Quantifying template signatures of center-of-mass motion during walking with ankle exoskeletons

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Abstract

Predicting ankle exoskeleton impacts on an individual's walking function, stability, and efficiency remains challenging. Characterizing how the dynamics underlying center-of-mass (COM) mechanics and energetics change with exoskeletons may improve predictions of exoskeleton responses. We evaluated changes in individual-specific COM dynamics in unimpaired adults and one individual with post-stroke hemiparesis while walking in shoes-only and with passive ankle exoskeletons. We leveraged hybrid sparse identification of nonlinear dynamics (Hybrid-SINDy) – an equation-free data-driven method for inferring nonlinear hybrid dynamics using a large library of candidate functional forms – to identify functional forms that best modelled physical mechanisms describing leg-specific COM dynamics, termed *template signatures*. Across participants, Hybrid-SINDy identified template signatures comprised of leg stiffness and resting length, similar to common spring-loaded inverted pendulum models. Rotary stiffness mechanisms were identified in only 40-50% of participants. Unimpaired template signatures did not change with ankle exoskeletons ($p > 0.13$). Conversely, post-stroke paretic leg and rotary stiffness increased by 11% with zero- and high-stiffness exoskeleton, respectively, suggesting that COM dynamics may be more sensitive to exoskeletons following neurological injury. Agreement between our automatically-identified template signatures and those found from decades of biomechanics research supports Hybrid-SINDy's potential to accelerate discovery of mechanisms underlying impaired locomotion and assistive device responses.

Introduction

Ankle exoskeletons are prescribed and designed to improve walking function and gait mechanics [1-3]. Personalized passive and powered ankle exoskeletons have shown promise to improve walking function and muscle coordination in unimpaired adults and individuals with neurological injuries [2, 4, 5]. However, changes in gait in response to ankle exoskeletons are highly individualized, especially following neurological injury, making device personalization critical to improving mobility. For example, in stroke survivors and children with cerebral palsy, ankle exoskeletons elicit diverse – and sometimes detrimental – impacts on gait mechanics, walking speed, step length, and the energetic cost of walking[1-4]. Capturing and characterizing these responses remains challenging and hinders clinicians and designers' abilities to customize exoskeletons to support walking function.

Despite observed changes in gait mechanics and energetics with ankle exoskeletons, little is known about how task-level mechanisms, such as the control of whole-body dynamics, impact exoskeleton responses [2, 5-7]. Specifically, controlling center-of-mass (COM) motion is known to impact gait energetics and stability, and is related to clinical metrics of walking function and speed [8-10]. Modulating COM dynamics in response to exoskeletons may, therefore, be critical to maintaining stable and efficient gait. Importantly, following neurological injuries (*e.g.*, a stroke), COM energetics are altered asymmetrically, with the paretic limb doing less work on the COM [11]. Similarly, when walking with ankle exoskeletons, stroke survivors struggle to modulate their gait in energetically efficient ways [2]. However, these experimental studies did not quantify changes in the underlying COM dynamics with exoskeletons. Even in unimpaired adults it is unclear whether individuals modulate COM dynamics similarly in response to exoskeletons. Characterizing the strategies used to modulate COM dynamics with ankle exoskeletons may provide a foundational step towards understanding and predicting exoskeleton responses.

Reduced-order representations of gait dynamics provide a foundation to quantify and understand complex exoskeleton responses [12-17]. For example, Full & Kodistchek (1999) used reduced-order representations of COM dynamics, such as the spring-loaded inverted pendulum (SLIP), to quantify strategies to achieve task-level goals including COM stabilization and perturbation responses during locomotion [15]. Such reduced-order representations (*i.e.*, models) of gait encode neural and biomechanical dynamics using a minimalist set of physics-based mechanisms. Specifically, common reduced-order models of COM dynamics employ a variety of mechanisms, such as the SLIP leg springs or the rigid legs of an inverted pendulum walker, describe COM accelerations during gait [9, 13, 15, 16, 18-22]. Each mechanism within a model, therefore, encodes a hypothesis about how neural and biomechanical subsystems interact to achieve task-level objectives. However

Reduced-order models of locomotion, often termed *Template* models, have provided influential insights into strategies to control COM motion to achieve stable and efficient walking. For example, inverted pendulum templates have been used to study COM energetics and the transition from walking to running [16, 18], as well as strategies for energetically-efficient COM acceleration [8, 22, 23] and lateral stabilization [9, 24]. Alternative template structures, such as the bipedal spring-loaded inverted pendulum (SLIP) and its variants further improved predictions of sagittal-plane ground reaction forces (GRFs) and revealed COM stabilization strategies across different speeds [13, 20, 21, 25, 26]. Extensions of the bipedal inverted pendulum and SLIP templates include leg damping elements, rotary springs, or curved feet. Note that each mechanism describes a unique relationship between leg kinematics and COM acceleration [19, 27]. Further, given the breadth of templates proposed for human gait, the mechanisms that best describe gait are likely individual-specific. Consequently, selecting an optimal template structure to describe an individual's gait is non-trivial, particularly following neurological injury [28].

Template models that describe COM dynamics during unimpaired gait may be useful to serve as normative archetypes, from which deviations can be used to characterize altered COM dynamics.

To emphasize the individualized nature of templates, we denote the individual-specific mechanisms that best describe COM dynamics as an individual's *template signature*. For example, differences in the template signatures in children with hemiparetic cerebral palsy differed from typically developing children and between legs, with increased stiffness in the paretic limb [28, 29]. These differences suggested that children with cerebral palsy may have reduced dynamic resources available to control COM motion during walking, providing insight into individualized strategies to accelerate the COM and their implications for walking function.

Similarly, we may expect that template signatures will reveal altered COM dynamics following other forms of neurological injury (*e.g.*, stroke) and with ankle exoskeletons. However, characterizing changes in template signatures in these contexts requires addressing a major methodological challenge: Manually identifying optimal template signatures mechanisms is a slow, *ad hoc* process that relies on first-principles knowledge of the system [19]. The candidate template signature increases combinatorially with the number of candidate mechanisms. Therefore, new methods are needed to select mechanisms rapidly and systematically from a literature-based library of candidate mechanisms.

Recent advances in data-driven modeling and machine learning provide powerful techniques to flexibly identify system dynamics from data [30-33]. **Hybrid-SINDy** (*SINDy: Sparse identification of nonlinear dynamics* [30]) identifies sparse nonlinear dynamics in hybrid systems from time-series data, making it particularly relevant to identifying template signatures of walking, which often have distinct dynamics based on foot contact configuration [28, 32]. Importantly, Hybrid-SINDy identifies a large number of candidate dynamical models (*e.g.*, template signatures) from an arbitrary library of possible functional forms (*i.e.*, mechanisms). The algorithm uses information criteria to determine levels of support for each candidate model and selects only those that are parsimonious *and* highly representative of the system [34]. Consequently, this approach enables rapid, systematic identification of individual-specific template signatures while leveraging first-principles knowledge of the physics of human gait.

The purpose of this study was to identify changes in template signatures in response to ankle exoskeletons and evaluate how neurological injuries may alter template signatures in response to exoskeletons. We used the Hybrid-SINDy algorithm to identify template signatures that encode COM dynamics during walking in unimpaired adults and how those signatures changed with hinged and stiff ankle exoskeletons. We hypothesized that ankle exoskeletons would primarily increase leg stiffness. Additionally, to examine the potential of template signatures to encode changes in COM dynamics for individuals with neurological injuries, we present a case study evaluating altered template signatures in an individual with post-stroke hemiparesis. To our knowledge, this work is the first to employ physics-informed data-driven modeling techniques to select low-dimensional, interpretable, representations of walking dynamics with ankle exoskeletons.

Methods

Data collection

We collected 3D marker trajectories using a ten-camera motion capture system (Qualisys AB, Gothenburg, SE) and ground reaction forces (GRFs) using an instrumented treadmill (Bertec Inc., Columbus, USA) in twelve unimpaired adults (6M/6F; age = 23.9 ± 1.8 years; height = 1.69 ± 0.10 m; mass = 66.5 ± 11.7 kg) and one stroke survivor with lower-limb hemiparesis (*sex not disclosed*; age = 24 years; height = 1.70 m; mass = 68.0 kg). Participants walked in shoes-only and with bilateral passive ankle exoskeletons under two conditions: zero resistance to ankle flexion (*i.e.*, zero stiffness; K_0) and high dorsiflexion resistance (*i.e.*, high stiffness; K_H = 5.1 Nm/deg; Figure 1). A detailed description of the experimental protocol and data preprocessing can be found in [35]. Briefly, following a practice session, data were collected while participants walked at their self-selected speed for four minutes per condition. To mitigate fatigue, the post-stroke participant walked for only three minutes per condition [36]. This study was approved by the University of Washington Institutional Review Board (#47744) and all participants provided written informed consent to participate in the study.

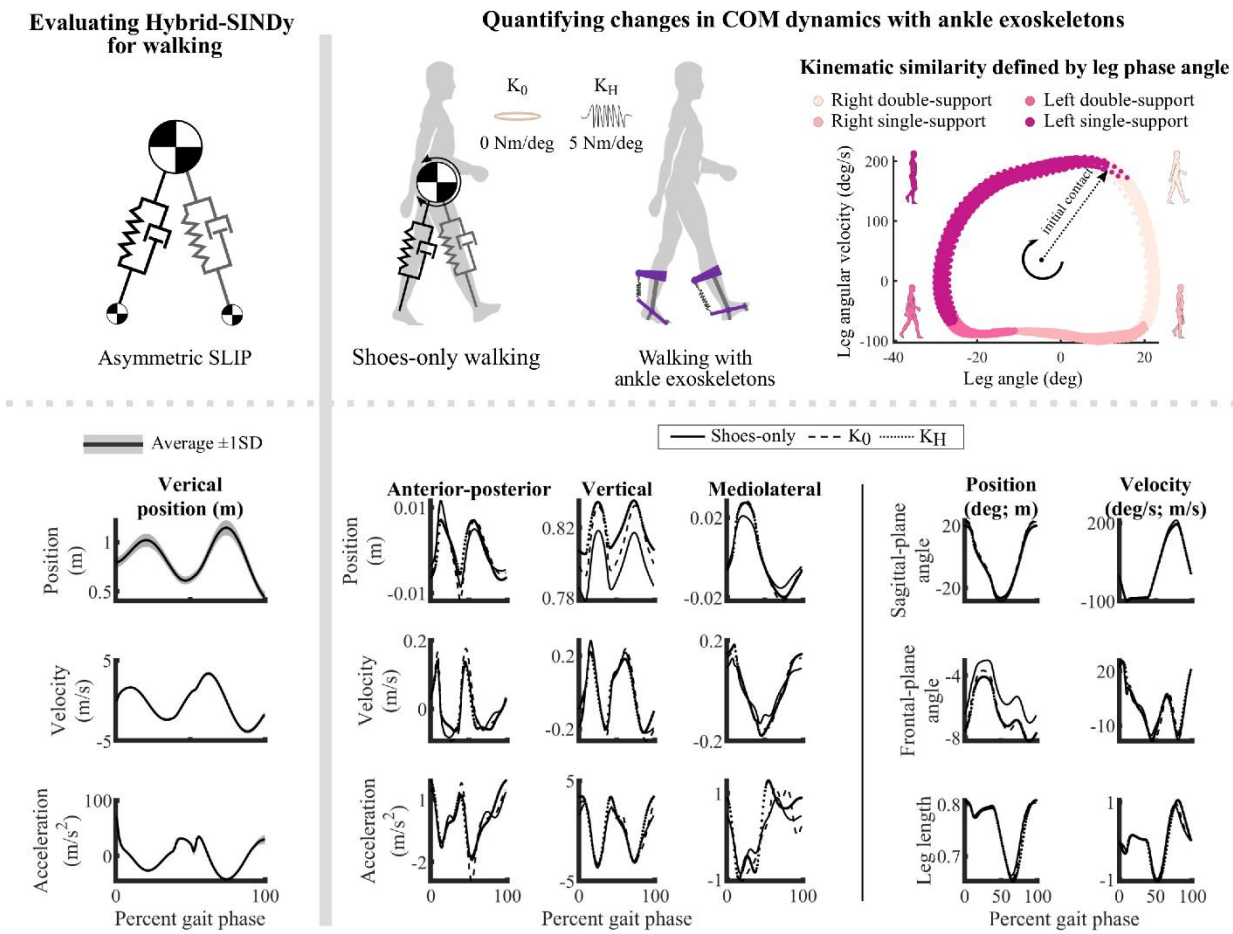

Figure 1 – Top: Two-dimensional depictions of the simulated SLIP (left) and human walking (right). The simulated SLIP used leg springs and dampers, as well as foot masses to simulate full strides. Participants walked on a treadmill in shoes-only and in zero- (K_0) and high-stiffness (K_H) ankle exoskeletons. The phase portrait (pink) defined a kinematic phase variable for clustering similar measurements. Colors denote gait phases corresponding to first and second double-limb support, single-limb support, and swing. Bottom: Time-series measurements of COM position, velocity, and acceleration for the simulated SLIP (left; average ± 1SD) and an exemplary unimpaired adult participant (right). For human gait, COM states (middle) and leg states (right) are shown for eahc walking condition. The leg length and velocity, and sagittal- and frontal-plane leg angles determined leg forcing, while COM position determined forcing direction.

Estimating template signatures with Hybrid-SINDy

We used the Hybrid-SINDy algorithm to identify template signatures (*i.e.*, mechanisms describing COM acceleration) with and without ankle exoskeletons. For each exoskeleton condition, we used OpenSim's *Body Kinematics* algorithm to estimate the COM and foot positions [37, 38]. In this section, we describe our implementation of the SINDy and Hybrid-SINDy algorithms to identify template signatures. Detailed descriptions of the algorithms can be found in [30] and [32, 33]. Let 3D COM linear accelerations, $\ddot{\boldsymbol{q}}(t) \in \mathbb{R}^{m\times n}$, be described as a continuous-time nonlinear system with dynamics $f\big(q(t), \dot{q}(t)\big)$, where m denotes the number of samples and $n = 3$ outputs:

$$\frac{d^2}{dt^2}\boldsymbol{q}(t) = \ddot{\boldsymbol{q}}(t) = f\big(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)\big), \qquad 1$$

where time is denoted by $t \in \mathbb{R}^{m\times 1}$, and $q(t)$ and $\dot{q}(t)$ denote COM positions and velocities relative to the feet, respectively, in $\mathbb{R}^{m\times n}$. We assume that only a small number of functional forms (*i.e.*, mechanisms) in $f(q(t), \dot{q}(t))$ describe most of the system's behavior. We omit the time notation in the remaining sections.

Sparse Identification of Nonlinear Dynamics (SINDy)

The SINDy algorithm ([30]) recovers sparse nonlinear dynamics from a library of candidate functional forms derived from arbitrary nonlinear functions of system measurements. We can rewrite the dynamics in equation 1 as:

$$\ddot{\boldsymbol{q}} = \boldsymbol{\Theta}(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{\Xi}, \qquad 2$$

where $\boldsymbol{\Xi} \in \mathbb{R}^{p\times n}$, is a linear map from the nonlinear function library encoding mechanisms potentially describing COM accelerations, $\boldsymbol{\Theta}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \mathbb{R}^{m\times p}$, to COM accelerations, $\ddot{\boldsymbol{q}}$. The coefficients in $\boldsymbol{\Xi}$, therefore, represent template signatures parameters, while $\boldsymbol{\Theta}$ encodes a template signature's structure. We included $p = 14$ functional forms in the function library, described below. The SINDy algorithm promotes sparsity in the identified dynamics using sequential least-squares regression with hard thresholding [32, 33].

Hybrid-SINDy

Hybrid-SINDy extends SINDy in two important ways. First, Hybrid-SINDy uses clustering to generalize SINDy to hybrid systems. For human walking data, clustering enables unique dynamics to be identified in each hybrid regime, such as distinct contact configurations (*i.e.*, single- and double-limb support) [10, 14, 18]. Second, Hybrid-SINDy uses information criteria to automatically select the system dynamics that best describe the data. This approach enables competing hypotheses about the mechanisms describing COM acceleration to be rapidly and systematically compared, and may highlight mechanisms describing COM accelerations that are individual-specific and those that are common across individuals.

Applying Hybrid-SINDy to walking

Here, we briefly describe our application of the Hybrid-SINDy algorithm to human gait, which we performed separately for each participant and walking condition (Figure 2). A detailed description of each step can be found in *Supplemental – S1*. First, for each sample in the training set, we clustered the 800 nearest-neighbors, corresponding to roughly 7.4% of a stride, using leg phase angle as a clustering variable [35]. For each cluster, we then used SINDy to identify multiple template signatures by sweeping 40 sparsity threshold values. This procedure typically produced 5-15 unique signatures per cluster [30]. We then evaluated the ability of each template signature to reconstruct COM accelerations in the anterior-posterior, vertical, and mediolateral directions in each hybrid regime using held-out data.

To identify the most plausible template signatures for an individual, we compared candidate signatures using the Akaike Information Criterion, corrected for finite sample sizes (AICc) [32, 39]. The AICc provides an unbiased estimate of the relative likelihood of competing system representations (*i.e.*, template signatures) by accounting for increases in model likelihood with the addition of each model parameter. Template signatures with lower AICc scores were considered more-plausible than those with higher scores. Specifically, template signature likelihood was determined by the relative AICc score ($\Delta\mathrm{AICc}_j = AICc_j - AICc_{min}$): the difference between the AICc score of the jth signature and the signature with the lowest AICc score [34]. Anderson and Burnham noted that models with $\Delta\mathrm{AICc}_j \leq 2$ have substantial support and those with $\Delta\mathrm{AICc}_j > 7$ have low support [34]. We adopted the threshold of [32], deeming template signatures with $\Delta\mathrm{AICc}_j \leq 3$ to be *plausible*. To define a final template signature, we performed multi-model inference, computing a weighted-average of the *plausible* template signatures in each hybrid regime using Akaike weights and a bootstrapping procedure to ensure template signature robustness (*Supplemental – S1*) [34].

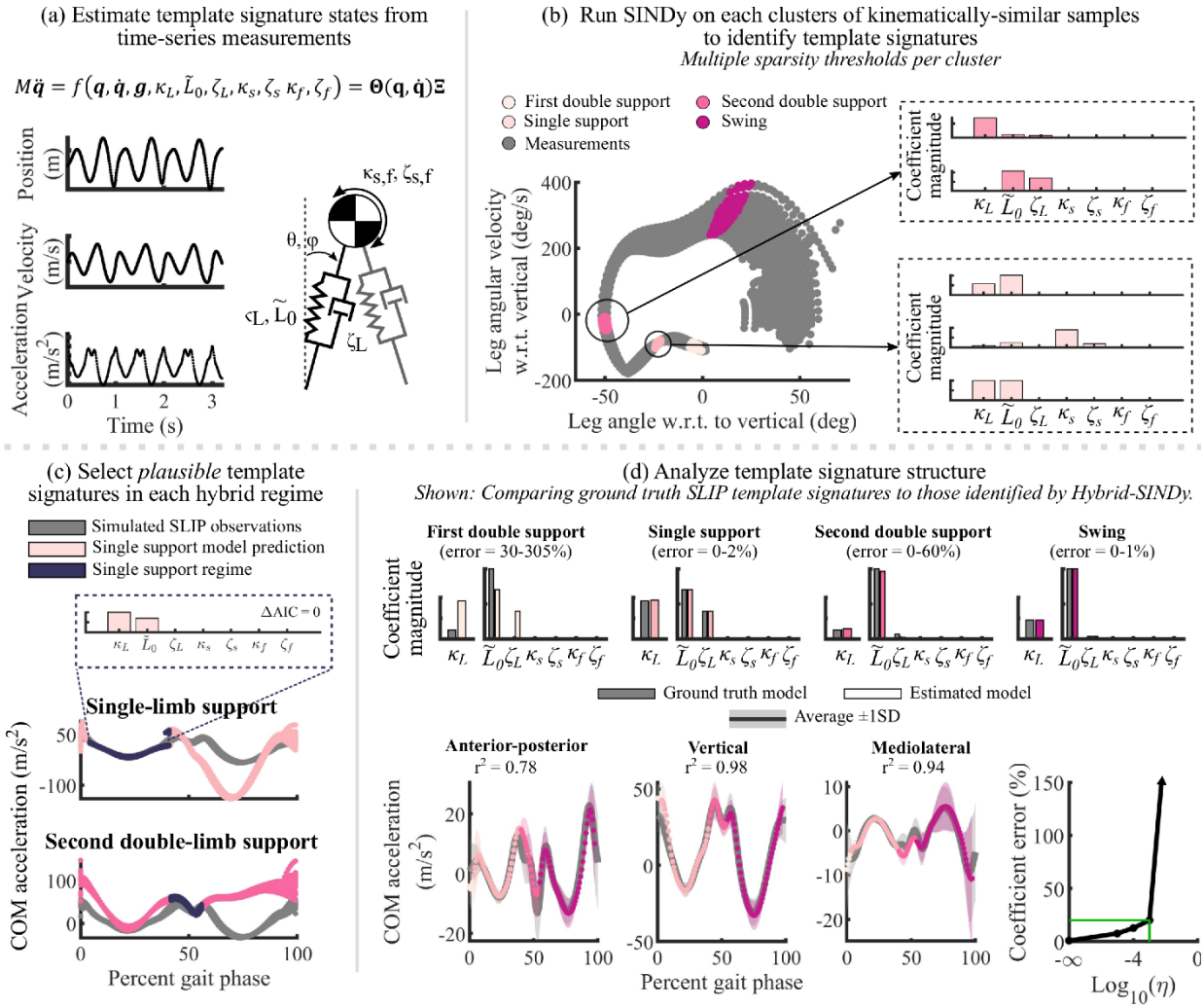

Figure 2 – The Hybrid-SINDy algorithm, applied to a simulated 3D-SLIP walking model. (a) COM and foot measurements from the simulated SLIP were used to approximate nonlinear SLIP states, $\boldsymbol{\Theta}(\boldsymbol{q}, \dot{\boldsymbol{q}})$, as shown in the equation and the SLIP diagram. Variables can be found in Table 1. (b) Kinematically-similar measurements were used to cluster the data (gray) and exemplary template signature parameters from two clusters. For each cluster, multiple signatures of different complexities were identified. (c) Each signature was evaluated in the hybrid regime containing the cluster centroid. Plots show COM accelerations and predictions in single-limb (top) and second double-limb (bottom) support. Error is low only in the correct hybrid regime for each signature (purple). (d) Top: Ground-truth (gray) and identified (colors) template signatures in each hybrid regime, with percent errors compared to ground-truth. Bottom: Average (±1SD) observed (gray) and predicted (colors) COM accelerations. Colors denote each hybrid regime. The rightmost plot shows the average percent error in template signature coefficients during single-limb support and swing with measurement noise (η). Green lines denote the approximate noise level of marker-based motion capture.

Template signatures mechanisms and dynamics

To model three-dimensional COM dynamics during walking, we created a function library of the following candidate mechanisms based on prior literature (Table 1):

- *Rigid legs*, which reflected pendular gait in stance [9, 12, 14, 18, 23]. Pendular gait is energetically passive and reflects kinetic and potential energy transfer during walking but cannot explain GRFs and lacks double-limb support phase [14, 21, 40].
- *Leg springs* ([13, 19-21, 25]) and *dampers* ([27]), which produce force along the leg. Leg springs are common energetically-conservative mechanisms used to describe walking and running dynamics, and enable a double-limb support phase. Leg dampers are less common, but have been used to capture non-conservative gait dynamics [27].
- *Rotary springs* ([13, 19]) and *dampers* in the sagittal and frontal planes, which enable forcing transverse to the leg axis. The addition of a rotary springs has been show to improve reconstructions of anterior-posterior GRFs in a bipedal SLIP [19]. We did not identify rotary damping elements in prior literature but include them as a candidate mechanism describing non-conservative transverse forcing.

The mechanisms selected by the Hybrid-SINDy algorithm defines a template signature's *structure*, which describes distinct strategies to accelerate the COM. The identified signature *parameters* describe each mechanism's contribution to COM accelerations. The dynamics of a three-dimensional bipedal SLIP augmented with damping and rotary mechanisms may be found in *Supplemental – S1*.

Normalized template mechanisms

To account for inter-individual differences in walking speed and body size during analysis, we normalized the template signatures during analysis (Table 1). Leg stiffness (k_L) was normalized as in [17, 19, 32]. Leg resting length (L_0) was normalized to the measured leg length [19, 27]. Rotary stiffness in the sagittal (k_s) and frontal (k_f) planes was normalized according to [19]. All damping terms (c_L, c_s, c_f) were converted to damping ratios [27]. The normalized leg, sagittal-plane, and frontal-plane stiffness mechanisms are denoted by κ_L, κ_s, and κ_f, respectively. The normalized leg, sagittal-plane, and frontal-plane damping mechanisms are denoted by ζ_L, ζ_s, and ζ, respectively. Normalized leg length is denoted $\tilde{L}_0$. The normalized coefficients for the right (R) and left (L) legs constitute $\boldsymbol{\Xi}$ in equation 2:

$$M(\ddot{\boldsymbol{q}} - \boldsymbol{g}) = [\kappa_L \quad \kappa_L\tilde{L}_0 \quad \zeta_L \quad \kappa_s \quad \zeta_s \quad \kappa_f \quad \zeta_f]_j\, \boldsymbol{\Theta}(\boldsymbol{q}, \dot{\boldsymbol{q}}), \quad j = \{Right, Left\}, \tag{3}$$

where M denotes body mass and we have subtracted gravity, g, from the dynamics. The COM position and velocity relative to the feet were used to compute candidate template signature states: leg lengths and lengthening velocities, sagittal-plane leg angles and angular velocities relative to vertical, and frontal-plane leg angles and angular velocities relative to vertical. The complete function library can be found in *Supplemental – S1*.

Table 1 – Template signature functional forms.

Term	Symbol	Normalized form
Leg stiffness	κ_L	$\frac{k_L L_{bio}}{Mg}$
Leg resting length	$\tilde{L}_0$	L_0/L_{bio}
Leg damping	ζ_L	$\frac{c_L}{2\sqrt{k_L M}}$
Sagittal-plane rotary stiffness	κ_s	$\frac{k_s}{MgL_{bio}}$
Sagittal-plane rotary damping	ζ_s	$\frac{c_s}{\sqrt{\kappa_s M L_{bio}^2}}$
Frontal-plane rotary stiffness	κ_f	$\frac{k_f}{MgL_{bio}}$

Frontal-plane rotary damping	ζ_f	$\frac{c_f}{\sqrt{\kappa_f M L_{bio}^2}}$

c_L=leg damping; c_s=sagittal-plane rotary damping; c_f=sagittal-plane rotary damping; g = gravitational acceleration; k_L=leg stiffness; k_s=sagittal-plane rotary stiffness; k_f=frontal-plane rotary stiffness; L_{bio} biological leg length; M = body mass

Evaluating Hybrid-SINDy's ability to select template signatures

To evaluate Hybrid-SINDy's ability to accurately identify and select walking dynamics, we evaluated the algorithm on synthetic data of a simulated bipedal SLIP walker with known dynamics (Figure 1 & Figure 2) [25]. The bipedal SLIP had asymmetric leg stiffness and damping, and masses representing the pelvis and feet to simulate a full gait cycle. Simulated SLIP parameters can be found in *Supplemental – S1*. We omitted rotary mechanisms from the SLIP dynamics but used the full function library described in equation 3. We simulated 120 gait cycles from randomly perturbed initial conditions with an average initial velocity of 1.20 m/s. We tested Hybrid-SINDy's ability to identify template signatures in the presence of sensor noise, by adding Gaussian noise, ranging logarithmically from $10\ nm$ to $10\ cm$, to the position measurements [30]. We quantified the Hybrid-SINDy's ability to reconstruct model dynamics using the number of correctly-selected mechanisms and the accuracy of estimated coefficients.

Identifying template signatures in human gait

We estimated unique template signatures that predicted COM accelerations using the Hybrid-SINDy algorithm for each participant and ankle exoskeleton condition. We used 10800 samples (90 seconds) were available for clustering and we identified template signatures for the first 3600 samples. An additional 3600 samples of held-out data were used for model evaluation and selection.

To evaluate the extent to which each mechanism described COM accelerations in unimpaired adults, we determined the proportion of participants for whom each template signature coefficient was selected. Signature terms identified across individuals represent common mechanisms describing COM dynamics, while infrequently identified mechanisms describe individual-specific features of COM dynamics.

To determine if unimpaired COM dynamics during shoes-only walking generalized to walking with ankle exoskeletons, we evaluated the ability of shoe-walking template signature structures to reconstruct COM accelerations in the K_0 and K_H conditions. We used least-squares regression to identify template signature parameters for the K_0 and K_H conditions using the shoes-only template signature structure. To determine if shoes-only template signatures were less plausible (*i.e.*, $\Delta AICc \geq 3$) than signature structures selected for the K_0 and K_H conditions, we compared shoes-only $\Delta AICc$ scores to those of the K_0 and K_H conditions using one-sample paired t-tests ($\alpha = 0.05$).

To determine if ankle exoskeleton frame or mass impacted COM dynamics, we compared template signature coefficients between the K_0 and Shoe conditions. Similarly, to evaluate the impacts of exoskeleton stiffness on COM dynamics, we compared template signature coefficients in the K_H and K_0 conditions. For both comparisons, we used paired two-sample t-tests with Holm-Sidak step-down corrections for multiple comparisons ($\alpha = 0.05$) [41]. Because shoes-only template signature structures were plausible for most unimpaired participants, we used the shoes-only template signatures structure to estimate signature coefficients across walking conditions.

To determine if COM dynamics may be altered post-stroke, we computed the percent difference in the non-paretic and paretic limb template signature parameters during shoes-only walking in one individual

with post-stroke hemiparesis. We also evaluated changes in post-stroke COM dynamics with ankle exoskeletons by computing percent changes in template signature coefficients for the K_0 condition compared to the shoes-only and K_H conditions.

Results

The Hybrid-SINDy algorithm correctly identified the salient features of the simulated SLIP's template signature during single-limb support (Figure 2). Specifically, Hybrid-SINDy identified non-zero leg stiffness (κ_L), resting length ($\tilde{L}$), and damping (ζ_L) terms only – consistent with the SLIP's dynamics. In the noise-free condition, 86% of terms were correctly selected and only one term, leg damping in first double-limb support, was incorrectly selected. Hybrid-SINDy incorrectly omitted only damping terms with small coefficients (see *Supplemental – S1*). Without measurement noise, in single-limb support and swing, coefficients were within 2% of the ground truth values. When measurement noise with a standard deviation of 1 mm – similar to that of our motion capture system – was added to the measurements, Hybrid-SINDy correctly selected only 63% of mechanisms. Incorrect mechanism selection occurred primarily in double-limb support, which spanned only 1.3-4.6% of a stride – a range smaller than the 7.4% of the training data in each cluster. With noise, template signature coefficients were estimated with an average error of less than 29% in single-limb support and swing (Figure 2). In single-limb support, Hybrid-SINDy was still able to estimate salient coefficients (*i.e.*, leg stiffness and resting length) with less than 1% error.

When walking in shoes-only, template signatures reveal individual-specific and common strategies to accelerate the COM. SLIP-like signatures best described COM dynamics across unimpaired participants. In both double-limb and single-limb support, SLIP mechanisms – leg stiffness and leg length – were selected in 96-100% of limbs (Figure 3). Unimpaired template signatures were not significantly different between legs (paired 2-sample t-test; $p > 0.01$). However, template signatures varied during double-limb support: Rotary stiffness terms were selected in 58-67% of limbs in the first double-limb support and 83-95% of limbs during second double-limb support. Except for sagittal-plane damping, damping terms were selected in less than 20% of limbs. As expected during swing, when the limb should not contribute to COM accelerations, no terms were selected. We omit the swing regime from remaining analyses.

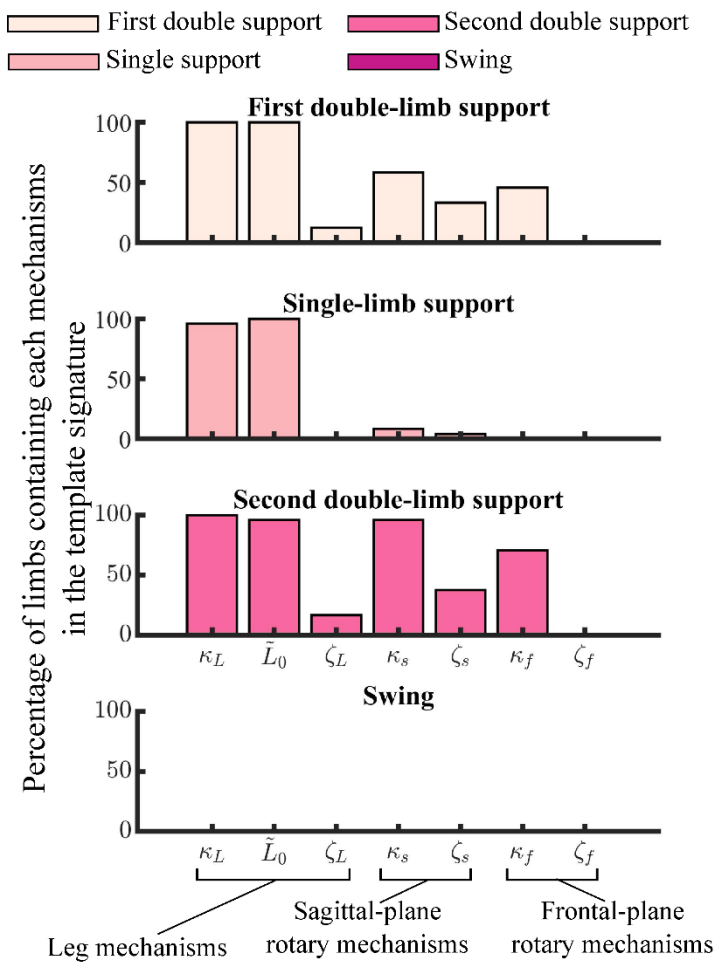

Figure 3 – The percentage of unimpaired limbs (24 limbs) whose template signatures contained each mechanism in each hybrid regime. Colors denote each hybrid regime. Mechanisms selected in a larger percentage of legs suggest common representations of COM dynamics, while less frequently-selected mechanisms reflect individual-specific features describing COM dynamics.

For each participant, template signatures were robust to stride-to-stride variations during single-limb support: stiffness and leg length exhibited coefficients of variation (CVs) less than 0.021 ± 0.015. In both double-limb support phases, CVs of stiffness and sagittal-plane rotary damping ranged from 0.11 – 0.26. Less frequently selected mechanisms in double-limb support – leg damping and frontal-plane damping – were highly variable, with CVs ranging from 0.082 – 0.442.

Across unimpaired participants, single-limb support leg length was most consistent (CV = 0.03), while leg stiffness was more variable (CV = 0.28; Figure 4; top). However, interindividual variability in double-limb support leg and rotary stiffness was larger (CV = 0.43 – 1.94). Despite this variability, template signatures reconstructed participants' COM accelerations well, 32-85% of the variance in 3D COM accelerations (Figure 4; bottom).

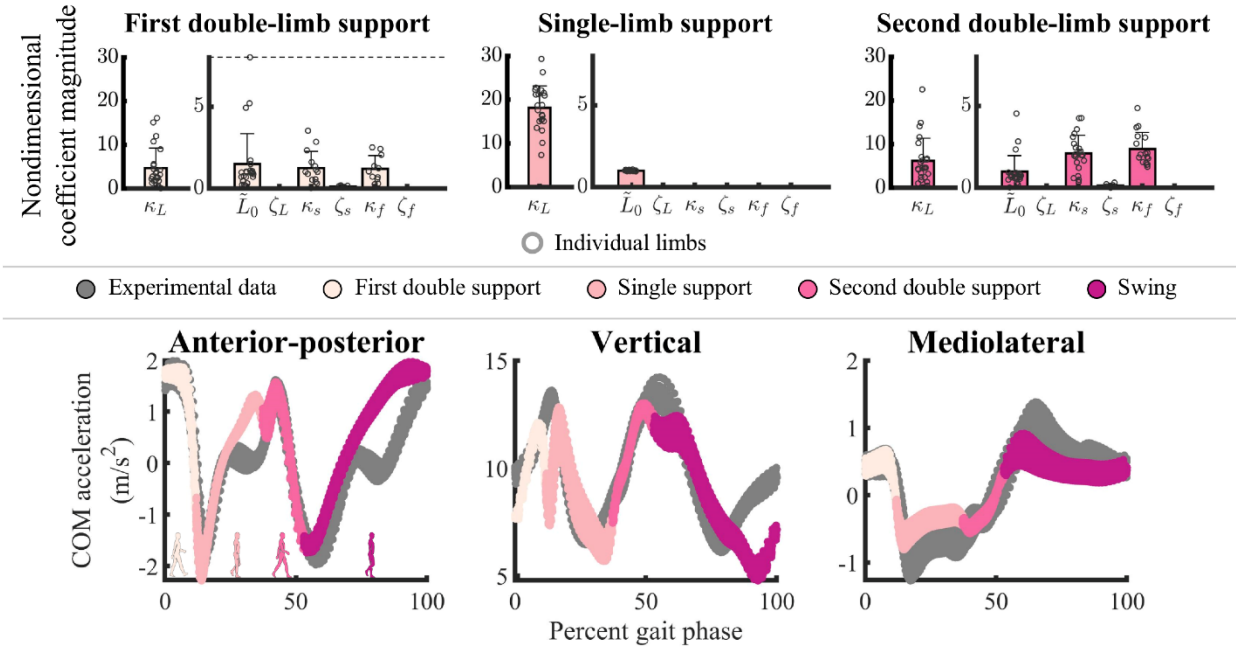

Figure 4 – Normalized template signatures (top) and reconstructed COM accelerations (bottom) for shoes-only walking in each hybrid regime. Top: Bars denote the average template signature (+1SD) in single- and double-limb support. The small circles represent individual limbs. Note that we omitted mechanisms that were selected in less than 25% of participants. The dashed lines truncate large terms for clarity. Bottom: Experimental and predicted COM accelerations from the test dataset of an exemplary unimpaired participant. The gray dots denote the experimental accelerations, while the colors correspond to predictions in each hybrid regime.

Passive ankle exoskeletons elicited only small changes in unimpaired COM dynamics. The shoes-only template signature structures reconstructed K_0 and K_H COM dynamics with similar accuracy to the K_0 ($\Delta AICc = 0.2 \pm 0.7$; p = 0.29) and K_H ($\Delta AICc = 0.7 \pm 2.9$; p = 0.24) signature structures, respectively, in single-limb support. Conversely, in double-limb support, shoes-only signatures did not consistently reconstruct K_0 or K_H COM dynamics, with relative AICc scores ranging from $-118 \leq \Delta AICc < 220$ (Figure 5; top). The negative relative AICc scores occurred when the shoes-only template signature structure was never identified by Hybrid-SINDy in the exoskeleton conditions. Therefore, to compare signature coefficients between exoskeleton conditions, we constrained each participant's template signatures to that selected in the shoes-only condition.

Using this approach, we found that no unimpaired template signature coefficients changed significantly when walking in shoes-only, zero-stiffness exoskeletons (K_0), and stiff exoskeletons (K_H) ($p > 0.05$). Specific to our hypothesis, we found that neither ankle exoskeleton mass and frame (K_0) nor stiffness (K_H) had a significant impact on leg stiffness in any gait phase ($p > 0.13$) (Figure 5; bottom).

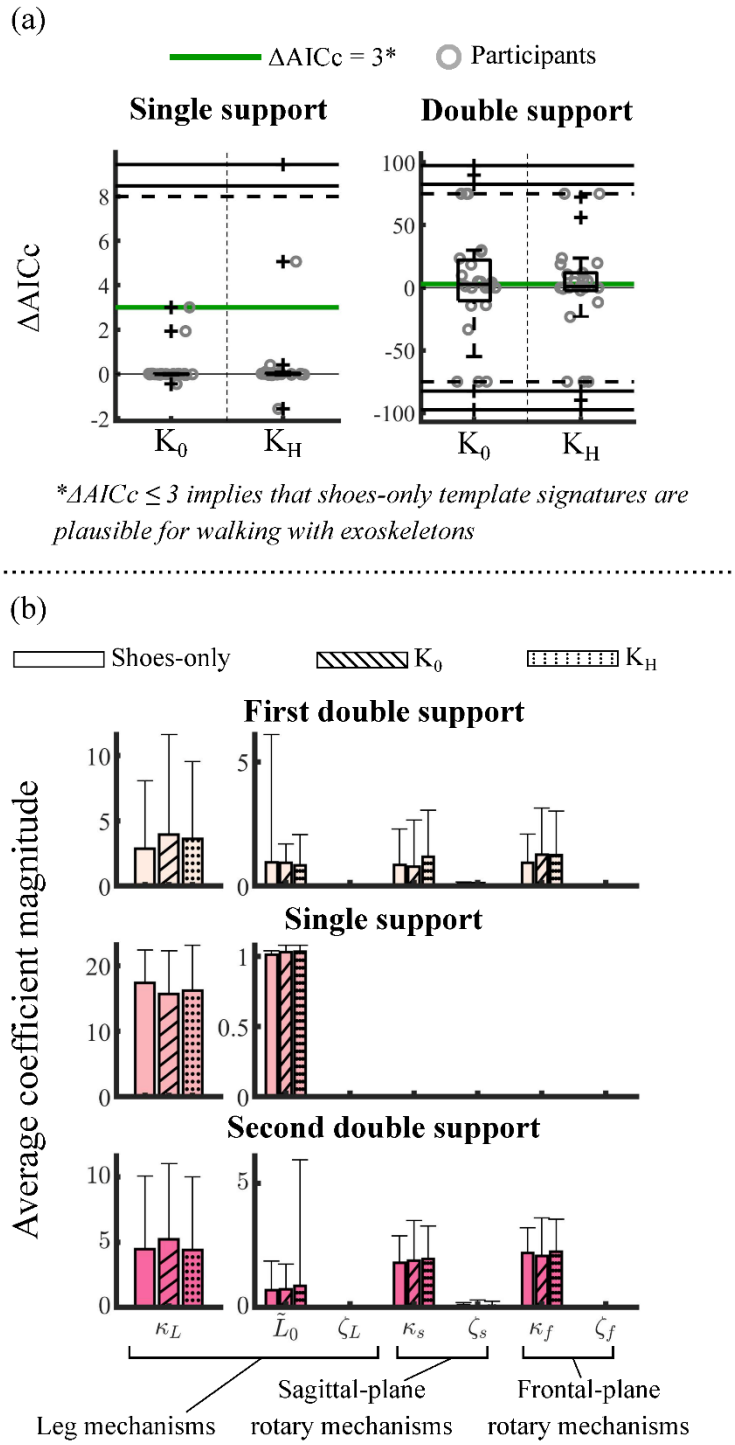

Figure 5 – (a) Relative AICc ($\Delta AICc$) between template signatures identified specifically for the K_0 and K_H conditions, and signatures constrained to the shoes-only signature structures. Positive $\Delta AICc$ values indicate that shoes-only signature structures were less plausible than those identified specifically for each condition. $\Delta AICc$ scores are shown for single-limb support (left) and double-limb support (right). $\Delta AICc \leq 3$ (green lines) denote plausible shoe template signature structures. For double-limb support, large $\Delta AICc$ scores are truncated at $\Delta AICc = \pm 75$ for clarity. (b) Template signatures of walking in

shoes only (solid bars), zero-stiffness (K_0; slashed bars) exoskeletons, and high-stiffness exoskeletons (K_H; dotted bars) during single- and double-limb support. Bars represent the average (+1SD) template signature across participants. Leg stiffness (κ_L) is included on a separate subplot for clarity. The large variability in leg length during double-limb support is due one limb with a high resting length.

During shoes-only walking, the stroke survivor's template signature revealed strategies for accelerating the COM that differed from unimpaired adults in double, but not single-limb support. The template signature structure was symmetric between the paretic and non-paretic limbs during single-limb support, consisting of only leg stiffness and resting length (Figure 6). However, the paretic limb was 31% stiffer than the non-paretic limb. Conversely, template signature structure differed in double-limb support, with rotary stiffness mechanisms improving descriptions of COM dynamics in only the paretic limb. During first double-limb support, leg damping and rotary stiffness were selected in only the paretic limb. In second double-limb support, both paretic and non-paretic rotary stiffness mechanisms were selected. Due to asymmetric template signature structures, we did not compare signature coefficients in double-limb support.

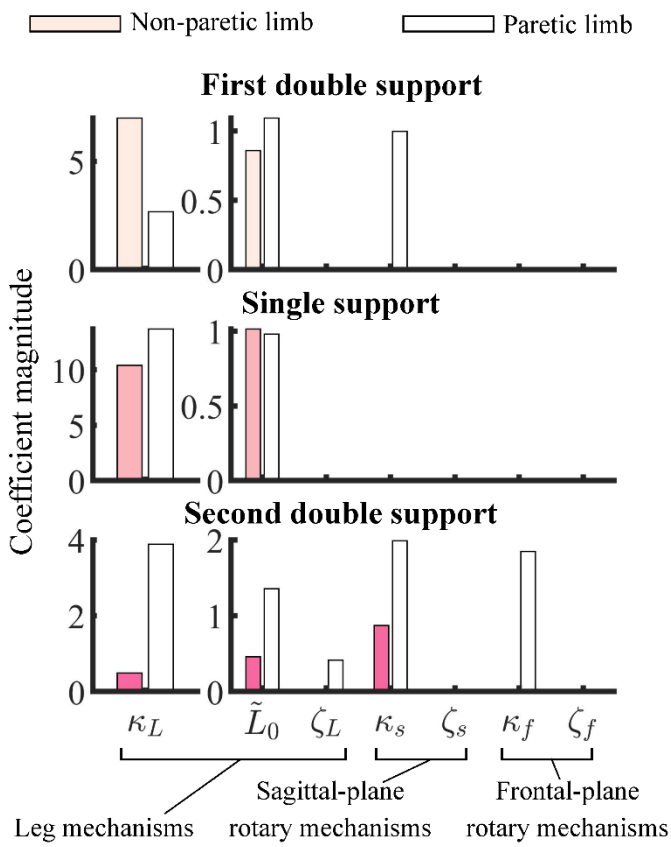

Figure 6 – Non-paretic (colored bars) and paretic (white bars) template signatures for one individual with post-stroke hemiparesis. This participant's template signatures differed in the paretic and non-paretic limbs in double-limb support, as shown by the zero-values for non-paretic sagittal-plane rotary stiffness in first double-limb support (top row) and the leg damping and frontal-plane stiffness in double-limb support (third row).

The ankle exoskeleton mass and frame (Shoe vs. K_0) impacted the stroke survivor's paretic leg stiffness and non-paretic leg rotary stiffness. Compared to walking in shoes-only, paretic leg stiffness was 11% greater in the zero-stiffness (K_0) exoskeleton condition in single-limb support and over twice as stiff in

double-limb support (Figure 7; slashed bars). Responses to the K_0 exoskeleton condition were asymmetric, with the non-paretic leg being slightly (3%) less stiff in the K_0 condition than the shoes-only condition in single-limb support, while sagittal-plane stiffness was 41% larger in second double-limb support. Conversely, exoskeleton stiffness (K_0 vs. K_H) had small impacts on paretic-leg template signatures. In the high-stiffness (K_H) exoskeleton condition, single-limb support paretic leg stiffness was only 1% larger than in the K_0 condition (Figure 7; dotted bars). Both paretic and non-paretic leg rotary stiffness were 11% greater in second double limb support than in the K_0 condition. Non-paretic leg stiffness was 49% smaller than with zero-stiffness (K_0) exoskeletons in single-limb support.

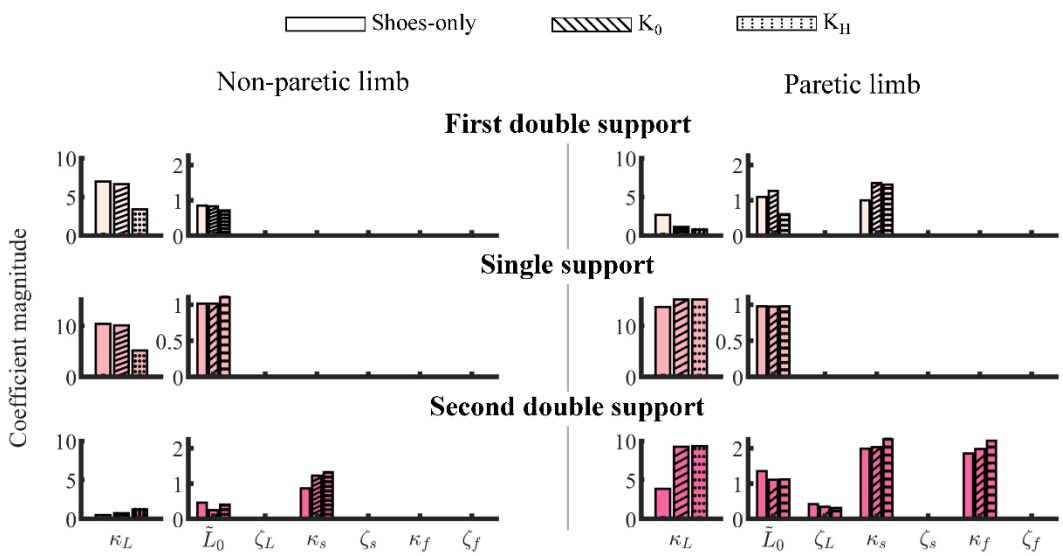

Figure 7 – Template signatures of a stroke survivor walking in the shoes-only (clear bars), zero-stiffness (K_0; slashed bars), and high-stiffness (K_H; dotted bars) ankle exoskeleton conditions for the non-paretic (left) and paretic (right) limbs. Bars represent the average template signatures over 200 bootstrapped model fitting iterations. Colors correspond to each hybrid regime.

Discussion

We evaluated the impacts of passive ankle exoskeletons on individual-specific COM dynamics – described by template signatures – using a recently-developed data-driven modeling framework, Hybrid-SINDy. To our knowledge, this is the first study to use data-driven identification of hybrid dynamics to characterize the impacts of ankle exoskeletons on COM dynamics. Using simulated SLIP dynamics, we showed that COM dynamics identified using Hybrid-SINDy were robust to measurement noise comparable to marker-based motion capture ($\eta = 1\ mm$). Therefore, the template signatures identified by Hybrid-SINDy for human gait likely encoded meaningful information about exoskeleton impacts on COM dynamics. Contrary to our hypothesis, leg stiffness did not change in response to the ankle exoskeletons in unimpaired adults. Further, because shoes-only template signatures were also plausible during walking with exoskeletons, it is likely that unimpaired COM dynamics are robust to altered ankle constraints due to passive ankle exoskeletons, despite changes in kinematics and muscle activity [35]. Similarly, Collins and colleagues (2015) observed small changes in total COM power with passive ankle exoskeletons compared to walking in shoes-only, but larger changes in biological contributions to COM power [5]. However, the authors did not explain how changes in COM energetics occurred. Our results suggest that these small changes in COM power were driven by altered COM *kinematics*, not *dynamics*. Maintaining COM dynamics with different ankle exoskeleton properties requires modulating joint or muscle-level (*i.e.*, execution-level) dynamics [5, 7, 35, 42]. Therefore, quantifying individual-specific trade-offs between the modulation of task-level and execution-level dynamics may improve our understanding of responses to exoskeletons or other assistive devices.

Unlike unimpaired adults, ankle exoskeletons impacted one stroke survivor's COM dynamics. The addition of exoskeleton mass and frame (K_0) increased paretic limb sagittal-plane rotary stiffness in first double-limb support and leg stiffness in single-limb and second double-limb support. Increased paretic-leg stiffness is likely due to the exoskeleton frame restricting ankle inversion, which the participant noted during data collection. The participant's paretic ankle was severely affected, such that restricting frontal-plane ankle motion may have increased leg stiffness by stiffening the ankle. This result emphasizes the importance of quantifying the impacts of ankle exoskeleton properties beyond flexion stiffness, such as restriction to frontal-plane ankle motion, in explaining exoskeleton impacts on gait following neurological injury [1, 5, 43].

The insensitivity of the paretic limb template signatures to exoskeleton stiffness during single-limb support suggests that, for this individual, exoskeleton stiffness did not alter body weight support strategies [44]. It is possible that restricting ankle inversion was the primary driver of changes in overall leg stiffness. That is, exoskeleton dorsiflexion stiffness did not further increase the paretic ankle's contribution to body-weight support. Conversely, during double-limb support, paretic-leg rotary stiffness increased compared to the zero-stiffness exoskeleton condition. Increased rotary stiffness implies that exoskeleton stiffness primarily altered the dynamics of forcing transverse to the leg, potentially altering propulsion and stabilization dynamics by serving a similar role to the paretic ankle plantarflexors [45-47]. Note that, while these findings suggest that exoskeleton stiffness impacted our participant's COM dynamics corresponding to stabilization and propulsion, this result is unlikely to generalize across stroke survivors [2]. Specifically, exoskeleton impacts on COM dynamics may depend on an individual's ability to convert exoskeleton assistance into functionally relevant forcing. While limited to a case study, our results highlight the potential contributions of ankle exoskeleton frame and stiffness on post-stroke COM dynamics, as described by template signatures.

Both unimpaired and post-stroke template signatures highlighted inter-individual and inter-limb differences in COM dynamics. While SLIP mechanisms described COM dynamics in all unimpaired limbs, rotary mechanisms were selected in only 50% of limbs during double-limb support. The uniform selection of SLIP mechanisms from data is consistent with common template walking models and

supports the common perspective that elastic legs are foundational mechanisms for describing COM accelerations during walking across individuals [12, 20, 21, 25, 48]. Our observation that rotary mechanisms were not critical to reconstructing COM accelerations for all individuals may explain their less frequent application in template walking models [13, 19]. Note that each template signature mechanism describes characteristic coordination patterns between leg kinematics and COM accelerations. It is possible SLIP mechanisms describe coordination patterns necessary for stable or efficient walking, while rotary mechanisms describe coordination patterns that have smaller, more individualized impacts on gait. Determining which individual-specific characteristic patterns are predictive of exoskeleton responses or walking function represents important areas of future research.

Template signatures also revealed compensation strategies in one stroke survivor. Consistent with template-based studies in children with cerebral palsy, the stroke participant's paretic leg was stiffer than the nonparetic leg in single- and second double-limb support [28, 29]. Increased paretic leg stiffness may reflect a more pendular gait post-stroke and increased paretic-leg co-contraction [49]. In first double-limb support, sagittal-plane rotary stiffness and damping suggest altered braking dynamics during loading response. This possibility is consistent with experimental observations that the paretic-leg does greater negative work than the non-paretic limb during loading response in individuals post-stroke [11]. These results highlight the functional interpretability of data-driven template signatures and the importance of personalized parameterizations of COM dynamics for elucidating inter-limb differences in COM dynamics.

Our use of the Hybrid-SINDy algorithm ([32, 33]) was essential to discovering individual-specific and limb-specific changes in COM dynamics with ankle exoskeletons. By automatically identifying and selecting different first principles-based representations of COM dynamics, we rapidly compared competing hypotheses about human gait dynamics. Conversely, manually comparing all possible template signatures for our fourteen-dimensional mechanism library would require a combinatorially-large number of models to be generated, fit, and compared. Using a principled approach to model identification and selection, Hybrid-SINDy selected dynamics that are consistent with literature in a fraction of the time required to manually compare each candidate model.

To our knowledge, only one other study in bipedal locomotion has taken a similar approach [50]. The authors performed a multi-layer optimization to select human-like template dynamics for walking robot controllers. Our approach extends their work by using information criteria to enable multiple highly-plausible representations of walking dynamics to be selected and analyzed for an individual [34, 39]. For most participants, the Hybrid-SINDy algorithm selected a unique template signature, indicating that alternative hypothetical representations of COM dynamics lacked strong statistical support. By identifying multiple plausible template signatures for an individual, future experiments may be designed to better differentiate between plausible signatures. To our knowledge, this work is the first to apply Hybrid-SINDy to real data and the first to apply it to human gait.

Since reduced-order models are often sensitive to measurement noise and inevitably incorrect model dynamics, we used a simulated SLIP with known dynamics to evaluate the extent to which Hybrid-SINDy correctly identified template signatures during walking. Hybrid-SINDy performed as expected in the noise-free condition, accurately identifying single-limb support and swing dynamics, except for small damping terms. Terms with small contributions to system behavior, such as damping in this study, may be omitted by the AICc in favor of parsimony [34, 39]. An important implication of this result is that our approach will not always select the true system dynamics but will select terms critical to describing salient behavior of the system. Therefore, designing experimental protocols capable of probing candidate mechanisms is critical to accurately identifying walking dynamics from data. For example, gait patterns that induced large leg velocities may result in damping terms having a larger impact on COM dynamics and being correctly selected by Hybrid-SINDy. Additionally, reduced simulated SLIP template signature

accuracy in double-limb support and with increasing noise magnitude highlights the importance of collecting long time-series datasets to ensure that clusters are small enough to span only a single hybrid regime but large enough to remain robust to sensor noise [32].

This study's limitations motivate future developments for data-driven modeling of human locomotion. First, human gait dynamics are continuously phase-varying rather than hybrid, such that template signature parameters varied continuously across clusters. However, preliminary analyses found candidate models' abilities to reconstruct COM dynamics diverged near the transitions between single and double-limb support, suggesting that changes in dynamics between foot contact configurations are larger than changes within configurations. Consequently, pre-defining hybrid regimes based on contact configuration was both appropriate and consistent with existing hybrid template models of walking [19, 20, 25, 32]. Second, we evaluated only a subset of mechanisms describing COM dynamics, which may not encode all important features of human gait. However, we chose a broad class of physically-meaningful mechanisms from literature ([17, 19-21, 25, 27]) that should motivate future studies using larger mechanism libraries describing COM dynamics. For example, our approach may be extended to include elements describing assistive devices or joint-level mechanisms. Finally, our limited sample size may have masked exoskeleton impacts on unimpaired gait. Particularly, rotary stiffness appeared to increase slightly with exoskeletons but did not reach significance. Similarly, our post-stroke case study only provides a proof of concept highlighting the potential of ankle exoskeletons to elicit changes in COM dynamics. Evaluating template signatures on larger unimpaired and post-stroke cohorts may provide additional insight into how COM dynamics change with ankle exoskeletons.

This work presents an early application of potentially transformative techniques in the study of human movement. Integrating principles from biomechanics and neuromechanics with state-of-the-art *interpretable* data-driven modeling techniques may accelerate the discovery of mechanisms underlying diverse movement patterns in human locomotion. Broadly, these techniques may be leveraged to study any intervention or walking condition and may inform the design of prosthetic limbs and other assistive devices. A natural extension of this work is to investigate the impacts of powered exoskeletons on COM dynamics. Changes in gait kinematics and kinetics with our passive ankle exoskeletons were small compared to changes with powered exoskeletons, which may elicit larger changes in COM dynamics [2, 6, 35]. As powered exoskeleton research shifts its focus from reducing metabolic demand to assisting other locomotion objectives, such as stability or agility, understanding changes in COM dynamics with exoskeletons may provide insight into how individuals leverage exoskeletons to realize these goals [51]. Particularly in a clinical context, quantifying changes in COM dynamics with exoskeletons may improve our understanding of the diverse responses to exoskeletons observed in individuals with neurological injuries [1-3]. For example, McCain and colleagues (2019) identified limb orientation as a potential mechanism impacting the conversion of exoskeleton torque to forward propulsion post-stroke [2]. Our results in one stroke survivor suggest that characterizing altered coordination between leg orientation and ground reaction forces (*i.e.*, template signatures) may unveil sub-classes of exoskeleton responses, enabling device customization to assist individuals in each sub-class. Finally, using similar physics-informed data-driven approaches to *anchoring* template signatures in physiological detail may reveal biological mechanisms that characterize responses to exoskeletons or other assistive devices [13, 15].

Conclusions

We quantified changes in individual-specific COM dynamics in response to passive ankle exoskeletons using a novel physics-informed data-driven modeling framework, Hybrid-SINDy. Hybrid-SINDy successfully constructed characteristic template signatures during walking by identifying salient mechanisms describing an individual's COM accelerations. Unimpaired adults maintained nearly invariant COM dynamics with ankle exoskeletons, while exoskeletons altered COM dynamics in an individual post-stroke. These findings provide insight into task-level control of walking dynamics with ankle exoskeletons and extend a powerful data-driven modeling tool to human locomotion.

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Data availability

All data and code used in this manuscript are available at: https://simtk.org/frs/admin/?group_id=1939.

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