1   **Title**

2   Towards a framework to unify the relationship between numerical abundance, biomass, and
3   quantitative eDNA
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19   Running Title: Correcting quantitative eDNA data to infer abundance

20   Classification: Biological Sciences; Ecology
21

22   *Keywords:* Environmental DNA; eDNA; abundance; biomass; *N*; population size; allometry;
23   allometric scaling; metabolic theory; fishes; biodiversity; Shannon Index

Abstract

Does environmental DNA (eDNA) correlate more closely with numerical abundance (N) or biomass in aquatic organisms? We hypothesize that the answer is 'neither': eDNA production likely scales allometrically, reflecting key physiological rates and surface area-to-body mass relationships. Building on individual-level frameworks developed from the Metabolic Theory of Ecology, we derive a framework through which quantitative eDNA data can be transformed to simultaneously reflect both population-level N and biomass. We then validated our framework using data from two previously published studies: (i) a marine eDNA metabarcoding dataset; and (ii) a freshwater single-species qPCR dataset. Using a Bayesian modeling approach, we estimated the value of the allometric scaling coefficient that jointly optimized the relationship between N, biomass, and 'corrected' eDNA data to be 0.82 and 0.77 in Case Studies (i) and (ii), respectively. These estimates closely match expected scaling coefficients estimated in previous work on Teleost fish metabolic rates. We also demonstrate that correcting quantitative eDNA can significantly improve correspondence between eDNA- and traditionally-derived quantitative community biodiversity metrics (e.g., Shannon index and Bray-Curtis dissimilarity) under some circumstances. Collectively, we show that quantitative eDNA data is unlikely to correspond exactly to either N or biomass, but can be 'corrected' to reflect both through our unifying joint modelling framework. This framework can also be further expanded to include other variables that might impact eDNA pseudo-steady-state concentrations in natural ecosystems (e.g., temperature, pH, and phenology), and is flexible enough to model these relationships across trophic levels.

Significance Statement

Aquatic animals release DNA (from shed cells, mucous, faeces, etc.) into water, which can be

detected via environmental DNA (eDNA) sampling. What is less clear is whether we can

estimate numerical abundance (N) or biomass from eDNA concentrations. We hypothesize that

eDNA production scales allometrically; that is, large animals release less DNA per unit mass

than smaller animals. Building from the Metabolic Theory of Ecology, we derived a framework

through which eDNA data can be transformed to simultaneously reflect both N and biomass. We

then validated the framework using two case studies in marine and freshwater systems. This

framework unifies discrepancies between eDNA, N, and biomass data, unlocking the potential of

eDNA to monitor population abundance/biomass and quantify biodiversity.

Introduction

Environmental DNA ('eDNA', referring to DNA found in environmental mediums like air, soil, or water) sampling represents a cost-efficient and highly sensitive method to detect aquatic species that often outperforms traditional sampling methods (1–3). The extent to which quantitative eDNA data (e.g., gene copy number concentrations or metabarcoding read counts) can provide quantitative inference regarding the 'unseen' numerical abundance (N) or biomass of organisms within an environment is less clear, and remains an active line of research (4, 5). Empirical studies typically show positive correlations between the concentration of organisms' DNA in an environment and their abundance (which we define as either N or biomass), but the extent to which observed eDNA concentrations will more closely reflect N or biomass remains a major outstanding question (6).

Numerical abundance and biomass represent two linked, but distinct parameters. Both metrics of abundance are often highly correlated (6), but also often exhibit subtle differences in their relationship with quantitative eDNA data (7–11). This is likely because eDNA production may not functionally relate to either N or biomass, but instead may scale allometrically; that is, mass-specific eDNA production rates may decline as organism mass increases (8, 9, 12, 13). Thus, the correlation between N, biomass, and quantitative eDNA data might be improved by modeling eDNA production using classic allometric frameworks derived from the Metabolic Theory of Ecology (MTE) (14), i.e., eDNA can be modeled as a function of the individual mass of an organism raised to the power of an allometric scaling coefficient ('b') multiplied by a normalization constant (8, 14).

A number of studies have supported this hypothesis by demonstrated that integrating allometry into metrics of organism abundance can strengthen correlations between quantitative

eDNA data and population/species abundance in natural ecosystems (8, 9, 15, 16). However,

these studies have integrated allometry by applying data 'corrections' to population-level

organism abundance variables, i.e., by expressing organism abundance as the sum of individual

mass values within a population raised to the power of an allometric scaling coefficient ('*b*'),

with the value of *b* depending on the rate at which eDNA production scales with individual body

size (8, 9, 15, 16). While such transformations reflect the functional relationships between eDNA

and organism abundance, to facilitate the inference of 'unseen' N and biomass from quantitative

eDNA data such 'corrections' must be applied to the 'eDNA-side' of equations (17).

We hypothesize that allometry represents the fundamental parameter linking population-

level eDNA-production to both N and biomass, and accounting for allometric scaling can (at

least partially) explain discrepancies observed for correlations between eDNA, N, and biomass.

Deriving quantitative 'corrections' to predict both abundance metrics from quantitative eDNA

data thus necessitates the derivation of a framework that jointly models both N and biomass from

quantitative eDNA data (e.g., $N = f(\text{eDNA})$ and biomass $= f(\text{eDNA})$) with a linking allometric

parameter shared by the two functions. This framework can then be applied to 'correct'

quantitative eDNA data based on species or population size structure data to *simultaneously* infer

both N and biomass as distinct-but-linked (by allometry) parameters, thus 'unifying' the

inference of N and biomass from eDNA.

Herein we present such a framework, derived from the core hypothesis that individual-

level eDNA production rate follows the same allometric relationship with body mass as other

key metabolically-linked rates (14, 18) and body surface area (19) in animals:

103   Equation 1.1: $I = I_0 \times M^b$

Where I = individual eDNA production rate, M = the individual body mass of an organism, I_0 = a

normalization constant, and b = an allometric scaling coefficient.

If eDNA production can be approximated using Equation 1.1, at an aggregate

'population'-level eDNA production can be expressed using the following formula:

$$\text{Equation 1.2: } eDNA_i = I_0 \times \sum_j^{N_i} x_{ij}^b$$

where $\sum_j^{N_i} x_{ij}^b$ is equal to the sum of the individual (j^{th}) mass values raised to the power of the

allometric scaling coefficient (b) for the ith population. This can alternatively be expressed as:

$$\text{Equation 1.3: } eDNA_i = I_0 \times (N_i \times \overline{x_i^b})$$

Where N_i is the numerical abundance of the ith 'population' and $\overline{x_i^b}$ is equal to the mean of

allometrically scaled individual mass values for the ith population. Equation 1.3 can be

rebalanced to express population N as a function of eDNA divided by $\overline{x_i^b}$ of the ith population:

$$\text{Equation 1.4: } N_i = \frac{1}{I_0} eDNA_i \div \overline{x_i^b}$$

Similarly, Equation 1.4 can be used to isolate population biomass as a function of eDNA,

population mean mass ($\bar{x}_i$), and an allometric scaling coefficient. Given:

$$\text{Equation 1.5: } Biomass_i = N_i \times \bar{x}_i$$

We can substitute Equation 1.4 for N_i and re-arrange to obtain:

$$\text{Equation 1.6: } Biomass_i = \frac{1}{I_0} (eDNA_i) \times \frac{\bar{x}_i}{\overline{x_i^b}}$$

It is also important to note that this framework can be applied both inter- and intra-

specifically, i.e., to model eDNA production across populations of species within an ecological

community or to model eDNA production across intra-specific experimental units (tank

replicates, stream or lake populations, etc.) that exhibit significant variation in body size

distribution. We thus validated our framework using two previously published case-studies: an

inter-specific eDNA dataset investigating the relationship between eDNA metabarcoding read

count and abundance data estimated from trawling (9) and an *intra*-specific eDNA dataset

investigating the relationship between eDNA concentration and Brook Trout (*Salvelinus*

fontinalis) abundance in nine lakes (8). We used a Bayesian approach in which we jointly

estimate models relating quantitative eDNA data to *N* and biomass in these two Case Studies

based on Equations 1.4 and 1.6, which share $\overline{x_i^b}$ and beta coefficient ($\frac{1}{I_0}$) parameters. We then

further demonstrate, using Case Study 1, that eDNA read-count data, after correcting for

allometry, can potentially be used as parameter inputs in common community diversity indices

that require relative *N* or biomass data (e.g., Shannon-index and Bray-Curtis dissimilarity).

Results

Case Study 1: *Inferring bony fish abundance from quantitative eDNA metabarcoding data*

Monthly beta-coefficients and model intercepts were included in our model to account for pseudo-replication across seasonal sampling periods in Stoeckle et al. (2021). All monthly beta-coefficients indicated a consistent positive relationship between metabarcoding read count, individuals-per-100-tows (an index of N), and biomass-per-100-tows (an index of biomass) (Figures S1a-d). Beta-coefficients also differed among seasonal sampling periods, with the coldest sampling period (January) exhibiting a significantly lower beta-coefficient value relative to the other three warmer months (Figure 2). Overall, estimates for the abundance of each species in each sampling period exhibited wide credible-intervals (Figures S2a-S2d). Nevertheless, it was possible to identify rare species within each time period, although distinguishing between moderate and high relative abundance species was challenging for either metric (N or biomass) likely due to heteroscedastic residual error that increased as corrected eDNA values increased. Directly estimating b (model *deviance* = 5115.1) represented a substantial improvement over joint-modelling approaches assuming a value of b equal to 0 (i.e., eDNA production scales linearly with N, *deviance* = 5372.2) and 1.00 (i.e., eDNA production scales linearly with biomass, *deviance* = 5158.6) (Figures S1a-S1d, Table 1).

Correcting quantitative eDNA data for eDNA production allometry in community biodiversity metrics

Comparison of eDNA and trawl-derived Shannon index values for N*, biomass*

We estimated bootstrapped 95% confidence intervals for the difference between traditional- and eDNA-derived Shannon index metrics for both N and biomass for values of b

equal to 0.00, 0.82 (the value of b inferred jointly from the previous Bayesian analysis), and 1.00

(Figure 3, Table 2). When $b = 0$, eDNA-derived Shannon-index estimates for N exhibited

consistent differences from traditional estimates, and confidence intervals for the value of the

Shannon index differences between eDNA and traditional methods did not overlap zero except

for the month of January. For biomass, confidence intervals for the differences (assuming $b = 0$)

all overlapped zero except for data aggregated across the entire year; however, mean point

estimates for the difference between eDNA-derived and trawl-derived biomass Shannon-index

values were greater assuming $b = 0$ than for $b = 0.82$ or 1.00 (mean differences = 0.78 vs. 0.22

and 0.25, respectively). When $b = 0.82$ or 1.00, bootstrapped confidence intervals for differences

between eDNA- and trawl-derived Shannon estimates for N and biomass overlapped zero for all

sampling time periods. Collectively, assuming $b = 0$ consistently overestimated Shannon

estimates based on species N and underestimated Shannon estimates based on species biomass

relative to traditionally-derived Shannon estimates, but estimates assuming $b = 0.82$ or 1.00

produced broadly comparable results for both N and biomass.

When Shannon-index values were estimated independently for N and biomass across a

gradient of b-values ranging from 0 to 1 by intervals of 0.01, the values of b that minimized the

absolute differences between eDNA- and trawl-derived Shannon index values were 0.80 and

0.65 for N and biomass, respectively (Figure 4). A b-value of 0.80 minimized the absolute value

of the sum-of-differences between eDNA and trawl-derived N and biomass metrics (minimum

sum-of-differences = 0.06, Figure 4), closely reflecting the inferred value of the allometric

scaling coefficient (0.82). Overall, eDNA-derived Shannon index values jointly 'corrected' for

biomass and N corresponded closely (i.e., within a |sum-of-differences| < 0.3) with traditional

trawl estimates for bony fish biomass and N when the value of b was between 0.67 and 0.93.

Comparison of Bray-Curtis distances between trawl and eDNA-derived metrics

Our Bray-Curtis dissimilarity analysis revealed that correcting for allometric scaling in

eDNA production rate can improve the similarity of the community inferred from transformed

quantitative eDNA data and from trawling data (Figure 5). Overall, correcting for allometry

assuming *b*-values of 0.82 or 1.00 significantly decreased distance to trawl data relative to

assuming a *b*-value of 0.00 ($t_{17} = 5.04$, $p < 0.001$ and $t_{17} = 4.56$, $p = 0.001$, respectively) (Table

3). Assuming a *b*-value of 0.82 and 1.00 were not significantly different across months ($t_{17} =$

0.47, $p = 0.88$).

Case Study 2: Brook Trout abundance and eDNA in nine study lakes

After correcting eDNA data using equations 1.4 and 1.6, our joint-modelling approach

demonstrated significant positive correlations between population *N,* biomass, and corrected

eDNA data, with a *b*-value estimate of 0.77 (0.44-1.13) (Table 1, Figure 6). The point-estimate

of *b* is broadly similar to predictions based on theory and the inferred value of the scaling

coefficient from Yates et al. (2021a) (~0.7). Directly estimating the value of *b* represented a

substantial improvement over joint-modeling approaches fixing *b* to 0.00 (i.e., eDNA production

scales linearly with *N*) (*deviance* = 306.7 vs 315.9, respectively) (Table 4). However, credible

intervals for *b* overlapped with 1.00 (i.e., assuming eDNA production scales linearly with

biomass) and, as a result, estimating the value of *b* represented comparatively marginal model

improvement over fixing *b* at 1.00 (*deviance* = 309.5) (Table 1, Figure 6). Credible intervals for

N permitted some differentiation between high- and low-*N* populations, but credible intervals for

biomass for each population were wide and overlapped (Figure 7).

Discussion

Unifying the inference of N *and biomass from quantitative eDNA data*

Building from metabolic frameworks describing eDNA production at the individual level, we derived and empirically validated a framework to correct for allometric scaling in eDNA production to infer 'unseen' organism N and biomass among populations and species, demonstrating that these principles can even be scaled to make community-level ecological inferences (e.g., quantitative community biodiversity metrics). This framework resolves the long-standing debate regarding whether eDNA more closely relates to N or biomass; the answer is *neither*, but that quantitative eDNA data can be corrected to simultaneously reflect *both*, depending on the assumed or inferred value of the allometric scaling coefficient associated with eDNA production. Allometry represents the key linking parameter between eDNA, N, and biomass; the relationship between eDNA and these abundance metrics can thus be unified by joint modeling with a shared allometric parameter and normalization constant.

To 'b', or not to 'b'?

Our framework demonstrates that directly correlating quantitative eDNA data to N or biomass makes inherent assumptions regarding the physiology of eDNA production. Correlating population N directly with quantitative eDNA data presumes eDNA production is invariable with individual body mass (i.e., $b = 0$); similarly, correlating biomass with quantitative eDNA read data assumes that eDNA production scales linearly with individual body mass (i.e., $b = 1$). For the brook trout Case Study, we observed a close correspondence between the value of b jointly estimated herein (0.77), and the value of the allometric scaling coefficient expected based on theory (~0.7) [8]. However, we will note that while the value of b was clearly different from '0'

(i.e., eDNA production scaled with numerical abundance), credible intervals overlapped with the

value of '1' (eDNA production scales linearly with biomass). Indeed, visual inspection of the

plots comparing models in which b was estimated vs. fixed at '1' demonstrates relatively

marginal modeling gains. For Case Study 2, while we can estimate that the interspecific b-value

for marine bony fish is less than '1', the upper credible limit (0.88) was also relatively close to

'1' and visual inspection of plots indicate comparable trends as observed for the brook trout (i.e.,

relatively marginal gains). Similarly, community diversity indexes estimated from quantitative

eDNA data assuming a value of b equal to '0' exhibited significant discrepancies from metrics

derived from traditional methods for both N and biomass; assuming that quantitative eDNA data

can be used as a 'proxy' for numerical species abundance would thus result in biased estimates

of community diversity. However, community biodiversity metrics estimated from eDNA data

with values of b equal to 0.82 and 1.00 (i.e., biomass) were both comparable to community

biodiversity values derived from traditional methods, with confidence intervals for differences

between eDNA- and trawl-derived Shannon values overlapping zero for both b-values.

Collectively, we can infer from both datasets that the value of b was much closer to '1'

than '0'. However, the extent to which directly estimating the value of b vs. assuming a value of

'1' (i.e., eDNA correlated to biomass) improves modeling efforts remains to be determined.

Nevertheless, our framework shows that assuming a value of 'b' equal to 0 (eDNA scales with

N) or 1 (eDNA scales with biomass) inherently necessitates a correction to infer the other metric

if populations/experimental units exhibit substantial body-size variation. Under the assumption

that b is equal to 0, Equation 1.4 shows that untransformed quantitative eDNA data should be

directly equivalent to abundance derived from individual counts (i.e., $\overline{x_i^b} = 1$, since $b = 0$), and

quantitative eDNA data can be 'corrected' to reflect biomass using Equation 1.6 by multiplying

quantitative eDNA data by the mean mass of individuals in the ith population (i.e., $\frac{\bar{x}_i}{x_i^b} = \bar{x}_i$,

since $b = 0$). Conversely, under the assumption that $b = 1$, Equation 1.6 shows that

untransformed quantitative eDNA data should be equivalent to population biomass (i.e., $\frac{\bar{x}_i}{x_i^b} = 1$,

since $b = 1$), and quantitative eDNA data can be 'corrected' to reflect N using Equation 1.4 by

dividing quantitative eDNA data by the mean mass of individuals in the ith population (i.e., $\overline{x_i^b} =$

$\bar{x}_i$, since $b = 1$). We also note that the framework we present for 'correcting' quantitative eDNA

based on the expected value of b should be relatively easy to apply even without detailed

population size structure data, at least in an interspecific context; all that is needed to

approximately correct eDNA data for allometry (in the absence of corresponding traditional data)

is the mean mass of detected species. Historical size structure data could also potentially be a

sufficient proxy for contemporary body mass distribution data, although we note that without

beta-coefficient estimates this will only provide *relative* abundance inferences, rather than

absolute estimates. Given that in many systems most of the variation in body mass likely occurs

across (rather than within) species, applying 'corrections' based on approximate mean body

masses of detected species could also potentially improve the application of quantitative eDNA

data to estimate community biodiversity in the absence of traditional abundance data. It is

important to note, however, that this simplifying assumption may be sensitive to systems or

species with substantial intraspecific variation in body mass.

Quantitative ecological inferences from eDNA

Managing expectations is critical for furthering the utility of eDNA to quantify

abundance (4). Even when integrating the 'ecology of eDNA', eDNA is likely only able to

facilitate 'coarse', but cost-effective, abundance inferences. This is reflected in the results of

applying our framework to both Case Studies. In Case Study 1 we were unable to generally

differentiate, at a species-specific level, between non-rare species due to high heteroscedastic

residual error in our models, highlighting key limitations associated specifically with

metabarcoding data (see 'limitations' section below for discussion). Nevertheless, it was possible

to confidently infer rare species (either N or biomass) based on the number of reads recovered.

We will note that this pattern persisted for N even if species had moderate biomass; integrating

allometry, for example, allowed us to identify that striped bass would likely occur in low

numerical abundance despite a 'moderate' population biomass in many sampling months due to

their large body size.

Despite only a 'coarse' capacity to infer species-specific abundance in Case Study 1, our

framework also substantially improved quantitative biodiversity inferences. Environmental

DNA-derived Shannon values tracked trawl-derived metrics very well as b approached 0.80

despite only moderate-strength relationships between eDNA, N, and biomass that possessed high

heteroscedastic residual error. A key strength of the Stoeckle et al. 2021 dataset is the large

numbers of species and datapoints; while relationships between quantitative eDNA data and

trawl abundance were weak-to-moderate in strength, we were nevertheless likely able to infer an

allometric effect due to the robust species sample size represented in the dataset. At the

community level, abundance patterns among species often follow log-normal distributions (9).

Community diversity metrics derived from corrected eDNA may be comparable to traditionally-

derived metrics as long as eDNA data 'on average' reflects high- and low-abundance species

(i.e., it is common to observe more eDNA from very abundant species and low levels of eDNA

from very rare species). There may be substantial variation and/or 'noise' around the 'regression

line' in the relationship between eDNA and abundance, making species-specific abundance

inferences challenging until metabarcoding methods are further refined. However, this residual

error may work out as 'random noise' in a community biodiversity metric calculation as long as

there is a sufficient sample size of species present in the ecosystem. High stochastic residual

error in the relationship between organism abundance and eDNA metabarcoding data may imply

that this may not be suitable to estimate biodiversity in a species-poor (i.e., low sample size)

environment.

It is worthwhile to contrast these results with Case Study 2, which was based on species-

specific qPCR data. Quantitative PCR data directly tracks template eDNA concentrations, and

thus is likely to quantitatively track abundance much better than metabarcoding approaches (4).

While we were unable to differentiate between population biomass based on corrected eDNA

data, these lakes are almost solely inhabited by brook trout and are thus likely close to their

maximum carrying capacity. Indeed, biomass only varied by 4 times across the lowest- and

highest-biomass populations. However, how that biomass was distributed across individuals

varied significantly among populations, with the lowest- and highest-N populations differing by

an order of magnitude. After applying our framework, we were able to differentiate between

high- and low-N populations based on a combination of quantitative eDNA data and population

size structure data, lending support to the assertion that eDNA may be able to provide abundance

inferences regarding critical conservation thresholds (20). Furthermore, on a broader species-

distribution level all of our ecosystems could be identified as 'high abundance' due to a lack of

predators/competitors facilitating population biomass levels likely close to ecosystem carrying

capacity. We suspect we would likely be able to further identify ecosystems where Brook Trout

are substantially rarer (e.g., in mixed-species ecosystems with other predatory/competitor fish

species) based on quantitative eDNA data.

Limitations and caveats

To evaluate the broad potential utility of this framework, it is crucial to first determine: i) how prevalent of an effect allometry has on observed concentrations of eDNA; and ii) if the value of the scaling coefficient tends to be consistent across systems, or if such values tend to be system/species/population specific. We will note that our Case Studies may represent a biased subsample of published eDNA studies – we selected these two studies for application of our framework precisely because previous analyses found an allometric effect on the distribution of quantitative eDNA data observed in these ecosystems. In particular, we note that body size varied substantially across populations/species in both studies: in Yates et al. 2021, mean body size among populations spanned an approximate order of magnitude, and in Stoeckle et al. 2021 mean body size varied from 1 g (bay anchovy, *Anchoa mitchilli*) to 44 kg (Atlantic sturgeon, *Acipenser oxyrhynchus*). As a result, these datasets provided illustrative examples for the application of our framework. We predict that the relative effect size improvement obtained by accounting for allometry will be positively correlated with body size variation among study populations and/or species, as our framework demonstrates that allometric 'corrections' approximate multiplying or dividing by a constant in the absence of substantial body size variation.

It is also important to note that different biases in detectability between methods may also imply that precise/exact correspondence may not be possible between diversity metrics estimated from eDNA and traditional surveys. Environmental DNA surveys tend to detect greater species richness and rarer species relative to traditional surveys (2, 21). The Shannon index, for example, can be sensitive to the presence of rare species relative to Simpson's index, which is sensitive to

abundant species (22). While we limited our dataset to species detected only by both methods

(thus facilitating direct comparisons), a generally greater sensitivity of eDNA to detection of rare

species could produce generally higher Shannon values from eDNA relative to traditionally

derived data. We note, however, that this should be an advantage to applying eDNA to quantify

ecosystem biodiversity, and that applying an appropriate correction to quantitative eDNA data

will likely more accurately reflect ecosystem biodiversity.

Finally, we want to emphasize that integrating allometry represents only 'one piece of the

puzzle' when interpreting the relationship between quantitative eDNA data and organism

abundance. It is important to draw a distinction between factors influencing the interpretation of

quantitative eDNA data that can affect its distribution in nature, vs. issues associated with

molecular methods that can affect the assessment of that quantification. Allometry represents an

'ecology-side' variable impacting the pseudo steady-state distribution of eDNA observed in

environments, similar to the impacts of factors such as transportation (17, 23, 24), temperature

(25, 26), pH (27), activity (28, 29), etc. A number of 'molecular-side' factors can also affect the

relationship between quantitative eDNA data and the fidelity of that data to the 'true'

concentration of template eDNA in an environment. Species-specific approaches that directly

quantify concentrations of template DNA in environmental samples (e.g., qPCR, ddPCR) likely

have high fidelity, although factors such as inhibitors can still interfere with accurate assessment

of template concentrations and thus potentially obfuscate relationships between organism

abundance and eDNA concentrations (4, 30, 31). Metabarcoding methods, however, are likely to

generate substantially more residual error in the relationship between quantitative eDNA data

(i.e., species read counts) and template eDNA concentrations because issues like primer bias

(and resulting preferential amplification) as well as community composition, can add substantial

'noise' to the relationship between quantitative metabarcoding eDNA data and species

abundance (32–34). In effect, metabarcoding read count is a variable that is correlated with, but

ultimately a proxy index of, template eDNA concentrations in environmental samples, in much

the same way that metrics such as CPUE or BPUE are indicators of N and biomass in natural

ecosystems. Research is progressing on how to disentangle relationships between quantitative

read count data and original template eDNA concentrations (35, 36), and we are confident that

the utility of applying our framework to metabarcoding data will improve as methods progress to

improve the fidelity of metabarcoding data to template concentrations. However, we nevertheless

suspect that currently applying allometric corrections to metabarcoding datasets will likely be

challenging in communities with low species numbers because stochastic primer amplification

bias may mask or overwhelm potential allometric effects.

Future directions

Expanding the framework to incorporate other parameters relevant to steady-state eDNA

concentrations

Classic bioenergetics frameworks that model key physiological rates like consumption

(shown below), excretion, egestion, often take the form of:

$$Consumption = I_0 \times M^b \times f(T)$$

where $f(T)$ equals a temperature dependency function (37). Temperature is a key parameter

influencing eDNA pseudo-steady-state concentrations (25, 26); such frameworks could likely be

extended to model the broad effect of temperature on quantitative eDNA data recovered from

natural ecosystems (4). The I_0 term (represented in our model by β) represents a normalization

parameter which describes, for a given size of fish, the expected pseudo-steady-state

concentration of eDNA (e.g., how much eDNA a single 100 g fish would produce under given

conditions) and would vary across ecosystems depending on the value of different parameters

affecting eDNA production. The $f(T)$ would traditionally apply a transformation to I_0 based on

the proportion of maximum eDNA concentration observed relative to the temperature at which

the concentration is *highest* (38). When applied through our framework, the associated

'correction' would represent the reciprocal of $f(T)$ and should be a shared parameter across both

the N and biomass models.

We observed evidence supporting the integration of $f(T)$ functions from β-coefficients

derived from the Stoeckle et al. (2021) model. While we do not know what the shape of $f(T)$

within the Stoeckle et al. (2021) system, we would expect it to follow a similar trajectory as

other temperature-dependency functions associated with physiological processes related to

eDNA production such as consumption (38), with observed eDNA concentrations decreasing at

lower temperatures (25, 26). In our analysis, we fit 'seasonal' beta-coefficients (i.e., I_0 terms) to

account for non-independence of datapoints associated with temporal sampling. In the absence of

an explicitly defined $f(T)$, we would expect the I_0 term in our analysis to absorb variation in

eDNA concentrations due to temperature differences across seasons. If temperature was a major

driver of pseudo-steady-state eDNA concentrations, we would expect the beta-coefficients

associated with different seasons to be lower as seasonal temperature declined. Models fit to the

January sampling period (by far the coldest sampling period, at 6˚C vs. the next highest at

15.3˚C) had significantly lower beta-coefficient values relative to other sampling periods

(Figure 2). These results were derived from just four sampling periods, an insufficient quantity to

derive the actual shape of $f(T)$ for eDNA. However, these results imply that it may be possible to

broadly derive $f(T)$ in natural ecosystems if data from a sufficient natural gradient in temperature

are collected.

A quantitative eDNA framework across ecosystems: I_0 and 'b' across taxa and trophic levels

419   We also note that our framework is flexible enough to model relationships between

420   organisms and eDNA across taxonomic groups and trophic levels. We hypothesize that we

421   would observe consistent differences in the joint regression slopes and b-values for our

422   framework for organisms that inhabit different trophic levels and (primarily for macro-

423   organismal taxa) different levels of metabolic activity (Figures 8 and 9).

424   Scaling coefficient values for extra-organismal eDNA produced by macro-organisms

would be expected to reflect excretory/surface area allometry, as discussed above. However,

taxonomic variation in the value of b exists for key physiological rates likely related to eDNA

production (39–41). Similarly, we would expect the value of I_0 to vary across macro-organisms

based on their metabolic rates and activity. Turtles, for example, are expected to have low mass-

specific eDNA production rates (42); this would manifest in a lower I_0 (and thus higher β slope

values) relative to other large-bodied macro-organisms (Figure 9). Aquatic plants/macro-algae,

which are notoriously difficult to detect using eDNA and for whom only weak relationships

between eDNA and biomass are often found (43), represent an even more extreme example.

Identifying the taxonomic level at which b and I_0 coefficients may vary across macro-organisms

is a critical next step.

435   We also note that this framework is agnostic as to the source of the DNA found in

environmental samples, i.e., it is flexible enough to apply to both extra-organismal DNA from

macro-organisms as well as 'eDNA' derived from whole-body multicellular and unicellular

micro-organisms captured in environmental samples. For whole-bodied multicellular plankton

(e.g., zooplankton) and multicellular micro-organisms, the amount of DNA in an environmental

sample would likely scale closely with the biomass of these organisms, as most 'eDNA' in the

sample would likely derive from captured whole organisms; we speculate that the expected value

of the scaling coefficient would thus likely be close to 1 (Figure 8). Identifying if, and at what

approximate body size, eDNA shifts from the being generated primarily by the excretion of

extra-organismal DNA to predominantly whole-body DNA from captured organisms is

important to apply this framework across trophic levels.

For unicellular organisms, we would expect that the eDNA present in the environmental

sample would scale with a b-value close to 0 (i.e., with numerical abundance): as many microbial

metagenomics studies have noted, eDNA copies should roughly scale with the number of

individuals of a given species present in the sample (44). Reads assigned to prokaryotes and

micro-eukaryotes, which are both captured 'whole organism' in environmental samples,

dominate metagenomic shotgun sequencing data (45, 46). When examining eDNA biomass

relationships, this would likely manifest in lower slope values for 'corrected eDNA'/biomass

relationships (Figure 9). We would also speculate that slope values between 'corrected' eDNA

data and biomass should be higher among multicellular plankton/microorganisms/protists

relative to prokaryotes, as differences in slope values between these groups may be largely

influenced by differences in mean cell size. Multicellular plankton/micro-organisms and protists,

which typically have much larger cell sizes relative to prokaryotes, may thus have higher slope

values.

It is important to note, however, that this framework models hypothesized relationships

between the eDNA concentrations within an environment and its relationship between organism

abundance and biomass. The hypothesized trends described above can be applied to single-

species approaches like qPCR that directly measure template concentrations in eDNA samples,

but the single-species nature of qPCR approaches are currently not practical or cost-effective for

multi-species or multi-trophic community-level inferences. Currently, the application of our

framework to interspecific eDNA approaches such as metabarcoding are limited by species-

specific amplification efficiency biases associated with the universal primers that are typically

used to study macro-organismal eDNA. As discussed previously, these amplification biases can

limit the fidelity of relative read count data to original template concentrations in eDNA samples

(32–34). This inherently limits (to a degree) the application of our framework to infer N and

biomass from quantitative metabarcoding data, which can only serve as a proxy for the 'true'

template concentrations (as discussed above in detail), although we again note that substantial

progress is being made to 'de-noise' read-count and template eDNA concentration relationships,

e.g., (34–36).

Amplicon-free approaches such as metagenomics, however, are not as affected by such

biases (44, 45). Metagenomic techniques have already been applied to study relative gene

abundances in microbial ecology for some time (44), although normalizing microbial

metagenomic eDNA data to reflect relative gene abundance remains a substantial analytical

challenge (mean genome length of individual species and the community can affect the number

of reads recovered, extraction protocol can affect efficiency of recovery of some taxa, etc.) (44,

47, 48). However, as sequencing technology and bioinformatics approaches improve, the

capacity to recover sufficient extra-organismal eDNA from non-microbial organisms to make

quantitative ecological inferences may be possible (46). For example, although reads were

dominated by prokaryotes (~64%) an environmental metagenomics study in Antarctic ocean

waters recovered substantial quantities of eDNA from a variety of metazoan taxa, although

inferences were potentially limited due to incomplete reference libraries (46). However, while

the assembly of reference genomes for traditionally underrepresented taxa lag vertebrate

reference genome availability, the last 25 years has seen substantial progress in assembling

reference sequences for earth's animals - and this progress is likely to accelerate in coming years

as sequencing costs and technology improve (49). Notably, this study found poor recovery of

echinoderm reads in metagenomic data, despite echinoderms representing diverse and high-

abundance taxa in this ecosystem (46); while this may be due to a general lack of reference

genomes for this taxonomic, it is notable that this taxonomic group is characterized by low

activity and resting metabolic rates relative to other chordate taxa (e.g., fish) (50). As a result, we

would predict higher slope relationships between the quantity of eDNA recovered and

echinoderm N and biomass relative to groups such as fish. Although substantial other

considerations may affect the interpretation of recovered metazoan eDNA from metagenomics

approaches (e.g. species/community genome length, total eDNA sequenced, etc.), the framework

described herein is likely flexible enough to model dynamics associated with eDNA source (i.e.,

extra-organismal or organismal eDNA, metabolic activity, etc.) across trophic levels, and thus

lays the foundation for broadly interpreting quantitative eDNA signals across an ecosystem.

Conclusions and Recommendations

We present a framework for integrating allometry when quantifying organism abundance from

eDNA. Most importantly, we demonstrate that N and biomass are distinct but correlated metrics,

and that assuming eDNA correlates directly with one metric *necessitates* a correction to derive

the other when body size variation among populations exists. We further note that any study with

both numerical abundance and biomass data could utilize our framework or its simplified

approximation based on mean mass. A meta-analysis of previously published datasets that report

eDNA, density, and biomass data could illuminate the conditions under which our framework

provides utility. Nevertheless, we suggest that, instead of implicitly assuming eDNA production

scales with a value of 0 or 1 (i.e., with N or biomass), future studies examining the relationship

between quantitative eDNA data and organism abundance should consider investigating

allometry, especially if study populations/species exhibit substantial body size variation. Finally,

we note that our framework could be expanded to include other relevant ecological or

environmental variables that might affect eDNA pseudo-steady-state concentrations (e.g., trophic

level, metabolic rates, temperature dependency functions).

Materials and Methods

Case Study 1: paired eDNA metabarcoding and trawl data for bony fish from Stoeckle et al.

(2021).

Trawl survey and eDNA collection, extraction, and analysis

For a full description of methodologies, please refer to the original manuscript (9). In

brief, eDNA water sampling was paired with fish trawl data collected off the northeastern coast

of the United States for 1-week periods in January, June, August, and November in 2019.

Environmental DNA was amplified using primers targeting a ~106-bp segment on the

mitochondrial 12S V5 region, and sequencing was conducted on an Illumina MiSeq. DNA

processing and bioinformatics were conducted as described in Stoeckle et al. (2020) and Stoeckle

et al. (2021).

While the Stoeckle et al. (2021) dataset included both bony fishes and chondrichthyans,

our analysis focused solely on the bony fish dataset for several reasons. First, bony fish and

Chondrichthyans exhibit substantially different physiology, with potential implications for

eDNA recovered for each taxonomic group (reviewed in Yates et al. (*in press*)). Second, bony

fishes were much more well represented relative to chondrichthyans, with 56 species and 160

datapoints versus only 13 species and 37 datapoints, respectively.

For details on metabarcoding dataset data curation, please see Yates et al. (*in press*). To

apply a correction to quantitative eDNA reads, the mean mass of a species must be known. In 14

instances, a species was detected in one month using eDNA sampling but was not captured

during that month in trawl surveys. These cases are problematic because we lacked body-size

data with which to 'correct' quantitative eDNA reads. For these months, we therefore substituted

mean body mass across all months in which these species were captured during trawling; this is

unlikely to have significantly impacted results as these cases formed a collective fraction of total

biomass observed across seasons (<0.01%, Stoeckle et al. 2021) and the majority of body size

variation occurred interspecifically (see below).

Integrating allometry into eDNA metabarcoding read count corrections

For the Stoeckle et al. 2021 dataset, we lacked individual size data; only total biomass per

species and species counts were available from tow sampling. As a result, it was only possible to

calculate the mean individual mass for each species (biomass-per-tow, BPT, divided by

individuals-per-tow, IPT). Given that only 'mean' species body mass data were available, several

simplifying assumptions can be made to our 'allometric eDNA correction'. All individual

members of a species were thus assumed to have a body mass equal to the mean body mass of

that species. When all members of a species are assumed to have identical body mass, the mean

of allometrically scaled individual mass values in the ith population or species is equal to the

allometrically-scaled mean mass of the ith population/species (note that this condition is not true

when intraspecific body size variation is present):

556
$$\overline{x_i^b} = \bar{x}_i^b$$

557   As a result, we can simplify equations 1.4 and 1.6b to:

558
$$\text{Equation 2.1: } N_i = \frac{1}{I_0}\, eDNA_i \div \bar{x}_i^b$$

559
$$\text{Equation 2.2a: } Biomass_i = \frac{1}{I_0}\, (eDNA_i \div \bar{x}_i^b) \times \bar{x}_i$$

560   And which rebalances to:

561
$$\text{Equation 2.2b: } Biomass_i = \frac{1}{I_0}\, eDNA_i \times \bar{x}_i^{1-b}$$

Where eDNA$_i$ denotes aggregate eDNA production for the ith species, I$_0$ denotes a normalization

constant, N_i is the numerical abundance of the ith species, $\bar{x}_i$ is the mean individual body mass of

the ith species, and b is the allometric scaling coefficient. While this is obviously an unrealistic

simplifying assumption, interspecific variation in body mass in this study system is likely much

greater than intra-specific variability in body mass (minimum mean mass = ~1 g (bay anchovy,

Anchoa mitchilli), maximum mean mass = 44 kg (Atlantic sturgeon, *Acipenser oxyrhynchus*); as

a result, this simplifying assumption is unlikely to strongly affect our analyses. We also

evaluated, using simulations, the potential bias introduced into allometrically scaled abundance

metrics from this simplifying assumption (see Yates et al. (*in press*), supplementary materials);

resulting bias was likely low ($< 5\%$).

We therefore applied eDNA corrections for abundance metrics and in calculations for

diversity indices by correcting eDNA data or abundance using the mean mass of a species raised

to the power of 'b' ($\bar{x}_i^b$) rather than the mean of allometrically scaled individual mass values of a

given species ($\overline{x_i^b}$) (see equations 2.1 and 2.2c above). It is important to note that the assumption

that bias resulting from this is low may not hold in all ecosystems, particularly ecosystems with

low interspecific diversity in body mass, and/or many species in which there is considerable

overlap in cohorts within habitats (e.g., small freshwater streams).

Inferring numerical abundance and biomass from quantitative eDNA data

The relationships between eDNA concentration, N, and biomass, however, are inherently

correlated, and our framework shows that allometry in eDNA production represents the linking

variable between the abundance metrics. As a result, the values of b and I$_0$ that 'optimizes' the

predictive utility of eDNA to infer both abundance metrics should be estimated from both

equations simultaneously; that is, the value of the b and I_0 parameters in equations 1.4 and 1.6

should be estimated *jointly*. To this end, we employed a Bayesian approach to jointly-estimate

the value of b and I_0 from the eDNA, population abundance, and size structure data observed in

Stoeckle et al. 2021. We assumed a negative binomial data model for abundance and a truncated

Gaussian data model (truncated below by zero) for biomass:

591
$$N_i \sim NB(p_i, r),$$

592
$$Biomass_i \sim TN(\mu_i, \tau_i),$$

593   where p_i is the probability parameter and r is the size parameter for the negative binomial model,

594   and μ_i and τ_i are the mean and precision parameters of the truncated Gaussian model. Since the

595   mean of a negative binomial distribution is $\lambda = r(1-p)/p$, we parameterized the abundance

596   model according to Equation 2.1 by the mean:

597
$$\lambda_i = \alpha_{N,m} + \beta_m \frac{eDNA_i}{\bar{x}_i{}^b},$$

598
$$p_i = r/(r + \lambda_i),$$

599   where $\alpha_{N,m}$ and β_m are the linear regression parameters for abundance, varying by month to

600   account for repeated observations of the same species across time. Note to aid in model

601   convergence and create integer abundance, we rounded individuals per tow to individuals per

602   100 tows. In the biomass model we used Equation 2.2c for mean biomass:

603
$$\mu_i = \alpha_{B,m} + \beta_m eDNA_i \bar{x}_i{}^{1-b},$$

where, as in the abundance model, $\alpha_{B,m}$ and β_m are the linear regression parameters for

biomass, varying by month. While we note that our framework predicts an intercept 'fixed' at

zero (i.e., the origin) for both abundance metrics, we allowed our model to estimate a monthly α

term for each abundance metric because (i) species abundance (e.g. individuals- or biomass-per-

tow) were estimated with error and (ii) the limit of detection for the metabarcoding assay was

likely variable across species due to variable amplification efficiency based on primer mismatch.

The intercept for each model should roughly correspond to the 'per-fish' or 'per-biomass' limit

of detection (LOD) for the metabarcoding assay, but for many species this value was likely

significantly different from the origin. Preliminary investigations demonstrated that variation in

biomass was also dependent on the linear predictor term. As a result, we parameterized the

precision accordingly:

615
$$\tau_i = \frac{1}{\sigma^2 (eDNA_i \bar{x}_i^{1-b})^a},$$

where σ^2 is the scalar variance multiplied by the linear predictor term ($eDNA_i \bar{x}_i^{1-b}$), which is

also scaled to the power of a so that the variance was not assumed to be monotonically

increasing with the linear predictor term. Since an eDNA value of zero would cause this term to

be undefined, we omitted eDNA detections of zero in this model (n = 27). Note that the monthly

regression coefficients, β_m, are shared between the abundance and biomass models, and are

equivalent to $\frac{1}{I_0}$ in the suite of equations 1 and 2 above.

We used global Gaussian priors truncated below by zero for the linear regression

coefficients (αs and βs) to ensure means were strictly positive, a uniform prior ranging from 0-2

for the allometric scaling coefficient (b), a uniform prior for the biomass variance term (σ), and a

gamma prior for the N variance and size terms:

626
$$b \sim Uniform(0,2)$$

627
$$\beta_m \sim TN(0,0.001)$$

628
$$\alpha_{N,m} \sim TN(0,0.001)$$

629
$$\alpha_{B,m} \sim TN(0,0.001)$$

630
$$r \sim Gamma(100,10)$$

631
$$\sigma \sim Uniform(0,200000)$$

We assessed model fit using Bayesian p-values based on the mean squared error (MSE) between

observed and simulated values ($p = 0.83$ and 0.75 for the N and biomass models, respectively),

visual assessment of traceplots (see Supplementary Appendix), and Gelman-Rubin statistics for

each parameter on 500000 number of iterations, five chains, and a 25-thinning rate with a burn-

in of 50000.

Deriving community biodiversity metrics from quantitative eDNA data

In addition to directly inferring organism abundance, eDNA data could potentially be

used to quantitatively monitor ecosystem biodiversity. Species diversity metrics are commonly

employed to quantify and monitor biodiversity in natural ecosystems, and several common

biodiversity metrics integrate relative species abundance. The Shannon Index (52), for example,

is commonly used to quantify biodiversity (53), and can be calculated using the following

equation:

$$\text{Shannon Index} = -\sum(p_i \times \log(p_i))$$

Where p_i is equal to the proportion of ith species in a whole community:

$$p_i = n_i/N_{All}$$

Notably, diversity metrics are traditionally calculated from either species biomass or numerical

abundance (N) data; n_i and N_{All} can be expressed as either the number of individuals of the ith

species and total individuals, or biomass of the ith species and total biomass, respectively.

By integrating the ecology of eDNA production, it may be possible to use quantitative

eDNA data to derive community biodiversity metrics, essentially employing relative eDNA read

count and/or concentrations as a 'proxy index' for species biomass or density. The general

equations described above can be used to transform and substitute quantitative eDNA data for

equivalent relative species numerical abundance or relative species biomass in community

biodiversity metrics (e.g., *H*). Community biodiversity metrics can be calculated from

quantitative eDNA data by substituting Equation 2.1 for species *N* or Equation 2.2c for species

biomass. A Shannon-index derived from quantitative eDNA data intended to approximate *N*, for

example, could substitute p_i as:

$$\text{Equation 3.2: } p_i = (eDNA_i/(\overline{x_i^b}))/\sum_i^{Spec}(eDNA_i/\overline{x_i^b})$$

Where the denominator represents the sum of $(eDNA_i/\overline{x_i^b})$ across all species (*Spec*). Similarly, a

Shannon diversity index derived from quantitative eDNA data intended to approximate species

biomass could substitute p_i as:

$$\text{Equation 3.3: } p_i = (eDNA_i \frac{\overline{x_i}}{x_{ij}^b})/\sum_i^{Spec}(eDNA_i \frac{\overline{x_i})}{x_i^b})$$

We evaluated the capacity of eDNA to infer community biodiversity after integrating

allometry by comparing eDNA-derived Shannon-index values to trawl-derived Shannon-index

values. We selected three specific values of *b*: 1) 0, corresponding to *N*; 2) the jointly-estimated

value of *b* inferred above from our previous Bayesian modelling; and 3) 1.00, corresponding to

biomass, and estimated Shannon-index diversity from quantitative metabarcoding reads across

all sampling seasons for both *N* and biomass data using equations 3.2 and 3.3, respectively. Note

that we used equations 2.1 and 2.2c as eDNA inputs instead of equations 1.4 and 1.6, as

discussed above. When estimating and pooling read-count data for each species for the 'yearly'

calculation, we also accounted for temporal variation among time-periods by multiplying

allometrically-scaled eDNA read data by the monthly beta-coefficients derived from the

Bayesian regression models described above. We then estimated Shannon index values for both

N and biomass from traditional trawl surveys, and calculated 95% confidence intervals for

differences between trawl and eDNA-derived Shannon indexes for the year and for each

sampling season for each value of *b*. Confidence intervals were estimated by non-parametric

bootstrap resampling eDNA and trawl datasets 10,000 times each and quantifying the difference

between bootstrap resampled metrics each iteration. All Shannon index values were estimated

using the R package *vegan* (54).

682   We then evaluated the sensitivity of Shannon index estimates to the value of the *b*-

parameter. We estimated yearly (i.e., pooled across all sampling seasons) Shannon index values

for *N* and biomass derived from 'corrected' metabarcoding read data for all *b*-values ranging

from 0 to 1.00, by intervals of 0.01; as above, we also accounted for temporal variation among

time-periods prior to pooling species eDNA reads by multiplying allometrically-scaled eDNA

read data by the monthly beta-coefficients derived from the joint Bayesian regression models.

For each abundance metric and each value of *b*, we then estimated the absolute difference

between the value of the 'corrected' eDNA-derived Shannon index values and trawl-derived

Shannon index values. To determine the value of *b* that jointly minimized differences between

eDNA- and both trawl-derived abundance metrics (i.e., *N* and biomass) across all sampling

seasons, we then estimated the sum of the absolute differences between eDNA and trawl-derived

Shannon index values for both *N*-based and biomass-based Shannon indices across the entire

year for each value of *b*.

Bray-Curtis dissimilarity metrics

697   We also calculated Bray-Curtis dissimilarity between community composition inferred

using eDNA sampling (with and without correction for allometric scaling) and trawl data across

the four sampling months using the three 'special' cases of the value of *b* described previously (*b*

= 0, the jointly-inferred value of *b* from our Bayesian analysis, and *b* = 1.00). First, so that we

could compare across sampling methods which report in different units (e.g., read count versus

organism count versus biomass), we converted each to proportion per taxon. For each abundance

metric, we then used the R package *vegan* to calculate Bray-Curtis dissimilarity between each

data type for each of the four months and visualized these differences using a non-metric

dimensional scaling (NMDS) plot. We assessed the results of this comparison visually where

points on the NMDS plot which are closer together represent community compositions that are

more similar, both in terms of species composition and relative abundance. We also evaluated

pairwise distances across months using a linear model with month, *b*-value, and abundance

metric (*N* vs. biomass) included as factorial independent variables; pairwise contrasts were

performed using *t*-tests.

Case Study 2: Brook trout abundance and eDNA – an intraspecific application of the framework

Yates et al. (2021a) examined the relationship between eDNA concentration and brook

trout (*Salvelinus fontinalis*) abundance and eDNA in nine study populations in the Rocky

Mountains, Canada. For a full description, please see the methods described therein. In brief,

brook trout abundance was assessed using fyke net capture-mark-recapture studies, and

population body size-structure was assessed from standardized mixed-mesh index netting. The

mean concentration of eDNA in each lake was assessed by collecting four littoral and four

pelagic water samples distributed evenly throughout the study lakes. Water samples were filtered

onsite, and qPCR (55) was used to assess brook trout eDNA concentration in each sample. Mean

lake eDNA concentration was then calculated by estimating the weighted average eDNA

concentrations observed in samples from the littoral and pelagic zone; samples from each zone

were assigned weights based on the respective proportion of each lake represented by the littoral

and pelagic zones.

Integrating allometry into eDNA corrections

Determining the optimal value of 'b' to jointly infer N and biomass was similar to Case

Study 1, with the exception that intraspecific eDNA concentration data for each lake was

associated with population-level abundance metrics (N, biomass), as opposed to monthly species

catch rates and mean body mass (as in Case Study 1). We estimated the value of b that jointly

optimized the b and I_0 values between 'corrected' eDNA data and our two abundance metrics.

Like in Case Study 1, we used negative binomial and truncated (below by zero) Gaussian data

distributions for abundance and biomass,

$$N_i \sim NB(p_i, r),$$

$$Biomass_i \sim TN(\mu_i, \tau_i),$$

$$\lambda_i = \alpha_N + \beta \frac{eDNA_i}{x_i^b},$$

$$p_i = r/(r + \lambda_i),$$

$$\mu_i = \alpha_B + \beta eDNA_i \frac{\overline{x_i}}{x_i^b}$$

where p_i is the probability parameter and r is the size parameter for the negative binomial model,

and μ_i is the mean of the truncated Gaussian model. Note that, like in the previous Case Study,

the regression coefficient for the eDNA predictor term, β, is shared between the abundance and

biomass models, and is equivalent to $1/I_0$ in the suite of equations 1 and 2 above. We used a

global Gaussian prior truncated below by zero for the β parameter and uniform priors for b and

the variance and size terms:

$$\beta \sim TN(0, 0.001)$$

$$r \sim Uniform(0, 50)$$

$$\tau = \frac{1}{\sigma^2}$$

$$\sigma \sim Uniform(0, 30000)$$

$$b \sim Uniform(0, 2)$$

Although the theoretical intercept based on our framework is approximately zero, very

low fish densities/biomass may be below the eDNA assay limit of detection; as a result, the

intercept value for our model is likely positive (albeit close to zero due to the high sensitivity of

the assay employed). To account for this, we used an informative prior for our model intercept

term based on laboratory tests of assay sensitivity. Using equations presented in Wilcox et al.

(2018); Appendix 1, we estimated a >99% probability of detecting eDNA at a concentration of

50 copies/L in the study lakes when using the sampling and analysis protocol described in (8).

Across lakes, we observed a mean of 10.6 DNA copies/L/fish and 0.09 DNA copies/L/g of fish.

With a limit of detection of 50 copies/L, these means correspond to correspond to an intercept of

5 and 531.7 for N and biomass regressions, respectively. For each model (*N* and biomass), we

specified a truncated Gaussian prior distribution for the intercept (α) with a mean corresponding

to the estimated per-fish and per-gram LoD, respectively:

$$\alpha_N \sim TN(5, 0.1)$$

$$\alpha_B \sim TN(531.7, 0.1)$$

As with the previous Case Study, we assessed model fit using Bayesian p-values based on the

mean squared error (MSE) between observed and simulated values ($p = 0.62$ and 0.55 for the *N*

and biomass models, respectively), visual assessment of traceplots (see Supplementary

Appendix), and Gelman-Rubin statistics for each parameter on 500000 iterations, five chains, a

25-thinning rate, and a burn-in of 50000. All analyses for both Case Studies were performed

within the statistical software R and JAGS using the packages *rjags* (57), *jagsUI* (58), and

MCMCvis (59).

Acknowledgements

MCY was supported by a CIGLR and NSERC post-doctoral fellowships. Comments from M. Young, A. Sepulveda and C. Jerde greatly improved previous versions of this manuscript. We thank A. Derry for her instrumental role in generating the data from Yates et al. (2021a), and we thank M. Stoeckle for allowing us to use data from Stoeckle et al. (2021) as well as substantial advice on its structure and interpretation.

Data accessibility statement:

No new data were generated for this manuscript; data from Case Studies 1 and 2 can be found in

Stoeckle et al. (2021) and Yates et al. (2021a), respectively.

Table 1: Model deviance for joint relationship between allometrically-corrected eDNA data and
N, biomass using data from Stoeckle et al. (2021) and Yates et al. (2021a). Results are shown for
the model in which 'b' was estimated and also for models assuming $b = 0$ (i.e., eDNA production
scales with numerical abundance) and $b = 1$ (i.e., assuming eDNA production scales linearly
with biomass. Values in brackets represent 95% credible intervals.

Stoeckle et al. 2021		*Yates et al. 2021a*	
b	Model Deviance	b	Model Deviance
0*	5372.2	0*	315.9
0.82 (0.76-0.88)	5115.1	0.77* (0.44-1.13)	306.7
1[+]	5158.6	1[+]	309.5

*Value fixed in model, assumes eDNA production scales with numerical abundance
[+]Value fixed in model, assumes eDNA production scales with biomass

Table 2: Corrections applied to quantitative eDNA data based on specific assumed values of the
allometric scaling coefficient (0, 0.82, and 1.00). Corrected eDNA data was used to calculate
Shannon-index values (including bootstrapped confidence intervals) and Bray Curtis
dissimilarity for N and biomass for comparison against metrics derived from traditional trawl
data. $(\bar{x}_{ij}^{(b)})$ corresponds to the mean mass of the ith species in the jth month, and β_j corresponds to
the beta-value estimated from the joint Bayesian model for the jth month. For the comparison
across years, total corrected eDNA reads across months and total species abundance (N and
biomass) were summed across months.

Assumed Scaling Coefficient Value	Biomass Correction	N Correction
0.00	$eDNA_{ij} \times (\bar{x}_{ij}^{(1)}) \times \beta_j$	$eDNA_{ij}/(\bar{x}_{ij}^{(0)}) \times \beta_j$
0.82	$eDNA_{ij} \times (\bar{x}_{ij}^{(0.18)}) \times \beta_j$	$eDNA_i/(\bar{x}_{ij}^{(0.82)}) \times \beta_j$
1.00	$eDNA_{ij} \times (\bar{x}_{ij}^{(0)}) \times \beta_j$	$eDNA_i/(\bar{x}_{ij}^{(1)}) \times \beta_j$

805 Table 3: Bray-Curtis distances between trawl and eDNA-derived abundance metrics, assuming
806 different values of the allometric scaling coefficient.

Assumed Scaling Coefficient Value	Mean Distance (across months) from trawl N	Mean Distance (across months) from trawl biomass
0.00	0.542	0.632
0.82	0.305	0.395
1.00	0.339	0.406

809 **Figure Captions**
810

811 Figure 1: Directed Acyclic Graph demonstrating how raw trawl and eDNA data (squares) are
812 used to estimate a latent parameter (allometric scaling coefficient 'b'; circle). Mean mass and
813 eDNA data, in combination with the latent 'b' parameter, are then used to derive community
814 biodiversity metrics (Shannon Index and Bray-Curtis).
815

816 Figure 2: Monthly beta coefficients for each sampling period estimated from Stoeckle et al.
817 (2021); temperature data presented reflects the mean ocean temperature during each sampling
818 period.
819

820 Figure 3: Forest plot for Shannon Index diversity metrics. Values correspond to Shannon index
821 values based on traditional estimates (either numerical abundance (N) or biomass) minus eDNA-
822 derived 'corrected' equivalents, given an assumed value of the allometric scaling coefficient (b).
823 'x' refers to the mean mass of the ith species. Error bars represent 95% bootstrap confidence
824 intervals for the differences between traditional vs. eDNA-derived metrics.
825

Figure 4: Non-metric Dimensional Scaling (NMDS) plot indicating inferred community
composition similarity (based on Bray-Curtis dissimilarity metric) for each month based on trawl
count data. Figure a) depicts dissimilarity metrics based on N, with b) depicting dissimilarity
metrics based on biomass. Colored polygons represent data for each month (January, July,
August, November). Closed squares represent trawl data, and open circles, diamonds, and
triangles represent eDNA data corrected assuming b-values of 0.00, 0.82, and 1.00
(respectively).
833

Figure 5: Comparison between eDNA- and trawl-derived Shannon-index values across all
seasons for values of b ranging from 0 to 1.00. The y-axis represents the absolute value of the
difference between 'corrected' eDNA Shannon index values and trawl-derived Shannon index
values. Figure (a) compares eDNA data corrected using equation 1.1, figure (b) compares eDNA
data corrected using equation 2.2c, and figure (c) compares the sum of the absolute differences in
eDNA- and trawl-derived Shannon indexes for both N and biomass (i.e. both equation 2.1 and
2.2c). Data from Stoeckle et al. 2021.
841

842 Figure 6: Linear regressions for Brook trout eDNA concentration, N (top row), and biomass
843 (bottom row), with eDNA data corrected for different values of the scaling coefficient ($b = 0$,
844 0.77, and 1.00) based on equations 1.4 and 1.6b. Data from Yates et al. 2021.
845

846 Figure 7: Brook Trout N and biomass point-estimates (black circle) and 95% credible intervals
847 based on eDNA data corrected using equations 1.4 and 1.6b. Blue asterisks represent the 'true' N
848 or biomass per population (estimated from mark-recapture). Data from Yates et al. 2021.
849

Figure 8: Allometric scaling coefficient is likely to vary based on combination of source of
'eDNA' and trophic level. Environmental DNA captured in filters from large metazoans (e.g.,
fish, amphibians, large invertebrates, etc.) is likely to be extra-organismal, and thus scale with
values similar to physiological rates and surface area (e.g., ~ 0.75). Environmental DNA from
small multicellular organisms may be predominantly derived from the captured of whole-bodied
organisms on water filters, and thus eDNA is likely to scale with biomass (e.g., $b \sim 1$).
Environmental DNA collected from single-celled organisms is likely to also be predominantly
derived from whole-bodied organisms on water filters, but will scale with gene copy per cell
(functionally numerical abundance, e.g., $b \sim 0$). Note that we define 'environmental DNA' as any
DNA (extra-organismal or organismal) captured in an environmental sample.
860
Figure 9: We would expect slope values between 'allometrically corrected' eDNA/biomass
relationships to vary based on metabolic activity and trophic level. Plants would be expected to
produce very little eDNA relative to biomass. Metazoans with low metabolic and activity rates
would also likely have high slope values relative to active taxonomic groups like fishes. Small
multicellular and single-celled organisms, which are captured 'whole-body' on eDNA filters,
would have very low slope values.
867
868 **Supplementary figures:**
869
870 Figure S1: Linear regressions for bony fish species' eDNA metabarcoding read count
871 (thousands), N (top row), and biomass (bottom row), with eDNA data corrected for different
872 values of the scaling coefficient (b = 0, 0.82, and 1.00) based on equations 2.1 and 2.2c. Panel a,
873 b, c, and d represent January, June, August, and November sampling seasons, respectively. Data
874 from Stoeckle et al. 2021.
875
876
877 Figure S2: Northwestern Atlantic bony fish species N and biomass point-estimates (black circle)
878 and 95% credible intervals based on eDNA data corrected using equations 2.1 and 2.2c. Blue
879 asterisks represent the 'true' N or biomass per population (estimated from trawling surveys).
880 Panel a, b, c, and d represent January, June, August, and November sampling seasons,
881 respectively. Data from Stoeckle et al. 2021.

References

1.   N. M. Sard, *et al.*, Comparison of fish detections, community diversity, and relative
abundance using environmental DNA metabarcoding and traditional gears. *Environ. DNA*
1, 368–384 (2019).

2.   D. Boivin-Delisle, *et al.*, Using environmental DNA for biomonitoring of freshwater fish
communities: Comparison with established gillnet surveys in a boreal hydroelectric
impoundment. *Environ. DNA* **3**, 105–120 (2021).

3.   N. T. Evans, P. D. Shirey, J. G. Wieringa, A. R. Mahon, G. A. Lamberti, Comparative
Cost and Effort of Fish Distribution Detection via Environmental DNA Analysis and
Electrofishing. *Fisheries* **42**, 90–99 (2017).

4.   M. C. Yates, M. E. Cristescu, A. M. Derry, Integrating physiology and environmental
dynamics to operationalize environmental DNA (eDNA) as a means to monitor freshwater
macro-organism abundance. *Mol. Ecol.* **30**, 6531–6550 (2021).

5.   M. L. Rourke, *et al.*, Environmental DNA (eDNA) as a tool for assessing fish biomass: A
review of approaches and future considerations for resource surveys. *Environ. DNA* **Early**
View, 1–25 (2021).

6.   M. C. Yates, D. J. Fraser, A. M. Derry, Meta-analysis supports further refinement of
eDNA for monitoring aquatic species-specific abundance in nature. *Environ. DNA* **1**, 5–13
(2019).

7.   B. P. Baldigo, L. A. Sporn, S. D. George, J. A. Ball, Efficacy of Environmental DNA to
Detect and Quantify Brook Trout Populations in Headwater Streams of the Adirondack
Mountains, New York. *Trans. Am. Fish. Soc.* **146**, 99–111 (2017).

8.   M. Yates, *et al.*, The relationship between eDNA particle concentration and organism
abundance in nature is strengthened by allometric scaling. *Mol. Ecol.* **30**, 3068–3082
(2021).

9.   M. Y. Stoeckle, *et al.*, Trawl and eDNA assessment of marine fish diversity, seasonality,
and relative abundance in coastal New Jersey, USA. *ICES J. Mar. Sci.* **78**, 293–304
(2021).

10.   A. Lacoursière-Roussel, M. Rosabal, L. Bernatchez, Estimating fish abundance and
biomass from eDNA concentrations: variability among capture methods and
environmental conditions. *Mol. Ecol. Resour.* **16**, 1401–1414 (2016).

11.   D. S. Pilliod, C. S. Goldberg, R. S. Arkle, L. P. Waits, Estimating occupancy and
abundance of stream amphibians using environmental DNA from filtered water samples.
CJFAS **70**, 1123–1130 (2013).

12.   A. Maruyama, K. Nakamura, H. Yamanaka, M. Kondoh, T. Minamoto, The release rate of
environmental DNA from juvenile and adult fish. *PLoS One* **9**, e114639 (2014).

13.   A. Takeuchi, T. Iijima, W. Kakuzen, S. Watanab, Y. Yamada, Release of eDNA by
different life history stages and during spawning activities of laboratory-reared Japanese
eels for interpretation of oceanic survey data. *Sci. Rep.* **9**, 6074 (2019).

14.   J. H. Brown, J. F. Gillooly, A. P. Allen, V. . Savage, G. B. West, Toward a metabolic
theory of ecology. *Ecology* **85**, 1771–1789 (2004).

15.   M. C. Yates, T. M. Wilcox, M. Y. Stoeckle, D. D. Heath, Interspecific allometric scaling
in eDNA production among Northwestern Atlantic bony fishes reflects physiological
allometric scaling. *Environ. DNA* **In Press** (2022).

16.   J. Skelton, A. Cauvin, M. E. Hunter, Environmental DNA metabarcoding read numbers
and their variability predict species abundance, but weakly in non-dominant species.
Environ. DNA **Early view**, 1–13 (2022).

17.   T. Levi, *et al.*, Environmental DNA for the enumeration and management of Pacific
salmon. *Mol. Ecol. Resour.* **19**, 597–608 (2019).

18.   M. J. Vanni, P. B. McIntyre, Predicting nutrient excretion of aquatic animals with
metabolic ecology and ecological stoichiometry: A global synthesis. *Ecology* **97**, 3460–
3471 (2016).

19.   B. O'Shea, A. J. Mordue-Luntz, R. J. Fryer, C. C. Pert, I. R. Bricknell, Determination of
the surface area of a fish. *J. Fish Dis.* **29**, 437–440 (2006).

20.   M. J. Spear, H. S. Embke, P. J. Krysan, M. J. Vander, Application of eDNA as a tool for
assessing fish population abundance. *Environ. DNA* **3**, 83–91 (2021).

21.   B. Hanfling, *et al.*, Environmental DNA metabarcoding of lake fish communities reflects
long-term data from established survey methods. *Mol. Ecol.* **25**, 3101–3119 (2016).

22.   E. K. Morris, *et al.*, Choosing and using diversity indices: Insights for ecological
applications from the German Biodiversity Exploratories. *Ecol. Evol.* **4**, 3514–3524
(2014).

23.   T. M. Wilcox, *et al.*, Understanding environmental DNA detection probabilities: A case
study using a stream-dwelling char Salvelinus fontinalis. *Biol. Conserv.* **194**, 209–216
(2016).

24.   A. J. Shogren, *et al.*, Controls on eDNA movement in streams: Transport, Retention, and
Resuspension. *Sci. Rep.* **7**, 5065 (2017).

25.   I. Caza-Allard, M. Laporte, G. Cote, J. April, L. Bernatchez, Effect of biotic and abiotic
factors on the production and degradation of fish environmental DNA: an experimental
evaluation. *Environ. DNA* **4**, 453–468 (2021).

26.   T. Jo, H. Murakami, S. Yamamoto, R. Masuda, T. Minamoto, Effect of water temperature
and fish biomass on environmental DNA shedding, degradation, and size distribution.
Ecol. Evol. **9**, 1135–1146 (2019).

27.   K. M. Strickler, A. K. Fremier, C. S. Goldberg, Quantifying effects of UV-B , temperature
, and pH on eDNA degradation in aquatic microcosms. *Biol. Conserv.* **183**, 85–92 (2015).

28.   B. Thalinger, *et al.*, The Effect of Activity, Energy Use, and Species Identity on
Environmental DNA Shedding of Freshwater Fish. *Front. Ecol. Evol.* **9**, 623718 (2021).

29.   K. E. Klymus, C. A. Richter, D. C. Chapman, C. Paukert, Quantification of eDNA

shedding rates from invasive bighead carp Hypophthalmichthys nobilis and silver carp
Hypophthalmichthys molitrix. *Biol. Conserv.* **183**, 77–84 (2015).

30. K. Uchii, *et al.*, Comparison of inhibition resistance among PCR reagents for detection
and quantification of environmental DNA. *Environ. DNA* **1**, 359–367 (2019).

31. E. Capo, S. Koizumi, I. Puts, F. Olajos, Droplet digital PCR applied to environmental
DNA, a promising method to estimate fish population abundance from humic-rich aquatic
ecosystems. *Environ. DNA* **3**, 343–352 (2021).

32. J. Piñol, M. A. Senar, W. O. C. Symondson, The choice of universal primers and the
characteristics of the species mixture determine when DNA metabarcoding can be
quantitative. *Mol. Ecol.* **28**, 407–419 (2019).

33. V. Elbrecht, F. Leese, Can DNA-based ecosystem assessments quantify species
abundance? Testing primer bias and biomass-sequence relationships with an innovative
metabarcoding protocol. *PLoS One* **10**, e0130324 (2015).

34. R. P. Kelly, A. O. Shelton, R. Gallego, Understanding PCR processes to draw meaningful
conclusions from environmental DNA studies. *Sci. Rep.* **9**, 1–14 (2019).

35. A. O. Shelton, *et al.*, Toward quantitative metabarcoding. *Ecology* **Accepted** (2022).

36. Z. Gold, *et al.*, Distinguishing Signal from Noise: Understanding Patterns of Non-
Detections to Inform Accurate Quantitative Metabarcoding. *bioRxiv Prepr.* (2022).

37. P. Hanson, T. Johnson, D. Schnindler, J. Kitchell, Fish Bioenergetics 3.0 Manual (1997).

38. D. J. Stewart, M. Ibarra, Predation and production by salmonine fishes in Lake Michigan
1978-88. *Can. J. Fish. Aquat. Sci.* **48**, 909–922 (1991).

39. C. L. Jerde, *et al.*, Strong evidence for an intraspecific metabolic scaling coefficient near
0.89 in fish. *Front. Physiol.* **10**, 1–17 (2019).

40. M. J. Vanni, P. B. McIntyre, Predicting nutrient excretion of aquatic animals with
metabolic ecology and ecological stoichiometry: A global synthesis. *Ecology* **97**, 3460–
3471 (2016).

41. J. E. Allegier, S. J. Wenger, A. D. Rosemond, D. E. Schindler, C. A. Layman, Metabolic
theory and taxonomic identity predict nutrient recycling in a diverse food web. *PNAS* **112**
(20) (2015).

42. C. I. M. Adams, L. A. Hoekstra, M. R. Muell, F. J. Janzen, A brief review of non-avian
reptile environmental DNA (eDNA), with a case study of painted turtle (Chrysemys picta)
eDNA under field conditions. *Diversity* **11** (2019).

43. L. M. Kuehne, C. O. Ostberg, D. M. Chase, J. J. Duda, J. D. Olden, Use of environmental
DNA to detect the invasive aquatic plants myriophyllum spicatum and Egeria Densa in
Lakes. *Freshw. Sci.* **39**, 521–533 (2020).

44. S. Nayfach, K. S. Pollard, Toward Accurate and Quantitative Comparative Metagenomics.
Cell **166**, 1103–1116 (2016).

45.   M. Stat, *et al.*, Ecosystem biomonitoring with eDNA: Metabarcoding across the tree of
life in a tropical marine environment. *Sci. Rep.* **7**, 1–11 (2017).

46.   D. A. Cowart, K. R. Murphy, C. H. C. Cheng, Metagenomic sequencing of environmental
DNA reveals marine faunal assemblages from the West Antarctic Peninsula. *Mar.*
Genomics **37**, 148–160 (2018).

47.   M. B. Pereira, M. Wallroth, V. Jonsson, E. Kristiansson, Comparison of normalization
methods for the analysis of metagenomic gene abundance data. *BMC Genomics* **19**, 1–17
(2018).

48.   A. Milanese, *et al.*, Microbial abundance, activity and population genomic profiling with
mOTUs2. *Nat. Commun.* **10** (2019).

49.   S. Hotaling, J. L. Kelley, P. B. Frandsen, Toward a genome sequence for every animal:
Where are we now? *Proc. Natl. Acad. Sci. U. S. A.* **118**, 1–8 (2021).

50.   B. A. Seibel, J. C. Drazen, The rate of metabolism in marine animals: Environmental
constraints, ecological demands and energetic opportunities. *Philos. Trans. R. Soc. B Biol.*
Sci. **362**, 2061–2078 (2007).

51.   M. Y. Stoeckle, M. Das Mishu, Z. Charlop-Powers, Improved environmental DNA
reference library detects overlooked marine fishes in New Jersey, United States. *Front.*
Mar. Sci. **7** (2020).

52.   C. E. Shannon, A Mathematical Theory of Communication. *Bell Syst. Tech. J.* **27**, 379–
423 (1948).

53.   C. J. Krebs, *Ecological Methodology*, 3rd Ed. (2014).

54.   J. Oksanen, *et al.*, vegan: community ecology package. (2020).

55.   T. M. Wilcox, K. J. Carim, K. S. McKelvey, M. K. Young, M. K. Schwartz, The dual
challenges of generality and specificity when developing environmental DNA markers for
species and subspecies of Oncorhynchus. *PLoS One* **10**, 1–13 (2015).

56.   T. M. Wilcox, *et al.*, Comment: The Importance of Sound Methodology in Environmental
DNA Sampling. *North Am. J. Fish. Manag.* **38**, 592–596 (2018).

57.   M. Plummer, rjags: Bayesian Graphical Models using MCMC. *R Packag. version 4.13*
(2003).

58.   K. Kellner, jagsUI: A Wrapper Around "rjags" to Streamline "JAGS" Analyses. *R*
Packag. Version 1.5.2 (2021).

59.   C. Youngflesh, MCMCvis: Tools to visualize, manipulate, and summarize MCMC output.
J. Open Source Softw. **3**, 640 (2018).

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Model specification

Case study 1: Stoeckle et al. 2021

$$N_i \sim NB(p_i, r)$$
$$\lambda_i = \alpha_{N,m} + \beta_m \frac{eDNA_i}{\overline{x_i}^b}$$
$$p_i = r/(r + \lambda_i)$$

$$r \sim Gamma(100, 10)$$
$$\alpha_{N,m} \sim TN(0, 0.001)$$

$$Biomass_i \sim TN(\mu_i, \tau_i)$$
$$\mu_i = \alpha_{B,m} + \beta_m eDNA_i \overline{x_i}^{1-b}$$
$$\tau_i = \frac{1}{\sigma^2(eDNA_i \overline{x_i}^{1-b})^a}$$
$$\sigma \sim Unif(0, 200000)$$
$$\alpha_{B,m} \sim TN(0, 0.001)$$

$$b \sim Unif(0, 2)$$
$$\beta_m \sim TN(0, 0.001)$$

Case study 2: Yates et al. 2021

$$N_i \sim NB(p_i, r)$$
$$\lambda_i = \alpha_N + \beta \frac{eDNA_i}{\overline{x_i^b}}$$
$$p_i = r/(r + \lambda_i)$$
$$r \sim Uniform(0, 50)$$

$$Biomass_i \sim TN(\mu_i, \tau_i)$$
$$\mu_i = \alpha_B + \beta eDNA_i \frac{\overline{x_i}}{x_i^b}$$
$$\tau = 1/\sigma^2$$
$$\sigma \sim Uniform(0, 30000).$$

$$b \sim Unif(0, 2)$$
$$\beta \sim TN(0, 0.001)$$

Directed acyclic graph

Observed quantities

Diversity Indices	Indices (Shannon, Bray-Curtis) estimated from eDNA-inferred N, Biomass. [†]Note that indices were estimated from Case Study 1 model output, not as part of hierarchical model
N_i	Abundance for population i
$Biomass_i$	Biomass for population i
$eDNA_i$	Quantitative eDNA data observed for population i
Size Structure_i	Size structure for population i

Estimated parameters

b	Allometric scaling coefficient
β	Coefficient for eDNA predictor term, equivalent to $1/I_0$ in general framework
α_N	Intercept for abundance model, differs by month in CS1
α_B	Intercept for biomass model, differs by month in CS1
r	Size parameter in abundance model
σ^2	Variance in biomass model, differs by population abundance in CS1

Sample Timing	Estimate [95% CI]

eDNA = f(N) (i.e. b = 0)

N - eDNA

Year	-1.15 [-1.52, -0.12]
January	-0.11 [-0.59, 0.21]
July	-0.87 [-1.43, -0.16]
August	-0.89 [-1.30, -0.01]
November	-1.54 [-1.92, -0.10]

Biomass - eDNA $\times$ $\bar{\mathbf{x}}$

Year	0.81 [0.13, 1.16]
January	0.13 [-0.32, 0.57]
July	1.08 [-0.86, 1.43]
August	0.19 [-0.26, 0.60]
November	1.00 [-0.17, 1.44]

eDNA = f(Biomass) (i.e. b = 1)

N - eDNA $\div$ $\bar{\mathbf{x}}$

Year	0.33 [-0.25, 0.56]
January	0.00 [-0.41, 0.33]
July	-0.40 [-1.01, 0.64]
August	0.61 [-0.50, 1.30]
November	0.14 [-0.36, 0.48]

Biomass - eDNA

Year	-0.07 [-0.24, 0.09]
January	-0.30 [-0.79, 0.02]
July	-0.33 [-1.28, 0.17]
August	-0.20 [-0.41, 0.09]
November	-0.08 [-0.38, 0.25]

eDNA = f(ASM) (i.e. b = 0.82)

N - eDNA $\div$ $\bar{\mathbf{x}}^{0.82}$

Year	0.05 [-0.38, 0.25]
January	-0.15 [-0.53, 0.13]
July	-0.45 [-1.05, 0.42]
August	0.35 [-0.48, 0.91]
November	-0.17 [-0.44, 0.34]

Biomass - eDNA $\times$ $\bar{\mathbf{x}}^{0.12}$

Year	-0.06 [-0.24, 0.11]
January	-0.21 [-0.64, 0.07]
July	-0.46 [-1.29, -0.00]
August	-0.18 [-0.40, 0.12]
November	-0.15 [-0.41, 0.15]

-3 -2 -1 0 1 2

Difference between trawl and
corrected eDNA Shannon Indices

b ≈ 0.75

b ≈ 1

b ≈ 0