Designing host-associated microbiomes using the consumer/resource model

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Abstract

A key step towards rational microbiome engineering is *in silico* sampling of realistic microbial communities that correspond to desired host phenotypes, and vice versa. This remains challenging due to a lack of generative models that simultaneously capture compositions of host-associated microbiomes and host phenotypes. To that end, we present a generative model based on the mechanistic consumer/resource (C/R) framework. In the model, variation in microbial ecosystem composition arises due to differences in the availability of *effective resources* (inferred latent variables) while species' resource preferences remain conserved. The same latent variables are used to model phenotypic states of hosts. *In silico* microbiomes generated by our model accurately reproduce universal and dataset-specific statistics of bacterial communities. The model allows us to address three salient questions in host-associated microbial ecologies: (1) which host phenotypes maximally constrain the composition of the host-associated microbiomes? (2) how context-specific are phenotype/microbiome associations, and (3) what are plausible microbiome compositions that correspond to desired host phenotypes? Our approach aids the analysis and design of microbial communities associated with host phenotypes of interest.

Introduction

Host-associated microbiomes play important roles in host health and disease [1] and, for livestock
species, feed efficiency and environmental footprint [2]. Unfortunately, however, rational design of
microbiomes to achieve desired host phenotypic states remains challenging due to extensive variability
[3-5], high dimensionality, and dynamic nature of microbiomes [4, 6, 7], co-evolution of hosts and
microbiome [8, 9], and numerous microbe ↔ microbe [10] and host ↔ microbe interactions [8, 9, 11].

A key step in rational design of high dimensional biological systems is generative modeling. Generative
models are machine learning models that probabilistically generate *in silico* data and reproduce key
features of experimental observations [12]. These models have been successfully used to model protein
sequences [13-15], genomes [16], presence/absence patterns in microbial ecosystems [17], and single
cell transcription profiles [18]. Generative models have been used for data augmentation and to identify
functional covariation in the data (e.g. residue-residue contacts in proteins [19]). Generative models can
assign probabilities to unseen observations and therefore can be used, for example, to identify disease
effect of genetic variants [20]. Importantly, generative models have been used to design biological
systems with desired properties, for example, proteins with higher enzymatic activity [13]. Generative
models can be extremely useful for microbiome engineering, for example, in identifying feasible
microbiome community structures that correspond to desired community functions or host phenotypes
or in identifying specific host phenotypes, environmental parameters, or microbial taxa that constrain
community composition.

Off-the-shelf generative models based on generative adversarial neural networks [21-24], maximum
entropy methods [25], and mixture Dirichlet distributions [26] have been used to study the composition
of microbiomes. However, many of these models tend to have a very large number of parameters and
run the risk of overparameterization when modeling microbiomes. This is because unlike other
biological data (protein sequences, RNAseq etc.), microbiome studies usually comprise significantly
smaller sample sizes ($N \sim 100$) and exhibit extensive variation. Additionally, these generative models
provide little insight into the environmental [27, 28] and host factors [29] that drive the diversity in
microbiomes. Notably, state-of-the-art generative models in microbiome studies have not modeled the
simultaneous variation in corresponding host phenotypes.

An alternative to data-driven generative modeling is mechanistic generative modeling. Here, variation in
microbiome composition is explained using variation in interpretable and mechanistic parameters of the
model. The consumer/resource (C/R) framework is one of the most popular mechanistic models to
understand variation in microbiome composition [28, 30-36]. In C/R models, variation in microbiomes
arises due to variation in resource availability while the preferences of microbial species (consumers)
towards these resources remain conserved across similar ecosystems. With an appropriate choice of
parameters, ecosystem compositions generated using the C/R model reproduce many universal
statistical features of real and synthetic microbiomes [28, 32, 36, 37] such as distributions of diversity

metrics [28] and scaling relationships [27]. Unfortunately, fitting the C/R model to data is difficult as it requires knowledge of high-resolution temporal dynamics of species abundances, or the concentrations of resources consumed by microbial species through time [38] or species-resource preferences [39]. Consequently, training these models on specific microbiome datasets is difficult even for controlled *in vitro* communities [36]. Therefore, studies using the C/R model cannot model covariation patterns observed in specific host-microbiome systems, instead randomly drawn parameters have been used to explain statistical features of the data [27, 28]. Finally, the C/R framework is not able to model variation in host phenotypes, except for the abundances of metabolites directly consumed/excreted by the microbiome.

Here, we develop a generative model for host-associated microbiomes using the mechanistic C/R framework. To that end, we identify a latent space induced from the C/R model that encodes the temporal histories of *effective resources* affecting microbial growth. Variation along this latent space results in variation in microbiome compositions. Using data from three host-associated microbiomes, we show that probabilistic modeling of this latent space reproduces both universal statistical features as well as system-specific covariations between bacterial taxa. Using the model and the bovine rumen microbiome as a test case, we investigate three salient questions in microbiome design and engineering: (1) which host phenotypes maximally constrain the composition of the host-associated microbiome? (2) how context-specific are phenotype/microbiome associations, and (3) what are plausible microbiome compositions that correspond to desired host phenotypes?

To the best of our knowledge, this is the first generative model that can simultaneously model microbiome compositions and the environmental factors, for example, phenotypic states of the host. Therefore, we believe that our generative model will be an important tool for the design of host associated microbiomes.

Results

The C/R framework identifies a latent space to build a machine learning model.

We use the C/R framework to identify a latent space to model microbiome compositions (SI Section 1, Figure 1). In the C/R model, the change in abundance of an organism (growth rate) depends on the availability of resources r_k and species preferences towards those resources θ_{ko}. The resources themselves are depleted according to their consumption by all organisms and net flow. As discussed above, fitting the C/R model to data requires not only the knowledge of microbial abundances but also the identities and abundances of those resources, either across samples or over time. Typically, these measurements are not available and, consequently, the C/R model is only used for its qualitative insights. Here, we show how to convert the C/R model to a latent variable model to describe species abundances. To do so, we integrate the equation describing the growth rate of a species o in the C/R model and obtain an expression for the species abundance $n_o(t)$ at time t:

$$n_o(t) \approx \exp\left(\int_0^t \mu_o(\tau)\, d\tau\right) = \exp\left(-\sum_{k=1}^K z_k \theta_{ko}\right) \tag{1}$$

where $\mu_o(\tau)$ is the time-dependent growth rate. We recognize:

$$z_k = -\int_0^t r_k(\tau)d\tau \tag{2}$$

as latent variables that represent the complex temporal dynamics of resources. We note that the latent variables thus defined are emergent variables; they correspond neither to the initial concentration nor to the steady state concentrations of the resources. Instead, they represent the temporal history of resource fluctuations which in turn may depend on flow rates, resource sharing, and cross-feeding [31, 37].

(A)

(B)

Figure 1. A generative model for host-associated microbiomes using the C/R framework. The C/R model defines changes in the abundance $n_o(t)$ of consumers "o" as a function of resource abundances $r_k(t)$ and consumer preferences θ_{ko} towards resources. Integrating the C/R dynamics over time shows that species abundances are a function of latent variables $z_k(t)$ that represent the sample-specific time history of resources, and universal species-specific preferences towards those latent variables. (B) To generate realistic microbiomes, sample embeddings in the latent space are modeled using a mixture Gaussian distribution. The model allows data augmentation via unbiased sampling in the latent space. Biased sampling from the model allows us to generate microbiome samples that correspond to desired host phenotypes.

Recognizing these latent variables allows us to circumvent the numerical difficulties in fitting the C/R model to microbiome data [39]. Observe that the relative abundances of organisms q_o are simple exponential functions of the latent variables:

$$q_o = \frac{1}{\Omega} \exp\left(-\sum_{k=1}^{K} z_k \theta_{ko} \right) . \tag{3}$$

Unlike the C/R model, Eq. 3 can be easily fit to cross sectional microbiome data [40]. To that end, consider that the relative abundances (relative counts) x_{so} of O OTUs (operational taxonomic units) are measured in S hosts. We assume that the resource preferences of OTUs are universal across hosts while resource dynamics are host dependent. That is, latent variables z_{sk} are host-specific but species-independent and θ_{ko} are species-specific but host-independent. For fitting Eq. 3 with a fixed K, we minimize the cross-entropy C [36], which is equivalent to maximizing a probabilistic model for species abundances that follows a multinomial distribution (Eq. 4).

$$C = \sum_{s,o} x_{so} \log q_{so} . \tag{4}$$

As we show in SI Section 1, this minimization is a nonlinear low rank matrix factorization wherein z and θ can be learnt using gradient descent.

Unlike other machine learning models, our model has clear mechanistic interpretation: microbial couplings to specific latent variables remain constant across different ecosystems, but the temporal histories of these latent variables vary from ecosystem-to-ecosystem, leading to observed variation in community compositions.

Host-associated microbiomes are surprisingly low-dimensional.

Figure 2. Low dimensionality of host-associated microbiomes. Panels (A), (B), and (C), the ratio of successive singular values $\frac{\lambda_k}{\lambda_{k+1}}$ of the $Z \times \Theta$ matrix for bovine, chicken, and human microbiomes respectively. Only the first 50 SVs are shown as later, less important SVs may be dominated by noise. The insets show the singular values arranged in a decreasing order. Panels (D), (E), and (F) show the average symmetric Jensen-Shannon distance(black) and the Bray-Curtis dissimilarity (gray) between observed community composition and the corresponding model fit as a function of the dimension of the latent space. The average is taken over all samples.

How accurate is Eq. 5 in modeling compositions of microbiomes? We studied microbiomes associated with three host species, the rumen of Holstein cows [41], the chicken cecum [42], and the human gut [43]. The bovine and the chicken microbiomes were characterized at the level of operational taxonomic units (OTUs) while the human microbiomes were characterized at the genus level. Based on our previous analysis on technical noise associated with sequencing of microbial samples, we only included taxa whose average abundance was higher than 0.1% [5]. In the three datasets, there were 790 samples and 157 OTUs (cows), 778 samples and 156 OTUs (chicken), and 127 samples and 95 genera (humans) respectively (SI Section 2).

The number of resources consumed by species in these ecosystems can potentially be very large ($K \sim 10^3 - 10^4$) [36]. To avoid overparameterization in our model, we seek a small number of *effective resources* (latent variables) [44] that capture the observed compositional variation of microbiomes. These latent variables do not correspond to specific nutrients present at a specific time. Instead, they represent a combination of factors that both positively and negatively affect the growth of microbial species aggregated over time. To determine the effective dimension of the latent space, we fit Eq. 5

where K is chosen as the smallest of the number of hosts S and the number of species O. We
determined the effective dimension by examining the singular values (SVs) of $Z \times \Theta$ (insets of Figure
2ABC). Singular values quantify the variance explained by low rank approximations to matrices and
therefore can identify coarse-grained latent variables that can accurately approximate the microbiome.
SVs for all three ecosystems showed two regimes. When arranged from largest to the smallest, SVs first
decreased rapidly, followed by a gradual power law-like decrease. This suggested that a small number of
latent variables captured the essential variation in microbial composition. To confirm this, we fit
microbiome compositions using different values of K. Indeed, as seen in Figure 2DEF, the symmetric
Kullback-Leibler divergence and the Bray-Curtis dissimilarity between data and model fits decreased
approximately logarithmically with increasing dimension of the latent space and at a significantly slower
rate for larger Ks.
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These results show that a low dimensional latent space constructed using the C/R model captures
compositional variation in microbiomes in three different host species. We note that this dimensionality
reduction approach is conceptually different from principal coordinate analysis (PCoA), a standard tool
in microbiome analysis. In PCoA, one approximates dissimilarity between compositions of pairs of
ecosystems using dimensionality reduction obtained via principal component analysis performed
directly on the pairwise distance matrix. The variance explained by PCoA is therefore the variance
explained in predicting pairwise distances using a lower dimensional model. In contrast, our approach
directly models (approximates) compositions of individual ecosystems using a lower dimensional model.
Notably, the accuracy of our approach quantified in terms of the variance explained in reconstructing
community composition as well as reconstructing pairwise Bray-Curtis dissimilarity shows that a small
number of components is sufficient to capture both community composition as well as $\beta-$diversity (SI
Figure 1).
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This low dimensionality may arise due to covariation in species that results from specific structure in
consumer/resource matrices [33, 35], due to covariation in resource flow rates [33], due to universal
factors such as pH [45], or due to coarse-grained structure/function relationships between the
microbiome and its environment [34] that affect broad classes of species in a similar manner. Notably,
as we show below, this low dimensionality was not due to low compositional diversity. Indeed, species
abundances were widely distributed with an abundance distribution (Figure 3C) that had an extended
power law scaling between abundances of 0.1% to 10%. In this work, we exploit this low
dimensionality for generative modeling. We leave it for future studies to explore its mechanistic
underpinnings.
278
279 Finally, we confirmed that the trends in community reconstruction accuracy did not depend on the
280 abundance cutoff used as the OTU inclusion criteria (SI Figure 2). Consequently, downstream analyses
281 using these latent variables are also expected to be insensitive to the abundance cutoff used.
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Probabilistic modeling reproduces dataset-specific statistics of microbiomes.

To endow generative capacity to our approach, we model the distribution $p(z)$ of data points over
latent variables using Gaussian mixture modeling (**SI Section 3**). Notably, Gaussian mixtures are versatile
models to fit multivariate data as they can in principle fit to distributions of arbitrary shapes. From this
distribution it is possible to generate realistic *in silico* microbiome compositions by first sampling their
latent space embeddings using the learnt distribution and then using the universal species preferences
learned during training and obtaining relative species abundances using Eq. 3. Additionally, the Gaussian
mixture model also enables to calculate the likelihood of any given latent space coordinate profile
relative to the data used to train the model.
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Can the generative model sample realistic ecosystem compositions? We tested this using comparison of
several lower-order statistics, metrics of distribution density, and embedding accuracy. This is a
common approach used in generative models. We trained a generative model using a latent space of
dimension $K = 16$ (Figure 2). We tested the accuracy of the generative model by sampling latent
variables from an inferred mixture Gaussian $p(z)$ and then generating *in silico* microbiomes using Eq. 3.
For clarity of presentation, we present results for chicken cecum microbiomes. The results for the other
datasets are shown in SI Figure 3 and 4. Figure 3A shows that the generative model accurately
reproduces OTU mean abundances and standard deviations. Notably, our approach is quite accurate at
predicting microbiome composition at all levels of taxonomy (SI Figure 5). Figure 3B (and inset) shows
that the model captures species abundance covariation and higher order co-occurrence patterns. Figure
3C shows that the model reproduces the species abundance distribution, including the power law-like
scaling between abundances of 10^{-3} and 10^{-1} with a power law exponent of $\gamma \sim -1.03$ ($\gamma \sim -1.12$
for the model) for the inverse cumulative distribution. Figure 3D shows that the model reproduces the
distribution of the α-diversity metric of Shannon diversity. Figure 3E shows that the model reproduces
distributions of the β-diversity metric Bray-Curtis dissimilarity between nearest neighbor samples as
well as pairs of randomly picked samples.
309
Are the species preferences towards effective resources θ_{ko} universal? If so, species preferences
inferred from a set of microbiome samples should be able to accurately model the composition of
dissimilar microbial communities of the same species. To test this hypothesis, we looked at bacterial
microbiomes associated with the cecum and ileum of broiler chickens. The cecum and the ileum are
distinct parts of the chicken's gastrointestinal track. Indeed, the abundances of microorganisms vary
substantially between the two organs [42].
316
We split the cecum microbiome samples randomly into an 80% training and 20% testing split and
trained the model only on the training data. As examples of dissimilar ecosystems, we used an
additional testing set comprising ileal microbiomes of chickens. As seen in Figure 3F, the species
preferences towards effective resources are indeed universal. Unsurprisingly, the learnt species
preferences embed testing data from the same organ site (cecum) with similar accuracy as the training
data (black versus gray dots, x-axis). Surprisingly, the accuracy of embedding testing data from another
organ using the same species preferences was quite similar to testing data from the same organ site (red
versus black dots, x-axis). In contrast, the accuracy of embedding the testing data using a scrambled

preference matrix was significantly lower (red versus blue dots, x-axis). Similarly, the inferred latent space locations of testing data microbiomes derived from ceca had similar likelihood as the training data. In contrast, the latent space locations of ileal microbiomes had significantly lower likelihood (y-axis). This is consistent with our interpretation of the consumer resource framework. Specifically, while the OTUs have consistent embeddings between the ileum and the cecum, leading to reasonably accurate reconstruction of the ileal microbiomes, the abundances of effective resources and therefore the values of the fitted latent variables zs are likely to be markedly different between the two organ sites. Consequently, the latent variables fitted for the ileums have significantly lower likelihood compared to the cecal latent variables. A similar result was observed when comparing embeddings of cattle from the Nordic Red breed using species preferences learnt from Holstein cows (SI Figure 3).

Figure 3. Latent space-based model generates accurate *in silico* communities. (A) A comparison between mean OTU abundances (filled circles) and standard deviations (open circles) and the corresponding model predictions (y-axis). The dashed red line represents $x = y$. (B) A comparison between OTU-OTU covariance $\langle \delta x_i \delta x_j \rangle$ as computed in the data and as predicted by the model. Absolute values are shown. (C) The inverse cumulative distribution representing the probability of observing an OTU abundance greater than a given abundance (x-axis) as computed from the data (black) and the corresponding model prediction (gray). The dashed lines show the best fit power law between relative abundances of 10^{-3} and 10^{-1}. The slope of the power law is -1.03 in the data and -1.12 in the model. (D) The distribution of Shannon diversity of community compositions as observed in the data (gray) and computed from the model generated communities (blue). (E) The distribution of Bray-Curtis dissimilarity between random pairs of community as well as closest pairs of communities in the data (gray) and the model (blue). Distributions are smoothed using Gaussian kernel density estimation. (F) a scatter plot of Kullback-Leibler

349 divergences between community composition in the data and the corresponding model fit (x-axis) and the log-
350 likelihood of the embedded latent variables (y-axis). Data are shown for the training data from chicken cecum
351 (black), testing data from chicken cecum (gray), testing data from chicken ileum (red), and testing data from
352 chicken ileum embedded using a scrambled preference matrix (blue).
353
Collectively, these results show that probabilistic modeling of the latent space defined by the C/R model
generates microbial compositions that reproduce both universal statistics such as species abundance
distributions and dataset specific statistics such as patterns of species covariances and α- and β-
diversity metrics. Moreover, the learnt species-resource preferences are universal; the statistics of
probability of observing unseen data and accuracy of embedding unseen data are similar to those of the
training data. In other words, the C/R model based generative model can sample compositions of
feasible microbiome community structures, an essential feature of accurate generative models.
361
Generative modeling identifies host phenotypes that maximally constrain the microbiome.
The coarse-grained latent space represents the history of effective resources. Therefore, it can be
informative of host phenotypes that affect microbial growth. Indeed, recent work has suggested that
many functions of microbial communities are coarse-grainable i.e. explained by a small number of
variables [35]. To test this, we investigated the quantitative relationship between phenotypes and
associated microbiomes of the rumens of ~ 800 bovine hosts. Here, ~ 50 phenotypes were measured,
including traits related to rumen chemistry (e.g. volatile fatty acids) and animal physiology (e.g. milk
production, feed conversion efficiency) (Supplementary Table 1).
370
Surprisingly, latent space embeddings inferred only using the microbiome accurately predicted most of
the measured phenotypes using linear regression (SI Figure 6). This suggests that the latent variables
fitted to the C/R model encode information about the environmental variables experienced by the
microbiome. Consequently, these *effective resources* may simultaneously capture microbial community
structure and host phenotypes that influence microbial growth. To directly incorporate host phenotypes
in our model, we employ a shared latent approach. Specifically, we require the latent variables to
simultaneously describe the composition of microbiomes and host phenotypes. To that end, we
introduce a new cost function:
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$$C = \alpha \sum_{s,o} x_{so} \log q_{so} + (1 - \alpha) \sum_{s,p} \left(m_{sp} - \sum_{k=1}^{K} z_{sk} C_{kp} \right)^2 \tag{5}$$

380
In Eq. 5 the first term captures the goodness of fit to the microbiome composition and the second term
is a model to fit the host phenotypes. Unlike the microbiome, the relationship between host phenotypes
and latent variables cannot be mechanistically derived. Therefore, we take a data-driven approach. We
approximate the host phenotypes using a linear low-rank model (SI Section 1). We note that other
functional forms, for example, feed forward neural networks can be employed as well. In Eq. (5),
$1 \geq \alpha \geq 0$ decides the relative importance of the two terms in the minimization process. We fit Eq. 7 to
the combined microbiome/phenotype data using $K = 24$ latent variables with $\alpha = 0.975$ (chosen to
minimize the compromise in accuracy of fitting the microbiome and phenotypes, SI Figure 7). Next, as

above, we trained a Gaussian mixture model on the shared latent space (SI Section 3). In this case, sampling latent variables using the learnt distribution $p(z)$ generated combined microbiome/phenotype data that preserved covariances between the microbiome and the phenotypes (SI Figure 8). Notably, the linear model employed in Eq. 5 may not be sufficient to model host phenotypes from the latent variables. However, at least in this case, nonlinear neural networks did not significantly improve the predictions (SI Figure 6). Therefore, going forward, we employ linear models to connect host phenotypes to latent variables.

Figure 4. Identifying phenotypes that constrain the microbiome. (A, B, and C) The average Bray-Curtis dissimilarity (y-axis, normalized by the Bray-Curtis dissimilarity between random pairs of communities) amongst *in silico* bovine rumen microbial communities constrained to have a specified single phenotype (x-axis). The top 3 phenotypes that constrained community composition the most are shown. Error bars represent standard errors of the mean. (D) Box plot of Bray-Curtis dissimilarity between predicted microbiome composition and the measured composition using top 3 constraining phenotypes, bottom 3 least constraining phenotypes, 3 randomly chosen phenotypes, and random pairs of communities in the testing samples for bovine rumen communities.

A central question in ecology is how strongly environmental parameters or ecosystem functions constrain the community composition. To answer this question in the context of host-associated microbiomes, we used generative modeling. Specifically, we sought to identify which host phenotypes constrain the space of possible microbial community structures? For example, host serum metabolomics may only weakly determine the microbiome while host dietary habits and gut chemistry may have a

more constraining impact. Indeed, our model finds that some phenotypes like rumen pH and propionate
levels can significantly constrain the space of feasible community structures while others such as organic
matter intake do not. Importantly, these dependencies are not always uniform across the range of host
phenotypic states. To quantify these relationships, we generated a large number of *in silico*
combinations of microbial communities and corresponding host phenotypes. For each phenotype, we
evaluated the average Bray-Curtis dissimilarity amongst pairs of communities that corresponded to a
narrow range of values of the phenotype. A low dissimilarity amongst communities corresponding to a
particular range indicates that specifying the phenotype in that range narrows the space of feasible
community compositions. We found that different phenotypes constrained community composition to a
different degree. Figures 4ABC show that specifying phenotypes such as levels of rumen short chain
fatty acids (propionate and isobutyrate) and rumen pH significantly constrained the microbiome
composition in the rumen. In contrast, phenotypes such as organic matter intake and protein intake
levels did not constrain the microbiome composition (Supplementary Table 2). Notably, these analyses
required *in silico* generation of thousands of feasible microbiome/host phenotype combinations and
therefore cannot be performed using the data alone.
426
To verify the importance of specific phenotypes in narrowing plausible microbiome compositions, we
investigated whether we could reconstruct the microbial composition in the rumens of hosts in the
testing set using partial phenotypic information about the host. To do so, we deeply sampled from the
mixture gaussian model and found *in silico* samples that had phenotype values in close proximity of the
data point of interest. We used the average microbiome composition of these *in silico* microbiomes as
our microbiome prediction (SI Section 4). We asked whether partial information about the phenotypes
identified in Figure 4ABC could identify the composition of the associated rumen microbiome.
Importantly, the predictions made using partial phenotypic information were significantly more accurate
compared to the dissimilarity between pairs of randomly picked samples in the testing data.
436
437 These results demonstrate that our generative model not only captures complex associations amongst
438 microbial species (Figures 3) but also the associations between the microbiome and corresponding host
439 phenotypes (Figure 4).
440
441
Generative modeling quantifies context-specificity of host-microbiome interactions.
Host-microbiome interactions are often context-specific, that is, the association between a taxon and a
host phenotype may depend on the composition of the rest of the community. We exploit the
generative capacity of our approach to identify subject-specific microbial correlates of methane
production (normalized by dry matter intake). Methane production by bovines is a major contributor to
greenhouse gasses and microbiome-based therapies to reduce methane production are a promising
intervention avenue. To identify host-specific associations, we densely sampled the space of combined
microbiome composition and host metadata using our generative model. Next, for each host, we
identify *in silico* microbiomes that are in its proximity (evaluated using Bray-Curtis dissimilarity) and
evaluate the correlation between microbial OTUs in this subset with methane production. As shown in
Figure 5, some OTUs associate with phenotypes in a universal manner, while others associate in a host-

specific manner. To illustrate this more clearly, we look at two OTUs classified at the genus level as *Prevotella* and *Moraxellaceae ge* that have similar association strength with methane production when correlated across all samples. In contrast, our model predicts that the *Moraxellaceae ge* OTU correlates with methane across all hosts with similar magnitude while the *Prevotella* OTU has a more heterogeneous association profile.

The phenotype/OTU associations calculated globally and on a per host basis (averaged over hosts) were correlated with each other. However, the average host-specific correlation was typically half as strong as the global correlation. Notably, some phenotypes were over-represented in host-specific local correlations. For example, of the top 100 host-specific correlated pairs, we found that certain phenotypes occurred more frequently than expected by chance. These include rumen propionate (22 out of 100, hypergeometric test p = 4.08e-16), rumen ammonia (11 out of 100, hypergeometric test p = 1.80e-5), fecal dry matter (11 out of 100, hypergeometric test p = 2.18e-3) and plasma creatinine (6 out of 100, hypergeometric test p = 2.86e-2).

Figure 5. **Quantifying context-specificity of host-microbiome associations.** (A) Pearson correlation coefficient between methane production (normalized by dry matter intake) and all OTUs evaluated in a sample-specific manner. Different colors in the sample-wise clustering tree represent different farms to which the cows belong. (B) Distribution of sample specific Pearson correlation coefficients for two OTUs (*Prevotella* and *Moraxellaceae ge*). The global (across sample) Pearson correlation for abundance of either OTUs with methane production is ~ 0.15. (C) The mean host-specific (local) correlation of an OTU with a phenotype is plotted against the global correlation

475 between that OTU and the phenotype, calculated *in silico*. The slope of the best fit line between local and global
476 correlations is 0.42.
477

Designing microbiomes that achieve desired host phenotypes.

479 The chief application of generative models is the design and identification of biological systems that
480 possess user-desired properties, for example, proteins with specific enzymatic activity or structural folds
481 [13]. Here, we illustrate with two examples how our generative modeling can be used to identify host
482 phenotypes and microbial ecosystem compositions that correspond to desired host states.
483

To validate that our generative model reproduces covariations observed in the data, we investigated the
relationship between rumen propionate levels and rumen pH. Propionate production is associated with
lower enteric methane emissions in ruminants due to its lower yield of molecular hydrogen [2]. The pH
of the rumen is an emergent property arising from the complex acid/base balance of multiple chemicals.
Notably, rumen pH negatively correlates with propionate levels (Spearman $r = -0.40, p = 1.5 \times$
10^{-31}), which was not true of all short chain fatty acids (for example, Spearman $r = 0.42, p = 3.0 \times$
10^{-34} for butyrate and Spearman $r = 0.10, p = 4.5 \times 10^{-3}$ for acetate). Therefore, we sought to
characterize host phenotypic states that corresponded with simultaneous high rumen pH and high
propionate levels. To achieve this, we performed biased sampling of the latent space using a Markov
chain Monte Carlo approach (SI Section 5, red contours in Figure 5A). Notably, these hosts were not
observed in the training population (black contours in Figure 5A). Next, we investigated how this shift in
propionate and pH changed the levels of other short chain fatty acids. Surprisingly, levels of both
acetate and butyrate, which positively correlated with pH, dropped in the biased samples. While not
apparent from the data, these results make physiological sense; increasing propionate concentration
may be compensated by decrease in acetate and butyrate to maintain mass balance and rumen pH.
499

Figure 6. Designing microbiomes for desired phenotypic states. (A) Contour plots of the 2-dimensional histograms of rumen pH and rumen propionate levels. Black contours represent *in silico* communities sampled from the inferred distribution over latent variables. Red contours represent the biased sampling of latent variables which encourages high propionate levels and high rumen pH. (B) Concentrations of volatile fatty acids in unbiased (black) and biased (red) samples. (C) Contour plots of the 2-dimensional histograms of rumen pH and daily starch intake levels. Black contours represent *in silico* communities sampled from the inferred distribution over latent variables. Red contours represent the biased sampling of latent variables which encourages high levels of rumen pH while maintaining a high starch intake. (D) Mean relative abundances of OTUs in the unbiased samples (x-axis) and the biased samples (y-axis). The OTUs with high abundance in the biased samples that also show a significant increase from unbiased samples are colored in red.

Our model can also identify the relationship between host phenotypes and the corresponding composition of the microbiome. Cows fed high amounts of starch often have acidic rumens due to its rapid fermentation compared to fiber rich diets. This may lead to sub-acute ruminal acidosis (SARA), an undesirable condition that leads to lowered economic productivity [46]. Indeed, in the dataset analyzed, starch intake was negatively correlated with pH (Spearman $r = -0.53, p = 1.6 \times 10^{-58}$). Notably, there were no hosts in the original dataset who could maintain a high pH on a starch-rich diet (black contours in Figure 5C). Given that the pH directly affects microbial growth and can also be controlled by it, we investigated microbial ecosystem compositions that would be present at a high rumen pH in hosts that have a high starch intake using biased sampling of the latent space (**SI Section 5**, red contours in Figure 6C). The rest of the host phenotypes (and the composition of the microbiome) were not specified but automatically sampled according to the biased distribution. The microbiome compositions obtained were significantly different for these desired host phenotypes (Figure 5D). Yet, these *in silico*

microbiomes have statistical properties (for example, $\alpha-$ and $\beta-$diversity) that are quite similar to real
microbiomes, suggesting that these microbiomes may exist in nature (SI Figure 9). Two OTUs were
consistently present in higher abundance in the *in silico* biased samples with high starch intake and a
high pH (Supplementary Table 3). These include an OTU from the genus *Ruminobacter* (1 out of 2,
hypergeometric test p = 0.025) and an OTU classified as *Succinivibrionaceae_UCG-002*. Both OTUs
belonged to the family *Succinivibrionaceae* (2 out of 12, hypergeometric test p = 0.005). Notably,
Ruminobacter bacteria are known to degrade starch [47]. However, their ability to do so while
maintaining a high rumen pH is not fully explored.
532
533 These studies show that our model can sample and thereby design host-microbiome metacommunities
534 with user-specified host phenotypes.
535

Discussion

Rationalizing the observed variation in complex and high dimensional biological systems using
mechanistic modeling is seldom possible. At the same time, we can now collect large amounts of data.
This has allowed building data driven generative models that describe possible variations in biological
systems. Unfortunately, except for a few examples, studies on host-associated microbiomes usually
operate with low sample sizes. Moreover, the context of the microbiome sample (abiotic environment
or the phenotypic states of the host organism) is of paramount importance in deciding ecosystem
composition. This context dependence likely contributes to a low reproducibility of microbiome studies
[47].
546

547 In this work, we presented a generative model based on the consumer/resource framework that can
548 model the simultaneous variation in host-associated microbiomes and host's phenotypes. Notably, our
549 model can assign probabilities to specific communities and phenotypes. This allows us to identify
550 realistic communities that are hypothesized to correspond to desired host states.
551

Importantly, although our model allows us to sample realistic microbial communities and associated
host phenotypes, it does not provide a causal connection between the two. We have shown that certain
host traits such as organ pH, nutrient intake, or the concentration of specific metabolites are associated
with more specific microbial communities within certain trait value ranges. This does not necessarily
mean that those parameters can be used to drive the microbiome towards a particular state, although
these are useful hypotheses. Instead, the ability to generate an arbitrary number of realistic *in silico*
samples facilitates the study of complex communities by enabling data stratification beyond what is
experimentally possible, even to cases that are seldom observed in experimental datasets. As a design
tool, the communities obtained by sampling from the model represent target communities for
microbiome manipulation strategies including pre and probiotic treatments, antibiotics, microbiome
transplants, phage therapies, etc. Knowing what these targets look like should provide clues as to how
to achieve them. Notably, however, the model can easily be made causal by incorporating a temporal
direction, for example, by training the model on pre- and post-intervention (drugs, probiotics, etc.)
microbiomes.

There are several ways to generalize the current framework. *First*, we learnt the latent space
embeddings zs of individual hosts first and then built a probabilistic model $p(z)$ that captured the
variation in the latent variables. This two-step process can be combined using Expectation-Maximization
like algorithms that simultaneously learn embedding for data and model the corresponding mixture
Gaussian distribution. *Second*, while the dependence of microbial abundances on latent variables was
mechanistically derived, the linear relationship between host phenotypes and latent variables was *ad*
hoc. This linear model can in principle be improved by incorporating more complex relationships
between the phenotypes and the latent variables, for example, those modeled using artificial neural
networks. *Third*, in the current approach we built a combined latent space model for host phenotypes
and the associated microbiomes. Similarly, phenotypic information about microorganisms, for example,
consumer/resource preferences inferred using genome-scale metabolic models [48] or species genomic
content, can also be incorporated in our framework by requiring the species-specific preferences θ_{ko} to
simultaneously model species abundances in microbiomes and species phenotypes using previously
developed latent variable models [17].

While this model was developed for host-associated microbiomes, it can be used to integrate different
types of sequencing and phenotypic information not only on animal hosts and corresponding microbial
ecosystems but also on cells and their phenotypic properties and potentially in a more general setting
when both sequencing and phenotypic information is available for the same entities.

We believe that the framework laid out in the manuscript will be of broad use to microbiome engineers
and scientists alike.

Materials and Methods

All data and code can be found at https://github.com/karthik-yale/host_microbe

Section 1. Model description and fit to data

A. Details of the latent variable model and performance
The consumer/resource (C/R) model [28] has been a popular model in understanding species dynamics
in natural and artificial ecosystems. The C/R model imagines that species abundances fluctuate
according to their dependence on resources. We have:

$$\frac{1}{n_o}\frac{dn_o}{dt} = \mu_o(t) = \sum_{k=1}^{K} r_k(t)\theta_{ko} - \delta_o \tag{S1}$$

601　and

$$\frac{dr_k}{dt} = J_k(t) - r_k(t)\sum_{o=1}^{O} n_o(t)\theta_{ko}\gamma_{ko}. \tag{S2}$$

602   In Eq. S1 and Eq. S2, $n_o(t) \geq 0$ are abundances of consumers (microbial species) "o", $\mu_o(t)$ is the time-
603   dependent growth rate, $r_k(t) \geq 0$ is the abundance of *resource* "k", $\theta_{ko} \geq 0$ are species-preferences
604   for resources, $\gamma_{ko} \geq 0$ are resource consumption efficiencies, and $J_k(t)$ are net inflow rates of
605   resources.
606
607   With these assumptions, we can formally integrate Eq. 1 to obtain:

$$n_o(t) \approx exp\left(\int_0^t \mu_o(\tau)d\tau\right) = \exp\left(-\sum_{k=1}^K z_k\theta_{ko} - \delta_o t\right) \tag{S3}$$

609   where we recognize

$$z_k = -\int_0^t r_k(\tau)d\tau \tag{S4}$$

610   as latent variables that represent the complex temporal dynamics of resources. We note that the
611   death/dilution term can be incorporated into the latent variable representation by identifying $z_{K+1} = t$
612   (the relevant time scale for the establishment of the current community composition) and $\theta_{K+1,o} = \delta_o$.
613   Shifting the indices from $K + 1$ to K for simplicity, the relative abundances of organisms q_o are simple
614   functions of the latent variables:

$$q_o = \frac{1}{\Omega} exp\left(-\sum_{k=1}^K z_k\theta_{ko}\right). \tag{S5}$$

615   Eq. S5 can be used either in a longitudinal context where latent variables z_k evolve over time inside a
616   single host or in a cross-sectional context where we imagine that microbiomes associated with similar
617   hosts (for example, same host organism) couple to the latent variables in the same manner and that
618   latent variables represent snapshots of individual hosts.
619
Here, we describe in detail the most general formulation of our approach; to model a collection of host-
associated ecosystems along with the phenotypes of the host. We start with the measurement of
relative abundances of species (or operational taxonomic units, OTUs) in microbial ecosystems
associated with hosts. We denote these abundances as x_{so} where $s \in [1, S]$ is the index of the subject,
o" is an organism. Additionally, we assume that measurements of P phenotypic metadata m_{sp} ($p \in$
$[1, P]$) that can potentially affect the microbiome are also measured.
626

B. Fitting the model to microbiome data

628   We write the model-predicted composition of the ecosystems as

$$q_{so} = \frac{1}{\Omega_s}\exp\left(-\sum_{k=1}^K z_{sk}\theta_{ko}\right). \tag{S6}$$

629   Since the latent variables are related to the properties of the ecosystem and the host, we further
630   hypothesize that the host's phenotypic metadata explain the latent variables. Unfortunately, however,
631   the relationship between latent variables and host phenotypes cannot be derived using mechanistic
632   models. To that end, we use data-driven methods to capture this dependence. Specifically, we use a
633   generalized low rank linear model:

$$m_{sp} \approx \sum_{k=1}^K z_{sk}L_{kp}. \tag{S7}$$

634 where $g(\cdot)$ is a user-specified function. We use $g(\cdot) = I(\cdot)$ (identity function). Importantly, we require
635 that the same host-specific latent variables explain the composition of all ecosystems as well as the
636 host's phenotypic information. To that end, we write a combined cost function:

$$C = (1 - \alpha)C_m + \alpha C_e \qquad (S8)$$

639 where for the bovine rumen data, C_m is the squared L_2 loss between the host metadata and the
640 corresponding model prediction. C_e is the Kullback-Leibler divergence between the measured microbial
641 composition in the ecosystem and the corresponding model prediction. $0 \leq \alpha \leq 1$ is a scaling factor
642 that determines the relative importance of the two terms. When fitting only the microbiome data, we
643 set $\alpha = 0$. We have the derivatives:

$$\frac{\partial C_m}{\partial z_{sk}} = -2 \sum_p \left(m_{sp} - \left(\sum_{k=1}^{K} z_{sk} L_{kp} \right) \right) L_{kp} \qquad (S9)$$

644 and

$$\frac{\partial C_m}{\partial L_{kp}} = -2 \sum_s \left(m_{sp} - \left(\sum_{k=1}^{K} z_{sk} L_{kp} \right) \right) z_{sk}. \qquad (S10)$$

645 The derivatives for the cost C_e are

$$\frac{\partial C_e}{\partial z_{sk}} = \sum_o \left(x_{so} - q_{so} \right) \theta_{ko} \qquad (S11)$$

646 and

$$\frac{\partial C_e}{\partial \theta_{ko}} = \sum_s \left(x_{so} - q_{so} \right) z_{sk} \qquad (S12)$$

647
We infer the model parameters using gradients in Eq. S9-S12 and constant step-size gradient descent
with learning rate η ranging between $\eta = 10^{-3} - 10^{-5}$ depending on the dataset. The learning rate was
set to ensure that the loss decreased smoothly with the number of iterations. We stopped the learning
process when the sum of L_2 norms of the gradients divided by L_2 norms of the corresponding matrices
was less than 10^{-2}. Notably, the inference problem is convex in zs, θs, and $\boldsymbol{L}$ when the other
parameters are kept constant [49]. Therefore, the inferred matrices were robust (up to a rotation) to
hyperparameters related to learning.
655

Section 2. Details of the datasets

Three datasets were used in our study that spanned multiple animal hosts. We used data collected on
(1) bacterial microbiome compositions in the bovine rumen as well as physiological information about
the bovine host, (2) bacterial ecosystems in chicken ceca, (3) fecal bacterial microbiomes in humans.
Below, we provide details of individual data sets.

Bovine rumen microbiome and host physiology

The data on rumen microbiomes and host physiology of Holstein cows was downloaded from the paper
by Wallace et al. [37]. For microbiome profiling, raw sequencing reads targeting the bacterial 16S rRNA
gene were downloaded from SRA, and processed with DADA2 (v.1.12.1) [50] to generate a matrix of
read counts per sample at the level of amplicon sequence variants (ASVs). ASVs were clustered into 97%
identity OTUs using CD-HIT [51] and OTU-level relative abundances were calculated from the ASV counts
using the corresponding clusters. The *assignTaxonomy* method of DADA2 was used to assign genus-level
taxonomic labels to the OTUs based on the Silva v. 138 database [52]. We only included bacterial OTUs
whose average relative abundance was larger than 0.1%. This cutoff was based on our previous work
where we had shown that OTU abundance variation below 0.1% is likely to be dominated by technical
noise. In the end, we had data on relative abundances of 156 bacterial OTUs in 790 cows. An additional
OTU represented the combined abundance of all OTUs whose average abundance was below 0.1%. For
training/testing purposes, the data was randomly split into a 80% training set and a 20% testing set. The
OTUs identified in Holstein cows were also used to define the microbiomes of Nordic red cows. There
were 156 bovine hosts belonging to the Nordic red species.
678

Chicken microbiomes
The data on chicken cecal microbiomes was downloaded from the paper by Johnson et al. [38]. Raw 16S
rRNA amplicon sequences were processed as described above. As above, we only included OTUs whose
average relative abundance was larger than 0.1%. In the end, we had a total of 425 host samples and
147 OTUs. As above, one OTU represented the combined abundance of all OTUs whose average
abundance was below 0.1%.
685

Human fecal microbiome
Metagenomics-derived genus-level microbial abundances were obtained from the Inflammatory Bowel
Disease database [39]. We considered 127 samples from the healthy controls. As above, genera with
average relative abundances across samples lower than 0.1% were aggregated into a single taxon,
yielding a total of 95 bacterial genera analyzed.

Section 3. Mixture Gaussian models
692

In our model, the compositions of the microbiomes and host metadata are direct functions of the latent
space variables. We make our model generative by approximating a distribution over the latent space.
To that end, we fit a mixture multivariate Gaussian model using Python's
sklearn.mixture.GaussianMixture to the inferred latent space descriptors. The number of Gaussians in
the mixture model is varied between 1 and 5 and the mixture with the lowest Bayesian information
criterion (BIC). Given that fitting a mixture Gaussian model typically leads to a locally optimum solution,
the number of Gaussians in the mixture and the mean values and covariances matrices of the mixtures
may change from fit to fit. Therefore, we ran the procedure 100 times and chose the Gaussian mixture
model with the lowest BIC.
702

703 **Section 4. Predicting microbiome composition from partial host phenotypic data**

To identify microbiome compositions corresponding to specified partial list of host phenotypes, we deeply sampled the inferred mixture Gaussian distribution and retained the samples with phenotypes of interest with values in proximity to the host values (within two tenths of the standard deviations of the measured phenotypes). We use the mean composition of the communities identified to have the required phenotype values as our predicted microbiome composition. The Bray-Curtis dissimilarity between this predicted microbiome composition and the host microbiome composition was used as the error/distance measure.

Section 5. Biased sampling of the latent space to identify microbiome compositions corresponding to desired host states

To identify microbiome compositions corresponding to desired host phenotypes, we sampled latent variables using the inferred mixture Gaussian distribution amended with a biasing term. Specifically, we modified the log probability of latent variables as

$$\log p(z) = \log p_0(z) - \gamma(st - 3)^2 - \gamma(pH - 3)^2 + const. \qquad (S13)$$

In Eq. S13, $p_0(z)$ is the inferred mixture Gaussian model, st denotes z-scored starch intake and pH denotes z-scored rumen pH. The starch intake and the pH were predicted from the latent variables using the inferred linear model (Eq. S7). $\gamma = 0$ corresponded to the unbiased simulation and biasing was achieved by setting $\gamma = 2$. To sample the latent space, we randomly chose 100 starting points from $p_0(z)$ and performed a Markov chain Monte Carlo simulation with the modified probability. Once the latent variables are sampled, microbiome compositions are predicted using Eq. S6.

Acknowledgments
PD is supported by NIGMS grant R35GM142547. PD and KS are supported by the Bill and Melinda Gates Foundation grant INV-062437 to BiomEdit.

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