Isometric Spiracular Scaling in Scarab Beetles: Implications for Diffusive and Advective

Oxygen Transport

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Short title: Spiracle scaling of beetles

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Abstract

The scaling of respiratory structures has been hypothesized to be a major driving factor in the evolution of many aspects of animal physiology. Here we provide the first assessment of the scaling of the spiracles in insects using ten scarab beetle species differing 180x in mass, including some of the most massive extant insect species. Using X-ray microtomography, we measured the cross-sectional area and depth of all eight spiracles, enabling the calculation of their diffusive and advective capacities. Each of these metrics scaled with geometric isometry. Because diffusive capacities scale with lower slopes than metabolic rates, the largest beetles measured require 10-fold higher PO_2 gradients across the spiracles to sustain metabolism by diffusion compared to the smallest species. Large beetles can exchange sufficient oxygen for resting metabolism by diffusion across the spiracles, but likely no scarab beetle can do so during flight. In contrast, spiracular advective capacities scale more steeply than metabolic rates, so larger beetles should require less pressure to drive sufficient ventilation to meet oxygen demands, assuming similar extraction efficiencies. These data illustrate a general principle of gas exchange: scaling of respiratory transport structures with geometric isometry diminishes the potential for diffusive gas exchange but enhances advective capacities; combining such structural scaling with muscle-driven ventilation allows larger animals to achieve high metabolic rates when active.

Keywords

Spiracle, scaling, Scarabaeidae, tracheal system, body size, oxygen transport

MAIN TEXT

Introduction

As animal species evolve different sizes, many aspects of their physiology and

morphology scale disproportionately with one another (allometrically) with consequences for

animal behavior, life history, evolution, and diversity (1-3). A driver of this disproportionality

lies in the nonlinear scaling of geometry: doubling the radius of a sphere gives quadruple the

surface area and octuple the volume; in a similar way, scaling up a small body plan gives

drastically altered ratios of surface area, volume, and body length. Since the challenges

associated with changes in body size have a geometric origin, they are ubiquitous. As a result,

understanding the mechanisms animals use to overcome the effects of changes in geometric

proportions remains a pervasive, important, and challenging biological problem. Three related

aspects of animal function modulated by allometry are scaling of animal metabolic rates,

generally scaling with mass$^{0.75}$ (4, 5), limits on the maximal body sizes of specific taxa (6, 7),

and gas exchange strategies (8). For gas exchange, volume of tissue and hence potential gas

exchange needs of animals scale with the cube of length (like the sphere), while surface areas

tend to scale with the square of length. This leads to a decline in the ratio of surface area to

volume with size. As a consequence, when animals evolve larger sizes, they may need to adapt

the proportions of their respiratory structures or increase the use of advection (bulk flow) to

avoid facing limitations based on processes that depend more on surface area such as diffusion.

Limitations on the capacity of larger animals to support oxygen delivery to tissues have

been proposed to drive the hypometric scaling of metabolic rates with size, as well as the

hypometric scaling of many physiological (e.g., heart and ventilation rates) and

behavioral/ecological traits (e.g., territory size, dispersal distance) (1-4, 9). However, competing

theories suggest that other factors, such as heat dissipation constraints, nutrient uptake

constraints, or performance-safety trade-offs, drive the hypometric scaling of metabolic rates and

correlated variables, and that evolutionary adaptations of respiratory systems to size allow

animals to match oxygen supply to need regardless of body size (10-13). One important step in

resolving this controversy is determining how respiratory structures and mechanisms scale. The

vast majority of prior studies of the scaling of gas exchange structures have focused on

vertebrates, especially mammals. In contrast, there is relatively limited information on the

scaling of gas exchange structures in invertebrates, despite the fact that most animal species are

invertebrates (5, 14). The scaling of the insect respiratory system is of particular interest, as

aspects of tracheal system structure have been reported to scale hypermetrically, in contrast to

the isometric or hypometric scaling of respiratory structures in vertebrates, supporting the

hypothesis that possession of a tracheal respiratory system limits insect body size (6, 15-18).

Here, we report the first study of the scaling of insect spiracles, the gateway of air into the body

and the first step in oxygen delivery from air to tissues, presenting new insight into a key

morphological pathway in this most biodiverse clade of terrestrial animals.

Gas exchange usually occurs in a series of steps, often a sequence of alternating diffusive

and advective processes. The capacity for a respiratory surface to conduct oxygen (diffusive

conductance, G_{diff}) can be described using Fick's law, that is

(1) $G_{diff} = \frac{area}{thickness} * K.$

where K is Krogh's diffusion constant for oxygen in the barrier. The diffusive oxygen exchange

across the surface (J_{diff}, mol sec^{-1}) is given by

(2) $J_{diff} = G_{diff} * \Delta P_{O2}$.

where ΔP_{O2} is the partial pressure gradient for oxygen across the exchanger. When gas exchange

relies on diffusion across a barrier, either G_{diff} or ΔP_{O2} must increase to match the increased

oxygen demand inherent in a larger body size (a larger relative tissue volume), or oxygen supply

will limit metabolic rate. Increases in G_{diff} may be accomplished by either a decrease in diffuser

thickness or increase in area. The ΔP_{O2} from air to mitochondria can be no greater than

atmospheric P_{O2} (approximately 21 kPa at sea level); this biophysical constraint sets an upper

limit on the ability of large animals to utilize increases in ΔP_{O2} to overcome a G_{diff} that does not

increase in proportion to oxygen consumption rate.

The scaling of surface area, barrier thickness, and ΔP_{O2} for gas exchangers across species

of animals varies with clade and developmental stage. In adult vertebrates, the scaling of the

passive diffusing capacity of the lung across species scales hypometrically, but matches the

scaling of metabolic rates (5). However, the scaling of respiratory morphology differs in

endotherms and ectotherms (5), as barrier thickness is constant with size in ectotherms, but

increases with size in endotherms. As a consequence, endotherms must scale surface area of the

lung more steeply than ectotherms to account for their increased barrier thickness and match the

scaling of G_{diff} to the scaling of metabolic rate. Bird eggs, which rely on diffusion through pores

for oxygen, employ a different strategy. Eggs of larger species have relatively thicker shells

(scaling with mass$^{0.45}$), increasing barrier thickness with size, likely to mitigate a higher

likelihood of mechanical damage (19). Pore area increases proportionally with shell thickness, so

G_{diff} per pore is relatively constant across egg size, and larger eggs have a higher density of pores

(20). The scaling of the G_{diff} of the shell overall matches the scaling of metabolic rate across

species, with both scaling hypometrically (19, 20). Pycnogonids (sea spiders) show yet another

pattern for the diffusing capacity of their respiratory structures (their legs). Unlike either bird

eggs or vertebrate lung membranes, pycnogonid barrier thickness scales isometrically (7). As in

bird eggs, there is an increase in the area-specific diffusing capacity of the leg cuticle of larger

pycnogonids, although the morphological basis remains unclear (7). However, the increases in

diffusive conductance of the respiratory exchanger are not sufficient to match increases in

metabolic rates with size, so the ΔP_{O2} across the leg cuticle increases in larger pycnogonid

species, which may limit maximal species size in this taxa (7).

Advective steps in gas exchange can occur using either air or fluid media and represent a

second broad strategy for delivering gases to tissues. The morphological capacity for a structure

to transport a fluid by advection can be described from Poiseulle's law,

$$(3)\ G_{adv} = \frac{area^2}{8 * dynamic\ viscosity * length}.$$

Given this relationship, the advective transport of oxygen through the structure is given by

$$(4)\ G_{adv} * [O_2].$$

where $[O_2]$ is the concentration of O_2 in the fluid. Some examples in mammals illustrate how

morphology scales for structures relying on advection. In mammals, the radius of the aorta scales

with $mass^{0.375}$, and the length of the aorta scales with $mass^{0.25}$, suggesting that G_{adv} of the aorta

scales with $mass^{1.25}$ (4 * 0.375 − 0.25) (21). The tracheal-bronchial system is the advective

structure for air transport in vertebrates; radius scales with $mass^{0.39}$ while lengths scale with

$mass^{0.27}$, suggesting that G_{adv} for mammalian aorta and bronchial systems scale with $mass^{1.29}$

(22). G_{diff} for these same structures, on the other hand, scale as $mass^{0.5}$. Thus, the morphological

structures of mammalian respiratory systems seem to scale such that advective capacities

increase more than metabolic rates in larger species, while diffusive capacities decline. Of

course, mammalian oxygen transport through the bronchial tree and circulatory system is thought

to rely on advection regardless of size.

The design of the insect tracheal system is fundamentally different from either the

vertebrate respiratory system or that of skin-breathing aquatic invertebrates; and it remains

unclear how the components of the system scales. In insects, spiracles provide a (usually) gated

opening to an air-filled conduit system that branches through the insect, with oxygen transported

in the gas phase to the most distal surface of the tracheoles, with diffusion then occurring from

tracheole to mitochondria (23). Since Krogh's demonstration that diffusion should suffice for

oxygen transport in a relatively large Lepidopteran larvae, diffusion has been considered to be an

important mechanism of gas exchange in insects (24, 25). However, many insects supplement

diffusion with advection, especially when active (23, 26, 27). The spiracles are potentially an

important step in insect gas exchange, since they are relatively small (difficult to see by eye in

most insects) and yet must sustain all gas flux. It appears that spiracle morphology just matches

gas exchange needs at peak metabolic performance with little additional capacity; for example,

sealing of just one thoracic spiracle reduces flight metabolic rate in *Drosophila* (28). At present it

is not clear whether the size of spiracles should best match G_{diff}, G_{adv} or some other physiological

capacity. To shed light on this question, we used micro-computed tomography (micro-CT) (29)

to provide the first interspecific examination of the scaling of spiracles, using ten species of

scarab beetles spanning two orders of magnitude in mass, including some of the most massive

extant species.

Methods

Acquisition of raw micro-CT images

Seventeen individuals of ten species (1-2 individuals per species) of scarab beetles (Fig.

1a) with a size range from 0.097 to 18 grams were obtained via breeders from online sources.

We examined the following species: *Goliathus goliathus*, *Coelorrhina hornimani*, *Dicronorrhina*

derbyana, *Mecynorrhina torquata*, *Eudicella euthalia*, *Protaetia orientalis*, *Popilia japonica*,

Trypoxylus dichotomus, *Dynastes hercules*, and *Cyclocephalis borealis*. Most species had both

male and females represented. Most specimens were scanned using a micro-CT scanner

(Skyscan 1172, Bruker, Bilerica, MA, USA) equipped with a Hammamatsu 1.3 MP camera and

Hammamatsu SkyScan Control software at Virginia Tech. To maintain tracheal structure in their

natural configuration, we used a minimal preparation of fresh samples (30). All beetles were

killed using ethyl acetate fumes, stored at 4 °C, and scanned within three days. They were

warmed back to room temperature to avoid motion artifacts from fluid flow, placed in X-ray

translucent polyimide tubing (Kapton, Dupont), and centered on a brass stage with putty. Power

was set at 10 W, voltage was adjusted for optimum brightness and contrast (70-96 kV), with

currents between 104-141 µA. Beetles were scanned with 0.4° rotation steps for 180° with frame

averaging. A flat-field correction was applied to all scans to account for subtract aberrations. All

raw projection images were collected a size of 1024 x 1280 pixels, yielding a scaling of 12-98

µm/pixel that was independent of beetle size. Average measured spiracle dimensions for

width/height/depth for the smallest beetles were around 10 pixels and hence resolvable with

minimally 10% resolution. Small beetles could be captured in a single scan, but larger beetles

were scanned sequentially in segments along their longitudinal axis by varying their position

relative to the beam.

Dynastes hercules were too large to be scanned with the same instrument, so these

beetles were imaged using an in-house-built bench-top micro-focus X-ray computed tomography

(micro-CT) platform (see (31, 32) for details) at Virginia Tech. The X-ray tube (Oxford

Instrument, Inc.) was operated at 70 kV and tube power was fixed at 20 W. Images were

collected with an X-ray flat-panel detector (model C7921, Hamamatsu, Inc.) operated at 1x1

binning mode, with a detector element size of 50 x 50 μm. The axial scanning field-of-view

(FOV) was 37.2 mm in diameter. In each scan, images were collected at 0.5° intervals as the

beetle was rotated through 360°, resulting in a total of 720 X-ray projections per scan. Because

the specimen was larger than the field of view, multiple scans were conducted sequentially along

the animal's anterior-posterior axis to image the entire body. The axial slice images were

reconstructed using the standard filtered back-projection (FBP) reconstruction algorithm, with an

image matrix of 1008 x 1012 px and an isotropic pixel size of 36.8 x 36.8 μm.

Image Reconstruction and Measurements

Raw micro-CT images were imported into NRecon reconstruction software from

SkyScan (Bruker, Bilerica, MA, USA). Ring artifact and beam hardening corrections were

applied where necessary, and contrast was optimized using the software's interactive histogram

feature. For large beetles that required multiple scans, reconstructions were set to align and fuse

automatically. Slices generated in NRecon were imported into Avizo 9 software (ThermoFisher

Scientific, Waltham, MA, USA) for 3D reconstructions.

Spiracles were identified by the characteristic slit-like shape of the opening, and the

bellows-shaped air sac behind it (Figure 1b, 1c, 1d). Spiracle locations were confirmed by

dissection on representative specimens. Measurements were taken for one of the paired six

abdominal and two thoracic spiracles for each beetle (Figure 1b,1c). A few scans had small

aberrant regions (e.g., blurriness) due to challenges in scanning, so which spiracle was measured

varied between the symmetric right and left side of an animal based on which region of the scan

was best resolved. Diameters of the spiracular opening were measured at the widest point of

opening to the outside air in the transverse and sagittal planes. Area of the opening was then

calculated assuming an elliptical shape, with the lengths of the semi-major/minor axes being the

diameters measured in the transverse and sagittal planes (Figure 1d). The depth of the spiracle

was measured from the outer opening to the interior valve connecting the spiracle to the tracheal

trunk (Figure 1d). The sex, mass and dimensions of all measured spiracles are provided in

supplementary tables 1 and 2.

Calculations and Statistical Analyses

We measured the scaling relationships for each spiracle separately, using log-log plots.

As dependent variables in these regressions, we tested $\log_{10}$ transformed spiracular depth, area,

area/depth (as an index of the diffusive capacity of the spiracle, see Eq. 1), and area^2/depth (as an

index of the resistance of the spiracle to advective flow, see Eq. 3 below).

We used two statistical approaches to assess the role of the phylogenetic relatedness of

the animals in scaling patterns: a phylogenetic generalized linear model (pGLS) and a generative

Bayesian model. We ran and plotted pGLS results in R (33-39). The goal of pGLS is to account

for non-independence of data points due to phylogenetic relatedness in construction of the linear

model, which requires a phylogeny of the study species (40). We spliced together such a

phylogeny from multiple published scarab phylogenies. The branch positions for beetle

subfamilies (Dynastinae, Rutelinae and Cetoniinae) were determined using Hunt et al. (41*). The

branches within Dynastinae were placed in the tree using work from Rowland and Miller (42),

and the branches of Cetoniinae determined with two trees, one from Mico et al. (43, 44) and the

other Holm (43, 44). Four of the genera in this study were present in the tree for Coleoptera

constructed by Bocak et al. (45), which indicated the same branch places as our spliced tree,

providing some positive confirmation for this tree structure. Branch lengths were set to a value of

one because actual branch lengths are not known. Similar to pGLS, we built a Bayesian model

assuming that the data were generated by a multivariate normal distribution with the covariance

matrix given by the amount of shared ancestry between species (amount of shared branch

length). See supplemental methods for the details of the model, selection of priors, and python

code. Detailed information on the analysis is available at the website for the paper here:

https://julianmwagner.github.io/spiracle_scaling/ and at the corresponding repository

(https://github.com/julianmwagner/spiracle_scaling). Analyses indicated that the parameter

characterizing the degree of phylogenetic signal in our data (λ) was non-identifiable

(supplemental figures 1,2); this result means that our data does not inform this parameter and it

could take on any value from zero (no phylogenetic signal) to one (strong phylogenetic signal)

with similar probability. Hence, we opted to omit the use of phylogenetic covariance from our

models since 1) the total non-identifiability made selecting a single λ via maximum likelihood

for the frequentist pGLS dubious, and 2) including it added no explanatory value to our Bayesian

regression (parameter samples for λ were essentially straight from the prior). We instead used

nonparametric bootstrapping (10,000 bootstrap replicates with ordinary least squares regression

slopes/intercepts/residual standard deviation as the summary statistics) to obtain confidence

intervals for our slope and intercept values. Additionally, we performed a Bayesian linear

regression. Our model was a normal likelihood with mean given by a line with slope and

intercept parameters. To obtain parameter estimates, we sampled using the Stan implementation

of Hamiltonian Monte Carlo (cmdstanpy) in Python. See supplemental methods for the details of

the model, selection of priors, and Python code and at the paper website/repository listed above.

No data were excluded.

We defined isometric scaling as scaling as follows:

$mass^{0.67}$ for areas

$mass^{0.33}$ for area/depth

$mass^{1}$ for $area^{2}$/depth,

according to basic principles of geometric similarity (assuming mass is proportional to volume).

We observed whether the 95% confidence interval given by bootstrapping/parameter samples for

the slope of our measures of spiracle morphology overlapped with the isometric prediction. To

produce any p values, we calculated the number of bootstrap replicates with test statistic at least

as extreme as a particular value of interest, e.g. slope compared to isometry.

The diffusive capacity of a spiracle (G_{diff}, nmol sec^{-1} kPa^{-1}) at 25°C was calculated as:

(1) $G_{diff} = \frac{area}{depth} * D * \beta$

with D (the diffusivity constant for O_2 in air) = 0.178 cm^2 sec^{-1} (46) and β (the capacitance

coefficient for oxygen in air) = 404 nmol cm^{-3} kPa^{-1} (47). For a diffusive system,

(2) Oxygen consumption rate (nmol s^{-1}) = Δ PO_2 * G_{diff},

with ΔPO_2 = the partial pressure gradient for oxygen across the spiracle (kPa). Advective

capacity (G_{adv}, m^3 s^{-1}•kPa^{-1}) was calculated as:

(3) $G_{adv} = \frac{area^2}{8 * dynamic\ viscosity * length}$

assuming a dynamic viscosity of air of 1.86 x 10^{-8} kPa sec (46). To calculate total diffusive or

advective capacity per beetle, the diffusive/advective capacity for all eight spiracles was summed

and doubled (to obtain the total for both sides of the animal).

To calculate the ΔPO_2 across the spiracles needed to supply the beetle's total metabolic

demand by diffusion, the metabolic rate for a quiescent beetle at a body temperature of 25 °C of

a given mass was estimated from (48) with the following equation: $\log_{10}(\text{metabolic rate } (\mu W)) =$

$3.2 + 0.75 \cdot \log_{10}(\text{mass (g)})$. Based on (49), the metabolic rate of small insects during flight is in

the range of 8x the resting value, whereas it is on the order of 32x resting metabolic rate in large

insects. Most scarab beetles are endothermic during flight, so flight metabolic rates of these

warm beetles could double with a 10 °C increase in thoracic temperature if the Q_{10} is 2, resulting

in a 64x total increase. Even this calculation may be a conservative estimate of hovering

metabolic rate, as oxygen consumption rose to 90x higher (compared to quiescent) in fig beetles

in one study (50), and maximal flight metabolic rates may be 1.25-2x higher than those reported

during hovering (51-54). Therefore, we conservatively estimated maximal aerobic metabolic rate

during flight as 90x those of quiescent beetles. The required PO_2 gradient across the spiracles to

support gas exchange by diffusion at rest and during flight was calculated by rearranging

equation 2 and performing unit conversions as follows:

$\quad$ (4) ΔPO_2 (kPa) $= \left(\dfrac{\left(10^{3.20+0.75*\log_{10}(\text{mass (g)})}\mu W\right)\left(\frac{1\frac{\mu J}{s}}{1\,\mu W}\right)\left(\frac{nL}{20.7\,\mu J}\right)\left(\frac{1\,nmol}{24.5\,nL}\right)}{\left(\frac{area}{depth}\,(cm)\right)\left(0.178\,\frac{cm^2}{sec}\right)\left(404\,\frac{nmol}{cm^3 kPa}\right)} \right).$

where conversion factors of 20.7 kJ/L and 24.5 mol/L were assumed for O_2 at 25 °C.

Results

$\quad$ All spiracles scaled with geometric isometry for area, depth, area/depth (which

corresponds to diffusive capacity), and $area^2$/depth (which corresponds to advective capacity)

(figure 2, supplemental figure 3-7). Some example regressions with confidence intervals for the

slopes are shown in figure 2, illustrating scaling isometry, the larger size of the mesothoracic

spiracle, and the tight size distribution of the more anterior spiracles as compared to the

posterior. The mesothoracic spiracle was much larger than any of the other spiracles, consistent

with the general trend of increasing spiracular area closer to the anterior of the animal

(supplemental figure 3). The area of the mesothoracic spiracles was approximately four times

larger than both the metathoracic spiracles, and abdominal spiracles 1-3, and abdominal spiracles

4-6 were approximately half the size of the more anterior abdominal spiracles (supplemental

figure 3). Not only were anterior spiracles larger than posterior, but they also had much lower

variability around the trend line within the species assayed in this study (figure 2a,b,c). In

comparison, the depth of the spiracles showed much less variability in tightness of the

distribution around the scaling trend lines (figure 2d,e,f).

As with individual spiracles, the combined diffusing capacity of all the spiracles scaled

isometrically (Fig. 3a). However, because this slope was less than the scaling exponent for

metabolic rate (slope of 0.75, shown as the repeated light grey background lines), the required

pO_2 gradient across spiracles necessary to supply oxygen by diffusion increased by an order of

magnitude from the small to the large scarabs (Fig. 3b). The advective capacity increased with a

positive slope that was greater than the scaling exponent for metabolic rate (slope of 0.75, light

grey lines) (Fig. 3c).

Discussion

Spiracles scaled with geometric isometry. Isometric scaling of diffusive capacities means

that diffusion becomes increasingly less able to meet oxygen demands in larger beetles, with the

required gradient for oxygen transport by diffusion through the spiracles increasing by an order

of magnitude over two orders of magnitude in body mass. Conversely, our data demonstrate that

the advective capacities of the spiracles scale more positively than do metabolic rates. This result

strongly suggests that, if insects switch from diffusive to advective gas transport, there is no

physical constraint associated with spiracular gas exchange that limits insect size and metabolic

rates.

It is important to note that geometric isometry does not occur universally for tracheal

structures, or consistently across clades. Comparing tenebrionid beetles inter-specifically, the leg

tracheae scale hypermetrically, but the head tracheae scale isometrically (6). Within a bumblebee

species, one spiracle scales isometrically (16). In the leg of growing locust (*Schistocerca*

americana), the diffusing capacity of the large longitudinal tracheae of the leg scales

hypometrically (17), whereas in a growing caterpillar (*Manduca sexta*), diameters of most

tracheae scale isometrically (55). Why different scaling patterns are observed in these different

cases is unclear; more in-depth analysis of the required gas transport and the mechanism of

transport are needed to evaluate the scaling of individual tracheal system structures.

Diffusive capacities of the spiracles scaled with mass$^{0.39}$, well below the scaling slope for

oxygen consumption rate (approximately 0.75); thus, diffusion across the spiracles becomes

more challenging for larger insects. The required O_2 gradient across the spiracles to supply the

metabolic demand by diffusion increases by approximately an order of magnitude from our

smallest to largest beetles, but the size effect on the required PO_2 gradient is less important than

the effect of activity. For quiescent beetles, the PO_2 gradients across the spiracles necessary for

diffusion are low (0.05-0.46 kPa depending on size). However, during endothermic flight, the

required PO_2 gradient across the spiracles increases from 5 to 41 kPa, which is impossible to

achieve because the maximum partial pressure of oxygen in air is only 21 kPa. Because the

distance required for oxygen diffusion from air to the flight muscle is much greater than the

distance through the spiracle, these data illustrate the impossibility of diffusion sustaining

oxygen consumption during flight in these beetles, regardless of size. With metabolic rates

scaling with mass$^{0.75}$ and spiracular depth with mass$^{0.33}$, spiracular area would need to scale with

mass$^{1.08}$ (0.75 + 0.33) to conserve the required PO$_2$ gradient to support diffusion across all insect

sizes.

By contrast, advective capacity scales with mass$^{1.1}$, much higher than the slope for the

metabolic rate, so oxygen delivery via advection becomes easier for larger beetles even as

diffusion becomes impossible. This effect might reflect a shift toward increasing reliance on

advection as scarab beetles increase in size; alternatively, this increasing advective capacity

relative to metabolic rate may allow larger beetles to require lower pressures to drive advection

through their spiracles, assuming no scaling of extraction efficiency.

We also observe much tighter distributions in the scaling pattern for the area of the large

anterior spiracles as compared to the smaller posterior ones. This result may suggest that the

large anterior spiracles are more constrained in their morphology, since they presumably provide

the gas exchange needed for metabolically demanding tissues like the flight muscle and brain.

There are some important caveats when interpreting our data. Insect spiracles are

morphologically complex structures. We made 3D measurements using tomographic imaging,

but analyzed air transport capacities by modeling the spiracle as a cylinder, which could over or

underestimate capacity depending on factors like valve position and the complex shape of the

spiracular atrium. Furthermore, our CT scans were conducted on sacrificed specimens; the

assessment of spiracles of living insects could offer insights not possible with static morphology.

For example, living insects might control the shape of the bellows-like atrium and valves in a

concerted way to promote air flow. As yet, we know little about how the tracheal system

structure and function might scale differently in different species. As an example of a fairly

dramatic difference in tracheal system function across beetle clades, some Cerambycid beetles

use draft inward ventilation through the mesothoracic spiracle during flight, whereas most scarab

beetles autoventilate the thorax using wing movements (56, 57). Dung beetle species vary

between exhaling nearly all to none of their air out the mesothoracic spiracles, with species from

more arid environments exhibiting more expiration via the mesothoracic spiracle (58). Multiple

beetle species collapse parts of their tracheal system to produce advective airflow, both in adults

and as pupae (59-61). Though it is unclear how respiration via tracheal collapse differs with size

and in different species, the prevalence of active breathing further highlights the need for

advective airflow for insect function. The phylogenetic, life history, and environmental

influences on tracheal system structure, function, and scaling seem likely to be a ripe area for

future research.

Our finding of isometric scaling of insect spiracles would appear to differ from reports

for tracheae of mammals, in which radius scales with mass$^{0.39}$ and lengths with mass$^{0.27}$ (62).

However, confidence limits from our study included these scaling slopes, suggesting that

respiratory scaling of tracheal morphology may be congruent across these disparate groups.

Tenney and Bartlett's study (62) had greater power, as it examined 43 species ranging over 5

orders of magnitude in body mass. However, it worth noting that they did not consider error in

their slope estimates, test for statistical differences in slopes between the radii and lengths, or

consider phylogeny, so the conclusion that mammalian tracheae scale non-isometrically (and

differently from insect spiracles) could benefit from rigorous comparative analysis.

Conclusions: Insect spiracles scale with geometric isometry in beetles, which means that

diffusive capacities increase much less than metabolic rates as body size increases, while

advective capacities increase more rapidly than do metabolic rates. These are general principles

of gas exchange that should apply to respiratory structures of any animal clade exhibiting

isometric scaling. Nonetheless, resting oxygen consumption rates of the largest beetles can likely

be supplied by diffusion, allowing even very large insects to recover from any form of anoxia it

might experience . In contrast, endothermic flight, and likely strong terrestrial locomotion,

require a strong ventilatory system, especially in larger insects.

Authors' contributions

JW helped with study design, collected spiracle measurements from the micro-CT data,

carried out statistical analyses, and drafted the manuscript and figures. JFH helped with study

design, coordinated the study, and drafting the manuscript. CJK and JJS collected the micro-CT

data; additionally, Dynastes data were collected in collaboration with HG and GC. MED helped

develop methods for analysis of micro-CT data, and helped with phylogenetic correction

analysis. JJS also helped draft the manuscript. All authors gave approval for publication.

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	Figure Legends

Figure 1. Scarab beetles include large bodied individuals and have eight spiracles. (a)

Phylogenetic tree for the scarab beetles used in this study showing size distribution among clades

(branch lengths are meaningless). (b) Location of the eight spiracles in the scarab body. (c) 3D

reconstruction of the tracheal trunks in the thorax, legs and abdomen of *Dicronorrhina derbyana*;

spiracles are shown in white. The larger images of spiracles show the size of the opening (dark in

color) compared to the mushroom-shaped (white) atrium behind and the differences in spiracle

shape. (d) Transverse x-ray slice through the third abdominal spiracle with diameter, α, and

depth, β, measures illustrated.

Figure 2. Isometric scaling of scarab beetle spiracles. Spiracle area scales isometrically (A, B),

with much tighter distribution about the isometric model for the large anterior spiracles (e.g. the

mesothoracic spiracle as seen in A) as compared to the smaller posterior spiracles that may be

less critical for gas exchange to metabolically demanding tissues. C shows estimates for the

variability for regression models for the various spiracles (S mesothoracic, T metathoracic, 1-6

abdominal), calculated as the standard deviation divided by $10^{\text{regression intercept}}$, which represents the

spiracle area for a 1g beetle. Black diamond and line show the median and 2.5th-97th residual

standard deviation divided by $10^{\text{regression intercept}}$ calculated on non-parametric bootstrap samples.

The white diamond and grey interval represent the median and 3rd-97th highest posterior density

interval for the standard deviation divided by $10^{\text{regression intercept}}$ calculated from parameter samples

from the Bayesian regression. We see a trend towards much higher variability in posterior

spiracle area as compared to anterior. In contrast to spiracle area, spiracle depth shows similar

variability in all spiracles (D-F) regardless of position.

Figure 3. Scaling of the spiracles is insufficient to conserve diffusive capacities but more than

sufficient to conserve advective capacities in large beetles. (a) The $\log_{10}$ of total spiracular

diffusive capacity per beetle (nmol s^{-1} kPa^{-1}) increases with beetle size, but with a slope

significantly less than the 0.75 slope of metabolic demand (slope greater than 0.75 in 0 of 10,000

bootstrap replicates). Metabolic rate slope is shown in light grey. (b) The $\log_{10}$ PO$_2$ gradient

(kPa) required to diffusively supply the oxygen demand of beetles increases with beetle size. The

upper band shows the range of PO$_2$ gradients across the spiracles needed for diffusive oxygen

supply in flying beetles, while the lower line shows the values for resting beetles at a body

temperature of 25°C. The scatter points at the bottom of the flight band represent required PO$_2$

gradients across the spiracles if metabolic rates during flight are eight times that of rest; the

upper scatter points indicate required PO$_2$ gradients across the spiracles if metabolic rates are

ninety times resting values, likely if thorax temperature warms to 40°C and beetles exhibit

maximal flight performance. (c) Hypermetric scaling of $\log_{10}$ summed advective capacity (m^3 s^{-1}·kPa^{-1}) versus $\log_{10}$(body mass), showing a decrease in resistance to advective gas exchange in

larger insects. The advective capacity scales significantly more steeply than the 0.75 slope of

metabolic rate (slope higher than 0.75 in all but 93 out of 10,000 bootstrap replicates). Metabolic

rate slope is shown in light grey. Equations of regression lines and R^2 shown for each plot.

Supplemental Materials

Supplemental figure 1. Non-identifiability of phylogenetic signal parameter in pGLS regression

model. Left-hand column shows both OLS regression and pGLS regression models plotted for a

given spiracle morphology (log of area, depth, area/depth, or area^2/depth) vs log of mass. The

Right-hand column shows the log likelihood space for the parameter λ, which scales the off-

diagonal covariance terms of the regression model thereby providing an indication of how

strongly phylogeny influences the pattern of data distribution. A λ of one indicates strong

phylogenetic signal whereas a value of zero indicates no sign of covariance in the data due to

phylogeny. The solid red line indicates the maximum likelihood for the parameter used in the

model on the left, the dashed red line indicates confidence interval estimates for the parameter

value. Note that the log likelihood for the parameter is fairly flat across large section of the

possible parameter space, and all confidence intervals on the parameter include zero (no

phylogenetic signal). Together, these indicate that many values for λ are similarly likely, and that

the strength of the phylogenetic signal is hence non-identifiable (our data does not strongly

inform the parameter). Given the weakly-to-non-peaked likelihood distributions, selecting a

particular value for λ to use in the model is a largely arbitrary choice, poorly supported by the

data.

Supplementary figure 2. Non-identifiability of the phylogenetic signal parameter as indicated by

Bayesian modeling. We constructed a generative model for our data (a multivariate normal

distribution with covariance matrix given by the phylogenetic signal) which is analogous to the

model assumed in pGLS regression. We included a parameter for λ that scaled the off-diagonal

terms of the covariance matrix, analogous to pGLS, and sampled from our model for the various

regressions of log spiracle morphology versus log mass. We then plotted the prior for the λ

parameter (a beta distribution with shape parameters 1.4 and 1.4) against the samples from HMC

sampling. This indicated that the samples for this parameter had no shrinkage: the posterior

samples for the parameter matched the prior. This indicates that the data does not inform this

parameter, a clear indication that the parameter is entirely non-identifiable given our data.

Together, with supplemental figure 1, this indicates that the phylogenetic signal is not

informative for our models and does not provide useful information or insight to our analysis.

Supplementary figure 3. Slopes, intercepts, standard deviations, and quasi-coefficient of

variation with confidence intervals generated via non-parametric bootstrapping or Bayesian

regression for all spiracles and area, depth, area/depth, or area^2/depth vs mass. All spiracles

scaled isometrically. The anterior spiracles are much larger than the posterior ones and hence

provide much of the gas exchange capacity for the animal. They also exhibit a much tighter

distribution for their area and gas exchange capacity than the posterior ones. The spiracle depth,

however, did not show the same level of variation in dimension; all spiracular depths had similar

degree of variation. This suggests there may be tighter selective regulation on the morphology of

the large spiracles that likely supply the airflow needed for highly metabolically demanding

tissue like the flight muscle, legs, and brain.

Supplementary figure 4. Regression plots for both Bayesian and non-parametric bootstrap

analyses. The top set of panels provide results for Bayesian linear regression; the light grey

bands provide the 80th and 95th percentile for the posterior predictive distribution for the

regression model, the dark grey interval shows the 95th percentile for regression line ranges given

609 by the slope and intercept samples, the black line the 10th percentile for these regression lines.

610 The bottom panels show regression results from non-parametric bootstrap replicates drawn from

611 the data, with ordinary least squares regression as the summary statistic applied to the

612 subsamples. The grey interval shows the 95th percentile for the regression lines generated by the

613 bootstrap sampling. The repeated light grey lines in both sets of figures indicate the theoretical

614 isometric slope value for the regression of a given morphological trait.

616 Supplementary table 1. Raw spiracle measurements.

618 Supplementary table 2. Spiracle measurements in a tidy format.

620 Supplementary video: Reconstruction of the spiracles and connecting tracheal trunks on one side

621 of the scarab beetle *Dicronorhina derbyana*.

A *Cyclocephala* · *Dynastes* · *Trypoxylus* · *Popillia* — Dynastinae / Rutelinae

Protaetia · *Coelorrhina* · *Dicronorrhina* · *Mecynorrhina* · *Eudicella* · *Goliathus* — Cetoniinae

32 mm

B
S
T

S→ Mesothoracic | T→ Metathoracic
1→ Abdominal 1, etc.

C 6 5 4 3 2 1 T S

D α β spiracle 3

air sac

0.4 mm

Diffusing Capacity

A

log G_{diff}

log body mass

slope 95% CI: [0.227, 0.505]
0/10,000 bootstrap slopes > 0.75

Required ΔPO₂

B

log ΔPO_2

ΔPO_2
kPa

90x

8x

log body mass

Advecting Capacity

C

log G_{adv}

slope 95% CI: [0.84, 1.353]
93/10,000 bootstrap slopes < 0.75

log body mass

Supplementary Figure 1

depth vs mass regression

Likelihood space, λ parameter

depth vs mass regression

Likelihood space, λ parameter

Spiracle 3 Slopes

Spiracle 4 Slopes

Spiracle 5 Slopes

Spiracle 6 Slopes

Data: comparative_data; Model: log_dist ~ log_mass
kappa 1.00; lambda 0.00; delta 1.00

area/depth vs mass regression

Likelihood space, λ parameter

area/depth vs mass regression

Likelihood space, λ parameter

Spiracle 3 Slopes

Data: comparative_data; Model: log_area_dist ~ log_mass
kappa 1.00; lambda 0.00; delta 1.00

Spiracle 4 Slopes

Data: comparative_data; Model: log_area_dist ~ log_mass
kappa 1.00; lambda 0.75; delta 1.00

Spiracle 5 Slopes

Data: comparative_data; Model: log_area_dist ~ log_mass
kappa 1.00; lambda 0.49; delta 1.00

Spiracle 6 Slopes

Data: comparative_data; Model: log_area_dist ~ log_mass
kappa 1.00; lambda 0.71; delta 1.00

area²/depth vs mass regression   Likelihood space, λ parameter

Spiracle S Slopes

Data: comparative_data; Model: log_area_2_dist ~ log_mass
kappa 1.00; lambda 0.74; delta 1.00

Spiracle T Slopes

Data: comparative_data; Model: log_area_2_dist ~ log_mass
kappa 1.00; lambda 0.00; delta 1.00

Spiracle 1 Slopes

Data: comparative_data; Model: log_area_2_dist ~ log_mass
kappa 1.00; lambda 0.00; delta 1.00

Spiracle 2 Slopes

Data: comparative_data; Model: log_area_2_dist ~ log_mass
kappa 1.00; lambda 0.00; delta 1.00

area²/depth vs mass regression   Likelihood space, λ parameter

Nonidentifiability of phylogenetic signal parameter, area vs mass

Nonidentifiability of phylogenetic signal parameter, depth vs mass

Nonidentifiability of phylogenetic signal parameter, area/depth vs mass

Nonidentifiability of phylogenetic signal parameter, area^2/depth vs mass

Supplementary Figure 4

Bayesian regression, area vs mass

OLS regression, bootstrap CI, area vs mass

Bayesian regression, depth vs mass

OLS regression, bootstrap CI, depth vs mass

Bayesian regression, area/depth vs mass

OLS regression, bootstrap CI, area/depth vs mass

Bayesian regression, area²⁄depth vs mass

OLS regression, bootstrap CI, area²⁄depth vs mass