The shirker's dilemma and the prospect of cooperation in large groups

Jorge Peña* Aviad Heifetz† Georg Nöldeke‡

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Abstract

Cooperation usually becomes harder to sustain as groups become larger because incentives to shirk increase with the number of potential contributors to a collective action. But is this always the case? Here we study a binary-action cooperative dilemma where a public good is provided only when at most a fixed number of players shirk from a costly, cooperative task. An example is a group of prey which succeeds to drive a predator away only if few group members refrain from engaging in conspicuous mobbing. We find that at the stable polymorphic equilibrium, which exists when the cost of cooperation is low enough, the probability of cooperating increases with group size and reaches a limit of one when the group size tends to infinity. Nevertheless, increasing the group size may increase or decrease the probability that the public good is provided at such equilibrium. We also prove that the expected payoff to individuals at the stable equilibrium (i.e., their fitness) decreases with group size. For costs of cooperation that are low enough, both the probability of provision of the public good and the expected payoff converge to positive values in the limit of large group sizes. However, we also find that the basin of attraction of the stable polymorphic equilibrium is a decreasing function of group size and shrinks to zero in the limit of very large groups. Overall, we demonstrate non-trivial comparative statics with respect to group size in a simple collective action problem.

1 Introduction

Living in groups, together with the possibilities and opportunities for conflict and cooperation entailed by communal life, is a widespread phenomenon in the natural world (Krause and Ruxton, 2002). Among the many factors affecting the evolution of cooperation in social groups, including relatedness and repeated interactions, group size has received considerable attention. In particular, how the strength of selection on cooperative behavior might increase or decrease

*Institute for Advanced Study in Toulouse, University of Toulouse Capitole, jorge.pena@iast.fr
†Department of Management and Economics, Open University of Israel, aviadhe@openu.ac.il
‡Faculty of Business and Economics, University of Basel, georg.noeldeke@unibas.ch

with group size has been a recurrent question in behavioral ecology and evolutionary biology (Beauchamp and Ruxton, 2003; MacNulty et al., 2012; Shen et al., 2014; Powers and Lehmann, 2017; Peña and Nöldeke, 2018). This question is paralleled in the social sciences by the question of how individual incentives to cooperate in a collective action problem are affected by group size (Olson, 1965; Chamberlin, 1974; Palfrey and Rosenthal, 1984; Oliver and Marwell, 1988). Although there is broad consensus that the effect of group size on cooperation is typically negative, with cooperation becoming more difficult to evolve or sustain as groups become larger (Olson, 1965; Boyd and Richerson, 1988), positive group size effects have been also documented in empirical studies (Isaac et al., 1994; Powers and Lehmann, 2017) and demonstrated in theoretical research (Esteban and Ray, 2001; Cheikbossian and Fayat, 2018; Peña and Nöldeke, 2018).

To fix ideas, consider a simple game-theoretic model of social interactions: the "volunteer's dilemma" (Diekmann, 1985). In this game, n players simultaneously become aware of a costly task needing at least one volunteer to be performed. If at least one of the players volunteers to pay the cost c of performing the task, a good of normalized value of one is created and enjoyed by all players. If nobody volunteers, the task is left undone and no public good is created. A more general version of this game (which, for simplicity, we will also refer to as a "volunteer's dilemma") considers the case where the cooperation of at least θ volunteers is needed for the collective good to be created, for $1 \leq \theta \leq n$ (Palfrey and Rosenthal, 1984). Such games have become influential in evolutionary biology, where they have been used to model a wide array of collective action problems ranging from the secretion of extracellular compounds in microbes and the construction of collective stalks in social amoeba (Archetti, 2009) to leadership in animal societies (Shen et al., 2010; Smith et al., 2016), confrontational scavenging in hominins (Bickerton and Szathmáry, 2011), and the costly punishment of free riders in humans (Raihani and Bshary, 2011; Przepiorka and Diekmann, 2013; Schoenmakers et al., 2014).

The volunteer's dilemma has also been used to provide firm theoretical underpinnings to the idea that larger groups are less conducive to cooperation. Indeed, it is well known that when only one volunteer is required ($\theta = 1$) both the proportion of volunteers and the probability that the collective action is successful decreases with group size (Archetti, 2009). More recently, Nöldeke and Peña (2020) have shown that this negative group-size effect extends to $1 < \theta < n$ for the best symmetric Nash equilibrium of the game (i.e., the equilibrium sustaining the highest probability of cooperation among the symmetric Nash equilibrium), while additionally proving that the expected payoff at such equilibrium is also a decreasing function of group size. Together, these results provide a mathematical proof of the fact that the larger the group is, the more difficult it is for cooperation to be achieved, at least for the kind of strategic interactions captured by the volunteer's dilemma. This casts doubts on the extent to which large-scale cooperation can be sustained without the presence of additional cooperation-enhancing mechanisms such as kin selection or reciprocity, and begs the question whether such a negative group-size effect is a general theoretical feature of a wide class of collective action problems or a peculiarity of the volunteer's dilemma.

In this paper, we consider the group-size effects of a related cooperative dilemma, that we will

call the "shirker's dilemma". As in the volunteer's dilemma, n players simultaneously become aware of a costly collective task. Yet, while in the volunteer's dilemma the collective task is successful (and a public good is produced) *if at least* a fixed number θ of players volunteer, in the shirker's dilemma the collective task is successful *if at most* a fixed number ζ of players shirk from volunteering. Obviously, for given group size n and cost of volunteering c the shirker's dilemma and the volunteer's dilemma are equivalent, and a shirker's dilemma with threshold $\zeta = n - \theta$ is the same as a volunteer's dilemma with threshold θ. However, and importantly, the two games capture different consequences of an increase in group size: While in the volunteer's dilemma the number of required volunteers is a constant independent of group size, in the shirker's dilemma the number of required volunteers θ increases with group size so as to keep the threshold maximum number of shirkers ζ fixed. Thus, for large group sizes (and relatively small thresholds), collective action is successful in the volunteer's dilemma provided that *some* individuals cooperate. In contrast, in the shirker's dilemma collective action is successful provided that *few* individuals shirk, and hence that *most* individuals cooperate. As it will be shown, the different payoff structure of the shirker's dilemma translates into different comparative statics with respect to group size and to a different answer to the question of whether or not larger groups are less conducive to cooperation.

The shirker's dilemma we study in this paper is relevant for situations in which the (potentially costly) cooperation of all or most individuals are required to produce a collective good. As an example, consider the sentinel behavior of Arabian babblers (*Argya squamiceps*)—a territorial, cooperatively breeding species of songbirds (Zahavi, 1990) living in arid areas along the Great Rift Valley. Much of the daytime, group members take turns as sentinels on treetops, while the other group members forage on the ground or on low branches. When a sentinel spots an approaching raptor, it emits an alarm call. Foragers then have two options. First, they can flee to shelter inside a thicket, within which they are temporarily protected from the raptor but unable to follow its moves. Alternatively, they can fly up and join the sentinel on treetops in calling towards the raptor. The latter is the typical choice of foragers, constituting more than 80% of foragers' reactions (Ostreiher and Heifetz, 2020). Moreover, even lone Arabian babblers devoid of territory (i.e., "floaters") engage in calling from treetops towards raptors when they spot them (Ostreiher and Heifetz, 2017). This signal is costly in terms of both the energy expended on calling and the fatal risk in case they temporarily lose track of the maneuvering raptor. The raptor, on its part, should decide whether to continue hovering above the group until one of its members becomes less vigilant or, alternatively, to move on in search of other prey. Hiding group members are the least vigilant when they emerge out of the thicket, so moving on is better for the raptor than lurking for hiding birds only if the number of hiding birds (i.e., the number of "shirkers") is small enough. Dissuading the raptor from attack is a public good enjoyed by all group members. This situation ressembles more a shirker's dilemma than a volunteer's dilemma because the predator's decision to hang around or to move away is likely to depend on the absolute number of non-vigilant individuals that it spots (i.e., the "shirkers"), rather than on the absolute number of vigilant individuals (i.e., the "volunteers")

that it is unlikely to catch.

Our analysis of group-size effects on the shirker's dilemma builds upon previous insights derived in Peña and Nöldeke (2018) for the general case of binary-action symmetric n-player games, and in Nöldeke and Peña (2020) for the particular case of the volunteer's dilemma. The question asked in Nöldeke and Peña (2020) was: What are the consequences of larger group sizes in a collective action that is successful if and only there are enough contributors? Here, we ask the dual question, namely: What are the consequences of larger group sizes in a collective action that is successful if and only if there are not too many shirkers? In particular, we are interested in the three group-size effects analyzed by Nöldeke and Peña (2020) for the volunteer's dilemma. First, what is the effect of group size on the probability that individuals cooperate at equilibrium? Second, what is the effect of group size on the expected payoff of individuals? Third, what is the effect of group size on the probability that collective action is successful at equilibrium? Finally, we also investigate what happens to these quantities as the groups are made arbitrarily large, that is, in the limit of infinitely large groups.

2 Model

2.1 The shirker's dilemma

A group of $n > 2$ players faces a task to be performed (e.g., predator vigilance, predator mobbing) that requires collective action for its success. Each player $i \in \{1, \ldots, n\}$ can either "volunteer" (or "cooperate", or "provide") at a cost $c \in (0, 1)$ or, alternatively, "shirk" from the task (or "defect"). All players, irrespective of their strategy, enjoy an additional payoff benefit normalized to one (i.e., a public good of value one is provided) if at most $\zeta \geq 1$ players shirk.

Throughout the following we take ζ and c as given and study the impact of group size n on equilibrium behavior. In doing so, we restrict attention to group sizes $n > \zeta + 1$, thereby excluding the trivial case in which the benefit arises even if all players shirk ($n = \zeta$) and the well understood case (Diekmann, 1985) in which exactly one volunteer is required for the benefit to arise ($n = \zeta + 1$).

As noted in the Introduction, for given n our game corresponds to a volunteer's dilemma in which $\theta = n - \zeta$ is the minimal number of volunteers. Nevertheless, we prefer the name "shirker's dilemma" as it draws attention to the essential feature of our analysis, namely that we study the case in which the maximal number of shirkers (rather than the minimal number of volunteers) compatible with the provision of the benefit is considered fixed.

2.2 Replicator dynamic and pivot probability

We assume that the population is infinitely large and comprised of at most two types of individuals: "shirkers" (or "defectors") and "volunteers" (or "cooperators"). Denoting by q the proportion of shirkers in the population, and assuming that groups of size n are matched to

play the game uniformly at random, the probability that there are exactly k shirkers in a group is binomial and given by

$$\binom{n}{k} q^k \left(1-q\right)^{n-k}.$$

Letting

$$\Pi_{\zeta,n}(q) \equiv \sum_{k=0}^{\zeta} \binom{n}{k} q^k (1-q)^{n-k} \tag{1}$$

denote the probability that there are at most ζ shirkers in a group, the expected payoff to a shirker and to a volunteer can then be respectively written as

$$w_{\mathrm{S}}(q) = \Pi_{\zeta-1,n-1}(q), \tag{2}$$

and

$$w_{\mathrm{V}}(q) = \Pi_{\zeta,n-1}(q) - c. \tag{3}$$

Indeed, a shirker gets a payoff of one if there are not more than $\zeta - 1$ shirkers among its $n-1$ co-players, and zero otherwise, which explains Eq. (2). In contrast, a volunteer gets a benefit of one in case there are not more than ζ shirkers among its co-players, but it always pays the cost c of volunteering, which explains Eq. (3).

We assume that the change in the proportion of shirkers over evolutionary time is given by the continuous-time two-strategy replicator dynamic (Taylor and Jonker, 1978; Weibull, 1995; Hofbauer and Sigmund, 1998)

$$\dot{q} = q\left(1-q\right) g_{n,\zeta,c}(q), \tag{4}$$

where $g_{n,\zeta,c}(q)$ corresponds to the "gain function" (Bach et al., 2006; Peña et al., 2014), the "incentive function" (Broom and Rychtář, 2022, p. 177-178), or the "private gain function" (Peña and Nöldeke, 2023). The gain function is the difference in expected payoffs between the two pure strategies in a population where the proportion of shirkers is q, and thus quantifies the benefits of being a shirker over being a volunteer. In our case the gain function is simply given by

$$g_{n,\zeta,c}(q) \equiv w_{\mathrm{S}}\left(q\right) - w_{\mathrm{V}}\left(q\right) = c - \pi_{\zeta,n}\left(q\right), \tag{5}$$

where

$$\pi_{\zeta,n}\left(q\right) = \binom{n-1}{\zeta} q^{\zeta} \left(1-q\right)^{n-1-\zeta} \tag{6}$$

is the *pivot probability*, that is, the probability that a given focal player is pivotal for the failure

of the collective action of its group when the proportion of shirkers in the population is equal to q. Indeed, since the shirking of a player will make a difference between provision and non-provision of the public good if and only if ζ out of $n-1$ players shirk, the pivot probability is given by the binomial expression on the right hand side of Eq. (6).

2.3 Rest points and their stability

We are interested in the asymptotically stable rest points of the replicator dynamic (4), as these indicate, for our model, the effects of selection in the long run. The rest points of the replicator dynamics correspond to all values of the share of shirkers q that nullify Eq. (4). There are two kinds of rest points. First, there are two trivial rest points $q = 0$ ("all volunteer") and $q = 1$ ("all shirk") at which the population is monomorphic, which make the type variance of the population given by $q(1-q)$ equal to zero. Second, there can be interior rest points $q \in (0, 1)$, at which the population is polymorphic but the payoffs to the two strategies are equal (i.e., $\Pi_{\mathrm{S}}(q) - \Pi_{\mathrm{V}}(q) = 0$ holds), so that the gain function vanishes. Setting Eq. (5) to zero, we find that the interior rest points correspond to the solutions of the pivotality condition

$$\pi_{\zeta,n}(q) = c. \tag{7}$$

The stability of both trivial and interior rest points can be verified by checking the sign pattern of the gain function (Bukowski and Miekisz, 2004; Peña et al., 2014; Peña and Nöldeke, 2023). In particular, $q = 0$ is stable if the initial sign of the gain function is negative, $q = 1$ is stable if the final sign of the gain function is positive, and an interior rest point is stable (resp. unstable) if the gain function changes sign from positive to negative (resp. negative to positive) at the rest point. In turn, the sign pattern of the gain function depends on how the pivot probability compares to the cost of volunteering as a function of the proportion of shirkers in the population. To make progress, we thus need a full characterization of the shape of the pivot probability as a function of the proportion of shirkers.

To this end, it can be verified that the pivot probability $\pi_{\zeta,n}(q)$ satisfies the following properties (Nöldeke and Peña, 2020; the left panel of Fig. 1 illustrates). First, the pivot probability is differentiable in q (differentiability). Second, $\pi_{\zeta,n}(0) = \pi_{\zeta,n}(1) = 0$ holds (end-points property). Third, the pivot probability is strictly increasing on the interval $[0, \zeta/(n-1)]$ and strictly decreasing on the interval $[\zeta/(n-1), 1]$ with non-zero derivative on the interiors of these intervals (unimodality). In particular, $\zeta/(n-1)$ is the unique maximizer of $\pi_{\zeta,n}(q)$ in the interval $[0, 1]$. Hence,

$$\bar{c}_{\zeta,n} \equiv \pi_{\zeta,n}(\zeta/(n-1)) \in (0, 1) \tag{8}$$

is the *critical cost* value such that (i) for $c > \bar{c}_{\zeta,n}$ the pivotality condition (7) has no solution, (ii) for $c = \bar{c}_{\zeta,n}$ it has a unique solution, and (iii) for $c < \bar{c}_{\zeta,n}$ it has two solutions.

The shape properties of the pivot probability allow us to fully characterize the evolutionary

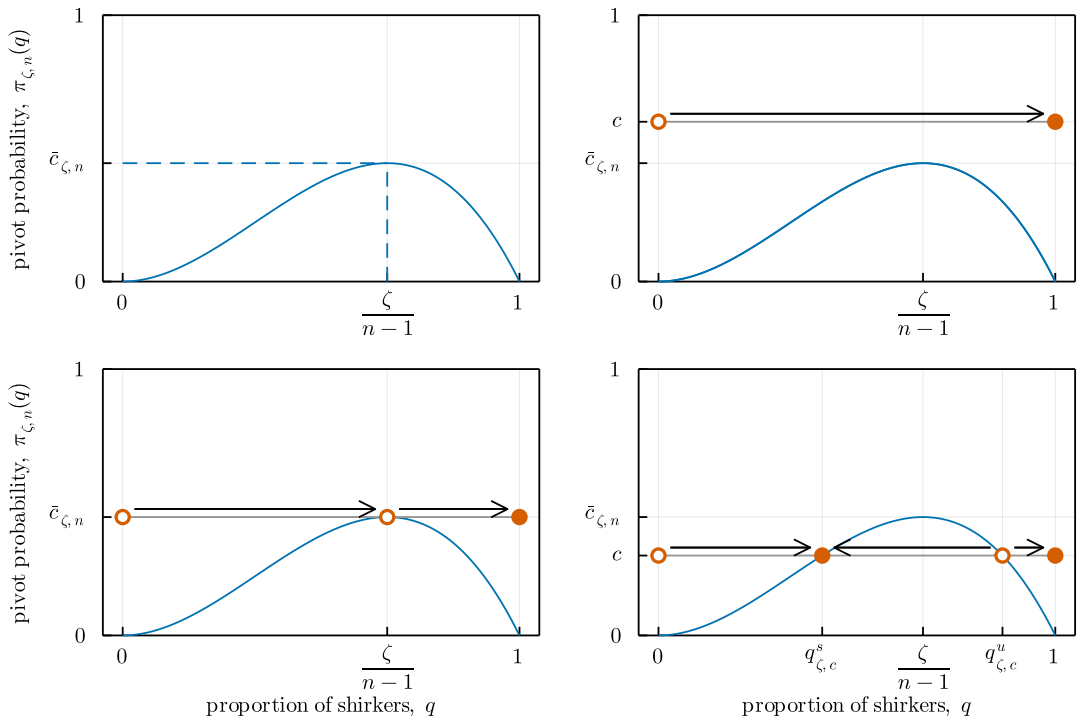

Figure 1: Pivot probability $\pi_{\zeta,n}(q)$ as a function of the proportion of shirkers q, and corresponding evolutionary dynamics, as given by Lemma 1, for $\zeta = 2$ and $n = 4$. *Top left:* The pivot probability is unimodal, with maximum $\bar{c}_{\zeta,n}$ at $q = \zeta/(n-1)$. Here, $\bar{c}_{2,4} \approx 0.444$, and $\zeta/(n-1) = 2/3$. *Top right:* For high costs ($c > \bar{c}_{\zeta,n}$; here $c = 0.6$) the replicator dynamic has no interior rest points. The trivial rest point $q = 0$ is unstable (*open circle*) while the trivial rest point $q = 1$ is stable (*full circle*). The evolutionary dynamics lead to $q = 1$ for all initial conditions (*arrow*). *Bottom left:* For a cost equal to the critical cost $\bar{c}_{\zeta,n}$, there is a unique interior rest point at $q = \zeta/(n-1)$ that is unstable (*open circle*). The evolutionary dynamics lead to $q = 1$ for all initial conditions in $[0,1] - \{\zeta/(n-1)\}$ (*arrows*). *Bottom right:* For low costs ($c < \bar{c}_{\zeta,n}$; here, $c = 0.3$), the replicator dynamic features two interior rest points: the leftmost ($q^s_{\zeta,c}$) is stable (*full circle*), and the rightmost ($q^u_{\zeta,c}$) is unstable (*open circle*). The dynamics (*arrows*) lead either to $q^s_{\zeta,c}$ or to $q = 1$, depending on initial conditions.

dynamics of the shirker's dilemma. Indeed, it follows from the end-points property that $q = 0$ ("all volunteer") is always an unstable rest point of the replicator dynamic (4), while $q = 1$ ("all shirk") is always a stable rest point. Additionally, these are the only rest points when the cost of volunteering is sufficiently large (i.e., when $c > \bar{c}_{\zeta,n}$ holds). When the cost is sufficiently low (i.e., when $c < \bar{c}_{\zeta,n}$ holds) the replicator dynamic has two additional rest points in the interval $(0, 1)$, corresponding to the two solutions to the pivotality condition (7). The derivative of the gain function is negative at the leftmost of these rest points, while it is positive at the rightmost of these rest points. Hence, the gain function changes sign from positive to negative at the leftmost of these rest points (that is then stable), while it changes sign from negative to positive at the rightmost of these rest points (that is then unstable). We collect these observations on the characterization of the replicator dynamic of the shirker's dilemma in the following Lemma (cf. Lemma 1 in Nöldeke and Peña, 2020; top right and bottom panels of Fig. 1 illustrate).

Lemma 1 *For any ζ, n, and c, $q = 0$ is an unstable rest point and $q = 1$ is a stable rest point of the replicator dynamic. Additionally, the number, location, and stability of interior rest points of the replicator dynamic depend on how the cost of volunteering c compares to the critical cost (8) as follows:*

1. *If $c > \bar{c}_{\zeta,n}$, there are no interior rest points.*
2. *If $c = \bar{c}_{\zeta,n}$, there is a unique unstable interior rest point, namely $q = \zeta/(n-1)$.*
3. *If $c < \bar{c}_{\zeta,n}$, there are two interior rest points: $q^s_{\zeta,c}(n)$, which is stable, and $q^u_{\zeta,c}(n)$, which is unstable. The two interior rest points satisfy $0 < q^s_{\zeta,c}(n) < q^u_{\zeta,c}(n) < 1$, with $q^s_{\zeta,c}(n)$ being the unique solution to (7) in the interval $(0, \zeta/(n-1))$, and $q^u_{\zeta,c}(n)$ being the unique solution to (7) in the interval $(\zeta/(n-1), 1)$.*

There are two cases of interest. First, for high costs (i.e., $c \geq \bar{c}_{\zeta,n}$) the replicator dynamic is such that the only stable rest point is $q = 1$, i.e., the population is characterized by full shirking at equilibrium. Second, for low costs (i.e., $c < \bar{c}_{\zeta,n}$), the replicator dynamic exhibit what can be called bistable coexistence (Peña et al., 2015), where an interior unstable rest point (located at $q^u_{\zeta,c}(n)$) separates the basins of attraction of two stable rest points: one where there is full shirking ($q = 1$) and another one where shirkers and volunteers coexist (i.e., $q^s_{\zeta,c}(n)$).

2.4 Proportion of volunteers, success probability, and expected payoff

We are interested in the effects of group size on two sets of different, but related quantities. First, we are interested in how the critical cost (8) (which determines the kind of evolutionary dynamics we obtain, as specified in Lemma 1) changes as the group size increases. Second, we look into the comparative statics with respect to group size (and the limits when group size goes to infinity) of (i) the proportion of volunteers, (ii) the probability of collective success, and (iii) the expected payoff, all evaluated at the stable rest point of the replicator dynamic sustaining

the smallest amount of shirking (i.e., the largest amount of volunteering). In the following, we refer to such rest point as the *minimal rest point*, and denote it by $q_{\zeta,c}(n)$.

Depending on the particular values of ζ, n, and c, the minimal rest point can be either the trivial rest point $q = 1$ or the interior rest point $q^s_{\zeta,c}(n)$. More precisely, and from Lemma 1, we have

$$q_{\zeta,c}(n) = [\![c \geq \bar{c}_{\zeta,n}]\!] + [\![c < \bar{c}_{\zeta,n}]\!] q^s_{\zeta,c}(n), \tag{9}$$

where we have used the Iverson bracket to write $[\![P]\!] = 1$ if P is true and $[\![P]\!] = 0$ if P is false. With this definition, we let

$$p_{\zeta,c}(n) = 1 - q_{\zeta,c}(n), \tag{10}$$

$$\phi_{\zeta,c}(n) = \Pi_{\zeta,n}(q_{\zeta,c}(n)), \tag{11}$$

$$u_{\zeta,c}(n) = \Pi_{\zeta-1,n-1}(q_{\zeta,c}(n)), \tag{12}$$

denote, respectively, the *proportion of volunteers*, the *success probability*, and the *expected payoff* at the minimal rest point. The proportion of volunteers is simply one minus the proportion of shirkers at the minimal rest point, and is thus given by Eq. (10). The success probability (11) is the probability that the collective good is produced, and hence that there are not more than ζ shirkers in a group of n players who are all shirking with probability (9), and is then given by Eq. (11). Finally, the expected payoff at the minimal rest point is most easily calculated from the perspective of a shirker (see Eq. (2)), and thus given by Eq. (12). Clearly, the equilibrium payoff can be also calculated from an ex ante perspective, and is thus given by the success probability minus the expected cost of volunteering. This observation provides a useful equation linking the three quantities, namely

$$u_{\zeta,c}(n) = \phi_{\zeta,c}(n) - p_{\zeta,c}(n) \cdot c. \tag{13}$$

3 Results

3.1 Preliminaries

3.1.1 The effect of group size on the critical cost

To proceed with our main results below, we first need to characterize how the critical cost $\bar{c}_{\zeta,n}$ from Eq. (8), which determines the replicator dynamic as specified in Lemma 1 and hence whether the minimal rest point $q_{\zeta,c}(n)$ from Eq. (9) is zero or a positive value, depends on group size. The following result provides such a characterization together with further properties of the sequence $\bar{c}_{\zeta,n}$ as a function of n. The proof is in Appendix A.1. Fig. 2 illustrates.

Lemma 2 *For any ζ, the critical cost $\bar{c}_{\zeta,n}$ is strictly decreasing in n with maximum*

$$\bar{c}_\zeta^\dagger \equiv \max_{n>\zeta+1} \bar{c}_{\zeta,n} = \bar{c}_{\zeta,\zeta+2} = \left(\frac{\zeta}{\zeta+1}\right)^\zeta, \tag{14}$$

and limit

$$\bar{c}_\zeta^* \equiv \lim_{n\to\infty} \bar{c}_{\zeta,n} = \rho_\zeta(\zeta) > 0, \tag{15}$$

where

$$\rho_k(\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}, \ k = 0, 1, \ldots \tag{16}$$

denotes the probability mass function of a Poisson distribution with parameter λ.

Moreover, both $\bar{c}_\zeta^\dagger$ and $\bar{c}_\zeta^*$ are decreasing in ζ, with limits $\lim_{\zeta\to\infty} \bar{c}_\zeta^\dagger = \bar{c}_1^* = 1/e$, and $\lim_{\zeta\to\infty} \bar{c}_\zeta^* = 0$.

Lemma 2 indicates a negative effect of group size on the evolution of cooperation in the shirker's dilemma. Namely, increasing the group size decreases the critical cost value $\bar{c}_{\zeta,n}$, and hence increases the range of cost levels for which full shirking is the unique stable rest point. In particular, for cost values satisfying $\bar{c}_{\zeta,n+1} < c < \bar{c}_{\zeta,n}$, cooperation can be sustained at equilibrium for the smaller group size n (i.e., $p_{\zeta,c}(n) > 0$), but it cannot be sustained for the larger group size $n+1$ (i.e., $p_{\zeta,c}(n+1) = 0$). An analogous negative group-size effect is present in the volunteer's dilemma (Nöldeke and Peña, 2020, Lemma 1).

3.1.2 Cost regions

An immediate consequence of Lemma 2 is that, for a given threshold $\zeta \geq 1$, the cost $c \in (0, 1)$ can fall into one of three different regions (see right panel of Fig. 2).

1. For $c \in [\bar{c}_\zeta^\dagger, 1)$, costs are so high that $c \geq \bar{c}_{\zeta,n}$ holds for all group sizes n. In this case, the minimal rest point is given by $q_{\zeta,c}(n) = 1$ for all n. Further, the proportion of volunteers, the success probability, and the expected payoff all reduce to

$$p_{\zeta,c}(n) = \phi_{\zeta,c}(n) = u_{\zeta,c}(n) = 0, \quad \text{for all } n. \tag{17}$$

2. For $c \in (\bar{c}_\zeta^*, \bar{c}_\zeta^\dagger)$, there exists a finite *critical group size* $\bar{n}_{\zeta,c}$, such that $c < \bar{c}_{\zeta,n}$ holds if and only if $n \leq \bar{n}_{\zeta,c}$ holds, while $c \geq \bar{c}_{\zeta,n}$ holds if and only if $n > \bar{n}_{\zeta,c}$ holds. In this case, the minimal rest point corresponds to the non-trivial rest point $q^s_{\zeta,c}(n)$ for $n \leq \bar{n}_{\zeta,c}$, and to the trivial rest point $q = 1$ for $n > \bar{n}_{\zeta,c}$. It follows that the proportion of volunteers, the success probability, and the expected payoff are all positive for group sizes smaller than or

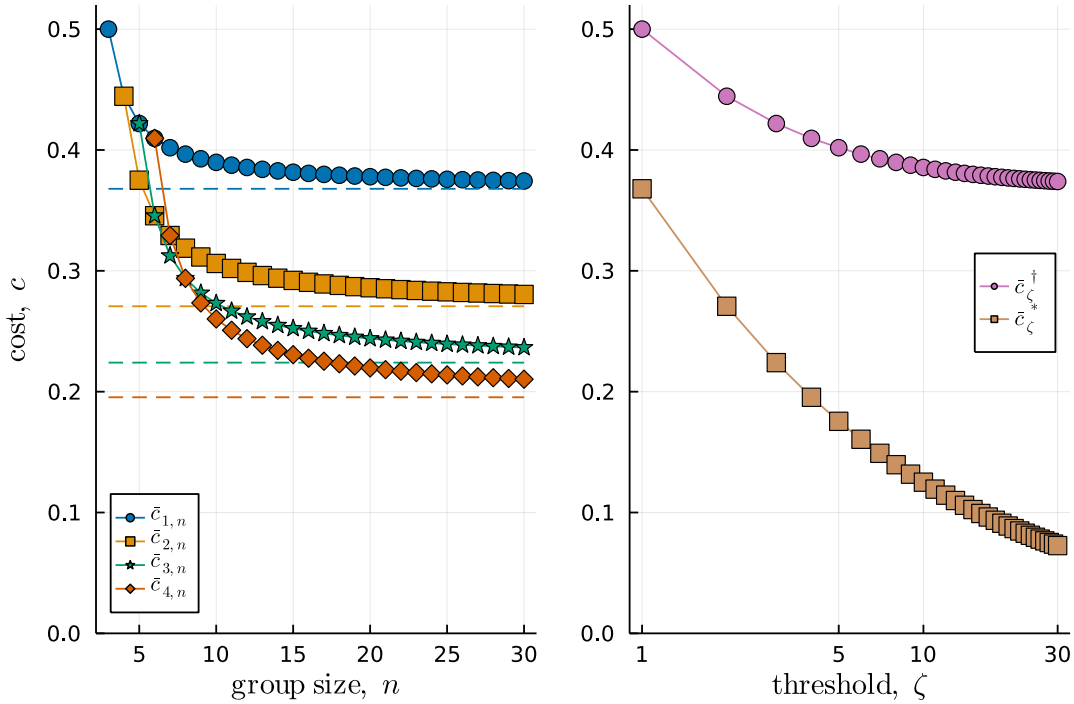

Figure 2: Critical cost $\bar{c}_{\zeta,n}$, maximum critical cost $\bar{c}^{\dagger}_{\zeta}$, and limit critical cost $\bar{c}^{*}_{\zeta}$. *Left:* Critical cost $\bar{c}_{\zeta,n}$ (*circles*) as a function of group size n for $\zeta \in \{1, 2, 3, 4\}$. As stated in Lemma 2, the sequence of critical costs is strictly decreasing for all ζ with limit given by $\bar{c}^{*}_{\zeta}$ (*dashed lines*). *Right:* Maximum critical cost $\bar{c}^{\dagger}_{\zeta}$ (*circles*) and limit critical cost $\bar{c}^{*}_{\zeta}$ (*squares*) as a function of threshold ζ.

equal to the critical group size, but they all drop to zero thereafter, namely

$$\begin{aligned} n \leq \bar{n}_{\zeta,c} &\Rightarrow p_{\zeta,c}(n) > 0, \phi_{\zeta,c}(n) > 0, u_{\zeta,c}(n) > 0, \\ n > \bar{n}_{\zeta,c} &\Rightarrow p_{\zeta,c}(n) = 0, \phi_{\zeta,c}(n) = 0, u_{\zeta,c}(n) = 0. \end{aligned} \tag{18}$$

3. For $c \in (0, \bar{c}^*_\zeta]$, costs are sufficiently low that $c < \bar{c}_{\zeta,n}$ holds for all group sizes n. In this case, the minimal rest point corresponds to the non-trivial rest point $q^s_{\zeta,c}(n)$, and the proportion of volunteers, the success probability, and the expected payoff are all positive for all group sizes n. That is, we have

$$p_{\zeta,c}(n) > 0, \phi_{\zeta,c}(n) > 0, u_{\zeta,c}(n) > 0, \quad \text{for all } n.$$

In the following, we restrict our analysis to costs satisfying $c \in (0, \bar{c}^\dagger_\zeta)$ (i.e., cases 2 and 3 above), thus excluding the uninteresting case in which the proportion of volunteers, the expected payoff, and the success probability are all zero for all group sizes.

3.1.3 Comparative statics with respect to cost

Before looking into the comparative statics with respect to group size and corresponding limit results, let us briefly state some straightforward comparative statics of the proportion of volunteers, the expected payoff, and the success probability with respect to the cost of volunteering c. The proof is in Appendix A.2.

Lemma 3 *For any ζ and n, the proportion of volunteers $p_{\zeta,c}(n)$, the expected payoff $u_{\zeta,c}(n)$, and the success probability $\phi_{\zeta,c}(n)$ are decreasing in the cost of volunteering c. More precisely:*

1. *If $\bar{c}^*_\zeta < c < \bar{c}^\dagger_\zeta$, then $p_{\zeta,c}(n)$, $u_{\zeta,c}(n)$, and $\phi_{\zeta,c}(n)$ are strictly decreasing in c for $c < \bar{c}_{\zeta,n}$, and equal to zero for $c \geq \bar{c}_{\zeta,n}$.*

2. *If $c \leq \bar{c}^*_\zeta$, then $p_{\zeta,c}(n)$, $u_{\zeta,c}(n)$, and $\phi_{\zeta,c}(n)$ are strictly decreasing in c.*

This result is intuitive: the larger the cost of volunteering, the larger the proportion of shirkers at equilibrium and hence the smaller the proportion of volunteers, the expected payoff, and the success probability at equilibrium.

3.2 The effect of group size on the proportion of volunteers

Our first main result on group-size effects pertains to the comparative statics of the probability of individual volunteering (or cooperation, or provision) (10) at the minimal rest point (9). To derive our result, we begin by stating the following lemma characterizing the ordering of the interior rest points for two consecutive group sizes (see Appendix A.3 for a proof):

Lemma 4 *For any ζ, n, and cost $c < \bar{c}_{\zeta,n+1}$, the interior rest points of the replicator dynamic for group size n and group size $n+1$ are ordered in such a way that*

$$q^s_{\zeta,c}(n+1) < q^s_{\zeta,c}(n) < q^u_{\zeta,c}(n+1) < q^u_{\zeta,c}(n) \tag{19}$$

holds.

Rewriting the first inequality in this lemma in terms of the proportion of volunteers, we have that

$$p_{\zeta,c}(n+1) > p_{\zeta,c}(n). \tag{20}$$

holds for any cost of volunteering $c < \bar{c}_{\zeta,n+1}$.

Combining this observation with Lemmas 1 and 2 we obtain the following characterization of the group-size effect on the proportion of volunteers at the minimal rest point. The top left panel of Fig. 3 and Fig. 4 illustrate.

Proposition 1 *For any ζ and $c \in (0, \bar{c}^\dagger_\zeta)$, the proportion of volunteers $p_{\zeta,c}(n)$ is either unimodal or strictly increasing in group size n. More precisely:*

1. *If $\bar{c}^*_\zeta < c < \bar{c}^\dagger_\zeta$, then $p_{\zeta,c}(n)$ is strictly increasing in n for $n \leq \bar{n}_{\zeta,c}$, and equal to zero for $n > \bar{n}_{\zeta,c}$.*

2. *If $c \leq \bar{c}^*_\zeta$, then $p_{\zeta,c}(n)$ is strictly increasing in n.*

This result demonstrates a positive group-size effect on cooperation in the shirker's dilemma. For sufficiently low costs (i.e., $c \in (0, \bar{c}^*_\zeta)$), the proportion of volunteers at the minimal rest point is strictly increasing in group size, so that the larger the group size the larger the proportion of volunteers in the population at equilibrium. For intermediate costs (i.e., $c \in [\bar{c}^*_\zeta, \bar{c}^\dagger_\zeta)$), the proportion of volunteers at the minimal rest point is strictly increasing with group size up to the critical group size (for group sizes $n \in \{\zeta+2, \ldots, \bar{n}_{\zeta,c}\}$) before falling to zero thereafter (for group sizes $n > \bar{n}_{\zeta,c}$). It follows that, from the perspective of maximizing the proportion of volunteers at the minimal equilibrium, the optimal group size is either intermediate and equal to the critical group size (for intermediate costs) or infinite (for low costs).

This said, note that Lemma 4 also demonstrates a negative group-size effect when looking into the comparative statics of the basin of attraction of the interior rest point $q^s_{\zeta,c}(n)$ sustaining non-zero volunteering at equilibrium. Since the size of this basin of attraction is determined by the proportion of shirkers at the unstable rest point $q^u_{\zeta,c}(n)$ and this proportion decreases with group size, it follows that the basin of attraction of $q^s_{\zeta,c}(n)$ decreases (and the basin of attraction of full shirking increases) with group size (see Fig. 5 for an illustration). Overall, the proportion of volunteers at equilibrium can increase with increasing group size, but such an increase is accompanied by a decrease in its basin of attraction.

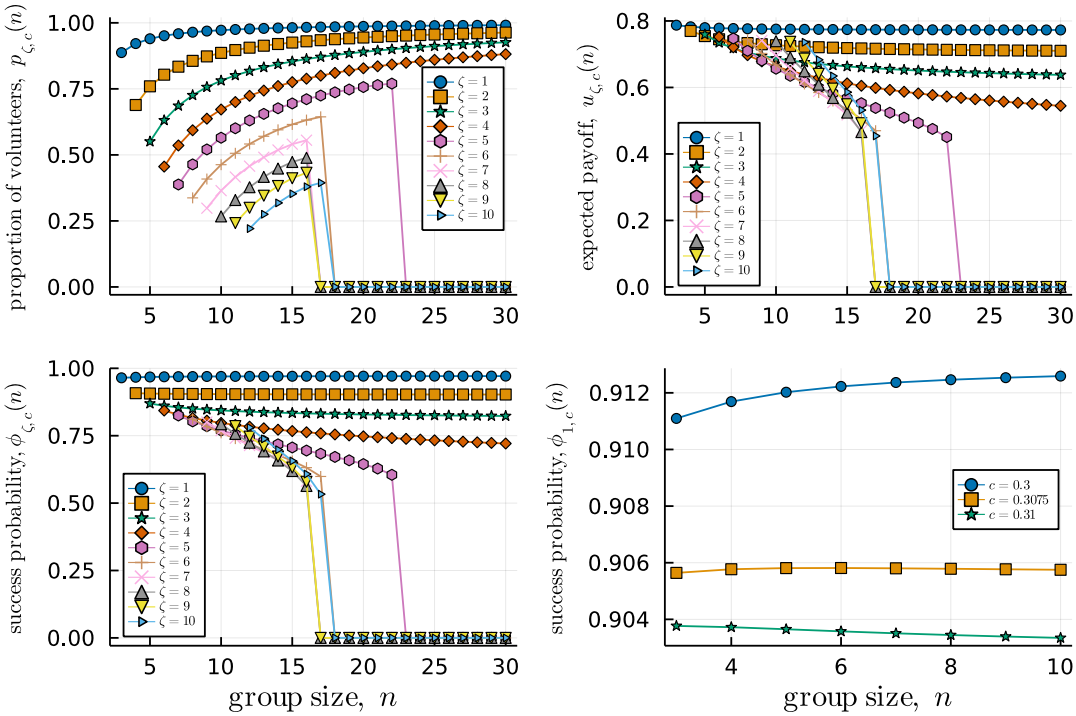

Figure 3: Volunteering probability $p_{\zeta,c}(n)$, expected payoff $u_{\zeta,c}(n)$, and success probability $\phi_{\zeta,c}(n)$ as functions of group size. *Top left, top right, bottom right*: proportion of volunteers $p_{\zeta,c}(n)$, expected payoff $u_{\zeta,c}(n)$, and success probability $\phi_{\zeta,c}(n)$ for $\zeta \in \{1, \ldots, 10\}$, $c = 0.2$, and $n \in \{\zeta + 2, \ldots, 30\}$. For $\zeta \geq 4$, all three quantities eventually fall to zero since $c > \bar{c}_4^* \approx 0.1954$, with critical group sizes given by $\bar{n}_{4,0.2} = 88$ (not shown), $\bar{n}_{5,0.2} = 22$, $\bar{n}_{6,0.2} = \bar{n}_{10,0.2} = 17$, and $\bar{n}_{7,0.2} = \bar{n}_{8,0.2} = \bar{n}_{9,0.2} = 16$. As stated in Proposition 1, the proportion of volunteers (resp. expected payoff) is strictly increasing (resp. decreasing) when it is positive. *Bottom right:* success probability $\phi_{\zeta,c}$ for $\zeta = 1$, three different values of cost c, and $n \in \{3, \ldots, 10\}$. For this range of group sizes, $\phi_{1,c}$ is strictly increasing for $c = 0.3$, unimodal (with local maximum at $n = 6$) for $c = 0.3075$, and strictly decreasing for $c = 0.31$.

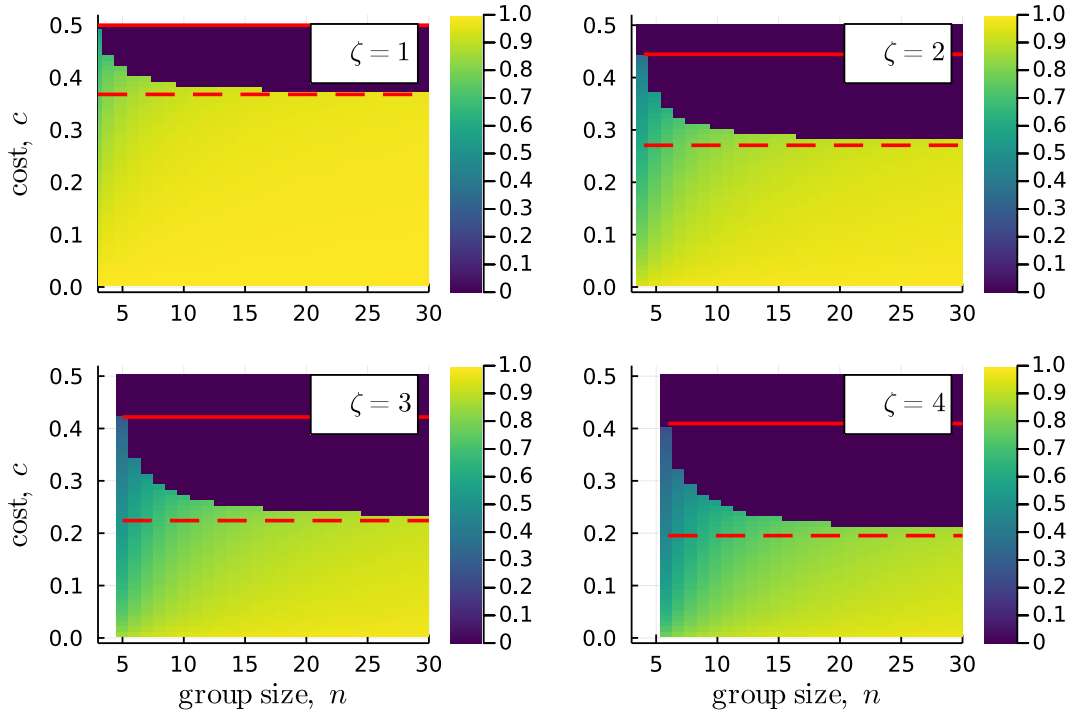

Figure 4: Proportion of volunteers $p_{\zeta,c}(n)$ for $\zeta \in \{1, 2, 3, 4\}$, $c \in \{0.01, 0.02, \ldots, 0.5\}$, and $n \in \{\zeta + 2, \ldots, 30\}$. The maximum and limit critical costs, $\bar{c}^{\dagger}_{\zeta}$ and $\bar{c}^{*}_{\zeta}$, are shown respectively as *red solid* and *red dashed* lines. As proven in Proposition 1, for costs between these two critical costs, the proportion of volunteers is first strictly increasing and then drops to zero. For costs below $\bar{c}^{*}_{\zeta}$ (*red dashed line*), the proportion of volunteers is strictly increasing.

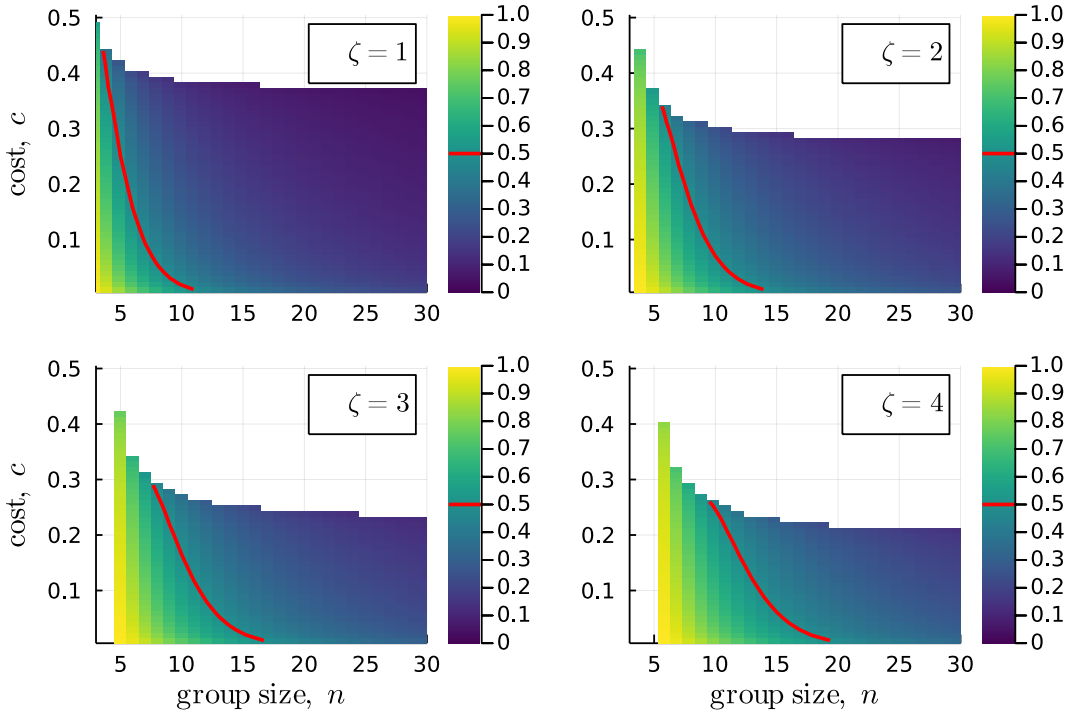

Figure 5: Location of the unstable rest point $q^u_{\zeta,c}(n)$, or equivalently, the size of the basin of attraction of the stable rest point $q^s_{\zeta,c}(n)$ for $\zeta \in \{1, 2, 3, 4\}$, $c \in \{0.01, 0.02, \ldots, 0.5\}$, and $n \in \{\zeta + 2, \ldots, 30\}$. The *red solid line* represents the contour line at which $q^u_{\zeta,c}(n) = 1/2$ holds. To the left of this line, $q^s_{\zeta,c}(n)$ has the largest basin of attraction. To the right of this line, full shirking has the largest basin of attraction. As indicated in the main text, $q^u_{\zeta,c}(n)$ decreases with group size n for fized threshold of shirkers ζ and cost of volunteering c.

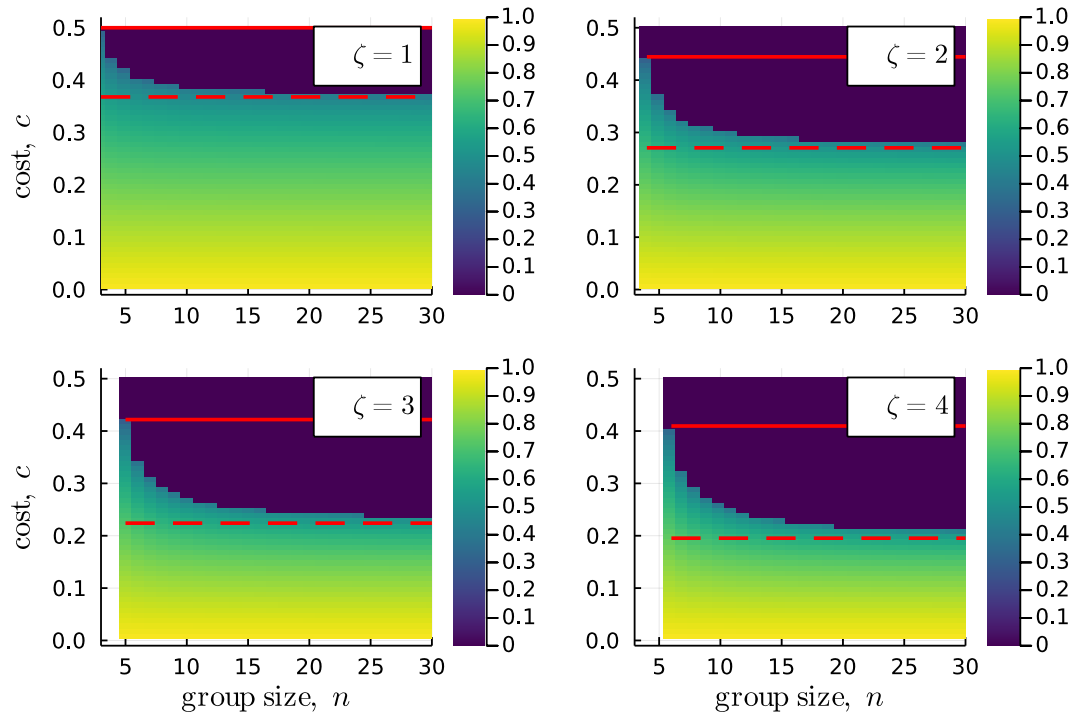

Figure 6: Expected payoff $u_{\zeta,c}(n)$ for $\zeta \in \{1, 2, 3, 4\}$, $c \in \{0.01, 0.02, \ldots, 0.5\}$, and $n \in \{\zeta + 2, \ldots, 30\}$. The maximum and limit critical costs, $\bar{c}^{\dagger}_{\zeta}$ and $\bar{c}^{*}_{\zeta}$, are shown respectively as *red solid* and *red dashed* lines. As proven in Proposition 2, for costs between these two critical costs, the expected payoff is strictly decreasing and drops to zero for finite but sufficiently large group sizes. For costs below $\bar{c}^{*}_{\zeta}$ (*red dashed line*), the expected payoff is strictly decreasing.

3.3 The effect of group size on the expected payoff

Next, we address the effect of group size on the expected payoff. The following result shows that the group-size effect on the expected payoff at equilibrium is negative. See Appendix A.4 for a proof, and the top right panel of Fig. 3 and Fig. 6 for an illustration.

Proposition 2 *For any ζ and $c \in (0, \bar{c}^{\dagger}_{\zeta})$, the expected payoff $u_{\zeta,c}(n)$ is decreasing in group size n. More precisely:*

1. *If $\bar{c}^{*}_{\zeta} < c < \bar{c}^{\dagger}_{\zeta}$, then $u_{\zeta,c}(n)$ is strictly decreasing in n for $n \leq \bar{n}_{\zeta,c}$, and equal to zero for $n > \bar{n}_{\zeta,c}$.*

2. *If $c \leq \bar{c}^{*}_{\zeta}$, then $u_{\zeta,c}(n)$ is strictly decreasing in n.*

This result contrasts with the positive group-size effect given in Proposition 1 when considering the proportion of volunteers. Together, Propositions 1 and 2 indicate that although there can be less shirking at equilibrium as the group size increases, the increased cooperation rate is not

enough to increase (via a concomitant increase in the success probability, see Eq. (13)) the expected payoff, and hence the fitness, of individuals in the population.

3.4 The effect of group size on the success probability

The previous two results naturally lead to the question of how the success probability that the public good is provided at equilibrium changes as group size increases. From Eq. (18), it is clear that the success probability is equal to zero if $\bar{c}^*_\zeta < c < \bar{c}^\dagger_\zeta$ and $n > \bar{n}_{\zeta,c}$. In the remaining and more interesting cases where either (i) $\bar{c}^*_\zeta < c < \bar{c}^\dagger_\zeta$ and $n \leq \bar{n}_{\zeta,c}$ or (ii) $c \leq \bar{c}^*_\zeta$, the effect of group size on the success probability cannot be signed unambiguously. To get a sense why this is so, notice that from Eq. (13) the success probability can be written as

$$\phi_{\zeta,c}(n) = u_{\zeta,c}(n) + p_{\zeta,c}(n) \cdot c. \tag{21}$$

Taking the first forward difference with respect to group size n, we then have:

$$\Delta_n \phi_{\zeta,c}(n) = \Delta_n u_{\zeta,c}(n) + \Delta_n p_{\zeta,c}(n) \cdot c, \tag{22}$$

where $\Delta_n f(n) \equiv f(n+1) - f(n)$. Eq. (22) decomposes the group-size effect on the success probability at the minimal rest point ($\Delta_n \phi_{\zeta,c}(n)$) into the group-size effect on the equilibrium payoff ($\Delta_n u_{\zeta,c}(n)$) and the group-size effect on the proportion of volunteers ($\Delta_n p_{\zeta,c}(n)$). For the cases we are interested in, Propositions 1 and 2 imply that these two effects have opposite signs. This not only precludes us from obtaining analytical results about the effect of group size on the success probability, but does indeed imply that whether or not the success probability increases or decreases with group size depends on parameter values. This is illustrated in the bottom right panel of Fig. 3, which shows that for some cost levels the success probability is increasing with group size; for other cost levels it is first increasing and then decreasing with group size; and for yet other cost levels it decreases with group size. Fig. 7 further illustrates the dependence of the success probability on the cost of volunteering and the group size, for $\zeta \leq 4$. Even though there is no general result (as illustrated by the bottom right panel of Fig. 3), Fig. 7 suggests that in most cases the success probability remains relatively high, and almost unchanged across group sizes. A similar effect can be seen in Fig 6, which illustrates the dependency of the expected payoff on group size. This prompt us to look into the limits of these two quantities, and of the volunteering probability, when group size tends to infinity. We do so in the following section.

3.5 The limit of infinitely large groups

From the results in Propositions 1 and 2 it is clear that the limits

$$p^*_{\zeta,c} = \lim_{n\to\infty} p_{\zeta,c}(n), \tag{23}$$

$$u^*_{\zeta,c} = \lim_{n\to\infty} u_{\zeta,c}(n) \tag{24}$$

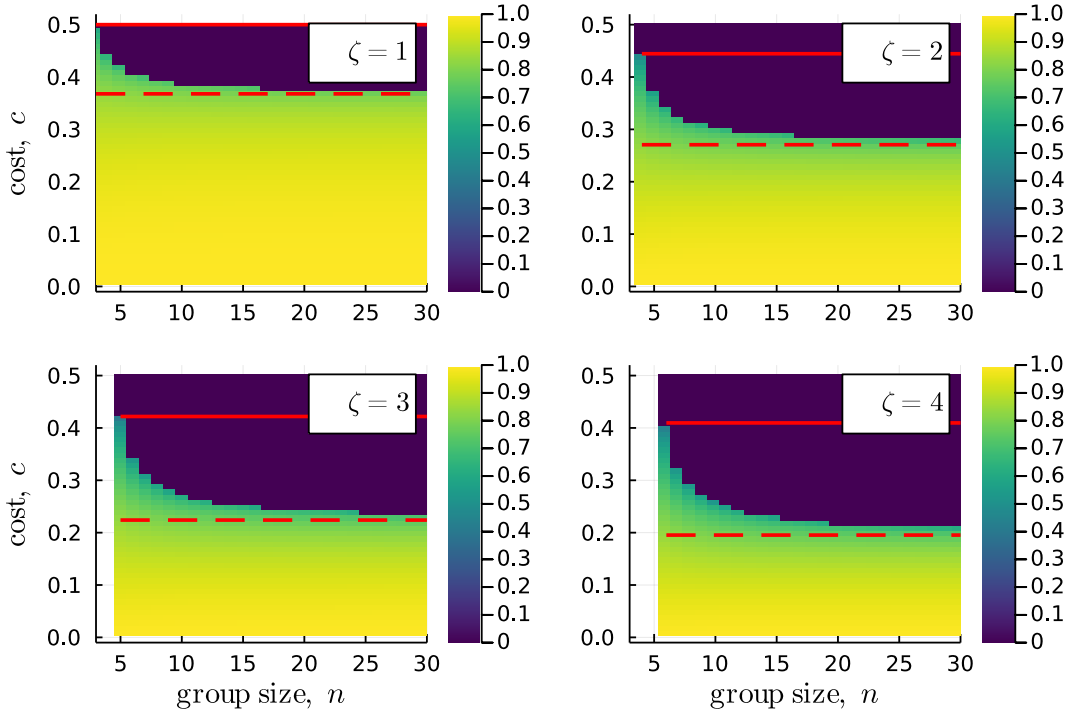

Figure 7: Success probability $\phi_{\zeta,c}(n)$ for $\zeta \in \{1, 2, 3, 4\}$, $c \in \{0.01, 0.02, \ldots, 0.5\}$, and $n \in \{\zeta + 2, \ldots, 30\}$. The maximum and limit critical costs, $\bar{c}_\zeta^\dagger$ and $\bar{c}_\zeta^*$, are shown respectively as *red solid* and *red dashed* lines.

are well defined. Furthermore, from Eq. (21), it follows that

$$\phi^*_{\zeta,c} = \lim_{n\to\infty} \phi_{\zeta,c}(n) \tag{25}$$

is also well defined and equal to

$$\phi^*_{\zeta,c} = u^*_{\zeta,c} + p^*_{\zeta,c} \cdot c.$$

The following results characterize the limits (23), (24), and (25) in the non-trivial case where $c \leq \bar{c}^*_\zeta$ (as for $c > \bar{c}^*_\zeta$ all limits are equal to zero). To obtain them, we first define the following quantities:

$$\begin{aligned} \lambda_{\zeta,c}(n) &= (n-1) \cdot q_{\zeta,c}(n), \\ \mu_{\zeta,c}(n) &= n \cdot q_{\zeta,c}(n). \end{aligned} \tag{26}$$

From the perspective of a given focal player, $\lambda_{\zeta,c}(n)$ is the expected number of co-players that are shirkers at the minimal rest point $q_{\zeta,c}(n)$. In contrast $\mu_{\zeta,c}(n)$ is the expected number of group members that are shirkers at $q_{\zeta,c}(n)$ from the perspective from an outside observer.

We begin by characterizing the limit

$$\lambda^*_{\zeta,c} = \lim_{n\to\infty} \lambda_{\zeta,c}(n).$$

Recalling that $\rho_k(\lambda)$ denotes the probability mass function of a Poisson distribution with parameter λ (see Eq. (16)), and by the Poisson approximation to the binomial distribution (using similar arguments as in Lemma 3 of Nöldeke and Peña, 2020) we have:

Lemma 5 *Let $c < \bar{c}^*_\zeta$. Then $\lambda^*_{\zeta,c}$ is given by the unique solution to*

$$\rho_\zeta(\lambda) = c \tag{27}$$

in the interval $\lambda \in (0, \zeta]$.

Eq. (27) can be regarded as the counterpart of the pivotality condition (7) when the number of shirkers among co-players follows a Poisson ditribution with mean value λ. For $c < \bar{c}^*_\zeta$, Eq. (27) has two solutions. Lemma 5 indicates that the limit value $\lambda^*_{\zeta,c}$ of the expected number of other shirkers in a group is given by the smaller of these two solutions. This is the natural counterpart of Lemma 1 identifying the smaller of the two solutions to the pivotality condition (7) as the (non-trivial) minimal rest point. See Fig. 8 for an illustration of these arguments and of the following Proposition.

We are now ready to state our limit results. Appendix A.5 proves:

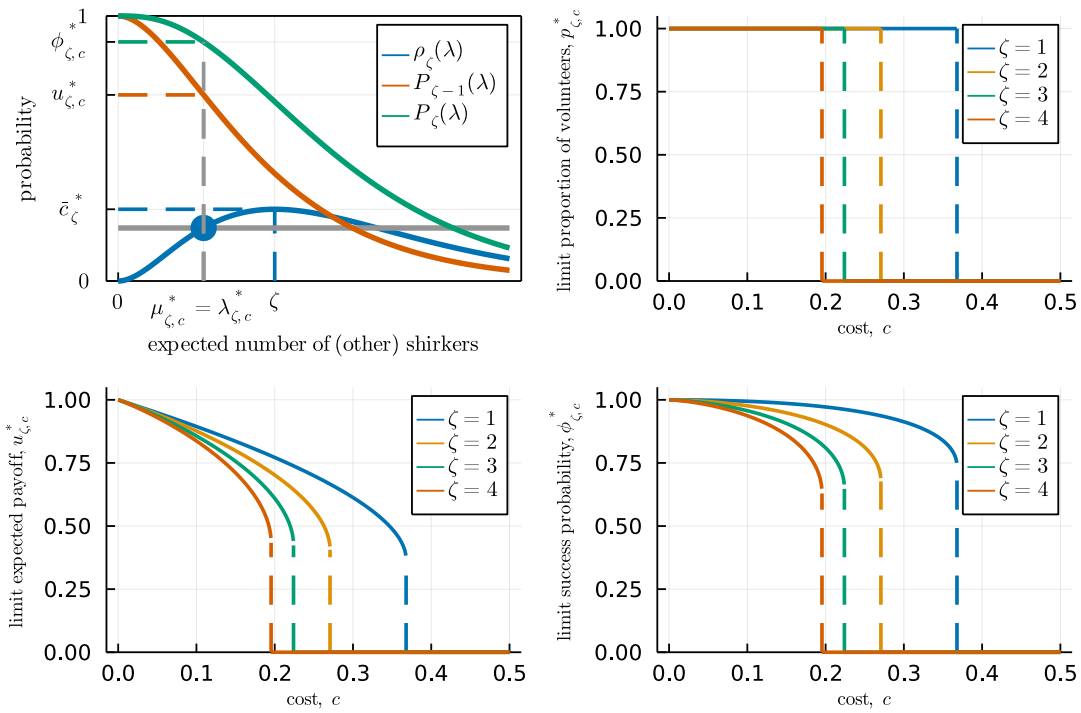

Figure 8: Illustration of the limit results given in Lemma 5 and Proposition 3. *Top left:* The limit of the expected number of other shirkers $\lambda^*_{\zeta,c}$, and the limit of the expected number of shirkers $\mu^*_{\zeta,c}$ coincide and are given by the smaller of the two solutions to the pivotality condition $\rho_\zeta(\lambda) = c$. The expected payoff $u^*_{\zeta,c}$ converges to the probability $P_{\zeta-1}(\mu^*_{\zeta,c})$ that there are at most $\zeta - 1$ shirkers given that the number of shirkers follows a Poisson distribution with expected value $\mu^*_{\zeta,c}$. Likewise, the success probability $\phi^*_{\zeta,c}$ converges to the probability $P_\zeta(\mu^*_{\zeta,c})$. Here, $\zeta = 2$ and $c = 0.2$. *Top right, bottom left, and bottom right:* Limit proportion of volunteers, $p^*_{\zeta,c}$, expected payoff $u^*_{\zeta,c}$, and success probability $\phi^*_{\zeta,c}$ as functions of cost c, for $\zeta \in \{1, 2, 3, 4\}$.

Proposition 3 *Let $c \leq \bar{c}^*_\zeta$. Then,*

$$p^*_{\zeta,c} = 1,\ \mu^*_{\zeta,c} = \lambda^*_{\zeta,c},\ \phi^*_{\zeta,c} = P_\zeta(\mu^*_{\zeta,c}) > 0,\ u^*_{\zeta,c} = P_{\zeta-1}(\mu^*_{\zeta,c}) > 0, \tag{28}$$

where

$$P_k(\lambda) = \sum_{\ell=0}^{k} \rho_\ell(\lambda),\ k = 0, 1, \ldots$$

is the cumulative distribution function of a Poisson distribution with parameter λ.

Proposition 3 demonstrates that, for sufficiently low costs of volunteering, the proportion of volunteers at the minimal rest point converges to one as the group size tends to infinity. In such a limit, both the success probability and the expected payoff at equilibrium are positive values that, as illustrated in Fig. 8, can be relatively large for small values of the threshold number of shirkers ζ. In particular, even though the expected payoff at the minimal rest point decreases with group size (as demonstrated in Proposition 2), the expected payoff can still be substantially larger than the expected payoff at the full shirking equilibrium (which is zero). Likewise, the probability that collective action is successful can be relatively high. For instance, for a cost of volunteering $c = 0.1$ (so that the cost is equal to one tenth of the benefit), the limiting success probabilities $\phi^*_{\zeta,0.1}$ for $\zeta \in \{1, 2, 3, 4\}$ are always greater than 90% and approximately given by $\phi^*_{1,0.1} \approx 0.994$, $\phi^*_{2,0.1} \approx 0.976$, $\phi^*_{3,0.1} \approx 0.957$, and $\phi^*_{4,0.1} \approx 0.937$. This said, note that a similar caveat concerning equilibrium selection as the one we pointed out for finite group sizes applies in the limit of infinitely large group sizes. In this case, it can be shown (by similar arguments demonstrating that $p^*_{\zeta,c} = 1$ holds) that the basin of attraction of the volunteering equilibrium $q^s_{\zeta,c}(n)$ effectively shrinks to zero.

4 Discussion

When the cost of volunteering is not too large, the evolutionary dynamics of the shirker's dilemma are characterized by two stable equilibria: a polymorphic equilibrium sustaining some cooperation (or a "volunteering" equilibrium) and a monomorphic equilibrium where everybody shirks (or a "full-shirking" equilibrium). We have investigated the group-size effects on the volunteering equilibrium and on a set of quantities related to it, namely (i) the probability of volunteering, (ii) the probability that the public good is provided (or that collective action is successful), and (iii) the expected payoff at such equilibrium, together with (iv) the size of its basin of attraction. Our analysis reveals non-trivial comparative statics. On the one hand, we have found that, for all thresholds $\zeta \geq 1$, and for sufficiently low costs, the probability of volunteering at the volunteering equilibrium increases with group size, and converges to one in the limit of large group sizes. We also found that the probability that the public good is provided at equilibrium can increase with group size and that it converges to a positive value.

On the other hand, the expected payoff, although positive, decreases with group size. Moreover, the basin of attraction of the volunteering equilibrium is also decreasing with group size, and converges to zero in the limit of infinitely large group sizes. These findings are in juxtaposition to those of the volunteer's dilemma (Diekmann, 1985) and its generalization to threshold games in which a minimal number θ of 'volunteers' are needed (Palfrey and Rosenthal, 1984), where the probability of cooperation and the overall probability that the public good is provided at equilibrium both decrease with group size $n > \theta$ (Nöldeke and Peña, 2020). In the limit, the corresponding "volunteering" equilibrium of the volunteer's dilemma tends to zero and its basin of attraction to one—a result that is the mirror image of what happens in the shirker's dilemma analyzed in this paper.

As it has been previously noted, a shirker's dilemma with maximum number of shirkers $\zeta < n - 1$ is equivalent to a (generalized) volunteer's dilemma with minimum number of cooperators $\theta = n - \zeta$. This equivalence allows us to make a connection between (i) our result on the comparative statics of the volunteering probability with respect to group size for the shirker's dilemma, and (ii) the comparative statics of the volunteering probability with respect to the threshold θ for the volunteer's dilemma, analyzed by De Jaegher (2020). To make this connection explicit, let us write $p_{\theta,c}(n)$ for the proportion of cooperators at the stable polymorphic equilibrium of a volunteer's dilemma with threshold $1 < \theta < n$ and cost of volunteering c. In terms of this notation, our comparative statics exercise for the shirker's dilemma is equivalent to considering *both* an increase in the group size n and in the threshold of contributors θ on this quantity, that is, we have asked about the sign of the difference $p_{\theta+1,c}(n+1) - p_{\theta,c}(n)$ for a given value of n. Adding and subtracting $p_{\theta+1,c}(n)$, and rearranging, we get

$$p_{\theta+1,c}(n+1) - p_{\theta,c}(n) = \underbrace{p_{\theta+1,c}(n+1) - p_{\theta+1,c}(n)}_{\Delta_n p_{\theta+1,c}(n)} + \underbrace{p_{\theta+1,c}(n) - p_{\theta,c}(n)}_{\Delta_\theta p_{\theta,c}(n)},$$

so that we can express the group-size effect on the shirker's dilemma (on the left hand side of the equation) as the sum of the group-size effect and the threshold-size effect on the volunteer's dilemma (on the right hand side). We have proved in Proposition 1 that sufficiently low costs of volunteering, the expression on the left hand side is positive. We also know (Nöldeke and Peña, 2020, Proposition 1) that the group-size effect on the volunteer's dilemma is negative, i.e., $\Delta_n p_{\theta+1,c}(n) < 0$ holds. It then follows that the threshold-size effect on the volunteer's dilemma must be positive, i.e., $\Delta_\theta p_{\theta,c}(n) > 0$ must hold. This result on higher thresholds increasing the proportion of cooperators at the interior stable rest point of the volunteer's dilemma when costs are low agrees with De Jaegher (2020, Result B1). This also provides an alternative explanation behind the positive group-size effects on the volunteering probability of the shirker's dilemma that we have analyzed in this paper: Although an increase in group size is detrimental to the evolution of cooperation in the volunteer's dilemma, in the shirker's dilemma such an increase in group size (while keeping fixed the threshold of shirkers ζ) is accompanied, by definition,

by a concomitant increase in the threshold of volunteers, which encourages the evolution of cooperation. As we have shown here, the second effect dominates the first, thus leading to different total comparative statics for the shirker's dilemma.

We motivated the shirker's dilemma in the Introduction with the example of the sentinel behavior of Arabian babblers. For a second example, consider the mobbing of terrestrial predators (such as snakes) by groups of birds. When one group member reveals a snake dug in the sand and lurking for prey, that bird emits an alarm call, approaches the snake, and engages in bodily displays that make the bird look as big as possible. Noticing this, all or most group members then follow suit. In most cases, the primary and most important benefit of such mobbing is that it causes the predator to recognize that it has been detected, and to leave the location (Caro, 2005). Here, too, as in the case of a raptor approaching a group of Arabian babblers, the predator presumably moves away when it perceives that most of its potential prey are vigilant, and that its outside options elsewhere might be more attractive.

In a similar vein, the shirker's dilemma may better capture the strategic nature of many collective action problems that have been traditionally conceptualized as volunteer's dilemmas. For instance, Archetti (2009) lists as an example of a volunteer's dilemma the case of the amoeba *Dictyostelium discoideum* that, when facing starvation, differentiates into a ball of reproductive spores (shirkers) and a sterile stalk (volunteers). However, it can be argued that what is needed for such collective action to be successful is not that enough cells become part of the stalk, but that there is a cap on the number of cells becoming reproductive that will have to be maintained at the top. Likewise, Raihani and Bshary (2011) argues that the punishment of free riders in an n-player social dilemma is best described (at least conceptually) by assuming that the benefits of punishment is a step function of the amount of punishment (e.g., the number of punishers in a group). For many social situations it might be more plausible that the inflexion point of such a step function (and hence the success of punishment as a collective action) is determined more by the maximum number of allowed second-order free riders (i.e., individuals that will refrain from punishing free riders) than by the minimum number of punishers. A maximal allowable number of shirkers instead of a minimal allowable number of volunteers (and hence the payoff structure of the shirker's dilemma in contrast to that of a volunteer's dilemma), may be also relevant in other human interactions. For example, a hacker may scan organizations' computer systems for vulnerable entry points, left open by employees who compromised computer security instructions. By itself, a particular entry point does not yet guarantee successful hacking of the organization's website or data for ransom purposes. The hacker may therefore decide to attack all vulnerable access points simultaneously only if their number exceeds a certain threshold and otherwise continue its search for a more vulnerable organization. In all of these situations, the shirker's dilemma, and its associated group-size effects, might be a better representation of the social dilemma at hand.

The model we explored is a benchmark for more elaborate, relevant models. For instance, we modeled interactions in a well-mixed population and hence among unrelated individuals. It would be of interest to see if our predictions are still supported when moving to spatially or

family-structured populations where interactants are related, and where relatedness itself can be a function of group size (Lehmann and Rousset, 2010; Peña et al., 2015). Another dimension for possible extensions arises by considering games with continuous levels of effort and smooth benefits, rather than the binary-choice game with sharp thresholds that we analyzed in this paper. The pure equilibria of such continuous-action games do not predict the independent randomization characteristic of mixed-strategy equilibria of binary-action games, which received only partial empirical support in the context of the sentinel behavior of Arabian babblers that has motivated our analysis (Heifetz et al., 2021). Further, such pure equilibria in continuous action games are not prone to the well-known peculiarity of mixed-strategy equilibria that individuals with higher costs or lower benefits cooperate with a higher probability at equilibrium (Diekmann, 1994). Considering continuous-action models would also allow us to explore if our result that shirking decreases with group size holds more generally or is a peculiarity of the binary-action models we have used here.

Acknowledgements

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A Proofs

A.1 Proof of Lemma 2

The equation $\pi_{\zeta,n+1}(q) = \pi_{\zeta,n}(q)$ has a unique solution in the interval $(0,1)$, given by $q = \zeta/n$. From the unimodality properties of the pivot probability $\pi_{\zeta,n}(q)$ listed in Section 2.2, $\zeta/(n-1)$ maximizes $\pi_{\zeta,n}(q)$ over $q \in (0,1)$. Thus, recalling the definition of the critical cost $\bar{c}_{\zeta,n}$ given in Eq. (8), we have $\bar{c}_{\zeta,n} = \pi_{\zeta,n}(\zeta/(n-1)) > \pi_{\zeta,n}(\zeta/n) = \pi_{\zeta,n+1}(\zeta/n) = \bar{c}_{\zeta,n+1}$. This proves that $\bar{c}_{\zeta,n}$ is strictly decreasing in n and thus maximized at the smallest possible value of n, which is $n = \zeta + 2$. After setting $n = \zeta + 2$ in (8) and simplifying this yields (14).

To prove the limit result, setting $m = n - 1$ in (6) and (8) and taking the limit we obtain

$$\bar{c}^*_\zeta = \lim_{m\to\infty} \binom{m}{\zeta} \left(\frac{\zeta}{m}\right)^\zeta \left(1 - \frac{\zeta}{m}\right)^{m-\zeta},$$

so that (15) follows from the Poisson approximation to the binomial distribution.

Finally, to prove that $\bar{c}^\dagger_\zeta$ and $\bar{c}^*_\zeta$ are decreasing in ζ, note that we can write $\bar{c}^\dagger_\zeta = 1/(1+1/\zeta)^\zeta$ and $\bar{c}^*_{\zeta+1}/\bar{c}^*_\zeta = (1/e)(1+1/\zeta)^\zeta$. The function $(1+1/x)^x$ increases with x for $x > 0$ (see, e.g., Hardy et al. 1952, Theorem 140) and its limit as x approaches infinity is e. Hence, $\bar{c}^\dagger_\zeta$ increases with ζ and $\bar{c}^*_{\zeta+1}/\bar{c}^*_\zeta < 1$ holds for all $\zeta \geq 1$.

A.2 Proof of Lemma 3

For $c \geq \bar{c}_{\zeta,n}$, the minimal rest point is the trivial rest point $q = 1$, and Eq. (18) applies. For $c < \bar{c}_{\zeta,n}$ the minimal rest point is the stable interior rest point, i.e., $q_{\zeta,c}(n) = q^s_{\zeta,c}(n)$ holds. As stated in Lemma 2.3 the stable interior rest point corresponds to the solution of the pivotality condition (7) in the interval $(0, \zeta/(n-1))$, where the pivot probability (6) is strictly increasing in q. It follows that $q_{\zeta,c}(n)$ is strictly increasing in c for $c < \bar{c}_{\zeta,n}$. From Eq. (10) it follows that the proportion of volunteers $p_{\zeta,c}(n)$ is strictly decreasing in c. That the success probability $\phi_{\zeta,c}(n)$ and the expected payoff $u_{\zeta,c}(n)$ are also strictly decreasing in c follows from Eq. (11) and (12) upon noting that the cumulative distribution defined in (1) is strictly decreasing in q for $q \in (0, 1)$.

A.3 Proof of Lemma 4

For $c < \bar{c}_{\zeta,n+1}$, the replicator dynamic (4) has two sign changes and hence two interior rest points for group sizes n and $n+1$. It can also be checked (see, e.g., Eq. 3 of Peña and Nöldeke (2018)) that the pivot probability (6) satisfies:

$$\pi_{\zeta,n}(q) = \pi_{\zeta,n+1}(q) - \frac{q}{n}\frac{\mathrm{d}\pi_{\zeta,n+1}}{\mathrm{d}q}(q),$$

which implies

$$g_{\zeta,n,c}(q) = g_{\zeta,n+1,c}(q) - \frac{q}{n}\frac{\mathrm{d}g_{\zeta,n+1,c}}{\mathrm{d}q}(q). \tag{29}$$

Evaluating Eq. (29) at $q^u_{\zeta,n+1}$, so that $g_{\zeta,n+1,c}(q^u_{\zeta,n+1}) = 0$, we obtain

$$g_{\zeta,n,c}(q^u_{\zeta,n+1}) = -\frac{q^u_{\zeta,n+1}}{n}\frac{\mathrm{d}g_{\zeta,n+1,c}}{\mathrm{d}q}(q^u_{\zeta,n+1}) < 0, \tag{30}$$

where the inequality holds because $\mathrm{d}g_{\zeta,n+1,c}(q^u_{\zeta,n+1})/\mathrm{d}q > 0$. Likewise, evaluating Eq. (29) at $q^s_{\zeta,n+1}$, so that $g_{\zeta,n+1,c}(q^s_{\zeta,n+1}) = 0$, we obtain

$$g_{\zeta,n,c}(q^s_{\zeta,n+1}) = -\frac{q^s_{\zeta,n+1}}{n}\frac{\mathrm{d}g_{\zeta,n+1,c}}{\mathrm{d}q}(q^s_{\zeta,n+1}) > 0, \tag{31}$$

where the inequality follows from $\mathrm{d}g_{\zeta,n+1,c}(q^u_{\zeta,n+1})/\mathrm{d}q < 0$. As noted above, the gain function $g_{\zeta,n,c}(q)$ has two sign changes. It follows from Eqs. (30) and (31), and from $g_{\zeta,n,c}(1) = c > 0$, that (i) the location of the first sign change, and hence of the rest point $q^s_{\zeta,n}$, is in the interval $(q^s_{\zeta,n+1}, q^u_{\zeta,n+1})$, and (ii) the location of the second sign change, and hence of the rest point $q^u_{\zeta,n}$, is in the interval $(q^u_{\zeta,n+1}, 1)$. This proves the inequalities in (19).

A.4 Proof of Proposition 2

That the sequence of expected payoffs is equal to zero if $\bar{c}^*_\zeta \leq c < \bar{c}^\dagger_\zeta$ and $n > \bar{n}_{\zeta,c}$ follows from Eq. (18). We thus need to show that $u_{\zeta,c}(n+1) < u_{\zeta,c}(n)$ holds in the remaining cases. This result is immediate for costs large enough such that $q_{\zeta,c}(n) < 1$ and $q_{\zeta,c}(n+1) = 1$ hold, because then we have $u_{\zeta,c}(n) > 0$ and $u_{\zeta,c}(n+1) = 0$. In the following, let us suppose $q_{\zeta,c}(n) < 1$ and $q_{\zeta,c}(n+1) < 1$.

Showing that $u_{\zeta,c}(n+1) < u_{\zeta,c}(n)$ holds is equivalent to showing that $1 - u_{\zeta,c}(n+1) > 1 - u_{\zeta,c}(n)$ holds. From (1) and (12) this inequality is in turn equivalent to

$$\sum_{k=\zeta}^{n} \binom{n}{k} \left(q_{\zeta,c}(n+1)\right)^k \left(1 - q_{\zeta,c}(n+1)\right)^{n-k} > \sum_{k=\zeta}^{n-1} \binom{n-1}{k} \left(q_{\zeta,c}(n)\right)^k \left(1 - q_{\zeta,c}(n)\right)^{n-1-k}. \quad (32)$$

Begin by noting that (32) holds if the inequality

$$\binom{n}{k} \left(q_{\zeta,c}(n+1)\right)^k \left(1 - q_{\zeta,c}(n+1)\right)^{n-k} \geq \binom{n-1}{k} \left(q_{\zeta,c}(n)\right)^k \left(1 - q_{\zeta,c}(n)\right)^{n-1-k} \quad (33)$$

holds for all $k \in \{\zeta, \ldots, n-1\}$, This is so because the last summand on the left side of (32), that is, $\left(q_{\zeta,c}(n+1)\right)^n$, is strictly positive.

From the pivotality condition (7) that determines $q_{\zeta,c}(n)$ and $q_{\zeta,c}(n)$, we know that for $k = \zeta$ the terms on the left side and the right side of (33) are both equal to c.

Let

$$L(k) = \frac{\binom{n-1}{k} \left(q_{\zeta,c}(n)\right)^k \left(1 - q_{\zeta,c}(n)\right)^{n-1-k}}{\binom{n}{k} \left(q_{\zeta,c}(n+1)\right)^k \left(1 - q_{\zeta,c}(n+1)\right)^{n-k}} \quad (34)$$

denote the likelihood ratio for having, among the co-players of a focal individual, exactly k shirkers at the minimal rest point with group sizes n and $n+1$. Obviously, (33) is equivalent to the claim that $L(k) \leq 1$ holds for $k \in \{\zeta, \ldots, n-1\}$. We have already argued $L(\zeta) = 1$.

Straightforward calculation shows that $L(k+1) = r(k)L(k)$, where

$$r(k) = \left(\frac{n-1-k}{n-k}\right) \cdot \left(\frac{q_{\zeta,c}(n)}{q_{\zeta,c}(n+1)}\right) \cdot \left(\frac{1 - q_{\zeta,c}(n+1)}{1 - q_{\zeta,c}(n)}\right)$$

is strictly decreasing in k. Hence, our claim holds, provided that we can establish $L(\zeta+1) \leq 1$ (as, from $L(\zeta) = 1$, this implies $r(\zeta) \leq 1$, which in turn implies $r(k) < 1$ for all $k > \zeta$, and thus $L(k) < 1$ for all $k > \zeta + 1$).

So, it remains to establish $L(\zeta+1) \leq 1$. This is easy to do for cost sufficiently close to (but below) the threshold $\bar{c}_{\zeta+1,n+1}$ at which the stable interior rest point $q^s_{\zeta,c}(n+1)$ disappears. Indeed, straightforward calculations (see the beginning of the Proof of Lemma 2) show that

$$\pi_{\zeta+1,n}\left(\frac{\zeta+1}{n}\right) = \pi_{\zeta+1,n+1}\left(\frac{\zeta+1}{n}\right)$$

and that $(\zeta+1)/n$ maximizes $\pi_{\zeta+1,n+1}(q)$ over $q \in (0,1)$. Since $(\zeta+1)/n < (\zeta+1)/(n-1)$ holds and $(\zeta+1)/(n-1)$ likewise maximizes $\pi_{\zeta+1,n}(q)$ over $q \in (0,1)$, for $c = \bar{c}_{\zeta+1,n+1}$ the stable interior rest point for group size n is $q_{\zeta,c}(n) = (\zeta+1)/n$, the smaller of the two values $q \in (0,1)$ for which $\pi_{\zeta+1,n}(q) = \bar{c}_{\zeta+1,n+1}$. By continuity, for $c \nearrow \bar{c}_{\zeta+1,n+1}$ we have that $q_{\zeta,c}(n+1)$ and $q_{\zeta,c}(n)$ converge to the same strictly positive limit $q^*_{\zeta,c} = (\zeta+1)/n \in (0,1)$. Hence, when $c \nearrow \bar{c}_{\zeta+1,n+1}$, $r(\zeta)$ converges to $(n-1-\zeta)/(n-\zeta) < 1$, which, from $L(\zeta) = 1$, implies $L(\zeta+1) < 1$. Thus, for all sufficiently high c, our claim is proven.

Now suppose there exists some $c \in (0, \bar{c}_{\zeta+1,n+1})$ such that $L(\zeta+1) > 1$ holds. As $q_{\zeta,c}(n)$ and $q_{\zeta,c}(n+1)$ are both continuous in c and the likelihood ratio (34) is also continuous in these probabilities, it follows that there exists some c such that $L(\zeta+1) = 1$ holds. Fix such c. From $L(\zeta) = 1$, we then have $r(\zeta) = 1$. Because $r(k)$ is strictly decreasing in k, we then have $r(k) > 1$ for all $k < \zeta$ and $r(k) < 1$ for all $k > \zeta$. This implies $L(k) < 1$ for all $k > \zeta+1$ and also $L(k) < 1$ for all $k < \zeta$. But this is impossible, as the probabilities for obtaining $k = 0, \ldots n+1$ shirkers need to sum to one for both group sizes. We conclude that $L(\zeta+1) \leq 1$ necessarily holds, thus finishing the proof.

A.5 Proof of Proposition 3

By Lemma 5, $\lambda_{\zeta,c}(n)$, as given by Eq. (26), converges to a finite limit. It follows that $q_{\zeta,c}(n)$ converges to zero, which implies (i) (using Eq. (10)) that $p^*_{\zeta,c} = 1$ holds, and (ii) that the expected number of shirkers $\mu_{\zeta,c}(n)$ converges to the same limit as $\lambda_{\zeta,c}(n)$, i.e., $\mu^*_{\zeta,c} = \lambda^*_{\zeta,c}$. To prove the proposition it remains to establish that the success probability $\phi_{\zeta,c}(n)$ (resp. expected payoff $u_{\zeta,c}(n)$) converges to the probability that there are at most ζ (resp. $\zeta-1$) shirkers given that the number of shirkers follows a Poisson distribution with expected value $\mu^*_{\zeta,c}$. To prove this, we infer from the generalization of the classical Poisson approximation (see, e.g., Billingsley 1995, Theorem 23.2) that (i) $\lambda_{\zeta,c}(n) \to \mu^*_{\zeta,c}$ implies $\Pi_{\zeta-1,n-1}(q_{\zeta,c}(n)) \to F(\zeta-1, \mu^*_{\zeta,c})$, and (ii) $\mu_{\zeta,c}(n) \to \mu^*_{\zeta,c}$ implies $\Pi_{\zeta,n}(q_{\zeta,c}(n)) \to F(\zeta, \mu^*_{\zeta,c})$. Using (11) and (12) this proves the final two equalities in (28). Moreover, since $\mu^*_{\zeta,c} > 0$ is implied by the equality $\mu^*_{\zeta,c} = \lambda^*_{\zeta,c}$ and Lemma 5, we have that, as asserted in (28), these two limits are positive values.

References

Archetti, M., 2009. The volunteer's dilemma and the optimal size of a social group. Journal of Theoretical Biology 261, 475–480. URL: https://www.sciencedirect.com/science/article/pii/S0022519309003890, doi:10.1016/j.jtbi.2009.08.018.

Bach, L.A., Helvik, T., Christiansen, F.B., 2006. The evolution of n-player cooperation—threshold games and ESS bifurcations. Journal of Theoretical Biology 238, 426–434. doi:10.1016/j.jtbi.2005.06.007. publisher: Elsevier.

Beauchamp, G., Ruxton, G., 2003. Changes in vigilance with group size under scramble

competition. The American Naturalist 161, 672–675. URL: https://doi.org/10.1086/368225, doi:10.1086/368225.

Bickerton, D., Szathmáry, E., 2011. Confrontational scavenging as a possible source for language and cooperation. BMC Evolutionary Biology 11, 261. URL: https://doi.org/10.1186/1471-2148-11-261, doi:10.1186/1471-2148-11-261.

Billingsley, P., 1995. Probability and Measure. 3rd ed., John Wiley and Sons, New York.

Boyd, R., Richerson, P.J., 1988. The evolution of reciprocity in sizable groups. Journal of Theoretical Biology 132, 337–356. URL: https://www.sciencedirect.com/science/article/pii/S0022519388802194, doi:10.1016/S0022-5193(88)80219-4.

Broom, M., Rychtář, J., 2022. Game-Theoretical Models in Biology. Chapman & Hall.

Bukowski, M., Miekisz, J., 2004. Evolutionary and asymptotic stability in symmetric multi-player games. International Journal of Game Theory 33, 41–54. URL: https://doi.org/10.1007/s001820400183, doi:10.1007/s001820400183.

Caro, T., 2005. Antipredator Defenses in Birds and Mammals. University of Chicago Press.

Chamberlin, J., 1974. Provision of collective goods as a function of group size. American Political Science Review 68, 707–716. URL: https://www.cambridge.org/core/article/provision-of-collective-goods-as-a-function-of-group-size/27838ADAE1E2C54F2A038687B53AFBF4, doi:10.2307/1959515.

Cheikbossian, G., Fayat, R., 2018. Group size, collective action and complementarities in efforts. Economics Letters 168, 77–81. URL: https://www.sciencedirect.com/science/article/pii/S016517651830140X, doi:10.1016/j.econlet.2018.04.011.

De Jaegher, K., 2020. High thresholds encouraging the evolution of cooperation in threshold public-good games. Scientific Reports 10, 5863. doi:10.1038/s41598-020-62626-3.

Diekmann, A., 1985. Volunteer's dilemma. Journal of Conflict Resolution 29, 605–610. URL: https://www.jstor.org/stable/174243.

Diekmann, A., 1994. Cooperation in an asymmetric volunteer's dilemma game theory and experimental evidence, in: Schulz, U., Albers, W., Mueller, U. (Eds.), Social Dilemmas and Cooperation, Springer Berlin Heidelberg, Berlin, Heidelberg. pp. 413–428.

Esteban, J., Ray, D., 2001. Collective action and the group size paradox. American Political Science Review 95, 663–672. URL: https://www.cambridge.org/core/article/collective-action-and-the-group-size-paradox/50A8854104F1667DA7C5B0B1704FD3A7, doi:10.1017/S0003055401003124.

Hardy, G.H., Littlewood, J.E., Pólya, G., Pólya, G., et al., 1952. Inequalities. Cambridge University Press.

Heifetz, A., Heller, R., Ostreiher, R., 2021. Do Arabian babblers play mixed strategies in a "volunteer's dilemma"? Journal of Behavioral and Experimental Economics 91, 101661. doi:10.1016/j.socec.2021.101661.

Hofbauer, J., Sigmund, K., 1998. Evolutionary Games and Population Dynamics. Cambridge University Press.

Isaac, R.M., Walker, J.M., Williams, A.W., 1994. Group size and the voluntary provision of public goods: Experimental evidence utilizing large groups. Journal of Public Economics 54, 1–36. URL: https://www.sciencedirect.com/science/article/pii/004727279490068X, doi:10.1016/0047-2727(94)90068-X.

Krause, J., Ruxton, G.D., 2002. Living in Groups. Oxford University Press.

Lehmann, L., Rousset, F., 2010. How life history and demography promote or inhibit the evolution of helping behaviours. Philosophical Transactions of the Royal Society B: Biological Sciences 365, 2599–2617. URL: https://doi.org/10.1098/rstb.2010.0138, doi:10.1098/rstb.2010.0138.

MacNulty, D.R., Smith, D.W., Mech, L.D., Vucetich, J.A., Packer, C., 2012. Nonlinear effects of group size on the success of wolves hunting elk. Behav Ecol 23, 75–82. URL: https://doi.org/10.1093/beheco/arr159, doi:10.1093/beheco/arr159.

Nöldeke, G., Peña, J., 2020. Group size and collective action in a binary contribution game. Journal of Mathematical Economics 88, 42–51. URL: https://www.sciencedirect.com/science/article/pii/S0304406820300288, doi:10.1016/j.jmateco.2020.02.003.

Oliver, P.E., Marwell, G., 1988. The paradox of group size in collective action: A theory of the critical mass. II. American Sociological Review 53, 1–8. URL: http://www.jstor.org/stable/2095728, doi:10.2307/2095728.

Olson, M., 1965. The Logic of Collective Action: Public Goods and the Theory of Groups. Harvard University Press.

Ostreiher, R., Heifetz, A., 2017. The sentinel behaviour of Arabian babbler floaters. Royal Society Open Science 4, 160738. URL: https://doi.org/10.1098/rsos.160738, doi:10.1098/rsos.160738.

Ostreiher, R., Heifetz, A., 2020. The function of sentinel alarm calls in the Arabian babbler. J Avian Biol 51. doi:10.1111/jav.02513.

Palfrey, T., Rosenthal, H., 1984. Participation and the provision of discrete public goods: a strategic analysis. Journal of Public Economics 24, 171–193. doi:10.1016/0047-2727(84)90023-9.

Peña, J., Nöldeke, G., 2018. Group size effects in social evolution. Journal of Theoretical Biology 457, 211–220. doi:10.1016/j.jtbi.2018.08.004.

Peña, J., Lehmann, L., Nöldeke, G., 2014. Gains from switching and evolutionary stability in multi-player matrix games. Journal of Theoretical Biology 346, 23–33. doi:10.1016/j.jtbi.2013.12.016.

Peña, J., Nöldeke, G., 2023. Cooperative dilemmas with binary actions and multiple players. OSF Preprints doi:10.31219/osf.io/8y2z5.

Peña, J., Nöldeke, G., Lehmann, L., 2015. Evolutionary dynamics of collective action in spatially structured populations. Journal of Theoretical Biology 382, 122–136. URL: https://www.sciencedirect.com/science/article/pii/S0022519315003185, doi:10.1016/j.jtbi.2015.06.039.

Powers, S.T., Lehmann, L., 2017. When is bigger better? The effects of group size on the evolution of helping behaviours. Biological Reviews 92, 902–920. URL: https://doi.org/10.1111/brv.12260, doi:10.1111/brv.12260.

Przepiorka, W., Diekmann, A., 2013. Individual heterogeneity and costly punishment: a volunteer's dilemma. Proceedings of the Royal Society B: Biological Sciences 280, 20130247. URL: https://doi.org/10.1098/rspb.2013.0247, doi:10.1098/rspb.2013.0247.

Raihani, N.J., Bshary, R., 2011. The evolution of punishment in n-player public goods games: a volunteer's dilemma. Evolution 65, 2725–2728. URL: https://doi.org/10.1111/j.1558-5646.2011.01383.x, doi:10.1111/j.1558-5646.2011.01383.x.

Schoenmakers, S., Hilbe, C., Blasius, B., Traulsen, A., 2014. Sanctions as honest signals – The evolution of pool punishment by public sanctioning institutions. Journal of Theoretical Biology 356, 36–46. URL: https://www.sciencedirect.com/science/article/pii/S0022519314002392, doi:10.1016/j.jtbi.2014.04.019.

Shen, S., Reeve, H., Herrnkind, W., 2010. The brave leader game and the timing of altruism among nonkin. The American Naturalist 176, 242–248. URL: https://doi.org/10.1086/653663, doi:10.1086/653663.

Shen, S.F., Akçay, E., Rubenstein, D.R., 2014. Group size and social conflict in complex societies. The American Naturalist 183, 301–310. URL: https://doi.org/10.1086/674378, doi:10.1086/674378.

Smith, J.E., Gavrilets, S., Mulder, M.B., Hooper, P.L., Mouden, C.E., Nettle, D., Hauert, C., Hill, K., Perry, S., Pusey, A.E., van Vugt, M., Smith, E.A., 2016. Leadership in Mammalian Societies: Emergence, Distribution, Power, and Payoff. Trends in Ecology & Evolution 31, 54–66. URL: https://www.sciencedirect.com/science/article/pii/S0169534715002499, doi:10.1016/j.tree.2015.09.013.

Taylor, P.D., Jonker, L.B., 1978. Evolutionary stable strategies and game dynamics. Mathematical Biosciences 40, 145–156. URL: https://www.sciencedirect.com/science/article/pii/0025556478900779, doi:10.1016/0025-5564(78)90077-9.

Weibull, J.W., 1995. Evolutionary Game Theory. MIT press.

Zahavi, A., 1990. Arabian babblers: the quest for social status in a cooperative breeder, in: Stacey, P.B., Koenig, W.D. (Eds.), Cooperative breeding in birds, Cambridge University Press. pp. 103–130.