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2	Higher-order dissimilarity in biodiversity:
3	Identifying dissimilarities of spatial or temporal dissimilarity structures
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5	Running title: Higher-order dissimilarity
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7	Ryosuke Nakadai ¹ , Keita Fukasawa ¹ , Taku Kadoya ^{1,2} , Fumiko Ishihama ¹
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9	* Correspondence: Ryosuke Nakadai
10	
11	¹ Biodiversity division, National Institute for Environmental Studies, Onogawa 16-2, Tsukuba,
12	Ibaraki, 305-8506, Japan
13	² Graduate School of Comprehensive Human Sciences, University of Tsukuba, Tsukuba,
14	Ibaraki, 305-8571, Japan
15	
16	ORCID
17	Ryosuke Nakadai 0000-0002-9512-8511
18	Keita Fukasawa 0000-0002-9563-457X
19	Taku Kadoya 0000-0002-7816-5484
20	Fumiko Ishihama 0000-0001-8515-5914
21	
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26	RN, FI, KF, and TK designed the study; RN conceived the idea of higher-order dissimilarity,
27	analysed the datasets, and wrote the first draft of the manuscript, with significant inputs from
28	KF, FI, and TK; KF and RN developed the methodology to quantitatively evaluate the idea

- 29 introduced in the present study; and FI conceived the basic concept of the methodology. All
- 30 authors have contributed substantially to the final version of this manuscript.

31 Abstract

32	Elucidating biodiversity patterns and their background processes is critical in biodiversity
33	science. Dissimilarity, which is calculated based on multivariate biological quantities, is a
34	major component of biodiversity. As the availability of spatial and temporal biodiversity
35	information increases, the scope of dissimilarity studies has been expanded to cover various
36	levels and types of spatio-temporal biodiversity facets (e.g. gene, community, and ecosystem
37	function), and diverse pairwise dissimilarity indices have been developed. However, further
38	development of the dissimilarity concept is required in comparative studies on spatio-
39	temporal structures of biodiversity compositional patterns, such as those exploring
40	commonalities of biogeographical boundaries among taxa, compared to the conventional
41	ones to consider higher dimensions of dissimilarity: dissimilarity of dissimilarity structures.
42	This study proposes a novel and general concept, high-order dissimilarity (HOD), for
43	quantitatively evaluating the dissimilarities of spatial or temporal dissimilarity structures
44	among different datasets, proposes specific implementation of HOD as operational indices,
45	and illustrates potential resolution of scientific and practical questions by means of HOD.
46	Our conceptual framework on HOD extends the existing framework of biodiversity science,
47	and is versatile, with many potential applications in the acquisition of more valuable
48	information from ever-increasing biodiversity data.
49	
50	Konworda

50 Keywords:

51 autocorrelation, beta diversity, dissimilarity matrix, genetic differentiation, multivariate

52 analysis

54 Introduction

55	Nature's patterns are ubiquitous regardless of the level of biological organisation, and the
56	elucidation of their determinants has been a long-standing and fundamental issue in
57	biodiversity science. Dissimilarity is a key element of such patterns, and has been studied in
58	terms of spatial and temporal differences and changes in various biological entities, mostly at
59	the same level of biological organisation (Anderson et al. 2011). This encompasses
60	differences and changes in genetic alleles, species composition, interaction networks, and
61	ecosystem functions (Whittaker 1960, 1972; Raymond and Rousset 1995; Miki et al. 2018;
62	Mori et al. 2015; Poisot et al. 2012). The scope of dissimilarity is continuously expanding,
63	covering various levels and types of biodiversity facets, which has stimulated the
64	development of conceptual and analytical frameworks for the analysis of the determinants of
65	dissimilarity patterns using various dissimilarity indices (Koleff et al. 2003; Lozupone and
66	Knight 2005; Baselga 2010; Rocchini et al. 2018; Mammola and Cardoso 2020). Notably,
67	despite the varieties, all these indices have a common form in that one dissimilarity value is
68	calculated from two vectors of multivariate biological quantities (i.e. pairwise dissimilarity
69	indices), and the calculation of dissimilarity values for all pairs of vectors yields a
69 70	indices), and the calculation of dissimilarity values for all pairs of vectors yields a dissimilarity matrix (Anderson et al. 2011).
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83	community composition (i.e. temporal beta diversity: Legendre 2019; Nakadai 2022) have
84	recently been expanded to multiple sites at large spatial scales (Magurran et al. 2019;
85	Nakadai 2020; Gotelli et al. 2022), thus necessitating quantitative evaluation of inter-site
86	dissimilarity between temporal dynamics of community composition measured using
87	pairwise comparison. Furthermore, loss and biotic homogenisation in species composition
88	and intraspecific haplotypes have been studied in-depth since the identification of human-
89	induced impacts on biodiversity within the context of conservation (Olden 2006; Valtonen et
90	al. 2017). Moreover, quantifying the temporal change of spatial dissimilarity structure from a
91	baseline at a time point is essential to assessing the impacts. Despite the apparent importance
92	and scientific need for higher-order consideration in dissimilarities, general concepts and
93	formal statistical methods for quantitative comparisons across conventional pairwise
94	dissimilarities have not been fully developed.
95	To address this limitation, we introduce a general framework for considering
96	dissimilarities at a higher order (hereafter referred to as higher-order dissimilarity; HOD)
97	than that of conventional pairwise dissimilarities. HOD is a natural extension of the pairwise
98	dissimilarity indices and considers differences between dissimilarity matrices. In addition, it
99	can be a novel building block in the studies of the dissimilarities of spatio-temporal structures
100	of biodiversity compositional patterns. First, in the present study, we formally define the
101	novel and general concept, HOD, to quantitatively evaluate dissimilarities of spatial or
102	temporal dissimilarity structures among different datasets (Fig. 1a). Second, we designed the
103	concept of HOD and proposed its implementation as a general statistical method. Third, we
104	demonstrated the advantages of the HOD concept by applying it to actual patterns, such as
105	long-term and/or large-spatial hypothetical monitoring datasets. Finally, we discuss the
106	potential impacts of the concept of "higher-order dissimilarity" and the developed analytical
107	framework on a wide range of research fields, including understanding complex spatio-
108	temporal structures in biodiversity and future methodological challenges.
109	

110 Higher-order dissimilarity

111 In biodiversity science, one dissimilarity value is generally calculated from two vectors of 112 multivariate biological quantities (e.g. community composition or allele frequency). 113 Specifically, a community dissimilarity index (e.g. Bray-Curtis dissimilarity; Odum 1950) 114 and a genetic distance index (e.g. Fst; Wright 1969) were calculated based on vectors of 115 species abundance in two target communities and vectors of genetic allele frequency in two 116 target populations, respectively. In addition, multiple vectors are summarised into a matrix 117 called a site-species (or time-species) matrix (e.g. compositional vectors for sites or time 118 points). The calculation of dissimilarity values for all pairs of vectors yields a dissimilarity 119 matrix, which is a conventional pairwise dissimilarity; we call this first-order dissimilarity 120 (Fig. 1a). Here, we extend the first-order dissimilarity to formally define HOD as 121 dissimilarities of spatial or temporal dissimilarity structures (Fig. 1a). We also define 'order' 122 as nested times for calculating dissimilarity, e.g. conventional pairwise dissimilarity is the 123 first order; and dissimilarity of dissimilarity matrices the second order. By definition, the 124 concept can be extended to a higher order where appropriate, such as to the third order (i.e. 125 dissimilarity of second-order dissimilarity matrices). 126 The concept of HOD is a generalisation of the traditional analytical approach to 127 identifying boundaries and their commonalities in the context of biogeography. In addition, 128 HOD targets the spatial structure, which has been the primary subject of biogeography, and 129 temporal structure. Specifically, to spatially and temporally distinguish HODs, we use the 130 terms 'spatial HOD' and 'temporal HOD' (Fig. 1b). Given the recent rapid growth of spatio-131 temporal monitoring data, HODs regarding temporal variation in spatial structure (i.e. 132 temporal change in spatial HOD, Fig. 1b) or spatial variation in temporal structure (i.e. 133 spatial change in temporal HOD, Fig. 1b) will be applicable in future analyses. To elucidate 134 the concept, the subsequent sections mainly focus on second-order dissimilarities, which are

135 similar to spatial and temporal HODs.

136

137 Implementation of the higher-order dissimilarity

138 When sampling points comprising two biodiversity datasets are obtained at the same spatial139 or temporal location and correspond precisely one-to-one, their dissimilarity matrix elements

140 also correspond one-to-one. However, this is not always true. For example, when comparing 141 two phylogeographic patterns, the sample points of each species are usually in different 142 locations. In such a case, the elements of dissimilarity matrices do not correspond one-to-one, 143 and temporal or spatial proximity information is required for comparison. To accommodate 144 the biodiversity dataset with different sampling designs, we classified the calculation 145 methods of HOD with and without spatial or temporal proximity information as types 1 and 2 146 of the HOD indices, respectively. Type 1 HOD indices were only adopted for exhaustive 147 datasets; therefore, pairs of datasets were collected at identical sampling points. Unlike type 1 148 indices, type 2 HOD was applicable to an 'irregular dataset' (i.e. a dataset where the two 149 compositional vectors to be compared were observed at different points spatially and 150 temporally, and did not correspond on a one-to-one basis). The most basic calculation of the 151 HOD index is determining the average value of the differences between the dissimilarity 152 matrices of the same pairs of sites or time points (mean absolute differences; Fig. 2), which is 153 a type 1 index. Technically, the HOD index can be calculated as the commonly used Mantel 154 correlation coefficient (Mantel correlation coefficient r; Fig. 2) calculated from two spatial or 155 temporal dissimilarity matrices as a type 1 index. While these two indices are technically 156 conventional methods, they are often impractical for calculating the dissimilarity of 157 dissimilarity structures owing to the complexity of the comprehensive collection of data from 158 all sampling points, as we assume for conceptualisation of the present study. Particularly, in 159 our comparison of the temporal HOD using a long-term monitoring dataset, we could not use 160 the two conventional indices if some places of the monitoring sites changed during the period 161 because perfectly matched datasets were assumed in the calculation of their dissimilarity. In 162 the case of spatial HOD, genetic samples of two species for comparison are often taken at 163 different sites, and we cannot apply the two indices in this case. This can be addressed using 164 spatial or temporal information. 165 Type 2 HOD indices are newly introduced analytical concepts in the present study, and are 166 calculated using two distance matrices of dissimilarity with spatial or temporal information.

167 Here, we introduce two HOD indices of type 2 based on bivariate Moran's I and Geary'C

- 168 indices (Eckardt and Mateu 2021). Our concept of HOD is general and is thus capable of any
- 169 metrics of first-order dissimilarity other than those employed here.

170 In the following section, we introduce a new calculation method that enables the

- 171 quantification of HOD by applying the indices of spatial cross-correlation developed in
- 172 previous studies to the distance space rather than the actual space. First, we define the spatial
- and temporal distance between two pairs of sampling points that correspond to elements of
- 174 two dissimilarity matrices to expand the indices of spatial statistics to calculate the HOD.
- 175 Considering a dissimilarity value from each of two dissimilarity matrices, X_{ij} and X'_{kl} , and
- 176 then the relevant two pairs of sampling points are at hand which we denote $(\mathbf{s}_i, \mathbf{s}_j)$ and $(\mathbf{s}'_k, \mathbf{s}'_l)$.
- 177 Between $(\mathbf{s}_i, \mathbf{s}_j)$ and $(\mathbf{s}'_k, \mathbf{s}'_l)$, there are two exhaustive sets of point-wise distances: $(d_{ik} = |\mathbf{s}_i \mathbf{s}_i|)$

178 $\mathbf{s}'_{k|}, d_{jl} = |\mathbf{s}_j - \mathbf{s}'_l|$ and $(d_{il} = |\mathbf{s}_i - \mathbf{s}'_l|, d_{jk} = |\mathbf{s}_j - \mathbf{s}'_k|)$ (Fig. 2). From these sets of distances, the

179 distance between the pairs of sampling points is expressed as follows:

180
$$\Delta_{ij,kl} = \min((d_{ik}^2 + d_{jl}^2)^{1/2}, (d_{il}^2 + d_{jk}^2)^{1/2})$$

181

182 For the special case where $\mathbf{s}_i = \mathbf{s}'_k$ and $\mathbf{s}_i = \mathbf{s}'_l$, this distance metric is equal to 0. Another special case is $\mathbf{s}_i = \mathbf{s}_j$ and $\mathbf{s}'_k = \mathbf{s}'_l$, where the distance becomes $\sqrt{2} \times$ (Euclidean distance 183 184 between \mathbf{s}_i and \mathbf{s}'_k). With this generalisation of distance for pairs of sampling points, we can 185 calculate the weight matrix for dissimilarity values and apply the conventional measures of 186 spatial cross-correlation to type 2 HOD. To simplify the calculation of the HOD, we 187 converted dissimilarity matrices into vectors $\mathbf{x} = vec(\mathbf{X})$ and $\mathbf{x'} = vec(\mathbf{X'})$, and denoted their 188 sizes as n and n', respectively. Correspondingly, the distance between the pair of sampling 189 points Δ was reshaped to a matrix with *n* rows and *n'* columns.

Bivariate Moran's *I* (Wartenberg 1985; Lee 2001; Eckardt and Mateu 2021) is a popular
measure of spatial cross-correlation. The specification for dissimilarity values is expressed as
follows:

193
$$I = \frac{nn'}{\sum_i \sum_j w(\Delta_{ij})} \times \frac{\sum_i \sum_j w(\Delta_{ij})(x_i - \mu_x)(x'_j - \mu_{x'})}{\sqrt{\sum_i (x_i - \mu_x)^2 \sum_j (x'_j - \mu_{x'})^2}}, (1)$$

where $w(\Delta_{ij})$ is an arbitrary spatial weight which is a function of Δ_{ij} and x_i and x'_j are the i^{th} and j^{th} elements of **x** and **x'**, respectively. μ_x and $\mu_{x'}$ are the mean values of **x** and **x'**,

- 196 respectively. Generally, $w(\Delta_{ij})$ is a decreasing or step function of Δ_{ij} . If the dissimilarity
- 197 values of the point pairs that are close in space (or time) show similar deviations from their
- 198 means, the positive values are calculated, which indicates that the two sets of data compared

199 have spatially and temporally similar dissimilarity structures, or vice versa.

200 Another option for a spatial cross-correlation measure alternative to Moran's *I* is bivariate

- 201 Geary's C (Eckardt and Mateu 2021):
- 202

203
$$C = \frac{(nn'-1)}{2\sum_{i}\sum_{j}w(\Delta_{ij})} \times \frac{\sum_{i}\sum_{j}w(\Delta_{ij})(x_{i}-x'_{j})^{2}}{\sqrt{\sum_{i}(x_{i}-\mu_{x})^{2}\sum_{j}(x'_{j}-\mu'_{x})^{2}}}.$$
 (2)

204

The definition of the parameters is the same as that in Eq. (1). The bivariate Geary's *C* tends to be large when the squared deviations between two dissimilarities are large, where w_{ij} is large, and correlates negatively with Moran's *I*.

208 To test the relationships among the HOD indices, we applied these four indices to both a 209 simple sample case and 100 simulated datasets, including 37 hexagon-grided communities 210 (Fig. S1) using the R package, 'mobsim' (May et al. 2018; see Supplementary text 1 for 211 details on simulation). The distance between adjacent hexagons was set to 1, and we applied 212 a step function where the weights were equal to 1 for pairs of sampling points where the 213 points in the pair were identical; or one was identical and the other adjacent (i.e. $\Delta_{ij} = 0$ or 1); 214 the distance between adjacent hexagons was set to 0 for all other site pairs. For all pairs of the 215 four HOD indices, we tested the relationships using Mantel tests, and found that all four 216 indices were highly correlated with each other (Fig. S2); specifically, all absolute values of 217 the Mantel correlation were above 0.9. 218 To check the properties of the HOD indices, we summarised the specific values of spatial 219 HOD for pairs of simple sample cases (Table 1). The spatial weight function for type 2 HODs 220 was identical to that in the simulation above. Within each type of HOD index, the indices are 221 correlated. Specifically, the correlations between mean absolute differences and mantle r, and 222 between bivariate Moran's I and bivariate Geary's C are positive and negative, respectively. 223 Conversely, the relationships between type 1 and type 2 HOD indices are inconsistent with

respect to the robustness of the spatial autocorrelation. Specifically, for pairs that contain at

225 least one dataset with low spatial autocorrelation, the calculated values of HOD differ

considerably between types 1 and 2. HOD type 2 is affected by spatial (or temporal)

227 autocorrelation because it is weighted in spatial or temporal distances, unlike HOD type 1,

228 which is based solely on simple dissimilarity values.

All the samples presented in the present study used unscaled values (i.e. absolute values)

230 of dissimilarity to allow for comparisons among similar datasets. When applying the HOD

approach to two different types of datasets, it would be appropriate to standardise values

232 within each dissimilarity matrix to compare dissimilarity structures. For example, when

233 calculating the HOD of the genetic structure between different species, a standardisation

234 process is necessary because the absolute value of the genetic distance between differentiated

and non-differentiated species is different.

236

237 **Resolvable questions through the higher-order dissimilarity approaches**

238 Theoretically, HOD concepts are applicable to any type of dissimilarity dataset if the relative 239 position (i.e. sites or time points) can be determined based on the absolute distance between 240 pairs of datasets. For example, the monitoring of the genetic structure is one of the most 241 urgent targets of biodiversity monitoring (Hoban et al. 2021; O'Brien et al. 2022) although no 242 effective monitoring framework has been developed. In this context, the HOD provides an 243 ideal framework for the quantitative evaluation of genetic structure temporally (Fig. 3). As 244 frequently probable examples, we show two hypothetical scenarios of genetic structure 245 changes: one is a stable scenario (orange line and dots) and the other is a genetically 246 homogenised scenario (blue line and dots) in Fig. 3 (see Supplementary Text 2 for detailed 247 information about construction for visualisation). Fig. 3(a) indicates temporal changes in the 248 values of the HOD type 1 index (i.e. mean absolute differences) to time 0 (starting point), and 249 the lower Fig. 2(b) represents mapped overviews of changes in genetic structure over time, 250 from which we calculated the HOD values. The scarcity of such spatially coordinated data on 251 genetic structures over a wide area will soon be addressed, particularly following the

development of environmental DNA technology (Tsuji et al. 2022), and our framework will

253 be an essential contribution to genetic diversity monitoring.

To show the advantages of HOD more comprehensively, we summarised the potential

applications of HOD in Table 2, including five types of datasets: community composition,

256 genetic frequency, ecosystem function, interaction network, and phylogeny. Most of the cases

are straightforward extensions of those shown in Fig. 1b. One special case is that HOD

approaches are applicable to phylogenetic patterns, despite the difficulty in extracting single-

dimensional values; for example, the output of non-metric multidimensional scaling based on

260 multiple traits. Evidently, HOD approaches can be applied to a wide variety of datasets in

261 community ecology, biogeography, phylogeography, and macroevolution. This fact further

262 emphasises the importance of developing a field of study for HOD.

263 The main focus has been on comparisons between dissimilarity matrices with similar

264 properties, but this is not necessarily the case. Examples include comparisons among

265 communities of different trophic levels and comparisons between community structures and

266 genetic structures in community members. In traditional approaches, revealing the relative

267 importance of the driving processes of local communities, such as abiotic vs. biotic factors

and/or top-down vs. bottom-up processes as drivers of community structures, has been a

269 major concern (Smith et al. 2010; HilleRisLambers et al. 2012). The HOD approach is a

271 considering the driving processes. Low values of HOD among data from different trophic

272 levels of communities and different hierarchies of genetic and community data allowed us to

different perspective that directly evaluates the similarity of patterns without explicitly

273 generate novel hypotheses with more direct evidence, as the processes shaping them are

274 considered common among the compared targets (taxonomic differences; Fig. 1b).

275 Conversely, high HOD values suggest that the process of constructing the overall system is

complex and contingent. From an application perspective, high values of spatial HOD also

suggest that achieving conservation through the establishment of protected areas, setting

278 focal species for conservation, or implementing a single conservation policy would be

279 difficult, as the driving processes differ among taxa and levels of biological entities.

280

270

281 Future challenges

282 Despite the significance of the concept of the HOD, there are several methodological 283 challenges. In this paper, we discussed HOD, assuming an ideal situation where the target 284 sites (or time points) have been exhaustively examined without any missing values. However, 285 in empirical studies, it is often unsatisfactory, and this is one of the reasons underlying the 286 failure to devise HOD in the past. To overcome this challenge, candidate approaches are 287 broadly divided into two categories. One is the interpolation of missing data through 288 modelling approaches, such as generalised dissimilarity modelling and species distribution 289 modelling. Generalised dissimilarity modelling (GDM) is an approach that specialises in 290 dissimilarities, differences, and distances (Ferrier et al. 2007). If target dissimilarities are 291 modelled by other variables (e.g. environment and geography) using GDM, the predicted 292 outputs can be applied to HOD. Another approach, type 2 HOD, which is weighted by spatial 293 or temporal distances, can theoretically consider missing values. However, further testing is 294 required to determine how many missing values are allowed for a sufficiently accurate 295 evaluation and what the most appropriate spatial or temporal weighting process is for each 296 target system.

297 In the HOD approaches proposed here, we cannot distinguish between parts of continuous 298 patterns and truly disjunctive patterns in dissimilarities; for example, either simple distance-299 decay patterns or patterns due to isolation by geographic barriers. Therefore, where the 300 commonality of spatial disconnections is the main concern, combining HOD approaches with 301 existing approaches would be more effective in the identification of disjunct barriers (e.g., 302 Soltis et al. 2006). In temporal structures, approaches separating trend components have been 303 used (Cowpertwait and Metcalife 2009), and potentially combined with such methods, it may 304 be possible to evaluate continuous and disjunctive dissimilarities separately.

In existing studies on dissimilarity, unique conceptual and analytical frameworks have been developed for dissimilarity datasets because of the non-independence problem between dissimilarity values which share an original value (Anderson et al. 2011). To overcome the problem of non-independence among values, statistical methods that deal with dissimilarity matrices, such as the Mantel test (Mantel 1967) and certain developed methods (Lichstein

310 2007, Ferrier et al. 2007, Legendre and Legendre 2012, Anderson and Walsh 2013), have 311 been used to calculate p-values by permutation procedures. Permutation-based hypothesis 312 testing approaches adjusted to the HOD framework need to be developed to test the statistical 313 significance of the similarity or dissimilarity between two dissimilarity matrices, which is a 314 critical issue for future research on HOD. 315 Although we mainly focused on the second-order dissimilarities in the present study, the 316 applicability of third- and higher-order dissimilarities would increase in situations where the 317 amount and dimension of biodiversity information continues to grow. To illustrate the benefit 318 of HOD, we showed temporal changes of genetic structure (Fig. 3). The figure shows a 319 pattern in a single species, and thus, second-order dissimilarity (i.e. space×time). If this type 320 of dataset is available for multiple species, the similarity and dissimilarity of temporal 321 changes in spatial structure in population genetics can be quantitatively evaluated among 322 species using third-order dissimilarities (i.e. space×time×species). This simple example tells 323 us that the breadth of areas that the HOD concept can adopt is rapidly expanding, aligning 324 with the forthcoming explosive increase in available dissimilarity information.

325

326 Conclusions

327 The rapid increase in the availability of spatial and temporal biodiversity information 328 necessitates comparative studies on spatio-temporal structures of biodiversity compositional 329 patterns require further development of the dissimilarity concept over conventional ones to 330 consider higher dimensions of dissimilarity: dissimilarity of dissimilarity structures. Our 331 study introduced the concept of higher-order dissimilarity (HOD), which can account for 332 dissimilarities of spatial or temporal dissimilarity structures. For example, this framework can 333 be applied to various types of broad-scale biodiversity monitoring datasets and enables the 334 evaluation of temporal changes in the spatial HOD to a baseline. Furthermore, the concept is 335 applicable even if the compared datasets have completely different origins (e.g. different 336 taxonomic groups and hierarchies of biological organisation, Fig. 1b), as long as they are 337 summarised into dissimilarity information at their own level. We mainly focused on the 338 biodiversity dataset in the present study; the concept of HOD is applicable to all kinds of

339	dissimilarity matrices, not just those based on biodiversity. For example, various
340	contemporary issues arise in the social ecological system, and this HOD will be useful in
341	examining the relationships between different layers, such as the economy, culture, and
342	biodiversity. Therefore, the HOD concept and related approaches pave the way for novel
343	dimensional approaches that deal with large amounts of dissimilarity information.
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345	Data accessibility statement
346	Simulated data and codes will be uploaded after acceptance of the manuscript to figshare.
347	
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449 **Figure legends**

450	Figure 1 Schematic of higher-order dissimilarity (HOD). A spatial case of HOD is shown in
451	(a). Potential analytical framework for HOD based on spatial and temporal comparisons (i.e.,
452	spatial and temporal HOD) are shown in (b). For visualization, only three compositional
453	types are hypothesized, which correspond to three level of darkness in each colour (e.g., blue,
454	green, and red). Specifically, we hypothesized that dissimilarities between sites (or time
455	points) of same level of darkness were zero (i.e., identical composition), those between sites
456	of one darker or lighter difference were 0.5, and those between the darkest and lightest sites
457	were 1.0. The grey arrows indicate conventional dissimilarity (i.e., first-order dissimilarity)
458	and the black arrows indicate HOD. Black star and white star among compared datasets
459	indicate identical sites or time points, respectively.
460	
461	Figure 2 Schematic of specific analytical procedures for two types of higher-order
462	dissimilarity (HOD), specifically one without spatial information and the other with the
463	information. For visualization, only three compositional types are hypothesized, which
464	correspond to three level of darkness in each colour (e.g., blue, green, and red). Specifically,
465	we hypothesized that dissimilarities between sites (or time points) of same level of darkness
466	were zero (i.e., identical composition), those between sites of one darker or lighter difference
467	were 0.5, and those between darkest and lightest sites were 1.0. The asterisk with $1(*1)$
468	showed the specific calculation of distance between pair of sites.
469	
470	Figure 3 Sample application of higher-order dissimilarity (HOD) to genetic monitoring using
471	two simulated datasets based on two types of scenarios from time 0 to time 100. In both
472	scenarios, the stable conditions are hypothesized by time 30 (shared section in light green).
473	After time 30, genetic structure is stable in one scenario (upper section in orange) but

474 homogenized in the other (lower section in blue). The change of calculated type 1 of HOD

475 index (i.e., mean absolute differences) compared to the value at time 0 is shown in (a). The

476 genetic structure is visualized in (b), and the colours were determined using the two axes of

477 NMDS (details see Supplementary text 2).

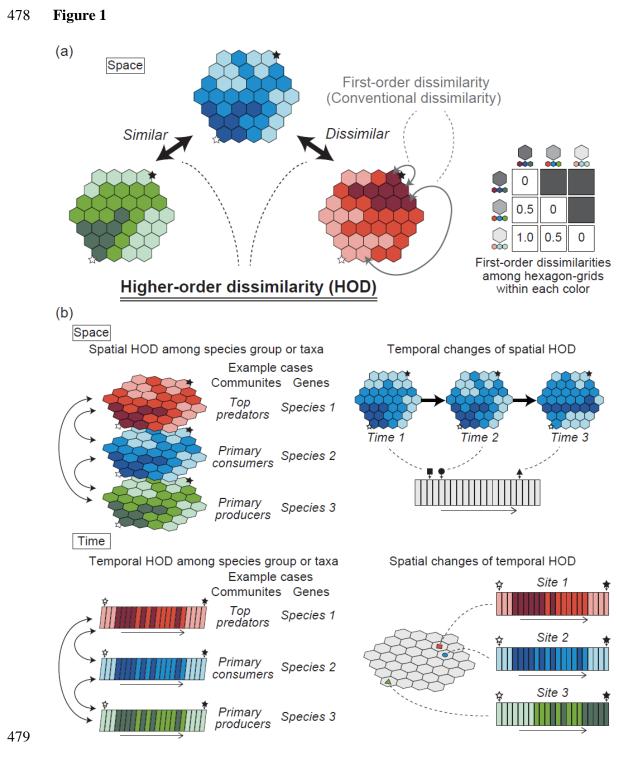
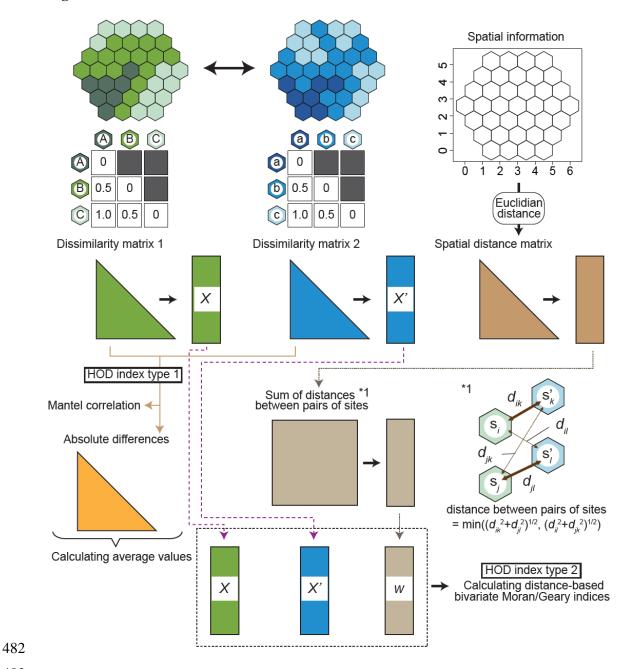
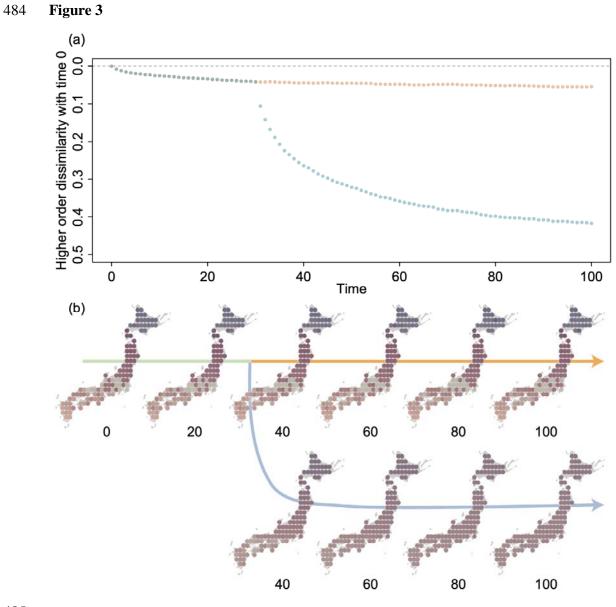


Figure 2





485

Table 1 Simple sample cases to show properties of HOD indices. Patterns X and Y represent pairs for calculating HODs. Mantel r (X) and (Y) indicate the spatial autocorrelation, thus correlation between spatial distances and dissimilarity values for each pattern X and Y. Four HOD indices are shown, which are mean absolute differences and mantel r (with P-value) as type 1 and bivariate Moran's I and Geary's C as type 2. For visualization purpose, only three compositional types are hypothesized, which correspond to three level of darkness in each colour (e.g., blue, green, and red). Specifically, we hypothesized that dissimilarities between sites (or time points) of the same level of darkness were zero (i.e., identical composition), those between sites of one darker or lighter difference were 0.5, and those between the darkest and lightest sites were 1.0.

Association	Pat	tern		Mantel	(Space)			HOD type 1		HOD	type 2
	Х	Y	<i>r</i> (X)	P-value	<i>r</i> (Y)	<i>P</i> -value	Absolute mean differences	Mantel r	<i>P</i> -value	Bivariate Moran I	Bivariate Geary C
A-A			0.0367	0.0013	0.0367	0.0013	0.0000	1.0000	0.0000	0.6539	0.3405
B-B			0.0263	0.0162	0.0263	0.0162	0.0000	1.0000	0.0000	0.4783	0.5107
C-C			-0.0173	0.1089	-0.0173	0.1089	0.0000	1.0000	0.0000	0.1594	0.8622
A-B			0.0367	0.0013	0.0263	0.0162	0.3108	0.1759	0.0036	0.1322	0.8594
B-C			0.0263	0.0162	-0.0173	0.1089	0.3649	-0.0886	0.0796	-0.0421	1.0473
C-A			-0.0173	0.1089	0.0367	0.0013	0.3423	0.0301	0.5580	0.0120	0.9959

A-A'		0.0367	0.0013	0.0367	0.0013	0.3183	0.1318	0.0183	0.1345	0.8599
B-B'		0.0263	0.0162	0.0263	0.0162	0.3453	0.0097	0.8685	-0.0075	0.9964
C-C'		-0.0173	0.1089	-0.0173	0.1089	0.3348	0.0405	0.3932	0.0131	1.0097
A-C'		0.0367	0.0013	-0.0173	0.1089	0.3288	0.0910	0.0809	0.0547	0.9548

.93

.94 **Table 2** Potential applications of higher-order dissimilarity (HOD) approach

Target axis of HOD	Targets of comparison	Question
(a) community composition		
Time	Space	How do temporal changes of community composition vary in space? Do closer sites show more similar community compositional change across time?
Time	Taxa	Which taxa share similar changes of community composition in time?
Space	Time	How does spatial composition change over time? When did large change of community compositional structure happen?
Space	Taxa	Which taxa share similar community composition structure in space?
(b) population genetics		
Time	Space	How does temporal genetic frequency change in space? Do closer sites show more similar changes in genetic frequency across time?
Time	Taxa	Which taxa share similar changes of genetic frequency over time?
Space	Time	How does the spatial genetic structure change over time? When did a large change in genetic structure occur?
Space	Taxa	Which taxa share similar changes of genetic frequency in space?
(c) ecosystem function		
Time	Space	How does temporal ecosystem function change in space? Do closer sites show more similar change of ecosystem function across time?
Space	Time	How does spatial genetic structure change over time? When did a large change in the genetic structure occur?
(d) interaction network		
Time	Space	How does the temporal network structure change in space? Do closer sites show more similar changes in network structure across time?

Space	Time	How does spatial network structure change in time? When did large change of network structure happened?
(e) macroevolution		
Phylogeny	Trait	Which traits share similar evolutionary changes across macroevolutionary history?
.95		

497 Appendices

- 498 **Figure S1** Mantel correlations among absolute mean differences and mantel *r* as type 1 and
- 499 bivariate Moran's *I* and Geary's *C* as type 2 (9,999 times permutation)
- 500 Figure S2 Hexagon grids used in simulation
- 501
- 502 Supplementary text 1 Detailed information on the simulation of community datasets and
- 503 calculation of dissimilarity matrices to test the properties of HOD indices.
- 504 **Supplementary text 2** Detailed information on simulated genetic structure across four major
- 505 islands in Japan