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Deep learning approaches to viral phylogeography are fast and as robust as likelihood methods to model misspecification

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Abstract.— Analysis of phylogenetic trees has become an essential tool in epidemiology. 12 Likelihood-based methods fit models to phylogenies to draw inferences about the 13 phylodynamics and history of viral transmission. However, these methods are 14 computationally expensive, which limits the complexity and realism of phylodynamic 15 models and makes them ill-suited for informing policy decisions in real-time during rapidly 16 developing outbreaks. Likelihood-free methods using deep learning are pushing the 17 boundaries of inference beyond these constraints. In this paper, we extend, compare and 18 contrast a recently developed deep learning method for likelihood-free inference from trees. 19 We trained multiple deep neural networks using phylogenies from simulated outbreaks that 20 spread among five locations and found they achieve similar levels of accuracy to Bayesian 21 inference under the true simulation model. We compared robustness to model 22 misspecification of a trained neural network to that of a Bayesian method. We found that 23 both models had comparable performance, converging on similar biases. We also trained 24 and tested a neural network against phylogeographic data from a recent study of the 25 SARS-Cov-2 pandemic in Europe and obtained similar estimates of epidemiological 26 parameters and the location of the common ancestor in Europe. Along with being as 27 accurate and robust as likelihood-based methods, our trained neural networks are on 28 average over 3 orders of magnitude faster. Our results support the notion that neural 29 networks can be trained with simulated data to accurately mimic the good and bad 30 statistical properties of the likelihood functions of generative phylogenetic models. 31 (Keywords: phylogeography, SSE, phylodynamics, machine learning, deep learning, 32

³³ epidemiology)

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INTRODUCTION

Viral phylodynamic models use genomes sampled from infected individuals to trace the 35 evolutionary history of a pathogen and its spread through a population (Holmes and 36 Garnett 1994; Volz et al. 2013). By linking genetic information to epidemiological data, 37 such as the location and time of sampling, these generative models can provide valuable 38 insights into the transmission dynamics of infectious diseases, especially in the early stages 39 of cryptic disease spread when it is more difficult to detect and track (Holmes et al. 1995; 40 Rambaut et al. 2008; Lemey et al. 2009; Pybus et al. 2012; Worobey et al. 2016, 2020; 41 Lemey et al. 2021; Washington et al. 2021; Pekar et al. 2022). This information can be 42 used to inform public health interventions and improve our understanding of the evolution 43 and spread of pathogens. Many phylodynamic models are adapted from state-dependent 44 birth-death (SDBD) processes or, equivalently, state-dependent speciation-extinction (SSE) 45 models (Maddison et al. 2007; FitzJohn 2012; Kühnert et al. 2014; Beaulieu and O'Meara 46 2016). Here, we will refer to the state as location and the models as location-dependent 47 birth-death (LDBDS) models which include serial sampling (Kühnert et al. 2016). 48

Epidemiologists are increasingly using LDBDS models to estimate transmission 49 rates, migration rates between locations, and variation in these rates amongst populations 50 (Nadeau et al. 2021) and species (Lu et al. 2021). Analysts fit data to these models with 51 likelihood-based inference methods, such as maximum likelihood (Maddison et al. 2007; 52 Richter et al. 2020) or Bayesian Markov chain Monte Carlo (Kühnert et al. 2016; Scire 53 et al. 2020). Likelihood-based inference relies upon a likelihood function to evaluate the 54 relative probability (likelihood) that a given phylogenetic pattern (i.e., topology, branch 55 lengths, and tip locations) was generated by a phylodynamic process with particular model 56 parameter values. In this sense the likelihood of any possible phylodynamic data set is 57 mathematically encoded into the likelihood as a function of (unknown) data-generating 58 model parameters. 59

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Computing the likelihood requires high-dimensional integration over a large and

complex space of evolutionary histories. Analytically integrated likelihood functions, 61 however, are not known for LDBDS models. Methods developers instead use ordinary 62 differential equation (ODE) solvers (Maddison et al. 2007; Kühnert et al. 2016) or data 63 augmentation (DA) methods (Maliet et al. 2019) to numerically approximate the 64 integrated likelihood. These clever approximations perform well statistically, but are too 65 computationally expensive to use with large epidemic-scale data sets. Thus, while 66 Nextstrain (Hadfield et al. 2018) and similar efforts provided useful visualizations to policy 67 makers during the COVID response, most phylogeographical methods are used forensically, 68 providing insight on the past, and are not used to provide parameter estimates in response 69 to emerging events to inform policy decisions in real-time due to the complexity and long 70 run-times of these models. 71

As phylodynamic models become more biologically realistic, they will necessarily grow more mathematically complex, and therefore less able to yield likelihood functions that can be approximated using ODE or DA methods. Because of this, phylodynamic model developers tend to explore only models for which a likelihood-based inference strategy is readily available. As a consequence, this impedes the design, study, and application of richer phylodynamic models of disease transmission.

To avoid the computational limitations associated with likelihood-based methods, 78 deep learning inference methods that are likelihood-free have emerged as a complementary 79 framework for fitting a wide variety of evolutionary models (Bokma 2006). Deep learning 80 methods rely on training many-layered neural networks to extract information from data 81 patterns. These neural networks can be trained with simulated data as another way to 82 approximate the latent likelihood function (Cranmer et al. 2020). Once trained, neural 83 networks have the benefit of being fast, easy to use, and scalable. Recently, likelihood-free 84 deep learning neural network methods have successfully been applied to phylogenetics 85 (Suvorov et al. 2020; Suvorov and Schrider 2022b; Nesterenko et al. 2022; Solis-Lemus 86 et al. 2022; da Fonseca et al. 2020), and phylodynamic inference (Voznica et al. 2022; 87

 $_{88}$ Lambert et al. 2022).

Here we extend new methods of deep learning from phylogenetic trees (Voznica 89 et al. 2021; Lambert et al. 2022) to explore their potential when applied to phylogeographic 90 problems in geospatial epidemiology. Phylodynamics of birth-death-sampling processes 91 that include migration among locations have been under development for more than a 92 decade (Stadler 2010; Stadler et al. 2012; Kühnert et al. 2014, 2016; Scire et al. 2020; Gao 93 et al. 2021, 2022). Given the added complexity of location specific dynamics (e.g. location 94 specific birth rates) and recent successes in deep learning with phylogenetic time trees 95 (Voznica et al. 2022) under state-dependent diversification models (Lambert et al. 2022), we 96 sought to evaluate this approach when applied to viral phylodynamics and phylogeography 97 by including location data when training deep neural networks with phylogenetic trees. 98

One important limitation of likelihood-free approaches is that it is unknown how 99 brittle the inference machinery is when the assumptions used for simulation and training 100 are violated (Schmitt et al. 2022). For example, a brittle deep learning method would be 101 more easily mislead by model misspecification when compared to a likelihood-based 102 method. Likelihood approaches may have some advantages because the simplifying 103 assumptions are explicit in the likelihood function while for trained neural networks it is 104 difficult to know how those assumptions are encoded for any given implementation. 105 However, with complex likelihood models, there may be unexpected interactions among 106 simplifying assumptions that result in large biases when applied to real-world data. 107 Characterizing the relative robustness and brittleness of these two inference paradigms is 108 essential for those who wish to confidently develop and deploy likelihood-free methods of 109 inference from real world data. 110

To explore relative robustness to model misspecification, we trained multiple deep convolutional neural networks (CNNs) with transmission trees generated from epidemic simulations. We show that simulation-trained CNNs are not only as accurate as likelihood-based approaches but are no more sensitive to model violations than the

likelihood approach. Both methods consistently show similar biases induced by model 115 violations in test data sets. We find that for the models tested here, the migration rate 116 estimates are highly sensitive to misspecification of infection rate and sampling rates, but 117 that estimates of the infection and sampling rates are fairly robust to misspecification of 118 the migration models. We also show that the rate parameter estimates are fairly robust to 119 misspecification of both the number of locations in the model and phylogenetic error. 120 Finally, we compared a simulation-trained neural network to a recent phylodynamic study 121 of the first wave of the COVID pandemic in Europe (Nadeau et al. 2021) and obtain 122 similar inferences about the dynamics and history of SARS-CoV-2 in the European clade. 123

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Methods

LDBDS processes are stochastic branching processes that define location-dependent rates 125 for birth, death, migration, and sampling events to randomly generate time-scaled 126 phylogenies where taxa are associated with various locations. With serial sampling, many 127 chains of the transmission tree go undetected. Consequently, in phylogeography an absence 128 of evidence is not evidence of absence in time and space. This fact requires simulation of 129 not just the sampled/observed phylogenetic tree but the evolution of the underlying 130 population from which it is sampled. This underlying population is divided into 131 compartments of Susceptible individuals, infectious individuals, and 132 recovered/non-susceptable individuals. The dynamics of these compartments are describe 133 by the Susceptible-Infectious-Recovered (SIR) compartmental model. 134 First, we define the SIR model we assume here that is approximately equivalent to 135

the LDBDS model (Kühnert et al. 2016). Following that, is a description of the simulation method to generate the training, validation, and test data sets of phylogenies under the model. We next describe our implementation of simulation-trained deep learning inference with convolutional neural networks (CNN) as well as a likelihood-based method using Bayesian inference. We then describe our methods for measuring and comparing their

performance when tested against data sets generated by simulations under the inference 141 model as well as several data sets simulated under models that violate assumptions of the 142 inference model. Finally, we describe how we tested our simulation-trained CNN against a 143 real-world data set. 144

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Model definition

We first define a general location-dependent SIR stochastic process used for simulations 146 and likelihood function derivation in the format of reaction equations we specified in 147 MASTER (Vaughan and Drummond 2013). Reaction equations 1 through 4 specify the 148 SIR compartment model with migration and serial sampling where S, I, and R denote the 149 number of individuals in each compartment. The S and I compartments are indexed by 150 geographic location using i and j. N_i is the total population size in location i and 151 $N_i = S_i + I_i + R_i$. The symbols for each rate parameter is placed above each reaction arrow. 152

$$S_i + I_i \xrightarrow{\beta_i/N_i} 2I_i$$
 infection (1)

$$I_i \xrightarrow{m_{ij}} I_j$$
 migration (2)

$$I_i \xrightarrow{\delta_i} R$$
 recovery (3)
 $I_i \xrightarrow{\delta_i} R$ sample and recovery. (4)

sample and recovery.
$$(4)$$

We parameterize the model with the basic reproduction number in location i, R_{0_i} , 153 which is related to β_i and δ_i by equation 5, 154

$$R_{0_i} = \frac{\beta_i}{\gamma + \delta_i}.\tag{5}$$

In particular, our study considers a location-independent SIR (LI-SIR) model with 155 sampling that assumes R_{0_i} was equal among all locations, and a location-dependent 156

(LD-SIR) model with sampling that assumes R_{0_i} varied among locations. During the 157 exponential growth phase of an outbreak, the LI-SIR and LD-SIR models are equivalent to 158 the location-independent birth-death-sampling (LIBDS) and location-dependent 159 birth-death-sampling (LDBDS) models, respectively, that are often used in viral 160 phylogeography (Kühnert et al. 2014, 2016; Douglas et al. 2021). 161 Each infectious individual transitions to recovered at rate γ . We assumed that 162 sampling a virus in an individual occurs at rate δ_i in location i and immediately removes 163 that individual from the infectious compartment and places them in the recovered 164 compartment. Thus the effective recovery rate in location i is $\gamma + \delta_i$. The above reactions 165 correspond to the following coupled ordinary differential equations. 166

$$\frac{dS_i}{dt} = -\frac{\beta_i}{N_i} S_i I_i$$

$$\frac{dI_i}{dt} = \frac{\beta_i}{N_i} S_i I_i + \sum_{j \neq i}^n m_{ij} I_j - \sum_{j \neq i}^n m_{ji} I_i - (\gamma + \delta_i) I_i$$

$$\frac{dR}{dt} = \gamma \sum_{i=1}^n \delta_i I_i$$
(6)

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When the migration rate is constant among locations and the model is a location-independent SIR model, or equivalently, LIBDS, equation set 6 reduces to 168

$$\begin{aligned} \frac{dS_i}{dt} &= -\frac{\beta}{N_i} S_i I_i \\ \frac{dI_i}{dt} &= \frac{\beta}{N_i} S_i I_i + m \left(\sum_{j \neq i}^n I_j - (n-1) I_i \right) - (\gamma + \delta) I_i \\ \frac{dR}{dt} &= (\gamma + \delta) \sum_{i=1}^n I_i \end{aligned}$$

The number of infections and the migration of susceptible individuals is at 169 negligible levels on the timescales investigated here. The infection rate is, therefore, 170

approximately constant and the migration of susceptible individuals can be safely ignored
requiring only migration of infectious individuals to be simulated.

At the beginning of an outbreak, it is often easier to know the recovery period from clinical data than the sampling rate which requires knowing the prevalence of the disease. Therefore, we treat the average recovery period as a known quantity and use it to make the other two parameters (the sampling rate and the basic reproduction number R_0) identifiable. This was done by fixing the corresponding rate parameter in the likelihood function to the true simulated value for each tree, and by adding the true simulated value to the training data for training the neural network.

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Simulated training and validation data sets

Epidemic simulations of the SIR+migration model that approximates the LIBDS process 181 were performed using the MASTER package v. 6.1.2 (Vaughan et al. 2014) in BEAST 2 v. 182 2.6.6 (Bouckaert et al. 2019). MASTER allows users to simulate phylodynamic data sets 183 under user-specified epidemiological scenarios, for which MASTER simultaneously 184 simulates the evolution of compartment (population type) sizes and tracks the branching 185 lineages (transmission trees in the case of viruses) from which it samples over time. We 186 trained neural networks with these simulated data to learn about latent populations from 187 the shape of sampled and subsampled phylogenies. In addition to the serial sampling 188 process, at the end of the simulation 1% of infected lineages were sampled. In MASTER 189 this was approximated by setting a very high sampling rate and very short sampling time 190 such that the expected number sampled was approximately 1%. This final sampling event 191 was required to make a 1-to-1 comparison of the likelihood function used for this study (see 192 Likelihood method description below) which assumes at least one extant individual was 193 sampled to end the process. Coverage statistics from our MCMC samples closely match 194 expectations (see Likelihood method description below; SI Figure S2). Simulation 195

parameters under LIBDS and LDBDS models for training the neural network under the
phylogeography model were drawn from the following distributions:

$$R_0 \sim \text{Uniform}(2, 8)$$

 $\delta \sim \text{Unif}(0.0001, 0.005)$
 $m \sim \text{Uniform}(0.0001, 0.005)$ (7)
 $\gamma \sim \text{Unif}(0.01, 0.05)$

root location ~ Multinomial(k = 1, $p_i = 1/n$), for n locations

All five locations had initial population sizes of 1,000,000 susceptible individuals 198 and one infected individual in one of the locations. Simulations were run for 100 time units 199 or until 50,000 individuals had been infected to restrict simulations to the approximate 200 exponential phase of the outbreak. For the experiments comparing the CNN to the 201 likelihood-based method under the LIBDS model, if this population threshold was reached 202 the simulation was rejected. This criterion was not enforced for simulations under the 203 LDBDS model. This ensured the LIBDS model used in the likelihood-based analyses are 204 equivalent to more complex density-dependent SIR models. After simulation, trees with 205 500 or more tips were uniformly and randomly downsampled to 499 tips and the sampling 206 proportion was recorded for training the neural networks and to adjust estimates of δ . 207

We simulated 410,000 outbreaks under these LIBDS settings to generate the training, validation, and test sets for deep learning. Any simulation that generated a tree with less than 20 tips was discarded, leaving a total of 111,157 simulated epidemiological data sets. Of these, 104,157 data sets were used to train and 7,000 were used to validate and test each CNN. A total of 193,110 LDBDS data sets were simulated, with 186,110 used to train and 7,000 used to validate and test the LDBDS CNNs.

Training simulation parameters for the LDBDS process used to analyze the real

data set (Nadeau et al. 2021) were drawn from the same distributions as LIBDS except R_0 was unique for each location and was drawn from a hierarchical distribution to narrow the magnitude of differences among locations within simulations to be within 8 of each other but expand the magnitude of differences between simulations to range from 0.9 to 15:

> $\alpha \sim \text{Uniform}(4.9, 11)$ $R_{0_i} \sim \text{Uniform}(\alpha - 4, \alpha + 4)$

For the empirical analysis, population sizes at each location were also set to 500,000 and instead of running the simulations for 100 time units, time was scaled by the recovery period, $1/\gamma$, and was drawn from a uniform distribution:

time ~ Uniform(1, 20)

222 Simulated test data sets with and without model misspecification

We first simulated a test set of 138 trees under the training model to compare the accuracy of the CNN and the likelihood-based estimates when the true model is specified. These data sets were simulated by random draws of parameter values from the same distributions described above for generating the training data set.

Sensitivity to model misspecification for each of the three rate parameters, R_0 , δ , and m, was tested. All sensitivity experiments used the same LIBDS model for inference for both the CNN and the Likelihood-based methods. Sensitivity experiments we conducted by simulating a test data set of trees that were generated by an epidemic process that was more complex than or different from the LIBDS model.

The tree data set for the misspecified R_0 experiment consisted of simulating 232 outbreaks where each location had a unique R_0 drawn from the same distribution as above. 233 Likewise, the misspecified sampling model test set was generated by simulating outbreaks 234 where each location had a unique sampling rate, δ , drawn from the same distribution used 235 for the global sampling rate described above. For the misspecified migration model, a 236 random pair of coordinates, each drawn from a uniform (0.5) distribution in a plane, were 237 generated for the five locations, and a pairwise migration rate was computed such that 238 pairwise migration rates were symmetric and proportional to the inverse of their euclidean 239 distances and the average pairwise migration rate was equal to a random scalar which was 240 also drawn from a uniform distribution (see equations 7 above). 241

The tree set for the misspecified number of locations experiment was generated by simulating outbreaks among ten locations instead of five. After simulations, six locations were chosen at random and re-coded as being sampled from the same location.

To generate a test set where the time tree used for inference has incorrect topology 245 and branch lengths, we implemented a basic pipeline of tree inference from simulated 246 genetic data to mimic a worst case real world scenario. We simulated trees under the same 247 settings as before. Phylogenetic error was introduced in two ways: the amount of site data 248 (short sequences) and misspecification of the DNA sequence evolution inference model (*i.e.*. 240 Using seq-gen V. 1.3.2 (Rambaut and Grassly 1997). We simulated the evolution of a 200 250 base-pair sequence under an HKY model with $\kappa = 2$, equal base frequencies and 4 251 discretized-gamma(2, 2) rate categories for among site rate variation. The simulated 252 alignment as well as the true tip dates (sampling times) was then used to infer test trees. 253 Test tree inference was done using iqtree v. 2.0.6 (Minh et al. 2020) assuming a 254 Jukes-Cantor model of evolution where all transition rates are equal. The inference model 255 also assumed no among-site rate variation. The number of shared branches between the 256 true transmission tree and the test tree inferred by IQ-Tree was measured using gotree v. 257 0.4.2 (Lemoine and Gascuel 2021). Polytomies were resolved using phytools (Revell 2012) 258

and a small, random number was added to each resolved branch. These trees were then
used for likelihood inference and CNN prediction.

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Deep learning inference method

The resulting trees and location metadata generated by our pipeline were converted to a 262 modified cblv format (Voznica et al. 2022), which we refer to as the cblv+S (+State of 263 character, e.g. location) format. The cblv format uses an in-order tree traversal to 264 translate the topology and branch lengths of the tree into an 2 x n matrix where n is the 265 number of tips in the tree. This representation gives each sampled tip a pair of coordinates 266 in 'tree-traversal space'. Our cblv+S format associates geographic information 267 corresponding with each sampled taxon by appending each vector column with a one-hot 268 encoding vector of length g states to yield a $(2+g) \times n$ cblv+S matrix. The cblv+S format 269 allows for multiple characters and/or states to be encoded, extending the single binary 270 character encoding format introduced by Lambert et al. (2022). Our study uses cblv+S to 27 encode a single character with g = 5 location-states. In addition to the the cblv+S data, 272 we also include a few tree summary statistics and known simulating parameters; the mean 273 branch length, the tree height and the recovery rate and the subsampling proportion. Trees 274 were rescaled such that their mean branch length was the default for phylodeep (Voznica 275 et al. 2021) before training and testing of the CNN. The mean pre-scaling branch length 276 and tree heights were also fed into the neural networks. Trees were not rescaled for the 277 likelihood-based analysis. Note that tree height did not vary for the LIBDS CNN training 278 set but did for the LDBDS training set. 270

Our CNNs were implemented in Python 3.8.10 using keras v. 2.6.0 and tensorflow-gpu v. 2.6.0. (Chollet; Abadi et al. 2016). For each model, LIBDS and LDBDS, we designed and trained two CNN architectures, one to predict epidemiological rate parameters and the other to predict the outbreak location resulting in four total CNNs trained by two training data sets (LIBDS and LDBDS). We used the mean-squared-error

for the regression neural loss function in the network trained to estimate epidemiological 285 rates, and the categorical cross-entropy loss function for the categorical network trained to 286 estimate outbreak location. We assessed the performance of the network by randomly 287 selecting 5,000 samples for validation before each round of training. We measured the 288 mean absolute error and accuracy using the validation sets. We used these measures to 289 compare architectures and determine early stopping times to avoid overfitting the model to 290 the training data. We also added more simulations to the training set until we could no 291 longer detect an improvement in error statistics. After comparing the performance of 292 several networks, we found that the CNN described in SI Figure S1 performed the best. In 293 brief, the networks have three parallel sets of sequential convolutional layers for the cblv+S 294 tensor and a parallel dense layer for the priors and tree statistics. The three sets of 295 convolution layers differed by dilation rate and stride lengths. These three segments and 296 the dense layer were concatenated and then fed into a segment consisting of a sequential 297 set of dense layers, each layer gradually narrowing to the output size to either three or five 298 for the rates and origin location networks, respectively, for the LIBDS model, and seven 299 and five for the seven rates and five locations, respectively, for the LDBDS model. 300

All layers of the CNN used rectified linear unit (ReLU) activation functions. We 301 used the Adam optimizer algorithm for batch stochastic gradient descent (Kingma and Ba 302 2017) with batch size of 128 and stopping after 15 epochs for the regression network and 303 ten epochs for the root location network. The output activation for the rates network was 304 linear with three nodes and for the outbreak location network was softmax with five nodes. 305 Otherwise the architecture was the same for all four networks. The LDBDS neural network 306 was trained with simulated trees where R_{0_i} varied among locations had output layer with 307 seven nodes; five for the each location's R_{0_i} and a node each for the sampling rate and the 308 migration rate. We tested networks with max-pooling layers between convolution layers as 309 well as dropout at several rates and found no improvement or a decrease in performance. 310

311

Likelihood-based method of inference

We compared the performance of our trained phylodynamic CNN to likelihood-based 312 Bayesian phylodynamic inferences. We specified LIBDS and LDBDS Bayesian models that 313 were identical to the LIBDS and LDBDS simulation models that we used to train our 314 CNNs. The most general phylodynamic model in the birth-death family applied to 315 epidemiological data is the state-dependent birth-death-sampling process (SDBDS; 316 (Kühnert et al. 2016; Scire et al. 2020)), where the state or type on which birth, death, and 317 sampling parameters are dependent is the location in this context. The basic model used 318 for experiments here is a phylogeographic model that is similar to the serially sampled 319 birth-death process (Stadler 2010) where rates do not depend on location, which we refer 320 to as the LDBDS model. The death rate, μ , is equivalent to the recovery rate, γ , in SIR 321 models. Standard phylogenetic birth-death models assume the birth and death rates, λ and 322 μ , are constant or time-homogeneous, while the SIR model's infection rate is proportional 323 to β and S and varies with time as S changes. However, when the number of infected is 324 small relative to susceptible people, as in the initial stages of an outbreak, the infection 325 rate, β , is approximately constant and approximately equal to the birth rate λ ; 326

$$\lambda = \frac{\beta S}{N} \approx \beta \tag{8}$$

The joint prior distribution was set to the same model parameter distributions that were used to simulate the training and test sets of phylogenetic trees in the first section with γ treated as known and the proportion of extant lineages sampled, ρ , set to 0.01 as in the simulations. The likelihood was conditioned on the tree having extant samples (*i.e.* the simulation ran for the allotted time without being rejected). All simulated trees in this study had a stem branch and the outbreak origins were inferred for the parent node of the stem branch.

We used Markov chain Monte Carlo (MCMC) to simulate random sampling from the posterior distribution implemented in the TensorPhylo package

(https://bitbucket.org/mrmay/tensorphylo/src/master/) in RevBayes (Höhna et al. 2016). 336 After a burnin phase, a single chain was run for 7,500 cycles with 4 proposals per cycle and 337 at least 100 effective sample size (ESS) for all parameters. If the effective sample size (ESS) 338 was less than 100, the MCMC was rerun with a higher number of cycles. We also analyzed 330 the coverage of the 5, 10, 25, 50, 75, 90, and 95% highest posterior density (HPD) intervals 340 to verify that our simulation model and inference model are the same and that the MCMC 341 simulated draws from the true posterior distribution. Bayesian phylogeographic analysis 342 recovered the true simulating parameters (SI Figure S2) at the expected frequencies, thus 343 validating the simulations were working as expected and confirming that the MCMC was 344 accurately simulating draws from the true posterior distribution. 345

346

Quantifying errors and error differences

We measure the absolute percent error (APE) of the predictions from the CNN and the mean posterior estimate (MPE) of the likelihood-based method. The formula for APE of a prediction/estimate, y^{estimate} , of y^{truth} is

$$APE = \left| \frac{y^{\text{estimate}} - y^{\text{truth}}}{y^{\text{truth}}} \right| \times 100$$

The Bayesian alternative to significance testing is to analyze the posterior distribution of parameter value differences between groups. In this framework, the probability that a difference is greater than zero can be easily interpreted. We therefore used Bayesian statistics to infer the median difference in error between the CNN and likelihood-based methods and the increase in median error of each method when analyzing misspecified data compared to when analyzing data simulated under the true inference model.

We used Bayesian inference to quantify population error by performing three sets of 357 analyses: (1) inferred the population median APE under the true model (this will be the 358 reference group for analysis 3), (2) the effect of inference method — CNN or 359 likelihood-based (Bayesian) — on error by inferring the median difference between the 360 CNN estimate and the likelihood-based estimate, (3) the effect of misspecification on error 361 for each parameter by comparing the median error of estimates under misspecified 362 experiments and the reference group defined by analysis 1. See SI Figures S3 - S8 and SI 363 Table S1 for summaries and figures for all analyses for this section. 364

To infer these differences between groups we used the R package BEST (Meredith 365 and Kruschke). BEST assumes the data follow a t-distribution parameterized by a location 366 parameter, μ , a scale parameter, σ , and a shape parameter, ν , which they call the 367 "normality parameter" (*i.e.* if ν is large the distribution is more Normal). Because the 368 posterior distribution does not have a closed form, BEST uses Gibbs sampling to simulate 369 draws from the posterior distribution. 20,000 samples were drawn from the posterior 370 distribution for each BEST analysis. BEST uses automatic posterior predictive checks to 371 indicate that a model adequately describes the data distributions. Posterior predictive 372 checks indicate the BEST model adequately fits each data set analyzed below. 373

Inferring the median APE.— Before inferring differences between groups, we inferred the population median APE for predictions of R_0 , δ , and m from test data simulated under the inference model using the CNN and likelihood-based methods. Histograms of the sampled log-transformed APE appears to be symmetric with heavy tails so we fit the log APE to the BEST model. This implies that the sampled APE scores are drawn from a log-t distribution. The log-t distribution has a mean of ∞ and median of e^{μ} , we therefore focus our inference on estimating posterior intervals for the population median APE from the sampled APE values for each parameter estimated by the CNN method and likelihood-based method which we denote APE^{CNN}, and APE^{Like} respectively. The data

analyzed here and likelihood assumed by BEST is

$$y = APE^{CNN}$$
 or APE^{Like}
log $y \mid \mu, \sigma, \nu \sim t_{\nu}(\mu, \sigma).$

The priors were set to the vague priors that BEST provides by default,

$$\mu \sim \text{Normal(mean(y), sd(y) \times 1000)}$$

$$\sigma \sim \text{Uniform(sd(y)/1000, sd(y) \times 1000)}$$

$$\nu \sim \text{Exponential}(1/29) + 1.$$

95% highest posterior intervals (HPI) for the median APE, $\tilde{\mu}$, was estimated by the following transformation of simulated draws from the posterior distribution

$$\tilde{\mu} = e^{\mu}.$$

In summary, the results we present are 95% HPI from the posterior distributions of the median error, $\tilde{\mu}$.

Inferring the relative accuracy of the CNN and likelihood-based method.— To quantify the difference in error between the CNN and the likelihood-based method, we fit the difference in sampled APE scores, Δ APE, between the CNN method and the likelihood-based method to the BEST model. Histograms of Δ APE appear symmetric with weak to strong outliers making the BEST model a good candidate for inference from this data. The data and likelihood are

$$\Delta y = APE^{CNN} - APE^{Like}$$
$$\Delta y \mid \mu, \sigma, \nu \sim t_{\nu}(\mu, \sigma)$$

³⁷⁶ We used the same default priors as above.

Because, Δy is not log-transformed, it is drawn from a t-distribution and the marginal posterior of the parameter μ is an estimate of the population mean, μ^d . Because the mean and the median are equivalent for a t-distribution, we again report the posterior distribution of the median difference, $\tilde{\mu}^d$ to simplify the results.

In summary, the results we present are 95% HPI from the posterior distribution of the median difference between the two methods, $\tilde{\mu}^d$.

³⁸³ When comparing CNN to the likelihood-based approach, positive values for $\tilde{\mu}^d$ ³⁸⁴ indicate the CNN is less accurate, and negative indicate the likelihood-based estimates less ³⁸⁵ accurate. We emphasise that this quantity is the median difference in contrast to the ³⁸⁶ difference in medians, $\Delta \tilde{\mu}$, reported in the next section.

Inferring sensitivity to model misspecification.— Finally, to quantify the overall sensitivity 387 of each rate parameter to model misspecification under each inference method, we infer the 388 difference in median APE, $\tilde{\mu}$ of predictions under a misspecified model relative to 389 predictions under the true model. In other words we are inferring differences in medians 390 between experiments. For example, to infer the sensitivity of the CNN's inference of the 391 sampling rate, δ , to phylogenetic error, we inferred the difference between the median APE 392 of the CNN's predictions for misspecified trees and the median APE of CNN predictions 393 for true trees. The data is concatenated as below. 394

> $(y_1, y_2) = (APE^{CNN}, APE^{CNN \text{ Ref}}) \text{ or}$ $(y_1, y_2) = (APE^{\text{Like}}, APE^{\text{Like Ref}})$

We inferred the difference between group median APE scores, denoted $\Delta \tilde{\mu}$, by assuming that the model parameters conditioned on the observed APE from the two groups, y_1 and y_2 , follow a posterior distribution that is proportional to

$$P(y_1 \mid \mu_1, \sigma_1, \nu) P(y_2 \mid \mu_2, \sigma_2, \nu) P(\mu_1, \mu_2, \sigma_1, \sigma_2, \nu),$$

where log y_1 and log y_2 follow t distributions with means μ_1 and μ_2 and standard deviations σ_1 and σ_2 , respectively while sharing a common normality parameter, ν . The posterior sample of $\Delta \tilde{\mu}$ is obtained by transforming samples from the joint marginal posterior distribution of μ_1 and μ_2 with the following equation,

$$\Delta \tilde{\mu} = e^{\mu_1} - e^{\mu_2}.$$

The two components of the likelihood are each t-distributed and share the ν parameter which means we assume both samples are drawn from a similarly shaped distribution (similarly heavy tails).

log $y_1 \mid \mu_1, \sigma_1, \nu \sim t_{\nu}(\mu_1, \sigma_1)$ log $y_2 \mid \mu_2, \sigma_2, \nu \sim t_{\nu}(\mu_2, \sigma_2)$

The prior distribution for the parameters of the model were set to the defaults for BEST,

$$\mu_1 \sim \text{Normal}(\text{mean}(\log y_1), \text{sd}(\log y_1) \times 1000)$$

$$\mu_2 \sim \text{Normal}(\text{mean}(\log y_2), \text{sd}(\log y_2) \times 1000)$$

$$\sigma_1 \sim \text{Uniform}(\text{sd}(\log y_1)/1000, \text{sd}(\log y_1) \times 1000)$$

$$\sigma_2 \sim \text{Uniform}(\text{sd}(\log y_2)/1000, \text{sd}(\log y_2) \times 1000)$$

$$\nu \sim \text{Exponential}(1/29) + 1$$

As before, interpretation of the posterior distribution of the difference in medians is straightforward: the more positive the difference in median APE from the misspecified model test set and the median APE from the true model test set, the more sensitive the parameter is to model misspecification in the experiment.

406

Real Data

We compared the inferences of a LDBDS simulation trained neural network to that of a 407 phylodynamic study of the first COVID wave in Europe (Nadeau et al. 2021). These 408 authors analyzed a phylogenetic tree of viruses sampled in Europe and Hubei, China using 400 a location-dependent birth-death-sampling model in a Bayesian framework using priors 410 informed by myriad other sources of information. We simulated a new training set of trees 411 under an LDBDS model where R_{0_i} depends on the geographic location, and the sampling 412 process only consists of serial sampling and no sampling of extant infected individuals. We 413 then analyzed the whole tree from Fig. 1 in (Nadeau et al. 2021) as well as the European 414 clade which Nadeau et al. (2021) labeled as A2 in the same figure. We note that our 415 simulating model is not identical to the inference model used in (Nadeau et al. 2021). We 416 model migration with a single parameter with symmetrical migration rates among 417 locations and all locations having the same sample rate. Nadeau and colleagues 418 parameterize the migration process with asymmetric pairwise migration rates and assume 419 location-specific sampling rates. We also do not include the information the authors used 420 to inform their priors as that requires an extra level of simulation and training on top of 421 simulations done here, and is thus beyond the scope of this study. 422

The time tree from (Nadeau et al. 2021) was downloaded from GitHub (https://github.com/SarahNadeau/cov-europe-bdmm). The recovery rate assumed in (Nadeau et al. 2021) was 0.1 days⁻¹ which was set to 0.05 to bring the recovery rate to within the range of simulating values used to train the CNN. Consequently, the branch lengths of the tree were then scaled by 2. The number of tips, tree height, and average

branch lengths were measured from the rescaled trees and fed into the network. The full
tree and A2 clade were then analyzed using the LDBD CNN and compared to the posterior
distributions from (Nadeau et al. 2021).

431

Hardware used

432 Simulations were run on a 16 core Intel(R) Xeon(R) Platinum 8175M CPU @ 2.50GHz.

⁴³³ For each simulation, an XML file with random parameter settings was generated using

434 custom scripts. These XML files were the inputs for MASTER which was run in the

⁴³⁵ BEAST2 platform. Neural network training and testing and predictions were conducted on ⁴³⁶ an 8 core Intel(R) Core(TM) i7-7820HQ CPU @ 2.90GHz laptop.

⁴³⁷ Data and code availability.— A repository containing data and code used in this study is
⁴³⁸ available here: Link to be provided soon.

RESULTS

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Comparing deep learning to likelihood

Our first goal in this study was to train a CNN that produced phylodynamic parameter 441 point estimates that were as accurate as likelihood-based Bayesian posterior mean 442 estimates under the true model. This will serve as a reference for quantifying level of 443 sensitivity in our misspecification experiments. We focused on estimating important 444 epidemiological parameters – the reproduction number, R_0 , the sampling rate, δ , and the 445 migration rate, m – as well as the outbreak origin from viral phylogenies like those typically 446 estimated from serially sampled DNA sequences that were obtained as the virus spread. 447 Our CNN produced estimates that are as accurate as the mean posterior estimates 448

(MPE) under the true simulating model. We compared the absolute percent error (APE)

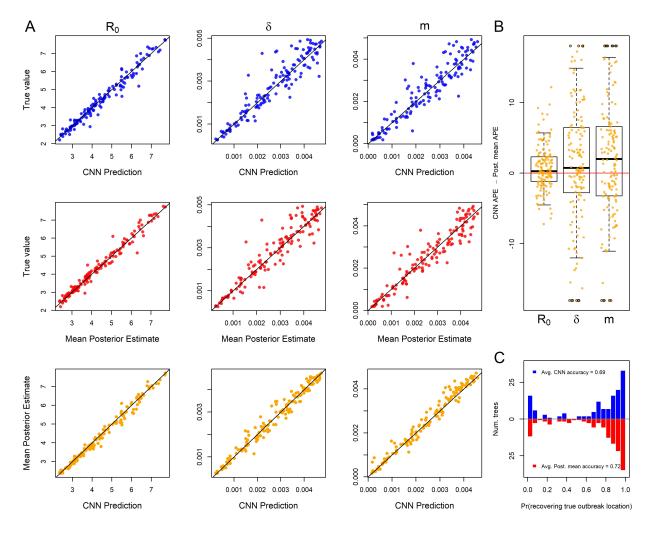


Figure 1: Inference under the true simulating model. (A) Scatterplot of CNN predictions and posterior mean estimates from Bayesian analyses against the true values (top two rows in blue and red respectively) of the basic reproduction number, R_0 , the sampling rate, δ , and the migration rate, m for 138 test trees. The bottom row in orange shows scatter plots of the CNN estimates against the posterior mean estimates for the same trees. (B) The difference in the absolute percent error (APE) of estimates for the two inference methods. Boxes show the inner 50% quantile of the data while whiskers extend 1.5 IQR. Dots with black circles show estimates that were truncated to the mean of the parameter with the most extreme outliers for visualization purposes. (C) Histograms of the probabilities of inferring the correct outbreak origin location for the same trees as in panels A and B.

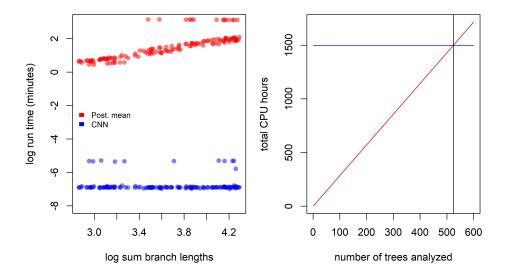


Figure 2: Left: Estimates of time to complete analysis of each of 138 trees relative to tree size. Right: The number of trees (524; gray vertical line) needed to analyze for total analysis time of Bayesian method (red line) to equal that of the entire simulation and CNN training and inference pipeline (blue line).

of the network predictions to the APE of the MPE of the Bayesian LIBDS model (Figure 450 1). The APE is straight-forward to interpret, e.g. an APE of < 10 means the estimate is 451 within 10 percentage points (ppts) of the true value. For the three epidemiological rate 452 parameters, R_0 , δ and m, both methods made very similar predictions for the 100 time tree 453 test set (Figure 1 panel A). The two methods appear to produce estimates that are more 454 similar to each other than to the ground truth labels (compare bottom row scatter plots in 455 orange to the blue and red scatter plots in panel A). Fig. 1 panel B shows that the inferred 456 median difference in APE, $\tilde{\mu}^d$, between the method's estimates for the three parameters is 457 close to zero (| $\tilde{\mu}^d$ | 95% highest posterior interval (HPI) is < 4 ppts; SI Table S1; SI Figure 458 S3). Fig. 1 Panel C shows that our predictions of the location of outbreak have similar 459 patterns of accuracy as those from the Bayesian method. 460

⁴⁶¹ Our trained CNN provides nearly instantaneous estimates of model parameters. ⁴⁶² While the run time of the likelihood approach employed in this study scales linearly with ⁴⁶³ the size of the tree, the neural network has virtually constant run times that are more than

three orders of magnitude faster. Because simulation-trained neural networks have a one-time cost of simulating the training data set and then training the neural network, these methods are often called amortized-approximators (Bürkner et al. 2022). This means the time savings aren't recouped until a certain number of trees have been analyzed. For example, here over 524 trees would need to be analyzed to realize the cost savings of simulating data and training our neural network (Figure 2). This illustrates the importance of simulation optimization and generality for likelihood-free approaches to inference.

471

Comparing robustness to model misspecification

To test the relative sensitivity of CNN estimates and the likelihood-based MPE to model misspecification, we simulated several test data sets under different, more complex epidemic scenarios and compared the decrease in accuracy (increase in APE).

Our first model misspecification experiment tested performance when assuming all 475 locations had the same R_0 when, in fact, each location had different R_{0_i} values. The 476 median APE for all three parameters increased to varying degrees (SI Fig. S4 Panel A) 477 compared to the median APE measured in Fig. S3. We found that both methods 478 converged on similar biased estimates for R_0 . In both the CNN and Bayesian method, 479 estimates of δ were relatively robust to misspecifying R₀. In contrast, the migration rate 480 showed much more sensitivity to this model violation in both methods with both methods 481 also converging on similarly biased estimates (Figure 3 A). The median difference in error 482 between the two methods is close to zero for all rate parameters (| $\tilde{\mu}^d$ | 95% HPI < 6 ppts; 483 SI Table S1) (SI Figure S4 Panel B). The CNN appears to be slightly more sensitive than 484 the Bayesian approach when predicting the outbreak location. Nevertheless, their 485 distributions are quite similar (Fig. 3 Panel C). 486

⁴⁸⁷ Next, we measured method sensitivity when the sampling process of the test trees ⁴⁸⁸ violates assumptions in the inference model. In this set, each location had a unique and ⁴⁸⁹ independent sampling rate, δ , rather than a single δ shared among locations. The median

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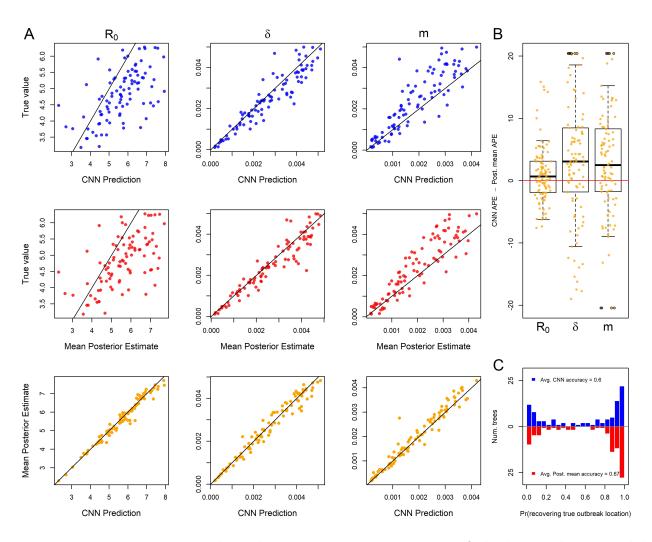


Figure 3: For 93 test trees where the R_0 parameter was misspecified: the simulating model for the test data specified 5 unique R_0 s among the five locations while the inference methods assumed one R_0 shared among locations. Because of this, the estimates for R_0 are plotted against mean of the five true R_0 values. See Figure 1 for general details about plots.

APE only increased for δ and m (SI Figure S5 Panel A). As expected, estimates of δ were 490 highly biased for both methods (Figure 4 panel A). Panel A also shows that R_0 is virtually 491 insensitive to sampling model misspecification, but that migration rate, again, is highly 492 sensitive in both the CNN and likelihood method. The median difference in error between 493 the two methods is close to zero for all the rate parameters ($|\tilde{\mu}^d|$ 95% HPI < 5 ppts; SI 494 Table S1, SI Figure S5) (Figure 4 panel B). The location of outbreak prediction is also 495 somewhat sensitive in both methods, with the CNN showing a slightly larger mean 496 difference, but the overall distribution of accuracy of all the test trees again is similar 497

⁴⁹⁸ (Figure 4 panel C).

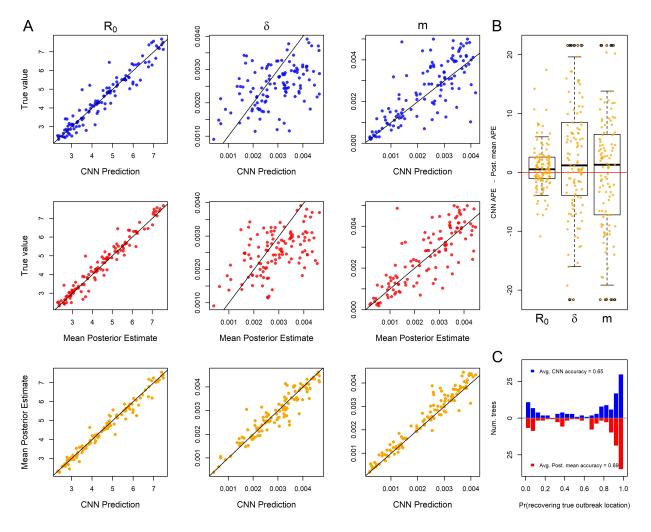


Figure 4: For 118 test trees where the sampling rate parameter was misspecified: the simulating model for the test data specified 5 unique sampling rates among the five locations while the inference methods assumed one sampling rate shared among locations. The estimates of δ are plotted against the mean true values of δ . See Figure 1 for general details about plots.

⁴⁹⁹ To explore sensitivity to migration model underspecification, we simulated a test set ⁵⁰⁰ where the migration rates between locations is free to vary rather than being the same ⁵⁰¹ among locations as in the inference model. This implies 5! unique location-pairs and thus ⁵⁰² unique migration rates in the test data set. Results show that for both methods the ⁵⁰³ parameters R_0 and δ are highly robust to this simplification (SI Fig. S6 Panel A). Though ⁵⁰⁴ estimates of a single migration rate had a high degree of error compared to a single pair of

⁵⁰⁵ locations migration rates (Figure 5 panel A), the two methods still had similar estimates ⁵⁰⁶ with the difference in APE centered near zero (Figure 5 panel B). The inferred median ⁵⁰⁷ difference in APE was close to zero ($| \tilde{\mu}^d | 95\%$ HPI < 3 ppts; SI Table S1; SI Figure S6 ⁵⁰⁸ Panel B). There was a slight but similar decrease in accuracy in predicting the outbreak ⁵⁰⁹ location for both methods (Figure 5 panel C).

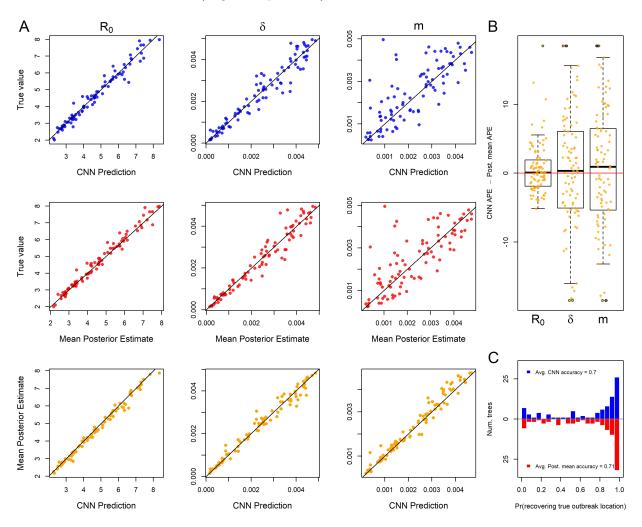


Figure 5: For 90 test trees where the migration rate parameter was misspecified: the simulating model for the test data specified 5! (120) unique migration rates among the unique pairs of the five locations while the inference methods assumed all migration rates were equal. The infered migration rate is plotted against the mean pairwise migraiton rates of test data set. See Figure 1 for general details about plots.

510 When testing the sensitivity of the two methods to arbitrary groupings of locations, 511 we found that both methods showed equal sensitivity to the same parameters (Fig. 6

Panels A and B). In particular, the migration rate showed a modest increase in median 512 APE and R_0 and sample rate showed virtually no sensitivity to arbitrary grouping of 513 locations (SI Figure S7 Panel A). The inferred median difference between method APE's 514 was again close to zero ($|\tilde{\mu}^d|$ 95% HPI < 4 ppts; SI Table S1; SI Figure S7 Panel B). This 515 suggests that for at least the exponential phase of outbreaks where rate parameters do not 516 vary among locations, these models have a fair amount of robustness to the decisions 517 leading to geographical division of continuous space into discrete space. The outbreak 518 location showed higher accuracy in both methods due to the fact that the test data was no 519 longer a flat distribution; the 6 combined locations should contain 60% of the outbreak 520 locations (Figure 6 panel C). 521

Finally, we explored the relative sensitivity of our CNN to amounts of phylogenetic 522 error that are present in typical phylogeographic analyses. Our simulated phylogenetic 523 error produced trees with average normalized Robinson-Foulds distances (Robinson and 524 Foulds 1981) between the inferred tree and the true tree of about 0.5 with 95% of 525 simulated trees having distances within 0.36 and 0.72. We again compared inferences 526 derived from the true tree and the tree with errors using the CNN and the Bayesian LIBDS 527 methods. Results show that migration rate was minimally affected but R_0 and δ were to a 528 some degree sensitive to phylogenetic error (Figure 7 panel A; SI Figure S8 Panel A), with 529 both methods again showing similar degrees of sensitivity (Figure 7 panel B). The inferred 530 median difference was, yet again, small ($|\tilde{\mu}^d|$ 95% HPI < 6 ppts. SI Table S1, SI Figure S8 531 Panel B). Inference of the origin location, were very similar for both methods (Fig. 7 Panel 532 C). 533

534

Analysis of SARS CoV-2 tree

We next compared our likelihood-free method to a recent study investigating the phylodynamics of the first wave of the SARS CoV-2 pandemic in Europe (Nadeau et al. 2021). Despite simulating the migration and the sampling processes differently from

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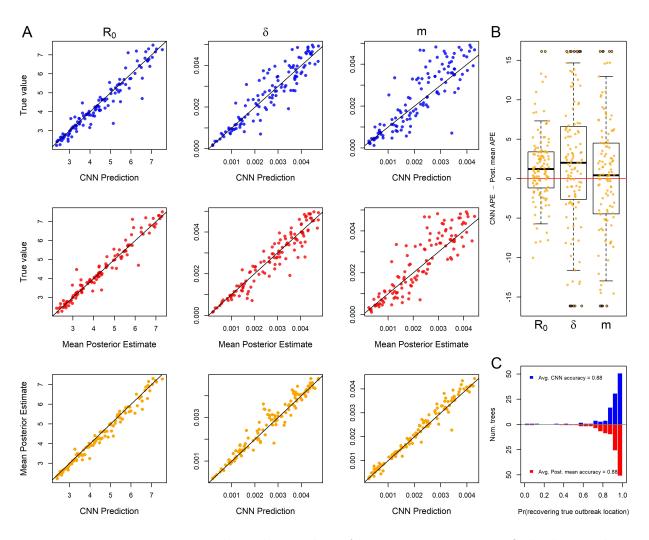


Figure 6: For 101 test trees where the number of locations was misspecified: the simulating model for the test data specified an outbreak among 10 locations with 6 locations subsequently combined into a single location while the inference methods assumed 5 locations with no arbitrary combining of locations. See Figure 1 for general details about plots.

Nadeau et al. (2021), our CNN produces similar estimates for the location-specific R_0 and 538 the origin of the A2 clade (Figure 8). Whether the full tree or just the A2 clade is fed into 539 the network, the predicted R_0 for each location was not far from the posterior estimates of 540 Nadeau et al. (2021). The only significant discrepancy in the origin prediction is that their 541 analysis suggests a much higher probability that the most recent common ancestor of the 542 A2 clade was in Hubei than our CNN predicts. This is likely because our CNN only used 543 the A2 clade to predict A2 origins which has no Hubei samples to infer the origin of the A2 544 clade while Nadeau et al. (2021) used the whole tree. Notwithstanding this difference, 545

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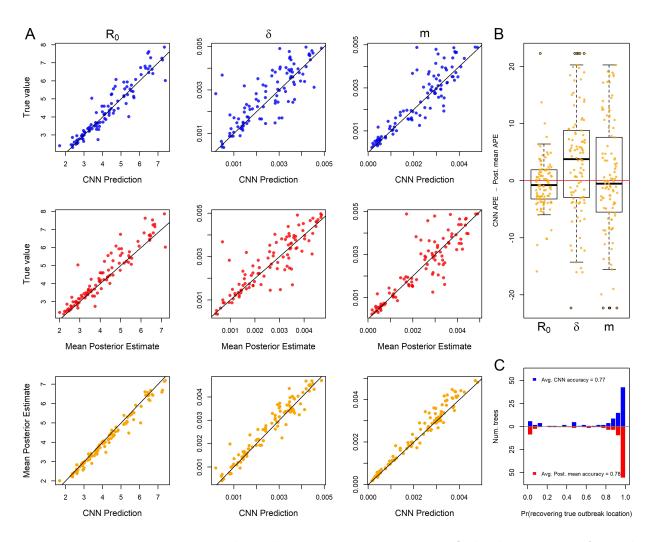


Figure 7: For 118 test trees where the time tree was misspecified: the true tree from the simulated test set was replaced with an inferred tree from simulated DNA alignments under the true tree. See Figure 1 for general details about plots.

⁵⁴⁶ among European locations, both methods predict Germany is the most likely location of

547 the most recent common ancestor followed by Italy.

548

DISCUSSION AND CONCLUSIONS

Inference models are necessarily a simplified approximation of the real world. Both
simulation-trained neural networks and likelihood-based inference approaches suffer from
model under-specification and/or misspecification. When comparing inference methods it is
important to assess the sensitivity of model inference to simplifying assumptions. In this

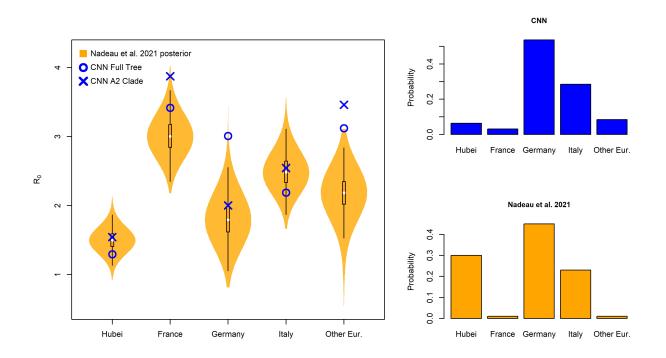


Figure 8: Location-dependent birth-death-sampling model (LDBDS) CNN comparison to (Nadeau et al. 2021) inference. Left violin plots show the posterior distributions of R_0 for each location in Europe as well as Hubei, China (orange). The blue X and O marks the MTBD CNN prediction from analyzing the full tree and the A2 (European) clade respectively. Right barplots show the LDBDS CNN prediction (blue) and posterior inference (orange) from (Nadeau et al. 2021) of the ancestral location of the A2 (European) clade (see Figure 1 (Nadeau et al. 2021)).

study we show that newer deep learning approaches and standard Bayesian approaches behave and misbehave in similar ways under a panel of phylodynamic estimation tasks where the inference model is correct as well as when it is misspecified.

By extending new approaches to encode phylogenetic trees in a compact data structure (Voznica et al. 2021; Lambert et al. 2022), we have developed the first application of phylodynamic deep learning applied to phylogeography with serial sampling. Our approach is similar to that of (Lambert et al. 2022) in which they analyzed a binary SSE model with exclusively extant sampling. By training a neural network on phylogenetic trees generated by simulated epidemics, we were able to accurately estimate key epidemiological parameters, such as the reproduction number and migration rate, in a fraction of the time it would take with likelihood-based methods. Like Voznica et al. (2021) and Lambert et al. (2022), we found that CNN estimators perform as well or nearly as well as likelihood-based estimators under conditions where the inference model is correctly specified to match the simulation model. The success of these separate applications of deep learning to different phylodynamic problems is a testament to the versatility of the cblv encoding of trees.

We compared the sensitivity of deep learning and likelihood-based inference to 568 model misspecification. Because deep-learning methods of phylogenetic and phylodynamic 560 inference are new, few studies compare how simulation-trained deep learning methods fail 570 in comparison to likelihood methods in this way (Flagel et al. 2019). We assume that when 571 the inference model is correctly specified to match the simulation model, the trained CNN 572 will, at best, produce noisy approximations of likelihood-based parameter estimates. In 573 reality, issues related to training data set size, learning efficiency, and network overfitting 574 may cause our CNN-based estimates to contain excess variance or bias when compared to 575 Bayesian likelihood-based estimators. Our results from five model misspecification 576 experiments show that both methods of inference perform similarly when the simulating 577 model and the inference model assumptions do not perfectly match. These similarities 578 exist not only in aggregate, when comparing method performance across datasets, but also 579 when comparing performance for each individual dataset. This suggests that the CNN and 580 likelihood methods are truly estimating parameters using isomorphic criteria, despite the 581 fact that CNN heuristically learns these criteria through data patterns, while likelihood 582 precisely and mathematically defines these criteria through the model definition itself. 583

Results of comparative sensitivity experiments like this are important because if likelihood-free methods using deep neural networks can easily be trained to yield estimates that are as robust to model misspecification as likelihood-based methods, then analysis of a large space of more complex outbreak scenarios for which tractable likelihood functions are not available can be developed and applied to real world data. Additionally, sufficiently realistic, pre-trained neural networks can yield nearly instantaneous inferences from data in

⁵⁹⁰ real time to inform analysts and policy makers.

⁵⁹¹ We also tested location-dependent SIR simulation trained neural network against a ⁵⁹² previous publication fitting a similar model – location-dependent birth-death-sampling ⁵⁹³ (LDBDS) model – on real-world data using a Bayesian method. Our CNN predicted ⁵⁹⁴ location-specific R_{0_i} and outbreak origin in Europe were similar to that inferred in (Nadeau ⁵⁹⁵ et al. 2021). This result and our model misspecification experiments suggest that ⁵⁹⁶ simulation-trained deep neural networks trained on phylogenetic trees can find patterns in ⁵⁹⁷ the training data that generalize well beyond the training data set.

Our study extends the results of Voznica et al. (2022) and Lambert et al. (2022) in 598 several important ways. This work showed that the new compact bijective ladderized 599 vector encoding of phylogenetic trees can easily be extended with one-hot encoding to 600 include metadata about viral samples. We extended it to include location data and were 60 able to train a neural network to not only predict important epidemiological parameters 602 such as R_{0_i} and the sampling rate, but also geographic parameters such as the migration 603 rate and the location of outbreak origination or spillover. We anticipate that much more 604 metadata can be added to train neural networks to bring more diverse and complex data to 605 make predictions about many important aspects of epidemiological spread such as the 606 relative roles of different demographic groups and the overlap of different species' ranges. 607

This approach can be readily applied to numerous compartment models used to 608 describe the spread of different pathogens among different species, locations, and 609 demographic groups, e.g. SEIR, SIRS, SIS, etc. (Ponciano and Capistrán 2011; Volz and 610 Siveroni 2018; Bjørnstad et al. 2020; Chang et al. 2020; O'Dea and Drake 2021) as well as 611 modeling super-spreader dynamics as in (Voznica et al. 2021). With fast, likelihood-free 612 inference afforded by deep learning, the technical challenges shift from exploring models for 613 which tractable likelihood functions can be derived towards models that produce realistic 614 empirical data patterns, have parameters that control variation of those patterns, and are 615 efficient enough to generate large training data sets. A growing number of advanced 616

simulators are rapidly expanding the possibilities for deep learning in phylogenetics. For 617 example, FAVITES (Moshiri et al. 2019) is a simulator of disease spread through large 618 contact networks that tracks transmission trees and simulates sequence evolution. Gen3sis, 619 MASTER, SLiM, and VGsim are flexible simulation engines for generating complex 620 ecological, evolutionary, and disease transmission simulations (Hagen et al. 2021; Vaughan 621 and Drummond 2013; Shchur et al. 2021; Haller and Messer 2019; Overcast et al. 2021). 622 Continued advances in epidemic simulation speed and flexibility will be essential for 623 likelihood-free methods to push the boundaries of epidemic modeling sophistication and 624 usefulness. 625

There are several avenues of development still needed to realize the potential of 626 likelihood-free inference in phylogeography using deep learning. The current setup is ideal 627 for simulation experiments, but it is more difficult to ensure that the optimal parameter 628 values for empirical data sets are within the range of training data parameters. 629 Standardizing input tree height, geographical distance, and other parameters help make 630 training data more universally applicable. Simulation-trained neural networks are often 631 called amortized methods (Bürkner et al. 2022; Schmitt et al. 2022) because the cost of 632 inference is front-loaded, *i.e.* it takes time to simulate a training set and train a neural 633 network. The total cost in time per phylogenetic tree amortizes as the number of trees 634 analyzed by the trained model increases. These methods are therefore important when a 635 model is intended to be widely deployed or be responsive to an emerging outbreak where 636 policy decisions must be formulated rapidly. Because amortized approximate methods 637 require multiple analyses to realize time savings, researchers need to generate training data 638 sets over a broad parameter and model space so that trained networks can be applied to 639 new and diverse data sets. Our research focuses on one phase of an outbreak (the 640 exponential phase), but there are many other scenarios to be investigated, such as when 641 the stage of an epidemic differs among locations (e.g. exponential, peaked, declining). 642 Quantifying uncertainty is also crucial to data analysis and decision making, and 643

Bayesian statistics provides a framework for doing so in a rigorous way. Quantifying uncertainty in predictions from deep neural networks is a difficult problem, as these models are trained to minimize prediction error, rather than to explicitly estimate uncertainty. In typical machine learning, uncertainty is ignored or measured using ad hoc methods in which interpretation requires care. Many of these approaches come with their own challenges and limitations, and there is still much ongoing research in this area to address the challenge of quantifying uncertainty in deep neural networks (Gal et al. 2022).

Another important challenge of inference with deep learning is the problem of convergence to a location on the loss function surface that approximates the maximum likelihood well. There are a number of basic heuristics that can help such as learning curves but more rigorous methods of ascertaining convergence is the subject of active research Bürkner et al. (2022); Schmitt et al. (2022).

With recent advances in deep learning in epidemiology, evolution, and ecology 656 (Battey et al. 2020; Schrider and Kern 2018; Voznica et al. 2022; Radev et al. 2021; 657 Lambert et al. 2022; Rosenzweig et al. 2022; Suvorov and Schrider 2022a) biologists can 658 now explore the behavior of entire classes of stochastic branching models that are 659 biologically interesting but mathematically or statistically prohibitive for use with 660 traditional likelihood-based inference techniques. Although we are cautiously optimistic 661 about the future of deep learning methods for phylogenetics, it will become increasingly 662 important for the field to diagnose the conditions where phylogenetic deep learning 663 underperforms relative to likelihood-based approaches, and to devise general solutions for 664 the field. 665

666

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670

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673

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SUPPLEMENTAL TABLES AND FIGURES

Table S1: BEST comparisons between CNN and Bayesian absolute percent errors (APEs) for model parameters across all experiments.

| 95% HPD Interv | als of average re | lative error from BEST | analysis |
|--|---|---|-------------------------------------|
| True inference model (Reference for misspecification experiments) | Median CNN APE | Median Likebased APE | median(CNN APE - Likebased APE) |
| R ₀ | 2.4, 3.5 | 2.1, 3.1 | 0.1,1.2 |
| δ | 7.0, 10.5 | 5.7, 8.9 | 0.2, 3.0 |
| m | 9.5, 14.1 | 8.4, 12.1 | 0.4, 3.2 |
| | | | |
| Misspecified R ₀ experiment | Median CNN APE - median CNN Reference APE | Median Like-based APE - median Reference Like-based APE | median(CNN APE - Like-based APE) |
| R _o | 11.8, 17.8 | 11.0, 16.9 | -0.1, 1.6 |
| δ | 0.8, 7.6 | -0.6, 5.3 | 1.3, 5.8 |
| m | 8.2, 17.9 | 6.5, 15.9 | 1.3, 4.7 |
| | • | | |
| Misspecified sample rate experiment | Median CNN APE - median CNN Reference APE | Median Like-based APE - median Reference Like-based APE | median(CNN APE - Like-based APE) |
| R _o | -0.3, 1.7 | 0.03, 1.7 | 0.1, 1.3 |
| δ | 12.0, 21.2 | 12.6, 21.4 | 0.1, 4.0 |
| m | 3.3, 12.0 | 5.6, 14.4 | -1.2, 2.7 |
| | | | |
| Misspecified migration rate experiment | Median CNN APE - median CNN Reference APE | Median Like-based APE - median Reference Like-based APE | median(CNN APE - Like-based APE) |
| R _o | -0.9, 0.8 | -0.6, 1.0 | -0.5, 0.8 |
| δ | -2.3, 3.3 | 0.1, 5,8 | -1.4, 2.3 |
| m | 4.0, 15.2 | 5.0, 16.2 | -1.3, 2.6 |
| | | · · · · · | |
| Misspecified number of locations experiment | Median CNN APE - median CNN Reference APE | Median Like-based APE - median Reference Like-based APE | median(CNN APE - Like-based APE) |
| R _o | -0.3, 1.5 | -0.7, 0.8 | 0.5, 1.9 |
| δ | -0.3, 4.9 | -0.5, 4.2 | 0.4, 3.5 |
| m | 3.4, 11.1 | 5.8, 13.5 | -0.9, 1.6 |
| | | | |
| Phylogenetic error experiment | Median CNN APE - median CNN Reference APE | Median Like-based APE - median Reference Like-based APE | median(CNN APE - Like-based APE) |
| R _o | 0.7, 3.0 | 1.7, 4.4 | -1.4, 0.1 |
| δ | 2.3, 9.6 | 1.5, 7.2 | 1.4, 5.3 |
| m | -1.2, 6.0 | -1.8, 5.4 | -1.7, 2.4 |

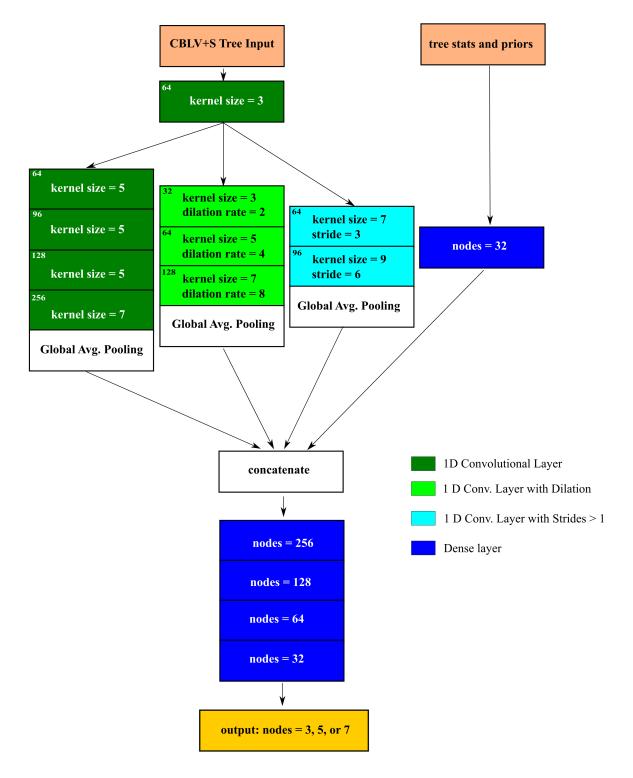


Figure S1: Diagram of deep neural network trained to make 2 kinds of predictions (rates and origin location) under two models (MTBD and SDMTBD).

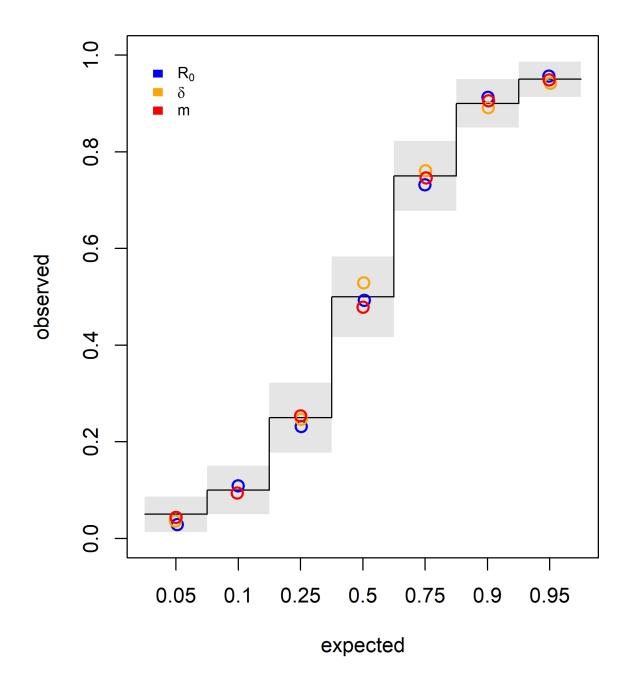


Figure S2: Coverage of posterior distributions simulated with TensorPhylo. Seven different HPD intervals were measured for coverage (labled horizontal). The expected frequency of coverage for each of the categories is shown at the black steps. Gray bands indicate the 95% confidence intervals for estimates of the binomial proportion at each of the expected values. The colored circles indicate the observed coverage of the three rate parameters at each of the expected values.

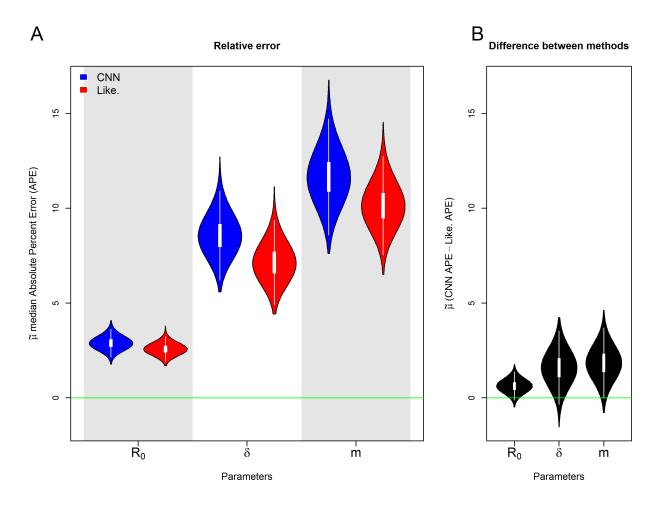


Figure S3: Posterior distributions of the population median, $\tilde{\mu}$, APE estimates of the rate parameters R_0 , δ , and m under the true model. A) shows posterior distribution of the median APE for each of the 3 rate parameters estimated by the CNN (blue) and the likelihood-based method (red). The green line indicates no error. B) shows the posterior distribution for the median difference between the CNN estimate's APE and the likelihood-based estimate's APE. The green line indicates the median APE difference is zero.

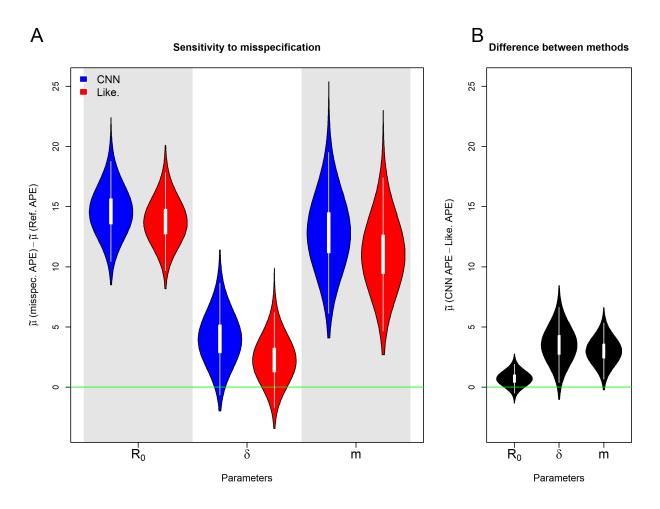


Figure S4: Posterior distributions of median, $\tilde{\mu}$, APE for the misspecified R₀ experiment. A) shows posterior distribution of the difference between the median error under the misspecified model and the the median error under the true, reference model. B) shows the posterior distribution for the population median difference between the CNN estimate's APE and the likelihood-based estimate's APE.

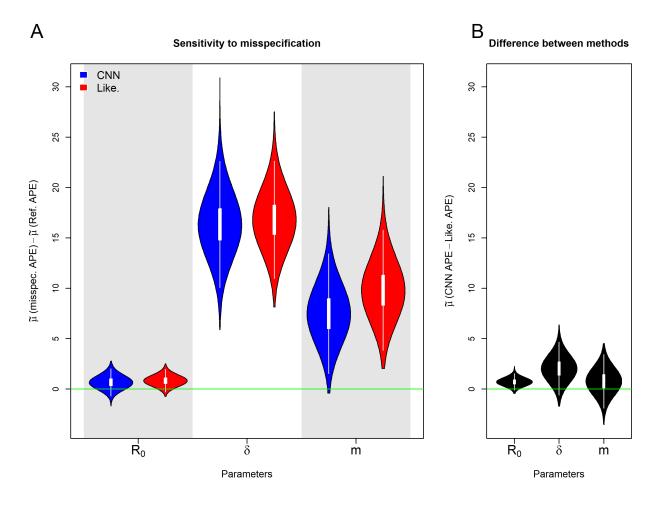


Figure S5: Posterior distributions of median, $\tilde{\mu}$, APE for the misspecified sampling rate, δ , experiment. Details are the same as in S4

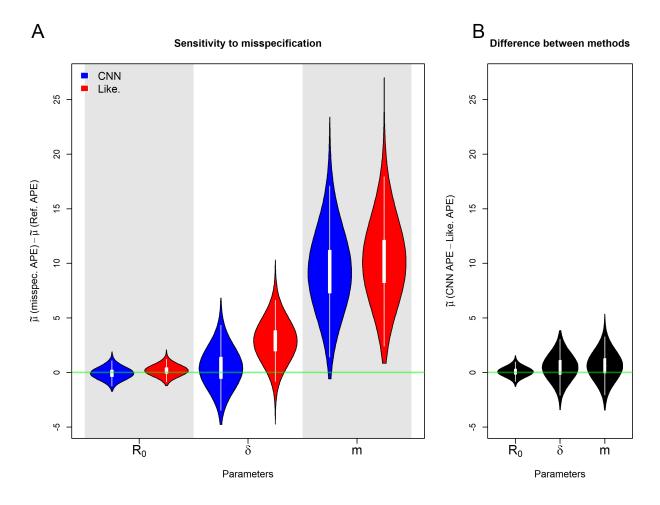


Figure S6: Posterior distributions of median, $\tilde{\mu}$, APE for the misspecified migration rate, m, experiment. Details are the same as in S4

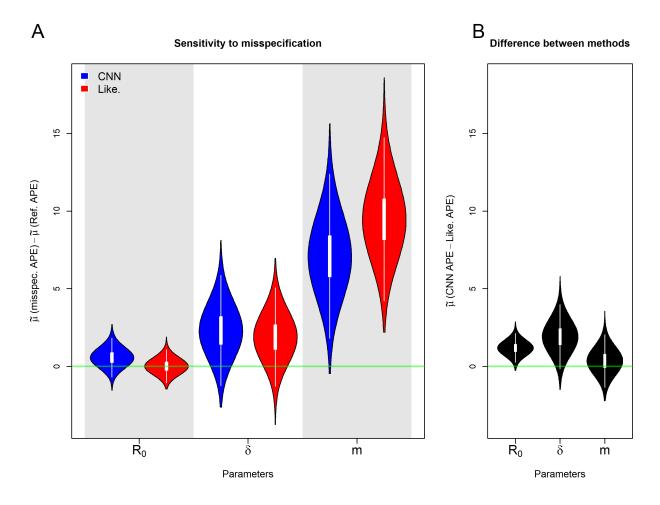


Figure S7: Posterior distributions of the median APE when the model is misspecified for the number of locations. Details are the same as in S4

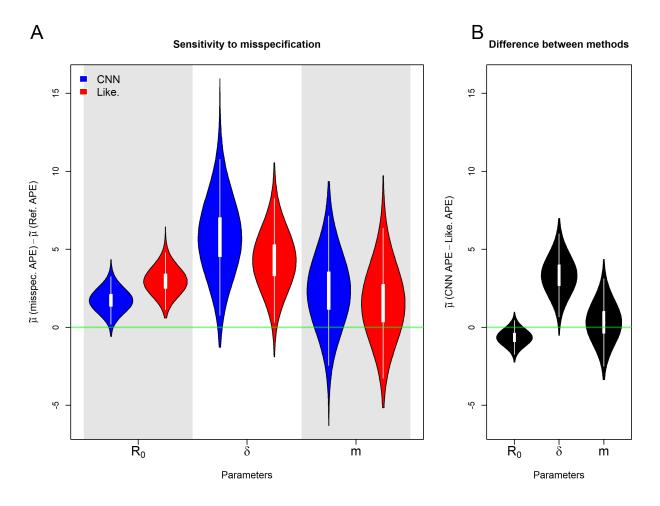


Figure S8: Posterior distributions of the median APE when the phylogenetic tree is incorrect. Details are the same as in S4 $\,$