

1 **Decision heuristics in contexts exploiting intrinsic skill**

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14 **Abstract**

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17 Heuristics can inform human decision making in complex environments through a reduction of
18 computational requirements and a robustness to overparameterisation. However, tasks capturing
19 the efficiency of reduced decision dimensionality typically ignore action proficiency in
20 determining rewards. The value of movement parameterisation in sensorimotor control questions
21 whether heuristics preserve efficiency when actions are non-trivial. We developed a novel
22 selection-execution task requiring joint optimisation of action selection and spatio-temporal skill.
23 Optimal choices could be determined by either a spatio-temporal forward simulation or a simpler
24 spatial heuristic. Sequential-sampling models of action-selection response times parsimoniously
25 distinguished human participants who adopted either strategy. Heuristics preserved broad
26 decisional advantages over forward simulations. In addition, heuristics aligned with greater action
27 proficiency, though predominantly through the core feature (spatial) shaping their decision policy.
28 We accordingly reveal evidence that the dimensionality of information guiding action selection
29 might be yoked to the granularity of plasticity in the motor system.

30 **Introduction**

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33 In naturalistic settings, our cognitive architecture for making goal-oriented decisions typically
34 resolves an ecological utility problem, integrating both extrinsic and intrinsic dynamics.
35 Extrinsicly, selected actions should maximise reward capture in line with a complex external
36 state - a soccer player in possession of the ball must select the most rewarding action (shoot or
37 pass) by incorporating such parameters as their location relative to the goalposts, availability of
38 teammates, wind direction, readiness of the opposition goalkeeper, and so on. While the player
39 might base their decision on exhaustive forward simulations across all possible actions, such a
40 highly dimensional external state likely favours some manner of goal-oriented heuristic, i.e., a
41 decision formed from a subset of all available external state information (e.g., if within 10 metres
42 of the goalposts, shoot). Behavioural evidence verifies that a human decision policy can span
43 different levels of planning complexity, with emerging neural evidence further suggesting that the
44 brain harbours separate neural controllers for heuristics¹.

46 The logic underscoring heuristic adoption is at least two-fold. Heuristics first offer a trade-off
47 between accuracy and available resources. That is, where exhaustive forward simulations might
48 exceed computational resources or decision deadlines, heuristics offer a less laborious means to
49 achieve a proxy for optimal action-selection policy^{1,2}. An alternative "less-is-more" rationale,
50 inspired by machine learning principles, considers heuristics as the optimal means to avoid
51 overfitting in uncertain environments. That is, in uncertain environments, an overparameterised
52 forward simulation will likely pick up on stochastic noise and create more prediction errors across
53 choices than a function that uses fewer parameters, even if the latter function produces a biased
54 estimate³.

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56 However, much like the areas of reinforcement learning and value-based decision making,
57 evidence that humans adopt goal-oriented heuristics has emerged predominantly in contexts that
58 do not consider intrinsic cost as a determining factor in reward yields. For example, in recent
59 work¹, simple button presses in a virtual task emulated foraging outcomes that probabilistically
60 imparted a positive (partial increase), negative (partial decrease) or nonlinearly negative (complete
61 erasure) impact on ongoing reward scores; human participants adopted a heuristic stimulus-driven
62 policy that primarily avoided the nonlinear outcome, consistent with accuracy-resource trade-offs.
63 Meanwhile, the less-is-more principle has been empirically supported in forecasting contexts such
64 as weather³, investments⁴ and sporting events⁵. Simple-action probabilistic emulations and
65 forecasting can innovatively replicate much of the extrinsic reward-oriented cognitive challenges
66 presented by dynamic naturalistic environments, however, they probe only one side of the
67 ecological utility dilemma. Lost in both paradigm formats are additional cost dimensions
68 associated with effort^{6,7}, motor plasticity^{8,9}, and a broader sense of agency¹⁰, all of which integrate
69 with external factors in the ultimate utility of selected actions in naturalistic settings^{11,12}.

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71 To our knowledge, no study has characterised heuristic adoption by humans when they select state-
72 appropriate actions in selection-execution contexts, i.e., not only is there a correct action for a
73 given state, but the proficiency of a selected action subsequently scales the level of reward and
74 generates independent intrinsic error distributions such as spatial and temporal motor skill.
75 According to sensorimotor control theory, such intrinsic error distributions are often attenuated by
76 increasing the parameterisation of movement, e.g., by implementing forward-models or
77 simulations^{27,28,29}. This raises the question: do heuristics preserve their efficiency when actions are
78 non-trivial; or do action values derived from parameterised utility assessments justify preserved
79 use of forward simulations? We additionally do not know how decision heuristics relate to
80 phenotypic variation in skill. On the one hand, higher skill should improve both the time to
81 generate, and the subsequent predictiveness of, forward simulations, potentially rendering
82 parameterisation a more rewarding option for higher-skilled individuals. However, an alternative
83 prediction stems from the computational underpinnings of how motor learning evolves. Here, the
84 commonly held view is that model-based deliberation dominates early in learning¹³, presumably
85 while skill levels are also at their lowest. Thus if heuristics reflect a "model-free" antipode to
86 model-based deliberation¹⁴, we might expect them to characterise planning in systems that have
87 reached greater proficiency with the plant's output.

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89 Given their dynamic, non-punctuative nature, naturalistic states likely require action-selection
90 deliberation to be a gradual process of evidence accumulation toward an action-deterministic
91 criterion. Indeed, for decades, such a sequential-sampling framework has guided joint modeling

92 of choice and the dynamics constraining its underlying response-time distribution, revealing
93 comprehensive accounts of decision formation in perceptual contexts¹⁵⁻¹⁸. A specific class of
94 sequential sampling, the drift-diffusion model (DDM), has more recently linked model-based
95 values to choice dynamics in reinforcement learning^{19,20} and value-based decision making^{21,22}. The
96 DDM approach has a potential benefit over discrete (e.g., logistic) models, stemming not just from
97 comprehensiveness²³, but also from increased reliability¹⁹ and robustness¹⁸. In the present work we
98 use a DDM framework for the first time with data from a selection-execution task to distinguish
99 people on their use of forward simulations or heuristics when selecting actions, and further
100 hypothesise that these two groups might differ in terms of how skillfully they perform them.

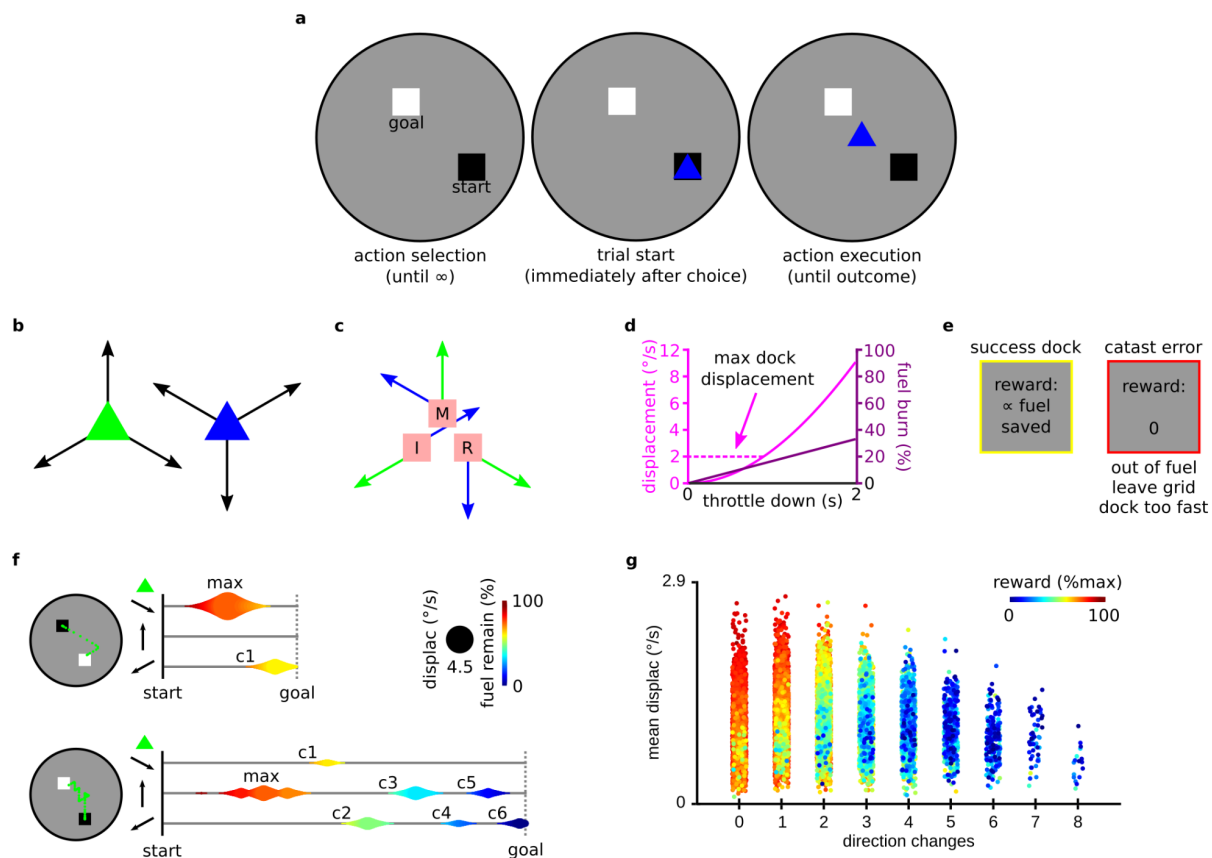
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102 The novel task we developed scales trialwise reward based on the joint optimisation of action
103 selection (between two possible actions) and subsequent action execution. Action execution is
104 indexed by a tractable reward function and action proficiency is also decomposable into spatial
105 and temporal error dimensions. Meanwhile, action selection can be guided by simulating reward
106 from each action (i.e., incorporating both spatial and temporal dynamics), or by a simpler spatial
107 heuristic. The forward and heuristic strategies therefore differ by a single, identifiable degree of
108 freedom (temporal dynamics). In addition, for each trial one possible action is low-cost, while the
109 other is high-cost. All environments can be solved with either action, however, certain
110 environments favour the high-cost action in terms of reward yield. These asymmetric action costs
111 first amplify the influence of effort, skill and agency. They further allow us to cull participants
112 who do not behave in a goal-oriented manner, i.e., rarely (or never) selecting the high-cost, high-
113 potential reward action. Thirdly, the degree of parametric bias toward the low-cost action offers a
114 novel complement to the commonly used approach of tracking intrinsic motor error as a means to
115 characterise low skill systems.

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117 Combining our novel task and seminal DDM framework, we first confirm that human participants
118 can be identified by whether they predominantly use forward simulations or heuristics, with
119 between-group parametric differences consistent with forward simulations requiring slower
120 evidence accumulation to more conservative decision criteria. We next uncover strong evidence
121 that heuristics remain efficient in action-execution contexts; heuristic planners made faster
122 decisions, reached choice optimality sooner and, overall, obtained more reward. In addition,
123 heuristic planners showed a striking skill profile. Overall, they demonstrated a higher level of
124 spatial skill, relative to participants using forward simulations. However, unlike the latter group,
125 heuristic participants demonstrated no learning across the task in the temporal domain. Their skill
126 learning therefore centred on the key dimension incorporated into their planning strategy.
127 Combined, our findings help unpack the broader dynamics of goal-oriented behaviour by revealing
128 the first evidence of heuristic efficiency in a selection-execution context and that a yoked
129 dimensionality might exist between planning and motor learning.

130 131 **Results**

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133 Fifty-three healthy human participants performed 360 trials (six runs of 60) of our novel
134 "boatdock" task (Figure 1), in which reward yields require joint optimisation of action selection
135 and action execution. On each trial, participants select one of two cursors to pilot between a
136 randomly drawn start-goal pairing (SG; Figure 1a). Each cursor accelerates continuously in three
137 unique orthogonal directions (Figure 1b), burning fuel any time an accelerator button (throttle -

138 Figure 1c) is down. One cursor imparts a higher motor execution cost via an incongruent key-
 139 mapping (Figure 1c). However, trialwise reward is contingent on fuel conservation, such that a
 140 selection policy that harnesses the relative orthogonality of the two cursors' directionality, and
 141 selects the cursor better suited to each SG, will yield higher reward. The two cursors accelerate
 142 with the same nonlinear function, and deplete fuel with the same linear function, i.e., faster
 143 displacement is more fuel efficient (Figure 1d). A maximum docking displacement rule (Figure
 144 1d), imposing a speed limit on arrival, imparts additional temporal control demands. Thus, in
 145 addition to fewer direction changes (spatial error), greater temporal control maximises reward
 146 (Figs. 1f, 1g). Finally, participants receive no reward for "catastrophic errors" (Figure 1e): when
 147 they run out of fuel, leave the grid, or attempt to dock above the maximum docking displacement.
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Figure 1 - Task outline. **a**, on each trial, participants pilot one of two cursors from
 152 a start to a goal (SG). **b**, each cursor can accelerate in three unique orthogonal
 153 directions. **c**, position of index (I), middle (M) and ring (R) finger of right hand on
 154 throttle buttons throughout the experiment, and cursor-specific throttle-vector
 155 mapping. Fuel burns any time a throttle is pushed down. Each trial allows six
 156 cumulative seconds of throttling before fuel depletes. **d**, throttle time linearly burns
 157 fuel, but nonlinearly increases displacement. Faster displacement is therefore more
 158 fuel efficient, however, a maximum dock displacement imparts additional temporal
 159 control requirements. **e**, successful docks yield a reward contingent on fuel
 160 conservation. This requires jointly maximising cursor choice for a given SG (action
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162 selection) in addition to spatial and temporal precision (action execution). Trials
163 containing catastrophic errors - running out of fuel, leaving the grid, or docking
164 above maximum displacement - yield no reward. **f**, schematic of two similar SGs
165 with the same cursor but different performance dynamics. Three horizontal lines in
166 each panel chart activity over time separately for each vector, while each vortex
167 relates to a single throttle pulse. Top panel utilises fewer direction changes (marked
168 with c_1, \dots, c_n), reaches a higher maximum displacement (depicted by diameter of
169 largest vortex) and yields higher reward (depicted by colour). **g**, reward (depicted
170 by colour), yielded on every successful trial across all participants (individual
171 markers), is a joint function of spatial and temporal precision.
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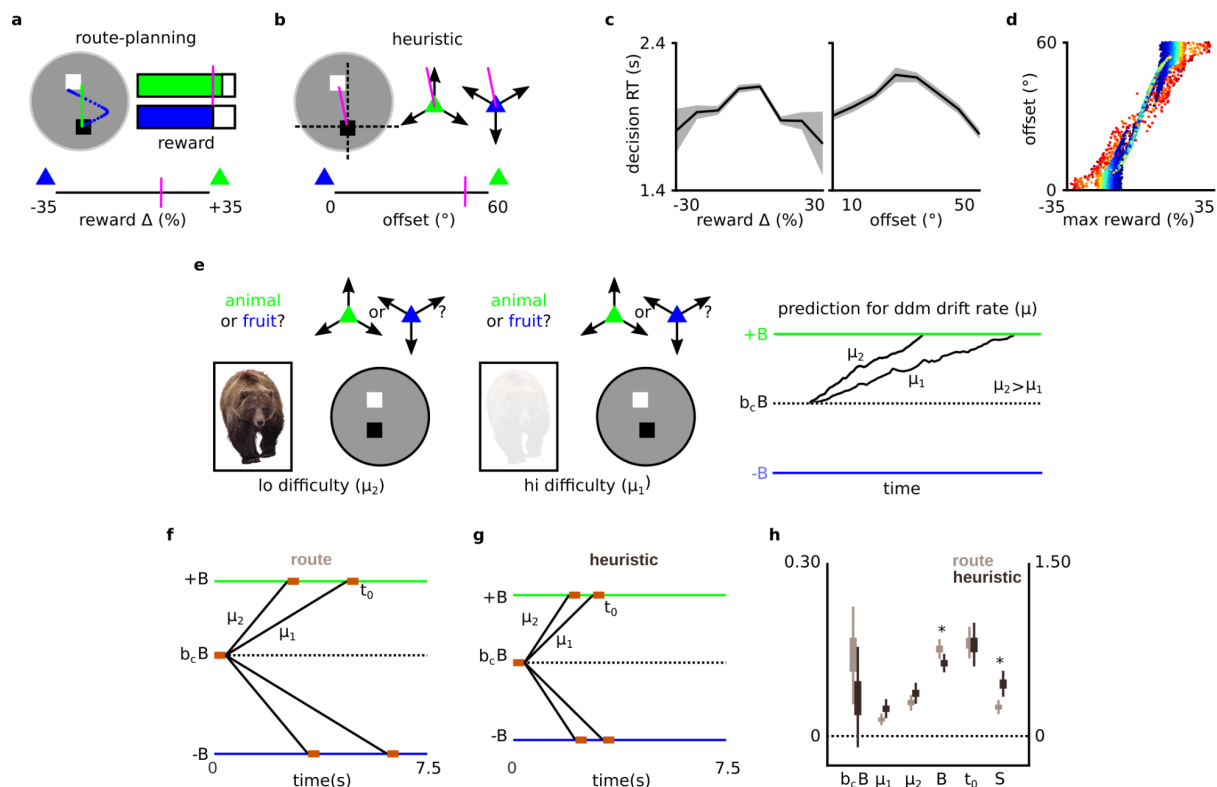
173 *Forward simulations vs heuristics*

174 We hypothesised that participants might select the cursor yielding the highest reward from forward
175 modeling of simulated routes, i.e., incorporating both the spatial and temporal constraints on
176 reward optimisation into their choice (route planning; Figure 2a). Alternatively, participants might
177 use a simple rule: select the cursor with a vector subtending the smallest angular offset to that of
178 the SG, i.e., incorporating only spatial constraints on reward optimisation into their choice
179 (heuristic; Figure 2b). Differences between these two approaches result in an imperfect correlation
180 of action value across all trials. Specifically, the nonlinear nature of the temporal dynamics
181 uniquely incorporated by route planning creates greater action-value deviance on trials where the
182 SG covers smaller Euclidean distances (Figure 2d). Across all participants, the reaction time (RT)
183 for action selection was slower in SGs where either strategy computed equivalent value for either
184 cursor (Figure 2c), suggesting first that these policies modulate decision formation, and secondly,
185 that difficulty or conflict (known to modulate drift parameter (μ) in perceptual contexts) could be
186 further characterised in a DDM model.
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188 *DDM framework distinguishes individual-participant strategy*

189 The DDM (Figure 2e) describes a noisy sequential sampling process, which originates at a starting
190 point ($b_c B$), and accumulates evidence at an average "drift" rate (μ) before reaching a decision
191 criterion or "boundary" (B or $-B$ for congruent or incongruent cursors, respectively). In perceptual
192 contexts, difficulty reduces the gradient of evidence accumulation, which can be verified
193 computationally when models containing two drift rates (e.g., μ_1 and μ_2) mapping respectively
194 onto decisions presenting a high or low degree of difficulty, provide better model fits (Figure 2e).
195 The goal of our DDM framework was to distinguish people based on whether they used a forward
196 simulation or heuristic to guide decisions (Figs. 2a-b). We therefore formally considered a
197 participant to be using a specific strategy if their evidence accumulation was best modulated by
198 difficulty arising from it. To each participant's set of trialwise choices and RTs, we fitted a total of
199 three DDMs, each containing free parameters for $b_c B$, B , and nondecision time t_0 . The null model
200 was constrained such that $\mu_1 = \mu_2 = 0$, while the route-planner and heuristic models had two free
201 parameters, μ_1 and μ_2 , mapping respectively onto high or low difficulty as calculated by either
202 forward simulations or angular offset (Figs. 2a-b.). Based on model fits²⁴ after adjusting for model
203 complexity according to the Akaike information criterion with correction for finite sample size
204 (AICc)^{25,26}, we confirmed that 19 participants' choice and RT data were best fitted by the route-
205 planner model, 14 participants' data were best fitted by the heuristic model, while a third group of
206 20 participants were best fitted by the null model, indicating neither strategy-specific difficulty
207 modulated the rate of their evidence accumulation (Table 1). We hence refer to these three groups

208 respectively as the "route", "heuristic" and "nonplanner" groups (see Table 1 for group-specific
 209 DDM parameter and behaviour summaries). The remaining portion of these results will describe
 210 hypothesis-driven contrasts between the route and heuristic groups, in terms of DDM parameters,
 211 choice behaviour and skill. However, for completeness, we include data from the nonplanner group
 212 in Figure 3 and Supplementary Table 1, and also include a supplementary section summarising
 213 their parametric and skill findings (see Supplementary Materials: *Nonplanner group*).
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217 **Figure 2 - Action-selection policy identified by sequential sampling**
 218 **framework.** **a-b**, route planning selects the cursor in accordance with the delta
 219 between reward yields estimated from forward simulations with both cursors, i.e.,
 220 incorporating both spatial and temporal task constraints into choice. A lower-
 221 dimensional heuristic instead selects the cursor with a displacement vector with the
 222 least angular offset to that described by the SG, i.e., incorporating only spatial
 223 information. **c**, across participants, both strategies create more difficult decisions,
 224 indexed by greater reaction time (RT) for action selection, on SGs where strategies
 225 ascribe equivalent value to both cursors. Here, solid black lines in each panel
 226 connect the means of nine RT bins after first sorting all participants' trials by
 227 relevant policy value. Gray shaded area depicts the standard error of the mean in
 228 each RT bin. **d**, strategy-specific action values are imperfectly positively correlated.
 229 Each individual marker describes the relation between individual action values
 230 derived from the route-planning and heuristic strategy across all trials from all
 231 participants. Hotter colors describe greater Euclidean distance between S and G,
 232 i.e., route-planning and heuristic strategy maximally deviates with shorter-distance

233 SGs. **e**, DDM framework, in which a noisy evidence accumulation process
 234 terminates at a decision criterion. We hypothesised that difficulty arising in our task
 235 would modulate the rate of evidence accumulation (drift rate μ) in a manner similar
 236 to that observed in perceptual tasks, and that comparison of strategy-specific
 237 difficulty modulation would identify individual-participant policy. For each
 238 participant's choice and RT data, two target models allowed separate drift rates μ_1
 239 and μ_2 for high and low difficulty, respectively as per route-planning and heuristic
 240 strategy. Three groups of participants emerged, based on whether their data were
 241 best fitted by the route-planning (n=19), heuristic (n=14) or null (n=20) models (see
 242 also Table 1). **f-g**, scaled schematic of DDM profile estimated for route-planning
 243 (route) and heuristic groups. **h**, comparison of DDM parameters between route and
 244 heuristic group consistent with the former integrating additional information (i.e.,
 245 temporal dynamics) into decision formation. Boxes and thin lines respectively
 246 represent the interquartile range (IQR) and highest density interval (HDI) of the
 247 posterior mean constraining individual-participant estimates of each parameter.
 248 Route group demonstrated a slower process of evidence accumulation toward a
 249 larger decision criterion; combining drift rates and decision criterion as part of a
 250 sensitivity metric $S=(\mu_1+\mu_2)/(2B(1+b_cB))$ we observe strong Bayesian evidence
 251 of group-level difference. This effect was primarily driven by the decision criterion,
 252 as the route group boundary (B) was also credibly higher than that of the heuristic
 253 group. In addition, the route group starting point (b_cB) was credibly above 0,
 254 indicating a bias toward the low-cost cursor. Asterisk indicates no overlap in
 255 groups' posterior HDIs for a given parameter. All parameters expressed in units of
 256 μ , except t_0 (in seconds). Note that t_0 and S parameters are aligned with the right
 257 axis for clarity.

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 259 **Table 1.** Participant groups and DDM parameters
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	non	route	heuristic
n (site1:site2)	20 (7:13)	19 (6:13)	14 (3:11)
p(optimal cursor)	[0.477,0.500]	[0.668,0.690]	[0.706,0.730]
p(congruent cursor)	[0.716,0.736]	[0.564,0.587]	[0.528,0.555]
p(catastrophic error)	[0.169,0.186]	[0.120,0.135]	[0.110,0.126]
median response time (s)	[0.779,1.351]	[1.873,2.672]	[1.485,2.191]
sensitivity - S	0	[0.192,0.316]	[0.330,0.564]
drift rate, high difficulty - μ_1	0	[0.019,0.040]	[0.030,0.062]
drift rate, low difficulty- μ_2	0	[0.044,0.073]	[0.054,0.093]

boundary - B	[0.065,0.146]	[0.134,0.168]	[0.108,0.143]
starting point - $b_c B$	[0.127,0.405]	[0.059,0.236]	[-0.024,0.164]
nondecision time - t_0 (s)	[0.380,0.661]	[0.666,0.961]	[0.585,0.977]
route-planning DDM model: residual deviance - D_2	0.2	213.3	202.3
heuristic DDM model: residual deviance - D_2	0.1	171.2	238.8

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Notes: values in square brackets for $p(\text{congruent cursor})$, $p(\text{optimal cursor})$ and $p(\text{catastrophic error})$ describe the bounds [upper,lower] of the HDI of group-level probability ($\theta_{v,g}$: see methods) from a summary binomial model. Values in square brackets for all other parameters describe HDI for group-level mean ($M_{v,g}$: see methods) from a summary Gaussian model. Drift rates (μ_1 and μ_2) describe rates of evidence accumulation per second toward Boundary (B), which is expressed in units of drift rate (μ). Starting point $b_c B$ is expressed as a proportion from $-B$ to B . Sensitivity, $S=(\mu_1+\mu_2)/(2B(1+|b_c B|))$. Higher values of residual deviances (D_2) reflect better model fits across a group's participants relative to the null model.

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We next tested whether individual-participant DDM parameters were consistent with group classifications ascribed by the model fit scores to the route and heuristic groups (Table 1, Figure 2h). By definition, route planning incorporates a larger volume of information into decisions, relative to the heuristic, which can be computationally indexed by more gradual evidence accumulation and or a broader decision criterion. We employed a Bayesian framework (see Methods) that estimated group-specific summaries of each DDM parameter listed in Table 1 in a single model, and further only considered strong Bayesian evidence of between-group parameter differences, i.e., where the highest density interval (HDI) of deterministic distributions of parameter differences did not contain zero. We first observed strong evidence of group difference for the sensitivity metric $S=(\mu_1+\mu_2)/(2B(1+|b_c B|))$ that combines the rate of evidence accumulation with the extent of the decision criterion ($S \Delta(\text{route-heuristic}) \text{ HDI}=[-0.322,-0.056]$). Credibly lower sensitivity amongst the route group is consistent with the above prediction that their employed policy integrates a greater volume of information, as this metric is low when decision formation is jointly constrained by a low rate of evidence accumulation and larger decision criterion (Figure 2f). This effect was primarily driven by the decision criterion, as we additionally observed the route group to have a credibly higher boundary (B) than the heuristic group ($B \Delta(\text{route-heuristic}) \text{ HDI}=[0.002,0.050]$), while not credibly differing across the two drift rates ($\mu_1 \Delta(\text{route-heuristic}) \text{ HDI}=[-0.036,0.002]$; $\mu_2 \Delta(\text{route-heuristic}) \text{ HDI}=[-0.040,0.008]$). Also of note, we observed a credible bias toward the low cost action amongst the the route ($b_c B_{\text{route}} \text{ HDI}=[0.057,0.233]$), but not amongst the heuristic group ($b_c B_{\text{heuristic}} \text{ HDI}=[-0.026,0.161]$). Together, these findings provide parametric plausibility to the DDM classifications, and argue that the route group's lengthier trialwise decision deliberations stemmed from a greater volume of information integration, potentially stemming from a stronger bias toward using the low-cost cursor.

296 We next tested whether, on a broader macroscopic level, the route planner's computationally-
297 intensive action selection delayed the emergence of optimality in their choice policy. For this, we
298 summarised choice optimality across time-on-task using a hierarchical binomial model. The model
299 estimated parameters of hierarchical Beta posterior distributions that constrained individual-
300 participant binomial posteriors summarising their likelihood of selecting the optimal cursor in a
301 run of trials, across a two-dimensional space described by group (route, heuristic, nonplanner) and
302 run (1-6). From hierarchical Beta posteriors we deterministically computed credible ranges of
303 group-specific choice optimality (θ) for each run, presented in Figure 3a. Consistent with
304 optimality delay, the route group's choice behaviour (i.e., $p(\text{optimal})$) did not credibly depart
305 chance (0.50) until the fourth run of trials ($\theta_{\text{route,run4}}$ HDI=[0.533,0.882]), while in contrast, the
306 heuristic group demonstrated above-chance choice optimality by the second run of trials ($\theta_{\text{heuristic,run2}}$
307 HDI=[0.501,0.898]; Figure 3a, see Supplementary Table 1 for each group-by-run θ HDI). This
308 relative delay-to-optimality of approximately 120 trials provides further support that the route
309 group mediated over a larger volume of evidence prior to action selection and further suggests a
310 trade-off between how quickly a policy produces state-relevant choices, and the dimensionality of
311 constituent planning.

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313 Finally, in a supplementary analysis (see: Supplementary Materials - *Hierarchical logistic choice*
314 *model*), we confirm that route and heuristic groups uniquely integrate extrinsic state information
315 into choices (relative to nonplanners), and also confirm that the route group had a more pronounced
316 bias toward the low-cost action, relative to the heuristic group, mirroring their bias revealed by the
317 DDM. This supplementary analysis further revealed speculative evidence that the route group's
318 planning strategy was not born purely out of risk-aversion, and that they instead potentially
319 reserved high-cost action usage early in the task to states with longer SGs, where, due to nonlinear
320 temporal task dynamics, state-appropriate choice offered disproportionately greater action value.

321 322 *Comparisons of skill between route and heuristic groups*

323 We have so far verified that a DDM framework parsimoniously distinguishes people on their likely
324 use of forward simulations or heuristics during action selection in a selection-execution task, in a
325 manner that is both parametrically consistent with the underlying characteristics of each strategy
326 and in line with trade-offs between the expediency and profundity of policy formation. We next
327 tested whether these two groups (identified solely using action-selection RT data) also differed in
328 terms of action-execution skill and skill learning. We again employed a Bayesian framework that
329 minimised the total number of fitted models and only considered strong evidence.

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331 We first enumerated performance on each trial in terms of three skill variables: reward, spatial
332 action execution and temporal action execution. Reward was the proportion of the fuel tank
333 conserved on each trial, i.e., higher values reflect better performance on this measure which is
334 modulated by the complete set of action-execution variables. Spatial action execution was the
335 number of direction changes on each trial, i.e., lower values reflect better performance on this
336 measure which is modulated specifically by spatial precision. Temporal action execution was the
337 normalised difference between the cursor's maximum and final velocity, i.e., higher values reflect
338 better performance on this measure that indexes proficiency in temporal task demands requiring
339 high max-velocities for more fuel-efficient displacement, while arriving at the goal below the
340 maximum threshold (Figure 1d), and ideally lower, to further preserve fuel. We next summarised
341 performance in these three variables across the task using three separate hierarchical Bayesian

342 models. Each model estimated hierarchical Gaussian (reward, temporal skill) or Poisson (spatial
343 skill) posterior distributions that constrained individual-participant posterior distributions. We
344 further fitted each model's hierarchical structure across a mixed three-dimensional space, i.e., first
345 between-group (route, heuristic, nonplanner) and then within-group, separately for each run (1-6)
346 and selected cursor (congruent, incongruent). In other words, each model estimated the credible
347 ranges of group-mean performance for its given variable, separately for each run, and separately
348 again for each cursor. Figures 3b-d and Supplementary Table 1 contain each group-by-cursor-by-
349 run μ HDI for each measure and deterministic HDIs collapsed across run.

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351 In terms of overall performance, the heuristic group garnered higher reward yields with the high-
352 cost (incongruent) cursor (Figure 3b), which were at least partially attributable to a higher level of
353 spatial skill (Figure 3c). Merging posteriors across runs, the route and heuristic groups showed no
354 credible differences in reward yielded using the congruent cursor ($\mu \Delta(\text{route-heuristic}) \text{HDI}=[-0.017,0.025]$,
355 Figure 3b), however the heuristic group amassed credibly higher yields using the
356 incongruent cursor ($\mu \Delta(\text{route-heuristic}) \text{HDI}=[-0.055,-0.006]$, Figure 3b). This result was
357 mirrored in spatial skill, where we again observed no credible between-group difference with the
358 congruent cursor ($\mu \Delta(\text{route-heuristic}) \text{HDI}=[-0.159,0.142]$, Figure 3c), but credibly fewer
359 direction changes amongst the heuristic group with the incongruent cursor ($\mu \Delta(\text{route-heuristic})$
360 $\text{HDI}=[0.011,0.358]$, Figure 3c). In contrast, we observed no between-group differences in
361 temporal skill, with either the congruent ($\mu \Delta(\text{route-heuristic}) \text{HDI}=[-0.018,0.056]$, Figure 3d)
362 or incongruent ($\mu \Delta(\text{route-heuristic}) \text{HDI}=[-0.068,0.007]$, Figure 3d) cursor. Given that the
363 heuristic group reached state optimal choices more quickly (Figure 3a; Supplementary Table 1),
364 i.e., they performed a higher volume of trials where their cursor selection theoretically reduced the
365 need for direction changes, we re-ran the model with trialwise direction changes adjusted by the
366 optimal solution for the cursor selected for each given trial (i.e., observed changes - ideal changes).
367 This model (see: Supplementary Materials - *Hierarchical Poisson with choice-normalised spatial*
368 *skill*) returned identical results, confirming that notwithstanding their better choices, the heuristic
369 group independently demonstrated greater spatial precision while piloting the incongruent cursor.
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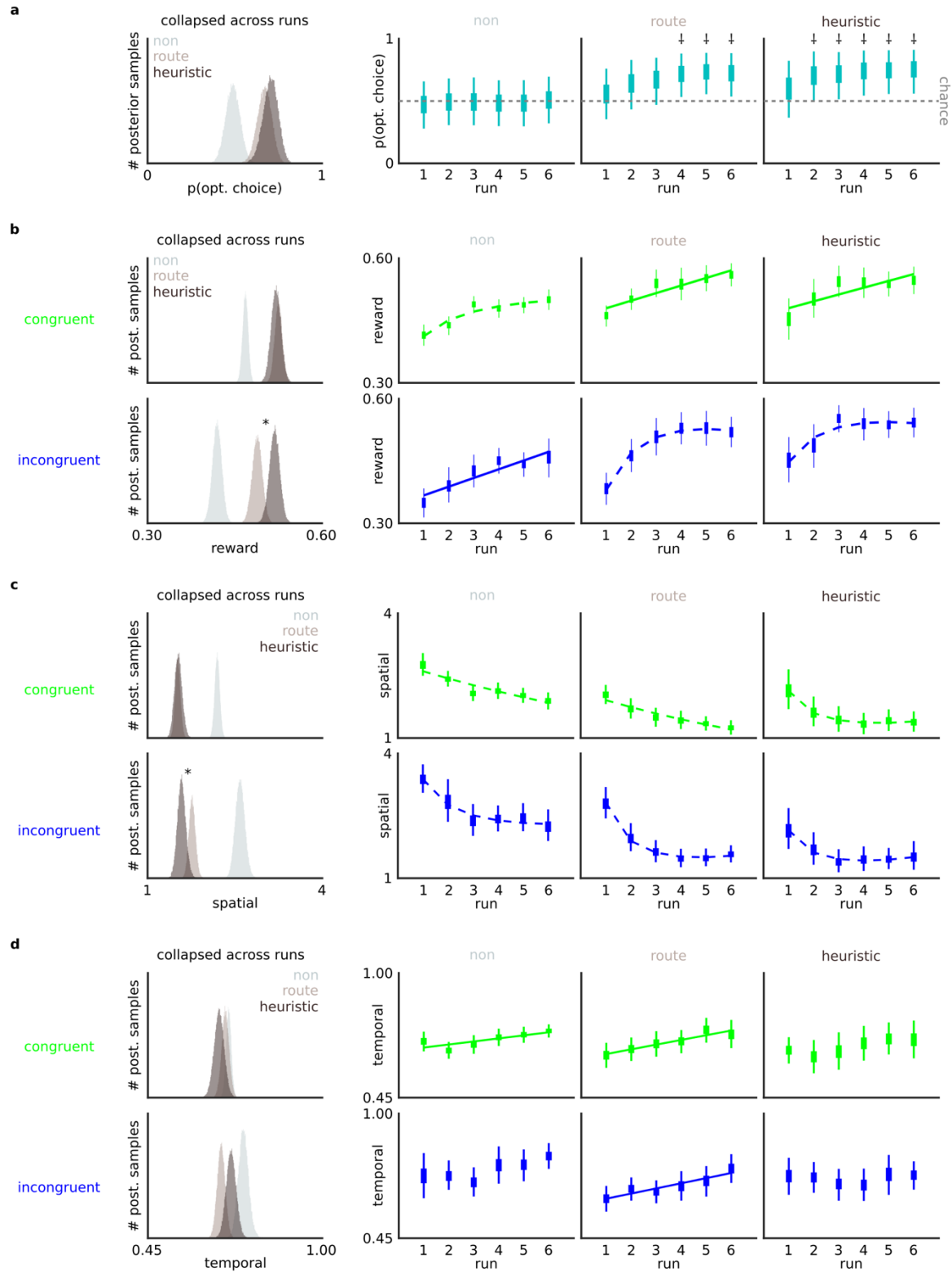


Figure 3 - Heuristics group reach choice optimality more quickly and show a spatial-specific skill advantage. a, consistent with classic decision-heuristic models, low dimensional planning aligns with faster trajectories toward choice

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376 optimality. Hierarchical binomial model of choice behaviour demonstrates trade-
377 off between the expediency and profundity of policy formation; heuristic group
378 exceeded chance optimality by run 2, earlier than route group (run 4). † reflects runs
379 where HDI of group-level θ posterior did not subtend 0.50, i.e., where group-level
380 proportion of choices were credibly above chance optimality. **b-d** skill and skill
381 learning consistent with yoked dimensionality between planning and motor
382 plasticity. Collapsing group-level posterior means across runs (skill) heuristic
383 group yielded more reward with the high-cost cursor (**b**, histograms bottom panel),
384 driven by superior spatial skill (**c**, histograms bottom panel), with no route-heuristic
385 difference in temporal skill (**d**, histograms). Asterisk relates to credible difference
386 between route and heuristic groups, i.e., that the HDI of the deterministic
387 distribution of their difference (route - heuristic) does not contain 0. Additionally,
388 while route and heuristic group demonstrated skill learning in terms of reward and
389 spatial skill (**b-c**, line plots), route group uniquely demonstrated learning in the
390 temporal domain (**d**, line plots). Boxes and thin lines in line plots respectively
391 represent IQR and HDI of hierarchical posterior means constraining individual-
392 participant posteriors for a given measure, run and cursor. In both histograms and
393 line plots, reward is the proportion of fuel preserved per trial (higher better), spatial
394 is the number of direction changes (fewer better) and temporal is the distance-
395 normalized difference between max and final velocity (higher better). Time-on-task
396 (skill learning) effects estimated from deterministic regression models fitted across
397 draws from each run's posterior; credible ($0 \notin$ coefficient HDI) effects depicted by
398 either a dashed (logarithmic) or solid (linear) line. Absence of any line reflects non-
399 credible effect.

400
401 We next probed whether the route and heuristic groups differed in terms of time-on-task
402 trajectories of specific skill features (skill learning). For this, we drew samples from each skill
403 variable's uncollapsed runwise posterior distributions and tested with a deterministic regression
404 model whether performance in each group-cursor dyad evolved across runs in either a linear or
405 logarithmic fashion (linear and nonlinear time-on-task effects). Here we again observed
406 divergence between the route and heuristic groups, with the route group's skill improvements
407 encompassing a broader range of motor control features (Figure 3b-d, see Supplementary Table 1
408 for each group-by-run skill measure μ HDI, and HDIs of time-on-task effects). Specifically, the
409 route group showed either linear or logarithmic improvement with each cursor across all variables,
410 while in contrast, the heuristic group only showed strong evidence of improvement for reward
411 yields and spatial skill, i.e., they demonstrated no time-on-task improvements with either cursor in
412 the realm of temporal skill. To further support this null result, we conducted a follow-up analysis.
413 This nonparametric analysis used more liberal criteria to establish temporal time-on-task effects at
414 the individual-participant-level, and a binomial design that could confirm different trajectories
415 between groups. Using summary values from individual-level posteriors, we quantified the number
416 of participants from each group that showed either logarithmic or linear temporal time-on-task
417 improvement, separately for each cursor. Confirmed via Bayesian binomial contrast, a greater
418 proportion of the route group improved in temporal skill relative to the heuristic group, both using
419 the congruent boat (10/19 vs 1/14; $\theta \Delta(\text{route-heuristic}) \text{ HDI}=[0.145,0.647]$) and using the
420 incongruent boat (10/19 vs 0/14; $\theta \Delta(\text{route-heuristic}) \text{ HDI}=[0.068,0.600]$). This additional
421 analysis confirmed that the route and heuristic groups diverged in the feature of temporal skill

422 learning. In addition, this group-distinguishing feature of skill-learning - temporal dynamics - was
423 also the feature which separated the two groups' planning styles. In other words, by definition, the
424 heuristic group did not incorporate temporal dynamics into their action selection, and uniquely
425 showed no skill learning in this domain. Together with the above findings relating to their overall
426 superiority in spatial skill, these findings suggest that a yoked dimensionality might exist between
427 planning and skill. I.e., in complex states, the number of features relevant (or not) for action-
428 selection policy may predict the number of features most likely to undergo learning (or not) during
429 action execution.

430

431 **Discussion**

432

433 At least two schools of thought lend plausibility to the idea that humans might achieve optimal
434 long-term ecological yields by basing goal-oriented decisions on subsets of information available
435 in complex states. On the one hand, if either time or computational resources are restricted, humans
436 might pragmatically trade off state-optimal parameterisation for reduced processing requirements
437 (accuracy-resource trade-off). On the other hand, decisions informed by fewer dimensions are
438 more robust to the influences of misleading stochastic noise (less-is-more principle). In either case,
439 extant knowledge on decision heuristics stems predominantly from action-trivial tasks that obviate
440 intrinsic motor proficiencies in determining choice outcomes. The present work directly addressed
441 this shortcoming.

442

443 We developed a novel task requiring joint optimisation of action planning (selecting state-
444 appropriate low-cost or high-cost cursors) and action execution (controlling cursors proficiently).
445 Focusing first on action planning, cursor selection could be shaped by either exhaustive forward
446 simulations, incorporating both spatial and temporal task dynamics, or a lower-dimension spatial
447 heuristic strategy. Using a between-group DDM framework we successfully parsed a wide pool of
448 human participants based on which strategy-specific planning difficulty best modulated their
449 evidence-accumulation rates. Exploring the dynamics of each group's choice behaviour in greater
450 detail, we revealed strong group-level Bayesian evidence supporting the model classifications.
451 Participants allocated to the route group (forward simulations) were constrained by a higher
452 decision criterion, consistent with the idea that they needed to incorporate a larger volume of
453 information into their choices. This group also showed an enduring bias toward low-cost actions,
454 revealed in both a DDM and logistic-choice context, and further demonstrated a more sluggish
455 trajectory toward choice optimality.

456

457 We next juxtaposed these two groups, who were classified solely on the basis of action-selection
458 RT, in the separate context of action execution. Specifically, we probed the relation between
459 decision-heuristic adoption and intrinsic skill, measuring the latter in terms of reward yielded per
460 trial, and in terms of additional independent skill dimensions of spatial and temporal precision. We
461 again revealed strong Bayesian group differences; heuristic adoption aligned with higher reward
462 yields underscored by better spatial skill with high-cost actions, even after correcting for the
463 influence of selections in state. Together with the route group showing a parametric bias toward
464 the low-cost cursor, we interpret the combined data across our task's action-execution contexts as
465 unambiguously supporting heuristic adoption in higher-skilled systems.

466

467 These core findings extend the remit of the accuracy-resource trade-off and less-is-more models
468 to include contexts involving non-trivial action. In a task requiring exquisite spatio-temporal
469 control of selected actions, decision heuristics nonetheless aligned with swifter individual action
470 selections, faster trajectories to choice optimality and, ultimately, better overall reward yields. The
471 skill advantage in the heuristic group additionally dissociates our findings from predictions that
472 integrate a compensatory role of heuristics with theories from the human motor literature. A core
473 rationale of sensorimotor-based forward models of action selection is that efference copies and
474 predicted sensorimotor costs provide improved efficiency and robustness in the face of noisy
475 sensory-prediction errors^{27,28,29}. A corollary is that a low-skill system will struggle to parameterise
476 forward models, increasing computational requirements (accuracy-resource) and/or generating a
477 high volume of online noise-driven corrective action (less-is-more)^{30,31}. In either case, a heuristic-
478 as-compensation model would predict that the low skill system is better served by adopting a
479 simpler heuristic decision strategy.

480
481 We observe that in a selection-execution context humans might not employ heuristics to
482 compensate for higher intrinsic motor noise. Instead, planning dimensionality might align with
483 progress along motor-learning trajectories previously observed in forced-choice contexts, (i.e., no
484 selection required)^{13,14}. Here, early in the acquisition of a novel motor skill, internal models that
485 simulate action outcomes can expedite learning in exchange for high computational cost¹³. As
486 participants then amass a wider cache of state transitions and successful experiences, control shifts
487 from deliberative model-based planning to less taxing draws of state-appropriate motor outputs
488 from memory¹³. Our findings are consistent with action selection following a similar qualitative
489 trajectory at the cross-phenotypic level; participants with superior skill, i.e., farther along motor-
490 learning trajectories, also used a less taxing policy to select actions.

491
492 In computational terms, the core difference between our task and paradigms previously exploring
493 heuristics is the source (internal vs external) of its generative model. Trial outcomes in our task
494 were determined solely by a joint function that integrated participants' cursor selection and its
495 subsequent execution. In other words, outcome variance was fully determined by parameters
496 (decision and performance) generated intrinsically by participants. In contrast, forecasting and
497 computerised emulations typically employ extrinsic generative models, where outcome variance
498 is a function of parameters beyond participants' control. Recent evidence from bandit tasks (a
499 computerised emulation with an extrinsic generative model) further suggests that humans might
500 overparameterise their choices when extrinsic forces determine their fate, resulting in apparently
501 irrational summary behaviour such as probability matching³². However, probability matching
502 dissipates as a function of increased agency, for example, with increased motor involvement in
503 choice execution³². While it is premature to conclude that increased agency will globally drive the
504 adoption of heuristics, our findings nonetheless predict that a low-skill system (i.e., low agency)
505 will more likely overparameterise choices in a selection-execution context, than revert to
506 heuristics. This inverse-agency-parameterisation framework is also consistent with emerging
507 associations in clinical computational work, where sequelae such as overthinking (in anxiety³³)
508 and rumination (in depression³⁴) align with excess deliberative model-based learning³⁵.

509
510 We reveal additional evidence that planning dimensionality and skill might not simply evolve
511 independently along separate strands of a learning manifold. In our task, common kinematic
512 variables parameterised both action value and the motor proficiency of subsequent execution. We

513 were therefore able to probe how the depth of planning qualitatively aligned with the motor
514 dimensions shaping both skill state and skill learning. As mentioned above, in terms of skill state,
515 we first observed that the heuristic group's skill advantage was localised to the spatial dimension,
516 with no overall group differences apparent in the temporal dimension. The heuristic group was
517 therefore more skilled solely in the core feature of their decision policy. In a series of time-on-task
518 analyses (skill learning) we additionally observed that the route group, employing the more
519 granular planning, demonstrated skill learning across a broad array of motor-control features,
520 including learning in temporal task dynamics. The heuristic group, in contrast, only showed skill
521 learning in either the spatial or overall reward realms, i.e., no skill learning in the temporal domain
522 (corroborated in a follow-up nonparametric analysis). Of note, a third nonplanner group, who
523 never incorporated any state parameters into choice, and largely exploited the low-cost action,
524 nonetheless improved across dimensions of motor-skill, including temporal skill (albeit only with
525 the congruent cursor). In other words, the only group not showing credible plasticity in any
526 temporal indices of skill learning was the heuristic group, i.e., the group who singularly used the
527 spatial dimension of information when selecting actions.

528
529 These combined findings support the idea that a yoked dimensionality might exist between plans
530 governing the selection of actions and the skill shaping their subsequent execution. In terms of a
531 bottom-up framework, the spatial dominance in the heuristic group's planning-policy and skill
532 advantage suggests that such dimensional yoking may be modulated by skill-first credit
533 assignment³⁶. In other words, higher execution proficiency stemming predominantly from spatial
534 precision may have overweighted this dimension during planning. Previous research has indeed
535 shown that human choice policy can be separately influenced by distinct dimensions of error
536 depending on the reliability of their signals^{37,38,39}, and that increased agency might determine
537 whether policy integrates either motor or reward-based errors¹⁰. A latter top-down framework is
538 also supported by the apparent absence of temporal learning in the heuristic group. Note that the
539 route and heuristic groups did not differ in terms of overall temporal skill, just that the heuristic
540 group uniquely showed no time-on-task evolution in this domain. An intriguing implication of this
541 pattern of results is that a controller that localises a cardinal subset of information for making state-
542 appropriate action selections might itself be able to influence controllers of what it considers
543 superfluous features of sensorimotor error.

544
545 Future behavioural enquiry into heuristics could employ advancements on our selection-execution
546 framework to investigate the yoked dimensionality hypothesis and investigate its potential bottom-
547 up and top-down underpinnings in more detail. An additional key outstanding question relates to
548 the robustness of heuristic adoption over time. Given the tendency for learning-related
549 configurations in the human brain to vary more across phenotypes than at the intra-subject level⁴⁰,
550 we employed a between-groups analytic approach inspired by an increasing body of work that uses
551 behavioural variance across the phenotype to increase robustness and reliability of hypothesis-
552 specific brain activity^{24,41}. While our DDM model parsimoniously distinguished human
553 participants based on planning dimensionality, for power reasons, parameter estimations utilized
554 all trials performed by participants. Our data therefore cannot inform any within-subject
555 hypotheses regarding heuristic adoption; whether, for example, the route group would eventually
556 reduce planning dimensionality with increased time-on-task. Though our logistic models revealed
557 the route group's bias toward the low-cost cursor endured in later runs, suggesting their planning
558 strategy may have held firm across the experiment, we cannot confirm whether they demonstrated

559 a robust phenotypic trait or a relatively slower evolution along a trajectory of policy formation
560 mutually traversed by both them and the heuristic group.

561

562 **Conclusion**

563

564 The association between decision heuristics and intrinsic skill has evaded description due to the
565 arbitrary nature of action in computerised goal-oriented tasks. Here we used a novel task emulating
566 both the decisional and motoric demands of goal-oriented behaviour in a dynamic environment.
567 DDM models parsimoniously identified human participants who adopted heuristics, and later
568 modeling unambiguously aligned this lower-dimensional planning strategy with higher skill,
569 consistent with an inverse-agency-parameterisation model. We additionally observe that
570 phenotypic variance in the intricacy of planning potentially maps onto the granularity of
571 improvement in motor ability. Advancements in the behavioural assays of actions selected and
572 executed will hopefully uncover the underlying causality, learning dynamics and neural
573 underpinnings giving rise to this possible yoked dimensionality.

574

575 **Materials and Methods**

576

577 *Participants and overview*

578 We report data from a multi-site experiment, with 53 right-handed human participants recruited in
579 total, via both word-of-mouth and the online participant-recruitment portal at the University of
580 California, Santa Barbara (UCSB). 34 participants reported as female and the group had an average
581 (standard deviation) age of 21.9 (3.05) years. Participants performed the experiment either in a
582 behavioural-testing suite (site 1, n=16) or an fMRI context (site 2, n=37). We report only
583 behavioural data in the present paper from both groups. Visual angle subtended by stimuli was
584 constant for the two testing sites and neither site differed in terms of eventual DDM group
585 classifications (see Supplementary Materials: *Site-specific DDM group classifications*).
586 Participant remuneration was \$10 (\$20, site 2) per hour baseline rate, with an additional \$10 (\$20,
587 site 2) contingent on performance. Testing at both sites took place during a single session. The
588 Institutional Review Board at UCSB approved all procedures. Prior to participating, participants
589 provided informed written consent. All stimuli were presented using freely available functions^{42,43}
590 written in MATLAB code, and unless otherwise stated all analyses were also conducted using
591 custom MATLAB scripts.

592

593 *Action selection-execution task: boatdock*

594 *Paradigm*

595 Our task was a continuous, nonlinear adaptation of the discrete grid-sail task¹³, extended such that
596 reward yields require joint optimisation of action selection and action execution. All visual stimuli
597 appear on a screen with a gray background ($RGB_{[0,1]}=[0.500,0.500,0.500]$). In each trial (Figure
598 1a), they select one of two cursors, depicted by equilateral triangles (side length=0.830 °), to pilot
599 from a start (S) to a goal (G), respectively depicted by a black ($RGB_{[0,1]}=[0,0,0]$) and white
600 ($RGB_{[0,1]}=[1,1,1]$) square (side length=1.37 °). The SG pair appears within a circular grid
601 (radius=3.82 °) centered on the screen center. Locations of the SG are drawn with uniform
602 probability on each trial, constrained such that neither element falls within 0.320 ° of the grid
603 perimeter, and their centres are at least 0.957 ° apart.

604

605 Each cursor displaces in three deterministic directions (Figure 1b.), mapping onto the same three
606 separate response buttons ("throttles") operated by the right hand for the duration of the experiment
607 (Figure 1c). One "congruent" cursor displaces at angles $7\pi/6$ (index finger), $\pi/2$ (middle finger),
608 and $11\pi/6$ (ring finger) in a reference frame where $\pi/2$ aligns with the vertical meridian of the
609 screen (Figure 1c). The other "incongruent" cursor displaces at angles $5\pi/6$, $\pi/6$ and $3\pi/2$, via one
610 of two sets of spatially incongruent throttle-mappings, selected with uniform ($p=0.500$) probability
611 for each subject (an example mapping is in Figure 1c). For the entire experiment, the congruent
612 and incongruent cursors are identified by a different color, green $RGB_{[0,1]}=[0,1,0]$ and blue
613 $RGB_{[0,1]}=[0,0,1]$, determined with uniform ($p=0.500$) probability before each participant's session.
614

615 For every frame a single throttle is down, the cursor will accelerate in that direction (see
616 Supplementary Materials for specific Acceleration dynamics) and a one unit of fuel is also
617 subtracted from an allocation of 360 units provided for each trial. Participants therefore have a
618 total 6 s throttle time on each trial before fuel depletes (refresh rate=60 Hz). Following a successful
619 "dock" (see below) a screen informs participants of the fuel conserved, expressed as a proportion
620 of the starting tank. No other exogenous cue is provided to participants regarding the size of the
621 initial fuel allocation, or its rate of depletion.
622

623 *Trial structure*

624 Each trial initiates with the action-selection period, signified by the appearance of an SG pair
625 within a grid ("action selection", Figure 1a). Participants have no time limit to select their desired
626 cursor with the middle or index finger of their left hand, respectively using "a" or "z" of a standard
627 keyboard (site 1) or buttons 1 and 2 (i.e., the two most leftward) of a six-button bimanual response
628 box⁴⁴ (site 2). Finger-cursor mapping (i.e., index→congruent, middle→incongruent, or vice versa)
629 is determined every twenty trials by uniform ($p=0.500$) probability, prompted throughout the
630 action-selection period by a silhouette of a hand (9.49° -by- 9.49°) below the grid, with the relevant
631 cursor above the relevant finger. Once an action is selected, the action-execution period
632 immediately begins, signified by the silhouette prompt disappearing and the selected cursor
633 spawning at the centre of S ("trial start", Figure 1a). Participants now pilot the cursor from S to G
634 with their right hand, using the "v" (index), "h" (middle) or "m" (ring) buttons on the keyboard
635 (site 1) or buttons 4-6 on the right side of the response box (site 2). Action execution lasts until
636 one of four possible trial outcomes. A successful "dock" is achieved if the cursor enters a 0.479° -
637 radius circular threshold (not visible to participants) centred on the centre of G, at a velocity no
638 greater than $1.920^\circ/s$. Alternatively, three catastrophic errors can occur if participants (i) run out
639 of fuel, i.e., cumulative throttle time greater than 6 s; (ii) leave the grid; or (iii) enter the circular
640 G threshold at a velocity greater than $1.920^\circ/s$. Once a trial outcome is achieved, a feedback screen
641 immediately informs participants of the outcome, respectively, "WELL DONE!", "OUT OF
642 GAS!", "LEFT THE GRID!" or "TOO FAST!", presented at the centre of the screen along with
643 "SCORE: \$", where \$ is either the proportion of fuel preserved (for successful docks) or 0 for all
644 catastrophic errors. The feedback remains on the screen for 1 s, followed by a blank grey inter-
645 trial-interval screen lasting one, two or three seconds (determined on each trial with uniform
646 probability $p=0.333$). Participants performed 360 choice trials in total, portioned into six runs of
647 60 trials. Interlaced between choice runs were 20 practice trials, on which scores do not count
648 toward the final bonus, forcing ten trials with both the congruent and incongruent cursor in
649 pseudorandom order.
650

651 *Dependent variables*

652 To enumerate action values derived from route planning we first computed forward simulations of
653 the optimal routes on each (simulation procedure described in Supplementary materials). We
654 subtract the total frames spent accelerating during the optimal route (λ) from the starting fuel bank
655 of 360 units to estimate the maximum reward obtainable on a given trial. To enumerate action
656 values derived from a simpler spatial heuristic we computed the angles (in $^\circ$) between the vector
657 of a trial's SG and each vector on the incongruent cursor. The vector creating the smallest angle
658 (which we term the "offset") quantifies action value from this heuristic on a raw scale where values
659 close to 0 reflect an SG perfectly aligning with one of the incongruent vectors and values close to
660 60 reflect an SG perfectly aligning with one of the congruent vectors.

661
662 We enumerated reaction time (RT) for action selection as the time elapsed between the time of the
663 first frame of the action-selection screen (described above) and the time of cursor selection. We
664 coded optimal selection as incongruent cursor on trials with $\text{offset} < 30$ and congruent cursor on
665 trials with $\text{offset} > 30$ (optimal cursor selection did not differ depending on which action value
666 (route vs heuristic) is computed).

667
668 We enumerated skill performance on each trial in terms of reward, spatial action execution and
669 temporal action execution. Reward was the amount of fuel conserved. All modeling of reward used
670 raw units (i.e., on a scale of 0 to 360) to allow Gaussian likelihood functions, however for clarity
671 in reported results we present findings as a proportion of the tank preserved (from 0 to 1). Spatial
672 action execution was the number of direction changes, i.e., a count of how many times a different
673 throttle was pressed relative to the one previous. Temporal action execution was the difference
674 between the cursor's maximum velocity recorded during action-execution (in $^\circ/\text{s}$), and the final
675 velocity (in $^\circ/\text{s}$) taken at the moment the cursor crossed the circular threshold around G, normalised
676 by the distance covered by the SG (in $^\circ$).

677
678 *Data analysis*

679 *Computational modeling*

680 We modeled action planning leading up to cursor selection with variants of a standard drift-
681 diffusion model^{45,46,47}. The full models included five free parameters: high-difficulty drift rate μ_1 ,
682 low-difficulty drift rate μ_2 , boundary B , starting point $b_c B$, and nondecision time t_0 . The boundaries
683 for congruent and incongruent choices were defined as B and $-B$, respectively. Hence a positive
684 $b_c B$ relates to a congruency bias. Parameters were necessarily constrained as follows: $0 \leq \mu_1 \leq \mu_2$,
685 $\mu_2 \geq 0$, $B > 0$, $-1 < b_c B < 1$, and $t_0 > 0$. Noise was represented as the standard deviation of diffusion
686 with a fixed scaling parameter $\sigma=0.1$.

687
688 We compared three types of models: two route-planning models (with one or two drift rates), two
689 heuristic models (with one or two drift rates), and the null (i.e., nonplanning) model. For route-
690 planning models, we determined difficulty by dividing trialwise differences in reward yields
691 (between the simulated optimal routes for either cursor) into five bins. For the heuristic models,
692 we determined difficulty by dividing trialwise offsets into five bins. The five difficulty bins
693 corresponded to drift rates of $-\mu_2$, $-\mu_1$, 0 , μ_1 , and μ_2 . We constrained single-drift-rate models such
694 that $\mu_1=\mu_2$ to minimise penalties for additional degrees of freedom, and the null model such that
695 $\mu_1=\mu_2=0$ to represent insensitivity to the onscreen information.

696

697 We fitted candidate models to empirical distributions of choices and RTs at the level of individual
698 subjects using maximum-likelihood estimation and the chi-square fitting method⁴⁸. We calculated
699 the frequencies of either choice and the 10, 30, 50, 70, and 90% quantiles (i.e., six bins) of their
700 respective RT distributions for each difficulty level. We optimised free parameters with respect to
701 overall goodness of fit for given subjects using iterations of the Nelder-Mead simplex algorithm
702 with randomised seeding⁴⁹. We adjusted for model complexity when comparing models that
703 differed in degrees of freedom using the Akaike information criterion with correction for finite
704 sample size (AICc)^{25,26}.

705
706 Three participant groups were defined by the results of model fitting following penalisation. The
707 "route" and "heuristic" groups included those who were best fitted by a route-planning or heuristic
708 model, respectively, according to the AICc. For assignment to the "nonplanner" group, adding free
709 parameters for planning would not yield a significant improvement in goodness of fit relative to
710 the null model without sensitivity to either route or heuristic information.

711 712 *Bayesian models*

713 We sampled all Bayesian posterior distributions using No U-Turn sampling (NUTS) Hamiltonian
714 Monte Carlo, implemented with the PyMC3 package⁵⁰ in custom Python scripts. Unless otherwise
715 specified, each model's posterior distributions were sampled across four chains of 10000 samples
716 (40000 total), with an additional initial 10000 samples per chain (40000 total) discarded after
717 tuning the sampler's step-size to an acceptance threshold of 0.95 (80000 samples combined), with
718 further convergence criteria that no chains contain any divergences and no posterior's \hat{R} value,
719 estimating the ratio of variance within the $n=4$ chains to the variance of the pooled chains, greater
720 than 1 (see:⁵¹). Unless otherwise stated, dependent variables were z-score normalised across
721 participants prior to fits. We calculated minimum-width Bayesian credible intervals of relevant
722 posteriors from their chains, using the default settings for Highest Density Interval (HDI)
723 calculation in the arviz package⁵².

724
725 A pair of models first estimated summaries of group-specific behaviour (reported in Table 1). A
726 single Gaussian model first summarised continuous variables, accounting for eight variables in
727 total. First, the four variables applicable to each group identified by the DDM framework,
728 specifically: starting point - $b_c B$, boundary - B , and nondecision time - t_0 , in addition to median
729 RT. In addition, the three variables applicable only to the route and heuristic groups, specifically:
730 drift rate, hi difficulty - μ_1 , drift rate, lo difficulty- μ_2 and sensitivity - S . This model assumed
731 individual participant (n) values (y) for each variable (v) were characterised by a separate Gaussian
732 likelihood function, further depending on n 's group-allocation ($g(n)$: route, heuristic or
733 nonplanner), i.e., $y_{n,v} \sim \mathcal{N}(M_{v,g(n)}, \Sigma_{v,g(n)})$. Each variable was z-score normalised separately (but across
734 all subjects) prior to fitting, and we respectively assigned each $M_{v,g(n)}$ and $\Sigma_{v,g(n)}$ an uninformed
735 Gaussian and half-Gaussian prior: $M_{v,g(n)} \sim \mathcal{N}(\mu=0, \sigma=10)$ and $\Sigma_{v,g(n)} \sim \text{half}\mathcal{N}(\sigma=10)$. Three separate
736 binomial models then estimated summaries of behaviour as measured by three binomial variables
737 that applied to all groups: $p(\text{congruent cursor})$, $p(\text{optimal cursor})$ and $p(\text{catastrophic error})$. For the
738 $n(g)$ participants in each group (g), each summary model used a Binomial likelihood function
739 $y_g \sim \text{Bin}(\theta_g, t_g)$, where y_g and t_g are $n(g)$ -element vectors, respectively enumerating the number of
740 observed instances reported by each individual participant in a group (y_g) and their total number
741 of trials (t_g). In each model, we assigned each θ_g an uninformed prior from the beta distribution:
742 $\theta_g \sim \text{Beta}(\alpha=1, \beta=1)$.

743

744 We used a hierarchical Bayesian binomial model to estimate the credible ranges of group-specific
745 choice optimality (p(optimal cursor)), separately for each run. The hierarchical structure used
746 Binomial likelihood functions to summarise the number of optimal cursor selections (y) made by
747 each participant (n) for all trials (t) in a given run (r), $y_{n,r} \sim \text{Bin}(\theta_{n,r}, t_{n,r})$. The model constrained $\theta_{n,r}$
748 posteriors with separate hierarchical group ($g(n)$) and run-specific Beta distributions, i.e.: $\theta_{n,r} \sim$
749 $\text{Beta}(\alpha_{g(n),r}, \beta_{g(n),r})$. Each $\alpha_{g(n),r}$ and $\beta_{g(n),r}$ were assigned uninformed priors from a half-Student's T
750 distribution, i.e.: $\alpha_{g(n),r} \sim \text{HalfStudentT}(\sigma=10, \nu=10)$ and $\beta_{g(n),r} \sim \text{HalfStudentT}(\sigma=10, \nu=10)$,
751 bounded to never draw values of $\alpha_{g(n),r}=0$ or $\beta_{g(n),r}=0$. Run-specific group-level deterministic
752 posterior estimates of optimal choice ($\theta_{g(n),r}$) were calculated by drawing 10,000 independent
753 samples (k) from relevant $\alpha_{g(n),r}$ and $\beta_{g(n),r}$ posteriors and computing the mean of the resulting k th
754 Beta distribution, i.e., $\theta_{g(n),r,k} = \alpha_{g(n),r,k} / (\alpha_{g(n),r,k} + \beta_{g(n),r,k})$.

755

756 We used two separate hierarchical Bayesian Gaussian models to estimate the credible ranges of
757 group-mean performance in the two continuous action-execution variables (reward and temporal
758 skill), separately for each run, and separately again for each cursor. In each model, the hierarchical
759 structure used Gaussian likelihood functions to summarise each (n) participant's trialwise measures
760 across all trials in a given run (r), separately for each cursor (c), i.e.: $y_{n,r,c} \sim \mathcal{N}(\mu_{n,r,c}, \exp(\sigma_{n,r,c}))$. The
761 model constrained $\mu_{n,r,c}$ and $\sigma_{n,r,c}$ posteriors with separate hierarchical group ($g(n)$), run (r) and
762 choice-specific (c) Gaussian distributions, i.e.: $\mu_{n,r,c} \sim \mathcal{N}(M(\mu)_{g(n),r,c}, \Sigma(\mu)_{g(n),r,c})$ and $\sigma_{n,r,c} \sim$
763 $\mathcal{N}(M(\sigma)_{g(n),r,c}, \Sigma(\sigma)_{g(n),r,c})$. Each $M(\mu)_{g(n),r,c}$ and $M(\sigma)_{g(n),r,c}$ were assigned uninformed Gaussian priors
764 ($\sim \mathcal{N}(\mu=0, \sigma=10)$), while each $\Sigma(\mu)_{g(n),r,c}$ and $\Sigma(\sigma)_{g(n),r,c}$ were assigned uninformed half-Gaussian
765 priors ($\sim \text{half}\mathcal{N}(\sigma=10)$). Note that the model for reward was fitted to a continuous measure, scoring
766 fuel conserved on a scale of 0 to 360, but for clarity, we adjusted runwise and collapsed HDIs
767 (division by 360), also prior to computing any HDIs related to between-comparisons, to express
768 results as a proportion of fuel preserved. Time-on-task betas, however, relate to unadjusted
769 posteriors.

770

771 We used a hierarchical Bayesian Poisson model to estimate the credible ranges of group-mean
772 performance in spatial skill, separately for each run, and separately again for each cursor. In each
773 model, the hierarchical structure used Poisson likelihood functions to summarise each (n)
774 participant's trialwise direction changes across all trials in a given run (r), separately for each cursor
775 (c), i.e.: $y_{n,r,c} \sim \text{Pois}(\exp(\mu_{n,r,c}))$. The model constrained $\mu_{n,r,c}$ posteriors with separate hierarchical
776 group ($g(n)$), run (r) and cursor-specific (c) Gaussian distributions, i.e.: $\mu_{n,r,c} \sim$
777 $\mathcal{N}(M(\mu)_{g(n),r,c}, \Sigma(\mu)_{g(n),r,c})$. $M(\mu)_{g(n),r,c}$ and $\Sigma(\mu)_{g(n),r,c}$ were respectively assigned uninformed Gaussian
778 ($\sim \mathcal{N}(\mu=0, \sigma=10)$) and half-Gaussian priors ($\sim \text{half}\mathcal{N}(\sigma=10)$). For clarity in reported results, we re-
779 adjusted runwise and collapsed HDIs (exponential transform), also prior to computing any HDIs
780 related to between-comparisons, to discount the use of $\exp(\mu_{n,r,c})$ in the likelihood function. Time-
781 on-task betas, however, relate to unadjusted posteriors.

782

783 For both the hierarchical Gaussian and Poisson skill models, separately for each group (g) and
784 choice (c), we enumerated deterministic posteriors of overall skill level by averaging each
785 posterior sample across runs, i.e., for each posterior sample, $M(\mu)_{g,c} = 1/6 \sum_{r=1}^6 M(\mu)_{g,r,c}$. We then
786 enumerated deterministic linear and logarithmic time-on-task effects $b_{g,c}$ by drawing posterior
787 samples from $M(\mu)_{g,r,c}$. Specifically, on each (k) of 40,000 draws, we computed the k th column of

788 $b_{g,c}$ ($b_{g,c,k}$), where $b_{g,c,k}=(X^T X)^{-1} X^T Y_{g,c,k}$. Here, $Y_{g,c,k}$ is a six-element column vector containing an
789 independent draw from each run (r) of $M(\mu)_{g,r,c}$ and matrix X is a three-column matrix respectively
790 containing six constant terms (1), z-scored linear $x \in (1, 2, \dots, 6)$ and z-scored logarithmic
791 $x \in (\ln(1), \ln(2), \dots, \ln(6))$ regressors. The second and third rows of resulting 3-by-40,000 matrix $b_{g,c}$
792 respectively contained deterministic posteriors for linear and logarithmic time-on-task effects.
793 Where logarithmic time-on-task effects were credible ($0 \notin \text{HDI}$), we considered that group-cursor
794 time-on-task effect to be logarithmic even if a linear effect was also observed. Note, as specified
795 above, that in reported results, we present the HDIs of time-on-task coefficients (linear and
796 logarithmic) fitted to unadjusted runwise posteriors, i.e., before we made any adjustment to
797 posteriors for intuitive presentation of runwise/collapsed HDIs.

798
799 For the individual-participant-level nonparametric analysis of temporal skill, we computed the
800 median of each $\mu_{n,r,c}$ posterior from the relevant Gaussian skill model. Separately for each cursor
801 we regressed the six-element vector of participant's run-specific median values, first as a function
802 of an intercept and a linear time-on-task regressor (z-scored linear $x \in (1, 2, \dots, 6)$), and then as a
803 function of an intercept and a z-scored logarithmic regressor $x \in (\ln(1), \ln(2), \dots, \ln(6))$. If either
804 model's regressor (x) was significant (determined by 95% coefficient confidence intervals not
805 containing 0), we considered that participant time-on-task+ for that cursor and skill variable. We
806 compared proportions of time-on-task+ participants (y) between groups (g), separately for each
807 cursor, by fitting Binomial likelihood function $y_g \sim \text{Binomial}(\theta_g, n_g)$, assigning each θ_g an
808 uninformed prior from the beta distribution: $\theta_g \sim \text{Beta}(\alpha=1, \beta=1)$.

809
810 In all above cases, we consider strong evidence of credible effects as follows: for comparison of
811 parameters to criterion values (e.g., a regression coefficient above 0, or a likelihood above 0.50,
812 etc.) we required the entire HDI of that parameter to not include the criterion value. For comparison
813 of two parameters we required the HDI of the deterministic distribution of their difference
814 (posterior A - posterior B) to not contain 0. Note that two HDIs might overlap, but that this
815 deterministic distribution of difference may yet still not contain 0.

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821 822 **Competing interests**

823 Authors declare no financial, and no non-financial, competing interests.

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938
 939 **Supplementary materials**

940
 941 *Nonplanner group*

942 In addition to the 33 participants identified by our DDM framework as likely employing one of
 943 two planning strategies, 20 participants were best fitted by the null model (skill variables of this
 944 group are included in Supplementary Table 1). While our hypotheses principally focused on
 945 parametric and skill differences between the two classes of planners (route and heuristic), we
 946 briefly comment here on the nonplanner group. Of all groups, nonplanners showed the fastest
 947 overall decision time and strongest DDM bias toward the congruent cursor (Table 1), consistent
 948 with an action-selection policy that did not integrate the external state. However, despite
 949 demonstrating no evidence of state-appropriate action selection (Figure 3b.), largely stemming
 950 from an over-reliance on the congruent cursor (Table 1), nonplanners nonetheless exhibited skill
 951 learning during the execution portion of our task (Figure 3a). They improved with both cursors in
 952 terms of reward yield and spatial precision, but only demonstrated improved temporal dynamics
 953 with the congruent cursor, i.e., the cursor they exploited to yield reward.

954
 955 **Supplementary Table 1:** group-by-run skill measure μ HDI, HDIs collapsed
 956 across runs and HDIs of time-on-task effects
 957

cursor/variable	HDI	non	route	heur
p(optimal choice)	θ run 1	[0.279,0.658]	[0.354,0.760]	[0.366,0.823]
	θ run 2	[0.307,0.681]	[0.433,0.830]	[0.501,0.898]+
	θ run 3	[0.306,0.689]	[0.469,0.848]	[0.514,0.893]+
	θ run 4	[0.299,0.667]	[0.533,0.882]+	[0.543,0.906]+
	θ run 5	[0.297,0.668]	[0.555,0.887]+	[0.557,0.908]+
	θ run 6	[0.321,0.696]	[0.536,0.885]+	[0.560,0.938]+

cong/reward	mu run 1	[0.415,0.468]	[0.463,0.515]	[0.430,0.532]
	mu run 2	[0.442,0.488]	[0.504,0.557]	[0.484,0.580]
	mu run 3	[0.495,0.538]	[0.535,0.603]	[0.535,0.613]
	mu run 4	[0.484,0.530]	[0.528,0.608]	[0.533,0.608]
	mu run 5	[0.496,0.535]	[0.550,0.614]	[0.537,0.598]
	mu run 6	[0.504,0.554]	[0.562,0.619]	[0.543,0.610]
	mu (runs coll)	[0.487,0.505]	[0.542,0.567]	[0.533,0.566]
	$\beta(\text{run})$	[0.091,0.191]*	[0.103,0.218]*	[0.058,0.230]*
$\beta(\log(\text{run}))$	[0.004,0.397]*	[-0.003,0.480]	[-0.004,0.714]	
incong/reward	mu run 1	[0.338,0.410]	[0.369,0.449]	[0.425,0.537]
	mu run 2	[0.375,0.462]	[0.450,0.532]	[0.461,0.568]
	mu run 3	[0.412,0.493]	[0.491,0.584]	[0.549,0.615]
	mu run 4	[0.446,0.510]	[0.519,0.584]	[0.528,0.608]
	mu run 5	[0.439,0.499]	[0.519,0.598]	[0.534,0.600]
	mu run 6	[0.437,0.533]	[0.515,0.601]	[0.536,0.690]
	mu (runs coll)	[0.431,0.462]	[0.500,0.534]	[0.530,0.565]
	$\beta(\text{run})$	[0.101,0.268]*	[0.146,0.308]*	[0.055,0.243]*
$\beta(\log(\text{run}))$	[-0.042,0.599]	[0.236,0.897]*	[0.008,0.763]*	
cong/spatial	mu run 1	[2.497,3.043]	[1.817,2.277]	[1.696,2.651]
	mu run 2	[2.237,2.614]	[1.470,1.956]	[1.284,2.000]
	mu run 3	[1.891,2.264]	[1.271,1.728]	[1.151,1.766]
	mu run 4	[1.939,2.335]	[1.218,1.657]	[1.092,1.605]
	mu run 5	[1.844,2.208]	[1.178,1.523]	[1.171,1.690]
	mu run 6	[1.687,2.098]	[1.082,1.428]	[1.153,1.640]
	mu (runs coll)	[2.114,2.279]	[1.433,1.605]	[1.406,1.656]
	$\beta(\text{run})$	[-0.156,-0.076]*	[-0.211,-0.105]*	[-0.212,-0.048]*
$\beta(\log(\text{run}))$	[-0.303,0.007]	[-0.378,0.054]	[-0.703,-0.024]*	

incong/spatial	mu run 1	[3.053,3.736]	[2.438,3.190]	[1.702,2.686]
	mu run 2	[2.354,3.380]	[1.650,2.323]	[1.324,2.106]
	mu run 3	[2.018,2.782]	[1.384,1.916]	[1.143,1.696]
	mu run 4	[2.123,2.748]	[1.265,1.706]	[1.174,1.766]
	mu run 5	[2.130,2.804]	[1.287,1.713]	[1.220,1.726]
	mu run 6	[1.895,2.654]	[1.380,1.791]	[1.204,1.893]
	mu (runs coll)	[2.439,2.748]	[1.662,1.874]	[1.452,1.726]
	$\beta(\text{run})$	[-0.180,-0.063]*	[-0.241,-0.128]*	[-0.195,-0.015]*
$\beta(\log(\text{run}))$	[-0.458,-0.008]*	[-0.728,-0.262]*	[-0.744,-0.035]*	
cong/temporal	mu run 1	[0.650,0.736]	[0.579,0.688]	[0.598,0.712]
	mu run 2	[0.619,0.692]	[0.609,0.711]	[0.555,0.700]
	mu run 3	[0.640,0.721]	[0.628,0.738]	[0.571,0.733]
	mu run 4	[0.674,0.749]	[0.643,0.743]	[0.610,0.761]
	mu run 5	[0.688,0.757]	[0.659,0.793]	[0.637,0.776]
	mu run 6	[0.710,0.768]	[0.665,0.787]	[0.621,0.784]
	mu (runs coll)	[0.685,0.716]	[0.667,0.712]	[0.641,0.700]
	$\beta(\text{run})$	[0.027 0.133]*	[0.043 0.205]*	[-0.018,0.187]
$\beta(\log(\text{run}))$	[-0.411,0.034]	[-0.250,0.385]	[-0.530,0.279]	
incong/temporal	mu run 1	[0.623,0.819]	[0.565,0.676]	[0.638,0.798]
	mu run 2	[0.660,0.787]	[0.612,0.713]	[0.647,0.780]
	mu run 3	[0.630,0.757]	[0.601,0.701]	[0.612,0.750]
	mu run 4	[0.686,0.848]	[0.612,0.741]	[0.611,0.751]
	mu run 5	[0.697,0.835]	[0.630,0.763]	[0.638,0.811]
	mu run 6	[0.750,0.862]	[0.689,0.814]	[0.660,0.783]
	mu (runs coll)	[0.716,0.776]	[0.654,0.701]	[0.678,0.737]
	$\beta(\text{run})$	[-0.004,0.219]	[0.048,0.217]*	[-0.096,0.114]
$\beta(\log(\text{run}))$	[-0.657,0.270]	[-0.389,0.256]	[-0.592,0.232]	

959 *Notes: non=nonplanner; heur=heuristic; cong=congruent cursor; incong=incongruent cursor;*
960 *spatial=spatial skill; temporal=temporal skill; coll.=collapsed across runs; runwise and*
961 *collapsed HDIs for reward have been re-adjusted (division by 360) to express reward as a*
962 *proportion of fuel preserved; *=time-on-task coefficient credibly dearts 0;+ proportion of choices*
963 *credibly above chance optimality (0.50).*

964

965 *Hierarchical logistic choice model*

966 We used a hierarchical Bayesian logistic regression model to assess the group-specific modulation
967 of choice (p(incongruent)) as a function of an intercept (β_0), trial offsets (β_1 ; i.e., the trialwise
968 enumeration of heuristic value) and the Euclidean distance (in screen pixels) of trial SGs (β_2). The
969 hierarchical structure used Bernoulli likelihood functions to characterise choice likelihood for each
970 individual participant (n) and trial (t), i.e.: $y_{n,t} \sim \text{Bernoulli}(p_{n,t})$, where $p_{n,t}$ is computed with a
971 deterministic logistic transition function $S(x_{n,t})$, where $x_{n,t} = \beta_0 + \beta_1 * \text{offset}_{n,t} + \beta_2 * \text{distance}_{n,t}$. The
972 model constrained coefficient posteriors fitted to each participant's set of trials with separate
973 hierarchical group-specific (g(n)) Gaussian distributions, i.e.: $\beta_0_n \sim \mathcal{N}(M(\beta_0)_{g(n)}, \Sigma(\beta_0)_{g(n)})$, $\beta_1_n \sim$
974 $\mathcal{N}(M(\beta_1)_{g(n)}, \Sigma(\beta_1)_{g(n)})$ and $\beta_2_n \sim \mathcal{N}(M(\beta_2)_{g(n)}, \Sigma(\beta_2)_{g(n)})$. Each $M(\beta_0)_{g(n)}$, $M(\beta_1)_{g(n)}$ and $M(\beta_2)_{g(n)}$
975 were assigned uninformed Gaussian priors ($\sim \mathcal{N}(\mu=0, \sigma=10)$), while each $\Sigma(\beta_0)_{g(n)}$, $\Sigma(\beta_1)_{g(n)}$ and
976 $\Sigma(\beta_2)_{g(n)}$ were assigned uninformed half-Gaussian priors ($\sim \text{half}\mathcal{N}(\sigma=10)$). Both regressors were z-
977 score normalised across all trials from all subjects prior to fitting. Finally, we fitted two iterations
978 of this model, one using trials from the early phase of the task (first three runs), and a second using
979 trials from the late phase of the task (final three runs).

980

981 Results of this hierarchical logistic regression model are summarised below in Supplementary
982 Table 2. This model first bolstered the DDM by demonstrating the route and heuristic group
983 uniquely integrated state information into action selection. During both early and late phases of
984 the task, the route ($\beta_{1_{\text{route}}}$ μ HDI early=[-0.865,-0.483]; $\beta_{1_{\text{route}}}$ μ HDI late=[-1.559,-1.014]) and
985 heuristic group ($\beta_{1_{\text{heuristic}}}$ μ HDI early=[-1.466,-0.601]; $\beta_{0_{\text{heuristic}}}$ μ HDI late=[-2.080,-1.201]),
986 incorporated route offsets optimally into choice; note that their credibly negative coefficient HDIs
987 reflect increased likelihood of selecting the incongruent cursor when offset angle was low, i.e.,
988 suited to the incongruent cursor (offset was normalised to vectors on the incongruent cursor; see:
989 Methods - Dependent variables). In addition, this model supported the finding from the DDM
990 relating to the route group's bias. The route group uniquely showed a bias to the congruent cursor
991 in both early and late phases of the task, ($\beta_{0_{\text{route}}}$ μ HDI early=[-0.682,-0.240]; $\beta_{0_{\text{route}}}$ μ HDI
992 late=[-0.423,-0.104]), which was not credibly evident in the heuristic group in either instance
993 ($\beta_{0_{\text{heuristic}}}$ μ HDI early=[-0.480,0.011]; $\beta_{0_{\text{heuristic}}}$ μ HDI late=[-0.401,0.073]). No groups credibly
994 modulated their choice by the distance covered by a route's start-goal pairing (SG), in either early
995 or late phases of the task (all HDIs for β_2 subtend 0 in Supplementary Table 2). However, of note,
996 the trending positive distance parameter estimate for the route group in the early phase suggests
997 first that their planning strategy was not born out of risk-aversion, (which instead would have been
998 characterised by incongruent selection on shorter SGs). Though we can only speculate on a non-
999 credible finding, if the route group selectively used the high-cost incongruent cursor early in
1000 primarily longer SGs, they may have been reserving its usage for situations where optimal choice
1001 was disproportionately beneficial, due to the nonlinear temporal task physics (Figure 1d, Results:
1002 *Forward simulations vs heuristics*, Figure 2d).

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Supplementary Table 2: Hierarchical logistic model of choice behaviour parameters

task phase	parameter	non	route	heur
early	β_0 (int.) mu	[-1.690,-0.780]*	[-0.682,-0.240]*	[-0.480,0.011]
late	β_0 (int.) mu	[-1.471,-0.672]*	[-0.423,-0.104]*	[-0.401,0.073]
early	β_1 (offset) mu	[0.016,0.196]*	[-0.865,-0.483]*	[-1.466,-0.601]*
late	β_1 (offset) mu	[-0.059,0.172]	[-1.559,-1.014]*	[-2.080,-1.201]*
early	β_2 (distance) mu	[-0.092,0.100]	[-0.015,0.136]	[-0.175,0.045]
late	β_2 (distance) mu	[-0.127,0.071]	[-0.083,0.087]	[-0.112,0.123]

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Notes: non=nonplanner; heur=heuristic; int.=intercept; *=coefficient credibly dearts 0;

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Hierarchical Poisson model with choice-normalised spatial skill

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To dissociate the heuristic group's superior spatial skill with the incongruent cursor from their overall more optimal choice behaviour, we used a hierarchical Bayesian Poisson model to estimate the credible ranges of group-mean performance in spatial skill, using a measure which had been normalised by the optimal number of direction changes in the simulated solution (see Supplementary Materials: *Optimal route simulations*). This normalisation took the number of direction changes made on each trial, and subtracted from that the number of direction changes made by the optimal solution for the specific cursor chosen on that trial (i.e., not necessarily normalised to the optimal cursor for a given route, but the selected cursor). Due to a small number of resulting trials (0.3%, across all subjects) containing a negative value (never lower than -1), we added a constant (1) to all trials, to ensure the lowest value was 0, suitable for a Poisson likelihood function. With this normalisation, higher values reflect worse spatial skill, i.e., more direction changes relative to cursor-optimal. As with the unnormalised model, we fitted the model separately for each run, and separately again for each cursor. In each model, the hierarchical structure used Poisson likelihood functions to summarise each (n) participant's trialwise direction changes across all trials in a given run (r), separately for each cursor (c), i.e.: $y_{n,r,c} \sim \text{Pois}(\exp(\mu_{n,r,c}))$. The model constrained $\mu_{n,r,c}$ posteriors with separate hierarchical group (g(n)), run (r) and cursor-specific (c) Gaussian distributions, i.e.: $\mu_{n,r,c} \sim \mathcal{N}(\mathbf{M}(\mu)_{g(n),r,c}, \Sigma(\mu)_{g(n),r,c})$. $\mathbf{M}(\mu)_{g(n),r,c}$ and $\Sigma(\mu)_{g(n),r,c}$ were respectively assigned uninformed Gaussian ($\sim \mathcal{N}(\mu=0, \sigma=10)$) and half-Gaussian priors ($\sim \text{half}\mathcal{N}(\sigma=10)$). For clarity in reported results, we re-adjusted runwise and collapsed HDIs (exponential transform, followed by subtraction of -1), also prior to computing any HDIs related to between-comparisons, to discount first the use of $\exp(\mu_{n,r,c})$ in the likelihood function, and then the constant added to all trials prior to fitting. Time-on-task betas, however, relate to unadjusted posteriors.

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Results of this model are summarised below in Supplementary Table 3. Crucially, collapsing across runs, we see the heuristic group demonstrating credibly fewer direction changes with the incongruent cursor ($\mu \Delta(\text{route-heuristic}) \text{ HDI}=[0.005,0.331]$), supporting the interpretation of

1038 the finding from the main paper (see: Results - *Comparisons of skill between route and heuristic*
 1039 *groups*) that their superior incongruent spatial skill is independent to the navigational
 1040 consequences of their choices.

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Supplementary Table 3: group-by-run spatial skill, normalised by cursor selection

cursor/skill	HDI	non	route	heur
cong/spatial	mu run 1	[1.664,2.216]	[1.085,1.522]	[1.000,1.886]
	mu run 2	[1.408,1.776]	[0.742,1.195]	[0.610,1.228]
	mu run 3	[1.092,1.430]	[0.555,0.972]	[0.481,1.046]
	mu run 4	[1.073,1.479]	[0.519,0.891]	[0.422,0.872]
	mu run 5	[0.982,1.323]	[0.464,0.758]	[0.477,0.889]
	mu run 6	[0.879,1.261]	[0.363,0.669]	[0.432,0.861]
	mu (runs coll.)	[1.276,1.436]	[0.707,0.861]	[0.704,0.926]
	β (run)	[-0.148,-0.076]*	[-0.173,-0.092]*	[-0.178,-0.052]*
β (log(run))	[-0.282,0.000]	[-0.308,0.029]	[-0.546,-0.016]*	
incong/spatial	mu run 1	[2.190,2.912]	[1.664,2.408]	[0.912,1.875]
	mu run 2	[1.568,2.593]	[0.943,1.578]	[0.629,1.347]
	mu run 3	[1.166,1.924]	[0.674,1.160]	[0.456,0.978]
	mu run 4	[1.266,1.915]	[0.640,1.018]	[0.534,1.100]
	mu run 5	[1.217,1.918]	[0.594,0.966]	[0.516,1.002]
	mu run 6	[1.032,1.782]	[0.611,0.994]	[0.489,1.113]
	mu (runs coll.)	[1.586,1.901]	[0.953,1.150]	[0.749,1.007]
	β (run)	[-0.178,-0.066]*	[-0.210,-0.115]*	[-0.157,-0.008]*
β (log(run))	[-0.421,0.007]	[-0.573,-0.185]*	[-0.567,0.022]	

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 1046 *Notes: non=nonplanner; heur=heuristic; cong=congruent cursor; incong=incongruent cursor;*
 1047 *spatial=cursor-normalised spatial skill; coll.=collapsed across runs; runwise and collapsed HDIs*
 1048 *have been re-adjusted (subtraction of -1) to discount the constant added to all trials prior to fitting;*
 1049 **=time-on-task coefficient credibly departs 0.*

1050
 1051 *Site-specific DDM group classifications*
 1052 To test group allocations from the DDM for each site, we fitted a summary Bayesian multinomial
 1053 model. The model used a k=3 multinomial likelihood function to characterise the counts (#) for

1054 each group classification $y=[\#(\text{route}) \#(\text{heuristic}) \#(\text{nonplanner})]$, separately for the participants
 1055 (n_s) each site (s) $y_s \sim \text{Multinomial}(\Theta_s, n_s)$. We assigned Θ_s an uninformed prior from a Dirichlet
 1056 distribution $\Theta_s \sim \text{Dirichlet}(\alpha=[1,1,1])$. Results from this model (summarised in Supplementary
 1057 Table 3 below) confirmed the DDM ascribed similar group allocations for both testing sites.

1058
 1059 **Supplementary Table 4: site-specific DDM group allocations**
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parameter	HDI (site 1)	HDI (site 2)
Θ p(route)	[0.167,0.569]	[0.213,0.493]
Θ p(heuristic)	[0.054,0.386]	[0.164,0.435]
Θ p(nonplanner)	[0.220,0.633]	[0.215,0.492]

1061
 1062 *Acceleration dynamics*
 1063 At a resolution of 60 Hz, cursor position during action execution is updated for each frame f by
 1064 adding a two-element vector (\mathbb{D}) to the cursor's position at frame $f-1$. \mathbb{D} is computed using $D =$
 1065 $\sum_{v=1}^3 \mathbb{P}_v * \mathbb{V}(v)$. Here, \mathbb{P}_v is the two-element vector (x,y) in screen coordinates describing a
 1066 Euclidean displacement of 0.320° in the direction of a given throttle (v). $\mathbb{V}(v)$ scales each
 1067 coordinate in \mathbb{P}_v in accordance with nonlinear acceleration by using $\mathbb{V}(v) = f(\mathbb{T}(v))$.
 1068 Where $f(x) = -0.011x^3 + 0.167x^2$. For every frame a given throttle (v) is down, the relevant
 1069 element of three-element vector \mathbb{T} (i.e., $\mathbb{T}(v)$) increases by 0.017 s, and for every frame a throttle
 1070 is released, $\mathbb{T}(v)$ decreases by 0.017 s until it reaches 0. Elements of \mathbb{T} therefore update separately
 1071 and gradually at this fixed rate, meaning non-zero momentum from one vector can continue
 1072 influencing the displacement of the cursor after its release and while another throttle is down,
 1073 allowing curvilinear two-dimensional displacement (see top panel of Figure 1f). However, if more
 1074 than one throttle is down for a given frame, each element of \mathbb{T} decreases by 0.017 s (unless already
 1075 at 0), precluding participants from using simultaneous throttle pulsing to create additional
 1076 displacement angles outside of the six afforded across the two cursors.

1077
 1078 *Optimal route simulations*
 1079 To enumerate action values derived from route planning we first computed forward simulations of
 1080 the optimal routes (i.e., with the highest reward yield) from S to G for each cursor on each trial.
 1081 Separately for each cursor, we first assessed whether the SG on each trial afforded a single linear
 1082 displacement with one of its vectors from S that would intersect the circular threshold around G
 1083 (point of intersection= G^*). If a cursor satisfied this requirement we computed the optimal throttle
 1084 sequence with that vector as a single pulse of length t_{opt} that accelerated the cursor to a maximum
 1085 speed at half the distance between S and G^* , followed by a release of the throttle to allow the
 1086 cursor's momentum to bring it to G^* , arriving at a velocity of 0. t_{opt} is estimated to the precision of
 1087 our (60 Hz) screen resolution by finding the lowest number of frames (λ), such that: $\sum_0^\lambda D_\lambda^* > D/2$,
 1088 where D is the Euclidean distance between S and G^* in screen coordinates and $D_\lambda^* =$
 1089 $\sqrt{SS(\mathbb{P}_v * f(\lambda * 0.017))}$, where SS denotes sum of squares and $f(x)$ and \mathbb{P}_v are from the above
 1090 section describing task physics. Expressing optimal pulse length (t_{opt}) in frames (λ) automatically
 1091 computes the number of units of fuel depleted by this optimal sequence. We subtract λ from 360
 1092 as our final estimate of the reward obtainable from the optimal route. (Note that we leave this score

1093 on a scale of 0 to 360 for modeling purposes, but present score feedback to participants on each
1094 trial as a more intuitive proportion of preserved fuel).

1095
1096 If a cursor does not provide a single linear displacement solution, its optimal route instead
1097 comprises a two-pulse sequence using its two vectors that most closely align with the trajectory of
1098 the SG, i.e., the two vectors (v_1 and v_2) with the smallest "offset" values (θ_1 and θ_2) as computed
1099 in Figure 2b. The shortest combined displacement of these two vectors that moves a cursor from
1100 S to its most nearby Euclidean point on the circular threshold around G (G^{**}) can be computed by
1101 first originating v_1 at S and v_2 at G^{**} and finding where they intersect (\cap). Forming an oblique
1102 triangle with lines $|S\cap|$, $|\cap G^{**}|$ and $|G^{**}S|$, the length of $|S\cap|$ and $|\cap G^{**}|$ (i.e., the singular
1103 displacements of v_1 and v_2) can then be solved using the law of sines, i.e., $|S\cap| = |G^{**}S| * \sin(\theta_2)/\sin(60)$
1104 and $|\cap G^{**}| = |G^{**}S| * \sin(\theta_1)/\sin(60)$. Optimal throttle sequence with these
1105 vectors is a vector of pulses (T_{opt}) containing $[t_{v1}, t_{v2}]$, respectively solved with the lowest $[\lambda_1, \lambda_2]$
1106 values such that $\sum_0^{\lambda_1} D_{\lambda}^* > D_{v1}/2$ and $\sum_0^{\lambda_2} D_{\lambda}^* > D_{v2}/2$, where D_{v1} is the Euclidean distance
1107 between S and \cap , and D_{v2} is the Euclidean distance between \cap and G^{**} . Given that λ_2 is calculated
1108 from 0 velocity, the optimal sequence pulses v_2 immediately upon the release of v_1 . We subtract
1109 λ_{total} from 360 as our final estimate of the reward obtainable from the optimal route, where
1110 $\lambda_{total} = \lambda_1 + \lambda_2$.

1111
1112 In most cases λ_{total} is the same value whether using the above order, or by originating v_2 at S and
1113 v_1 at G^{**} , and estimating $[t_{v2}, t_{v1}]$ relative to the resulting intersection (\cap'). The exception occurs
1114 when one intersection (\cap or \cap') falls outside the grid, requiring more than one direction change to
1115 avoid catastrophic error with this sequence. However, all trials had at least one sequence with an
1116 intersection inside the grid for each cursor, i.e., at least one optimal path involving a single
1117 direction change. Our modeling framework simply required the lowest λ_{total} for each cursor on each
1118 trial, i.e., either λ from a single linear displacement, λ_{total} for either route if both intersections fall
1119 within the grid, or λ_{total} corresponding to the route with its intersection inside the grid, if one fell
1120 outside it.