Which Variable Should Be Dependent in Phylogenetic Generalized Least

Squares Regression Analysis Under Pagel's Lambda Model

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ABSTRACT

Phylogenetic generalized least squares (PGLS) regression is widely used to analyze evolutionary correlations between two traits. In contrast to the equal treatment of analyzed traits in conventional correlation methods such as Pearson and Spearman's rank tests, PGLS regression accounts for phylogenetic non-independence unbalanced between independent and dependent variables when estimating some evolutionary parameters. We investigated the correlations of CRISPR-Cas and prophage contents with optimal growth temperature and minimal doubling time using Pagel's λ model and found that switching independent and dependent variables resulted in conflicting results in 26.3% of cases. To better understand this phenomenon, we conducted 12,000 simulations of the evolution of two traits (X_1 and X_2) along a binary tree with 100 terminal nodes, using different models and variances. We found that switching dependent and independent variables in PGLS regression analysis using Pagel's λ Model resulted in conflicting outcomes in 17.2% of cases. To assess correlations in each simulation, we established a gold standard by analyzing changes in traits along phylogenetic branches. Next, we tested seven potential criteria for dependent variable selection: log-likelihood, Akaike information criterion, R^2, p-value, Pagel's λ, Blomberg et al.'s K, and the estimated λ in Pagel's λ model. We determined that the last three criteria performed equally well in selecting the dependent variable and were superior to the other four. As Pagel's λ and Blomberg et al.'s K values are indicators of phylogenetic signals and are commonly calculated at the beginning of phylogenetic comparative studies, we suggest using the trait with a higher λ or K value as the dependent variable for practicality in future PGLS regressions. Logical cause and effect analysis should be conducted after establishing a significant correlation by PGLS regression, rather than providing an indicator for the dependent variable.

Keywords

Evolutionary correlation; simulation; phylogenetic signal; phylogenetic generalized least

squares (PGLS); regression; dependent variable; Akaike information criterion (AIC).

Through mutations, genetic drift, and natural selection, some biological characteristics

tend to co-evolve throughout evolution (Felsenstein 1985, Revell and Collar 2009, Goswami

et al. 2014, Caetano and Harmon 2019, Revell et al. 2022). Analyzing correlations between

such traits can reveal the extent and direction of changes in one characteristic in response to

evolutionary changes in another, providing supporting evidence or contradictions for

hypotheses (Pearman et al. 2014). This in-depth analysis of biological processes contributes

to a better understanding and reconstruction of evolutionary history (Bartomeus et al. 2018,

Bawa et al. 2019, Suarez-Castro et al. 2020).

The relationship between the two variables is typically assessed by calculating Pearson,

Spearman's rank, or Kendall's rank correlation coefficient. Conventional statistical methods,

however, have limitations in dealing with evolutionary traits because they violate the

assumption of independence and are interconnected through the evolutionary relationships of

the species studied (Felsenstein 1985). Neglecting phylogenetic dependence may lead to

distorted trait correlations (Revell 2010, Whitney and Garland 2010). Various techniques

have been developed to address the issue of phylogenetic non-independence (Felsenstein

1985, Grafen 1989, Lynch 1991, Garamszegi 2014). Among them, the phylogenetic

generalized least squares (PGLS) approach, first proposed by Grafen (1989) and further

refined by Martins and Hansen (1997), Pagel (1997, 1999), and Rohlf (2001), has become the

most commonly used method for determining the relationship between multiple traits. In

interpreting trait correlations using PGLS regression, positive and negative slope values

correspond to significant positive and negative correlations.

Both traits occupy the same position in standard correlation analyses, such as Pearson

and Spearman's rank tests. However, in a regression model, one trait is designated as the

independent variable and the other as the dependent variable. When exploring trait

correlations through regression models, there is a common misconception that the positions

of independent and dependent variables are not equivalent. A causal relationship is often

assumed in regression analysis (Waugh 1943). For example, when assessing the relationship

between GC content and ecological factors, researchers often choose ecological factors as the

independent variable and GC content as the dependent variable (Travnicek et al. 2019, Hu et

al. 2022), assuming that environmental factors may have driven the evolution of GC content.

However, according to statistical principles, regression analysis only examines whether there

is a significant positive or negative correlation between the two traits. In standard regression

analysis, the independent and dependent variables are interchangeable; switching them will

not affect the sign (positive or negative) of the regression coefficient or the significance test

results ($p <$ or ≥ 0.05). Therefore, we could arbitrarily select one trait as the independent

variable and designate the other as the dependent variable in standard regression analysis.

To control for the amount of phylogenetic signal in the data, PGLS assumes that the

model residual error is distributed according to the variance-covariance matrix representing

the residual variance and the evolutionary relationships among species. In its simplest form,

PGLS assumes a Brownian Motion model of evolution at a single rate. By parameter

estimation and altering the properties of the variance-covariance matrix, PGLS can be

extended to various evolutionary models, like Pagel's λ model (Freckleton et al. 2002),

Ornstein–Uhlenbeck model (Hansen 1997, O'Meara and Beaulieu 2014) and the early burst

model (Harmon et al. 2010). At this point, PGLS regression analysis becomes different from

standard regression analysis. Switching independent and dependent variables in the PGLS

analysis will result in inconsistent estimates of the parameters (like Pagel's λ) and so

inconsistent correction for the phylogenetic non-independence of the trait values. As a result,

conflicting results may appear. In principle, only when using evolutionary models that do not

require parameter estimation, such as the Brownian Motion model, will switching

independent and dependent variables in PGLS analysis not affect results.

In examining the correlations presented in a recent publication from our laboratory

(Liu et al. 2023), we observed that switching independent and dependent variables can

significantly impact the results and conclusions of correlation analysis (see Results section).

A significant correlation between two traits ($p < 0.05$) can potentially become non-significant

($p \geq 0.05$) after such variable swaps. In these situations, how can we determine which

regression model to accept?

Using Pagel's λ model as an example, we further assessed the frequency of inconsistent

outcomes caused by interchanging independent and dependent variables in the PGLS analysis

of simulated data. In addition, we compared seven criteria to determine the appropriate

designation of independent and dependent variables in PGLS analysis.

MATERIALS AND METHODS

Phylogenetic Generalized Least Squares

Phylogenetic generalized least squares, as proposed by Martins and Hansen (1997),

rely on the generalized least squares model and can be written as

$$\boldsymbol{y} = \boldsymbol{X\alpha} + \boldsymbol{\varepsilon}, \ \boldsymbol{\varepsilon} \sim MVN(0, \sigma^2 \boldsymbol{\Sigma}), \tag{1}$$

where $\boldsymbol{y}$ is an n-dimensional vector of values of trait y, considered as a response variable in

the regression model, n is the sample size. $\boldsymbol{X}$ is an $n \times (1 + p)$ matrix of a column of

ones and p columns of explanatory variables; the first column of ones can be interpreted as

the intercept. $\boldsymbol{\alpha}$ is a column vector of regression coefficients. $\boldsymbol{\varepsilon}$ is a column vector of

residuals, under the multivariate normal distribution, with a mean of 0 and a

variance-covariance matrix of $\sigma^2 \boldsymbol{\Sigma}$. $\boldsymbol{\Sigma}$ is a matrix describing phylogenetic relationships

(topology and branch length). Consequently, $\boldsymbol{\varepsilon}$ will have a multinormal probability density

given by

$$p(\boldsymbol{\varepsilon}) = \frac{1}{(2\pi\sigma^2)^{n/2}} |\boldsymbol{\Sigma}|^{-1/2} \times exp\left[-\frac{1}{2\sigma^2}\boldsymbol{\varepsilon}^T \boldsymbol{\Sigma}^{-1}\boldsymbol{\varepsilon}\right]. \tag{2}$$

According to equation (1), equation (2) can be expressed by

$p(\boldsymbol{\varepsilon}) = \frac{1}{(2\pi\sigma^2)^{n/2}} |\boldsymbol{\Sigma}|^{-1/2} \times exp\left[-\frac{1}{2\sigma^2}(\boldsymbol{y} - \boldsymbol{X\alpha})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{y} - \boldsymbol{X\alpha})\right],$ (3)

and the log-likelihood function can be given by

$L(\widehat{\boldsymbol{\alpha}}, \widehat{\sigma}^2) = -\frac{(\boldsymbol{y}-\boldsymbol{X}\widehat{\boldsymbol{\alpha}})^T(\widehat{\sigma}^2\boldsymbol{\Sigma})^{-1}(\boldsymbol{y}-\boldsymbol{X}\widehat{\boldsymbol{\alpha}})}{2} - \frac{\log(|\widehat{\sigma}^2\boldsymbol{\Sigma}|)}{2} - \frac{n\log(2\pi)}{2},$ (4)

The parameters $\boldsymbol{\alpha}$ and σ^2 can be estimated by maximizing the value of $L(\widehat{\boldsymbol{\alpha}}, \widehat{\sigma}^2)$ via a

maximum likelihood approach.

All PGLS regressions in this study were performed using the R (version 4.0.2) package

phylolm (version 2.6.2) (Ho and Ane 2014).

Evolutionary Models

The Brownian motion (BM) on a phylogeny is like a "random walk" model, in which

the trait value changes with a constant probability σ^2 in any unit of time. In this model,

$\boldsymbol{\Sigma}_{ij} = \boldsymbol{C}_{ij}$, where $\boldsymbol{C}_{ij}$ is the distance from the root node to the most recent common ancestor

of tip i and j. If $i = j$, $\boldsymbol{C}_{ij}$ will be the distance from the root node to the tip i. Pagel (1997)

introduced the parameter λ to represent different evolution rates on branches and perform

length transformations. In Pagel's λ model, the off-diagonal elements of the

variance-covariance matrix are multiplied by λ. We have noted the variance-covariance

matrix modified by λ as $\boldsymbol{\Sigma}(\lambda)$. Multinormal probability density can be written as:

$p(\boldsymbol{\varepsilon}) = \frac{1}{(2\pi\sigma^2)^{n/2}} |\boldsymbol{\Sigma}(\lambda)|^{-1/2} \times exp\left[-\frac{1}{2\sigma^2}(\boldsymbol{y} - \boldsymbol{X\alpha})^T\boldsymbol{\Sigma}(\lambda)^{-1}(\boldsymbol{y} - \boldsymbol{X\alpha})\right],$ (5)

and the log-likelihood function can be given by

$L(\lambda, \widehat{\boldsymbol{\alpha}}, \widehat{\sigma}^2) = -\frac{(\boldsymbol{y}-\boldsymbol{X}\widehat{\boldsymbol{\alpha}})^T(\widehat{\sigma}^2\boldsymbol{\Sigma}(\lambda))^{-1}(\boldsymbol{y}-\boldsymbol{X}\widehat{\boldsymbol{\alpha}})}{2} - \frac{\log(|\widehat{\sigma}^2\boldsymbol{\Sigma}(\lambda)|)}{2} - \frac{n\log(2\pi)}{2}.$ (6)

The parameter λ, lying between 0 and 1, is estimated by a search procedure to maximize the

likelihood of equation (6) (Freckleton et al. 2002). The higher the λ value, the stronger the

phylogenetic dependence. If $\lambda = 0$, there is no phylogenetic dependence between residuals. If

$\lambda = 1$, $\boldsymbol{\Sigma}(\lambda) = \boldsymbol{C}$, Pagel's λ model and Brownian motion model are equivalent.

The phylogenetic signals (λ) were estimated using the R (version 4.0.2) package

phytools (version 1.0-3) (Revell 2012).

Empirical Datasets

A dataset including the empirical minimal doubling times, CRISPR spacer numbers,

optimal growth temperature, and prophage numbers of 262 bacteria was extracted from the

Supplemental Material Table S1 of Liu et al. (2023). The phylogenetic tree of these 262

bacteria was retrieved from the Genome Taxonomy Database (GTDB; accessed April 8, 2022)

(Parks et al. 2022).

Simulation Data

First, we generated a binary tree containing 100 terminal nodes using the R package

ape (Paradis and Schliep 2019). The trait X_1 evolved according to a Brownian motion model

(using the R package ape) with a variance rate $\sigma^2_{BM} = 4$ along the tree, and then the trait

X_2 was simulated according to

$X_2 = X_1 + \varepsilon,$ (7)

where ε was a normally distributed random noise term with a mean of 0 and a variance of 1,

4, 16, 64, 256 to 1024. The term ε introduced noise to the dependent variable X_2 and a

gradient variance of this noise term (from 1 to 1024) changed the correlation between X_1

and X_2, from strong to weak. We named this case "BM & BM + Norm."

Then, we simulated the trait X_1 according to a normal distribution with a mean of 0

and a variance of 4. The trait X_2 was simulated according to equation (7), where ε was

simulated according to a Brownian motion model with σ^2_{BM} in a range of 1, 4, 16, 64, 256,

1024. We named this case "Norm & Norm + BM."

We ran 1000 simulations for each parameter condition, and a total of 12,000

simulations were performed.

RESULTS

Switching Dependent and Independent Variables in PGLS Analyses of Empirical Data

Sometimes Gave Conflicting Outcomes

In a recent publication from our laboratory (Liu et al. 2023), a dataset comprising a

range of traits from 262 bacteria was deposited as Supplemental Material Table S1. We

analyzed the correlations of empirical minimal doubling time and optimal growth

temperature with genomic characteristics. Thirty-eight pairs of traits were re-examined using

Pagel's λ model in the PGLS approach. However, switching dependent and independent

variables in 10 pairs resulted in conflicting outcomes, leading to different conclusions (Table

1). For instance, when considering the average prophage number as the dependent variable

and the optimal growth temperature as the independent variable, the PGLS analysis

demonstrated a significant negative correlation between the two traits ($p = 9 \times 10^{-4}$, Table 1).

In contrast, switching dependent and independent variables showed no statistically significant

correlation ($p = 0.242$, Table 1). Identifying and relying on accurate findings rather than

conflicting results is necessary. However, the question is how one can determine which one is

correct.

Table 1. Switching dependent and independent variables in PGLS may give conflicting

results[a]

Dependent Variable ~ Independent Variable	Slope	*p*	R^2

EMDT ~ min_CRISPR_spacer	0.010	0.049	0.015
min_CRISPR_spacer ~ EMDT	1.376	0.053	0.014
EMDT ~ mean_prophage_number	0.565	0.013	0.024
mean_prophage_number ~ EMDT	0.008	0.558	0.001
EMDT ~ min_prophage_number	0.622	0.007	0.028
min_prophage_number ~ EMDT	0.011	0.427	0.002
EMDT ~ max_prophage_number	0.448	0.037	0.017
max_prophage_number ~ EMDT	0.005	0.722	5×10^{-4}
OPT ~ min_CRISPR_array	0.308	0.061	0.014
min_CRISPR_array ~ OPT	0.053	8×10^{-5}	0.058
OPT ~ mean_prophage_number	−0.197	0.242	0.005
mean_prophage_number ~ OPT	−0.031	9×10^{-4}	0.042
OPT ~ min_prophage_number	−0.154	0.374	0.003
min_prophage_number ~ OPT	−0.026	0.004	0.031
OPT ~ max_prophage_number	−0.183	0.214	0.006
max_prophage_number ~ OPT	−0.036	4×10^{-4}	0.047
OPT ~ prophage_existence	−1.046	0.182	0.007
prophage_existence ~ OPT	−0.009	8×10^{-4}	0.043
OPT ~ Average_Cas_gene_number	0.197	0.610	0.002
Average_Cas_gene_number ~ OPT	0.023	0.012	0.043

[a]Pagel's λ model was used in these analyses. The dataset containing 262 bacteria was extracted from the Supplemental Material Table S1 of Liu et al. (2023), and the phylogenetic tree used in PGLS analysis was retrieved from the Genome Taxonomy Database (Parks et al. 2022). EMDT, empirical_minimal_doubling_times; OPT,

mean_optimal_growth_temperature.

Prevalence of Conflicting Resulting from Switching Dependent and Independent Variables

In the above survey on correlations within empirical data, we found that switching

dependent and independent variables resulted in conflicting outcomes in 26.3% of cases. To

determine the prevalence of conflicting results from switching dependent and independent

variables, we simulated the progression of two traits along a binary tree of 100 species.

Different distributions of the traits were simulated. In the case of "BM & BM + Norm", we

constructed the trait X_1 under the Brownian motion model ("BM") and the trait X_2, as "BM +

Norm", equals to X_1 plus a noise term "Norm." In the case of "Norm & Norm + BM," the

noise term is "BM." To account for varying levels of correlation, we set the gradient variance

of the noise term, with the variance σ^2_{Norm} varying from 1, 4, 16, 64, 256, to 1024. For

each parameter condition, we simulated 1000 times.

For the data from each simulation, we performed two rounds of PGLS analysis,

$X_1 \sim X_2$ and $X_2 \sim X_1$. The results are presented in Supplementary Table S1 and summarized in

Table 2. In 3768 simulations, neither $X_1 \sim X_2$ nor $X_2 \sim X_1$.gave significant correlations ($p \geq$

0.05 for all of them). In 6150 simulations, both $X_1 \sim X_2$ and $X_2 \sim X_1$.gave significant

correlations ($p < 0.05$). In each of these 6150 cases, the regression coefficients of $X_1 \sim X_2$

and $X_2 \sim X_1$ have the same sign. In the other 2064 simulations, only one of $X_1 \sim X_2$ and

$X_2 \sim X_1$.gave significant correlations ($p < 0.05$ for one and $p \geq 0.05$ for the other). That is, the

frequency of conflicting results caused by switching dependent and independent variables in

our simulated dataset is 17.2%.

Table 2 shows a relationship between the variance in simulations and the frequency of

conflicting results caused by switching dependent and independent variables. When the

variance of the noise term is slight (like 1 and 4), i.e., there is a strong correlation between

X_1 and X_2, switching independent and dependent variables gives almost the same results. As

the variance in the term noise increases, i.e., when the correlation between X_1 and X_2

becomes weak, switching independent and dependent variables leads to many conflicting

results. In cases where the variance is 16 in "BM~BM+Norm" and 64 in "Norm~Norm+BM,"

there are even close to 50% of cases with conflicting results. However, as the variance

becomes much more pronounced and the correlation between X_1 and X_2 becomes weaker,

the frequency of conflicting results caused by switching dependent and independent variables

decreases (Table 2).

Table 2. Frequency of conflicting results caused by switching dependent and independent variables in PGLS analyses

X_1[a]	BM						Norm					
X_2[b]	BM+Norm						Norm+BM					
Noise Term Variance	1	4	16	64	256	1024	1	4	16	64	256	1024
Simulation times	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
Times of $X_1 \sim X_2$ giving $p < 0.05$	1000	982	538	187	68	53	1000	1000	925	386	133	74
Times of $X_2 \sim X_1$ giving $p < 0.05$	1000	1000	954	525	170	80	1000	1000	1000	844	317	128
Both $X_1 \sim X_2$ and $X_2 \sim X_1$ giving $p < 0.05$	1000	982	529	166	34	14	1000	1000	925	374	87	39
Both $X_1 \sim X_2$ and $X_2 \sim X_1$ giving $p \geq 0.05$	0	0	37	454	796	881	0	0	0	144	637	837
Times of conflicting results between $X_1 \sim X_2$ and $X_2 \sim X_1$[c]	0	18	434	380	170	105	0	0	75	482	276	124

[a]In 6000 simulations, the trait X_1 evolved according to a Brownian motion (BM) model, and in the other 6000 simulations, the trait X_1 evolved according to

a normal distribution. For details, see the MATERIALS AND METHODS section.

[b]The trait X_2 was simulated by adding a noise term to X_1. The term noise was normally distributed in the first 6000 simulations and generated according to a

Brownian motion model in the other 6000 simulations. For details, see the MATERIALS AND METHODS section.

[c]When one of $X_1 \sim X_2$ and $X_2 \sim X_1$ gives a $p < 0.05$ result, and the other gives a $p \geq 0.05$ result, we define these results as conflicting results.

Establish a Golden Standard for Evaluating the Correlations Between Simulated Traits

When performing PGLS analysis on both empirical and simulated datasets, avoiding

the arbitrary selection of the dependent variable is essential. Switching the positions of

dependent and independent variables leads to a significant frequency of conflicting results. In

order to select a better dependent variable, we will use simulated data to establish a selection

criterion.

In empirical phylogenetic data, we only have access to trait values at terminal nodes,

and potential correlations between traits can be estimated using methods such as PGLS.

Conversely, we also have access to trait values at internal nodes in simulated phylogenetic

data. Since changes along different phylogenetic branches are independent, we can measure

the evolutionary correlation between two traits by analyzing their changes along phylogenetic

branches using standard statistical methods. First of all, we calculated the changes in the

traits X_1 and X_2 along evolutionary branches per unit time, $\Delta X_1/L$ and $\Delta X_2/L$, where L

is the length of the branch. Then, we did Shapiro-Wilk test on $\Delta X_1/L$ and $\Delta X_2/L$ to see if

they had normal distributions. If both of them satisfy normality, we used Pearson correlation

to detect the correlation between $\Delta X_1/L$ and $\Delta X_2/L$. Otherwise, we used Spearman's rank

correlation. These analyses revealed significant correlations between the two traits, X_1 and

X_2, in 7902 simulations (7099 positive and 803 negative, $p < 0.05$ for all these cases) but not

in the other 4098 simulations ($p \geq 0.05$ for all these cases) (Supplementary Table S1). These

results provide "golden standards" for judging whether PGLS analyses of trait values on

terminal nodes (X_1 and X_2) give correct results.

PGLS analyses of $X_1 \sim X_2$ show that there are significant positive correlations in 6317

simulations ($p < 0.05$ for all these cases), significant negative correlations in 29 simulations

($p < 0.05$ for all these cases), and no significant correlations in 5654 simulations ($p \geq 0.05$ for

all these cases) (Supplementary Table S1). The same analyses of $X_2 \sim X_1$ show that there are

254 significant positive correlations in 7999 simulations ($p < 0.05$ for all these cases), significant

255 negative correlations in 19 simulations ($p < 0.05$ for all these cases), and no significant

256 correlations in 3982 simulations ($p \geq 0.05$ for all these cases) (Supplementary Table S1). By

257 comparing with the golden standards, we found that PGLS analyses gave correct results for

258 both $X_1 \sim X_2$ and $X_2 \sim X_1$ in 7475 simulations (62.29%) and incorrect results for both

259 $X_1 \sim X_2$ and $X_2 \sim X_1$ in 2461 simulations (20.51%). In 2064 (17.20%) simulations, only one

260 of the two competing models, $X_1 \sim X_2$ or $X_2 \sim X_1$, yielded correct results. Therefore, limited

261 by the performance of PGLS regression analysis, we could at most obtain an accuracy of

262 79.49% in analyzing the data of our 12000 simulations by PGLS regression. However, if we

263 arbitrarily select one trait (X_1 or X_2) as the independent variable with the worst luck, we

264 could get an accuracy of only 62.29%. In the following attempt to find an accurate criterion

265 for the selection of dependent variables, we hope to perceive more correct cases from the

266 2064 simulations, where $X_1 \sim X_2$ and $X_2 \sim X_1$ gave conflicting results.

268 *Looking for an Accurate Criterion for Dependent Variable Selection*

269  Referring to the golden standards, we evaluated the potential performance of seven

270 criteria for selecting a better model from $X_1 \sim X_2$ and $X_2 \sim X_1$.

271  In statistics, the suitability of two competing statistical models is often assessed by

272 calculating each model's log-likelihood (LLK) value. First, we examined whether a higher (or

273 lower) LLK could give an accurate prediction of the better model between $X_1 \sim X_2$ and

274 $X_2 \sim X_1$. By calculating the LLKs of the two models for the 2064 simulations (Supplementary

275 Table S1), we found that the models selected from $X_1 \sim X_2$ and $X_2 \sim X_1$.by a lower LLK

276 (denoted as $Model_{LLK,lower}$) have more correct outcomes than the alternative model

277 (denoted as $Model_{LLK,higher}$), 1079 *vs.* 985. A χ^2 test showed a statistically significant

278 difference ($p = 0.004$, Table 3).

Akaike information criterion (AIC) is a widely used estimator of the quality of

statistical models for a given dataset (Akaike 1974). It balances the excellent fit of the model

with the complexity of the model. Calculating the AIC values of the two competing models

for the 2064 simulations (Supplementary Table S1), we found that the models selected with a

higher AIC value (denoted as $Model_{AIC,higher}$) had significantly more correct outcomes than

the alternative model (denoted as $Model_{AIC,lower}$), 1079 $vs.$ 985, $p = 0.004$ (Table 3).

R^2 describes the proportion of total variation in the dependent variable explained by

independent variables in the regression model, and the p-value of the regression coefficient is

the probability of observing the test statistic value under the assumption that the null

hypothesis is true, where the regression coefficient equals 0. These two parameters are widely

used indicators of the suitability of regression models. The models selected by a higher R^2

(denoted as $Model_{R^2,higher}$) and a lower p-value (denoted as $Model_{p,lower}$) have

significantly more correct outcomes than the alternative models ($p = 4 \times 10^{-4}$ for both cases,

Table 3).

The phylogenetic signal measures how much phylogenetic structure influences species

trait values. Pagel's λ and Blomberg et al.'s K are the most commonly used indicators of the

phylogenetic signal (Pagel 1999, Blomberg et al. 2003). The estimated λ ($\hat{\lambda}$) is a parameter in

Pagel's λ model (Pagel 1997) that measures the relatedness of the regression residuals with

the phylogenetic structure. We found that the models that use the trait with a higher λ value or

a higher K value as the dependent variable (denoted as $Model_{\lambda_y,higher}$ and $Model_{K_y,higher}$,

respectively) have significantly more correct outcomes than the alternate models (denoted as

$Model_{\lambda_y,lower}$ and $Model_{K_y,lower}$, respectively) ($p < 2.2 \times 10^{-16}$ for both case Table 3). And

the model chosen by a higher $\hat{\lambda}$ (denoted as $Model_{\hat{\lambda},higher}$) also had significantly more

correct results than the alternative models (denoted as $Model_{\hat{\lambda},lower}$) ($p < 2.2 \times 10^{-16}$, Table

3).

Table 3. Differences in the performance of models selected by six criteria.

Criteria	Models	Correct	Incorrect	χ^2	p^a
LLK	$Model_{LLK,higher}$	985	1079	8.381	0.004
	$Model_{LLK,lower}$	1079	985		
AIC	$Model_{AIC,higher}$	1079	985	8.381	0.004
	$Model_{AIC,lower}$	985	1079		
R^2	$Model_{R^2,higher}$	1089	975	12.37	4×10^{-4}
	$Model_{R^2,lower}$	975	1089		
p	$Model_{p,higher}$	975	1089	12.37	4×10^{-4}
	$Model_{p,lower}$	1089	975		
λ_y	$Model_{\lambda_y,higher}$	1736	328	1918.3	$< 2.2 \times 10^{-16}$
	$Model_{\lambda_y,lower}$	328	1736		
K_y	$Model_{K_y,higher}$	1736	328	1918.3	$< 2.2 \times 10^{-16}$
	$Model_{K_y,lower}$	328	1736		
$\hat{\lambda}$	$Model_{\hat{\lambda},higher}$	1736	328	1918.3	$< 2.2 \times 10^{-16}$
	$Model_{\hat{\lambda},lower}$	328	1736		

[a]The minimal significance value that the R environment (version 4.1.0) gives is 2.2×10^{-16}.

Llk, log-likelihood; AIC, Akaike information criterion. When the χ^2 tests were extended to all

12000 simulations, λ_y and $\hat{\lambda}$ are also the best criteria (Supplementary Table S2).

In addition, we defined a virtual criterion where one model was randomly selected

from the two competing models, $X_1 \sim X_2$ and $X_2 \sim X_1$. The results of the models selected by

this virtual criterion ($Model_{rc}$) were compared with the better models selected by the above

seven criteria. Although the better models selected by the above seven criteria consistently

have more correct results than $Model_{rc}$ (Table 4), the differences of $Model_{LLK,lower}$ and

$Model_{AIC,higher}$ with $Model_{rc}$ are not statistically significant ($p = 0.093$ for both cases),

the differences of $Model_{R^2,higher}$ and $Model_{p,lower}$ with $Model_{rc}$ are marginally

significant ($p = 0.046$ for both cases), but the differences of $Model_{\lambda_y,higher}$, $Model_{K_y,higher}$,

and $Model_{\hat{\lambda},higher}$ with $Model_{rc}$ are highly significant ($p < 2.2 \times 10^{-16}$ for all the three

cases).

Table 4. Comparing the performance of six criteria with random selection.

Criteria	Models	Correct	Incorrect	χ^2	p^a
$\hat{\lambda}$	$Model_{\hat{\lambda},higher}$	1736	328	552.7	$< 2.2 \times 10^{-16}$
	$Model_{rc}$	1024	1040		
K_y	$Model_{K_y,higher}$	1736	328	552.7	$< 2.2 \times 10^{-16}$
	$Model_{rc}$	1024	1040		
λ_y	$Model_{\lambda_y,higher}$	1736	328	552.7	$< 2.2 \times 10^{-16}$
	$Model_{rc}$	1024	1040		
LLK	$Model_{LLK,lower}$	1079	985	2.827	0.093
	$Model_{rc}$	1024	1040		
AIC	$Model_{AIC,higher}$	1079	985	2.827	0.093
	$Model_{rc}$	1024	1040		
R^2	$Model_{R^2,higher}$	1089	975	3.971	0.046
	$Model_{rc}$	1024	1040		

p	$Model_{p,lower}$	1089	975	3.971	0.046
	$Model_{rc}$	1024	1040		

[a]The minimal significance value that the R environment (version 4.1.0) gives is 2.2×10^{-16}.

LLK, log-likelihood; AIC, Akaike information criterion.

From Tables 3-4 and S2, we can see three distinct groups of criteria: LLK and AIC, R^2 and p-value, Pagel's λ, Blomberg et al.'s K, and $\hat{\lambda}$. Within each group, the criteria gave the same results. Among the three groups, Pagel's λ, Blomberg et al.'s K, and $\hat{\lambda}$ seem to be the best criterion for dependent variable selection. For a quantitative evaluation of these impressions, we performed the χ^2 tests to compare Pagel's λ with Blomberg et al.'s K, $\hat{\lambda}$, LLK, AIC, R^2, and p-value using the 2064 simulations where $X_1 \sim X_2$ and $X_2 \sim X_1$ gave conflicting results. As shown in Table 5, the equivalency between Pagel's λ, Blomberg et al.'s K, and $\hat{\lambda}$ and the superiority of λ to LLK, AIC, R^2, and p-value have been statistically confirmed.

Table 5. Comparing the performance of λ_y with other criteria for dependent variable selection

Criteria	Models	Correct	Incorrect	χ^2	p[a]
λ_y vs. $\hat{\lambda}$	$Model_{\lambda_y,higher}$	1736	328	0	1
	$Model_{\hat{\lambda},higher}$	1736	328		
λ_y vs. K_y	$Model_{\lambda_y,higher}$	1736	328	0	1
	$Model_{K_y,higher}$	1736	328		
λ_y vs. LLK	$Model_{\lambda_y,higher}$	1736	328	480.6	$< 2.2 \times 10^{-16}$

	$Model_{LLK,lower}$	1079	985		
λ_y *vs.* AIC	$Model_{\lambda y,higher}$	1736	328	480.6	$< 2.2 \times 10^{-16}$
	$Model_{AIC,higher}$	1079	985		
λ_y *vs.* R^2	$Model_{\lambda y,higher}$	1736	328	468	$< 2.2 \times 10^{-16}$
	$Model_{R^2,higher}$	1089	975		
λ_y *vs.* p	$Model_{\lambda y,higher}$	1736	328	468	$< 2.2 \times 10^{-16}$
	$Model_{p,lower}$	1089	975		

[a]The minimal significance value that the R environment (version 4.1.0) gives is 2.2×10^{-16}.

LLK, log-likelihood; AIC, Akaike information criterion.

In summary, Pagel's λ, Blomberg et al.'s K, and $\hat{\lambda}$ are the best criteria for dependent

variable selection. Of the 2064 simulations that $X_1 \sim X_2$ and $X_2 \sim X_1$ gave conflicting results,

these criteria led to correct results in 1736 simulations. Combined with the 7475 simulations

where both $X_1 \sim X_2$ and $X_2 \sim X_1$ gave correct results, PGLS analysis was able to achieve

76.8% accuracy when the trait with a stronger phylogenetic signal was selected as the

dependent variable. As this accuracy is still 2.7% below the upper limit of PGLS analysis

(79.49%), a much better criterion may be found in the future. PGLS regression analysis itself

should also be improved.

DISCUSSION

In the PGLS correlation analysis of an empirical dataset (Liu et al. 2023), we found

that switching dependent and independent variables could lead to a significant incidence of

conflicting results. To investigate this further, we simulated the evolution of two traits (X_1

and X_2) along a binary tree with 100 terminal nodes, using different models and variances

for 12000 iterations. Our PGLS analysis of these simulated datasets showed a 17.2%

frequency of conflicting results when the dependent and independent variables were swapped.

Using simulation, we established a golden standard for whether the two traits, X_1 and X_2,

are correlated in each simulation by conventional correlation analysis of the changes of the

two traits along the branches of the phylogenetic tree. With this golden standard, we can tell

which model, $X_1 \sim X_2$ or $X_2 \sim X_1$, is correct. Seven potential criteria for dependent variable

selection, LLK, AIC, R^2, p-value, Pagel's λ, Blomberg et al.'s K, and $\hat{\lambda}$, have been compared.

The last three criteria are equivalent in selecting dependent variables and have shown their

superiority over the other four criteria. Pagel's λ and Blomberg et al.'s K values are generally

calculated at the beginning of the phylogenetic comparative analysis, so they are already

known before the PGLS analysis. If we can choose the trait with a higher λ or K value as the

dependent variable, two rounds of PGLS analysis, like the $X_1 \sim X_2$ and $X_2 \sim X_1$, are not

required. Considering the practical convenience, Pagel's λ and Blomberg et al.'s K are

superior to $\hat{\lambda}$.

This study primarily focuses on the issue of dependent variable selection in PGLS

analyses using Pagel's λ model (Freckleton et al. 2002). Arbitrary choosing a dependent

variable does not affect the correlation results in models that do not require parameter

estimation, such as Brownian motion. However, in other models involving parameter

estimation, such as Ornstein–Uhlenbeck (Hansen 1997, O'Meara and Beaulieu 2014) and the

early burst model (Harmon et al. 2010), swapping independent and dependent variables may

result in different estimations of their respective parameters and potentially give conflicting

results. Further research is needed to establish criteria for selecting the dependent variable in

these models.

It is important to note that independent and dependent variables are potentially

misleading when used in evolutionary correlation studies. These terms should not be

interpreted literally. In a PGLS regression analysis, the independent variable does not

necessarily mean the causes of changes in the dependent variable. This analysis replaces

conventional correlation methods in phylogenetic comparative studies, such as Pearson and

Spearman's rank tests. When selecting a dependent variable for PGLS regression analysis,

focusing on an accurate perception of the relationship rather than a predetermined assumption

of cause and effect between analyzed traits is critical.

SUPPLEMENTARY MATERIAL

Supplementary material is available on GitHub at

https://github.com/BNU-Genome-Evolution/dependent-variable-selection.

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