarb H, Eskofier BM, Koelewijn AD, et al. Efficient  
636 trajectory optimization for curved running using a 3D musculoskeletal model with implicit  
637 dynamics. *Scientific Reports.* 2020;10(1):17655.
- 638 40. De Groote F, Falisse A. Perspective on musculoskeletal modelling and predictive simulations of  
639 human movement to assess the neuromechanics of gait. *Proc Biol Sci.* 2021 Mar  
640 10;288(1946):20202432.

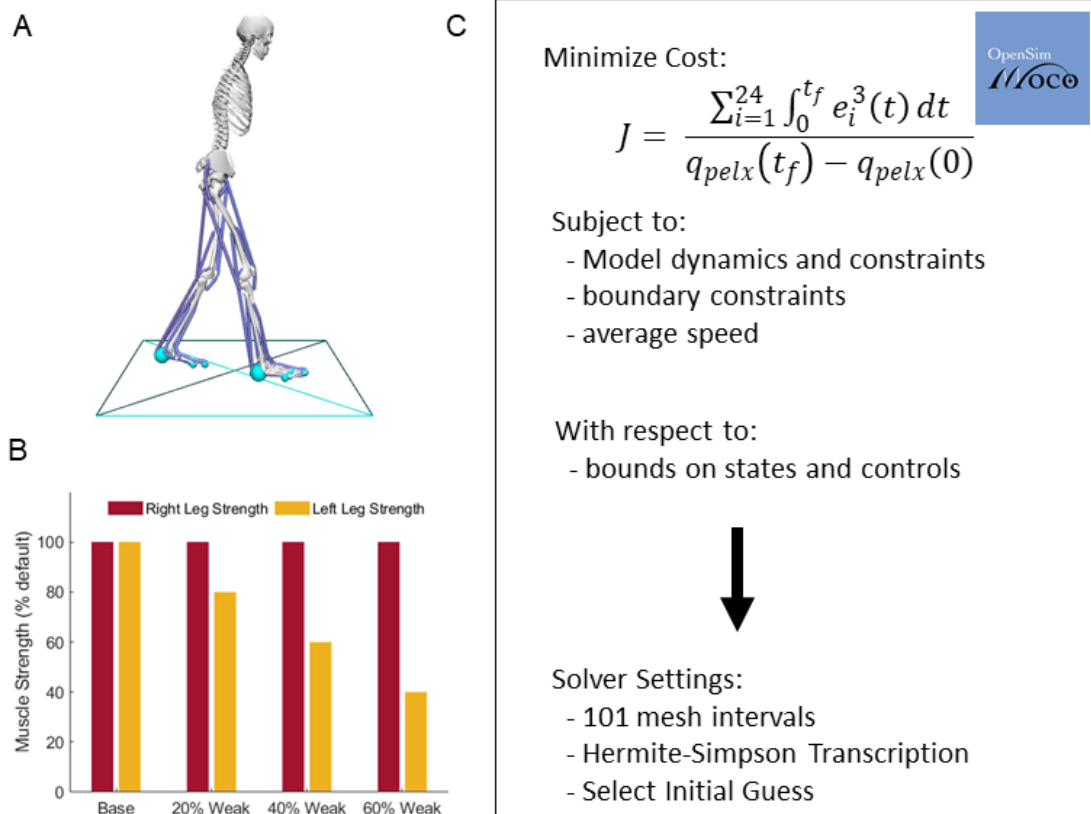
- 641 41. De Groot F, Kinney AL, Rao AV, Fregly BJ. Evaluation of Direct Collocation Optimal Control  
642 Problem Formulations for Solving the Muscle Redundancy Problem. *Annals of biomedical*  
643 *engineering*. 2016 Oct;44(10):2922–36.
- 644 42. Serrancolí G, Falisse A, Dembia C, Vantilt J, Tanghe K, Lefeber D, et al. Subject-Exoskeleton  
645 Contact Model Calibration Leads to Accurate Interaction Force Predictions. *IEEE Transactions on*  
646 *Neural Systems and Rehabilitation Engineering*. 2019;27(8):1597–605.
- 647 43. Crowninshield RD, Brand RA. A physiologically based criterion of muscle force prediction in  
648 locomotion. *Journal of Biomechanics*. 1981;
- 649 44. Bertram JE, Ruina A. Multiple walking speed-frequency relations are predicted by constrained  
650 optimization. *Journal of theoretical biology*. 2001 Apr;209(4):445–53.
- 651 45. Donelan JM, Kram R, Kuo AD. Mechanical and metabolic determinants of the preferred step width  
652 in human walking. *Proceedings of the Royal Society B: Biological Sciences*. 2001  
653 Oct;268(1480):1985–92.
- 654 46. Kuo AD, Donelan JM. *Dynamic Principles of Gait and Their Clinical Implications*. *Physical*  
655 *Therapy*. 2010 Feb;90(2):157–74.
- 656 47. Ralston HJ. Energy-speed relation and optimal speed during level walking. *Internationale Zeitschrift*  
657 *fur angewandte Physiologie, einschliesslich Arbeitsphysiologie*. 1958;17(4):277–83.
- 658 48. Sánchez N, Simha SN, Donelan JM, Finley JM. Using asymmetry to your advantage: learning to  
659 acquire and accept external assistance during prolonged split-belt walking. *Journal of*  
660 *Neurophysiology*. 2021 Feb 1;125(2):344–57.

- 661 49. Seethapathi N, Srinivasan M. The metabolic cost of changing walking speeds is significant, implies  
662 lower optimal speeds for shorter distances, and increases daily energy estimates. *Biol Lett*. 2015  
663 Sep;11(9):20150486.
- 664 50. Bohannon RW, Andrews AW, Smith MB. Rehabilitation goals of patients with hemiplegia.  
665 *International Journal of Rehabilitation Research* [Internet]. 1988;11(2). Available from:  
666 [https://journals.lww.com/intjrehabilres/Fulltext/1988/06000/Rehabilitation\\_goals\\_of\\_patients\\_with\\_](https://journals.lww.com/intjrehabilres/Fulltext/1988/06000/Rehabilitation_goals_of_patients_with_hemiplegia.12.aspx)  
667 [hemiplegia.12.aspx](https://journals.lww.com/intjrehabilres/Fulltext/1988/06000/Rehabilitation_goals_of_patients_with_hemiplegia.12.aspx)
- 668 51. Betts JT. *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming*,  
669 *Second Edition* [Internet]. Society for Industrial and Applied Mathematics; 2010 [cited 2021 Jul  
670 29]. 442 p. (*Advances in Design and Control*). Available from:  
671 <https://doi.org/10.1137/1.9780898718577>
- 672 52. Andersson JAE, Gillis J, Horn G, Rawlings JB, Diehl M. CasADi: a software framework for  
673 nonlinear optimization and optimal control. *Mathematical Programming Computation*. 2019 Mar  
674 1;11(1):1–36.
- 675 53. Wächter A, Biegler LT. On the implementation of an interior-point filter line-search algorithm for  
676 large-scale nonlinear programming. *Mathematical Programming*. 2006 Mar 1;106(1):25–57.
- 677 54. Park S, Liu C, Sánchez N, Tilson JK, Mulroy SJ, Finley JM. Using Biofeedback to Reduce Step  
678 Length Asymmetry Impairs Dynamic Balance in People Poststroke. *Neurorehabil Neural Repair*.  
679 2021 Jun 1;15459683211019346.
- 680 55. Umberger BR. Stance and swing phase costs in human walking. *Journal of the Royal Society*,  
681 *Interface / the Royal Society*. 2010 Sep;7(50):1329–40.

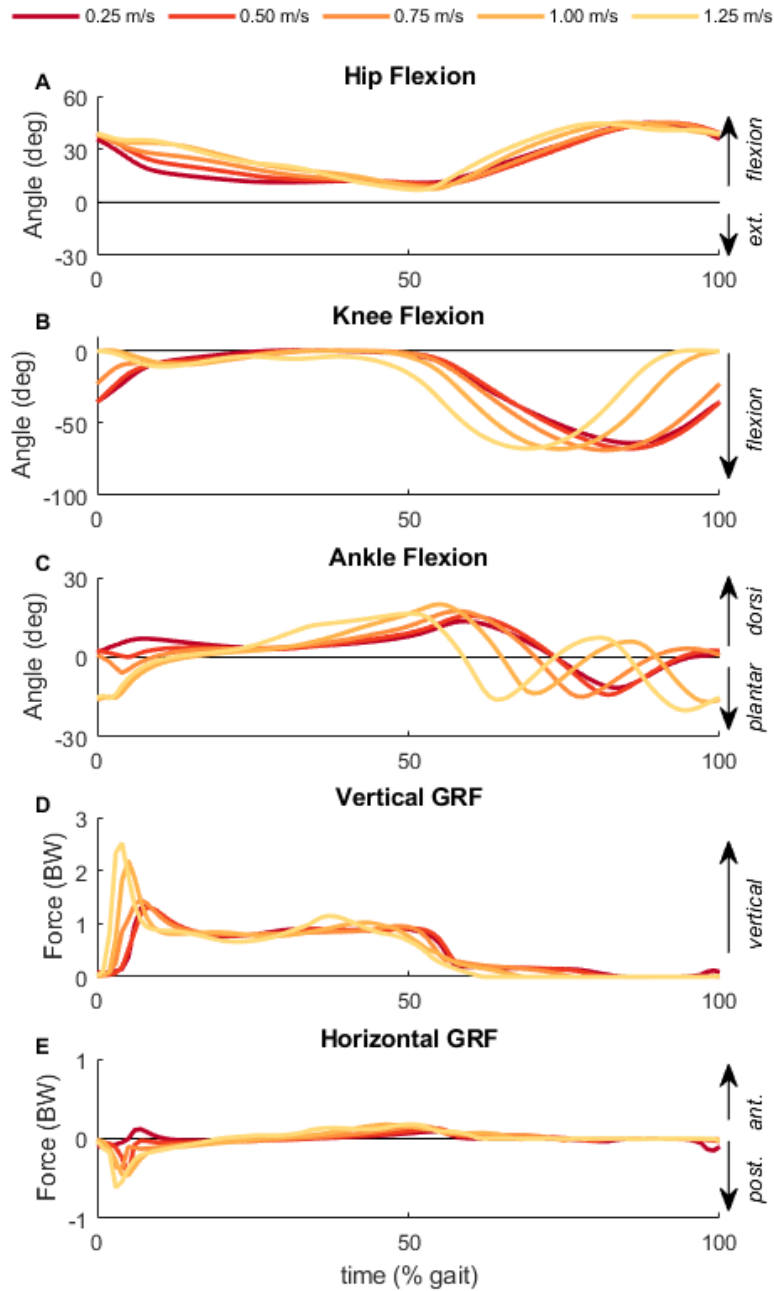
- 682 56. Umberger BR, Gerritsen KG, Martin PE. A model of human muscle energy expenditure. Computer  
683 methods in biomechanics and biomedical engineering. 2003 Apr;6(2):99–111.
- 684 57. Koelewijn AD, Heinrich D, van den Bogert AJ. Metabolic cost calculations of gait using  
685 musculoskeletal energy models, a comparison study. PLOS ONE. 2019 Sep 18;14(9):e0222037.
- 686 58. Bhargava LJ, Pandy MG, Anderson FC. A phenomenological model for estimating metabolic  
687 energy consumption in muscle contraction. Journal of Biomechanics. 2004;37(1):81–8.
- 688 59. Stoquart G, Detrembleur C, Lejeune T. Effect of speed on kinematic, kinetic, electromyographic  
689 and energetic reference values during treadmill walking. Neurophysiologie Clinique/Clinical  
690 Neurophysiology. 2008 Apr 1;38(2):105–16.
- 691 60. Nilsson J, Thorstensson A. Ground reaction forces at different speeds of human walking and  
692 running. Acta Physiologica Scandinavica. 1989 Jun;136(2):217–27.
- 693 61. Margaria R. Positive and negative work performances and their efficiencies in human locomotion.  
694 Internationale Zeitschrift für angewandte Physiologie, einschliesslich Arbeitsphysiologie. 1968  
695 May;25(4):339–51.
- 696 62. Umberger BR, Martin PE. Mechanical power and efficiency of level walking with different stride  
697 rates. The Journal of experimental biology. 2007 Sep;210(Pt 18):3255–65.
- 698 63. Zarrugh MY, Todd FN, Ralston HJ. Optimization of energy expenditure during level walking.  
699 European journal of applied physiology and occupational physiology. 1974;33(4):293–306.
- 700 64. Zarrugh MY, Radcliffe CW. Predicting metabolic cost of level walking. European journal of  
701 applied physiology and occupational physiology. 1978 Apr;38(3):215–23.



- 702 65. Sánchez N, Schweighofer N, Finley J. Different Biomechanical Variables Explain Within-Subjects  
703 Versus Between-Subjects Variance in Step Length Asymmetry Post-Stroke. *IEEE Transactions on*  
704 *Neural Systems and Rehabilitation Engineering*. 2021;29:1188–98.
- 705 66. Roerdink M, Beek PJ. Understanding inconsistent step-length asymmetries across hemiplegic stroke  
706 patients: impairments and compensatory gait. *Neurorehabil Neural Repair*. 2011 Apr;25(3):253–8.
- 707 67. Awad LN, Palmer JA, Pohlig RT, Binder-Macleod SA, Reisman DS. Walking speed and step length  
708 asymmetry modify the energy cost of walking after stroke. *Neurorehabil Neural Repair*. 2015  
709 Jun;29(5):416–23.
- 710 68. Chen G, Patten C, Kothari DH, Zajac FE. Gait differences between individuals with post-stroke  
711 hemiparesis and non-disabled controls at matched speeds. *Gait Posture*. 2005 Aug;22(1):51–6.
- 712 69. Stoquart G, Detrembleur C, Lejeune TM. The reasons why stroke patients expend so much energy  
713 to walk slowly. *Gait Posture*. 2012 Jul;36(3):409–13.
- 714 70. Dons B, Bollerup K, Bonde-Petersen F, Hancke S. The effect of weight-lifting exercise related to  
715 muscle fiber composition and muscle cross-sectional area in humans. *Eur J Appl Physiol Occup*  
716 *Physiol*. 1979 Jan 10;40(2):95–106.
- 717 71. Miller RH, Hamill J. Optimal footfall patterns for cost minimization in running. *Journal of*  
718 *Biomechanics*. 2015 Aug 20;48(11):2858–64.
- 719

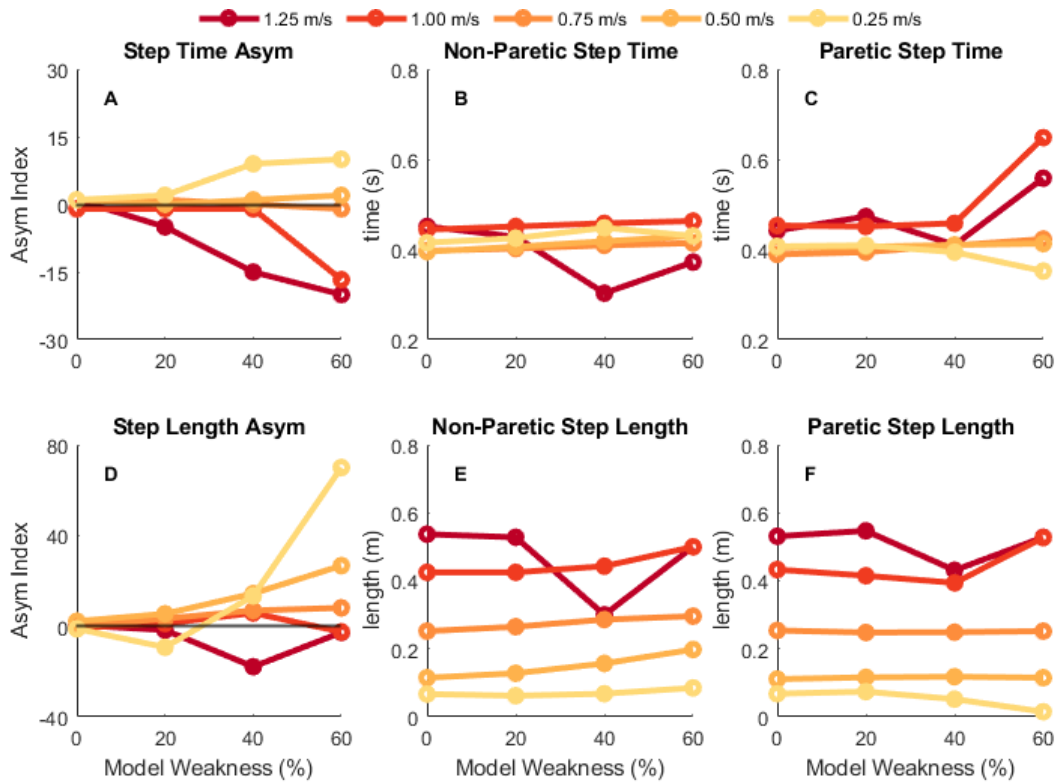


**Fig 1: Summary of Set Up.** A) Two-dimensional musculoskeletal model with 11 mechanical degrees-of-freedom and 24 muscle-tendon units, B) Maximum isometric muscle forces relative to base model for the 20%, 40% and 60% weak models, muscle strength was uniformly reduced on the left limb, C) Overview of optimization process in Moco adapted from (37), where the objective function (Eqs 1, 5, or 6 respectively from methods) and constraints (Eqs 2 and 3 from methods) for the problem are sent to the solver, with specified settings. Note that the objective function value seen here is for the nominal optimizations, see the methods for further details.



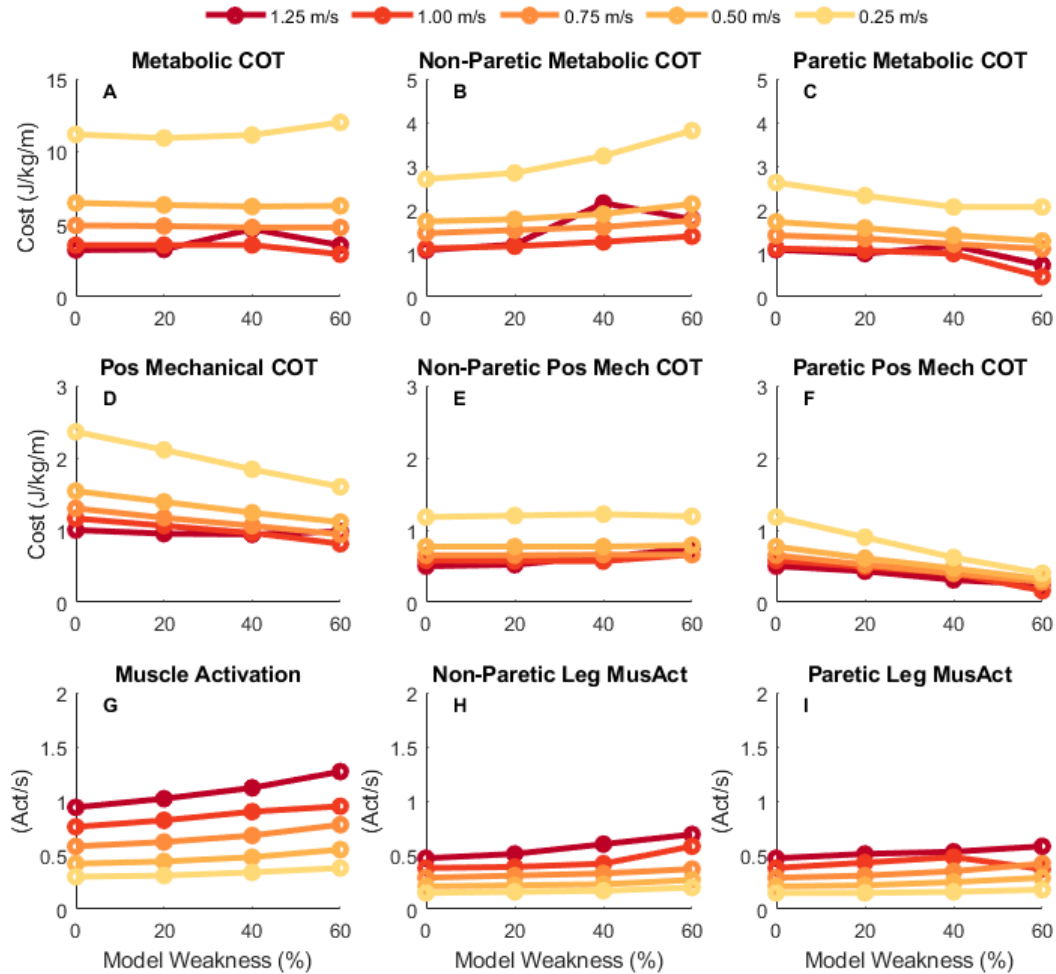
**Fig 2: Baseline Kinematic and Ground Reaction Forces.** The (A) hip flexion, (B) knee flexion, (C) ankle flexion angles and (D) vertical GRF and (E) horizontal GRF for the base model across each of the five gait speeds. Time is normalized to the gait cycle with 0% being the time of right heel strike.

722

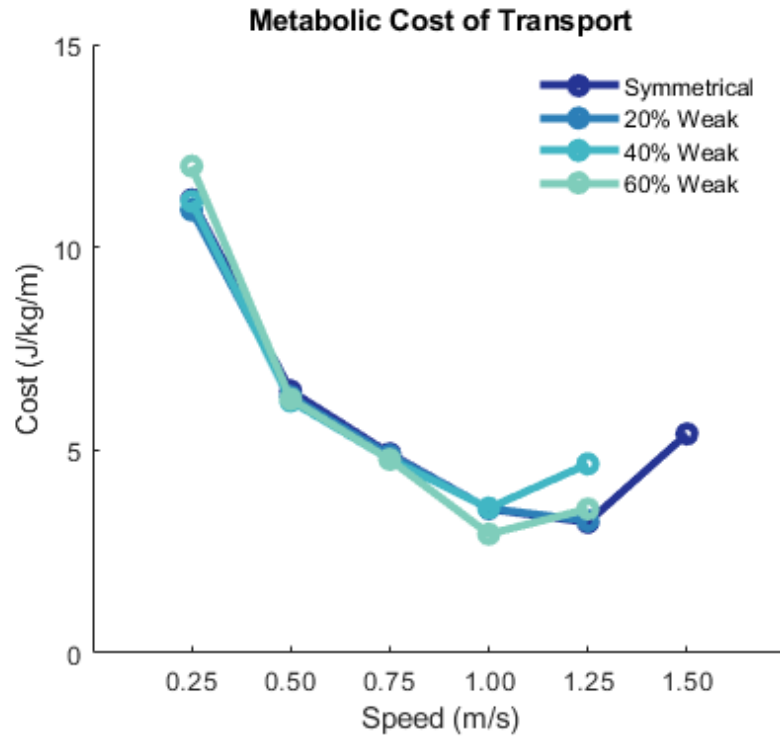


**Fig 3: Step time and step length results.** Spatiotemporal results for step time (top row) and step length (bottom row) for each condition and each speed. The leftmost column is the step time or step length asymmetry index (A, D), middle column is the non-paretic (right) step time or step length value (B, E), and rightmost column is the paretic (left) step time or step length value (C, F). Darker, red colors depict results for faster speeds while lighter, yellow colors depict results for slower speeds. Positive asymmetry indices indicate that the non-paretic value is greater than the paretic value, whereas negative asymmetry indices indicate that the paretic value is greater than the non-paretic value.

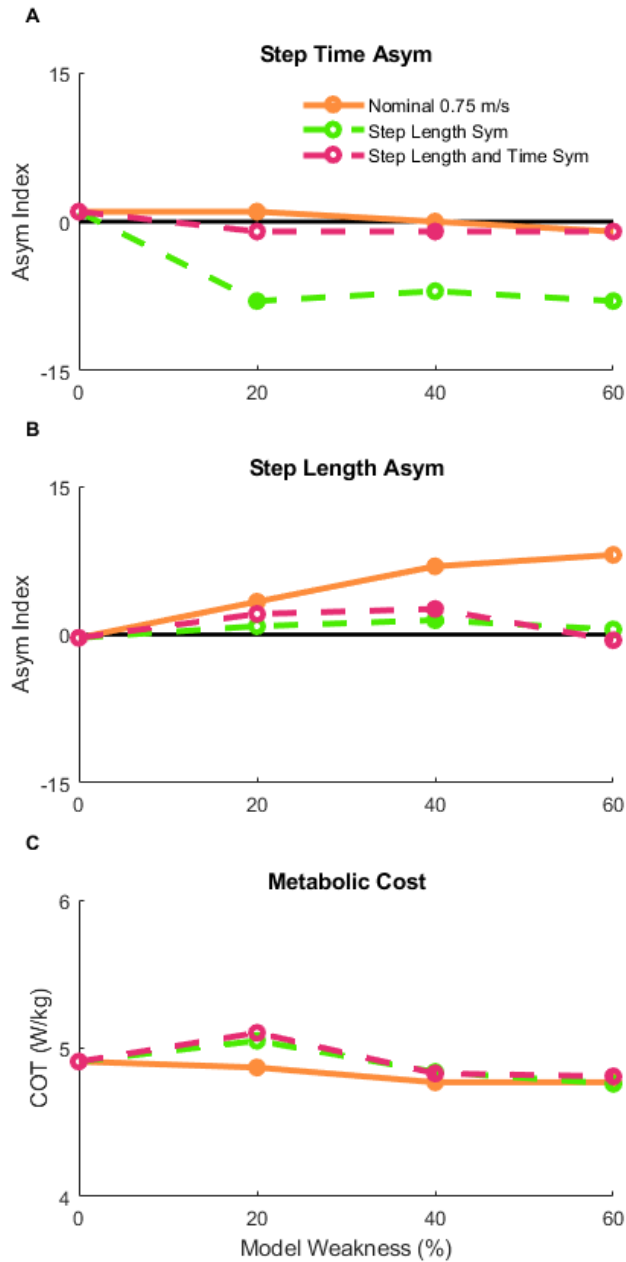
723



**Fig 4: Metabolic and Mechanical Cost of Transport.** Metabolic cost of transport (COT; top row), positive mechanical COT (middle row), and sum of the integrated muscle activation (bottom row) for each condition. The leftmost column shows the result for the sum across both limbs, while the middle column shows the results for the non-paretic (right) limb and the rightmost column shows the result for the paretic (left) limb. Darker, red colors are for faster speeds while lighter, yellow colors are for slower speeds. Metabolic COT remained relatively consistent within a speed across the different weakness models, partially a result of an increase in mechanical work done by the right (non-paretic) limb with a proportional decrease in mechanical work done by the left (paretic) limb.



**Fig 5: Metabolic cost of transport across speeds.** Dark blue represents results for the symmetrical (or base) model, while blue-green lighter colors represent the asymmetrical models. The symmetrical condition included an optimization at 1.50 m/s so that we could establish the U-shaped curve of metabolic cost (where 1.25 is approximately the metabolic-optimal speed for the symmetrical model). We did not solve the optimal control problem for other models at 1.50 m/s because the weakness in the 40% and 60% models prevented convergence at speeds faster than 1.25 m/s.



**Fig 6: Secondary Analysis of Spatiotemporal asymmetry.** After the nominal conditions (orange solid line), additional optimizations were performed across the three hemiparetic models with the goal of reducing the step length asymmetry (green dashed line) to assess the effect of enforced step length (B) symmetry on step time asymmetry (A) and metabolic cost (C). Finally, a set of optimizations were performed with the goal of reducing both step length and step time asymmetry (purple dashed line).