Adaptive planning depth in human problem solving

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Abstract

We humans are capable of solving challenging planning problems, but the range of adaptive strategies that we use to address them are not yet fully characterized. Here, we designed a series of problem-solving tasks that require planning at different depths. After systematically comparing the performance of participants and planning models, we found that when facing manageable problems that require planning to a certain number of subgoals (from 1 to 6), participants make an adaptive use of their cognitive resources – namely, they tend to select an initial plan having the minimum required depth, rather than selecting the same depth for all problems. When facing more challenging problems that require planning up to 7 or 8 subgoals, participants tend to select a simpler (greedy) strategy wherein plans consider only the next 1 or 2 subgoals. Furthermore, we found a strong similarity between how different participants solve the same problems. These results support the view of problem solving as a bounded rational process, which adapts costly cognitive resources to task demands.

Keywords: planning; problem solving; cognitive search; bounded rationality

Introduction

Since the early days of cognitive science, researchers have asked how we solve challenging problems that engage planning abilities, such as the Tower of Hanoi and Traveling Salesman as well as popular games such as chess or go (Newell & Simon, 1972; Shallice, 1982; Wiener et al., 2009). Most cognitive theories assume that planning requires a form of cognitive *tree search* over an internal model or mental map of the task (Craik, 1943; Daw & Dayan, 2014; Hunt et al., 2021; Kolling et al., 2012; Tolman, 1948). Following this view, several planning studies in humans and other animals used tree-like tasks, but mostly focused on simple (e.g., two-step) problems which could be searched exhaustively (Akam et al., 2015; Daw et al., 2011; Hasz & Redish, 2018; Miller et al., 2017). It is still unclear how we solve more complex problems, which – given our limited resources – defy exhaustive search.

From a normative perspective, planning with limited resources could be described as a *bounded* rational processes, which balances the accuracy of the solution and the cognitive resources invested, e.g., memory and time (Bhui et al., 2021; Lieder & Griffiths, 2020; Simon, 1957). One way to lower cognitive resources is using *heuristics* to alleviate the burden of exhaustive search (Geffner & Bonet, 2013; Russell & Norvig, 1995). For example, it has been proposed that people use a pruning heuristic during mental search: if they encounter a tree node that seems unpromising, they discard the whole branch of the tree (Huys et al., 2012, 2015; Van Opheusden et al., 2017). Other heuristics consist of sampling only a few promising routes, or many routes but only up to a certain depth (Keramati et al., 2016; Pezzulo et al., 2013). Furthermore, it is possible to alleviate the burden of planning by using a hierarchical approach to split the problem into more manageable subproblems (Balaguer et al., 2016; Donnarumma et al., 2016; Ribas-Fernandes et al., 2011; Solway & Botvinick, 2012; Tomov et al., 2020) or by interleaving planning and execution; for example, plan until a certain subgoal, then revise and complete the plan along the way, as one moves toward the chosen subgoal (Geffner & Bonet, 2013; Russell & Norvig, 1995). Despite this progress, we still have incomplete knowledge of the (approximate) planning methods that humans and other animals might adopt during problem solving, as well as their neuronal underpinning (Mattar & Lengyel, 2022; Miller & Venditto, 2021; Pezzulo et al., 2019).

Another stream of research explored the limitations (e.g., the maximum depth) of our planning abilities. Various studies have shown that with sufficient time, people are able to find near-optimal solutions to challenging problems, such as the Traveling Salesman, which requires finding the shortest possible closed path that connects a fixed number of "cities" (MacGregor & Ormerod, 1996; Vickers et al., 2001). Other studies have tried to quantify planning depth in chess (Chase & Simon, 1973; De Groot, 1946) and other games (Krusche et al., 2018), see (van Opheusden & Ma, 2019) for a recent review. Classical studies reported that top chess players can plan ahead (on average) a relatively small number of moves, between 3.6 and 5.4 and their maximum planning depth is between 6.8 and 9.1 moves (De Groot, 1946; Gobet, 1998; Saariluoma, 1995) and a recent large-scale study reported that planning depth in games increases with expertise up to a ceiling at a similar level (van Opheusden et al., 2023); but see (Campitelli & Gobet, 2004) for evidence that chess grand masters can plan ahead (on average) 13.8 moves.

Planning – especially when done at greater depths – requires engaging significant cognitive resources and hence a key question is whether people make an adaptive (Anderson, 1990) or resource-rational (Callaway et al., 2022, 2018) use of these resources. One possibility is that

people flexibly adapt their planning depth to the demands of the problem. In other words, rather than using a fixed planning depth for all problems, people might use the minimum planning depth necessary to find a solution.

To test this hypothesis, we asked participants to solve a series of planning problems that required finding a path to connect all the "gems" in a grid, without passing through the same node twice (though "backtracking" to un-select nodes was allowed). Participants solved the problems by "navigating" with a finger in a grid that was fully visible on their mobile phones. They had 60 seconds to solve each problem and earned more points if they solved it faster. Figure 1 shows an example problem, which requires finding a path from the home location (yellow node) through all the gems (red nodes). The six panels show six representative timesteps of the solution, with the azure line indicating the path taken (visible to the participant) and the small red dots showing the actual finger positions at different times (not visible to participants). In the example illustrated in Figure 1, the participants see on their mobile phone the configuration shown in Panel A. They first select an incorrect path towards the two gems to the right (Panel B), then they backtrack to the home location (Panels C-D) and finally select a correct path that connects all the gems (Panels E-F) – therefore solving the problem.

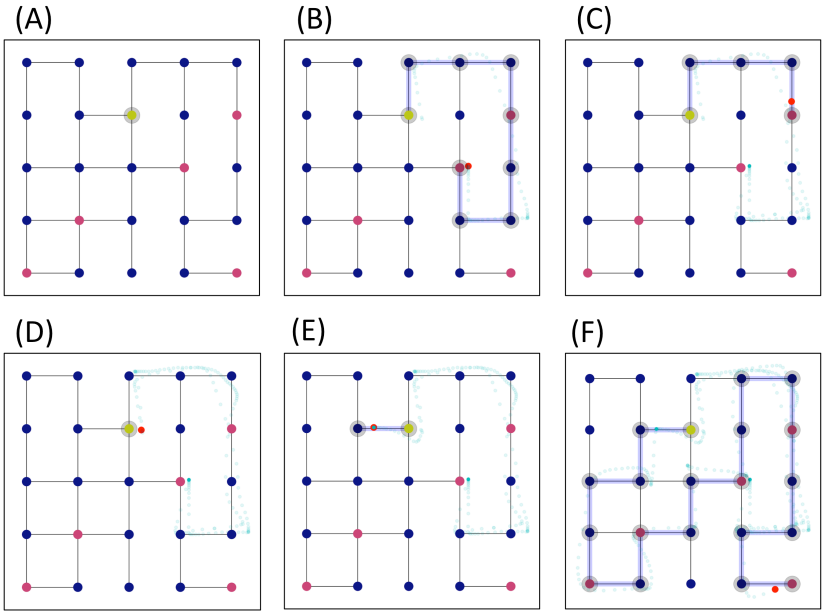

Figure 1. An example problem in the experiment. *The problem requires finding a path in the grid that starts from the home location (yellow node) and collects all the "gems" (red nodes), without passing through the same node twice. Participants solved the problem by navigating with their finger on the (fully visible) grid, on their mobile phones. The figure shows six time steps of the solution, with the azure path indicating the path taken by one of the participants, the azure dots the actual finger trajectory (sampled at 60Hz) and the small red dot the current finger position. Note that solving this particular problem requires planning 5 gems in advance.*

Crucially, we designed problems that require different planning depths to be solved, from 1 to 8. Some problems could be solved using a "greedy" strategy to always move to the closest gem (i.e.,

planning depth 1), whereas other problems required optimizing the plan through the next 2 to 8 gems. For example, the problem shown in Figure 1 requires a planning depth of 5. In other words, finding the correct path (Figure 1F) requires planning 5 gems in advance and any optimal (shortest) path to a lower number of gems (from 1 to 4) makes the solution impossible.

This design allows us to study the planning depth that participants use to solve each problem, by systematically comparing their behaviour with the behaviour of 8 planning models, which use planning depths from 1 to 8. In our analysis, we focused solely on the paths chosen by participants before they made their first backtracks, as these paths serve as valuable indicators of their initial plans. This approach also allows for easier alignment with findings from studies on other games like chess, where movements cannot be reversed.

If people always use the same planning depth (e.g., depth 3) to solve all the problems, then their behaviour should always match the same planning model (e.g., the one at depth 3) across all problems. If instead people recruit resources according to task demands, they will adapt their planning depth to the minimum depth required to solve the problem, and then their behaviour should match a different planner for each set of problems; namely, the planner that uses the (set-specific) minimum depth.

To preview our results, we found that people tend to use the minimum depth required to find the solution, when the depth is between 1 and 6. In contrast, people address problems that require greater planning depth (7 or 8) by using a lower depth – usually 1. Furthermore, we found that the above results are not an artefact of averaging across multiple participants; rather, for each particular problem, most participants tend to select the same planning depth. These results indicate that people flexibly adapt their planning depth – or how much they look ahead – to task demands, unless the problem is too hard, in which case planning costs are minimized.

Methods

Data Collection

The experiment was conducted with the support of ThinkAhead, an Android application developed to study navigational planning and problem solving (https://play.google.com/store/apps/details?id=com.GraphGame.Conan). We recruited 160 participants online and all gave informed consent to our procedures which were approved by the Ethical Committee of the National Research Council. Participants were free to leave the experiment at any moment. In the analysis, we consider the 65 players (42 male, age = 34 +- 11 years; 19 female, age = 35 +- 10; 4 participants who preferred not to specify their gender, age = 34 +- 10 years) who tried at least 80 of the 90 problems of the experiment, although they do not necessarily solve all of them.

Experiment design

The experiment comprised 90 problems, each requiring participants to collect all the "gems" (i.e., colored dots) in a grid, without passing twice through the same node, in 60 seconds; see Figure 1 for an example problem and Figure S1 for other example problems of the three levels and planning depths. Participants were instructed that they would earn points proportional to

the time left to solve the problems and that the points were doubled in problems where the gems were red (which happened in half of the trials) compared to those where the gems were blue (in the other half of the trials). As soon as the problem was shown to the participants, the time countdown started. If the participants did not solve a problem within the deadline, it counted as a failure; participants were allowed to either complete it (without getting any points) or to skip it and pass to the next one.

We generated a range of problems that required different planning depths, from 1 to 8 (see the next section for the exact definition of planning depth). The problem grids were all unique and were generated with an average level of connectivity of .75 +- .2, which we found in a pilot study to afford a good range of planning solutions.

Before the experiment, participants performed a short practice session, in which they had to solve 4 problems, whose results were not analysed. The 90 problems were divided into 3 interleaved blocks (henceforth, "levels"), with 30 problems for each level. We varied planning demands both within and between levels; see Table 1 and Supplementary Figure S1. To vary planning demands between levels, the 3 levels were characterized by increasingly large maps and more gems to be collected, making the higher-level problems (on average) more challenging. To vary planning demands within levels, we divided each level into 3 sublevels of 10 problems each. The 3 sublevels comprised problems that could be solved using planning depth 1, 2-3, or 4-8, respectively; see Table 1. Note that the levels have (on average) increasing difficulty but include a fixed proportion of easier trials (i.e., requiring lower planning depth). This design permits increasing the challenge and simultaneously maintaining a high frequency of success across levels, which is important for intrinsic motivation (van der Kooij et al., 2021).

Table 1. Experiment design. *The experiment is structured into 3 levels of increasing (average) difficulty, each including 30 problems. See the main text for explanation.*

	Level 1	**Level 2**	**Level 3**
Sublevel 1	Depth = 1 Size = 5x5 Gems = 4-5	Depth = 1 Size = 6x6 Gems =6-7	Depth = 1 Size = 6x8 Gems = 8
Sublevel 2	Depth = 2-3 Size = 5x5 Gems = 4-5	Depth = 2-3 Size = 6x6 Gems = 6-7	Depth = 2-3 Size = 6x8 Gems = 8
Sublevel 3	Depth >=4 Size = 5x5 Gems = 4-5	Depth >=4 Size = 6x6 Gems = 6-7	Depth >=4 Size = 6x8 Gems = 8

Analysis of success probability, problem completion time and total number of backtracks

We used multiple logistic and linear regressions and the R library lme4 (Bates et al., 2015) to analyse participants' success probability (i.e., the probability that they solved the problems before the deadline of 60 seconds), problem completion time (i.e., the average time they needed to complete the problem, in seconds) and total number of backtracks that they executed during the experiment. The 3x3 design considers the 3 levels of the experiment (Levels 1-3) and the 3 planning depths (DepthID 1-3, corresponding to depth 1, depths 2-3, and depths 4-8, respectively).

Planning models and assessment of the minimum planning depth required for each problem

To assess participants' planning depth during the experiment, we designed 8 planning models that use a depth-first search strategy over all the simple paths that start from the current position and reach *n* gems, with *n* being the planning depth (from 1 to 8) and the only difference between the planners (Geffner & Bonet, 2013; Russell & Norvig, 1995). For example, a "greedy" planner with $n = 1$ will look for all the simple paths (i.e., paths that do not pass twice on the same node) that reach one and only one gem. A planner with $n = 2$ will look for all the simple paths crossing only 2 gems, and so on. After the computation, the shortest path is selected. If there are many shortest paths the choice will be uniformly random among them. From the new position (i.e., the end of the last selected path), and with the nodes from the chosen paths being removed to comply with task rules, the computation of the next (partial) paths is repeated, until there are no more gems to collect, or the planner has reached a dead-end. The planners cannot backtrack, so once a dead-end is reached, the simulated trial ends. Note that since there might be various paths to choose among, multiple runs of the same planner on the same problem might have different outputs. Please note that the use of the depth-first strategy for the search of the paths was just a method to enumerate all the possible paths (compatible with the constraints of the task and the instantaneous configuration) and any other exhaustive search algorithm (e.g., breadth-first) would produce the same output. See the Supplementary Materials for the pseudocode of the planning algorithm.

We used the 8 planning models to assess the minimum planning depth required to solve each problem. For this, we classified each problem according to the minimum value of *n* for which a solution could be found with non-null probability, from 1 to 8; for example, problems of depth 5 can only be solved by planning models with *at least* 5 as planning depth, but not by planning models at depth 1 to 4. This permits us to group the problems into 8 groups, with the index denoting the minimum planning depth required to solve them. Interestingly, the fact that a problem can be solved by a planner with depth *n* does not imply that deeper planners can necessarily solve the same problem, or that they solve it in the same way. Let's suppose, for example, that a solution to a problem exists for a planning depth of 5: then part of the solution must be made of the shortest path connecting 5 gems. However, a planner with a planning depth of 6 (or higher) looks for the shortest way of connecting 6 gems, and there is no guarantee that this constraint considers the solution found when planning at depth 5 (for example because a longer path to the first 5 gems is needed in order to reach the sixth one with a globally shorter solution). By the same token, planners having different planning depths could solve the same problem in different ways, which permitted us to compare human participants and planners at different depths (see Supplementary Figures S4 and S5).

Similarity between participants and planning models, for each of the 8 problem groups

The comparison between human participants and planning models involved assessing the similarity in planning depth across the eight problem groups that required minimum planning depths ranging from 1 to 8. For each of the 90 problems, we determined the likelihood of the data (number of gems collected by the participants before the first backtrack) given the models (the eight planning models). To ensure comprehensive coverage of potential solutions at varying depths, we utilized 200 instances of each planner for every problem. Subsequently, we identified which planner model, across the range of depths from 1 to 8, exhibited the highest probability (i.e., was the 'winner') within each problem group.

Stereotypy of planning depth across participants on each problem

Furthermore, we devised a measure of stereotypy of the planning depth that the participants select to solve the same problems. A high stereotypy indicates a strong similarity between how different participants solve the same problems. Conversely, a low stereotypy indicates that participants solve the same problems in different ways. To calculate stereotypy, we first needed to assess a similarity between participants and planning models, at the level of single participants, rather than at the group level as above. For this, we compared the number of gems collected by each participant in a given trial, before the first backtrack, with the *average* number of gems collected from 200 instances of each planning models. We picked the planning models whose average number of gems was the closest to that of the participant as the most similar planner. Then, to estimate the variability of planning depth for each problem, we computed the mode of the most similar planning models and then estimated the proportion of participants who chose their planning depth according to the mode. Finally, we averaged this number across the 8 groups of problems sharing the same minimum planning depth.

Results

Participants are sensitive to both problem level and planning depth, but in different ways

The multiple logistic regression shows a main effect of the DepthID (p-value<0.05) and no main effect of the Level (p-value>0.5) on participants' success probability (i.e., the probability that they solved the problems before the deadline of 60 seconds), see Figure 2. Post-hoc analysis, based on the comparisons of the coefficients of the dummy variables and their interactions (values for the dummy variables were defined with respect to the reference condition Level = 1, DepthID = 1) reveal that Level by Depth interactions result significant for all Level by DepthID interactions (p-value <0.01 for Level = 3 and DepthID = 3, and Level = 3, DepthID = 2; p-value <0.05 for Level = 2 and DepthID = 3, and Level = 2, DepthID = 2), with a mean value of probability of success for the task smaller than the reference condition.

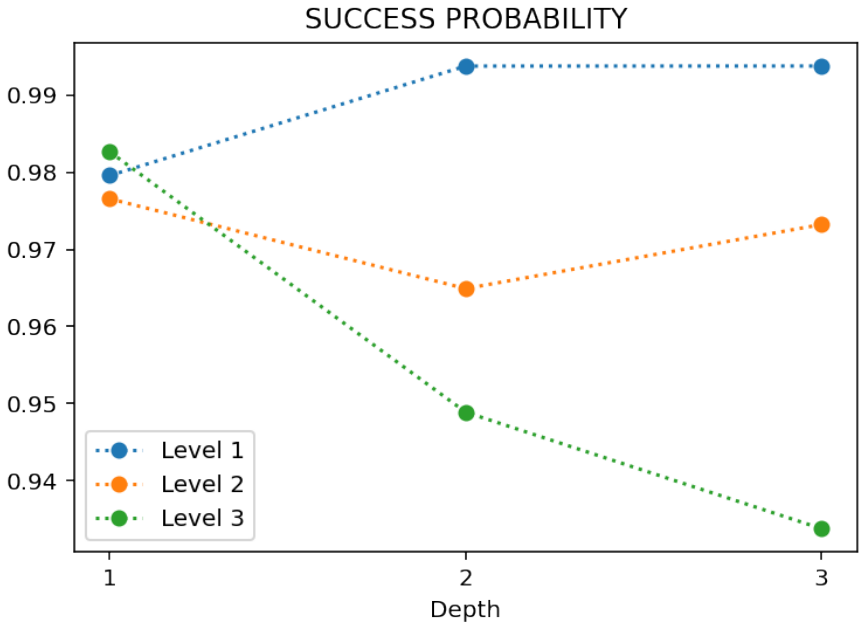

Figure 2. Success probability as a function of the 3 levels of the experiment and of the 3 depthIDs. *The figure permits appreciating that participants' performance decereases as a function of levels, but remains stable across levels when the problem planning depth is 1. See the main text for details.*

The multiple linear regression shows a main effect of both the Level and of the Depth (p-value<0.01 for both) on participants' completion time (i.e., the average time they needed to complete the problem, in seconds), see Figure 3. Post-hoc analysis, based on the comparisons of the coefficients of the dummy variables and their interactions (values for the dummy variables were defined with respect to the reference condition Level = 1, Depth = 1) reveal that: Level by Depth interactions result significant for Level = 3 and Depth = 3 (p-value <0.01), with a mean value of the total time for the task larger than the reference condition.

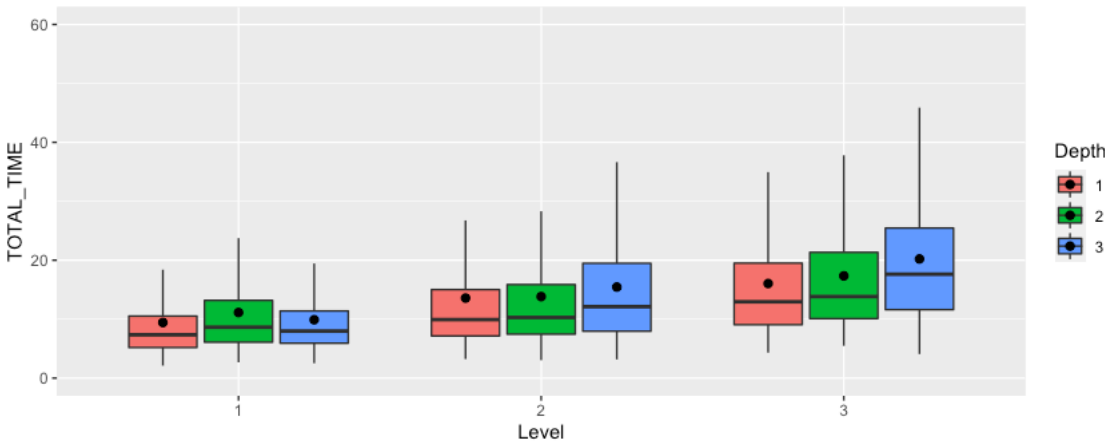

Figure 3. Problem completion time (in seconds) as a function of the 3 levels of the experiment and of the 3 planning depths. *Results are organized by level and depth. The plot shows the total time, which is bounded to 60 seconds in our experiment.*

The multiple linear regression shows a main effect of the Level (p-value<0.01) and no main effect of the Depth (p-value>0.1) on the total number of backtracks that participants executed, see Figure 4. Post-hoc analysis, based on the comparisons of the coefficients of the dummy variables and their interactions (values for the dummy variables were defined with respect to the reference condition Level = 1, Depth = 1) reveal that: Level by Depth interactions result significant for Level= 2 and Level = 3, and Depth = 3 (p-value <0.01), with a mean value of the number of backtrack higher than the reference condition.

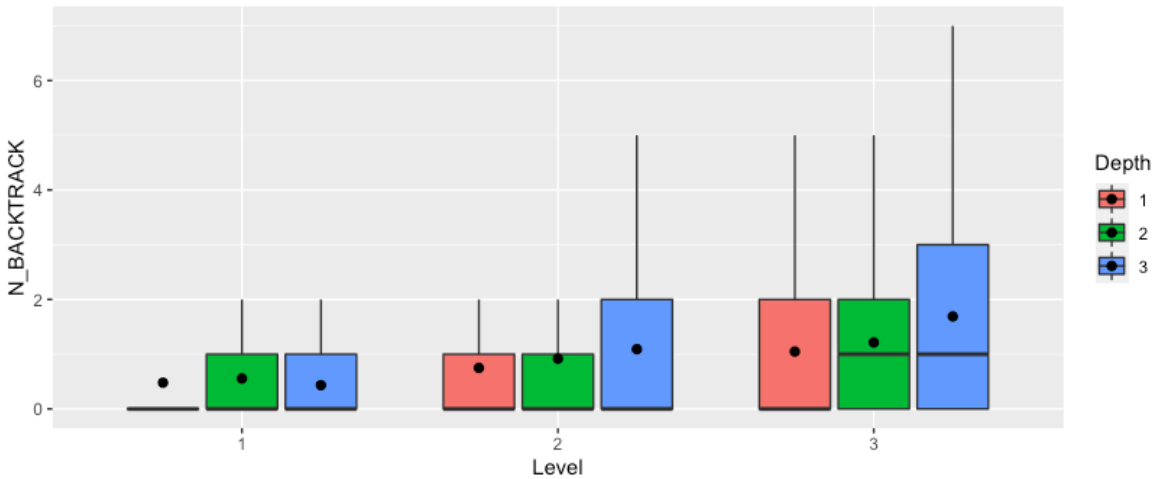

Figure 4. Total number of backtracks as a function of the 3 levels of the experiment and of the 3 planning depths. *Results are organized by level and depth.*

Participants' initial planning depth is adaptive and matches task demands, unless the problem is too hard.

We assessed the *similarity* between the initial plans (i.e., the plans before the first backtrack) of participants and planning models, for each of the 8 problem groups. The results of this analysis, aggregated across the three levels and for red and blue gems, are shown in Figure 5. Our results indicate that in the 6 simpler sets of problems (from 1 to 6), the planner that best explains the behaviour of participants (i.e., has the maximum likelihood for the majority of problems) is the one having the minimum required planning depth. For example, the planner that best explains the behaviour of participants during the solution of problems requiring minimum planning depth of 5 is the planning models having planning depth 5. However, in the two more challenging sets of problems (rows 7 and 8), participants are best aligned with greedy planners that use depth 1.

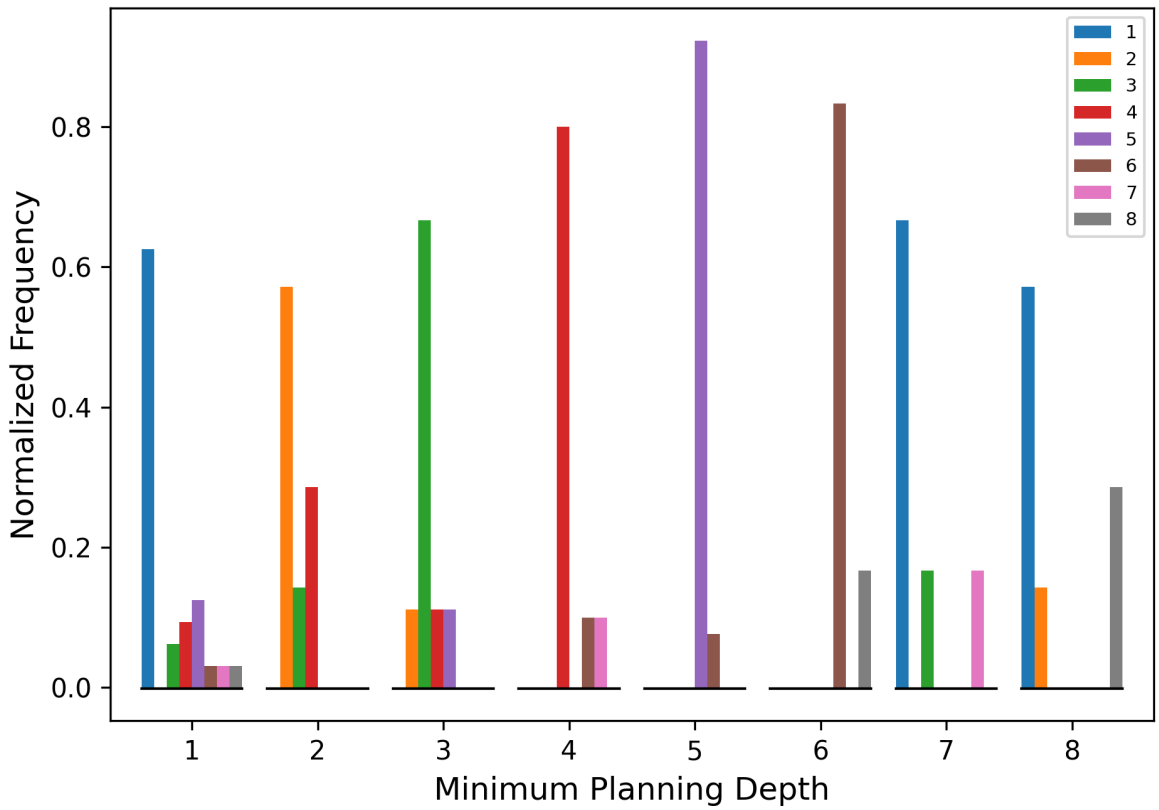

Figure 5. Comparison between participants and planning models. *The figure shows the number of times that each planning model from depth 1 to 8 had the maximum likelihood of the gems collected before the first backtrack by the participants, in the problems of each of the 8 problem groups. Problems are grouped according to the minimum planning depth required to solve them, from 1 to 8, and color coded (see the legend). The frequency of each planner is normalized within each group. For the first 6 groups of problems, the planner that wins most frequently (i.e., that has the greatest likelihood in the plurality of simulations) is the one having the minimum planning depth. In contrast, for the last 2 groups of problems, the winning planner has depth 1.*

We obtain the same results when considering the Kullback–Leibler (KL) divergence between the distribution of the number of gems collected by the whole group of participants for any given problem before the first backtrack, and the distributions of the number of gems collected by 200 instances of each of the 8 planning models (see Supplementary Figure S2). Moreover, we achieve identical outcomes when examining the problems separately for the 3 levels of the experiment (Supplementary Figure S3), separately for red or blue gems (Supplementary Figure S4) and grouping them into classes reflecting their minimum required depth and maximum number of gems (Supplementary Figures S5 and S6). These control analyses indicate that our main findings do not depend on map size (which varies across levels and class groups) or incentives (which are different for red and blue gems). Furthermore, these control analyses highlight that for all problem groups, the 8 planning models collect different average gem distributions and are therefore distinguishable (Supplementary Figures S5 and S6).

Strong similarity between how different participants solve the same problems

We next assessed the stereotypy of participants' initial planning depth, in the 8 problem groups. The results of the analysis are shown in Figure 6. There is a decreasing linear trend of the behavioural stereotypy as the planning depth increases (orange line), which was estimated through Pearson test (coefficient: -0.94, p-value = 3*10^-4). The figure shows that the proportion of problems that satisfy the behaviour stereotypy criterion (blue dot-dash line) is very high and beyond chance level (black dot-dash line), across all problem groups. Indeed, the slope of participants' stereotypy (m = -3.5*10^-2, std = 4.9*10^-3) was significantly smaller than the slope of the random case (m = -6,4*10^-3, Z-test = -4,89, p-value <10^-5). We confirmed this statement by evaluating the statistical significance of the interaction between the depth and the group (Participants vs. Random case) with an ANOVA test (F = 2.98, p-value = 5.8*10^-3).

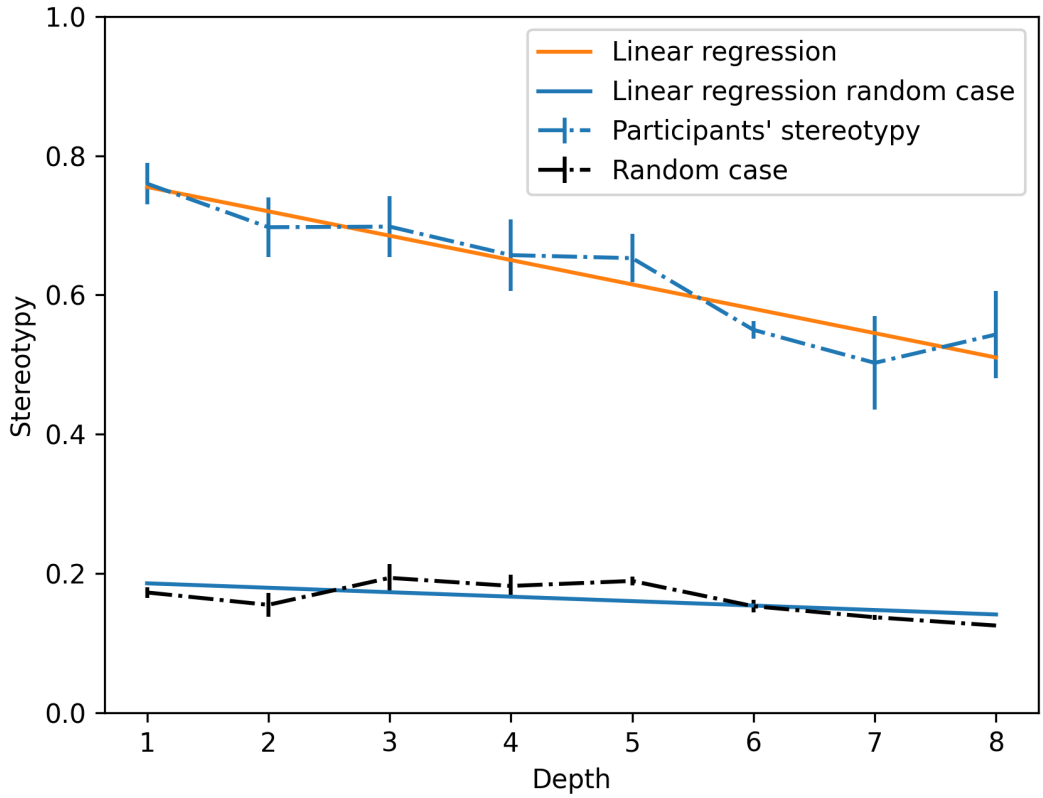

Figure 6. Stereotypy of participants' initial planning depth, in the 8 problem groups. *The figure shows the mean stereotypy across problems having minimum planning depth from 1 to 8. The blue dotted bar shows the data trend, with error bars indicating the standard error of the mean, obtained by averaging problem stereotypy in each problem set. The orange line indicates the linear fit. The black dashed line indicates chance level. Note that since the number of possible planners is bounded by the maximum number or gems in the problem, the expectation for the null case (random across the possible depths) varies across the 8 problem groups. See the main text for explanation.*

Figure 7 further unpacks the results illustrated in Figure 6, by showing the results for each participant and problem. Each vertical block in Figure 7 indexes one of the 8 groups of problems having planning depth (D) 1 to 8, each row indexes one participant and the colours indicate the depth of the planner that best resembles the performance of each participant in each problem (white dots are missing points, e.g., skipped problems). The figure helps appreciate that many participants tend to select the same planning depth to address the same problem, as evident by the numerous vertical bands of Figure 7. Notably, this happens also in "misleading" trials, such as problems 10 and 23 in the first column (D = 1), which were addressed using high planning depths, despite its minimum depth was 1.

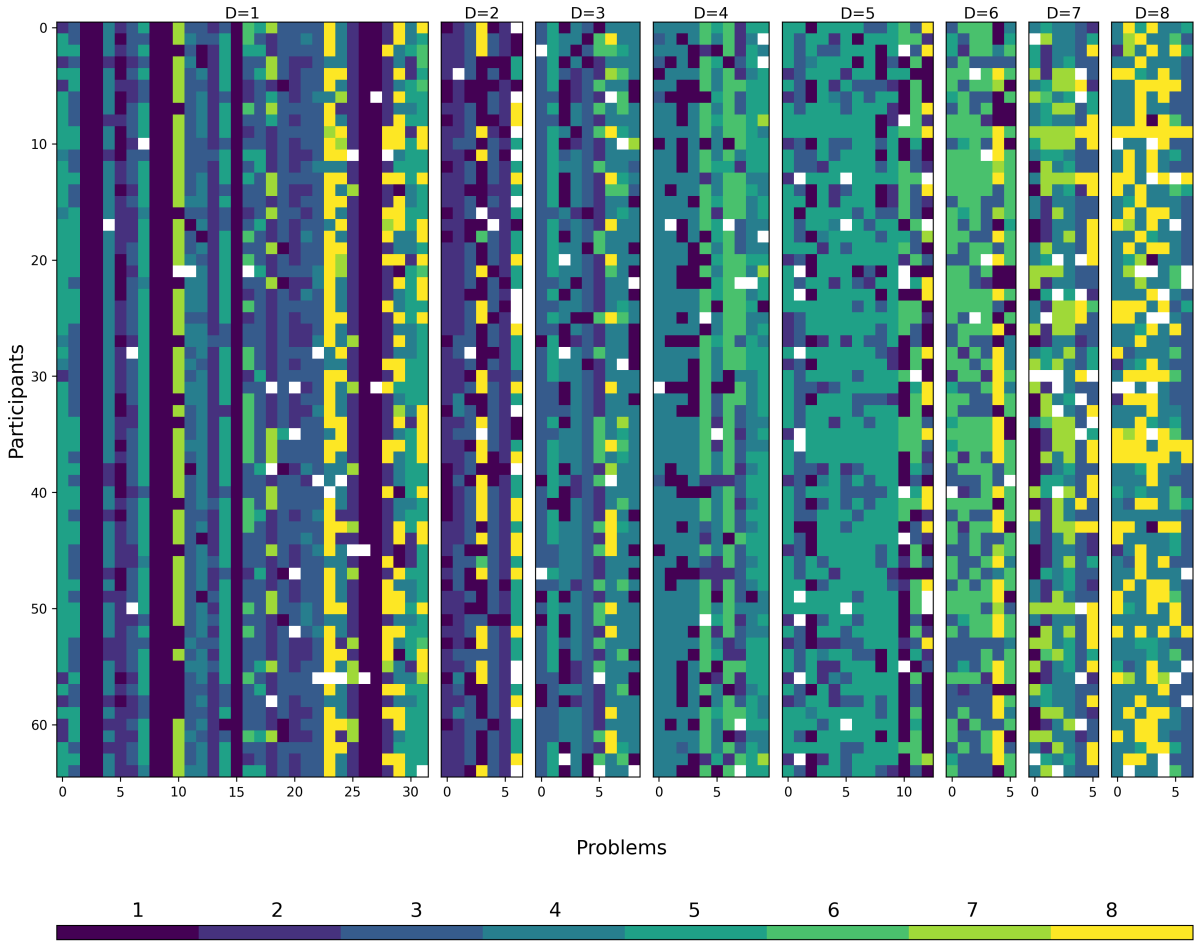

Figure 7. Variability of planning depth across participants. *Each row corresponds to a different participant (y-axis) and each column to a specific problem (x-axis), so each square is a different trial. Problems are grouped according to the minimum required planning depth (D, in the figure), ranging from 1 to 8. The color bar, going from 1 (dark blue) to 8 (yellow), assigns to each trial the color of the most similar planning models. White squares are missing or discarded trials. The resulting vertical "bands" of the same colors suggest the presence of problems for which several participants used the same planning depth (see the main text for explanation).*

Discussion

Since the early days of cognitive science, researchers have asked how we solve complex planning problems that defy the exhaustive evaluation of all possible choices (Newell & Simon, 1972). It is commonly assumed that planning is a form of cognitive tree search using a mental map (Craik, 1943; Daw & Dayan, 2014; Hunt et al., 2021; Kolling et al., 2012; Tolman, 1948). However, except in the simplest cases, exhaustive search is infeasible, hence pointing to *bounded* forms of planning that adapt cognitive resources to task demands (Bhui et al., 2021; Lieder & Griffiths, 2020; Simon, 1957).

In this study, we asked whether participants adapt their planning resources to the demands posed by problems of different complexity. For this, we asked participants to solve a series of planning problems that required finding a path to connect all the "gems" in a grid, without passing through the same node twice. The problems were divided into three levels, characterized by increased map size and number of gems. We varied planning demands both within and between levels, therefore designing 8 groups of problems that required a minimum planning depth from 1 to 8. The planning depth required for each problem was unknown to the participants.

Our results indicate that participants' performance is influenced by both the level of complexity and the planning depth, albeit in distinct ways. The probability of problem-solving depends predominantly on planning depth rather than the level of difficulty, underscoring that the challenge lies more in the cognitive demands imposed by a problem rather than its spatial scale. The time required for problem-solving is not only affected by planning depth but also by the level of complexity, illustrating that addressing problems within larger maps inherently demands more time. Lastly, the frequency of backtracking correlates more with the level of difficulty than the planning depth. Particularly in large maps with numerous gems, participants often need to change their initial plans because they can't fully solve these complex problems in their minds.

Furthermore, and crucially, our results show that participants flexibly adapt their initial planning depth (i.e., the depth of their plans before the first backtrack) to the minimum depth required to solve the problems, as estimated using planning models. However, we only observe this flexible adaptation during the solution of relatively manageable problems, requiring planning depth from 1 to 6. In contrast, participants tend to use a simpler, "greedy" strategy to plan at depth 1 when dealing with the most challenging problems, requiring planning depth 7 or 8. Finally, we found a strong correlation between how different participants solve the same problem, which permits ruling out the possibility that our results are an artefact of averaging across multiple participants.

This study adds to a large literature showing that during problem solving, people adapt to the complexity and structure of the environment (Simon, 1996), make an adaptive use of their cognitive resources (Anderson, 1990) and adopt various simplifications when the problems are too challenging. For example, people prune unpromising branches of the search tree (Huys et al., 2012, 2015) and reduce tree search under time pressure (Van Opheusden et al., 2017). An emerging idea is that these (and other) simplifications might be "rational", in the sense that they entail a flexible and efficient use of limited resources– i.e., *bounded* or *resource-rational* planning (Callaway et al., 2022, 2018). For example, during problem solving, people might spend more time planning ahead when the benefits of investing cognitive resources are greater (Russek et al., 2022).

In keeping with this perspective, our study shows that when participants address manageable problems, they use their cognitive resources efficiently, by selecting initial plans that have the minimum required depth (Figure 5). These initial plans therefore tend to use the appropriate depth, in keeping with previous reports that the initial moves of participants during problem solving are usually "good enough" (Klein et al., 1995). However, the above pattern of results breaks down when addressing the two more challenging sets of problems – because to address these problems, participants often use a much lower planning depth. Note that neither the levels nor the economic incentives (i.e., the different values of red and blue gems) significantly modulate planning depth, plausibly because the task is challenging and participants might always perform at ceiling level (see Supplementary figures S2, S3, and S4).

Furthermore, we found a strong similarity between how different participants solve the same problem. This stereotypy was partially unexpected, given that previous studies of navigational planning showed a greater variability of strategies (Krichmar & He, 2021; Lancia et al., 2023). However, please note that our experiment required lower memory demands than the previous studies, since the participants could observe the whole problem graph. It is possible that part of the variability reported in the previous studies was due to incorrect map memorization, not planning; but this hypothesis remains to be tested in future studies.

Taken together, these results indicate that during problem solving, people make an adaptive use of their cognitive resources by selecting the appropriate level of planning depth – or a much simpler (and cheaper) planning strategy if the problem is too challenging.

This study has various limitations that need to be addressed in future work. First, this study indicates that participants adapt their initial planning depth to task demands but does not clarify how they do that. There are multiple alternative strategies that could explain our findings. For example, before navigation begins, participants might use a "gist" of the maze to decide planning depth – or how much cognitive resources they would need to invest to plan ahead – and then use such planning depth to find a solution (note that selecting an appropriate planning depth does not necessarily entail solving a particular problem, because there are usually several alternative paths at the same planning depth). In the planning literature, there is a key distinction between an "encoding" phase in which participants form a mental representation of the problem and the "planning" phase, in which they form a plan based on the mental representation. Previous results suggest that during the encoding phase, participants might form simplified mental representations of the problems, which omit many details (Ho et al., 2022). It is possible that this simplified (or gist) representation could be sufficient to guide the selection of an appropriate planning depth, but future studies are required to understand whether and how this is possible. Alternatively, before navigation begins, participants might start searching for a solution at low planning depth and then increase the depth progressively, until they find a (satisficing) solution. This procedure, which is akin to "iterative deepening depth-first search" in AI (De Groot, 1946; Korf, 1985) and to algorithms that start from an initial "cheap" plan and progressively refine it (Lancia et al., 2023; Todorov, 2009), would impose a filter on insufficient planning depth, which could result in the over-investing bias apparent in Figure 5, when considering that for most problem groups, the second-best planner has a greater depth than the minimum required depth. In other words, it could be faster to recognize that your strategy is too poor for the problem rather than recognizing that you are investing too much. However, when the required planning depth is deemed excessive, participants might give up and rather select the least costly initial plan (i.e., a plan at low depth) – and possibly revise it

along the way. The plausibility of these or alternative strategies for adapt planning depth remains to be fully investigated in future studies.

A related limitation is that this study does not disambiguate the "algorithm" used by the brain to solve the problems. When establishing a similarity between the planning models and human participants, we are not necessarily claiming that human participants use depth-first planning, but only that they appear to adapt their planning depth across problems – regardless of mechanism (e.g., a different planner for each problem, "iterative deepening depth-first search", or other methods). Having said this, our specific setting, in which the problem maps are novel and fully visible, epitomizes the use of model-based planners (Daw & Dayan, 2014; Dickinson & Balleine, 1994; Dolan & Dayan, 2013; Friston et al., 2016; Parr et al., 2022). While in principle the problems considered in this experiment could be solved using model-free and successor representation algorithms that dispense from planning (Dayan, 1993; Gershman, 2018; Sutton & Barto, 1998), these are unlikely candidates, since they would require an extensive learning phase, whereas in our experiment the participants never see the same map twice. Future studies might compare more directly various planning (or even non-planning) methods. Apart for those discussed above, another relevant class of algorithms is Monte Carlo planning. These algorithms offer an approximate solution to the problem of sampling in large or continuous state spaces (Silver & Veness, 2010) and have been linked to human cognitive search (Jensen et al., 2023; Mattar & Lengyel, 2022; Pezzulo et al., 2019), but would behave similarly to depth-first planners in our (relatively small scale) problems. Another possibility is using hierarchical planners, which split large problems into smaller, more manageable ones (Donnarumma et al., 2016; Solway et al., 2014; Tomov et al., 2020). Given that the focus of this paper is on the first plans that people form, not on their entire plans, the use of hierarchical planners seems less compelling; but future studies could use hierarchical planners to extend the results of this study to the entire plan selected by participants.

This leads us to another limitation of this study: the fact that it only focuses on the initial planning phase. Focusing on the first part of the plan is meaningful, since previous research has established that the initial moves of participants during problem solving are revelatory of their strategy (Klein et al., 1995). Furthermore, from a methodological perspective, considering only the first part of the plan drastically simplifies the assessment of planning depth, since it does not require making assumptions about the algorithm used to decide (for example) when to backtrack or whether to change plan or plan depth along the way, as some planning models do (Weld, 1994). In other words, this choice does not require delving into the full complexity of planning, acting and replanning dynamics that can occur in our setup. However, a more complete analysis of planning dynamics in our experiments should consider that backtracking is part and parcel of human problem solving (De Groot, 1946) and that people might generate longer plans when they are forced to do so – for example, because they cannot backtrack (Delaney et al., 2004). We hope to address these open issues in future studies.

Finally, another limitation of the study is that by restricting our analysis to participants who downloaded the game and completed at least 80 problems, we could have selected those having sufficient skill and/or engagement levels. Previous studies with a much larger pool of participants reported significant individual differences in navigation ability (Coutrot et al., 2022), suggesting that weaker navigators might not show the same adaptive use of resources that we report here. Therefore, the possible differences in adaptive planning depth between good and weak navigators remains to be tested in future studies.

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Software note

Giovanni Pezzulo devised the ThinkAhead App concept. The App has been implemented in Unity by Antonella Maselli on a general-purpose architecture for maps navigation games previously implemented by Massimiliano Schembri. Marco D'Alessandro and Jeremy Gordon contributed with setting the online communication with a dedicated database used to store users' progresses and solutions. Gian Luca Lancia edited the video tutorial. Mattia Eluchans prepared the list of problems shown to participants for the current study.

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