TITLE:

A Novel Simulation Framework for Validation of Ecological Network Inference

RUNNING TITLE:

Network Inference Validation

AUTHORS:

Erik Kusch[1,2,3,*], Anna C. Vinton[4,*]

AFFILIATIONS:

* … Authors have contributed equally to this research

[1]Machine Readable Nature Research Group, Natural History Museum, University of Oslo,
Oslo, Norway, Norway

[2]Center for Biodiversity Dynamics in a Changing World (BIOCHANGE), Department of
Biology, Arhus University, Aarhus, Denmark

[3]Section for Ecoinformatics & Biodiversity, Department of Biology, Arhus University,
Aarhus, Denmark

[4]Department of Biology, University of Oxford, Oxford, UK

CORRESPONDING AUTHOR:

Erik Kusch; erik.kusch@nhm.uio.no

KEYWORDS

Ecological Networks, Network Inference, Demographic Stochasticity, Biological Interaction,

Interaction Simulation, Model Validation

ABSTRACT

1. Understanding how the differential magnitude and sign of ecological interactions vary
across space is vital to assessing ecosystem resilience to biodiversity loss and predict
community assemblies. This necessity for ecological network knowledge and their
labour-intensive sampling requirements has spurred the creation of ecological network
inference methodology. Recent research has identified inconsistencies in networks
inferred using different approaches thus necessitating quantification of inference
performance to facilitate choice of network inference approach.

2. Here we develop a data simulation method to generate data products fit for network
inference and subsequently quantify the validity of two well-established ecological
interaction network inference methods – HMSC and COOCCUR. The simulation
framework we present here can be parameterised using real-world information (e.g.,
biological interactions observed in-situ and bioclimatic niche preferences) thus
representing network inference capabilities in real-world applications. Using this
framework, it is thus possible to evaluate the performance of any ecological network
inference approach.

3. We identify a concerningly large range in accuracy of inferred networks as compared
to true, realisable association networks. These differences in inference accuracy are
governed by a paradigm of input data types and environmental parameter estimation as
previously suggested. To establish a workflow for quantification of network inference
reliability, we suggest analysis procedures with which to explore inference and
detection probabilities of association types of different identity and sign with respect to
bioclimatic niche preferences and association strength of association partner-species.

4. With this study, we provide the groundwork with which to validate and compare
ecological network inference methods, and ultimately vastly increase our ability to
understand and predict species biodiversity across space and time.

INTRODUCTION

Despite their complexity (e.g., in directionality, sign, and strength), biological interactions (e.g., competition, mutualism, predation) can be described readily – even among and across large assemblages of interacting species – via ecological networks. Ecological networks quantify the direction and sign of interactions (i.e., links) between species (i.e., nodes) as either positive, negative, or absent while also describing their strength/magnitude (Morales-Castilla et al., 2015). Real-world counterparts of such abstracted biological interactions and their respective networks are undergoing changes at local and macroecological scales simultaneously, but traditional in-situ quantifications become intractable across large scales due to sampling effort requirements (Jordano, 2016). Nevertheless, it is at these macroecological scales that climate change impacts ought to be evaluated (Lavorel & Garnier, 2002; Rapacciuolo & Blois, 2019; Wang et al., 2016) and so macroecological networks have to be inferred from proxies. This has led to the development of interaction inference approaches, most of which utilize spatial patterns of species co-occurrence (e.g.; Griffith, Veech, & Marsh, 2016; Blonder & Morueta-Holme, 2017; Damgaard, Ehlers, Ransijn, Schmidt, & Svenning, 2018; Lamonica, Pagel, & Schurr, 2021; Swain et al., 2021; Tikhonov et al., 2021; Weiss-Lehman et al., 2021; Bimler, Mayfield, Martyn, & Stouffer, 2022) to identify ecological networks. Due to their demonstrated usefulness in studying and predicting (1) community assembly patterns (Freilich et al., 2018; Ovaskainen et al., 2017), (2) ecosystem processes (Dongli et al., 2022) and services (Keyes et al., 2021) as well as (3) ecosystem stability (Korkmazhan & Dunn, 2022; Thébault & Fontaine, 2010), it is pivotal that networks inferred via these approaches are accurate representations of realised biological interactions.

A recent comparison of network inference outcomes has demonstrated that – even when analysing the same ecosystem – inferred networks are inconsistent across inference approaches and geographic scales (Kusch et al., 2023). Thus, practitioners of ecological network inference

need to carefully consider which network inference approach to use as inferred networks differ

considerably across scales and approaches. To streamline choice of inference methodology for

specific study purposes, criteria of inference approach applicability have been proposed (Kusch

et al., 2023). However, objective measurements of inference performance to judge quality of

ecological network inference remain absent. To account for the ongoing development of

network inference approaches, a generalizable inference validation framework that can take

into account ongoing developments is vital.

To study performance of ecological network inference approaches, we require a framework

which produces testable networks and data with which to carry out inference of networks which

can then be compared to the underlying testable networks. While one previous attempt has

been made at creating such a comparison framework (Lavender et al., 2019), this comparison

arguably suffers from a lack of biological realism which has also been identified as a set of

pitfalls in a recent overview of ecological network inference practices (Blanchet et al., 2020).

First, limiting data simulations and network inference to undirected links (i.e., biological

associations) fails to represent a large part of the ecological interaction type spectrum (Harris,

2016). To study the effect of link directionality on ecological network inference, a network

inference performance assessment framework must therefore be capable of generating

synthetic networks containing directed (i.e., interactions) or undirected (i.e., associations)

links. Second, neglecting to simulate and infer data according to bioclimatic niche preferences

may lead to overestimations of the impact of ecological interactions (Turnbull et al., 2013).

Thus, a simulation framework for network inference validation must explicitly consider

bioclimatic preferences of ecological network-contained species to represent more realistic

biodiversity products for network inference. Third, spatial biodiversity products for network

inference beyond co-occurrence records (i.e., abundance or performance matrices) have been

demonstrated to lead to vastly different inference outcomes even when using the same

underlying statistical framework for network inference (Kusch et al., 2023). Conclusively, to

quantify the accuracy-changes in network inference owing to differences in spatial input types,

the simulation framework must be capable of generating biodiversity products beyond co-

occurrence. Finally, to mirror real-world data as closely as possible, such a simulation

framework ought to be capable of incorporating considerations of population dynamics such

as density dependence and dispersal as these factors have been demonstrated to shape spatial

patterns of biodiversity alongside biological interactions (Saupe et al., 2015).

Here we present a novel simulation-based toolbox for ecological network inference

performance assessments overcoming the above-mentioned limitations. This framework

centres on a spatially explicit, individual-based simulation for the generation of data products

ready for network inference. With this framework, we address the impact of bioclimatic niches

and biological interactions as well as density dependence and dispersal dynamics in shaping

species distributions and abundance through time and space. This data generation framework

can be parameterised either with real-world network and species data (e.g., species-specific

bioclimatic niches and dispersal potential), or randomly generated network and species data

(for which we provide supporting functionality). While the former will allow for precise

quantification of network inference performance given specific study systems, the latter

enables a more general assessment of network inference reliability across a range of systems

and species. Because of the flexibility of the data input and robustness of the mathematical

simulation approach, this framework is generalisable across study ecosystems and species

pools. Moreover, this simulation framework provides a flexible toolbox with which additional

considerations beyond the ones presented here may be addressed (e.g., different types of

species interactions, environmental change impacts). Using this simulation framework, we

subsequently carry out ecological network inference using two well-established approaches of

ecological network inference – HMSC (Ovaskainen et al., 2017) and COOCCUR (Veech,

2013). While the former allows us to assess the performance of network inference while accounting for bioclimatic preferences and spatial products beyond simple co-occurrence records, the latter inference approach is blind to climatological gradients and restricted in input data to co-occurrence matrices. We expect that network inference will more accurately identify true pairwise associations when bioclimatic preferences are accounted for and data products corresponding to co-abundance or co-performance rather than co-occurrence are utilised, thus making HMSC inference more accurate. Finally, we propose a generalisable framework with which to study the likelihood of inference and detection (i.e., correct inference) of different types on pairwise links. Ultimtely, the framework for data generation and process of network inference assessments we present here represent an advancement in our understanding of performance and applicability of ecological interaction inference methodology.

MATERIAL & METHODS

ECOLOGICAL NETWORK MATRICES

When encoding ecological networks, it is common practice to represent nodes (i.e., species) and their links (i.e., interactions or associations) as matrices (Ávila-Thieme et al., 2023; Opedal & Hegland, 2020). Within these matrices, columns and rows represent species-identities while cells contain the link weight (interaction/association strength) between corresponding species pairs. Network inference approaches can detect network matrices as either representing directed (Bimler et al., 2022) or undirected networks (Morueta-Holme et al., 2016; Tikhonov et al., 2020; Veech, 2013).

Directedness of ecological network links is identified through arrangement of values within the corresponding network matrix. Undirected, association matrix entries are contained either in the upper or lower triangle of the species-by-species matrix (i.e., every species in an association pair affects the partner equally). Directed, interaction matrices, on the other hand, contain the

interaction strength and sign of an actor species (stored in columns) onto a subject species

(stored in rows) and vice versa. While undirected network matrices identify biological

associations (e.g., probability of co-occurrence of two species), directed network matrices

allow for the identification of more complex biological processes like predator-prey

interactions. Conclusively, for our framework of network inference validation presented here,

we have implemented support for interaction and association matrices.

While our simulation framework can be parameterised with ecological network matrices as

observed in nature, we provide functionality for creation of random network interaction or

association matrices. Optional arguments of this helper function control maximum value of

link strengths and sparsity of the matrix (i.e., how many of all possible values should be exactly

0) representing an absence of interaction or association between species.

SIMULATION FRAMEWORK

In order to generate data to assess network inference performance, we developed a spatially

explicit, individual-based birth-death model. Here, we iteratively simulate birth and death

events of individuals belonging to interacting species (as defined via a network matrix) across

a two-dimensional (x and y), continuous landscape.

THE MODEL

Individuals in this model have constant birth rates (b_0), while their death rate over time ($d(t)$)

is determined by their (1) population density and carrying capacity (P), (2) environmental

maladaptation (E), and (3) interaction effects of neighbouring individuals (Ω).

$$d_i(t) = d_0 + P_i \times E_i - \Omega_i \qquad (1)$$

Where $d_i(t)$ is the dynamic death rate for individual i at time t and d_0 denotes the background

death rate.

Population Dynamics - P_i

To account for logistic growth of individual populations, our simulation evaluates the impact

of density and carrying capacity (P_i) within the dynamic death rate calculation (equation 1).

Here, P_i is determined through species-specific (S) population sizes and carrying capacities

thus ensuring $P_i = P_S$ for all individuals i belonging to the same species S:

$$P_S(t) = \frac{N_S(t) \times (b_0 - d_0)}{K_S} \tag{2}$$

Where d_0 and b_0 represents the background rate of death and birth respectively. K_S and $N_S(t)$

represent the carrying capacity and population size at time t for an individual of species S. The

higher the value of P_S, the higher the dynamic death rate of individuals of species S.

Impact of Environment and Trait - E_i

The maladaptation of an individual with trait μ at location (x, y), $E_\mu(x, y)$ is related to the

difference between the individual's trait value μ and the optimal trait value, or the environment

at its location ($u_0(x, y)$):

$$E_\mu(x, y) = M_{\sigma_E}(\mu_i - u_0(x, y)) \tag{3}$$

180   The larger $E_\mu(x, y)$, the worse the individual is adapted to its local abiotic conditions and the

181   higher its dynamic death rate will be. $M_{\sigma_E}(z)$ is a Gaussian function that determines the effect

182   of being maladapted in the current environment. This maladaptedness is weighted by the model

183   parameter σ_E which controls the severity by which the difference between an individual's trait

184   and their environment affects the dynamic death rate:

$$M_{\sigma_E}(z) = e^{\left(\frac{z}{\sigma_E}\right)^2} \tag{4}$$

185   The optimal phenotype, or $u_0(x, y)$, is determined via a simulation-user defined function. For

186   the study presented here, we assigned a linear function of the one-dimensional environmental

187 gradient (see Figure S1 for a visual representation) but note that this framework can be adapted

188 to more complex landscapes by simply changing the following formula:

$$u_0(x, y) = x \tag{5}$$

By evaluating environmental maladaptedness as an outcome only of the x dimension our

simulations enable the formation of alternative community assemblies where species of similar

environmental preference, but with competitive association between them may both persevere.

<u>Species Interactions/Associations</u> - Ω_i

In our framework, the impact of species interactions/associations an individual experiences is

a product of the number and species identity of individuals within an area around the target

individual. These interactions vary between species according to network matrices.

Our framework quantifies the effect of interactions Ω_i for a user-defined distance around each

individual i as follows. The interaction coefficient, $l_{i,j}$ denotes the impact of species j on

species i drawn from the $i \, X \, j$ network matrix. To account for effects of neighbouring

individuals by their abundance, the interaction coefficients are multiplied by the respective

abundances of the j-th species in the user-defined area surrounding the i-th individual (a_j). The

resulting sum of abundance-weighted interaction coefficients is subsequently standardised by

dividing its value by the number of individuals in the i-neighbourhood (a_j):

$$\Omega_i = \frac{\sum_1^J l_{i,j} * a_j}{\sum_1^J a_j} \tag{6}$$

203 Positive values of Ω_i reflect beneficial interaction effects and thus decrease the dynamic death

204 rate while negative Ω_i values increase the dynamic death rate.

 STOCHASTIC SIMULATION ALGORITHM

Our simulation framework tracks birth and death events of individuals belonging to interacting

species through time and space as directed by Gillespie, 1977. At each time step t, the

simulation framework computes the dynamic death rate $d_i(t)$ for each individual as described

above. A birth or death event is selected to occur at random (demographic stochasticity),

weighted by the individual's static birth and dynamic death rates.

If a death event is selected, the chosen individual is removed from the population. When a birth

event occurs, the offspring is a clone of the parent individual in their trait value μ_i. Thus,

reproduction in our framework is asexual. Natal dispersal is the only form of movement in our

framework with offspring location coordinates (x, y) given as the parental x and y location,

with the addition of a dispersal magnitude drawn from a normal distribution with mean equal

to 0 and a user-defined standard deviation. The boundary conditions of the continuous

landscape in either dimension are defined by the simulation user. Any dispersal attempt beyond

these limits will result in dispersal to the limit.

Next, the time until the subsequent event is determined by randomly sampling from an

exponential distribution with a mean of the inverse of the sum of the probabilities all possible

events. This is done to ensure that simulation time represents realistic passage of time in

biological systems (i.e., more individual birth and/or death events happen in shorter time in

larger populations). Finally, the simulation starts over with the updated pool of individuals.

This framework is similarly developed in Vinton & Vasseur, 2020.

Lastly, simulations are initialised with a user-defined or randomly generated (1) network

matrix, (2) table of initial individuals containing spatial location and trait values for each

individual, (3) a vector of carrying capacities for each species, and (4) a function which defines

how coordinates relate to environmental values/optimal phenotype (like in equation 5).

We coded and executed our simulation framework in R version 4.0.2. (R Core Team, 2021).

The simulation inputs (see below) parameterize the simulation framework described above. To

ensure reproducibility, we set a seed for random computation processes. For faster computation

of several simulation instances at once we enable parallel processing of several simulation runs

using the doParallel (Microsoft & Weston, 2020a) and foreach (Microsoft & Weston, 2020b)

libraries in R and carry out updates to dynamic death rate values only to those individuals who

have been birthed or who have experience a birth or death event in their a_j-neighbourhood.

Network Matrices

Using the randcorr R package (Schmidt & Makalic, 2018), we created supporting functionality

with which we establish a set of 1000 random association matrices containing 30 species with

a maximum strength of association of 10 to create networks in which associations dominate

birth and death probabilities over environmental effects and a matrix-sparsity of 10% to

simulate well-balanced networks of diverse association motifs. We focus here on association

matrices as most network inference approaches to date aim to infer undirected links.

Initial Individuals & Carrying Capacities

Using each random network we have created, additional functionality of our simulation

framework creates a user-defined number of individuals (either in total or for each species) and

allocates trait values to each individual which are drawn from a random distribution around a

randomly selected trait mean for each species given a user-defined standard deviation for these

random trait distributions. Additionally, individuals are placed randomly within a user-defined

spatial extent in x and y dimensions. Lastly, carrying capacities for each species are chosen

from a uniform distribution with user-defined boundaries.

For the analyses presented here, we chose random trait mean values between 0 and 10 for each

species and set carrying capacities of 300 for all species. We subsequently initialised each

simulation with 500 individuals per species in random locations ranging between 0 and 10 in

both x and y dimensions. Individual trait values were drawn from random distributions around

individual-species-specific trait mean values and a standard deviation of one.

Simulation Parameters

To use the initialising data (i.e., network matrices, data tables of individuals and carrying

capacities) to generate spatially explicit products ready for network inference, we executed our

simulation framework with the following parameters. Background rates of birth (b_0) and death

(d_0) were set to 0.4 and 0.6 respectively. To calculate local environmental effects on the

dynamic death rate, we defined σ_E as 2.5 (a value at which we found resulting species

distributions to be spread, on average, enough for substantial overlap so that species

associations could be realised between species-pairs) and supplied a linear function which

translates x-values directly into environmental values/optimal phenotypes (see equation 5).

When calculating association effects on the dynamic death rate, we defined the size of

bounding box around each individual by which association effects and respective species

abundances are calculated as 0.5 resulting in a quadratic extent where each side measures 1

unit in x and y dimensions. This was done to correspond with later gridding of the continuous

space for network inference although not aligning strictly with each other. Dispersal distance

of individuals originating from a birth event were randomly drawn for each dimension (x and

y), for each birth event, from a normal distribution centred on 0 with a standard deviation of

0.2. Finally, we ran each simulation until simulation time 20 was reached, recording individual

locations and trait values in time intervals of 0.1 (see Figure S2 for a visual representation of

how these individual-time-step information can be used for studying time-series of abundance

which a number of network inference approaches may use; e.g.; Damgaard & Weiner, 2021).

 INFERENCE APPROACHES

We carried out ecological network inference using two approaches – HMSC and COOCCUR.

We chose these two approaches for their unique placements along a co-occurrence to

performance spectrum which characterises network inference input data types and which has

been demonstrated to affect network inference outcomes (Kusch et al., 2023). While

COOCCUR incorporates co-occurrence data, HMSC can consider species performance

implicitly through abundance products as well as explicitly through, for example, population

growth rates. Additionally, while COOCCUR is blind to environmental gradients and

preferences, HMSC can identify these as part of its inference approach. Lastly, COOCCUR

has also been part of a previous simulation-based assessment (Lavender et al., 2019).

COOCCUR

COOCCUR is a probabilistic framework first introduced by Veech, 2013 which we

implemented in our analyses via the cooccur R package (Griffith et al., 2016). COOCCUR

identifies species associations in sign and strength through comparison of analytical estimates

of species co-occurrence probabilities to purely random species distributions and resulting co-

occurrence patterns. Probabilities of observed co-occurrence rates being higher or lower than

expected at random are used to denote statistical significance of identified associations. Using

these p-values and the traditional cut-off of 0.05, we retained only those associations inferred

with statistical significance, setting all other associations in inferred networks with COOCCUR

as absent associations for the sake of inference performance assessments.

HMSC

We executed the HMSC method (Ovaskainen et al., 2017) through the hmsc R package

(Tikhonov et al., 2021). HMSC is designed as a hierarchical species distribution modelling

framework which aims to explain inputs of site-by-species matrices through environmental

niches of species and site-specific environmental characteristics, functional traits of species,

and phylogenetic relatedness. Residual covariance of species is then treated as association

inference (Ovaskainen et al., 2017). The statistical significance of these inferred associations

is available via the credible intervals of their posterior distributions (Opedal & Hegland, 2020).

For all subsequent analyses, we only retained those associations per HMSC model which were

identified with 95% credible interval not overlapping 0, again assigning absence of association

to all remaining inferred associations. To study the effect of informing network inference with

environmental conditions, we executed HMSC models in two modes – naïve and informed

which represent no consideration of environmental information and consideration thereof,

respectively. Furthermore, to assess the effect of data inputs along the co-occurrence-

performance spectrum, we executed each naïve and informed HMSC models with three

different sets of spatial inputs corresponding to occurrence, abundance and performance data.

DATA PREPARATION FOR NETWORK INFERENCE

To prepare the simulation outputs for network inference, for each final spatial output, we

extracted trait means for each species which correspond to species- but not individual-specific

optimal location along the x dimension. We then identified species abundances across the

continuous two-dimensional landscape in 100 evenly spaced grids (i.e., 10 segments along each

axis resulting in grid cells with a length of 1 on either side). Correspondingly, we also extracted

for each grid cell population growth (an indicator of species performance) as the change in

abundance between the last two simulation time steps. This information was subsequently

stored as a species-by-site abundance matrix (see Figure S3 for a visual representation) as well

as a species-by-site performance matrix. Lastly, we noted the coordinates of each grid cell and

assigned a unique ID to each grid cell. This product was used as a study design identifier and

to establish random effect levels with respect to cell ID in HMSC models. For COOCCUR as

well as occurrence-informed HMSC analyses, the species-by-site abundance matrix was

transformed into a matrix of presence/absence.

INTERACTION/ASSOCIATION REALISATION

As a product of the random generation of environmental preferences and association motifs per species, some of the simulated associations may never be realised due to association partners not being able to survive in each other's presence due to their environmental preferences. As such, the simulated ecological networks represent "potential" association patterns which network inference approaches cannot reasonably be expected to identify. Therefore, we distinguish between true potential ecological networks (i.e., the randomly generated input networks for our population dynamics simulations) and true realisable ecological networks. The latter are established by assigning as absent (i.e., magnitude of 0) all associations whose association partners exhibit environmental niche preferences (i.e., initialising mean per species in data simulation) further apart than the radius of the association effect bounding box (i.e., 0.5) plus the niche breadth parameter (σ_E; i.e., 2.5). At a carrying capacity of 300 and a species abundance of 1 without any associations present, an individual 3 units removed from its optimal environment receives a dynamic death rate of ~1 (~1.6 at species abundance 300) as compared to the static birth rate of 0.4, thus making death of this individual much more likely than survival. Thus, this condition ought to adequately mask associations which could never affect their involved species-pairs. See Figure 2 for a visual representation of how this process works.

PERFORMANCE ASSESSMENT

To facilitate quantification of network inference accuracy, we identified whether individual true realisable associations were inferred correctly. Since species may go extinct throughout our simulation runs either due to maladaptation to the environment or competitive exclusion in their preferred environment, we focus on the dissimilarity in links between nodes shared by the true realisable networks including only extant species by the end of the simulation procedure and their inferred counterparts. Expressing the rate of correctly inferred associations divided

by the total number of possible links between nodes of the final species pool allows us to

express interaction accuracy as a proportion thus making it comparable across different

simulation settings and inference approaches. We multiply these scores by 100 to express them

as intuitive percentages of whole-network inference accuracy.

To assess sub-network inference accuracy with respect to association identity (present/absent)

and sign (negative/positive), we also calculate rates of (1) correct, (2) incorrect, and (3) failure

of identification of positive, negative and absent associations.

Finally, we provide a generalisable framework with which to quantify inference behaviour of

any network inference approach following our simulation procedures. To do so, we establish

two sets of logistic models using the brms R package (Bürkner, 2021) which model two target

variables as an outcome of the difference in trait expression per pair of association partners as

well as the true strength of associations. These two target variables are each split according to

association types (i.e., negative, positive, absent) and denote (1) inference and (2) detection

(i.e., correct inference) of individual associations. In doing so, we generate valuable insight

into what conditions lead to individual network inference approaches inferring a specific

interaction type and under which conditions such inference can be assumed to be accurate.

RESULTS

Our simulations yielded 1000 distinct random association networks containing 30 species each

with random mean trait/phenotype expressions per species per simulation product. These are

accompanied by their corresponding 1000 lists of data tables containing individual IDs,

phenotypes/traits and locational information which we use for network inference. Here, we

focus first on a reduced conceptual simulation run to demonstrate the functionality of the data

generation simulation framework and subsequent analysis procedures before we explore

validation efforts of network inference at scale.

SIMULATION OUTPUTS

To demonstrate our simulation framework, we established a conceptual random association

matrix containing five species (with a maximum association strength of 10 and a network

sparsity of 0.1). See Figure 2 for a visual representation of the conceptual true potential

association network matrix as well as the process by which we establish an understanding of

the true realisable version of this network. The true potential random association network

matrix is then used to calculate the interaction effect on dynamic death rates at each simulation

step and shapes the final spatial product of species abundances across the artificial continuous

landscape as can be seen in Figure 1.

FIGURE 1 - CONCEPTUAL SPATIAL PRODUCTS. Simulations are initialised (left panel) with individuals belonging to interacting

species which are placed randomly across the continuous landscape. This placement is irrespective of trait expressions specific to each species

(coloured lines showing the mean trait expression per species in either panel). The simulation terminates at a user-defined simulation-time

and returns the final output of individuals across the landscape (right panel) which have shifted with respect to (1) population dynamics, (2)

individual environmental maladaptation, and (3) interaction effects.

If desired, our simulation framework also returns products containing all extant individuals as

well as their traits and locational information at pre-defined simulation-time intervals. This

enables the analyses of time-series of species abundances as shown in Figure S2.

FIGURE 2 - CONCEPTUAL RANDOM NETWORK MATRIX AND REALISATION OF ASSOCIATIONS. A) A true potential association matrix for five distinct species. Note that maximum association strength

was set to 10 and network sparsity to 0.1 resulting in 1 out of the possible ten associations having a strength of exactly 0 while all other associations may range between -10 and +10. B) The pairwise difference in

environmental difference as identified through mean and standard deviation of species-specific trait values at the final simulation step (see vertical lines in left-hand panel of Figure 1). Black squares identify associations

which can be realised due to shared environmental preferences. C) The true realisable association matrix corresponding to A). All associations which cannot be realised have been set to a magnitude of 0 (i.e., absent

associations).

 PERFORMANCE OF NETWORK INFERENCE METHODS

401 Using COOCCUR and HMSC in our conceptual example, the inferred networks are notably

402 different to the true, original networks which we used to drive our simulations (Figure 3) with

403 the exception of absent associations in most HMSC inferences.

FIGURE 3 - CONCEPTUAL INFERRED NETWORKS. Inferred networks of statistically significant pairwise associations (whose association strengths have been set to -1 and +1 respective to association inference sign) show great dissimilarity to original network (figure 2, c) which affected the simulation framework. Black squares identify correctly identified positive, negative, and absent associations. Abbreviations: O; A; P denote co-occurrence, co-abundance, and co-performance data inputs. Clim identifies models informed with environmental data. Black squares denote correctly inferred associations.

Assessing network inference accuracy for all 1000 simulation outputs of 30-species networks,

we find that, within our analyses network inference accuracy varies largely across our two

network inference approaches and their sub-specifications (Figure 4). We find that COOCCUR

performs worse (a mean average network inference accuracy of roughly 23%) and is outdone

by HMSC inference irrespective of how HMSC models are specified or informed. However,

within HMSC, we find that network inference accuracy is governed by spatial input type and

environmental parameterisation.

FIGURE 4 - NETWORK INFERENCE ACCURACY. When analysing similarity links of shared species between inferred networks and

the true realisable association networks for each inference approach across all simulation runs, we find that COOCCUR produces less accurate

inference than HMSC. Meanwhile, HMSC specification and inputs determine HMSC network inference accuracy. Abbreviations: O; A; P

denote co-occurrence, co-abundance, and co-performance data inputs. Clim identifies models informed with environmental data.

While we find that inclusion of environmental information in HMSC models generally leads

to greater network inference accuracy, this effect is not present when utilising co-performance

inputs. Similarly, while we find a general trend of greater network inference accuracy for

HMSC models when moving from occurrence to abundance and performance data, HMSC

models informed with abundance data but not accounting for environmental effects perform

substantially worse than other HMSC specifications. Most strikingly, however, we find that

occurrence-based HMSC models which account for environmental effects perform on-par with

performance-informed HMSC models. For a breakdown of network inference similarity across

approaches and HMSC specifications see Figure S4.

Further sub-network deconstruction of network dissimilarities into rates of true, false and not-

inferred (i.e., missed) positive, negative and absent associations identified in each inferred

network (see Figure S5) under our simulation settings reveals that (1) COOCCUR and HMSC

(irrespective of specification) perform comparably at correctly identifying positive and

negative associations with both approaches struggling to correctly identify negative

association, (2) HMSC, irrespective of specification, performs better at correctly identifying

association absence than COOCCUR, (3) both COCCUR and HMSC infer positive, negative

and absent associations incorrectly at high rates with absent associations assigned by

COOCCUR being wrong at markedly higher rates than those rendered by HMSC inference,

(4) COOCCUR and abundance-informed HMSC models fail to identify positive and negative

associations the least. Meanwhile, (5) these two specifications fail at inferring true, realised

absent associations and are far outdone by HMSC model informed through performance data.

This highlights a stark contrast in network inference reliability given the underlying true

realisable association type.

<u>NETWORK INFERENCE APPROACH BEHAVIOUR</u>

To further understand network inference approach behaviour, our logistic regression models

using the difference in trait expression per pair of association partners as well as the strength

and sign of associations to predict the probability of (1) assigning (i.e., inference) and (2)

assigning correctly (i.e., detection) either positive, negative, or absent associations to species-

pairs for COOCCUR and HMSC. For clarity and brevity, we focus only on models of positive

association inference and detection using COOCCUR as well as abundance-informed HMSC

models due to their clear distinction in average network inference accuracy (Figure 4).

INFERENCE PROBABILITY OF ASSOCIATIONS

We find that the inference of positive associations between species pairs is unaffected by the

true realisable association between species pairs for COOCCUR as well as abundance-

informed HMSC models. However, we do find that decreasing difference in niche preference

of species-pairs (which corresponds to their separation in space) leads to a rapidly increasing

probability of inference of positive associations (Figure 5). While this pattern persists across

all three network inference approaches assessed here, this behaviour is strongest for

COOCCUR, followed closely by the naïve implementation of abundance-informed HMSC

while the informed HMSC model shows the weakest signal of niche-preference differences.

DETECTION PROBABILITY OF ASSOCIATIONS

Detection of positive associations is driven, for the three evaluated network inference

approaches, by a combination of the magnitude of the underlying true realisable positive

association as well as environmental niche preference difference of species-pairs. Detection

probability increases with underlying association magnitude and decreasing environmental

niche preference differences (Figure 6). All three approaches behave similarly as was to be

expected from their indistinguishable true positive association detection rates (Figure S5).

B) COOCUR

C) HMSC – [A]

A) HMSC – [A] Clim

FIGURE 5 - PROBABILITY OF CORRECT IDENTIFICATION OF TRUE REALISED ASSOCIATIONS. Logistic regression coefficients (left column) show clear effects of environmental difference in species which translate to distinct probabilities of inferring a pairwise associations (right column). However, this effect is diminished for abundance-informed, environmentally-informed HMSC models as compared to their naïve HMSC counterparts and COOCCUR. Abbreviations: A denotes co-abundance data inputs; Clim identifies models informed with environmental data.

A) COOCUR

B) HMSC – [A]

D) HMSC – [A] Clim

FIGURE 6 - PROBABILITY OF CORRECT IDENTIFICATION OF TRUE REALISED ASSOCIATIONS. Logistic regression

coefficients (left column) show clear effects of association magnitude, environmental difference in species niches as well as the interaction of

these two which translate to distinct probabilities of correctly identifying a pairwise associations (right column). Abbreviations: A denotes co-

abundance data inputs; Clim identifies models informed with environmental data.

<u>**DISCUSSION**</u>

Aligning with previous recommendations for choice of network inference approach (Kusch et

al., 2023), we find that network inference accuracy is governed by (1) spatial biodiversity input

type along a spectrum of co-occurrence, abundance, and performance which confers increasing

accuracy onto networks inferred therefrom, and (2) environmental considerations in network

inference approaches with network inference approaches which account for environmental

conditions rendering more accurate inferences of true networks than those approaches who

don't. These findings confirm previous criticisms of co-occurrence based network inference

approaches failing to capitalise on existing information in abundance and performance data

(Bimler et al., 2022; Kusch et al., 2023; Ovaskainen et al., 2017). In addition, our findings also

highlight the previously suggested fallacy of neglecting environmental gradients in network

inference leading to erroneous assignment of positive and negative because of differences in

environmental preferences rather than actual interactions (Blanchet et al., 2020). This is

particularly demonstrated via our inference model outputs in Figure 5 where inclusion of

environmental information leads to a diminished impact of spatial separation on inference

likelihood of positive associations. Lastly, we also find that the underlying inference

approaches themselves (i.e., COOCCUR and HMSC) affect accuracy of network inference

with both naïve occurrence-informed HMSC models and COOCCUR using the same inputs,

but HMSC performing far better at accurately inferring ecological networks (Figure 4).

However, these findings do not hold for all approach specifications in our simulation set-up as

described here. Notably, there seem to exist saturation effects to inference accuracy gains with

there being no difference in inference accuracy of naïve and informed performance-HMSC

models as well as informed occurrence-HMSC models. While this may suggest that the

inclusion of either performance data or environmental data may suffice in maximising network

inference accuracy, the markedly worse performance of naïve abundance-HMSC models when

compared to their informed abundance-HMSC counterparts should serve as a cautionary tale

to neglecting environmental data inclusion.

While assessing whole-network inference accuracy revealed a clear and strong effect of choice

of network inference approach as well as input data selection, less generality can be identified

at sub-network scale. We find that different inference approaches and specifications thereof

have different likelihood of rendering correctly positive, negative, and absent associations.

Absent associations are particularly well-rendered by performance-HMSC models. However,

the assignment of absent associations for these models was so frequent that they failed largely

to identify associations belonging to other types. Meanwhile, all other specifications of HMSC

as well as COOCCUR perform similarly well at detecting true positive associations, but also

infer their presence incorrectly at high rates (Figure S4). Informed abundance-HMSC models

performed the best at detecting true negative associations, but do so nevertheless at worryingly

low rates of success. Owing to this varied association-type-specific detection performance of

our tested approaches, we suggest that it may be useful to establish inference of ecological

networks via ensemble approaches which prioritise association types inferred with one

approach over the same association type inferred by different approaches. Caution should be

exerted in transferring these findings for use in other studies however as our simulation settings

will affect these performances. Thus, for judging how to best build an association-type specific

network inference ensemble approach, it is pivotal that network inference performance is first

validated with simulation parameters approximating study-specific conditions.

We expect that using our simulation framework in conjunction with real-world association

networks and bioclimatic niche preferences and environmental suitability constraints will yield

a more realistic overview of network inference performances. However, such a study will

require extensive data collection to allow more precise parameterisation of the simulation

framework we are presenting here. To circumvent the tedious data collection and costly

simulation effort this undertaking would entail we propose that the logistic regressions we

present may help, in first instance, to investigate under which conditions association detection

is likely. Instead of selecting one "best-performing" network inference approach, we believe

that it may be more prudent to quantify capabilities of network inference approaches to identify

association types with respect to association sign, magnitude, and differences in environmental

preferences of association partner-species as we have done with our logistic models in Figure

6. Doing so will create ranges of environmental and biological constellations within which

individual network inference approaches can be treated as reliable.

Even so, we do not claim our network inference performance scores nor our logistic regressions

to be necessarily representative of real-world applications, but merely that they remain accurate

for our specific simulation parameters. For example, increasing σ_E (i.e., making the

environmental constraint more forgiving) or increasing association magnitudes and introducing

a lower limit to association magnitude would likely increase the performance of either network

inference approach as the effects of associations will become greater in comparison to

environmental preference and location-specific environmental mismatches. Additionally, we

suspect that increasing the distance which defines the i-neighbourhood may make

identification of associations more accurate as interaction effects will have greater reach, thus

affecting each individual site in the site-by-species matrix used for network inference more

than before. Alternatively, restricting interactive effects not to the i-neighbourhood but strictly

to the grid cells which define sites should improve performance of either network inference

approach and be more representative of study setups with distinct sites (López-Segoviano et

al., 2021) rather than continuous megaplot designs (Lutz et al., 2012). Such considerations

would incur additional development of the simulation framework we present here.

While our simulation framework represents a major advancement of our capability of

identifying network inference performance and validate inferred networks, we also realise that

the simulation framework we present here may be regarded as an oversimplification of natural processes. For examples, non-natal movement, which is absent from our framework, has been identified as a confounding effect to interactions studies of animals (Blanchet et al., 2020; Matthiopoulos et al., 2015). Subsequently, we expect the current version of our simulation framework to be most useful to studies of plant-plant interactions/associations or those of other sessile organisms. Furthermore, our simulation framework only includes one functional trait per individual despite organisms inhabiting a multi-dimensional space of functional trait expressions (Blonder, 2018). Our framework neglects the impacts of functional trait expression on association/interaction outcomes which have been demonstrated particularly among tree species (Kunstler et al., 2012). Thus, we believe that our simulation may be further enhanced by the inclusion of additional functional trait metrics which determine the dynamical death rate through the interaction/association coefficient (Ω). Additionally, many species reproduce sexually rather than clonally and there may thus exist considerable variation in such functional traits between parents and offspring. Despite these avenues for future development, our simulation framework remains the most realistic for network inference data generation purposes particularly when compared to previous attempts at quantifying pairwise association inference performance (Lavender et al., 2019).

CONCLUSION

Network inference approaches have been developed to address the labour intensity of

ecological network research which requires impractical sampling efforts particularly at large

geographical scales or for large species pools. Despite the large number and diversity in

approaches of network inference frameworks, no general framework for network inference

validation or performance quantification has been established yet. Here, we have presented

an easy-to-use, mathematically robust simulation framework which can be used for data

generation with which to carry out network inference validation. While we find promising

accuracy of network inference under specific conditions (i.e., use of site-species performance

matrices and environmental information), we argue that these performance scores may be

subject to change given further parametrising of our simulation with more realistic real-world

data. Additionally, further explorations of network inference accuracy of directed links (i.e.,

interactions) or using the capacity of our simulation framework to output time-series of

individuals across space may prove particularly useful in aiding to the overview of network

inference capabilities and current practices thereof.

Nevertheless, using our simulation framework and a novel network inference performance

and behaviour quantification pipeline, we have confirmed previous criticisms of ecological

network inference practices and added to existing guidelines for choice of appropriate

network inference approaches. Thus, our simulation framework for network inference

validation represents an important addition to the toolbox of network inference method

development which fills a crucial knowledge gap of network inference performance

assessments.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

ACKNOWLEDGMENTS

This research was supported by the Aarhus University Research Foundation Start-up Grant

(grant no. AUFF-2018-7-8) to Alejandro Ordonez.

AUTHORS' CONTRIBUTIONS

E.K. conceptualised the study. A.V. developed the simulation framework used in this study.

E.K developed the network inference and network validation framework used in this study.

E.K. created all R scripts necessary for analyses. E.K. led the analysis and manuscript writing.

All authors contributed critically to the drafts and gave final approval for publication.

DATA AVAILABILITY

All code required for data generation, network inference, and inference performance

quantification is available at https://github.com/ErikKusch/Scaleable-Inference-of-Ecological-

Networks (please note that this is currently a private repository until submission to a relevant

journal so that the simulation framework specification remains protected). We are ready to

submit code and data to dryad upon acceptance of the manuscript.

REFERENCES

Ávila-Thieme, M. I., Kusch, E., Corcoran, D., Castillo, S. P., Valdovinos, F. S., Navarrete, S. A., & Marquet, P. A. (2023). NetworkExtinction: An R package to simulate extinction propagation and rewiring potential in ecological networks. *Methods in Ecology and Evolution*, 2020.10.17.305391. https://doi.org/10.1111/2041-210X.14126

Bimler, M. D., Mayfield, M. M., Martyn, T. E., & Stouffer, D. B. (2022). *Estimating interaction matrices from performance data for diverse systems*. https://doi.org/doi.org/10.1101/2022.03.28.486154

Blanchet, F. G., Cazelles, K., & Gravel, D. (2020). Co-occurrence is not evidence of ecological interactions. *Ecology Letters*, ele.13525. https://doi.org/10.1111/ele.13525

Blonder, B. (2018). Hypervolume concepts in niche- and trait-based ecology. *Ecography*, *41*(9), 1441–1455. https://doi.org/10.1111/ecog.03187

Blonder, B., & Morueta-Holme, N. (2017). *netassoc: Inference of Species Associations from Co-Occurrence Data* (p. 1).

Bürkner, P. C. (2021). Bayesian Item Response Modeling in R with brms and Stan. *Journal of Statistical Software*, *100*(5). https://doi.org/10.18637/JSS.V100.I05

Damgaard, C., Ehlers, B. K., Ransijn, J. C. G., Schmidt, I. K., & Svenning, J. C. (2018). Insights on plant interaction between dominating species from patterns of plant association: expected covariance of pin-point cover measurements of two species. *Environmental and Ecological Statistics*, *25*(2), 221–235. https://doi.org/10.1007/S10651-018-0400-Z/FIGURES/6

Damgaard, C., & Weiner, J. (2021). The need for alternative plant species interaction models. *Journal of Plant Ecology*, *14*(5), 771–780. https://doi.org/10.1093/jpe/rtab030

Dongli, D., Chengxing, W., Yuchen, Z., Changchun, L., & Ning, W. (2022). Coexistence mechanism of alien species and local ecosystem based on network dimensionality reduction method. *Chaos, Solitons & Fractals*, *159*, 112077. https://doi.org/10.1016/j.chaos.2022.112077

Freilich, M. A., Wieters, E., Broitman, B. R., Marquet, P. A., & Navarrete, S. A. (2018). Species co-occurrence networks: Can they reveal trophic and non-trophic interactions in ecological communities? *Ecology*, *99*(3), 690–699. https://doi.org/10.1002/ecy.2142

Gillespie, D. T. (1977). Exact stochastic simulation of coupled chemical reactions. *The Journal of Physical Chemistry*, *81*(25), 2340–2361. https://doi.org/10.1021/j100540a008

Griffith, D. M., Veech, J. A., & Marsh, C. J. (2016). cooccur: Probabilistic species co-occurrence analysis in R. *Journal of Statistical Software*, *69*(1), 1–17. https://doi.org/10.18637/jss.v069.c02

Harris, D. J. (2016). Inferring species interactions from co-occurrence data with Markov networks. *Ecology*, *97*(12), 3308–3314. https://doi.org/10.1002/ecy.1605

Jordano, P. (2016). Sampling networks of ecological interactions. *Functional Ecology*, *30*(12), 1883–1893. https://doi.org/10.1111/1365-2435.12763

Keyes, A. A., McLaughlin, J. P., Barner, A. K., & Dee, L. E. (2021). An ecological network approach to predict ecosystem service vulnerability to species losses. *Nature Communications*, *12*(1), 1586. https://doi.org/10.1038/s41467-021-21824-x

Korkmazhan, E., & Dunn, A. R. (2022). High-order correlations in species interactions lead to complex diversity-stability relationships for ecosystems. *Physical Review E*, *105*(1), 1–10. https://doi.org/10.1103/physreve.105.014406

Kunstler, G., Lavergne, S., Courbaud, B., Thuiller, W., Vieilledent, G., Zimmermann, N. E., Kattge, J., & Coomes, D. A. (2012). Competitive interactions between forest trees are driven by species' trait hierarchy, not phylogenetic or functional similarity: Implications for forest community assembly. *Ecology Letters*, *15*(8), 831–840.

660 https://doi.org/10.1111/j.1461-0248.2012.01803.x

661 Kusch, E., Bimler, M. D., Lutz, J. A., & Ordonez, A. (2023). *Ecological network inference is*
662 *not consistent across scales or approaches approaches*.
663 https://doi.org/10.1101/2023.07.13.548816

664 Lamonica, D., Pagel, J., & Schurr, F. M. (2021). Predicting the dynamics of establishing tree
665 populations: a framework for statistical inference and lessons for data collection. *Methods*
666 *in Ecology and Evolution*, 2041-210X.13656. https://doi.org/10.1111/2041-210x.13656

667 Lavender, T. M., Schamp, B. S., Arnott, S. E., & Rusak, J. A. (2019). A comparative evaluation
668 of five common pairwise tests of species association. *Ecology*, *100*(4).
669 https://doi.org/10.1002/ecy.2640

Lavorel, S., & Garnier, E. (2002). Predicting changes in community composition and
ecosystem functioning from plant traits: revisting the Holy Grail. *Functional Ecology*,
16(Essay Review), 545–556. https://doi.org/Doi 10.1046/J.1365-2435.2002.00664.X

López-Segoviano, G., Arenas-Navarro, M., Villa-Galaviz, E., Díaz-Infante, S., & Arizmendi,
M. del C. (2021). Hummingbird-plant interactions along an altitudinal gradient in
northwestern Mexico. *Acta Oecologica*, *112*, 103762.
676 https://doi.org/10.1016/j.actao.2021.103762

677 Lutz, J. A., Larson, A. J., Swanson, M. E., & Freund, J. A. (2012). Ecological importance of
678 large-diameter trees in a temperate mixed-conifer forest. *PLoS ONE*, *7*(5).
679 https://doi.org/10.1371/journal.pone.0036131

Matthiopoulos, J., Fieberg, J., Aarts, G., Beyer, H. L., Morales, J. M., & Haydon, D. T. (2015).
Establishing the link between habitat selection and animal population dynamics.
Ecological Monographs, *85*(3), 413–436. https://doi.org/10.1890/14-2244.1

Microsoft, & Weston, S. (2020a). *doParallel: Foreach Parallel*.

Microsoft, & Weston, S. (2020b). *foreach: Provides Foreach Looping Construct*.

Morales-Castilla, I., Matias, M. G., Gravel, D., & Araújo, M. B. (2015). Inferring biotic
interactions from proxies. *Trends in Ecology and Evolution*, *30*(6), 347–356.
687 https://doi.org/10.1016/j.tree.2015.03.014

688 Morueta-Holme, N., Blonder, B., Sandel, B., McGill, B. J., Peet, R. K., Ott, J. E., Violle, C.,
689 Enquist, B. J., Jørgensen, P. M., & Svenning, J. C. (2016). A network approach for
690 inferring species associations from co-occurrence data. *Ecography*, *39*(12), 1139–1150.
691 https://doi.org/10.1111/ecog.01892

692 Opedal, Ø. H., & Hegland, S. J. (2020). Using hierarchical joint models to study reproductive
693 interactions in plant communities. *Journal of Ecology*, *108*(2), 485–495.
694 https://doi.org/10.1111/1365-2745.13301

Ovaskainen, O., Tikhonov, G., Norberg, A., Guillaume Blanchet, F., Duan, L., Dunson, D.,
Roslin, T., & Abrego, N. (2017). How to make more out of community data? A conceptual
framework and its implementation as models and software. *Ecology Letters*, *20*(5), 561–
576. https://doi.org/10.1111/ele.12757

R Core Team. (2021). *R: A Language and Environment for Statistical Computing* (4.0.5). R
Foundation for Statistical Computing.

Rapacciuolo, G., & Blois, J. L. (2019). Understanding ecological change across large spatial,
temporal and taxonomic scales: integrating data and methods in light of theory.
Ecography, *42*(7), 1247–1266. https://doi.org/10.1111/ecog.04616

Saupe, E. E., Portell, R. W., Qiao, H., Soberón, J., Hendricks, J. R., Lieberman, B. S., & Hunter,
S. J. (2015). Niche breadth and geographic range size as determinants of species survival
on geological time scales. *Global Ecology and Biogeography*, *24*(10), 1159–1169.
707 https://doi.org/10.1111/geb.12333

Schmidt, D. F., & Makalic, E. (2018). *randcorr: Generate a Random p x p Correlation Matrix*
(1.0).

Swain, A., Byrum, T., Zhuang, Z., Perry, L., Lin, M., & Fagan, W. (2021). *Inferring,*
comparing and exploring ecological networks from time-series data through R packages
constructnet, disgraph and dynet. 1–15.
Thébault, E., & Fontaine, C. (2010). Stability of ecological communities and the architecture
of mutualistic and trophic networks. *Science*, *329*(5993), 853–856.
https://doi.org/10.1126/science.1188321
Tikhonov, G., Opedal, Ø. H., Abrego, N., Lehikoinen, A., Jonge, M. M. J., Oksanen, J., &
Ovaskainen, O. (2020). Joint species distribution modelling with the R-package Hmsc.
Methods in Ecology and Evolution, *11*(3), 442–447. https://doi.org/10.1111/2041-
210X.13345
Tikhonov, G., Ovaskainen, O., Oksanen, J., de Jonge, M., Opedal, O., & Dallas, T. (2021).
hmsc: Hierarchical Model of Species Communities. https://doi.org/10.1111/2041-
210X.13345
Turnbull, L. A., Levine, J. M., Loreau, M., & Hector, A. (2013). Coexistence, niches and
biodiversity effects on ecosystem functioning. *Ecology Letters*, *16*(SUPPL.1), 116–127.
https://doi.org/10.1111/ele.12056
Veech, J. A. (2013). A probabilistic model for analysing species co-occurrence. *Global*
Ecology and Biogeography, *22*(2), 252–260. https://doi.org/10.1111/j.1466-
8238.2012.00789.x
Vinton, A. C., & Vasseur, D. A. (2020). Evolutionary tracking is determined by differential
selection on demographic rates and density dependence. *Ecology and Evolution*, *10*(12),
5725–5736. https://doi.org/10.1002/ece3.6311
Wang, D., Heckathorn, S. A., Mainali, K., & Tripathee, R. (2016). Timing effects of heat-stress
on plant ecophysiological characteristics and growth. *Frontiers in Plant Science*,
7(NOVEMBER2016), 1–11. https://doi.org/10.3389/fpls.2016.01629
Weiss-Lehman, C. P., Werner, C. M., Bowler, C. H., Hallett, L. M., Mayfield, M. M., Godoy,
O., Aoyama, L., Barabás, G., Chu, C., Ladouceur, E., Larios, L., & Shoemaker, L. G.
(2021). Disentangling key species interactions in diverse and heterogeneous communities:
A Bayesian sparse modeling approach. In *bioRxiv*. Cold Spring Harbor Laboratory.
https://doi.org/10.1101/2021.07.23.453227
Zakharova, L., Meyer, K. M., & Seifan, M. (2019). Trait-based modelling in ecology: A review
of two decades of research. *Ecological Modelling*, *407*(April), 108703.
https://doi.org/10.1016/j.ecolmodel.2019.05.008
743
744