Is phase-dependent stability related to phase-dependent gait robustness?

Jian Jin[1], Dinant Kistemaker[1], Jaap H. van Dieën[1], Andreas Daffertshofer[1,2], Sjoerd M. Bruijn[1,2]

[1] Department of Human Movement Sciences, Faculty of Behavioural and Movement Sciences, Vrije Universiteit Amsterdam, Amsterdam Movement Sciences, Amsterdam, The Netherlands

[2] Institute of Brain and Behavior Amsterdam, Amsterdam, The Netherlands

Corresponding Author:

Sjoerd M. Bruijn

Van der Boechorststraat 9, 1081 BT Amsterdam, The Netherlands

Email address: s.m.bruijn@vu.nl

Keywords: stability measure, gait robustness, phase-dependent.

Word count: 1700

I. ABSTRACT

Predicting gait robustness is useful for targeting interventions to prevent falls. A first step towards this is to properly quantify gait robustness. However, this step already comes with challenges, as humans can withstand different magnitudes of perturbations at different phases in a gait cycle. Earlier, we showed using a simple model that phase-dependent stability measures are limited to predict gait robustness. However, phase-dependent stability measures might relate to phase-dependent gait robustness. To study this, we simulated a 'simple' walker model that walks stably and periodically. We applied push and pull perturbations to the stance or swing leg at each phase of the single stance phase and evaluated how phase-dependent stability measures correlate with phase-dependent gait robustness. The latter was quantified via the maximum energy deviation induced by a perturbation that the walker could withstand without falling within 50 steps. Phase-dependent stability measures were obtained, after linearizing the system in a rotating hypersurface perpendicular to the periodic trajectory, via the maximum and the sum of the eigenvalues of the reduced Jacobian matrix, i.e., the trajectory-normal divergence rate. We did not find any strong association between phase-dependent stability measures and phase-dependent robustness. Combining this with our previous assessment of gait robustness, we conclude that phase-dependent stability does not allow for predicting gait robustness, let alone predicting fall risk.

II. INTRODUCTION

Predicting fall risk is important for targeting interventions to prevent falls. It has been proposed that predictors of fall risk should be theoretically sound, have predictive value in 'simple' models, and be validated in experimental and observational studies (Bruijn et al., 2013). Several studies have used phase-dependent stability measures to evaluate local stability in human gait and ultimately predict fall risk (Ihlen et al., 2012a; Ihlen et al., 2012b; Mahmoudian et al., 2016). They defined stability via the inverse of a dynamical system's rate of divergence from the unperturbed trajectory under infinitesimal perturbations at different phases (Ali & Menzinger, 1999). In a previous study, we have shown that these phase-dependent stability measures do not correlate with gait robustness in simple dynamic walking models (Jin et al., 2021). One reason for this could be that we did evaluate the overall gait robustness rather than a phase-dependent one. Humans (Cordero, 2003; Eng et al., 1994; Golyski et al., 2022; Tang & Woollacott, 1999) and simple walking models (de Boer et al., 2010; Williams & Martin, 2021) alike can withstand different magnitudes of perturbations at different phases of a gait cycle. Put differently, the robustness of gait is phase-dependent.

Accordingly, here, we asked whether phase-dependent stability measures are related to phase-dependent robustness. A positive answer to this question may have practical implications for identification of fall-prone individuals, i.e., by 'just' estimating the phase-dependent stability. A negative answer, however, would reveal further limitations of phase-dependent stability measures, i.e., one should look for alternatives when seeking to predict (phase-dependent) gait robustness and ultimately fall risk.

III. METHODS

We built on our previous study (Jin et al., 2021) and employed a simple walking model (Garcia, 1998; Norris et al., 2008). This model consists of two massless legs connecting the hip point mass M and foot point masses m, see Figure 1. It can walk stably and periodically down a slope, with each step consisting of a single stance phase and an instantaneous double stance phase (foot strike). We assumed the walker to have a fixed configuration (mass ratio $\beta = m/M = 0.002$, and slope $\gamma = 0.001$) yielding stable periodic gait. The state variables were represented in Hamiltonian form as $s = \left(\theta, \varphi, p_\theta, p_\varphi\right)$, where θ denotes the angle of the stance leg with respect to the normal of the inclined floor, φ is the angle between the legs, and p_θ and p_φ are the canonical conjugate momenta. For a detailed description of the equations of motion including the relations between $\left(p_\theta, p_\varphi\right)$ and $\left(\dot{\theta}, \dot{\varphi}\right)$, we refer to (Jin et al., 2021; Norris et al., 2008). The numerical simulations to search for the stable period-one solutions (Hurmuzlu & Moskowitz, 1987) were conducted as in Jin et al., (2021).

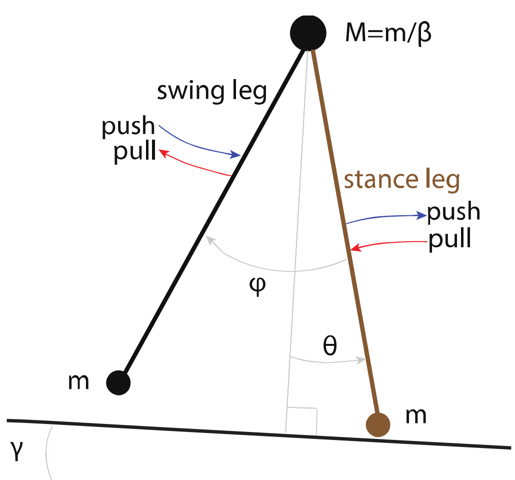


Fig. 1. Compass walker model with point feet. This model consists of two massless legs connected by a hip point mass M and foot point masses m. State variable θ is the (directional) angle of the stance leg with respect to the normal of the inclined plane, and φ is the (directional) angle between the legs. We applied push and pull perturbations to the stance and swing leg by instantaneously changing the angular velocities $\dot{\theta}$ and $\dot{\varphi}$.

Phase-dependent stability measures were obtained by linearizing the system in a rotating hypersurface perpendicular to the periodic trajectory; see (Jin et al., 2021; Norris et al., 2008) for details. This coordinate transformation yields stability estimates invariant against phase-shifting perturbations (perturbations along the periodic trajectory), where the system stays on the periodic trajectory and the gait cycle is merely advanced or delayed. Then, we determined the eigenvalues of the so-reduced Jacobian matrix, and computed the trajectory-normal divergence rate (sum of these eigenvalues) as well as the maximum eigenvalue as phase-dependent stability measures. The former quantifies the mean volume contraction or expansion rate of all infinitesimal perturbations that are perpendicular to the periodic trajectory (i.e., the larger the trajectory-normal divergence rate, the faster the mean volume of all perturbations perpendicular to the periodic trajectory expands), and the latter quantifies the highest rate of divergence of infinitesimal perturbations along the corresponding eigenvector direction (i.e., the smaller the eigenvalue, the faster the perturbations along the eigenvector direction decay). Figure 2 shows that the trajectory-normal divergence rate peaks at the beginning and drops towards the end of the single stance phase, similar to (Jin et al., 2021; Norris et al., 2008). In line, the maximal eigenvalue of the reduced Jacobian matrix is highest at the beginning and lowest at the end of the single stance phase.

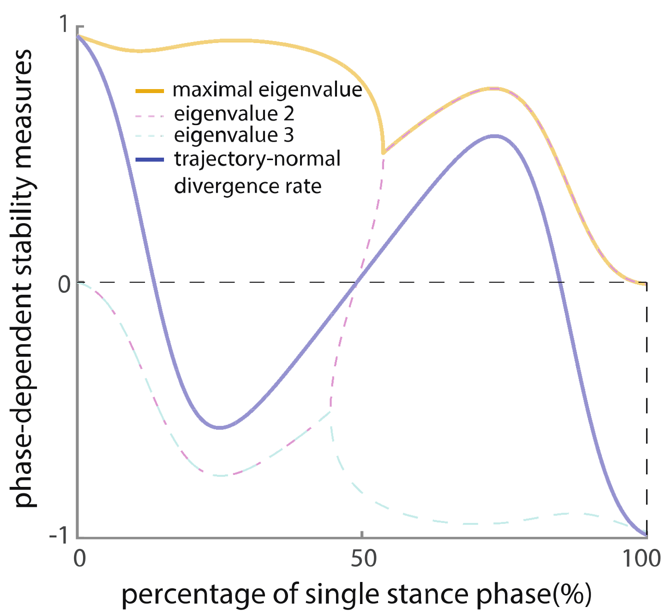


Fig. 2. Maximal eigenvalue (yellow curve) and trajectory-normal divergence rate (blue solid curve) over the single stance phase for the given configuration of the walker. Positive maximal eigenvalue indicates that infinitesimal perturbations are diverging and negative maximal eigenvalue indicates that infinitesimal perturbations are converging in the direction of the corresponding eigenvector. The trajectory-normal divergence rate indicates the mean volume contraction or expansion rate of all infinitesimal perturbations perpendicular to the periodic trajectory in phase space.

To estimate phase-dependent gait robustness, we applied one-time stance leg or swing leg perturbations at consecutive points in the gait cycle separated by a non-dimensional time of 0.1 (non-dimensional step time ≈ 3.83). Stance leg and swing leg perturbations were realized by instantaneously increasing or decreasing the angular velocity of the stance leg $\dot{\theta}$ and of the hip $\dot{\varphi}$ respectively. Since the total energy of a passive walker is constant until the instant the walker is perturbed, and this perturbed energy remains constant afterwards until foot strike, we quantified phase-dependent gait robustness using the maximum allowable energy deviation from the unperturbed gait that could be induced from these perturbations without falling within 50 steps.

Finally, the trajectory-normal divergence rate and maximal eigenvalue during the single stance phase were correlated to the phase-dependent gait robustness. To quantify the correlations, we used Kendal rank correlation coefficient (Kendall, 1938) instead of Pearson correlations, because the latter cannot quantify nonlinear relations, which could be indicators of good prediction accuracy (see also Jin et al., 2021). In short, the nonparametric correlation coefficient (between -1 and 1) evaluates the similarities in the ordering of the data ranked by two variables, with 1 (-1) indicating monotonic increasing (decreasing) relations, and 0 implying the absence of any association between the two variables. In line with our previous paper, we considered Kendall rank correlation coefficients larger than 0.7 (or smaller than -0.7) to be indicator of strong correlations.

IV. RESULTS

As shown in Figure 3, the model's robustness against perturbations to the stance and swing leg depended on the phase of the gait cycle. The maximum robustness against stance leg perturbations was around mid-stance, resisting larger push than pull perturbations (Fig. 3a). The maximal robustness against swing leg perturbations was largest for pull perturbations just before collision (Fig. 3b). All other swing leg perturbations displayed much lower robustness.

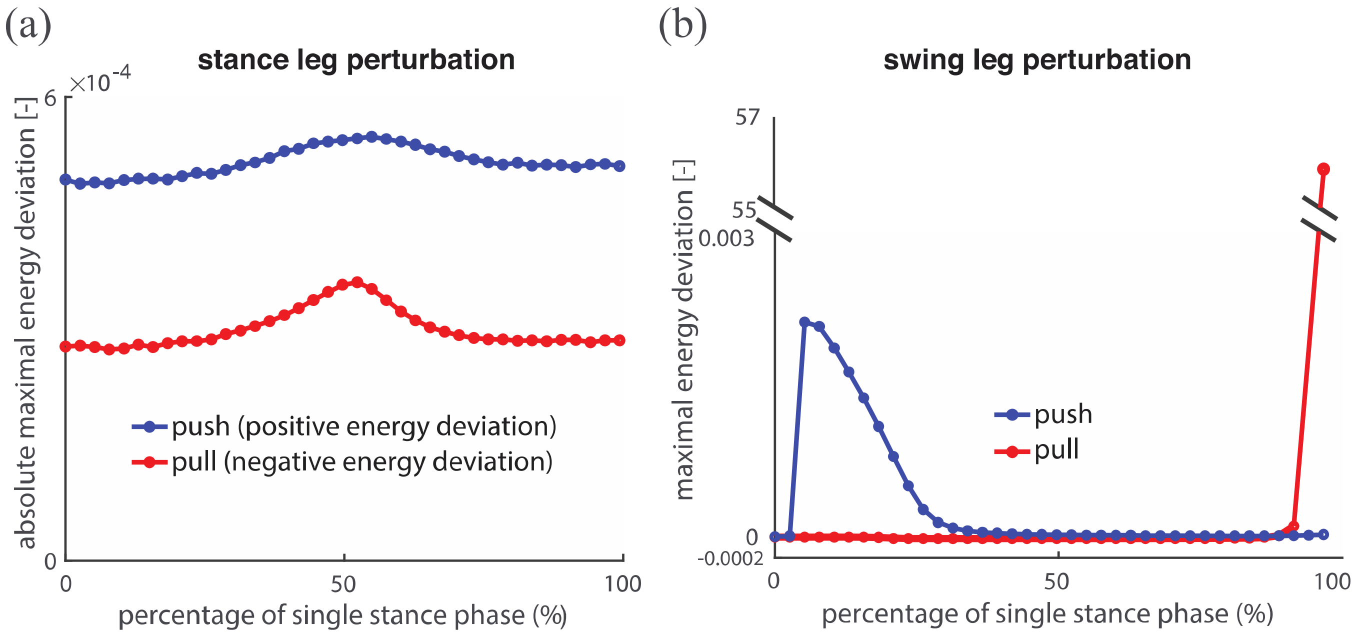


Fig. 3. (a) The absolute of maximal energy deviation for push (blue) and pull (red) perturbations applied to the stance leg in different phases of the single stance phase. Note here that for pull perturbations, there is a reduction in energy, which has been plotted positive to increase readability of the figure. Robustness against stance leg perturbations was largest around mid-stance, resisting larger push than pull perturbations; (b) The maximal energy deviation for push and pull perturbations applied to the swing leg in different phases of the single stance phase. Note here that the maximal energy deviation at some phases was negative. Robustness against swing leg perturbations was largest for pull perturbations just before collision.

Figure 4 shows that none of the four types of phase-dependent robustness had strong correlations with either the trajectory-normal divergence rate or the maximal eigenvalue. Put differently, neither the trajectory-normal divergence rate nor the maximal eigenvalue appeared to be related to the phase-dependent robustness of the walker.

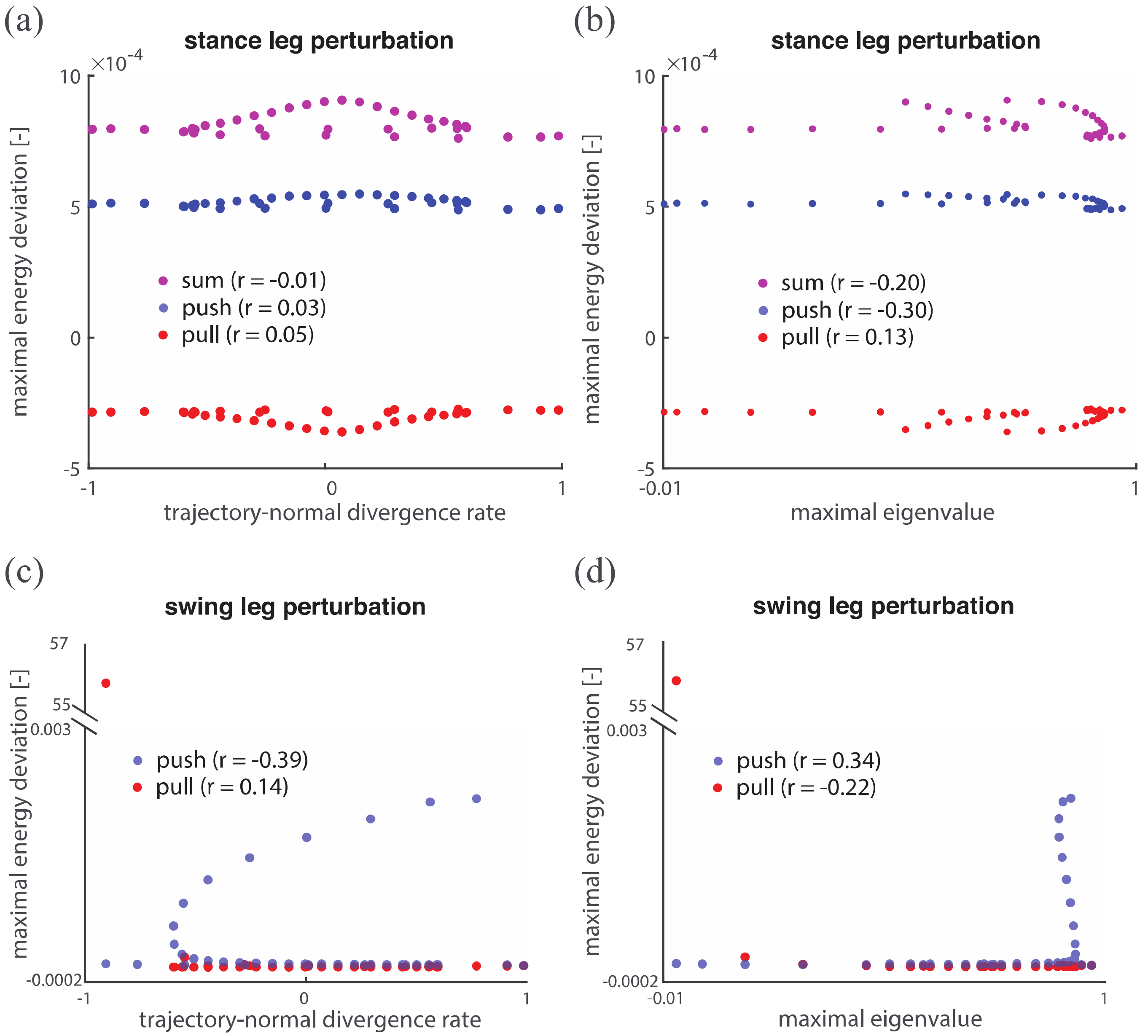


Fig. 4. (a)(b) For stance leg perturbations, the relations between trajectory-normal divergence rate or maximal eigenvalue with maximal energy deviation for push (blue) and pull (red) perturbations applied to the stance leg; (c)(d) For swing leg perturbations, the relations between trajectory-normal divergence rate or maximal eigenvalue with maximal energy deviation for push and pull perturbations. Kendal rank correlation coefficients (r value) were used to quantify correlations.

V. DISCUSSION

We studied whether phase-dependent gait robustness is correlated to phase-dependent stability measures. We used a 'simple' walker model at a fixed configuration that yields stable periodic motion. For stable periodic motion, locally stable and/or locally unstable phases can (co-)exist (Guckenheimer et al., 1984;

Norris et al., 2008; Tallapragada & Sudarsanam, 2017). Yet, infinitesimal perturbations that can be rejected at one phase can be rejected at any phase regardless of its local (in)stability (Guckenheimer et al., 1984). Our walker was able to reject push and pull perturbations to both the stance and swing leg. No strong correlations were found between either of the phase-dependent stability measures (trajectory-normal divergence rate and maximal eigenvalue) and any of the phase-dependent robustness measures (push/pull/sum of push and pull on the stance/swing leg). Specifically, phase-dependent stability measures failed to identify that robustness to stance leg perturbations is largest around mid-stance, and that robustness to swing leg pull perturbations is largest before foot strike. Interestingly, Williams & Martin (2021) found that in a kneed and circular feet walker, robustness was also largest against horizontal impulse perturbations at the hip applied at mid-stance, regardless of the type of feedback controller. Our results also showed that robustness to different types of perturbations is distinct and phase-dependent, making their (separate or overall) prediction from a single (phase-dependent) stability measure difficult and unlikely.

The trajectory-normal divergence rate quantifies the rate of divergence of infinitesimal perturbations from the *unperturbed* trajectory. A trajectory deviating from the *unperturbed* trajectory (e.g., due to local instability) may still settle on a steady-state gait, if, after a push perturbation on the stance leg (which increases the energy of the walker) there is a larger collision loss which prevents forward falling, and after a pull perturbation on the stance leg there is a smaller collision loss which prevents backward falling. This may explain why robustness to stance leg push perturbations is largest around midstance: a midstance push is more effective to increase *both* legs' angular velocities, leading to larger step lengths (and thus larger collision loss) compared to (similar magnitude) push perturbations added earlier or later in the gait cycle, when the angular acceleration of the swing leg is limited. Likewise, robustness to stance leg pull perturbations is larger around midstance compared to the end of the gait cycle because the earlier the (similar magnitude) pull perturbations, the smaller the step lengths and collision loss. Stance leg pull perturbations at the very beginning of the gait cycle, on the other hand, lead to too small step lengths such that the walker falls forward, which results in a lower robustness for early pull perturbations. The large robustness to late swing leg pull perturbations can be explained by the fact that (generalized) hip momentum and step length hardly changes due to perturbation, and the additional energy is dissipated by similarly large collision loss. Since foot strike is the only phase at which energy is dissipated in our model, the foot strike dissipation is likely to be of greater importance in predicting overall gait robustness than the single stance phase and the measures based on this phase. Our previous work (Jin et al., 2021) indeed showed that the divergence of foot strike in a periodic gait was reasonably well correlated to gait robustness in terms of stepup/step-down perturbations for different configuration parameters (mass ratio and slope) in the compass walker model. As the divergence of foot strike can also be computed on a step-to-step basis from the perturbed trajectories, and is only a function of step length in our model, future work should validate whether stability measures estimated at foot strike can accurately predict gait robustness.

VI. CONCLUSION

While phase-dependent stability measures are known for their capacity to quantify local stability in human gait, their potential for predicting phase-dependent gait robustness was unclear. In the current study we failed to establish any significant correlation between phase-dependent stability measures and phase-dependent robustness. Combined with our previous assessment of overall gait robustness, we conclude that phase-dependent stability does not allow for predicting gait robustness, let alone to predict fall risk.

CREDIT AUTHORSHIP CONTRIBUTION STATEMENT

J. Jin: Conceptualization, Design, Interpretation, Writing - original draft, Writing - review & editing, Visualization, Formal analysis. **D. Kistemaker:** Conceptualization, Design, Interpretation, Writing - original draft, Writing - review & editing, Visualization, Formal analysis. **J.H. van Dieën:** Interpretation, Writing - original draft, Writing - review & editing, Visualization, Formal analysis. **A. Daffertshofer:** Interpretation, Writing - original draft, Writing - review & editing, Visualization, Formal analysis. **S.M. Bruijn:** Conceptualization, Design, Interpretation, Writing - original draft, Writing - review & editing, Visualization, Formal analysis.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

ACKNOWLEDGEMENT

S.M.B. and J.J. are funded by a VIDI grant no. (016.Vidi.178.014) from the Dutch Organization for Scientific Research (NWO).

DATA AVAILABILITY

All code and simulated data can be accessed via the link https://drive.google.com/drive/folders/1igRFDPc0Ct39mflvDd1oR4x999SJ-Vn9?usp=sharing.

REFERENCES

Ali, F., & Menzinger, M. (1999). On the local stability of limit cycles. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, *348*(9). https://doi.org/10.1063/1.166412

Bruijn, S. M., Meijer, O. G., Beek, P. J., & van Dieën, J. H. (2013). Assessing the stability of human locomotion: a review of current measures. *Journal of The Royal Society Interface*, *10*(83).

Cordero, A. (2003). Human gait, stumble and... fall. *American Journal of Respiratory and Critical Care Medicine*, *168*(4).

De Boer, T., Wisse, M., & Van Der Helm, F. C. T. (2010). Mechanical analysis of the preferred strategy selection in human stumble recovery. *Journal of Biomechanical Engineering*, *132*(7). https://doi.org/10.1115/1.4001281

Eng, J. J., Winter, D. A., & Patla, A. E. (1994). Strategies for recovery from a trip in early and late swing during human walking. *Experimental Brain Research*, *102*(2). https://doi.org/10.1007/BF00227520

Garcia, M. (1998). The Simplest Walking Model: Stability, Complexity, and Scaling. *Journal of Biomechanical Engineering*, *120*(2), 281. https://doi.org/10.1115/1.2798313

Golyski, P. R., Vazquez, E., Leestma, J. K., & Sawicki, G. S. (2022). Onset timing of treadmill belt perturbations influences stability during walking. *Journal of Biomechanics*, *130*. https://doi.org/10.1016/j.jbiomech.2021.110800

Guckenheimer, J., Holmes, P., & Slemrod, M. (1984). Nonlinear Oscillations Dynamical Systems, and Bifurcations of Vector Fields. *Journal of Applied Mechanics*, *51*(4). https://doi.org/10.1115/1.3167759

Hurmuzlu, Y., & Moskowitz, G. D. (1987). Bipedal Locomotion Stabilized by Impact and Switching: II. Structural Stability Analysis of a Four-Element Model. *Dynamics and Stability of Systems*, *2*(2), 99–112. https://doi.org/10.1080/02681118708806030

Ihlen, E. A. F., Goihl, T., Wik, P. B., Sletvold, O., Helbostad, J., & Vereijken, B. (2012). Phase-dependent changes in local dynamic stability of human gait. *Journal of Biomechanics*, *45*(13), 2208–2214. https://doi.org/10.1016/j.jbiomech.2012.06.022

Ihlen, E. A. F., Sletvold, O., Goihl, T., Wik, P. B., Vereijken, B., & Helbostad, J. (2012). Older adults have unstable gait kinematics during weight transfer. *Journal of Biomechanics*, *45*(9). https://doi.org/10.1016/j.jbiomech.2012.04.021

Jin, J., Kistemaker, D., van Dieën, J. H., Daffertshofer, A., & Bruijn, S. M. (2021). The validation of new phase-dependent gait stability measures: a modelling approach. *Dryad Digital Repository*. https://doi.org/10.5061/dryad.s4mw6m94r

Kendall, M. G. (1938). A New Measure of Rank Correlation. *Biometrika*, *30*(1/2). https://doi.org/10.2307/2332226

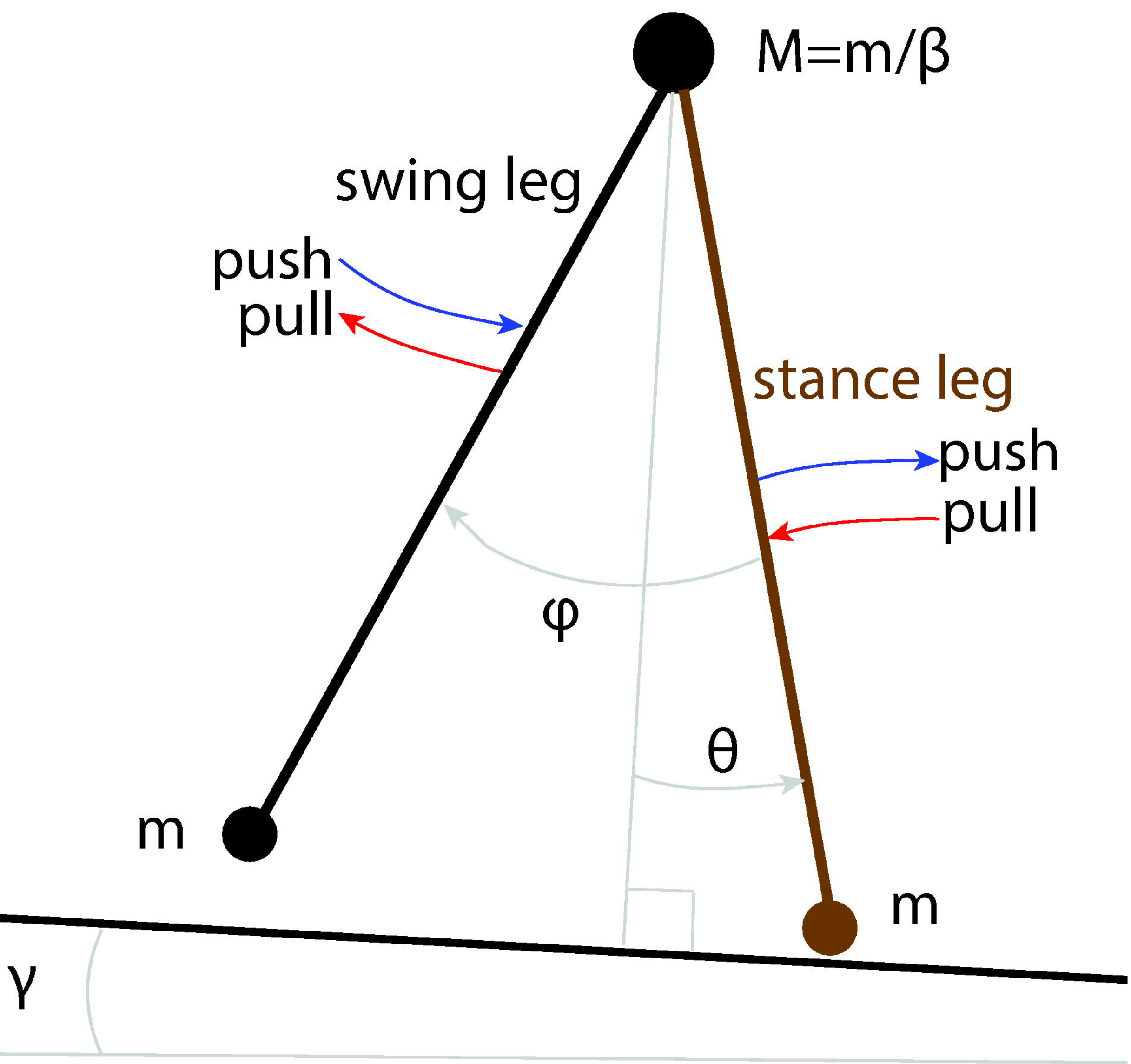
Mahmoudian, A., Bruijn, S. M., Yakhdani, H. R. F., Meijer, O. G., Verschueren, S. M. P., & van Dieen, J. H. (2016). Phase-dependent changes in local dynamic stability during walking in elderly with and without knee osteoarthritis. *Journal of Biomechanics*, *49*(1), 80–86. https://doi.org/10.1016/J.JBIOMECH.2015.11.018

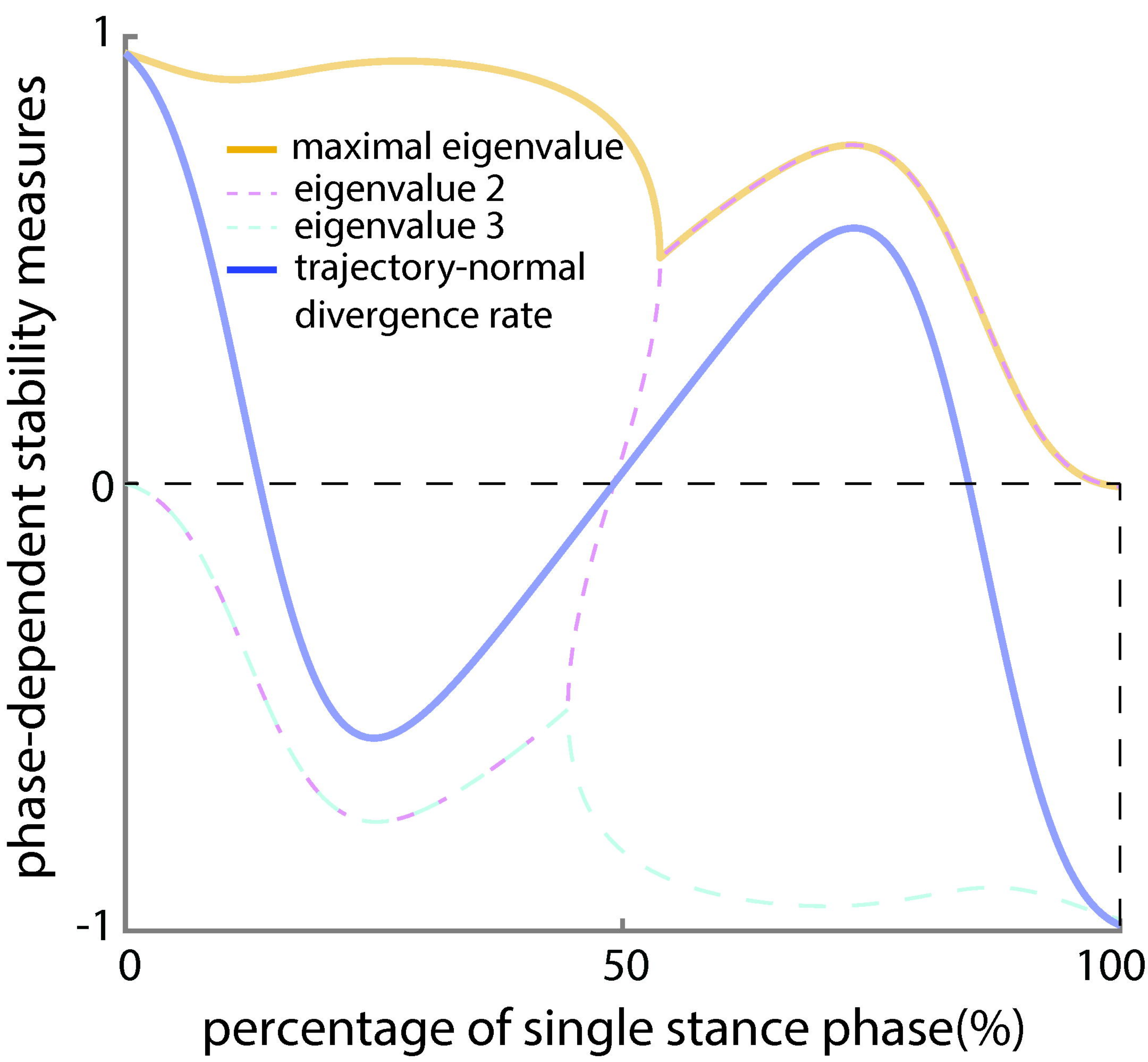
Norris, J. A., Marsh, A. P., Granata, K. P., & Ross, S. D. (2008). Revisiting the stability of 2D passive biped walking: Local behavior. *Physica D: Nonlinear Phenomena*, *237*(23), 3038–3045. https://doi.org/10.1016/j.physd.2008.07.008

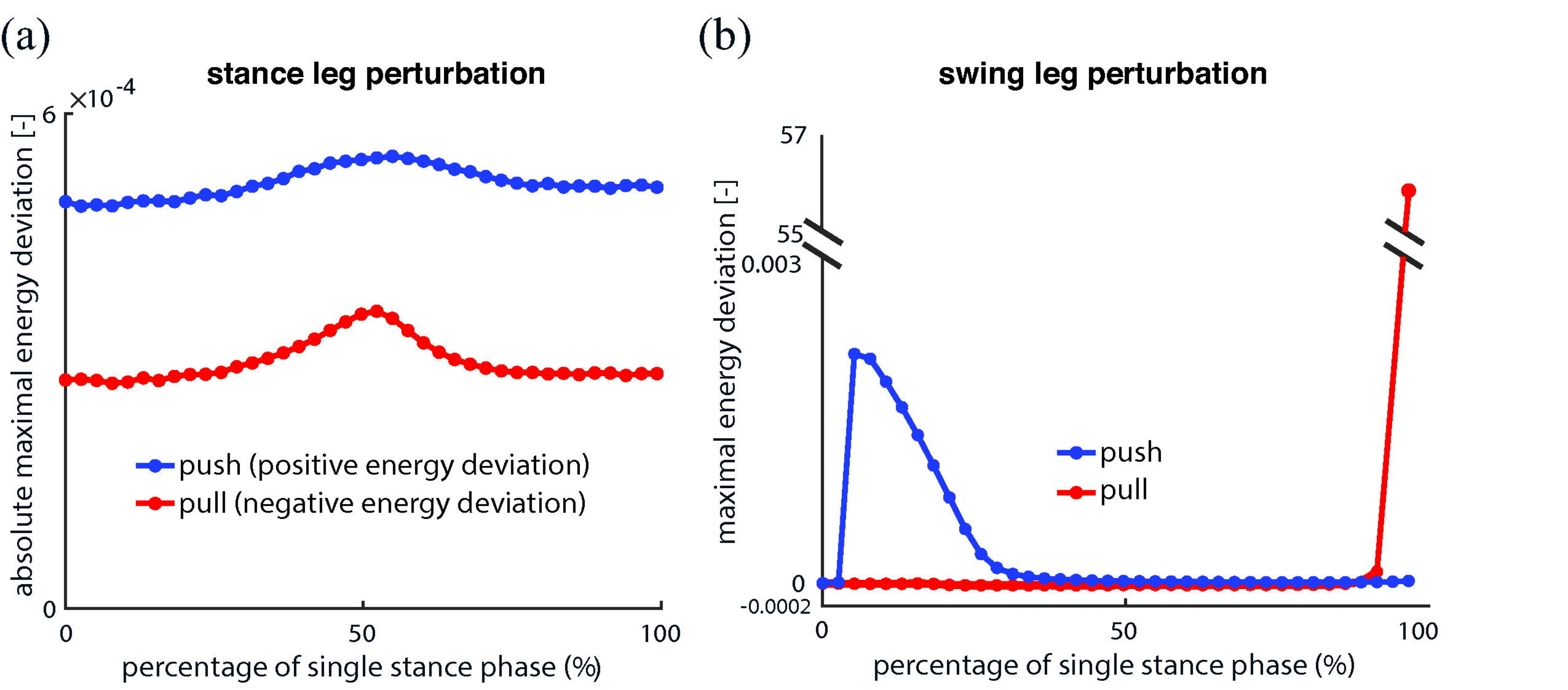
Tallapragada, P., & Sudarsanam, S. (2017). A globally stable attractor that is locally unstable everywhere. *AIP Advances*, *7*(12). https://doi.org/10.1063/1.5016214

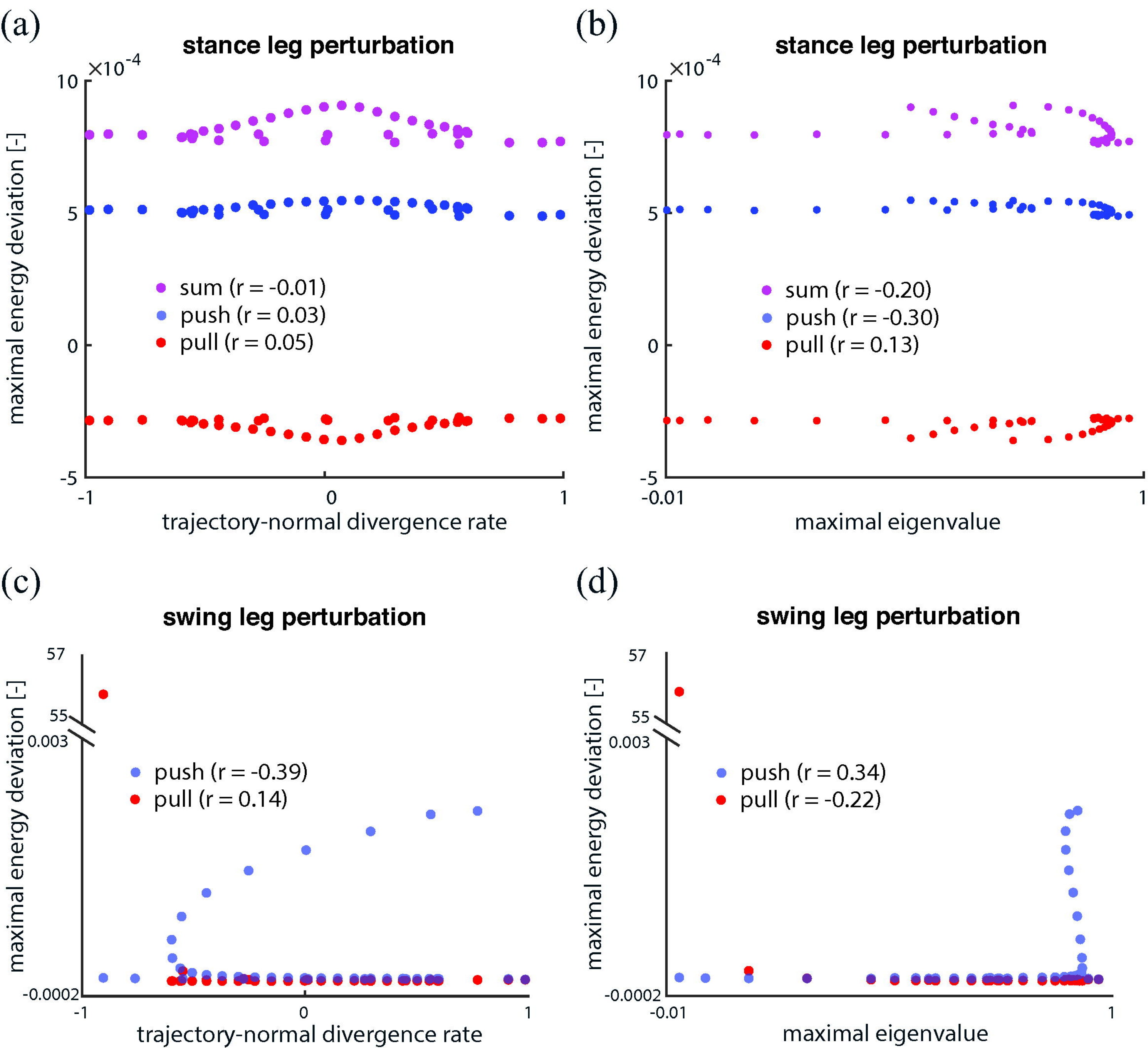
Tang, P. F., & Woollacott, M. H. (1999). Phase-dependent modulation of proximal and distal postural responses to slips in young and older adults. *Journals of Gerontology - Series A Biological Sciences and Medical Sciences*, *54*(2). https://doi.org/10.1093/gerona/54.2.M89

Williams, D. S., & Martin, A. E. (2021). Does a finite-time double support period increase walking stability for planar bipeds? *Journal of Mechanisms and Robotics*, *13*(1). https://doi.org/10.1115/1.4048832


M=m/β
swing leg
push
pull
stance leg
push
pull
φ
θ
m
m
γ


1
0
-1
0
50
100
phase-dependent stability measures
percentage of single stance phase(%)
maximal eigenvalue
eigenvalue 2
eigenvalue 3
trajectory-normal
divergence rate


(a)
stance leg perturbation
×10^-4
6
0
absolute maximal energy deviation [-]
push (positive energy deviation)
pull (negative energy deviation)
0
50
100
percentage of single stance phase (%)
(b)
swing leg perturbation
57
55
0.003
0
-0.0002
maximal energy deviation [-]
push
pull
0
50
100
percentage of single stance phase (%)


(a)
stance leg perturbation
×10^-4
maximal energy deviation [-]
sum (r = -0.01)
push (r = 0.03)
pull (r = 0.05)
trajectory-normal divergence rate
(b)
stance leg perturbation
×10^-4
maximal energy deviation [-]
sum (r = -0.20)
push (r = -0.30)
pull (r = 0.13)
maximal eigenvalue
(c)
swing leg perturbation
maximal energy deviation [-]
push (r = -0.39)
pull (r = 0.14)
trajectory-normal divergence rate
(d)
swing leg perturbation
maximal energy deviation [-]
push (r = 0.34)
pull (r = -0.22)
maximal eigenvalue