Extrinsic mortality and senescence: a guide for the perplexed

Abstract

Do environments or species traits that lower the mortality of individuals create selection for
delaying senescence? Reading the literature creates an impression that mathematically
oriented biologists cannot agree on the validity of George Williams' prediction (who claimed
'yes'). The abundance of models and opinions may bewilder those that are new to the field.
Here we provide heuristics as well as simple models that outline when the Williams prediction
holds, why there is a 'null model' where extrinsic mortality does not matter, and why it is also
possible to expect the opposite of William's prediction: increased extrinsic mortality favours
slower senescence. While most existing theory focuses on interpreting differences in selection
gradients, we hope to offer intuition by quantifying how much delaying the 'placement' of an
offspring into the population reduces its expected contribution to the gene pool of the future.
Our first example shows why the null result arises and why the null can stop being valid in
models that consider population regulation. Thereafter, a model with 10 different choices for
density dependence shows why extrinsic mortality has the power to favour slow life histories
on the fast-slow continuum, but this requires that density-dependent effects harm juveniles.

Introduction

"It is not the case that additional mortality automatically favours the evolution of senescence."

Caswell and Shyu, 2017

"Reports of the death of extrinsic mortality moulding senescence have been greatly exaggerated."

Jack da Silva, 2018

"Williams' intuition about extrinsic mortality is irrelevant"

Moorad et al. 2020b

The above quotes lay bare a rather odd state of affairs: more than six decades after Williams (1957) presented his argument for the relationship between adult mortality rates and the evolution of senescence, mathematically trained biologists still cannot seem to agree on what patterns theory actually predicts. Williams' seminal work argued that populations experiencing different rates of mortality (as adults) should senesce at different rates (Gaillard & Lemaître 2017). The intuitive message is that if life is bound to be short 'anyway' (due to, e.g., high predation risk), it makes little sense to invest in a robust body able to resist 'wearing out' for a long time (Medawar 1952, Williams 1957).

William's work has since been interpreted to mean that an increase in age-independent extrinsic mortality — typically defined as the result of hazards from the environment which are constant throughout life (Koopman et al. 2015, see Moorad et al. 2020a for definitional issues) — should select for faster senescence (Da Silva 2018, Dańko et al. 2018, André and Rousset 2020). Others have argued against this idea, stating that age-independent extrinsic mortality cannot affect the evolution of senescence (Gadgil and Bossert 1970, Taylor et al. 1974, Abrams 1993, Caswell 2007, Caswell and Shyu 2017, Wensink et al. 2017, Moorad et al. 2019). Recent work, while aiming to clarify, has simultaneously led to a large number of different models and opinions, which as a whole may be confusing to those that are new to the field (André and Rousset 2020, Dańko et al. 2017,2018, and the debate started by Moorad et al. 2019 and continued in Day & Abrams 2020, da Silva 2020, Moorad et al. 2020a,b). Here our aim is to explain what happens in models of senescence in relation to extrinsic mortality, and outline when the prediction made by Williams holds and why it is also possible to state an expectation of it not holding.

In the following, we call, for the sake of conciseness, the prediction that age-independent
extrinsic mortality does not impact senescence 'the null result'. The null result can be
interpreted to mean that 'Williams was wrong', but it is useful to distinguish the null from an
even stronger way for a prediction to disagree with the Williams hypothesis: it is logically
possible that higher extrinsic mortality associates with *slower* (not faster) senescence (Abrams
1993). Thus, we have a range of potential results which we, for brevity, call 'Williams' (is right),
'null', and 'anti-Williams'.

The null result is typically explained using selection gradients in an approach that derives the
strength of selection for a trait that is assumed to improve a vital rate (e.g. survival), and asks
whether selection differs between organisms where age-independent extrinsic mortality rates
are *a priori* high or low (Caswell 2007, Caswell & Shuy 2017). The result can be summarized as
additional mortality having, perhaps surprisingly, no effect on selection gradients unless the
additional mortality is itself age-dependent (Caswell 2007). Here we hope to provide an
intuitive explanation for this result by focusing explicitly on the time that a newborn is placed
into a population. Delaying the 'placement' of an offspring into the population reduces its
expected contribution to the gene pool of the future — but only if a population is growing
exponentially.

We first show the utility of this approach with a simple example that shows why the null result
arises and why the null can stop being valid in models that consider population regulation (as
opposed to unlimited exponential growth). This model is intentionally kept simple and
analytically tractable, e.g., we assume no tradeoffs between reproduction and survival. We
thereafter introduce such tradeoffs by linking the ideas of fast and slow life histories (Stearns
1989, Promislow and Harvey 1990) with Gompertz-Makeham survival curves (Gompertz 1825,
Makeham 1860, Missov and Lenart 2013), together with a total of ten different styles of density
regulation.

The advantage of a simulation approach is that it allows linking senescence to the
'understudied territory' identified by Moorad et al. (2019): what happens when a population
does not stabilize to zero growth but fluctuates, so that there are years (or, more generally,
time steps) with increasing and others with decreasing population sizes (see also Caswell &
Shyu 2017)? Fluctuations in population abundance due to continually occurring stochastic
fluctuations in the vital rates are a common way to model such situations (Tuljapurkar 2013,
Caswell & Shyu 2017), but populations might also fluctuate due to events that occur less often
and cause large mortality in a pulsed manner, a scenario that we include. These events may
impose age-dependent or stage-dependent mortality. A population may be regulated via these
events if they happen more often at high density (e.g. a disease spreads), and the population

may then spend much of its time growing towards high density rather than remaining near an
equilibrium (in other words, transient dynamics become important). In this case predictions
based on selection gradients might not apply (Capdevila et al. 2020), since their calculation
requires demographic stability or small stochastic and age-independent fluctuations around a
demographic equilibrium (Caswell and Shyu 2017).

Our results provide several examples yielding intuition as to why regulation that operates via
fecundity or recruitment can be expected to have a different impact on senescence than
regulation that operates via declining survival (across all ages) with increasing population
densities. While such results do not overturn previous insights already gained (Abrams 1993,
Caswell and Shyu 2017, Wensink et al. 2017, Dańko et al. 2017,2018, and other papers cited
above), we hope that our presentation will make the issues more heuristically transparent.

An example free of tradeoffs: why does the null result arise?

Being able to fly is often quoted as an example of reduced extrinsic mortality (Austad and
Fischer 1991, Holmes and Austad 1994, Healy et al. 2014). Although this is clearly not the only
reason for e.g. bat lifespans exceeding those of similarly sized rodents (for complexities, e.g.,
hibernation, see Wilkinson and Adams 2019), we take the dichotomy 'volant or not' as a way to
conceptualize extrinsic mortality differences in our first, trade-off-free model. We ask whether
a bat, assumed to experience relatively low extrinsic mortality, will be selected more strongly to
delay senescence than a similar-sized non-volant organism, such as a mouse. Note that ignoring
trade-offs means that we are in this first exercise not interested in the fact that litter sizes are
smaller in bats than in rodents; we wish to consider the effect of mortality in isolation of
everything else. Reproductive effort and its potential trade-offs with senescence will be
considered in the second part of our paper.

We further simplify the situation (away from real life, but helpful for heuristic understanding)
by assuming a finite lifespan that does not permit more than one or two breeding attempts.
Both the bat and the mouse have two competing types that differ in their rates of senescence
in a simple and dramatic fashion: a 'fast-senescer' can only breed once and always dies
thereafter, while a 'slow-senescer' can breed up to two times (we also include survival up to
each breeding event). Clearly, both mice and bats will benefit from adding an extra breeding
attempt to their lifespan, if all else is equal (i.e. in the absence of any trade-offs). The sign of
selection is therefore clear, and our aim here is to compare its relative strength for the two
species. If bats benefit much more from the extra breeding attempt than mice, then selection
on bats to reduce senescence is stronger and the result is in line with Williams' hypothesis.

Each mouse individual survives with probability s_M from birth to first reproduction, and slow-
senescers additionally have the same probability of surviving after their first breeding to reach
their second attempt. For bats, the rules are the same, but the survival probabilities equal s_B.
Since we assume all else is equal, we assign the same fecundity F to mice and to bats. F also
does not change between the first and the second breeding attempt. Since there are already
many analytical results available (e.g. Day & Abrams 2020), and our aim is to aid intuition
maximally with a single example, we will make use of a numerical example where
s_B=3 s_M, i.e., bat survival is three times that of mice, and we show results assuming 20% survival
in mice, 60% in bats (Table 1 gives an overview of bat and mice life-history parameters).

The lifetime reproductive success (LRS) of slow-senescing bats is increased by 4.8/3 = 1.6
relative to the fast-senescing bats, i.e. an improvement of 60%. The LRS of slow-senescing mice
is increased by 1.2/1 = 1.2 relative to fast-senescing mice, i.e. an improvement of 20%. It is not
a coincidence that 60% and 20% are identical to the survival values we assigned to the two
species: $(sF+s^2F)/(sF) = 1+s$, thus s is a direct measure of the expected improvement over the
baseline. Since bat LRS increased by a factor three times that of the mice LRS when gaining the
ability to breed twice, one might be tempted to conclude that bats are selected to reap the
benefits of a long life much more strongly than mice, based on the extrinsic mortality argument
$(s_B > s_M)$.

However, this conclusion is premature, and this illustrates a key argument in the debate. In the
absence of density regulation, the superior survival of bats compared with mice also makes
their population grow much faster than that of mice – in our example, their growth rate is
precisely threefold (Table 1). This result applies for any positive value of F: the terms containing
F in the calculation of the growth rate λ are identical for bats and for mice. It does, however,
require that bat fecundity does not differ from mouse fecundity, which is simply a reminder
that we are focusing here on the effect of extrinsic mortality alone, and leave fecundity
considerations for later.

In growing populations, LRS ceases to be a good fitness measure. If two individuals both have
the same LRS but one produces its offspring earlier, these (and their descendants) form a larger
proportion of the gene pool. To quantify precisely how important it is to reproduce early in a
fast growing population, we calculate the relative contribution to the total population at some
later time t of an early produced offspring and a late produced offspring in both the mouse and
the bat population.

This point is easiest to make with populations of slow senescers in both bats and mice, simply
because contrasting fast-senescers only would not allow us to specify how an individual's

fitness accrues from early and late produced offspring. Also, for there to be any late-produced
offspring, we focus on an initial parent assumed to be among the lucky ones who survive to
breed twice. The offspring themselves are examples of slow-senescing life histories with the
appropriate survival rates. These initial founding offspring, of which we consider 1 each (early
and late produced) in both species, are placed into a population that is growing exponentially at
the appropriate species-specific rate (Figure 1, with growth rates from Table 1). The differing
timing of offspring placement into the population is graphically illustrated as an earlier and a
later star symbol in Figure 1), and since the populations of both mice and bats are assumed to
grow, the late-produced offspring form a smaller proportion of the population at the time of
production than the early-produced offspring. This initial disadvantage has consequences ever
after. Measured at a later time point, the proportion of the population that descends from the
early-produced offspring is far larger than the proportion descending from the late-produced
offspring in the (well-surviving and hence fast growing) bat population. This difference also
exists in the mouse population, but it is much less extreme in this species (the widths of the two
'stripes' denoting lineages show only moderate differences in Figure 1a, and strong differences
in Figure 1b).

These differences can be quantified. The descendants of an early-produced offspring, $N_{B,early}$,
as well as a late-produced offspring, $N_{B,late}$, will eventually reach a stable proportion of adults
and juveniles, forming two lineages that both grow at a rate identical to the population growth
rate (Caswell 2001). It follows that the lineage arising from the early-produced offspring,
measured at some time t after both lineages have been initiated, is larger than the lineage
arising from the population of descendants of the late-born offspring, by a factor of λ_B. That is,

$$N_{B,early}(t) = \lambda_B N_{B,late}(t). \tag{1a}$$

Likewise for the descendants of the early and late offspring of a focal mouse individual,

$$N_{M,early}(t) = \lambda_M N_{M,late}(t). \tag{1b}$$

If we divide both sides by the total population (this total contains additionally all other
descendants from this or other parents), we obtain the proportion of the total population at
time t that represents descendants of early and late offspring, respectively. The proportion of
the population descended from the original bat parent's early offspring is larger than that of
her late offspring by a factor λ_B, and for mice, this is λ_M. Since $\lambda_M = \frac{1}{3}\lambda_B$ (Table 1), the early
produced bats are worth three times more (relative to their later produced siblings) than early
produced mice (relative to theirs). In other words, in a population that is growing at a threefold
rate compared with another, the importance of reproducing early is also elevated by the exact

same factor (threefold). This is analogous to investing money into a growing economy: the
faster the growth, the better off are those who were able to invest early; the penalty
(discounting) of late investments is visible in Figure 1 as the trumpet shaped pale stripes
(descendant lineages) being more unequal in height for the bat than for the mouse.

To conclude, even though being able to delay senescence until after the 2nd breeding attempt
(instead of dying after the 1st) benefits bat LRS much more than mice if surviving to breed is
more likely for bats, LRS fails as a predictor of selection because it does not take into account
that late-produced offspring are also less valuable than the early-produced ones — and this
decline in value is much faster for the species that, by virtue of its high survival, has faster
population growth. Since we assumed that higher survival directly translates into a higher
growth rate, the 'penalty' of placing offspring late into the population is far greater for bats
than for mice. These two effects (better improvement in LRS, and the larger penalty) cancel
each other out exactly. The outcome is the 'null result': selection for slow life-histories (against
senescence) is equally strong in the bat and the mouse population.

This result can also be confirmed by comparing population growth rates of entire populations
of fast-senescers versus slow-senescers. Calculating the population growth rate improvement
of slow-senescing bats and mice relative to their fast-senescing competitors yields the same
answer for both species: both improvements are 17% (with data from Table 1, note that 1.17 /
1 = 3.51 / 3). Since population growth rate is the correct fitness proxy for exponentially growing
or declining populations (Charlesworth, 1994; Mylius and Diekmann, 1995; Caswell, 2001), not
the LRS (or its namesake R_0), this section has confirmed that age-independent extrinsic
mortality does not affect the relative benefit of reduced senescence for species experiencing
different levels of extrinsic mortality, *in the absence of density-dependence.*

Beyond the null: what cancels out under density dependence, what does not?

Above, we intentionally considered an unrealistic comparison, to be able to show what happens
if survival is the only difference between two populations. Real bat populations do not show
threefold growth compared with mice, and neither can sustain exponential growth forever.
Intuition (to some at least) suggests that the slow-senescing bats can begin to truly reap the
benefits of a long life if density dependence makes the 'penalty' of having to discount the value
of late-produced offspring less severe. Why? If the population does not in reality expand as fast
as predicted by density-independent growth rules "5 offspring per year and 60% survival for all
who aren't scheduled to die of old age yet", then the trumpet shapes of Figure 1 do not expand
as fast as they did before; mathematically, slower growth means that the value of late-placed
offspring is not devalued as strongly compared to the early-placed ones, and as a whole

density-dependence offers a potential for a smaller penalty for a lineage of descendants
appearing late into a population. If population growth ceases altogether, the penalty vanishes
as well. In other words: *if we assume the slow bats can reach old age just as often as they did in*
the density-independent case, and now their late offspring are not nearly as bad investments as
they were under unlimited population growth, then selection is now much freer to reward slow
life histories (Figure 2 illustrates the idea graphically).

This intuition can be correct, but it comes with a strong caveat: the *if* clause in the sentence
above. The argument relies on the assumption that bats really can reach old age just like they
did under unlimited growth. The crux of the issue is that population growth cannot be reduced
'just like that'; that is, while keeping all vital rates unchanged. Something has to change for the
growth to be lower. Perhaps fewer young are born, or perhaps some are never born because
their potential mothers had already died. There are many possibilities, and this matters.

If slowed down population growth is achieved by making reproduction somewhat harder for
everyone, then it is indeed possible that the chances that a slow-senescing bat reaches old age
remain the same ($s_B^2 = 0.36$ in our example) across all densities (Abrams 1993, Day & Abrams
2020). In this situation, the slowing down and eventual stabilization of population growth can
begin favouring delaying senescence in those organisms that are relatively likely to reach old
age in the first place (i.e. bats as opposed to mice in our example). Slow-senescing mice, too,
enjoy some of this advantage, but only 4% of them do, because high extrinsic mortality ($s_M^2 =$
0.04) means most (96%) do not live to enjoy their intrinsic ability to breed twice.

But, importantly, slowing down (the tendency of $r = \ln(\lambda)$ to decline towards 0) can also be
achieved via other mechanisms. High densities could, for various ecological reasons, make it
very hard for adult bats (or mice) to survive while the fecundity of survivors is left intact. Now it
is quite hard to be convinced that those who *in principle* have good prospects for reaching old
age (bats, as opposed to mice, in our example) would also *in practice* achieve this benefit. If
density regulation effectively prevents slow-senescing types from translating their intrinsic
survival ability to actual survival (and subsequent reproduction), selection will be blind to their
slow-senescing phenotypes.

This can explain why the 'null' result sometimes happens even when density dependence is
included (e.g. Caswell and Shyu 2017). Typically, in these cases, a range of extrinsic mortality
values are compared between hypothetical populations, but each population is also forced to
have zero growth ($r = 0$). If the condition $r = 0$ is achieved by adjusting mortality rates at all ages
equally, then, effectively, the initial elevation of extrinsic mortality (for those populations in the
comparison who were supposed to have high extrinsic mortality) is removed again from the

model by density-dependent adjustments of the mortality itself. Some people argue that this is
a fundamental and exciting proof that helps us understand why extrinsic mortality cannot
matter (Moorad et al. 2019); Moorad et al. 2020a make their preference for including the total
effects of a mortality adjustment more explicit still. Others might reason that this particular
exercise is somewhat pointless, as it assumes that underlying variations in extrinsic mortality
will not be visible in the mortality schedules experienced by individuals at equilibrium. Phrased
in the context of our example, they would never be measurable as real bats having lower
mortality than real mice.

Because this example is important, we repeat the message in the context of an experiment.
While our example is hypothetical, it is inspired by experimentally imposed high and low adult
mortality regimes in *Drosophila* populations (Stearns et al. 2000). Imagine that an empiricist is
applying random (age-independent) mortality to the population in the high-mortality
treatment, but ends up realizing that the remaining individuals respond with improved survival,
so that total mortality (considering both the treatment and its subsequent effect) remains
unchanged. Any measures of senescence rates remain unchanged as well. Did the researcher
recover a deep insight, confirming Moorad et al's message? Or will she instead respond by
stating "my experiment didn't work - it remained uninformative because the manipulation
failed to produce an actual difference in the mortality actually experienced by the population,
making the subsequent finding that senescence didn't change a trivial one"? We leave it to the
reader to form their own opinions about this matter, as we believe both viewpoints have their
merits. It is of interest to note that Moorad et al. 2020a identify a difference in Day & Abrams'
(2020) thinking compared to theirs based on whether the label 'extrinsic mortality' is applied
before or after various consequences, such as those in our hypothetical experiment, have been
allowed to act on the population.

To sum up, by now, we have achieved some intuition as to why it is important to identify who
precisely fails to survive, or fails to be born, when increasing densities reduce population
growth. The key question is: can a slow-senescing phenotype reap the benefits of its long life
across all relevant population densities, or are its survival prospects themselves affected by
density? If adult survival is left relatively intact and so is the value of late-placed young (due to
the population no longer growing so fast), then we can expect the Williams prediction to hold.
If the slow-senescer, on the other hand, itself suffers from density increases, we may enter the
realm of the null, or even an anti-Williams region (see Abrams 1993 for examples), if the
survival of old slow-senescers is disproportionately targeted by density regulation.

Ten case studies of slow and fast life histories

To make our thought experiment above as simple to follow as possible, we focused on an 'all
else being equal' comparison where the two species did not differ in fecundity and there were
no trade-offs: an ability to delay senescence required no lowering of reproductive effort. We
next turn to examples that are considerably more realistic than the above comparison between
hypothetical species that only differ in one respect (survival) and cannot ever breed more than
twice. We now sacrifice analytical tractability to achieve three goals: (i) we consider a wide
variety of density-dependent scenarios; (ii) we link senescence to the ideas of fast and slow life
histories (which is argued to underlie e.g. the mammal-bird dichotomy in senescence rates,
Jones et al. 2008, relationships between senescence and latitude across bird species, Møller
2007, all the way to within-species patterns, Cayuela et al. 2020), taking into account that a
slow senescence rate may involve 'accepting' lower fecundity; and (iii) we see if the intuition
remains robust in non-equilibrium situations.

We explore 10 different kinds of density regulation, of which nine are organized in a 3×3 setup
(Table 2) and an additional one (density dependence acting on recruitment probability) added
for the reason that this form of population regulation is often discussed in territorial species
(Newton, 1992; Sæther et al. 2002; López-Sepulcre and Kokko, 2005; Krüger et al. 2012; Grant
et al. 2017). In the 3×3 scheme, we have three examples each of density-dependence acting on
(1) survival in an age-independent manner, (2) on adult survival (neither the number of
juveniles nor adults impacts juvenile survival) or (3) on fecundity (noting also that fecundity
regulation in this case is mathematically indistinguishable from newborns dying, or having
trouble recruiting into the population; see Discussion). Each of these is investigated in three
different ways: density dependence may be absent for a while until it acts in a pulsed
('catastrophic') manner via sudden decreases in the vital rates either (a) deterministically above
a certain density or (b) stochastically, or (c) density-dependence may exert a continuously
increasing pressure on the relevant vital rate. The additional scenario of density dependence
acting via recruitment limitation is closest to the case that combines (3) with (c). Obviously, the
ten scenarios we consider do not represent an exhaustive list of all (infinitely many)
possibilities, but are helpful for highlighting what is common and what is different between
fecundity and survival regulation.

We implement the same type of trade-off between fecundity and survival in all ten scenarios,
where we contrast the success of a 'fast' life history with one 'slow' one that does not senesce
at all. In the above, trade-off-free section, the slow type always had an advantage, but now we

switch to a trade-off: absence of senescence can only be achieved by 'accepting' a lower

fecundity than that achieved by the fast-senescer. The 'fast' type has thus superior fecundity

but also experiences senescence according to the Gompertz-Makeham model, where mortality

has an intrinsic component that increases exponentially with age (Gompertz 1825, Makeham

1860, Missov and Lenart 2013). For simplicity, we only consider survival senescence, and

fecundities do not depend on the age of the reproducing individual (while they depend on

population density in 3 out of the 10 scenarios). We use subscript 0 to denote slow (using the

mnemonic that 'no senescence' is indicated with a 0), and 1 denotes the fast type.

We describe here what we call the 'standard procedure' (Figure 4), which are the steps that are

shared among all our regulation scenarios; Table 2 then describes what differs between each

scenario.

Each step begins with a census of all individuals, whose ages are integers 1, 2, … with no upper

limit (in practice, we worked with an upper limit of 200, without ever observing a significant

number of individuals reaching this age). The life cycle continues with reproduction, where slow

and fast individuals' fecundities relate to each other as a *per capita* fecundity ratio $F_0 : F_1$. In the

standard scenario, this is achieved simply by letting slow types produce F_0 offspring, while fast

types produce F_1. In case of fecundity regulation (3 of the 10 cases), the fecundities need to

respond to density; we then interpret F_0 and F_1 as maximal fecundities in the absence of

competitors, and use realized fecundities when letting strategies compete: realized fecundities

are αF_0 and αF_1 where $\alpha < 1$ takes smaller values with increasing density (Table 2).

Next in the life cycle, survival is applied deterministically, such that a proportion of individuals

remain to be part of the next census (Figure 4). Survival for slow life histories equals

$$s_0 = e^{-\mu}, \tag{2}$$

where the subscript 0 refers to slow, μ is extrinsic mortality (interpreted as a constant hazard,

which means that 1 year is survived with $e^{-\mu t}$ where $t = 1$, hence $e^{-\mu}$), and there is no age-

dependency since slow individuals do not senesce. Note that the 'no age-dependency'

statement applies to the standard procedure; density-dependent adjustments may mean that

survival is adjusted (multiplication with a factor α) for some age classes $s_0(i)$ but not others

(Table 2).

Fast individuals' survival is age-dependent to begin with (even in the standard procedure; age-

dependency may become additionally modified by density dependence). In the standard

procedure, we model senescence of fast individuals using the commonly used Gompertz-

Makeham model of mortality which assumes mortality has a constant age-independent

component μ and a component that increases exponentially with age i,

$$\mu_{\mathrm{GM}}(i) = \mu + \frac{1}{d} e^{\frac{(i-a)}{d}}. \tag{3}$$

357 It follows that the probability that a newborn reaches age 1 (and becomes part of its first

358 census) is $s_1(1) = P_1(1) = e^{-\mu + e^{-\frac{a}{d}} - e^{\frac{1-a}{d}}}$. Here $s_1(1)$ denotes survival over 1 unit of time from

359 0 to 1, which here is the same as $P_1(1)$, the proportion of individuals still alive at age 1 (the 1 in

360 brackets denotes age, the subscripted 1 indicates this applies to the fast strategy). For the case

361 of newborns these are the same value ($s_1(1) = P_1(1)$). For later ages, they are not. Generally

$$P_1(i) = e^{-\mu i + e^{-\frac{a}{d}} - e^{\frac{i-a}{d}}}, \tag{4}$$

363 Since in our notation $s_1(i)$ captures survival from $i-1$ to i, it equals $P_1(i)/P_1(i-1)$, which

364 yields

$$s_1(i) = e^{-\mu + e^{\frac{i-1-a}{d}} - e^{\frac{i-a}{d}}}. \tag{5}$$

In the absence of extrinsic mortality ($\mu = 0$), senescence is the only cause of death, and under

this (unlikely) scenario the parameter a gives the modal age of death. In the presence of

extrinsic mortality, a alone no longer translates into the modal age of death; across all values of

$\mu \geq 0$, a is better interpreted as the age at which senescence acts strongly to limit lifespan

(Figure 3) — loosely put, it measures how long an individual is 'built for'. The parameter d

impacts the variance in lifespan: at low d values most individuals die around the same age, at

higher d values there is more variation in the age at death. As before (with s_0), the values $s_1(i)$

can be further modified by density dependence (Table 2).

Clearly, we do not claim that nature offers only two life history options available for a

population to choose from, or that a completely non-senescing phenotype is within the range

of evolvable possibilities for many organisms (but see Roper et al. 2021 for a recent discussion

on the topic). We focus on the simple contrast between an ageing high-fecundity and a non-

ageing low-fecundity strategy because it serves our general aim of improving intuition about

why density dependence has its known effects on the general applicability of the null result. For

each of the ten types of population regulation (density dependence), we report the outcome of

competition between the slow and the fast type for a range of values of extrinsic mortality μ

(which acts on both types equally).

Whatever the value of μ, the outcome obviously depends on just how much lower the
fecundity of the slow type is (the ratio F_0:F_1). Intuition suggests that there is always some
intermediate value where the fates of the two strategies switch. At the one extreme, if F_0 = 0,
the lack of senescence of slow individuals cannot help them in competition against F_1
individuals, as the former are infertile; while at the other extreme, where $F_0 = F_1$, slow
individuals have a longer lifespan with no cost in fecundity, and the slow life history is
guaranteed to take over. In between, there is a value of F_0:F_1 where selection switches from
favouring fast to favouring slow. Therefore we show all results in the form of an answer to the
following question: what is the lowest fecundity (F_0) that allows the slow strategy to
outcompete the fast strategy (with fecundity F_1)? And, how does this threshold depend on
extrinsic mortality? If it increases with μ, then the 'Williams' prediction holds: low-μ conditions
make it easy for slow life histories to evolve, even if building a robust body (high a) means
sacrificing fecundity by a lot.

The results of the 10 different regulation styles are clearly categorizable in three rows (Figure
5). When density-dependence causes all individuals to suffer diminished survival, extrinsic
mortality has no effect on the threshold value, i.e. the 'null' result holds (Figures 5: 1A-C) — and
it does so regardless of whether we chose a 'pulsed' type of regulation acting occasionally
(cases 1A,B) or one of a more continuous nature (case 1C). When juveniles are shielded from
the negative effects of density, however (ecologically, such a result might arise if their niche
differs from that of the adults, and the adult niche is the limiting one), then an increase in
extrinsic mortality makes it easier for the slow strategy to invade (the threshold reduces), and
we find an anti-Williams pattern (Figure 4: 2A-C). Finally, when density-dependence acts on
fecundity or on juvenile recruitment, an increase in extrinsic mortality makes it harder for the
slow strategy to invade (Figure 4: 3A-C), in line with Williams' hypothesis and predictions made
by later models (Abrams 1993).

Discussion

Williams' hypothesis has triggered lively debates among theoreticians for decades. Previous
work has generally focused on selection gradients. Our approach offers an alternative analysis,
focusing on an intuitive explanation by considering the relative importance of placing offspring
into a population earlier rather than later. Our results do not contradict earlier work (e.g.
Hamilton 1966, Charlesworth 1993, Wensink et al. 2017, Dańko et al. 2018, Day & Abrams
2020), but we hope that our examples provide intuition for why age-independent extrinsic
mortality does not affect the evolution of senescence in the absence of density-regulation, or in

the presence of density-regulation that depresses survival to an equivalent degree across all
individuals.

Also, our results are fully in line with earlier findings (Abrams 1991, Day & Abrams 2020) that
emphasize that whenever density dependence 'hurts juveniles' (either by making it difficult for
adults to produce them in the first place, or making their survival or recruitment low), then
Williams' prediction is likely to hold). The general pattern that density-dependence acting on
juvenile production or recruitment leads to a Williams result is strikingly consistent, and given
that there is rather broad empirical support for Williams-type patterns across species (e.g.
Ricklefs 2008), it may be seen as indirect evidence that population regulation often operates via
this mode.

To understand why density regulation affecting fecundity leads to a Williams-like result, it is
useful to go back to our first, trade-off-free result: the comparison of bats and mice. There, we
showed that the intuition behind the Williams prediction is that the benefits of a potentially
long life (little senescence) can only materialize if the organism also avoids all other causes of
death that do not directly relate to senescence. Populations grow, and growth must become
limited at some point; the crucial question is where in the life cycle the effects are 'felt'. One
possibility is that juvenile production suffers. This allows the slow-senescing type to keep
reaping the benefits of its robust body even when population regulation is acting. In a typical
assumption set (like ours), it survives just as well as under low density, and while its fecundity
now suffers, this effect is felt by parents of all ages, and does not translate into a reduction in
relative productivity of old parents (the crucial tacit assumption here is that juveniles feel the
negative effects of density equally across all ages of their *parents*).

Our 10 examples, that themselves are based on a specific comparison where a no-senescing
type competes with a Gompertz-Makeham-type senescer, obviously do not constitute proof
that deviant patterns could never be found, should one consider other comparisons. Our
approach provided its simple graphical contrasts (flat, decreasing and increasing curves in
Figure 5 corresponding to null, anti-Williams and Williams, respectively) by pitting a senescing
type against an ideal type that is immune to senescence. The biological interpretation of the
latter is somewhat challenging in some cases, especially our setting 2A where only oldest age
classes are removed at high density. The idealized type of a slow life history here combines the
assumption of intrinsically age-independent survival (in the sense of eq. 1) with an inability to
withstand high density situations that applies from a certain age onwards; the physiological
interpretation of such a case is challenging. A general point, however, remains: given that
different population regulation modes definitely exist (e.g., Drury & Dwyer 2005, Sæther et al.

2016, Dánko et al. 2017, Lee et al. 2021), variance in senescence (and lifespan) among taxa
cannot be solely attributed to differences in extrinsic mortality.

Because our results show that comparative predictions ideally require an understanding of
causes of shorter lifespans in one population compared with another, as well as general
information on the mode of population regulation, it may be premature to make statements
about individual case studies. It is nevertheless interesting that e.g. predation has been shown
to impact senescence either positively or negatively. Insular populations of opossums are under
lower predation pressure and senesce at a lower rate compared to mainland populations
(Austad 1993). Reznick et al. (2004) on the other hand showed that guppy populations subject
to higher predation rates senesced at lower rates than populations under lower predation risk.
The latter authors speculate about possible mechanisms explaining this anti-Williams type
pattern, but data is still lacking to show possible density-dependent effects on older age
classes. More generally, empirical studies of the effect of extrinsic mortality on senescence
usually lack evaluations of density-dependent effects on vital rates (but see Stearns et al. 2000),
hindering interpretations about causal factors behind observed patterns. There is also indirect
evidence for the effect of predation on senescence; patterns of senescence are compared
among species with different modes of life, under the assumption that the ability to fly or to
live underground decrease exposure to predation (Austad and Fischer 1991, Holmes and Austad
1994, Healy et al. 2014).

Since our focus was on making the theory easy to understand, we do not claim that our study
encompasses all the mechanisms by which extrinsic mortality affects senescence. It is
interesting that our results, in line with earlier theory (Abrams 1991, André and Rousset 2020,
Day & Abrams 2020), emphasize the importance of understanding population regulation, while
in experimental (Stearns et al. 2000) and observational (e.g., desiccating ponds) data, high
mortality or high risks of habitat disappearance are often stated to lead to faster life histories
(*Daphnia:* Dudycha and Tessier 1999, killifish: Tozzini et al. 2013). This may appear to be at
odds with our predictions, as desiccation typically kills adults and the next generation hatches
from eggs once the water returns. Similarly, grasshoppers living at higher altitudes are subject
to higher risks of freezing episodes and accordingly show faster life-histories and earlier
senescence compared to populations at lower altitudes (Tatar et al. 1997). Note, however, that
abiotic causes behind mass mortality do not involve a causal link from high density to mortality,
a link that is incorporated in density-dependent senescence models (like ours). In other words,
although *Daphnia* populations are more dense just before a desiccation event than when the
hatching first began, and there may be more grasshoppers late in the season than early, this is
correlation, not causation: an abundance of *Daphnia* does not cause ponds to dry and winter

does not happen because grasshoppers became abundant. Ephemeral habitats therefore
require models of their own; one possibility that our models did not address is a timescale
where ephemeral habitats may cut individual lives short before maturity is reached. Speeding
up maturation time may be an adaptive response in such situations, with effects felt
throughout the life cycle.

Models that include processes not included by us may highlight other reasons for finding
specific patterns. Anti-Williams patterns may, for example, be found when explicitly considering
condition-dependence impacting susceptibility to extrinsic mortality (the definition of 'extrinsic'
is then subtly different: it ceases to be 'unavoidable' as an organism's traits now influence its
susceptibility to it). In brief, when being frail or senescent increases an organism's susceptibility
to extrinsic mortality, high extrinsic mortality leads to stronger selection on slow senescence
(Abrams 1993, Williams & Day 2003). Fitting this pattern, salmon populations senesce at lower
rates when predation rates by bears are high and directed towards senescing individuals
specifically (Carlsson et al. 2007). Selection for heat resistance is associated with increase in
lifespan in *Caenorhabditis elegans*, such that populations under higher temperature-related
mortality risks also senesce at slower rates (Chen and Maklakov 2012). We have also chosen to
model trade-offs (or lack thereof) in a stylized way, leaving subtleties such as the difficulty of
optimizing function simultaneously for early and late life (Maklakov & Chapman 2019) for later
studies. For additional viewpoints see e.g. the system reliability approach (Gavrilov & Gavrilova
2001, Laird & Sherratt 2009, 2010a,b) as well as selection that relates to the possibility of
indeterminate growth (Vaupel et al. 2004, Caswell & Salguero-Gómez 2013, Purchase et al.
2022). While the multitude of factors listed above suggests that wide diversity in senescence
patterns and lifespans (Jones et al. 2014) is be the expectation, we hope that our conceptual
examples help to see why a specific feature of life cycles — the diversity in modes of population
regulation — continue to play a very important role.

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Figure 1

Figure 1. A visualization of why the bat, with its threefold growth rate, is penalized more for a delay in placing offspring into a growing population. For purely aesthetic reasons, population growth is depicted over continuous time ($r = \ln(\lambda)$). Values of r correspond to λ values given in Table 1. Populations are assumed to consist of slow senescers (Table 1) and stars depict the placement of one offspring at time $t=1$ or $t=2$ into a growing population. Both species consist of 50 individuals at time $t = 1$ (shown at a different scale as indicated, to fit the entire growth into picture, as bats, with their higher survival, increase their numbers much faster than mice. For both species, the lineage (pale stripes) that starts with an offspring placed into the population at $t = 2$ is thinner than the lineage that had its start at $t = 1$, but this difference is much more marked if population growth is (b) fast than if it is (a) slow. In (b) both stripes appear narrow, because the vertical scale has to differ between (a) and (b) to allow the entire bat population to be depicted.

Figure 2.

(a) No density dependence

(b) Density dependence, zero growth

time

Figure 2. A visualization of the difference between placing offspring early or late in an exponentially growing population (on the left) or in a stable population (on the right). In a growing population, the late placed offspring will be a smaller fraction of the future population than the early placed offspring. In a stationary population, there is no such discounting of early vs late placed offspring.

Figure 3

Figure 3. (a,b) Gompertz-Makeham survivorship function (the probability that an individual is still alive) and (c, d) age-dependent survival for example combinations of a and d as indicated. All example use $\mu = 0.05$.

Figure 4: The standard procedure, describing a life cycle used to create the 10 different modes of density regulation. Note that there is no implication that completing each of the three arrows takes equally much time (in reality, census is immediately before breeding, to allow mortality rates the appropriately long time to apply before the next year). Generations are overlapping, therefore at each point the population will consist of individuals of different integer ages. Details about regulation are briefly summarized next to the loop; for full see details in Table 2.

Figure 5

Density-dependence affects

↓

1. Age-independent survival

2. Survival age>1

3. Recruitment/ fertility

Deterministic pulsed

Probabilistic pulsed

Continuous

Competition for territories

1A   **1B**   **1C**

2A   **2B**   **2C**

3A   **3B**   **3C**   **3D**

$F_1=1$
$F_1=0.75$
$F_1=0.5$

Null

anti-Williams

Williams

Figure 5. How much fertility does a non-senescing (slow) strategy need to beat the senescing (fast) strategy? The lines indicate a threshold fecundity F_0 for the slow life history: above this threshold slow types win, below this threshold the fast strategy wins. Parameters used for the Gompertz-Makeham (equation 3): a=4, d=1, K=100 000.

| | Variable name, or expression

 | Numerical example | Variable name, or expression

Numerical example
Fertility	F	5	F	5
Survival (to breed, or to breed again after breeding once)	s_M	0.2	s_B	0.6
Lifetime reproductive success				
Fast scenescers	$s_M F$	1	$s_B F$	3
Slow scenescers	$s_M F + s_M^2 F$	1.2	$s_B F + s_B^2 F$	4.8
Growth rate (λ)				
Fast scenescers	$s_M F$	1	$s_B F$	3
Slow senescers	$\frac{1}{2} s_M (F + \sqrt{F^2 + 4F})$	1.17	$\frac{1}{2} s_B (F + \sqrt{F^2 + 4F})$	3.51

Table 1: List of variables and their values used in the trade-off free example. To calculate λ, start from the Euler-Lotka equation for a species with ω age classes,

$$\sum_{a=1}^{\omega} \lambda^{-a} l(a) b(a) = 1,$$

where $l(a)$ is the proportion of individuals surviving until age a, and $b(a)$ is the mean reproductive output of these survivors. For example, for the slow senescing mouse the Euler-Lotka equation becomes

$$\frac{1}{\lambda} s_M F + \frac{1}{\lambda^2} s_M^2 F = 1.$$

Solving for λ, gives the equation in the table.

Table 2: Description of each of the ten scenarios used to create Figure 5.

Scenario	Description
1A	*Exponential growth & marked excess mortality when carrying capacity is exceeded.* Standard procedure (see main text) except in generations where the pre-breeding census yields $\sum_i N_0(i) + \sum_i N_1(i) > K$, where $N_0(i)$ and $N_1(i)$ are the number of slow-senescing and fast-senescing individuals in age class i, and K is a predefined carrying capacity. These high-density census events lead to the pre-breeding population experiencing 90% mortality across all ages of both life-history strategies (slow and fast). Thereafter, breeding and survival proceed normally (standard procedure) among the survivors. This type of regulation features exponential growth with a 'resetting' to small population sizes at regular intervals, with the cull impacting all ages equally.
1B	*Excess mortality events become more common at high density.* Like (1A), but resetting the population to small sizes occurs in a stochastic manner. Whether the 90% pre-breeding mortality occurs is decided probabilistically at each census. The probability p that 90% pre-breeding mortality is applied increases with density and happens with certainty if density K is reached or exceeded: $$p = \begin{cases} \frac{\sum_i N_0(i) + \sum_i N_1(i)}{K} & \text{if } \sum_i N_0(i) + \sum_i N_1(i) < K \\ 1 & \text{if } \sum_i N_0(i) + \sum_i N_1(i) \geq K \end{cases}$$
1C	*Continuous decline of survival with density.* At each census, we compute the value of a density-dependent factor $$\alpha = \begin{cases} 1 - \frac{\sum_i N_0(i) + \sum_i N_1(i)}{K} & \text{if } \sum_i N_0(i) + \sum_i N_1(i) < K \\ 0 & \text{if } \sum_i N_0(i) + \sum_i N_1(i) \geq K \end{cases}.$$ The population follows the standard procedure as defined in the main text, but modified such that every survival value is multiplied by α. Note that this multiplication is also applied to juveniles born in the census year, who did not yet contribute to the census.
2A	*High density removes all old individuals.* The standard procedure is applied except when the pre-breeding census reveals a total population size above K, i.e. $\sum_i N_0(i) + \sum_i N_1(i) > K$. If the threshold is exceeded, all individuals above a certain age j, irrespective of being type 0 or 1, die before breeding begins. The value of j is chosen as the largest possible age threshold (leading to the smallest possible number of age classes removed) that yields $\sum_{i=1}^{j} N_0(i) + \sum_{i=1}^{j} N_1(i) < K$, i.e. brings the pre-breeding population back to < K: If some individuals from age class j need to be removed, everyone in that age class is removed.

	Afterwards, the remaining population reproduce and survive as normal (standard procedure).
2B	*High density leads stochastically to an event of removing all old individuals*. The removal event described in (2A) occurs probabilistically, with a probability that behaves like *p* in (1B).
2C	*Continuous decline of adult survival with density.* Like (1C), but only survival of parents is negatively impacted by high density. The production and survival of newborns of the current year is unaffected, as their parents' survival is only negatively impacted after breeding occurred (Figure 4).
3A	*Crowding stops reproduction entirely.* If the census yields $\sum_i N_0(i) + \sum_i N_1(i) > K$, there is no reproduction in the given year. Extant individuals survive according to the standard procedure.
3B	*High density leads stochastically to an event of reproductive failure*. As in 3A but now a no-reproduction year occurs with probability *p*, defined as in 1A.
3C	*Continuous decline of fecundity with density*. At each census, we compute α as in 1C, but it is now applied to fecundities. Multiplication with α, when performed both for F_0 and F_1, keeps the ratio F_0:F_1 intact.
3D	*Territoriality.* The standard procedure is applied in all other respects than juvenile survival. Adults die, which implies that there are $K - (\sum_i N_0(i)s_0(i) + \sum_i N_1(i)\,s_1(i))$ vacancies available once the population is proceeding towards the new census; survived juveniles are recruited to vacant territories, and should there not be sufficiently many vacancies available, the rest die.

Table 3: Classification of the nine scenarios according to the type of density-dependence and according to which vital rates are affected by the density-dependence. The tenth scenario clearly has density-dependence acting on recruitment

Density-dependence acts on	Pulsed DD		Continuous DD	
	Deterministic	Stochastic	Deterministic	Competition for territories
Survival in an age-independent way	1A	1B	1C	
Adult survival in an age-dependent way	2A	2B	2C	
Recruitment, fecundity, or newborn survival	3A	3B	3C	3D