

# 1 Quantifying Drift-Fitness Balance Using an Agent-Based Biofilm Model of 2 Identical Heterotrophs Under Low Nutrient Conditions

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## 7 **Abstract**

8 Both deterministic and stochastic forces shape biofilm communities, but the  
9 balance between those forces is variable. Quantifying the balance is both  
10 desirable and challenging. For example, negative drift selection, a stochastic  
11 force, can be thought of as an organism experiencing ‘bad luck’ and  
12 manipulating ‘luck’ as a factor in real world systems is difficult. We used an  
13 agent-based model to manipulate luck by controlling seed values governing  
14 random number generation. We determined which organism among identical  
15 competitors experienced the greatest negative drift selection, gave it a  
16 deterministic growth advantage, and re-ran the simulation with the same seed.  
17 This enabled quantifying the growth advantage required to overcome drift, *e.g.*, a  
18 50% chance to thrive may require a 10-20% improved growth rate. Further, we  
19 found that crowding intensity affected that balance. At moderate spacings, there  
20 were wide ranges where neither drift nor growth dominated. Those ranges shrank  
21 at extreme spacings; close and loose crowding respectively favoured drift and  
22 growth. We explain how these results may partially illuminate two conundrums:  
23 the difference between taxa and functional stability in wastewater treatment

24 plans and the difference between equivalent and total community size in neutral  
25 community assembly models.

26 **Keywords:** agent-based model, biofilm, drift, neutral assembly, community  
27 assembly, individual based model

## 28 1 Introduction

29 Both stochastic and deterministic assembly processes can shape biofilm populations.<sup>1,2</sup> Those  
30 processes, however, rarely act equally and the balance between them is determined by many  
31 conditions related to competition intensity. Such conditions include population size,<sup>3,4</sup> available  
32 space,<sup>5</sup> and resource availability.<sup>6</sup> Understanding how this balance shifts under differing conditions  
33 provides insights into biofilm-associated systems such as environmental bioreactors, healthcare,  
34 industrial production, and natural ecosystems.

35 Here, we attempt to quantify the balance between drift, a pure stochastic process,<sup>1,3</sup> and a more  
36 deterministic kinetic advantage. Under this balance, even if losing the ‘drift lottery,’ an individual’s  
37 progeny may thrive if their maximum growth rate ( $\mu_{max}$ ) or substrate affinity ( $K_s$ ) confers increased  
38 fitness over their competitors.

39 Such quantification is challenging. Drift is an inherently random process and experimental  
40 manipulation of a random process, distinct from simply controlling for it, is difficult. Despite that  
41 difficulty, there have been some physical experiments in which drift is isolated as an experimental  
42 factor,<sup>4,7,8</sup> often requiring subtle statistical analyses or extremely precise experimental work.

43 An alternative approach, used here, is to perform the experiments *in silico* where drift may be  
44 directly manipulated via random number generation. We used an agent-based model (NUFEB)<sup>9,10</sup> to  
45 simulate spatially competing bacteria under low nutrient conditions. The bacteria were identical and  
46 evenly spaced, differentiated only by random growth directions and biomass allocations during  
47 division. Drift was therefore the only selection process and was controlled by the seed value  
48 initializing the random number generator.

49 Our goal was to determine the degree to which a deterministic factor (here, Monod kinetics) must  
50 improve to overcome negative drift selection, so subsequent simulations using identical seeds were  
51 run. The difference was that the ‘biggest loser’, the lineage with the lowest relative abundance, was

52 assigned different kinetics. This approach allowed us to relate quantifiable fitness changes to the  
53 likelihood that the failing lineage would overcome negative drift and thrive. We also determined how  
54 the required degree of fitness advantage varied under differing crowding intensities (*e.g.*, closer  
55 spacing and increased initial population size).

56 We found that under purely stochastic conditions the losing lineage varied unpredictably between  
57 runs, showing the expected effects of drift. Further, altered fitness did enable losing lineages to  
58 overcome drift. For example, for an initial population of 9 cells evenly spaced 10 diameters apart  
59 either  $K_s$  or  $\mu_{max}$  had to improve by at least 10-20% for a 50% chance of thriving. Crowding affected  
60 both the improvement needed for a 50% chance of thriving and the ranges over which both drift and  
61 fitness co-dominated. The strong and sometimes non-linear interactions between terms could not be  
62 adequately reproduced using simple linear estimators but could be adequately expressed with a  
63 generalized additive model.

## 64 2 Methods

### 65 2.1 Agent Based Model

66 The agent-based model employed NUFEB (Newcastle University Frontiers in Engineering  
67 Biology),<sup>9,10</sup> which is based on the LAMMPS<sup>9</sup> molecular dynamics simulation framework and has  
68 successfully been used to model multi-species biofilms,<sup>10</sup> including development and detachment,<sup>7</sup>  
69 trade-offs in extracellular polymeric substance production,<sup>11</sup> and phototroph-heterotroph metabolic  
70 interactions.<sup>12</sup>

71 NUFEB is not lattice based, cells were positioned in three dimensions and had individual dynamic  
72 sizes. The directions in which cells divided and biomass allocations (40 to 60%) during division were  
73 randomly determined using a Park-Miller pseudorandom number generator and were the two factors  
74 contributing to drift.

75 The individually simulated bacterial cells physically interacted using realistic physics and grew  
76 according to Monod-style models described by Equation (1) where  $\mu$  is the substrate-dependent  
77 growth rate (1/hr),  $\mu_{max}$  is the maximum specific growth rate (1/hr),  $[S]$  is the concentration of the  
78 relevant substrate ( $\text{kg}/\text{m}^3$ ), and  $K_s$  is the affinity constant for the substrate ( $\text{kg}/\text{m}^3$ ). Additional  
79 descriptions of NUFEBs mechanics are detailed in previous publications.<sup>9,10</sup>

$$\mu = \mu_{max} \frac{[S]}{K_s + [S]} \quad (1)$$

80 The simulation volume height ( $2 \times 10^{-4}$  m) was defined to be in the Z-dimension, the bulk substrate  
81 concentration boundary condition at the top of the simulation volume was  $1 \times 10^{-4}$   $\text{kg}/\text{m}^3$  and the  
82 initial substrate concentration throughout the volume was set to the same value. The X and Y  
83 dimensions were equal and varied based on spacing and number of initial cells. Additionally, the X  
84 and Y boundaries were periodic, allowing biomass and substrates to wrap from one side of the  
85 simulation to the other.

## 86 2.2 Experimental Approach

87 The base experimental unit was an agent-based simulation initially seeded with identical bacterial  
88 cells with starting diameters of  $1 \times 10^{-6}$  m,  $K_s$  of  $3.5 \times 10^{-5}$   $\text{kg}/\text{m}^3$ ,  $\mu_{max}$  of  $1 \text{ h}^{-1}$ , and yield 0.61 kg  
89 biomass per kg substrate consumed. The initial cells (total population 4, 9, or 16) were arranged  
90 along evenly spaced (2.5, 5, or 10 cell diameters)  $M \times M$  points at the base of the simulation volume.  
91 Bacteria were allowed to grow and compete until 20% of the simulation volume consisted of  
92 heterotrophic biomass.

93 Each combination of populations sizes and spacings was run 120 times using different seed values to  
94 initialize the random number generator and the ‘biggest loser’ from each run was identified (see 2.3).  
95 Those simulations were then run again, but with the failed lineage given altered kinetic values (see  
96 2.4). The results of the runs were used to determine how the altered kinetics contributed to the

97 probability of transitioning from drift-driven failure to a thriving state (see 2.5) under various  
98 crowding intensities.

99 All combinations of the factor levels listed in **Table 1** (1089 combinations) were simulated for each  
100 of the 120 seeds, resulting in a total of 130680 runs. Each run required between 2 to 36 hours to  
101 complete, so the simulations were carried out on a high-performance computing cluster (see 2.6).

102 **Table 1:** Experimental factors and levels

| <u>Factor</u>            | <u>Values</u> |     |     |     |     |   |    |    |    |    |    |
|--------------------------|---------------|-----|-----|-----|-----|---|----|----|----|----|----|
| Spacing (cell diameters) | 2.5           | 5   | 10  |     |     |   |    |    |    |    |    |
| Initial Population Size  | 4             | 9   | 16  |     |     |   |    |    |    |    |    |
| % Change in $K_s$        | -50           | -40 | -30 | -20 | -10 | 0 | 10 | 20 | 30 | 40 | 50 |
| % Change in $\mu_{max}$  | -50           | -40 | -30 | -20 | -10 | 0 | 10 | 20 | 30 | 40 | 50 |

### 103 2.3 Determining Failed Lineages

104 For a system initialized with  $N$  bacterial lineages, the total biomass  $X_t$  is the sum of the biomass for  
105 each lineage  $X_i$ , as expressed by equation(2).

$$X_t = \sum_i^N X_i \quad (2)$$

106 In a system where each initial cell is identical, with no competition, and with no random effects, all  
107  $X_i$  are expected to be equal, thus the expected relevant abundance of any lineage ( $X_E$ ) is given as:

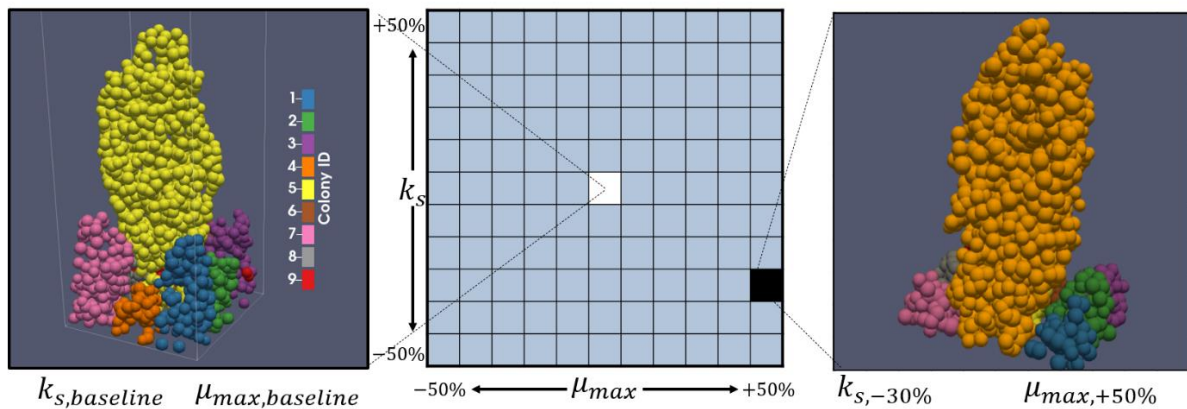
$$X_E = X_T/N \quad (3)$$

108 In the first round of simulations, all initial cells were identical and evenly spaced, but cell division  
109 directions and biomass allocations during division were determined randomly. As a result, the final  
110 biomass for any lineage was often not equal to the expected relevant abundance,  $X_i \neq X_E$ . In  
111 practice, there were often one or two lineages which strongly dominated with  $X_i \gg X_E$ , one or two  
112 lineages which became vanishingly small with  $X_i \ll X_E$  (the ‘biggest losers’), and the rest persisted at  
113 some noticeable abundance that was however below  $X_E$ . Moreover, the outcomes appeared to be  
114 determined early in the simulation, especially for the best and worst performing lineages. (Supporting  
115 Information Figure S1, Table S1, and Video SV1). We have defined three classifications of lineage

116 survival based on the difference between  $X_E$  and  $X_i$ : *languishing* ( $X_i < 0.3 X_E$ ), *thriving* ( $X_i >$   
117  $0.9X_E$ ), and *barely surviving* ( $0.3 X_E \leq X_i \leq 0.9X_E$ ).

## 118 2.4 Fitness Alteration

119 The worst-performing bacterial lineages from each of the initial homogenous runs were given a  
120 potential competitive advantage by altering their individual maximum specific growth rate ( $\mu_{max}$ )  
121 and/or their substrate affinity ( $K_s$ ) (**Figure 1**). The altered values were selected as described in **Table**  
122 **1**. We acknowledge that not all combinations of  $\mu_{max}$  and  $K_s$  were advantageous and that  $\mu_{max}$  and  $K_s$   
123 are often strongly correlated; here our goal was to thoroughly explore the parameter space.



**Figure 1:** Illustration of a parameter sweep. Under baseline conditions when all bacteria are identical (left hand side), colony 4 was the worst performing lineage. When colony 4 was given a competitive advantage (right hand side) via reduced  $K_s$  and increased  $\mu_{max}$ , colony 4 transitioned to thriving. This result along with all other parameter combinations across 120 random seeds was used to estimate  $p_{thrive}$ , the probability that the worst-performing colony would transition to thriving under given altered kinetics.

## 129 2.5 Probability Map Generation

130 The kinetic parameter sweeps were used to generate tables for each combination of factors which  
131 listed the final relative biomass of each bacterial lineage, that lineage's status as the 'biggest loser',  
132 and the lineage's success under each run. The percentage of failing lineages across all random seeds  
133 which transitioned to thriving was calculated for each combination of initial population size, spacing,  
134  $\mu_{max}$ , and  $K_s$ . Those percentages represent the probabilities that the fitness advantage (if any)  
135 conferred by altered kinetics would outweigh negative drift selection under the given conditions.

## 136 2.6 Simulation Management

137 Simulations were run and their results tabulated on the Newcastle University Rocket High  
138 Performance Computing environment and managed using Snakemake<sup>13,14</sup> workflows populating a  
139 SLURM<sup>15</sup> queue. Each simulation was run on a single core, with multiple hundreds of simulations  
140 run in parallel. Job submissions encompassed all kinetic parameter sweeps for each combination of  
141 other parameters, *e.g.*, a single batch submission would consist of all combinations of  $\mu_{max}$  and  $K_s$  for  
142 4 bacteria, spaced 5 diameters apart.

## 143 2.7 Data Analysis

144 Simulation results were saved as tabular comma separated value (CSV) text files and aggregated  
145 using BASH<sup>16</sup> (v. 4.2) shell and Python<sup>17</sup> (v. 3.8) scripts which included the NumPy<sup>18</sup> and pandas<sup>19</sup>  
146 libraries. Further processing of the data was performed off the cluster and used R<sup>20</sup> (v. 4.2) scripts  
147 incorporating various Tidyverse<sup>21</sup> and other supporting packages.<sup>22-43</sup>

### 148 2.7.1 Parameters Quantifying the Balance Between Drift and Fitness

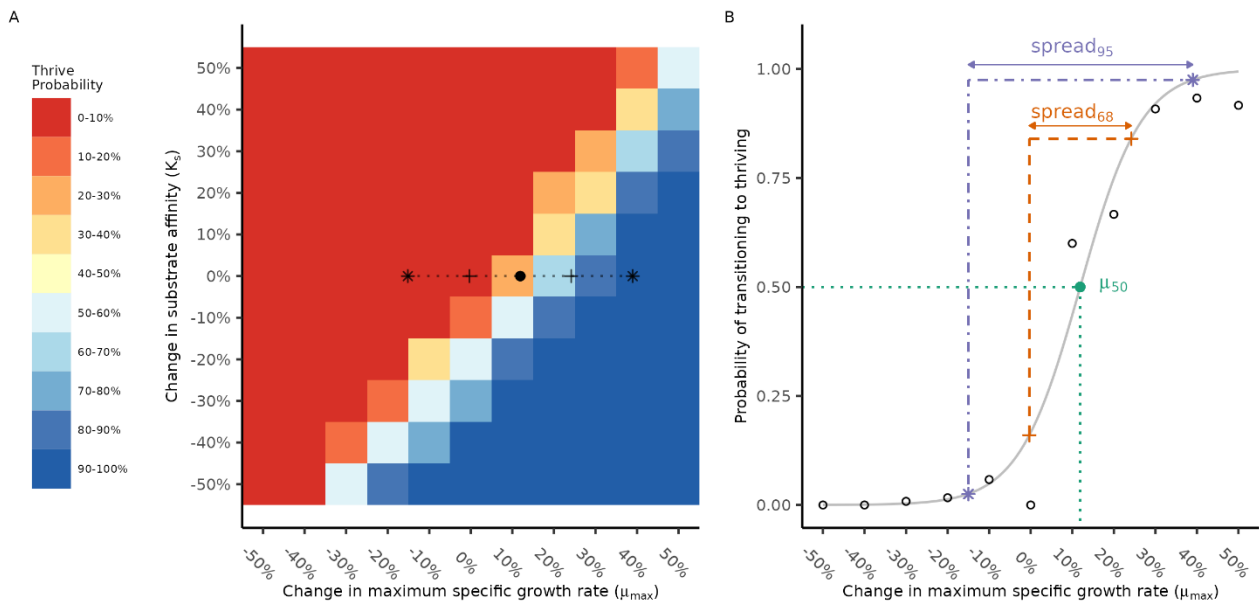
149 Each probability map was conceptually analogous to a cliffside; a continuous sharp probability  
150 threshold gradient separated by two flat regions of either 100% lineage success or failure (**Figure 2**  
151 **A**). We wished to quantify the midpoint and steepness of the gradient along lines of constant  $K_s$  for  
152 each crowding condition. A cross-section of the probabilities along  $\mu_{max}$  for any constant  $K_s$  produces  
153 a sigmoid-shaped profile (**Figure 2 B**). The profiles were fit to a logistic function of  $\mu_{max}$  with a  
154 maximum value of 1 given by equation (4), where  $p_{thrive}$  is the probability of transitioning to a  
155 thriving colony,  $k$  is a parameter affecting the steepness of the curve, and  $\mu_{50}$  is the  $\mu_{max}$  value at  
156 which there is a 50% probability of thriving.

$$p_{thrive} = \frac{1}{1 + e^{-k*(\mu_{50} - \mu_{max})}} \quad (4)$$

157 The relevant  $k$  and  $\mu_{50}$  parameters from each fit were recorded. We also determined the domains of  
158  $\mu_{max}$  values associated with the  $p_{thrive}$  ranges covering either a 2.5-95% or 16-84% chance of



159 thriving. These domains, respectively named *spread<sub>95</sub>* and *spread<sub>68</sub>* quantified the regions over which  
 160 neither drift nor fitness dominated.



161  
 162 **Figure 2:** Illustration of how the  $\mu_{50}$  and *spread* parameters were calculated. In this example, the probability map corresponding to 4  
 163 initial organisms placed 5 diameters apart is shown (A), and the dashed line is drawn along a line of constant  $K_s$ . The full length of the  
 164 line denotes the *spread<sub>95</sub>* region, the portion between crosses denotes *spread<sub>68</sub>*, and the solid point represents the  $\mu_{50}$  mark. When the  
 165  $p_{thrive}$  values are plotted as a function of  $\mu_{max}$  along the line of constant  $K_s$ , (B) it is apparent that a logistic function (grey solid line)  
 166 may be fitted to the points (black rings). The fitted function was used to estimate both the value of  $\mu$  corresponding to  $\mu_{50}$  and the  
 167 widths of the *spread* regions. This analysis was repeated for all crowding conditions along all lines of constant  $K_s$ .

168 The results of all sigmoid fits are shown in Supporting Information Figures S2-S10.

## 169 2.7.2 Analysing Balance Parameters

170 Within each crowding scenario, the extracted parameters were analysed using simple linear  
 171 regression models of the parameters as functions of  $K_s$ . The effect of crowding pressure (spacing and  
 172 total population) was then analysed by comparing the results of the fits between scenarios.

173 We note that although the linear fits for a 2<sup>nd</sup> order polynomial on  $\mu_{50}$  generally resulted in  
 174 marginally improved  $R^2$  scores and removed parabolic patterns from the residuals, the simple linear  
 175 regressions were still excellent and more interpretable; care should be taken if extending this work to  
 176 larger ranges of kinetic values.

### 177 2.7.3 Modelling the Effect of Competitive Pressure and Altered Kinetics

178 We wished to determine if a model based on the simulation results could accurately reproduce the  
179 transition probabilities for each crowding scenario. The ultimate goal of these models was not  
180 prediction, but to provide a descriptive framework<sup>44</sup> showing which factors, interactions, and  
181 potential non-linearities were important. Variations on both multiple linear regression models (MLR)  
182 and Generalized Additive Models (GAMs)<sup>45</sup> were fitted to either the log-likelihood of  $p_{thrive}$  (for  
183 MLRs) or directly to  $p_{thrive}$  (GAMs).

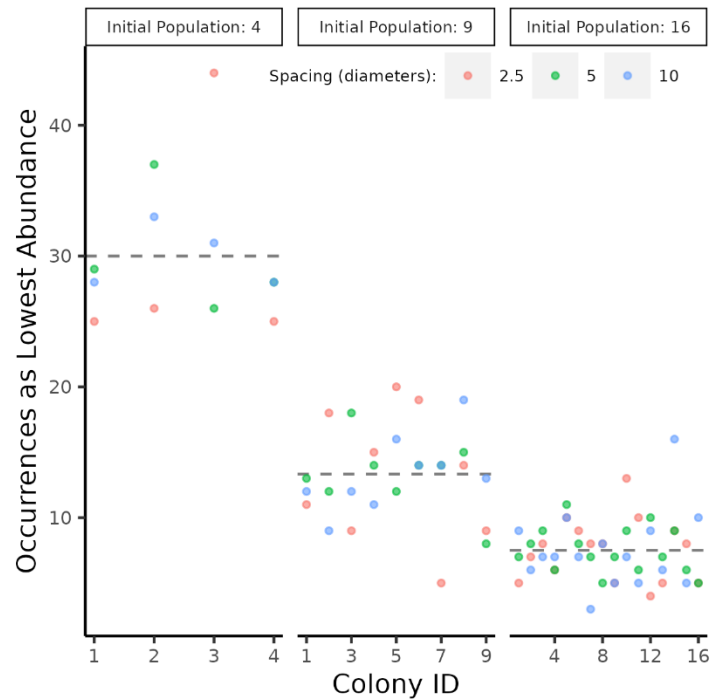
184 In both cases, backward step selection from factorial models incorporating up to three-way  
185 interactions was performed to select the final model. Non-significant ( $p > 0.05$ ) terms were  
186 iteratively removed from the model starting with the highest order interactions. Main effects were  
187 retained even if non-significant when they were part of a significant interaction term.

188 The final models were selected based on  $R^2$  and Akaike Information Criterion (AIC) values as well  
189 as interpretability. The potential models and the associated fit criteria are included in Supporting  
190 Information Tables S2-S5.

## 191 3 Results

### 192 3.1 Drift Occurred When All Cells Were Identical

193 A foundational assumption of this approach is that even in a system with equally spaced, identical  
194 microbes, random growth will lead to drift. We tested this assumption for crowding scenarios where  
195 all microbes had identical base  $K_s$  and  $\mu_{max}$  parameters by determining the number of times each  
196 lineage was the ‘biggest loser’ over 120 simulations (**Figure 3**) and, similar to testing  $m$  dice for  
197 fairness, applied a Chi-Square test ( $\alpha=0.05/m$ ) where  $m$  is a Bonferroni correction for multiple testing  
198 ( $m=9$  at 3x3 initial spacings and population sizes). Each initial site was statistically as likely as any  
199 other to be the biggest loser (Supporting Information Table S6).



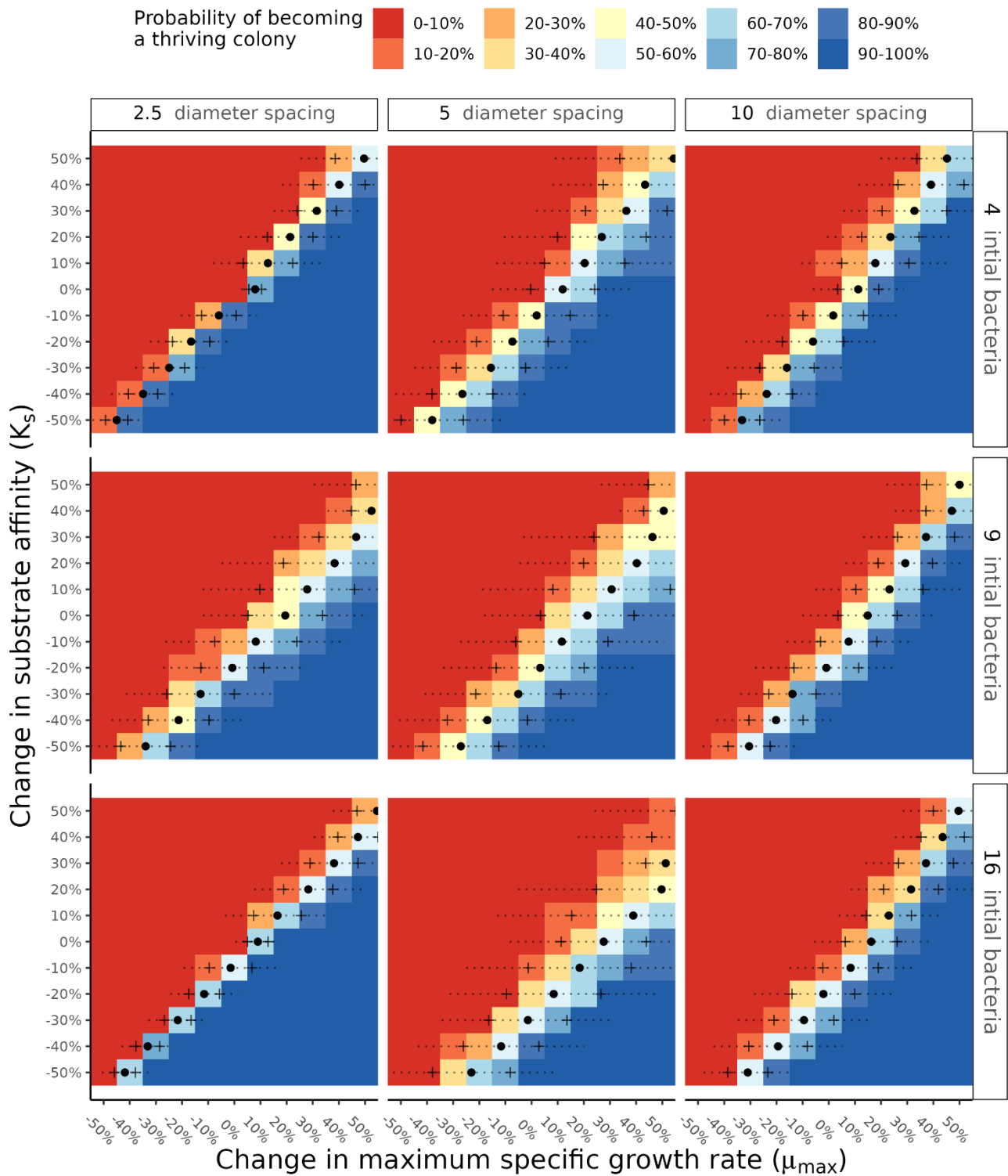
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203

**Figure 3:** The number of times each colony was the least successful performer during all 120 runs of the baseline simulation where all bacteria were identical. Dashed grey lines indicate the expected value. Points are colored based on spacings between initial sites. For each set of initial populations, no colony appeared biased away from the expected number of failures.

204 Additionally, the relative proportion of lineages which languished, survived, or thrived for each set  
205 of crowding conditions was determined. Simulations, on average, had between one and two thriving  
206 lineages, with the rest languishing (65-75% for 4 initial sites, 80-88% others), and a few (0-5%)  
207 which did not thrive but grew to non-negligible abundance (Supporting Information Table S1). When  
208 4 organisms were initially present, only languishing and thriving lineages existed, there was  
209 otherwise no clear trend between these ratios and either the number or spacing of initial bacteria.

### 210 3.2 The Least Successful Lineages Could Overcome Drift with Altered Kinetics

211 As expected, improving the relative fitness of an organism gave it a chance to overcome negative  
212 selection via drift (**Figure 4**).



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**Figure 4:** Changing the  $\mu_{max}$  and  $K_s$  of the least successful lineage was associated with a probability of transitioning to a thriving status. Solid dots represent  $\mu_{50}$ , the percent change in  $\mu_{max}$  at a given  $K_s$  associated with 50-50 odds of thriving. Dashed lines show the range of  $\mu_{max}$  corresponding to a  $p_{thrive}$  of 2.5 to 97.5 (i.e.,  $spread_{95}$ ). Crosses indicate the analogous  $spread_{68}$  region.

217 The increases in  $\mu_{max}$  corresponding to the least successful lineage having a 50% chance to become  
218 thriving, which we denote as  $\mu_{50}$ , are represented by the dark circles in **Figure 4**. At the baseline  $K_s$  a  
219 typical  $\mu_{50}$  is in the range of 10-30%, with the exact value affected by initial spacing and population

220 size (*i.e.*, crowding). Decreasing  $K_s$ , as expected, reduces  $\mu_{50}$  – even to the point where so long as  
221 substrate uptake affinities are ‘good enough’, the initially failing organism may have excellent odds  
222 despite having a  $\mu_{max}$  notably lower than its peers. The overall effect, for a given crowding  
223 condition, is a semi-linear ‘cliff’ of  $\mu_{50}$  values where  $\mu_{50}$  changes inversely with  $K_s$ . Qualitatively  
224 speaking, the location of that ‘cliff’ was shifted to the right (higher  $\mu_{50}$ ) when crowding was  
225 increased either via initial population size.

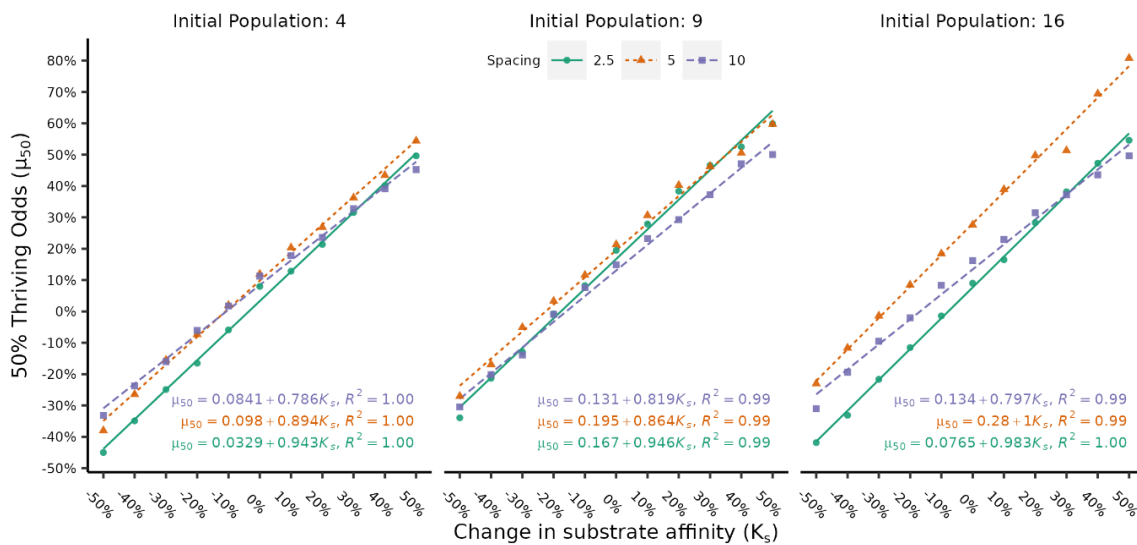
226 Areas where the probability of thriving is neither 0 (drift dominated) nor 1 (fitness dominated), are,  
227 by definition, areas where drift and fitness both determine success. The widths of these areas are  
228 denoted as *spread* and are indicated by the dotted horizontal lines and crosses in **Figure 4**. The full  
229 length of the line denotes the *spread*<sub>95</sub> area, which is the range of  $\mu_{max}$  for a given  $K_s$  which  
230 corresponds to a 2.5% to 97.5% chance of thriving. The crosses represent a similar range, *spread*<sub>68</sub>,  
231 which corresponds to a 16% to 85% chance of thriving.

232 Because the  $\mu_{50}$  values are also the centre point of the *spread* regions, *spread* shifted in the same  
233 manner as  $\mu_{50}$ . However, the actual magnitudes of *spread* did not necessarily follow the same  
234 patterns. First, there was no guaranteed symmetry about  $K_s$ . For example, for 9 initial organisms  
235 separated by 5 diameters, the *spread*<sub>95</sub> for  $K_s$  of -30% and 30% are visibly different (**Figure 4**, row 2  
236 column 2). Though the asymmetry varied between crowding conditions, it generally manifested as  
237 *spread* widening with increasing  $K_s$ . Second, there was no clear monotonic trend with *spread* values  
238 corresponding to crowding. A spacing of 5 diameters appeared to produce the widest *spreads*,  
239 *ceteris paribus*. Further, there was no clear rule determining which of the two spacing extremes  
240 would have a larger *spread*. For example, with 4 initial bacteria a spacing of 10 diameters resulted in  
241 larger *spreads* than in 2.5 diameters, but the opposite occurred with 16 initial bacteria.

### 242 3.3 Quantitative Effect of Crowding on $\mu_{50}$ and *spread*

243 The qualitative effects of crowding described in the previous section were quantified via simple  
 244 linear regression as described in section 2.7.2.

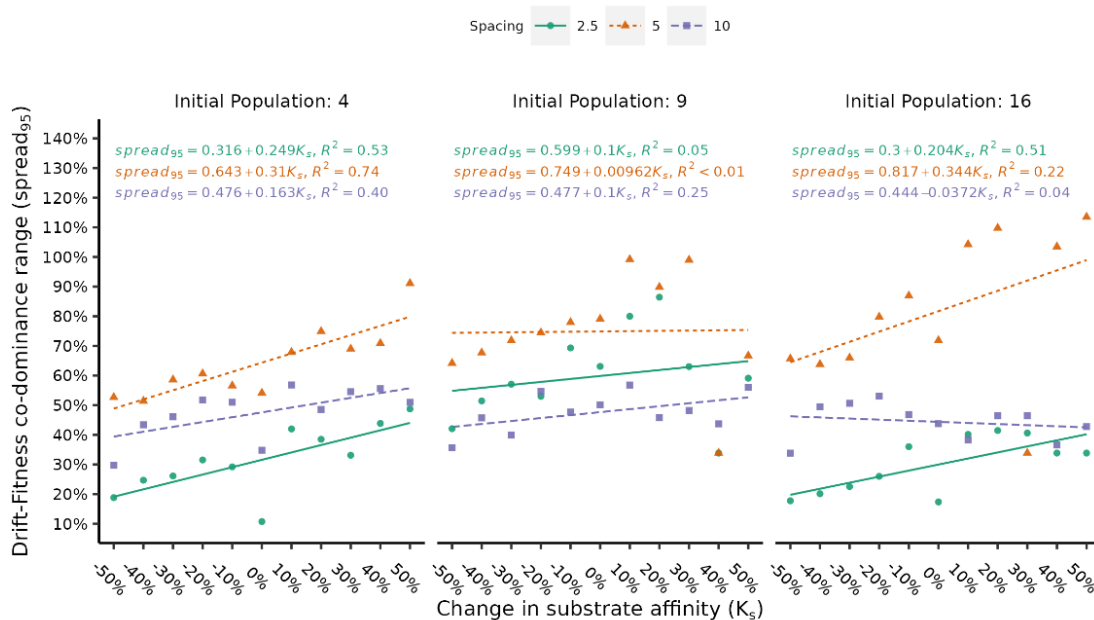
245 For any given crowding condition  $\mu_{50}$ , the relative change of  $\mu_{max}$  at which the worst performing  
 246 lineage had a 50% chance to transition towards thriving, was essentially linear with respect to  $K_S$  and  
 247 the correlation coefficient was uniformly high (**Figure 5**). The slopes of these relationships indicate  
 248 the change in  $\mu_{50}$  required to compensate for a change in  $K_S$ . At the tightest spacing,  $\mu_{50}$  had to  
 249 change the most, with a ratio of essentially 1:1 and a slight monotonic increase corresponding to  
 250 initial population size. As initial spacings widened, the ratio almost always decreased for any initial  
 251 population size. Across initial population sizes, the ratio for 5 and 10 diameter spacings appeared to  
 252 follow a general trend of increasing, but this was not monotonic.



253 **Figure 5:** Under each crowding condition,  $\mu_{50}$  changed linearly with  $K_S$ . Large initial population sizes increased the differences  
 254 between spacings, moderate spacings generally required the largest absolute  $\mu_{50}$ , but the tightest spacings required the largest change  
 255  $\mu_{50}$  in per unit change in  $K_S$ .  
 256

257 The absolute value of  $\mu_{50}$  was strongly affected by differences between the fitted intercepts. For  
 258 example, a 2.5 diameter spacing under an initial population size of 16 had a high slope (0.983) but  
 259 also the lowest required  $\mu_{50}$  of all spacings under the same conditions until a 30% change in  $K_S$ . The  
 260 practical difference between spacing was largest at high initial population size, indicating a potential  
 261 interaction between these factors.

262 Unlike  $\mu_{50}$ , the range over which both drift and fitness effects co-dominated,  $spread_{95}$  did not have a  
 263 simple linear relationship with  $K_s$ , with many poor  $R^2$  values, residual patterns, and high leverage  
 264 datapoints (**Figure 6**). There was also no clear, consistent relationship applicable across factors. In  
 265 general, linear fits became worse with increasing population size which appeared to produce higher  
 266 variance and generated more high-leverage points, especially at separation distances of 5 diameters.  
 267 These issues were largely the same when the analysis was repeated for  $spread_{68}$  (Supporting  
 268 Information Figure S13). There is little to concretely say except that the  $spread$  was most often  
 269 widest at moderate spacings, generally increased with  $K_s$ , and had a noisy, complicated relationship  
 270 with initial population size and spacing.



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**Figure 6:** Under each crowding condition,  $spread_{95}$  changed with  $K_s$ . Insofar as trends were present, moderate spacing produced the widest  $spread_{95}$  and the differences between spacings increased with population size.

### 274 3.4 Description via Multiple Linear Regression and Generalized Additive Models

275 The simulation results were modelled using both multiple linear regression (MLR) and a generalized  
 276 additive model (GAM) respectively described by equations (5) and (6) where:  $p_{thrive}$  is the probability  
 277 of transitioning to a thriving status,  $\mu_p$  and  $K_p$  are the respective percent changes from the baseline  
 278  $\mu_{max}$  and  $K_s$ ,  $N_0$  is the initial population size,  $s_i$  is the initial spacing (in diameters) between  
 279 organisms, and  $\epsilon$  is a small pseudo-probability ( $1 \times 10^{-6}$ ) added to avoid division by 0 and issues with

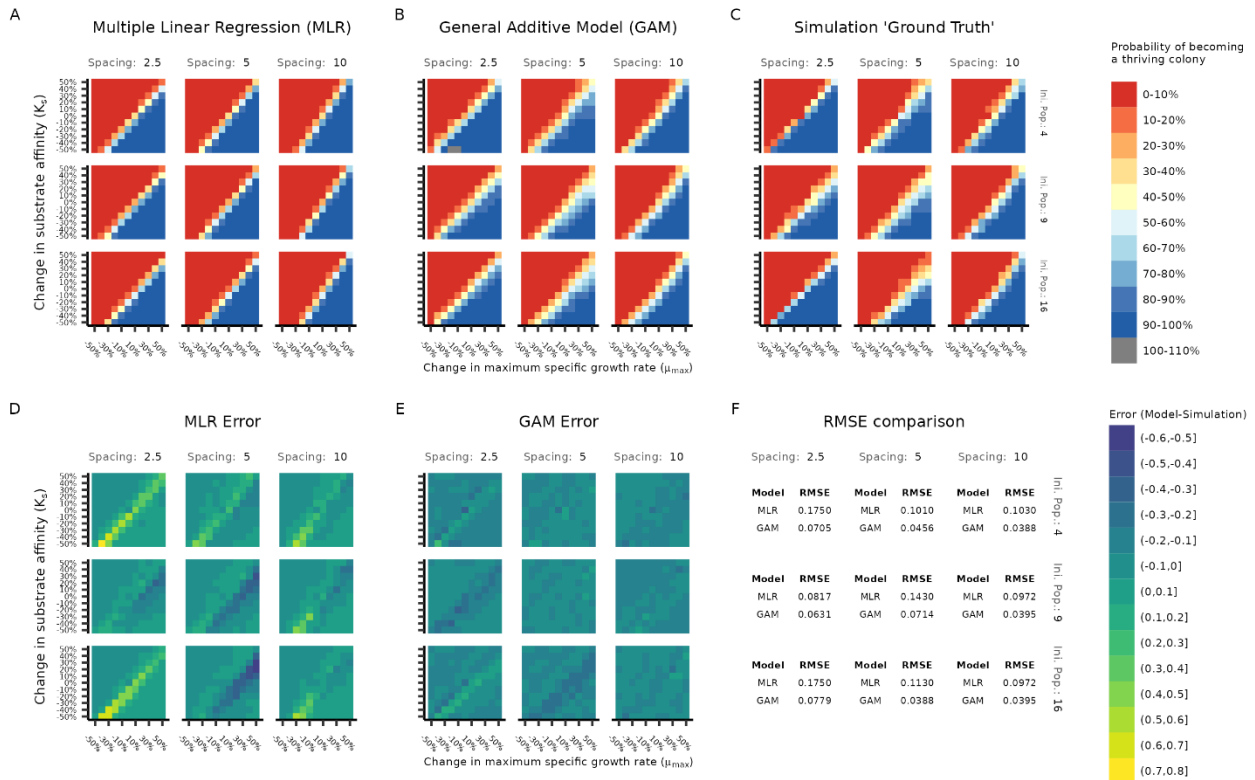
280 log transformation. For linear terms in equations (5) and (6),  $\beta_i$  denotes the fitted coefficient for term  
281  $i$  with  $i=0$  representing the intercept. Terms to which GAM smoothing was applied are represented  
282 by  $s(\dots)$  in equation (6) with interactions between a smoothed variable  $x$  and linear variable  $y$   
283 denoted as  $s(x, \text{by } y)$ . Significant terms ( $p < 0.05$ ) are highlighted in bold. The associated  
284 coefficients, significance values, and other relevant fitting information are included in Supporting  
285 Information Tables S2-S5.

$$\log\left(\frac{p_{thrive}}{1 - p_{thrive} + \varepsilon} + \varepsilon\right) = \beta_0 + \beta_1\mu_p + \beta_2\mathbf{K}_p + \beta_3\mathbf{N}_0 + \beta_4s_i + \beta_5\mu_p s_i + \beta_6\mathbf{K}_p s_i \quad (5)$$

$$p_{thrive} = \beta_0 + s(\mu_p) + s(\mathbf{K}_p) + \mathbf{s}(\mathbf{N}_0) + s(s_i) + s(\mu_p \mathbf{K}_p) + s(\mu_p s_i) + s(\mathbf{K}_p s_i) \\ + s(\mathbf{N}_0, \text{by } s_i) + s(\mu_p \mathbf{K}_p \mathbf{N}_0) + s(\mu_p \mathbf{K}_p s_i) \quad (6)$$

286 The MLR model captured the general behaviour of the shift in the boundary between low and high  
287 thriving probabilities but did not adequately reproduce changes in *spread* (**Figure 7 A vs. C**). The  
288 overall root-mean-squared error (RMSE) of the model was 0.125. While most predicted probabilities  
289 differed from the simulation by no more than  $\pm 0.1$ , some predictions were subject to large error  
290 (**Figure 7 A, D, F** and Supporting Information Figures S11 and S14-S15). The largest errors  
291 unsurprisingly appear closest to the boundary between low and high  $p_{thrive}$  regions with the MLR  
292 model over-optimistic at the extremes of spacing and lower initial population size. Conversely, the  
293 model tended towards overly pessimistic at moderate spacing.





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**Figure 7:** Predictions of MLR model (A) and GAM (B). Simulation results in (C) are presented for ease of comparison. The model errors for the MLR (D) and GAM (E) are presented visually as well as quantified per-crowding condition in (F). The GAM outperformed the MLR, which particularly failed to capture *spread*, was overly optimistic at spacing extremes, and pessimistic at moderate spacing. The small region of greater than 100% odds occurred because the GAM was not constrained to predicting values in the range of [0,1]. Larger individual plots of panels A, B, D, and E are available in Supporting Information figures S14-S17.

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In comparison to the MLR model, the GAM not only captured the general boundary shift but also the

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changes in *spread* (**Figure 7 B vs. C** in contrast to **A vs. C**). The overall RMSE of the GAM was

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0.0563, or somewhat better than half the RMSE of the MLR model. As with the MLR model, most

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predicted probabilities differed from the simulation by no more than  $\pm 0.1$ . Unlike the MLR model,

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there were fewer exceptionally large errors and those which did occur were of smaller magnitude

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(**Figure 7 B, E, F** and Figures S12 and S16-S17). The GAM followed the same trends in over- and

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under-prediction as the MLR.

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## 4 Discussion

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### 4.1 Crowding Affects the Balance Between Drift and Need for Fitness

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The two parameters describing the balance between drift and fitness,  $\mu_{50}$  and *spread*, were both

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affected as crowding became more intense due to either decreased initial spacing or increased initial

311 population size. It was originally expected that as crowding intensity increased, greater fitness would  
312 be required ( $\mu_{50}$ ) along with a decrease in the range of values over which both drift and fitness co-  
313 dominate (*spread*). That was not the case.

314 Instead, the largest *spread* values predominately occurred at moderate (5 diameter) initial spacing.  
315 We suggest the cause is physical competition for space, specifically the practical significance of  
316 single ‘bad’ random choices in division direction and biomass allocation. When bunched tightly  
317 together, competition for space is intense and even a few poor random events can consign a lineage  
318 to languishing despite a moderate growth advantage. At the other extreme, spatial competition is  
319 lessened sufficiently that a few missteps do not guarantee ruin, allowing a lineage to take the full  
320 benefit of any growth advantage. Meanwhile, at moderate spacing, immediate neighbours are close  
321 enough so that poor random events are harmful but not necessarily disastrous and, at the same time,  
322 growth advantages are somewhat hindered, but still helpful. Remembering that *spread* quantifies the  
323 region where neither fitness nor drift dominate, it then makes sense that we observed the largest  
324 *spread* values at moderate spacing.

325 The 50-50 odds point,  $\mu_{50}$ , was also slightly larger at moderate spacings, although not consistently  
326 and the effect size was not practically different except at large population sizes. The underlying basis  
327 for why is not entirely clear, numerically it was due to the consistently larger intercept (Figure 5).  
328 The trend of the slopes is, however, more easily explained and we attribute it to competition for  
329 substrate. For any initial population size, smaller spacings resulted in higher slopes. In other words,  
330 to maintain the 50-50 odds when  $K_s$  was poor,  $\mu_{50}$  had to change more at closer spacing. This makes  
331 intuitive sense – closer spacings result in lower local substrate concentrations, and any deficit to  $K_s$  is  
332 more deleterious to fitness.

333 Increased initial population sizes had more straightforward, secondary, effects on  $\mu_{50}$  and  $K_s$ . As the  
334 initial population size increased, the differences between spacings became more pronounced, but the

335 general trends remained unchanged. In other words, more competitors are problematic, especially as  
336 it relates to diffusible substrate, but the major influence on success is competition for space between  
337 immediate neighbours.

#### 338 4.2 Interactions Between Factors Incorporating Non-Linear Effects are Important

339 In the MLR a main-effects only model (RMSE 0.125,  $R^2$  of 0.820) performed essentially identically  
340 to the MLR model with interactions (RMSE 0.127, and  $R^2$  of 0.820), however neither adequately  
341 reproduced simulation results and were especially poor at representing the regions where both fitness  
342 and drift co-dominated. A GAM which incorporated only main effects using non-linear smoothing  
343 quantitatively performed slightly worse than either MLR main-effects model (RMSE 0.197 and  $R^2$  of  
344 78.1), but drastically and uniformly overpredicted spread. Only when both interactions and  
345 smoothing were incorporated did a model adequately reproduce the simulation results (**Figure 7** and  
346 Supporting Information Figure S17). It is visually apparent in the simulation results and quantified in  
347 the fitting results (Supporting Information Table S4-5) that interactions are important, particularly  
348 those involving spacing. Further, the non-linearity of the interactions (measured as the departure of  
349 the term's extended degrees of freedom from a value of 1), is particularly high for any interaction  
350 incorporating both  $\mu_p$  and  $K_p$  and less so but still notably for interactions incorporating spacing  
351 (Supporting Information Table S5).

#### 352 4.3 Limitations and Extensions

353 The simulated conditions were deliberately chosen to isolate the effect of drift. While this made the  
354 work tractable, a system wherein every organism is completely identical, starts growing at the same  
355 time, and is initially evenly spaced on a grid does not frequently occur in nature. Although we  
356 believe the general themes uncovered translate to real ecological systems, the exact quantification  
357 does not and is not meant to apply to all situations. Future work should focus on stochastically placed  
358 (in time and space) populations with natural variability in Monod parameters.

359 Extending the work so that the simulated community reflects a more natural distribution would also  
360 enable validation of the model, as, despite promising advances,<sup>46</sup> it is currently infeasible to exactly  
361 place essentially identical bacteria at the resolution required.

362 Additional parameters affecting drift and fitness should also be evaluated – especially the influence  
363 of nutrient-rich conditions<sup>47</sup> and how a change to yield, rather than growth rate, alters success.<sup>48</sup>  
364 Adding these factors requires however overcoming the curse of dimensionality, the current  
365 simulations took over 1 year of real-world time and 175 years' worth of CPU time. Given the large  
366 areas where 'nothing interesting' happens, designing further experiments to incorporate adaptive  
367 sampling<sup>49</sup> is a promising solution. Further, adaptive sampling would enable, at the same  
368 computational cost, exploring a larger range of  $\mu_{max}$  and  $K_S$  variation (which may vary by orders of  
369 magnitude in real-world conditions<sup>50</sup>) and at a greater degree of resolution than 10% changes in the  
370 region where the probabilities rapidly change.

## 371 5 Conclusion And Relevance to Real World Systems

372 It is apparent that during biofilm formation in low nutrient conditions, drift strongly determines  
373 which organisms thrive and which organisms fail, so long as they have similar growth rates and  
374 substrate affinities. Even when those parameters differ between individuals by  $\pm 50\%$ , there are still  
375 large regions where a fitness advantage does not guarantee overcoming negative drift selection.

376 In fact, we observed the lineage fates were determined very early in the simulations and for these  
377 systems 'well-begun is half done'. We speculate that this may be a piece to the puzzle explaining the  
378 apparent contradiction between actual and effective community size in neutral modelling<sup>4</sup> – the  
379 bacteria are not in competition with the full steady-state community but only the immediate smaller,  
380 community near the beginning of biofilm growth. However, the conditions studied here violate the  
381 steady state assumption of that work, so a more careful analysis is warranted.

382 The conditions we have described are not dissimilar from those within an aerated portion of a  
383 wastewater treatment plant, where tightly packed bacterial aggregates are suspended in a bulk liquid  
384 and where substrate concentrations are often quite low, especially during operation as a completely  
385 mixed stirred reactor (albeit somewhat higher than simulated here). Further, these bacteria are  
386 recirculated through the system and relatively well-adapted to domestic wastewater, thus already  
387 selected for similarity. Based on the results presented here, we would expect to see a system in which  
388 there is a high degree of random turnover in organism identity, but relatively stable functional and  
389 biological activity, which is exactly what has been observed in wastewater treatment plants.<sup>51,52</sup>

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395 understanding the NUFEB software, and countless helpful comments from many interested parties  
396 during poster sessions and talks.

## 397 7 Competing Interests

398 The author has no competing interests.

## 399 8 Data Availability Statement

400 The data analysis code, data from the simulations, and exact NUFEB variant are respectively located  
401 in the following repositories:

402

403 Analysis: [https://github.com/joeweaver/agent\\_based\\_biofilm\\_drift/](https://github.com/joeweaver/agent_based_biofilm_drift/)

404 Data: <https://osf.io/fch3z/>

405 NUFEB variant: [https://github.com/nufeb/NUFEB-dev/tree/compute\\_vol\\_group](https://github.com/nufeb/NUFEB-dev/tree/compute_vol_group)

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