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2 Clustering Deviation Index (CDI): A robust and  
3 accurate unsupervised measure for evaluating  
4 scRNA-seq data clustering

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16 January 3, 2022

## Abstract

Single-cell RNA-sequencing (scRNA-seq) technology allows us to explore cellular heterogeneity in the transcriptome. Because most scRNA-seq data analyses begin with cell clustering, its accuracy considerably impacts the validity of downstream analyses. Although many clustering methods have been developed, few tools are available to evaluate the clustering “goodness-of-fit” to the scRNA-seq data. In this paper, we propose a new Clustering Deviation Index (CDI) that measures the deviation of any clustering label set from the observed single-cell data. We conduct *in silico* and experimental scRNA-seq studies to show that CDI can select the optimal clustering label set. Particularly, CDI also informs the optimal tuning parameters for any given clustering method and the correct number of cluster components.

## 1 Introduction

Single-cell RNA-sequencing (scRNA-seq) quantifies the transcriptome of individual cells, allowing us to explore the biological heterogeneity among cells (Shapiro et al. 2013). Thus, scRNA-seq analysis usually begins with cell type clustering. Over the past five years, many methods have been developed or re-purposed for scRNA-seq clusterings, such as K-means, hierarchical clustering, RaceID (Grün et al. 2015), CIDR (Lin et al. 2017), SIMLR (Wang et al. 2017), SCANPY (Louvain algorithm) (Wolf et al. 2018), and Seurat (Louvain algorithm) (Stuart et al. 2019).

The outputs of these clustering methods are cell label sets that assign each cell to a cluster. Different methods usually yield different label sets. Even if we use a given clustering method, we still obtain different label sets by setting different tuning parameters. These different label sets lead to the challenge of choosing an “optimal” label set. To address the challenge, one approach is to apply a consensus method on these label sets to derive an ensemble label set. However, the ensemble label set is not guaranteed to reflect the underlying cell type structure better than any input label set. Furthermore, different consensus methods (Yang et al. 2019; Kiselev et al. 2017) often generate different ensemble label sets; thus, the challenge of choosing the optimal label set remains. Therefore, we need a reasonable evaluating index to score the “goodness-of-fit” or the deviation of each label set to the data. The evaluating index will help us select the optimal label set among the candidates.

In general, the evaluating indices can be divided into two categories. The first category

48 consists of unsupervised indices. Calculating unsupervised indices does not depend on the  
49 knowledge of the actual label set; instead, the unsupervised indices usually use geometric or  
50 statistical properties to evaluate the quality of a label set. For example, the Calinski-Harabaz  
51 index (Caliński and Harabasz 1974) selects the label set with the minimum ratio between the  
52 within-cluster and between-cluster variances; similarly, the Silhouette coefficient (Rousseeuw  
53 1987) and the Davies-Bouldin index (Davies and Bouldin 1979) selects the label set with the  
54 minimum ratio between the within-cluster and between-cluster distances. However, in the  
55 context of scRNA-seq clustering, we found that these methods often selected different label  
56 sets. None of their selected label sets matches the benchmark label set (Fig. 5).

57 The second category consists of supervised indices, whose accuracy is determined by the  
58 benchmark label set. Examples of supervised indices include the Adjusted Rand Index (ARI)  
59 (Hubert and Arabie 1985), the Normalized Mutual Information (Vinh et al. 2010), Fowlkes-  
60 Mallows scores (Fowlkes and Mallows 1983), and the weighted Rand Index and Mutual Infor-  
61 mation (Z Wu and H Wu 2020). One of the most commonly used supervised indices is ARI, a  
62 corrected-for-chance version of the Rand index that measures the agreement between two label  
63 sets: if they are similar, ARI is close to 1; otherwise close to 0. Because calculating supervised  
64 indices relies on the benchmark label set, the supervised indices are more accurate than unsu-  
65 pervised indices when the benchmark label set is accurate or close to the truth. Unfortunately,  
66 the benchmark label set is usually unavailable or challenging to generate. Moreover, even if it  
67 is available, it could be biased or incorrect because of outdated domain knowledge, leading to  
68 poor performances of supervised indices.

69 In this study, we developed a new unsupervised index, Clustering Deviation Index (CDI),  
70 to quantify the deviation of single cell data from the the data distribution based on the given  
71 label set. CDI is an unsupervised evaluation index whose calculation does not rely on the  
72 actual unobserved label set. However, its performance on scRNA-seq is consistent with ARI  
73 (Fig. 5). We applied CDI to multiple experimental scRNA-seq data sets and demonstrated  
74 that it successfully selected biologically meaningful clustering labels in each case. Because  
75 CDI is unsupervised, it is much more broadly applicable than supervised indices.

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## 2 Results

	Dataset	# cells	# genes	# cell types (subtypes)	Protocols	Reference
Real datasets	CT26.WT	9,621	11,710	1(-)	10X v3	-
	T-CELL	2,989	7,893	5(-)	10X	Christian et al. 2021
	CORTEX	7,390	12,887	8(33)	inDrop	Hrvatin et al. 2018
	RETINA	26,830	13,118	6(18)	Drop-seq	Shekhar et al. 2016
Simulated datasets	SD1	4,000	10,000	10(-)	-	-
	SD2	4,200	10,000	4(-)	-	-
	SD3	2,800	10,000	2(4)	-	-
	SD4	3,000	4,887	5(-)	-	-

**Table 1.** Dataset summary. More details on the experimental and simulated datasets are provided in Sections 4.5 and 4.6.

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### 2.1 Characterizing UMI count distributions of monoclonal cells

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We built our model on a rigorous characterization of the statistical properties of the scRNA-seq data, CT26.WT, from a pure monoclonal population. We generated CT26.WT for a murine colon carcinoma cell line derived through monoclonal expansion to eliminate cell type heterogeneity and avoid experimental biases (Section Discussion). Then, we extracted the unique molecular identifier (UMI) count of each cell. UMI is barcoded for each transcript before amplification in many scRNA-seq protocols, leading to more accurate quantification of the transcript count (Klein et al. 2015; Zheng et al. 2017). Based on CT26.WT, we evaluated the “goodness-of-fit” of the following four families of gene-specific UMI count distributions to the actual gene-specific UMI count distributions. All four families consist of negative binomial (NB) distributions or zero-inflated NB distributions. The difference among these families lies in their dispersion and zero-inflation parameter modeling, and their mean parameter modeling is similar (Supplemental Note 1).

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- *Gene-common NB*: negative binomial (NB) distributions with gene-common dispersion parameters;

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- *Gene-common ZINB*: zero-inflated negative binomial (ZINB) distributions with gene-common dispersion parameters;

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- *Gene-specific NB*: NB distributions with gene-specific dispersion parameters;

- *Gene-specific ZINB*: ZINB distributions with gene-specific dispersion parameters.

We used the Pearson’s chi-squared test (Chernoff and Lehmann 1954) to evaluate the “goodness-of-fit” of the distributions in the four distribution families to the actual gene-specific UMI count distributions in CT26.WT (Supplemental Fig. S2). The test rejected 34.3% poorly fitted genes for the gene-common NB family and 34.5% for the gene-common ZINB family. The rejection rates are high, indicating that these distribution families do not fit the actual UMI count distributions. In contrast, when applied to the gene-specific NB and ZINB families, the “goodness-of-fit” tests only rejected 9.7% and 6.1% poorly fitted genes, respectively. The rejection rates are not far from the preset type I error rate 5%, suggesting an overall good fit of these models.

The test results suggest that the well-fitted distribution family should include the gene-specific dispersion parameters, but including the zero-inflation parameters might not be necessary. To demonstrate the latter point, we split CT26.WT into two datasets - a half for training and the other half for testing. We fitted the unknown parameters in these four distribution families in the training dataset, estimated the zero UMI count proportions in the test dataset, and then compared the estimated and the observed zero UMI count proportions. The results (Fig. 1A) show that the gene-common NB and ZINB families underestimated the zero UMI count proportions in CT26.WT. In contrast, the gene-specific NB and ZINB families yielded reasonable and similar estimates of the zero UMI count proportions. Thus, adding additional ZINB parameters does not further improve the fitting. Hence, for the remainder of this study, we used the gene-specific-dispersion NB model without zero inflation.

## 2.2 Characterizing UMI count distributions of polyclonal cells

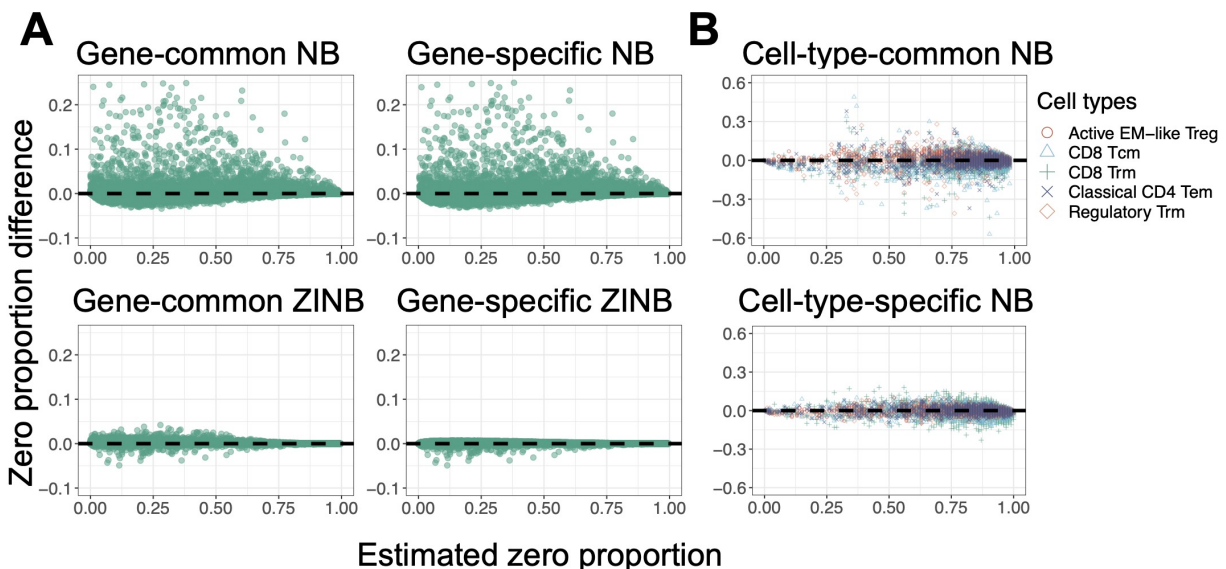
Polyclonal cell populations consist of cells from multiple cell types. To fit their UMI count distributions, we consider the following two NB distribution families. The difference between the two families lies in whether the mean and dispersion parameters are the same across cell types (Supplemental Note 1).

- *Cell-type-common NB*: NB distributions with the cell-type-common but gene-specific mean and dispersion parameters;
- *Cell-type-specific NB*: NB distributions with the cell-type-specific and gene-specific mean and dispersion parameters.

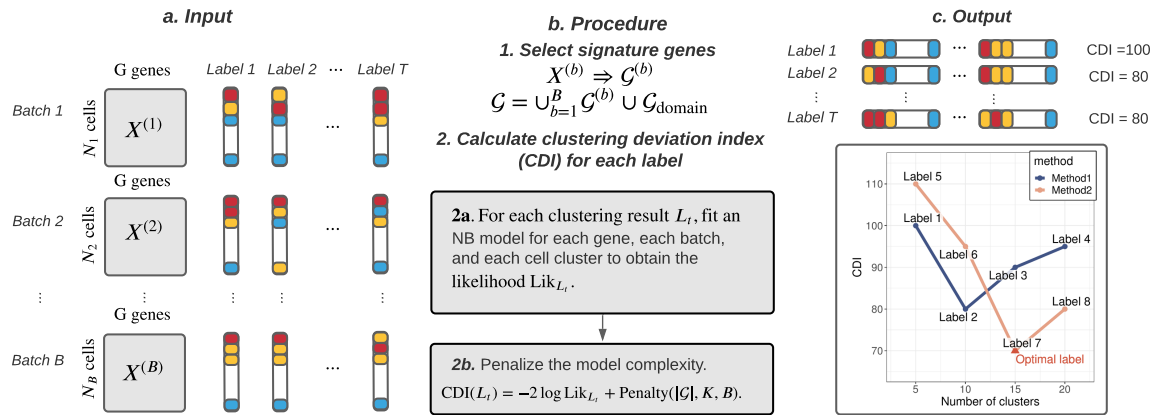
We used the cell-type-specific “goodness-of-fit” tests (Section 4.2) to check whether these

126 two families fit the observed UMI counts well in the T-CELL dataset (Table 1). The tests  
127 rejected 34.27% of the genes for the cell-type-common NB family and 2.27% for the cell-  
128 type-specific NB family. Thus, adding cell-type-specific parameters substantially improved  
129 the fitting to the UMI count distributions.

130 The cell-type-specific NB family consists of more free-changing parameters, so we need  
131 to show that the improved fit does not stem from overfitting. Therefore, we did a 50:50  
132 train/test split on the T-CELL dataset. We used the training dataset to fit the gene-specific  
133 distributions in each family, estimated the zero UMI count proportions in the test dataset, and  
134 then compared the estimated and the observed zero UMI count proportions in the test dataset.  
135 In the test dataset, we found that the cell-type-specific NB family still provides much better  
136 estimates to the zero UMI count proportions (Fig. 1B). We performed similar analyses on the  
137 CORTEX and RETINA datasets and observed similar results (Supplemental Fig. S5–S8).



**Figure 1.** Zero UMI count proportion fitting under six models in monoclonal and polyclonal datasets. In both (A) and (B), cells in each type were randomly divided into two datasets: half for training and half for testing. The x-axis is the estimated zero UMI count probabilities based on the training dataset, and the y-axis is the difference between observed proportions in the test dataset and the estimated zero probabilities based on the training dataset. (A) The monoclonal CT26.WT dataset ( $n = 9,621$  cells). Each point represents a gene. (B) The polyclonal T-CELL dataset ( $n = 2,989$  cells). Each point represents a gene in a specific cell type, while the point colors and shapes represent five benchmark cell types (Christian et al. 2021). The CD8 Tcm cell type in this study is defined as  $IL17RA^+CD28^+$ .



**Figure 2.** CDI calculation illustration. CDI inputs are  $T$  candidate label sets and the raw UMI count matrices from  $B$  batches. CDI contains two steps. CDI first selects the feature genes for each batch and takes their union. Other known feature genes can also be manually included. Then, CDI is calculated for all candidate label sets. CDI outputs are  $T$  CDI values corresponding to  $T$  candidate label sets. The label set with the lowest CDI is optimal.

### 2.3 CDI Overview

We developed CDI as an unsupervised index to evaluate the fitting of the observed UMI counts to the UMI count distribution based on the candidate label set. It calculates the negative penalized maximum log-likelihood of the selected feature genes based on the candidate label set. We have shown that the raw UMI counts follow gene-specific and cell-type-specific NB distributions given the actual cell-type labels. CDI is low if the candidate label set and the actual label set are similar; otherwise, CDI is high. The CDI calculation involves the following two steps (Fig. 2).

- 1. Feature gene selection.** Feature genes are those differentially expressed across cell types. Therefore, many scRNA-seq clustering methods rely on feature genes to cluster cells: selecting feature genes could substantially reduce data dimensions and possibly boost the signal in clustering. We also selected feature genes before calculating CDIs because of similar reasons. Many existing feature gene selection methods are available (Brennecke et al. 2013; Townes et al. 2019; Stuart et al. 2019). Here, we derive a new approach using a working dispersion score (WDS). WDS estimates the working dispersion for each gene. For single-batch datasets, we select genes with the largest average sample dispersion estimates as the feature genes. For multi-batch datasets, we rank genes in each batch by their average sample dispersion, combine rankings across batches by taking the

156 minimum, and select top-ranked genes as the feature genes. Compared with other feature  
157 selection methods, our approach is derived from the parametric NB mixture model,  
158 capturing the fold change of mean parameters across cell types (Section S2.2). This  
159 property allows WDS to improve the performance of CDI, as described in Section 2.4.1,  
160 and Section 4.3.

161 **2. Optimal clustering label set selection.** If the candidate label set is close to the  
162 actual cell label set, the UMI count of each feature gene follows a gene-specific and cell-  
163 type-specific NB distribution. We calculated CDI as the sum of the negative penalized  
164 maximum log-likelihood for all the feature genes. The penalties are based on either the  
165 Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC) (Akaike  
166 1974; Schwarz et al. 1978; Konishi and Kitagawa 2008) (see details in Section 4.4). The  
167 choice of AIC or BIC depends on users' scientific goals. Because BIC puts a higher  
168 penalty on model complexity, CDI with BIC (CDI-BIC) favors label sets with fewer  
169 clusters. Thus, we recommend using CDI-BIC to select the optimal label set on main  
170 cell types. Conversely, we recommend using CDI with AIC (CDI-AIC) to select the  
171 optimal subtype label set to depict the heterogeneity with a higher resolution.

## 172 **2.4 Performance Evaluation**

173 We evaluated the performance of WDS and CDI on four simulated datasets (SD1–SD4) and  
174 three experimental datasets (T-CELL, CORTEX, and RETINA). All datasets have the bench-  
175 mark label sets. For the simulated datasets, the benchmark label sets are the actual cell label  
176 sets. For the experimental scRNA-seq datasets, the benchmark label sets were obtained by the  
177 multi-step process, including fluorescence-activated cell sorting (FACS), known feature gene  
178 checking, cell screening, and clustering; thus, these benchmark label sets may not be accurate  
179 but reflect our best knowledge of the cell types (see Section 4.6 for details).

### 180 **2.4.1 Performance of WDS in selecting feature genes for CDI**

181 We compared WDS against another feature selection method, VST, the default for Seurat V3  
182 (Stuart et al. 2019) and V4 (Hao et al. 2021). First, we selected the top 500 feature genes using  
183 WDS and VST, respectively. Second, we normalized the UMI counts of the selected feature  
184 genes by the  $\log(\max(\text{count}, 0.1))$  transformation. Third, we calculated the top 50 principal  
185 components (PCs) of the normalized UMI counts. Finally, we plotted the two-dimensional



186 uniform manifold approximation and projection (UMAP) (Becht et al. 2018) based on the top  
187 50 PCs (Fig. 3).

- 188 • For all datasets except for SD3 and SD4, the UMAPs based on both the WDS-selected  
189 feature genes and the VST-selected feature genes separate different cell clusters well.
- 190 • For SD3, WDS selected 74/75 of the actual feature genes while VST only selected 48/75.  
191 Most of the feature genes missed by VST but not WDS are highly expressed in three  
192 similar subtypes but lowly expressed in the less similar main types. Consequently, the  
193 UMAP based on the WDS-selected feature genes reflects the cell type structure better  
194 than the VST-selected feature genes.
- 195 • For SD4, VST performed better than WDS. SD4 was generated from Splatter (Zappia  
196 et al. 2017), a scRNA-seq data simulator that imposes strong mean-dispersion trends on  
197 gene expressions – that is, highly expressed genes are forced to have a lower dispersion.  
198 Such trends are commonly seen in bulk RNA-seq data, but we did not observe them in  
199 the UMI counts of the scRNA-seq datasets (Supplemental Fig. S9). On datasets with  
200 such trends, WDS will select genes with lower average UMI counts. These genes contain  
201 little information on cell types; thus, the resulting UMAP cannot separate the cells from  
202 different cell types. Because splatter is a commonly used scRNA-seq data simulator, we  
203 included SD4 to check the robustness of the subsequent procedure of CDI. In practice,  
204 when such mean-dispersion trends exist for the UMI counts, we should not use WDS to  
205 select feature genes; however, when such mean-dispersion trends do not exist (as in all  
206 of our experimental datasets), WDS works well.

207 When WDS and VST select different feature gene sets, even if their resulting UMAPs  
208 separate cell types similarly well, CDI based on the two gene sets could select different label  
209 sets. For example, for T-CELL, both UMAPs look similar (Fig. 3E). However, CDI following  
210 VST selected the six-cluster label set generated by the spectral clustering (ARI=0.39); CDI  
211 following WDS selected the five-cluster label set generated by Seurat (ARI=0.87). For refer-  
212 ence, T-CELL’s benchmark label set has five clusters, similar to the five-cluster label selected  
213 by CDI following WDS (T-CELL panel in Fig. 4A, B). Thus, CDI following WDS is more  
214 robust and accurate.

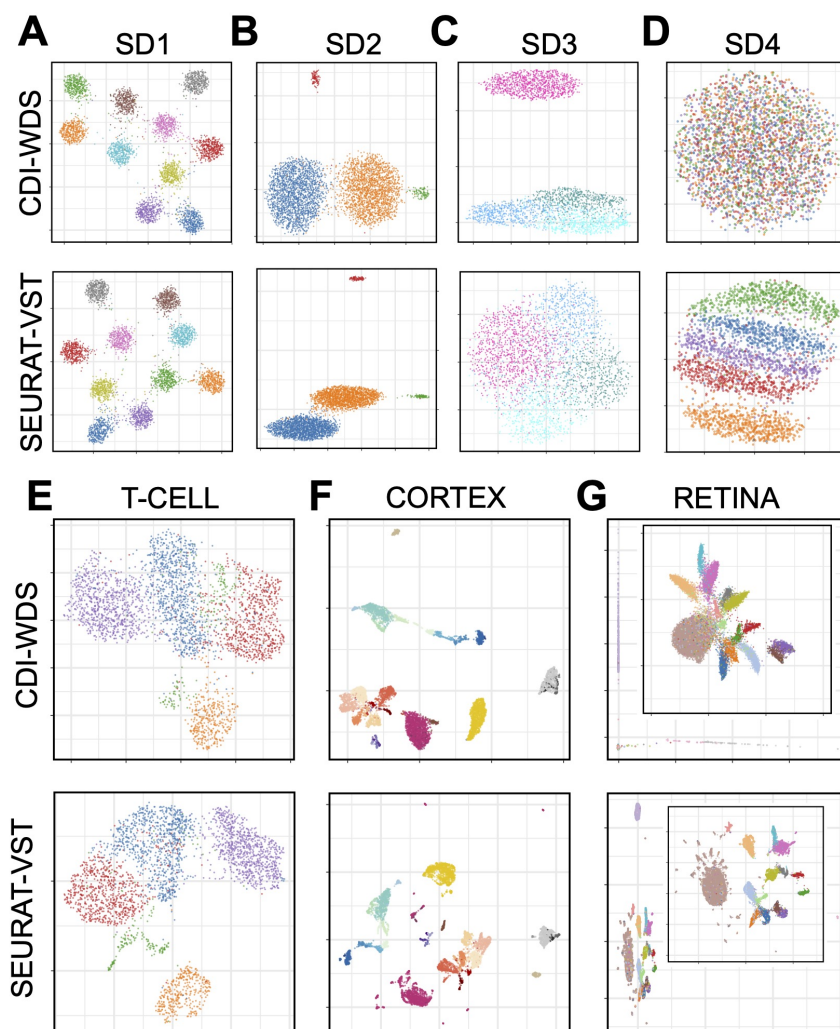
215 When WDS selects different numbers of feature genes, CDI based on these different feature  
216 gene sets has robust performance. For example, for T-CELL, based on the 200 WDS-selected  
217 feature genes, CDI selected a six-cluster label set generated by Seurat; the second-best was

218 the five-cluster label set generated by Seurat. Based on the 300 WDS-selected feature genes,  
219 CDI selected the five-cluster label set generated by Seurat. Finally, based on the 400 or  
220 500 WDS-selected feature genes, CDI selected the five-cluster label set generated by SC3.  
221 (Supplemental Fig. S14). These label sets were all similar to the benchmark label set (ARIs  
222 between 0.80 and 0.87). Thus, CDI's performance is robust to the number of WDS-selected  
223 feature genes.

## 224 **2.4.2 Performance of CDI in selecting the optimal label set**

225 We evaluated the performance of CDI using the candidate label sets generated by multiple  
226 clustering methods, each with a wide range of tuning parameters. The clustering methods  
227 we applied include hierarchical clustering, K-means clustering, spectral clustering, CIDR (Lin  
228 et al. 2017), Seurat V3 (Stuart et al. 2019), and an ensemble clustering method called SC3  
229 (Kiselev et al. 2017).

230 *A. Data containing no rare cell types.* We define a cell type as *rare* if its proportion is below  
231 3%. We evaluated the performance of CDI on the datasets where none of the cell types are  
232 rare. These datasets include SD1, SD4, and T-CELL. SD1 contains ten equally proportional  
233 cell types simulated from the NB model verified in Section 2.2; SD4 contains five unequally  
234 proportional cell types simulated from Splatter; T-CELL contains a mixture of five cells types  
235 of T cells. We selected their feature genes with either WDS (SD1, T-CELL) or VST (SD4) and  
236 then applied CDI-BIC to select the optimal label set marking their main cell types. CDI-BIC  
237 performed very well on all three datasets. It selected the label sets with the correct numbers of  
238 clusters (Fig. 4A); moreover, the selected label sets are very similar to the benchmark label sets  
239 (Fig. 4B). Of note, some other candidate label sets have the correct numbers of clusters, but  
240 their cell labels are very different from the benchmarks. For example, for T-CELL, two label  
241 sets with the lowest CDI are the five-cluster label sets generated by Seurat and SC3. These  
242 two label sets have similarly low CDIs ( $1.2744 \times 10^6$  for Seurat and  $1.2743 \times 10^6$  for SC3) and  
243 similarly high ARIs (0.875 for Seurat and 0.870 for SC3). However, the five-cluster label sets  
244 generated by other methods have much higher CDIs ( $1.3356 \times 10^6$ ,  $1.3095 \times 10^6$ ,  $1.2918 \times 10^6$ ,  
245  $1.2812 \times 10^6$ ) and much lower ARIs (0.002, 0.116, 0.132, and 0.388). Apparently, the label sets  
246 with lower CDIs have higher ARIs. The heatmaps to compare the candidate label sets with  
247 the benchmark label sets also verified that the label sets with lower CDIs are more similar to  
248 the benchmark label set (Supplemental Fig. S10). These results suggest that CDI has similar  
249 performance with ARI in selecting the optimal label set when the data contain no rare cell



**Figure 3.** Comparisons between WDS and VST. (A)–(D) UMAPs of the simulated datasets (SD1–SD4); (E)–(G) UMAPs of the experimental datasets (T-CELL, CORTEX, and RETINA). In all plots, cells from different cell types are marked with different colors.

250 types. Moreover, CDI has a significant advantage over ARI because its calculation does not  
251 rely on the knowledge of the benchmark label set.

252 *B. Data containing rare cell types.* We evaluated the ability of CDI-AIC and CDI-BIC to  
253 detect rare cell types. For example, SD2 simulated a cell population with two normal-sized  
254 (47.62% of all cells each) and two rare (2.38% of all cells each) cell types. Two normal-sized  
255 cell types and one rare cell types have different but similar feature gene UMI count distribu-  
256 tions. This rare cell type is called RC1; the other rare cell type is called RC2 (Fig. 3A SD2  
257 panel). For SD2, CDI-BIC selected a label set similar to the benchmark, suggesting that it  
258 can differentiate both rare cell types. Next, we reduced the cell proportion of RC1 further to  
259 challenge CDI with more difficult tasks. When the cell proportion of RC1 reduced to 2.03%  
260 (85/4185), CDI-BIC selected the three-cluster label set generated by spectral clustering; how-  
261 ever, CDI-AIC still selected the four-cluster label set including RC1 (Supplemental Fig. S13B,  
262 S13D). When the cell proportion of RC1 reduced to 0.49% (20/4120), neither CDI-AIC nor  
263 CDI-BIC distinguished RC1; instead, they both selected the three-cluster label set generated  
264 by Seurat. Although this label set misses RC1, it has a very high ARI (0.98) (Supplemen-  
265 tal Fig. S13G, S13I). Also, if we put the benchmark label set to the candidate pool, CDI-BIC  
266 would rank it as the fourth among all label sets, and CDI-AIC would select it as the optimal  
267 (Supplemental Fig. S13I). These results suggest that both CDI-BIC and CDI-AIC perform well  
268 in detecting rare cell types; compared with CDI-BIC, CDI-AIC is more sensitive in detecting  
269 rare cell types.

270 *C. Data with hierarchical cell type structures.* In scRNA-seq data, some main cell types  
271 can be further divided into subtypes. For example, SD3 simulated a cell population with two  
272 main cell types; one main cell type contains three subtypes, and the other is homogeneous.  
273 We used CDI-BIC to select the main type label set and CDI-AIC to select the subtype label  
274 set. As a result, CDI-BIC selected the two-cluster label set similar to the benchmark main  
275 type label set; CDI-AIC selected the four-cluster label set similar to the benchmark subtype  
276 label set (Fig. 4A SD3 panel, 4B SD3 panel). Another dataset, CORTEX, has eight main  
277 types and 33 subtypes. CDI-BIC selected a 20-cluster label set (Fig. 4A CORTEX panel).  
278 These 20 clusters correspond to the partitions of eight benchmark main types: clusters 4-10  
279 correspond to excitatory neurons, clusters 11 and 12 correspond to microglia cells, and clusters  
280 15-20 correspond to oligodendrocytes (Fig. 4B CORTEX panel). CDI-AIC selected a label set  
281 with 36 clusters (Fig. 4A CORTEX (AIC) panel). Some of these 36 clusters were partitions  
282 of the benchmark subtypes: they further partitioned Endothelial cells subtype 1, Astrocytes,

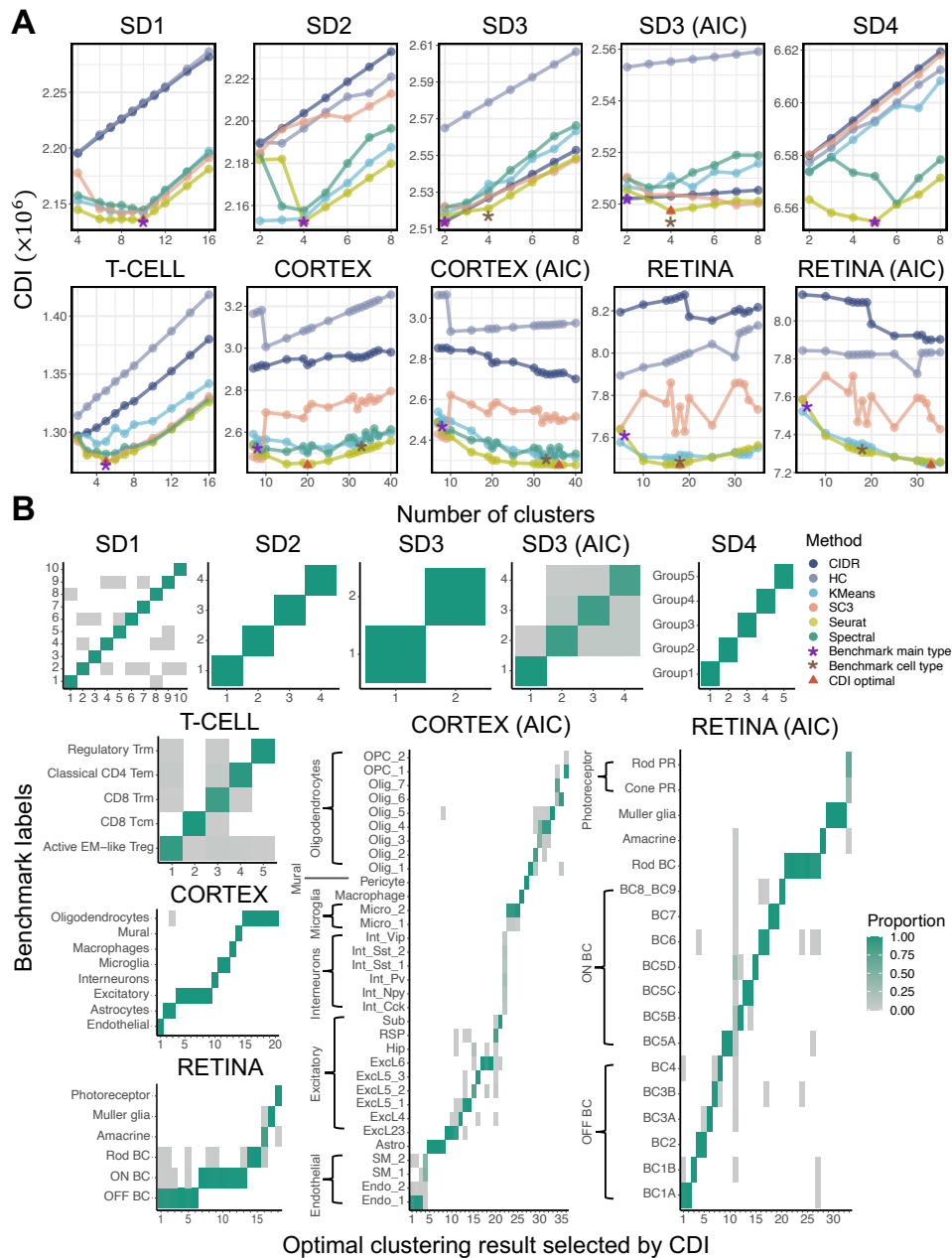
283 Excitatory cells subtype 23, Excitatory neuron 5\_1, Excitatory neuron 6, Oligodendrocytes  
284 subtype 5, and microglia subtype 2 into 21 clusters. Other clusters in the 36-cluster label set  
285 were mixtures of rare cell types, including a cluster mixing all interneuron subtypes (taking  
286 up 1.8% of all cells), a cluster mixing two Endothelial subtypes, and a cluster mixing two  
287 Microglia subtypes (Fig. 4B CORTEX (AIC) panel) In fact, not only the selected label set  
288 but also all other candidate label sets cannot separate these rare cell types. These results  
289 suggest that when the data have hierarchical structures, applying CDI-BIC in combination  
290 with CDI-AIC is an excellent strategy to reveal its hierarchical structure. When the subtypes  
291 contain too few cells, CDI-AIC may fail to identify the rare subtype but can still cluster them  
292 with other similar subtypes.

293 *D. Data from multiple batches.* RETINA had two batches with six main types. Among  
294 them, photoreceptors were further divided into rod photoreceptors (0.34%) and core photore-  
295 ceptors (0.18%. The ON cone bipolar cells (BCs) had seven subtypes; the OFF cone BCs had  
296 six. CDI-BIC selected an 18-cluster label set that classified the main types well. It further  
297 partitioned the rod BCs, ON cone BCs, and OFF BCs into several subtypes. On the other  
298 hand, CDI-AIC selected a label set with 33 clusters. This label set separated all the subtypes  
299 well. Besides, it provided a more exemplary partition on the benchmark Müller glia cells and  
300 the rod bipolar cells (RBCs). Some subtypes of ON cone BCs (BC5A, BC5C, BC6, BC7)  
301 and OFF cone BCs (BC1A, BC2) were also further partitioned into sub-clusters. Cluster 11  
302 spanned many cell types; however, they are mainly BCs. These results suggest that CDI works  
303 well on the multi-batch scRNA-seq data by incorporating hypothesis tests for significant batch  
304 effect and subsequent adaptive modeling (Section 4.1.2).

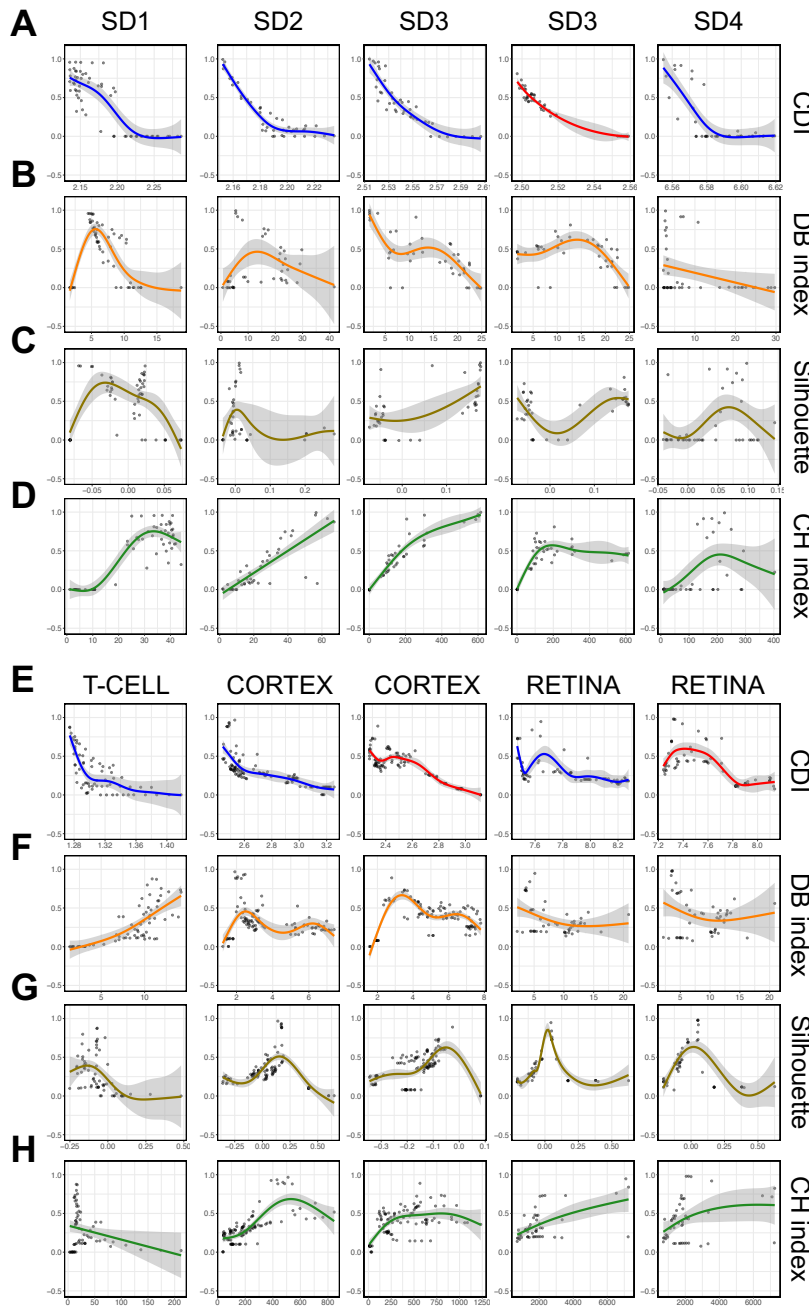
### 305 **2.4.3 Comparison of CDI with other unsupervised indices**

306 For general clustering problems, many other unsupervised indices are developed, including  
307 Davies-Bouldin index (Davies and Bouldin 1979), Silhouette coefficient (Rousseeuw 1987),  
308 and Calinski-Harabasz index (Caliński and Harabasz 1974). Although these methods are not  
309 customized for scRNA-seq data clustering, they have been applied in multiple publications to  
310 select clustering label sets (H Jiang et al. 2018; Peyvandipour et al. 2020; Liu et al. 2020).  
311 We compared the performance of these unsupervised indices and CDI by using the benchmark  
312 supervised index ARI. An unsupervised index performs well if it is a monotone function of  
313 ARI.

314 Because CDI-BIC aims to select main type label sets, we compared it with the ARI using



**Figure 4.** CDI's performance evaluation. CDI was applied to four simulated datasets (SD1-SD4) and three experimental datasets (T-CELL, CORTEX, RETINA). All datasets have benchmark label sets. If the subplot title contains AIC, CDI-AIC was applied; otherwise, CDI-BIC was applied. (A) CDIs of different candidate label sets. The x-axis labels the cluster number in the candidate label sets, and the y-axis labels their corresponding CDIs. For datasets with hierarchical structures (SD3, CORTEX, and RETINA), both CDI-BIC and CDI-AIC were applied. The line color represents the clustering method. The red triangle marks the CDI-selected label set with the corresponding clustering method, the number of clusters, and the CDI value. The purple star marks the benchmark main type label set. The brown star marks the benchmark subtype label set (if available). (B) Benchmark cell proportion heatmaps of different candidate label sets. The x-axis labels the clusters in the candidate labels set, and the y-axis labels the benchmark cell types. The color of each rectangle represents the benchmark cell type proportions in each candidate cluster. Each column adds up to 1. PR, photoreceptor.



**Figure 5.** Comparison between CDI and other unsupervised indices. In each panel, the x-axis is an unsupervised index; the y-axis is ARI; each dot represents a candidate label set as introduced in Fig. 4A. (A)–(D) are for simulated datasets, and (E)–(F) are for experimental datasets. (A) and (E) ARI vs. CDI (blue for CDI-BIC and red for CDI-AIC). (B) and (F) ARI vs. Davies-Bouldin index. (C) and (G) ARI vs. Silhouette coefficient. (D) and (H) ARI vs. Calinski-Harabasz index. The curves are fitted using these dots based on the generalized additive model (Hastie 2017), and their 95% confidence bands are shown using the grey regions around the curves.



315 the benchmark main type label sets. Similarly, we compared CDI-AIC with the ARI using  
316 the benchmark subtype label sets. Both CDI-AIC and CDI-BIC show clear monotone trends  
317 with ARI in all the datasets (Fig. 5). Therefore, CDI has matching performance with ARI,  
318 which requires the prerequisite knowledge of the benchmark label sets.

319 In contrast, other unsupervised indices did not exhibit any apparent monotone relationships  
320 with ARI. They sometimes assigned similar scores to two label sets with very different ARIs.  
321 For example, the Davies-Bouldin index assigned similar scores to many varying-quality label  
322 sets in SD1 and CORTEX (Fig. 5B, F). Same for the Silhouette coefficient in SD1, SD2, and  
323 T-CELL (Fig. 5C, G) and the Calinski-Harabaz index in T-CELL (Fig. 5H). Therefore, these  
324 general unsupervised indices are inferior to CDI in selecting the optimal clustering label set  
325 for scRNA-seq data.

### 326 **3 Discussion**

327 In this study, we develop a new index, CDI, to calculate the deviation between the candi-  
328 date label set and the observed UMI counts. For each candidate label set, CDI calculates the  
329 negative penalized maximum log-likelihood of the feature gene UMI counts. The likelihood  
330 function is calculated based on the gene-specific cell-type-specific NB distribution family veri-  
331 fied in both monoclonal and polyclonal scRNA-seq data. We recommend using WDS to select  
332 the feature genes for CDI because CDI following WDS has robust and satisfying performances.

333 Because calculating CDI relies on the gene-specific cell-type-specific NB distribution family,  
334 we would like to elaborate on two major innovations of our approach to ensure the distribution  
335 family is reliable.

336 First, to generate a monoclonal dataset, we cloned a single mother cell to derive a cell line  
337 whose components can be considered identical. This strategy is better than the existing artifi-  
338 cial External RNA Control Consortium (ERCC) or fluorescence-activated cell sorting (FACS)  
339 strategies for the following reason. For scRNA-seq data, variations in UMI counts could come  
340 from the variations in the efficiency of library construction, sequencing depths, cell cycles,  
341 and cell types. The first three variations cannot be avoided even in a monoclonal scRNA-  
342 seq dataset, while the last one is successfully eliminated. Monoclonal single cell datasets are  
343 essential to characterize the UMI count distributions in scRNA-seq data: with the well char-  
344 acterized distributions, we can use the model validation tools such as AIC or BIC to evaluate  
345 the deviation from the data to the model given the candidate label set. Previously, such



346 monoclonal datasets were generated by either piking in the ERCC RNA or purifying cells by  
347 FACS. The ERCC RNA samples contain the synthesized RNAs differing from the endogenous  
348 transcripts in many aspects (Zheng et al. 2017; L Jiang et al. 2011) (such as length, guanine-  
349 cytosine content, 5' cap, polyA length, and ribosome binding). These structural disparities  
350 lead to different conversion efficiencies of mRNA into cDNA. Thus, while ERCC eliminates  
351 the cell type variations, they also eliminate or distort the variations in library construction, se-  
352 quencing depths, and cell cycles. Another choice, FACS, can keep these variations; however, it  
353 only purifies cells based on a limited number of protein markers and therefore can only reduce  
354 but not eliminate the cell type heterogeneity in a cell population. Different from these two  
355 existing strategies, we used the single-cell expansion strategy to ensure an ideal monoclonal  
356 population: it keeps the variations in library construction, sequencing depths, and cell cycles;  
357 on the other hand, it also eliminates cell type heterogeneity.

358 Second, although we are not among the first to suggest that the UMI count distributions  
359 are not necessarily zero-inflated, we characterized the UMI count distributions with more  
360 specific gene-specific and cell-type-specific models. Previous studies model the UMI count  
361 distributions differently (Townes et al. 2019; Svensson 2020). Townes et al. (2019) mod-  
362 els the distributions of cellular gene UMI counts as an over-dispersed Dirichlet-multinomial  
363 distribution, which can be approximated by the independent NB models with gene-common  
364 dispersion parameters. Svensson (2020) proposes to use the NB models with gene-common  
365 and cell-type-common dispersion parameters. Neither evaluated or compared the proposed  
366 model with other candidate NB models. To derive a reliable NB distribution family, we used  
367 both monoclonal and polyclonal datasets to evaluate the fitting of distribution families to  
368 the UMI count distributions. Eventually, we found that the NB distribution family with the  
369 gene-specific cell-type-specific mean and dispersion parameters is the best.

370 Next, we would like to elaborate on the key features and limitations of CDI.

371 First, calculating CDI relies on the likelihood of the raw UMI counts. If only the normalized  
372 UMI counts are available, CDI cannot be applied.

373 Second, CDI is not a clustering method; instead, it is an index to evaluate the quality of  
374 the candidate label set: the label set with the lowest CDI will be selected. Thus, the quality  
375 of the selected label set highly depends on the quality of the candidate label sets. If none of  
376 the candidate labels fits the data well, the CDI-selected label set will not improve the fitting.  
377 Thus, providing a large pool of candidate label sets is crucial. We suggest using at least three  
378 methods, each with at least five labels with different cluster numbers.

379 Third, CDI-AIC puts fewer penalties on cluster numbers and usually selects the label with  
380 more clusters than CDI-BIC. These clusters often correspond to cell subtypes, and some could  
381 mark the cell transition stages. In many cases, the collected cells have different developmental  
382 stages so that the scRNA-seq data exhibit trajectory patterns. In those datasets, we can  
383 consider the cell types as “continuous”. When discrete clustering methods are applied to  
384 these datasets, the resulting clusters often represent a local stage on the trajectory. Although  
385 CDI is not designed to select the optimal trajectory, we can use it to select the label sets  
386 corresponding to the optimal local stages. For example, in RETINA, the subtypes of ON  
387 cone BCs and OFF cone BCs represent different development stages of those BCs. The  
388 CDI-AIC successfully selected a satisfying discrete label set that approximates the continuous  
389 trajectories of ON cone BCs and OFF cone BCs (Fig, 4B. RETINA (AIC)).

390 In summary, finding an optimal clustering label set for scRNA-seq data is critical because  
391 clustering impacts all downstream analyses; CDI provides a robust and accurate unsupervised  
392 method to select cluster label sets and hence contributes to the reliability of downstream  
393 scRNA-seq analysis.

## 394 4 Methods

395 We develop CDI in two stages. In stage I, we characterize the unique molecular identifiers  
396 (UMIs) count distributions using the experimentally generated monoclonal and polyclonal cell  
397 populations. In stage II, based on the UMI count distributions in stage I, we develop the CDI  
398 and select the label set with the lowest value as the optimal label set.

### 399 4.1 Models to characterize the UMI counts

#### 400 4.1.1 Single-batch UMI count distributions

401 In a single batch scRNA-seq dataset, denote the UMI count for gene  $g$  and cell  $c$  by  $X_{gc}$ ,  $g \in$   
402  $\{1, \dots, G\}$ , and  $c \in \{1, \dots, N\}$ . Suppose the *true* label of the  $N$  cells is  $\mathbf{L}_0 = (L_{0,1}, L_{0,2}, \dots, L_{0,N})'$   
403 with  $L_{0,c} \in \{1, 2, \dots, K_0\}$ . Here  $K_0$  is the number of the underlying true cell types.

404 Based on our experiments and modeling, assume

$$405 X_{gc} \mid (L_{0,c} = k) \sim \text{NB}(s_c \mu_{gk}, \phi_{gk}), \quad (1)$$

406 where  $s_c$  is a scale factor to adjust for the imbalance of the cellular total UMI counts in

407 the scRNA-seq data (Supplementary Note 1), and  $\mu_{gk}$  and  $\phi_{gk}$  are the mean and dispersion  
408 parameters of gene  $g$  in cell type  $k$ . The probability mass function of negative binomial  
409 distribution is shown in Supplementary Note 1.

410 The cellular scale factor  $s_c$  only impacts the mean but not the dispersion of the NB dis-  
411 tribution. From a Bayesian’s perspective, we can view  $X_{gc}$  as a Poisson-gamma distribution  
412 where

$$413 \quad X_{gc} \mid (L_{0,c} = k, \lambda_{gk}) \sim \text{Poisson}(s_c \lambda_{gk}), \quad \lambda_{gk} \sim \text{Gamma}(r_{gk}, \theta_{gk}).$$

414 Then, (1) holds with  $\mu_{gk} = r_{gk} \theta_{gk}$  and  $\phi_{gk} = 1/r_{gk}$ .

#### 415 4.1.2 Multi-batch UMI count distribution

416 The UMI count distributions may differ across batches. When multiple batches exist, we  
417 model the UMI count distribution in batch  $b$  as

$$418 \quad X_{gc}^{(b)} \mid (L_{0,c}^{(b)} = k) \sim \text{NB}(s_c^{(b)} \mu_{gk}^{(b)}, \phi_{gk}^{(b)}). \quad (2)$$

419 We assume batch effect lies in a low-dimensional space. Thus, for some genes or some cell  
420 types, introducing batch-specific parameters might not be necessary.

421 To decide whether to introduce batch-specific parameters for a gene in a cell type, we first  
422 test the hypothesis:

$$423 \quad H_{0,gk} : \forall b \in \{1, \dots, B\}, \mu_{gk}^{(b)} = \mu_{gk}, \phi_{gk}^{(b)} = \phi_{gk}. \quad (3)$$

424 We used the likelihood ratio test to test this hypothesis. If the hypothesis is rejected, it  
425 indicates that the batch effect is significant for the gene in this cell type; thus we will introduce  
426 the batch-specific parameters like in (2) to model the UMI count of this gene in this cell type.  
427 Otherwise, even if the batch effect exists, it is not significant; thus we will not use the batch-  
428 common model (1) to characterize the UMI count distribution of this gene in this cell type.

## 429 4.2 Cell-type-specific “goodness-of-fit” tests

430 We filtered the genes and the cells as described in Section 4.6. We derived a “goodness-of-fit”  
431 test for each gene. First, we assigned the cells into  $5K_0$  bins, where 5 is the number of UMI  
432 count categories and  $K_0$  is the number of the benchmark cell type. Based on the values of

433  $(X_{gc}, L_c)$ , we assigned cell  $c$  into one of the following bins:

434 
$$\mathcal{U} \times \mathcal{K}, \quad \mathcal{U} = \{\{0\}, \{1\}, \{2\}, \{3\}, [4, \infty)\}, \quad \mathcal{K} = \{1, \dots, K\},$$

435 where  $\times$  is the Cartesian product of two sets. Second, we computed the test statistic as

436 
$$T_g = \sum_{(U,k) \in \mathcal{U} \times \mathcal{K}} \frac{(n_{U,k} - N\hat{\pi}_{U,k})^2}{N\hat{\pi}_{U,k}}.$$

437 Here,  $n_{U,k} = \sum_c I(X_{gc} \in \mathcal{U}, L_c = k)$ , and  $\pi_{U,k} = P(X_{gc} \in \mathcal{U}, L_c = k)$ , which are the parameters of the multinomial distributions on  $(n_{U,k} : U \in \mathcal{U}, k \in \mathcal{K})$ . Because  $\pi_{U,k}$  is unknown, we  
438 estimated  $\pi_{U,k}$  by first expressing it as a function of  $(\mu_{gk}, \phi_{gk})$  in the corresponding NB distribution family and then derived the maximum likelihood estimator (MLE) in the multinomial  
439 likelihood (Chernoff and Lehmann 1954). Third, if  $T_g$  is larger than the 95% quantile of the  
440 chi-square distribution with the degree of freedom  $5K_0 - p - 1$ , we rejected the “goodness-of-fit” hypothesis for gene  $g$ . Here,  $p$  is the number of parameters in the corresponding NB  
441 distributions: for cell-type-common NB distributions,  $p = 2$ ; for cell-type-specific NB distributions,  $p = 2K_0$ . We used the chi-square quantile as the threshold because when the UMI  
442 count of gene  $g$  follows the corresponding NB model and  $L_c$  all match the true cell types,  $T_g$   
443 asymptotically follows  $\chi^2(5K_0 - p - 1)$  (Chernoff and Lehmann 1954). Finally, we performed  
444 the test for all the genes and calculated the rejection proportion. We used the proportion as  
445 the criterion to assess the overall fitting of the corresponding NB distribution family and the  
446 UMI count distribution.  
447  
448  
449  
450

### 451 4.3 Working dispersion score (WDS) to select feature genes

452 WDS is a score to measure the average sample (cell) dispersion of each gene.

453 
$$\hat{\xi}_g = \frac{\hat{\sigma}_g^2 - \hat{\mu}_g}{\hat{\mu}_g^2}, \quad (4)$$

454 where  $\hat{\mu}_g$  and  $\hat{\sigma}_g^2$  are the sample mean and variance of gene  $g$ 's UMI counts in the pooled data.  
455 It can be viewed as the estimator of  $\xi_g = (\sigma_g^2 - \mu_g)/\mu_g^2$ , where  $\mu_g$  and  $\sigma_g^2$  are the mean and  
456 variance of gene  $g$ ' UMI counts. The rationale of WDS is provided in the supplementary note  
457 2.

458 Before selecting the feature genes, we filtered the genes with more than 95% zero counts.  
459 We computed WDS for each gene and selected the top  $G_1$  (default  $G_1 = 500$ ) genes with the

460 highest WDS. For datasets with multiple batches, we calculated and ranked WDS for genes  
 461 in each batch: the minimum rank across the batches is set as the overall rank for each gene.  
 462 In the end, we selected the top  $G_1$  (default  $G_1 = 500$ ) genes with the highest overall rank as  
 463 the feature genes.

464 In our software, feature genes can also be added manually. For example, the users can use  
 465 the feature genes provided by other approaches or domain knowledge.

#### 466 4.4 CDI optimal label selection

467 Denote a label set with  $K$  cell types by  $\mathbf{L} = (L_1, \dots, L_N)'$  with  $L_c \in \{1, \dots, K\}$ . This label  
 468 set is not necessarily the true label set. It could be derived from any clustering method.

469 Let  $\mathcal{S}_b$  be the set of cells that belong to batch  $b$ . Let  $\mathcal{K} = \{1, \dots, K\}$ , and  $\mathcal{B} = \{1, \dots, B\}$ .

470 Based on model 2, if label sets are accurate, then the likelihood function of all genes and  
 471 all cells are

$$472 \ell(\mu_{gk}^{(b)}, \phi_{gk}^{(b)} : g \in \mathcal{G}, k \in \mathcal{K}, b \in \mathcal{B}) = \sum_{g \in \mathcal{G}} \sum_{b \in \mathcal{B}} \sum_{c \in \mathcal{S}_b} \log\{F(X_{gc} | s_c \mu_{gL_c}^{(b)}, \phi_{gL_c}^{(b)})\}, \quad (5)$$

473 where  $F$  is the probability mass function of negative binomial distribution (Supplementary  
 474 Note 1). Across  $B$  batches, for each feature gene  $g$  and cell type  $k$ , we use the score tests  
 475 to check if batch-specific mean and dispersion parameters need to be introduced. If  $H_{0,gk}$  is  
 476 accepted, we will use a batch-common but cell-type-specific mean and dispersion parameters  
 477 for this feature gene, *i.e.*,

$$478 \mu_{gk}^{(1)} = \dots = \mu_{gk}^{(B)}, \quad \phi_{gk}^{(1)} = \dots = \phi_{gk}^{(B)}.$$

479 If  $H_{0,gk}$  is rejected, we will introduce batch specific and cell-type-specific mean and dispersion  
 480 parameters.

481 Next, we obtain the MLE  $(\hat{\mu}_{gk}^{(b)}, \hat{\phi}_{gk}^{(b)} : g \in \mathcal{G}, k \in \mathcal{K}, b \in \mathcal{B})$  based on (5). The corresponding  
 482 maximum likelihood is  $\hat{\ell} = \ell(\hat{\mu}_{gk}^{(b)}, \hat{\phi}_{gk}^{(b)} : g \in \mathcal{G}, k \in \mathcal{K}, b \in \mathcal{B})$ .

483 To adjust for the model complexity, we use the penalized negative log-likelihood function  
 484 as CDI

$$485 \tilde{\ell} = -2\hat{\ell} + c_{\text{pen}} \cdot d,$$

486 where  $c_{\text{pen}}$  is the scale of penalty, and  $d$  is the overall degree of freedom of the model. For  
 487 AIC,  $c_{\text{pen}} = 2$ ; for BIC,  $c_{\text{pen}} = \log(N)$ . The overall degree of freedom  $d = \sum_{g \in \mathcal{G}} \sum_{k \in \mathcal{K}} d_{gk}$ . If

488  $H_{0, gk}$  is rejected,  $d_{gk} = 2B$ ; otherwise,  $d_{gk} = 2$ .

489 In practice, when  $T$  label sets  $\{\mathbf{L}_t : t \in \{1, \dots, T\}\}$  are given, we calculate the CDI  $\tilde{\ell}(\mathbf{L}_t)$   
490 for each label set  $\mathbf{L}_t$ . If  $\mathbf{L}_t$  is accurate,  $\tilde{\ell}(\mathbf{L}_t)$  tends to be small. The optimal label set is chosen  
491 by  $\hat{\mathbf{L}} = \arg \min_t \tilde{\ell}(\mathbf{L}_t)$ .

492 Because BIC adds more penalty on the model degree of freedom, it prefers the models  
493 with fewer numbers of clusters. Thus, we use CDI-BIC to select the main type label set and  
494 CDI-AIC to select the subtype label set.

## 495 4.5 Simulation setting

496 We simulated three sets of single-cell data (SD1-SD3) from the Negative Binomial distribution  
497 with gene-specific and cell-type-specific parameters. More specifically for SD1-SD3, the gene  
498 expression level for cells in cell type  $k$  and gene  $g$  were randomly sampled from  $\text{NB}(\mu_{gk}, \phi_{gk})$ ,  
499 where  $\mu_{gk}$  represented the mean parameter and  $\phi_{gk}$  represented the dispersion parameter.  
500 Each dataset contained 10,000 genes, and the number of cells ranged from 2,800 to 4,200. To  
501 test the robustness of CDI, we generated SD4, which contained many outliers, and the UMI  
502 count distributions no longer followed the verified NB distribution.

503 **SD1.** We generated ten equal-sized cell groups. Each group contained 400 cells; thus, in total  
504 there are 4,000 cells. One cell type was treated as the baseline type with  $\mu_{gk}$  generated  
505 from the truncated normal distribution with mean 0.2 and standard deviation 0.1, and  
506  $\phi_{gk}$  generated from the truncated normal distribution with mean 0.5 and standard deviation  
507 0.1. For each of the other nine groups, 25 genes had mean parameters shifted from  
508 the baseline group with log2 fold change 2.4. The feature gene dispersion parameters  
509 were shifted by a Gaussian-distributed factor with mean 0 and standard deviation 0.05.

510 **SD2.** We generated two normal-sized cell types with 2,000 cells each and two rare cell types  
511 with 100 cells each. One normal-sized cell type was treated as the baseline group with  $\mu_{gk}$   
512 generated from the truncated normal distribution with mean 0.2 and standard deviation  
513 0.1. The other normal-sized cell type contained 40 feature genes with log2 fold change of  
514 mean 1.5. One rare cell type, RC1, contained 50 feature genes with log2 fold change of  
515 mean 2.8; the other rare cell type, RC2, contained 50 feature genes with log2 fold change  
516 of mean 3.2. Because the log2 fold change of RC1 is smaller than RC2, RC1 is considered  
517 to be more similar to the two normal-sized cell types. The dispersion parameters  $\phi_{gk}$   
518 were set in the same way as in SD1.

519 **SD3.** We generated two main-types: C1 contains 1,000 cells from a homogeneous cell type,  
520 and C2 contains 1,800 cells from 3 subtypes. C1 is the baseline group with  $\mu_{gk}$  generated  
521 from the truncated normal distribution with mean 0.4 and standard deviation 0.1 and  $\phi_{gk}$   
522 generated from the truncated normal distribution with mean 1 and standard deviation  
523 0.1. Each subtype of C2 contains 600 cells and 40 feature genes. Among these 40 feature  
524 genes, 30 were shared by all subtypes, and the rest 10 were exclusive for each subtype.  
525 The log2 fold change in the means was 4 for the main type and 1.8 for the subtype. The  
526 dispersion parameters  $\phi_{gk}$  were set in the same way as in SD1.

527 **SD4.** We generated five common cell types using the R package Splatter (Zappia et al. 2017)  
528 with 3,000 cells and 5,000 genes. The probabilities that a cell belongs to any cell group  
529 were 0.2 for all groups. The proportion of differentially expressed genes were 1% per cell  
530 group. The location and scale parameters of the log-normal distribution for these feature  
531 genes were (0.4, 0.1). In addition, we followed the default option to add 5% of outliers.  
532 After we filtered the cells with less than 1% non-zero counts and the genes with less than  
533 1% of non-zero cells, the dataset had 4,887 remaining genes. See Supplemental Fig. S12  
534 for all the parameters used in this setting.

## 535 **4.6 Description of the experimental scRNA-seq datasets and their** 536 **preprocessing**

537 **CT26.WT.** This dataset was generated in Dr. Qi-Jing Li's lab. Dr. Li is one of the co-  
538 authors of this paper. The wild-type CT26 cells from the murine colorectal carcinoma  
539 cell line were single-cell-diluted, and a clone was picked and cultured for 220 days. For  
540 the single-cell RNA-seq library preparation, 10,000 cells of each clone were processed  
541 with the protocol of Chromium Single Cell 3' Reagent kits v3 from 10X Genomics to  
542 make the single-cell RNA sequence library. Cells with more than 10% mitochondrially  
543 derived transcripts were removed. Among these cells, we selected those with non-zero  
544 gene proportions greater than 3% or the number of non-zero genes greater than 300 (at  
545 least one of the two conditions needed to hold). We further selected genes with non-zero  
546 count proportions greater than 1% or the number of non-zero cells greater than 50. This  
547 dataset was of high quality, with 24,208 median UMI counts per cell and 4,376 median  
548 genes per cell. Since this dataset was highly homogenous, we used CT26.WT to evaluate  
549 the Pearson's chi-squared "goodness-of-fit" of different models to the UMI counts in the

550 monoclonal scRNA-seq data.

551 **T-CELL.** The T-CELL dataset was generated in our previous study (Christian et al. 2021).

552 The benchmark clustering labels of the T-CELL population were generated as a combi-  
553 nation of protein-marker-based flow sorting labels and bioinformatics labels from Seurat  
554 v2. For evaluation purpose, we selected 5 distinct cell types: Regulatory Tm cells, Clas-  
555 sical CD4 Tem cells, CD8 Tm cells, CD8 Tcm cells, and Active EM-like Treg cells. In  
556 this study, tumors were firstly collected from the female mice after 3 weeks since the mice  
557 were injected by 4T1 tumors. Tissues were then disassociated into single cells and ho-  
558 mogenized. T cells were separated out by flow sorting with a stringent gating threshold  
559 and sequenced on the 10X platform. For preprocessing, we filtered out genes with less  
560 than 2% non-zero cells and removed cells with less than 2% non-zero genes. Eventually,  
561 2,989 cells from five cell types with 7,893 genes were retained.

562 **CORTEX.** The visual cortex dataset was generated by Hrvatin et al. (Hrvatin et al. 2018)  
563 using inDrop to study the diversity of activity-dependent responses across cortical cell  
564 types. We obtained the labeled scRNA-seq dataset from Huang et al. 2018, which con-  
565 tained 10,000 cells with 19,155 genes. Among these 10,000 cells, 7,390 cells were identi-  
566 fied to 33 cell types as an intersection of Seurat v1 and a density-based method (Rodriguez  
567 and Laio 2014). In addition, eight main cell types (excitatory neurons, oligodendrocytes,  
568 astrocytes, interneurons, etc.) were annotated with known feature genes. We selected  
569 cells with at least 300 or 3% of non-zero genes, and genes with at least 50 or 1% non-zero  
570 cells. After preprocessing, 7,376 cells with 12,887 genes were included in clustering.

571 **RETINA.** The mouse retina dataset was generated by Shekhar et al. 2016 using Drop-seq to  
572 classify retinal bipolar neurons. This dataset contained 27,499 cells with 13,166 genes.  
573 Among these 27,499 cells, 26,830 cells were labeled with 18 cell types by the assembled  
574 pipeline: they first used Louvain-Jaccard (Blondel et al. 2008) method to cluster the  
575 cells and then annotated the clusters with known feature genes. This 18-cluster label  
576 set was treated as the benchmark subtype label set. We further grouped the cell types  
577 into 6 main types based on the original paper. These cells came from two experimental  
578 batches of FAC sorted Vsx2-GFP positive cells on different days. We selected cells with  
579 at least 300 or 3% of non-zero genes, and genes with at least 100 or 2% non-zero cells.  
580 After preprocessing, all 26,830 cells with 13,118 genes were selected. The preprocessing  
581 step removed very few genes and cells because the dataset obtained from the original  
582 paper was filtered before cell type annotation.



## 583 **5 Data access**

584 ScRNA-seq for the murine colorectal carcinoma cell line CT26.WT is available at [https://](https://data.mendeley.com/datasets/cb2cb8f8mp/2)  
585 [data.mendeley.com/datasets/cb2cb8f8mp/2](https://data.mendeley.com/datasets/cb2cb8f8mp/2). Three other public scRNA-seq datasets were  
586 also used in this study: T-CELL (<https://data.mendeley.com/datasets/3f4rsk96kf/3>),  
587 CORTEX (GSE102827), and RETINA (GSE81905).

588 We used the CDI R packager version 0.99.2 in this study. This R package has been  
589 submitted to the Bioconductor project. The development version of CDI is available from  
590 <https://github.com/jichunxie/CDI>. The scripts for to reproduce figures of the manuscript  
591 using this package are available at [https://github.com/jfanglovestats/CDI\\_figures](https://github.com/jfanglovestats/CDI_figures).

## 592 **6 Competing interest statement**

593 The authors declare no competing interests.

## 594 **7 Acknowledgements**

595 Ms. Jiyuan Fang's research was partially supported by Duke Center for Human System Im-  
596 munology (CHSI). Dr. Cliburn Chan's research was partially supported by the Duke Uni-  
597 versity Center for AIDS Research (NIH fund 5P30 AI064518) and the Duke Senescent Cell  
598 Evaluations in Normal Tissues (SCENT) Mapping Center (NIH fund 1U54AG075936-01). Dr.  
599 Kouros Owzar and Dr. Liuyang's research was supported by Duke University. Dr. Diyuan  
600 Qin's research was supposed by Duke University. Dr. Qi-Jing Li's research was partially  
601 supported by NIH Award R33 CA225328. Dr. Jichun Xie's research was partially supported  
602 by SCENT (NIH fund 1U54AG075936-01). In addition, Drs. Cliburn Chan, Qi-Jing Li, and  
603 Jichun Xie's research was partially supported by the Translating Duke Health (TDH) Control-  
604 ling the Immune System grant. We also thank the Duke CHSI for providing the computation  
605 resources and administrative support.

606 The content is solely the responsibility of the authors and does not necessarily represent  
607 the official views of the National Institutes of Health.

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